Computer algebra independent integration tests

Summer 2022 edition

4-Trig-functions/4.5-Secant/119-4.5.1.3-d-sin-^n-a+b-sec-^m

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [306]. This is test number [119].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	99.67 (305)	0.33 (1)
Mathematica	99.35 (304)	0.65 (2)
Maple	87.25 (267)	12.75 (39)
Fricas	77.45 (237)	22.55 (69)
Mupad	63.07 (193)	36.93 (113)
Giac	62.42 (191)	37.58 (115)
Maxima	57.19 (175)	42.81 (131)
Sympy	2.61 (8)	97.39 (298)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

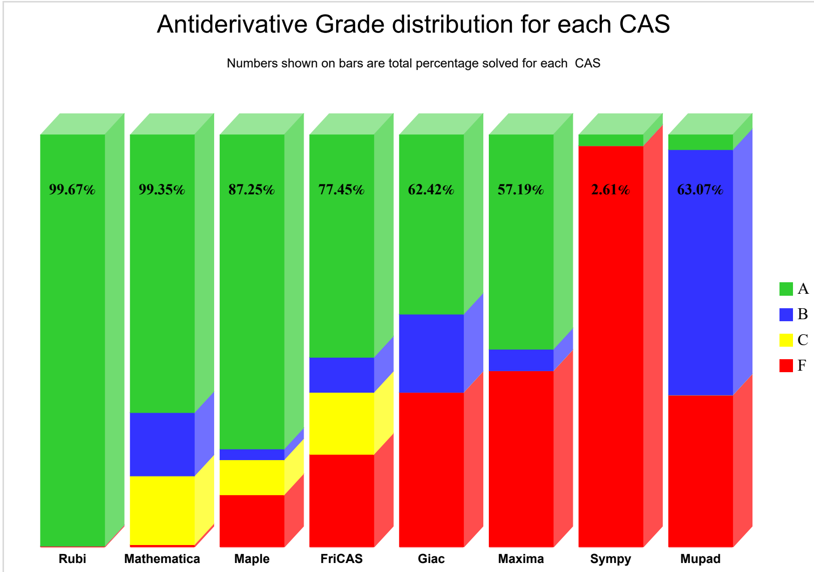
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

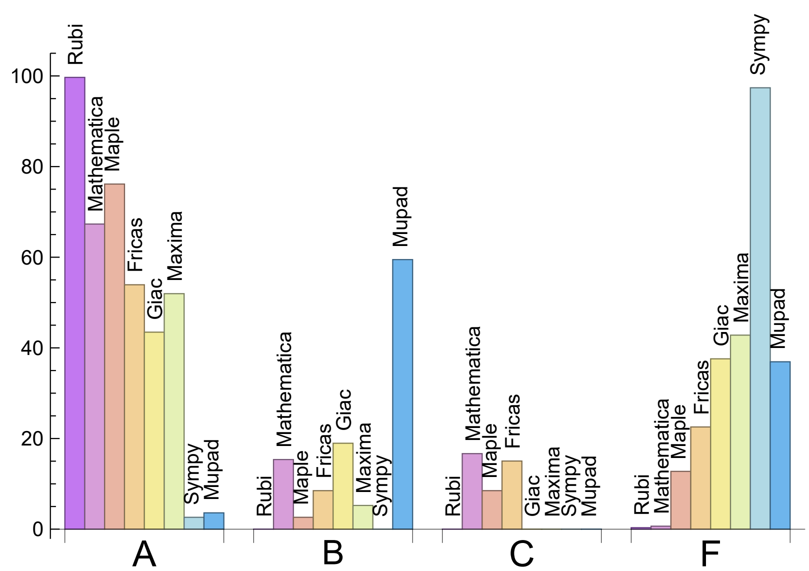
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.67	0.00	0.00	0.33
Maple	76.14	2.61	8.50	12.75
Mathematica	67.32	15.36	16.67	0.65
Fricas	53.92	8.50	15.03	22.55
Maxima	51.96	5.23	0.00	42.81
Giac	43.46	18.95	0.00	37.58
Mupad	N/A	59.48	0.00	36.93
Sympy	2.61	0.00	0.00	97.39

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	1	100.00 %	0.00 %	0.00 %
Mathematica	2	100.00 %	0.00 %	0.00 %
Maple	39	100.00 %	0.00 %	0.00 %
Fricas	69	66.67 %	33.33 %	0.00 %
Giac	115	97.39 %	0.00 %	2.61 %
Maxima	131	72.52 %	15.27 %	12.21 %
Sympy	298	61.41 %	22.82 %	15.77 %
Mupad	113	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

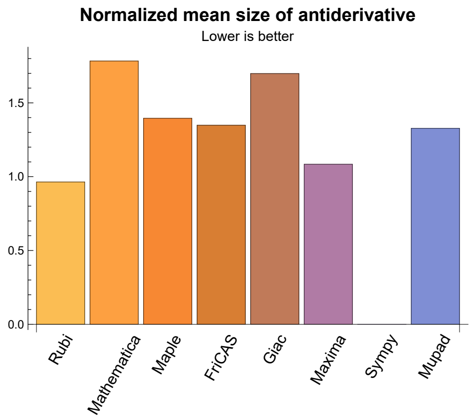
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

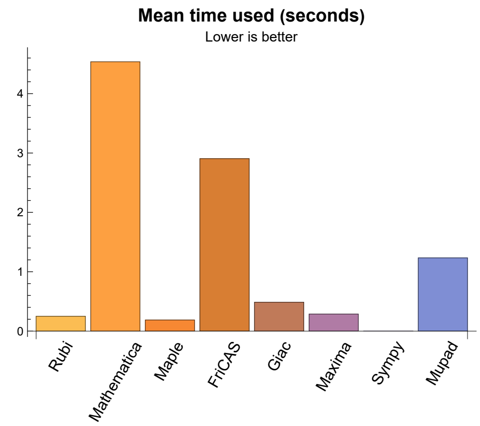
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.25	169.10	0.96	127.00	1.00
Mathematica	4.54	325.64	1.78	137.00	1.10
Maple	0.19	274.37	1.40	128.00	1.02
Maxima	0.29	127.21	1.08	106.00	0.97
Fricas	2.91	203.73	1.35	133.00	1.16
Sympy	0.00	0.00	0.00	0.00	0.00
Giac	0.49	248.10	1.70	149.00	1.42
Mupad	1.23	217.40	1.33	107.00	0.98

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
1.5
1.0
0.5
0.0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
4
3
2
1
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

$\{263, 264, 265, 266, 267, 273, 274, 277, 278, 279, 280\}$

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {114, 115, 116, 117, 118, 119, 138, 140, 141, 142, 143, 144, 152, 153, 154, 155, 156, 157, 158, 159, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 254, 260, 261, 262, 272, 275, 276, 287, 288, 289, 290, 291, 292}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```python
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

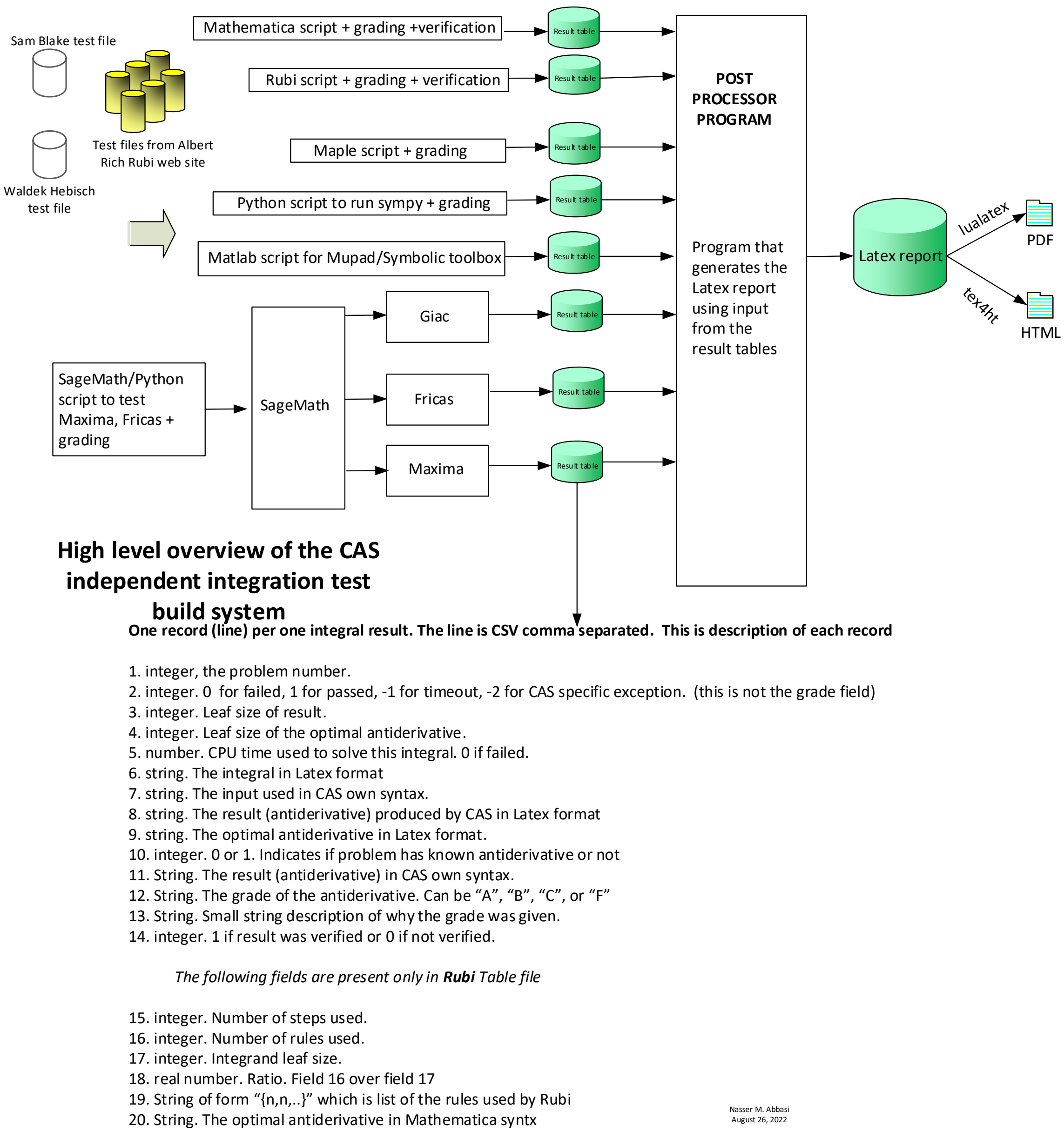
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306 }

B grade: { }

C grade: { }

F grade: { 276 }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 122, 124, 126, 127, 129, 131, 133, 136, 145, 146, 147, 148, 150, 154, 155, 160, 161, 162, 163, 165, 166, 167, 168, 169, 170, 174, 175, 176, 177, 179, 180, 181, 185, 186, 187, 188, 189, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 228, 230, 231, 232, 249, 250, 252, 253, 254, 256, 257, 258, 259, 263, 264, 265, 266, 267, 269, 270, 271, 273, 274, 277, 278, 279, 280, 281, 283, 285, 287, 289, 291, 293, 295, 297, 299, 300, 302, 304, 306 }

B grade: { 6, 32, 33, 34, 35, 36, 37, 51, 52, 53, 54, 55, 56, 70, 71, 72, 73, 140, 141, 142, 143, 144, 149, 151, 156, 157, 158, 159, 164, 178, 182, 183, 184, 190, 191, 192, 193, 194, 195, 229, 260, 261, 262, 268, 272, 275, 276 }

C grade: { 14, 15, 16, 17, 18, 121, 123, 125, 128, 130, 132, 134, 135, 138, 152, 153, 171, 172, 173, 226, 227, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 251, 255, 282, 284, 286, 288, 290, 292, 294, 296, 298, 301, 303, 305 }

F grade: { 137, 139 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 253, 263, 264, 265, 266, 267, 273, 274, 277, 278, 279, 280 }

B grade: { 203, 233, 250, 251, 252, 254, 255, 256 }

C grade: { 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306 }

F grade: { 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 257, 258, 259, 260, 261, 262, 268, 269, 270, 271, 272, 275, 276 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 69, 70, 74, 75, 76, 77, 78, 79, 80, 81, 82, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 103, 104, 105, 106, 107, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 209, 210, 211, 212, 213, 214, 221, 222, 223, 224, 225, 226, 263, 264, 265, 266, 267, 273, 274, 277, 278, 279, 280 }

B grade: { 65, 66, 67, 68, 71, 72, 73, 83, 84, 85, 86, 100, 101, 102, 215, 227 }

C grade: { }

F grade: { 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 203, 204, 205, 206, 207, 208, 216, 217, 218, 219, 220, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 268, 269, 270, 271, 272, 275, 276, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 35, 36, 38, 39, 40, 41, 42, 43, 44, 45, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 100, 101, 102, 103, 104, 105, 106, 107, 160, 161, 162, 163, 164, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 203, 204, 205, 206, 207, 209, 210, 211, 212, 213, 216, 217, 218, 219, 220, 221, 222, 223, 224, 228, 229, 230, 231, 232, 263, 264, 265, 266, 267, 273, 274, 277, 278, 279, 280 }

B grade: { 9, 16, 17, 18, 28, 37, 46, 47, 64, 71, 72, 73, 82, 97, 98, 99, 165, 166, 167, 202, 208, 214, 215, 225, 226, 227 }

C grade: { 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 282, 284, 286, 288, 290, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306 }

F grade: { 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 268, 269, 270, 271, 272, 275, 276, 281, 283, 285, 287, 289, 291 }

2.1.6 Sympy

A grade: { 264, 265, 266, 267, 274, 278, 279, 280 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 268, 269, 270, 271, 272, 273, 275, 276, 277, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306 }

2.1.7 Giac

A grade: { 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 22, 23, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 41, 42, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 78, 79, 80, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 162, 163, 168, 175, 176, 177, 180, 186, 187, 191, 193, 194, 195, 198, 199, 200, 201, 206, 211, 212, 216, 217, 218, 219, 220, 223, 224, 229, 231, 232, 263, 264, 265, 266, 267, 273, 274, 277, 278, 279, 280 }

B grade: { 1, 2, 3, 19, 20, 21, 24, 38, 40, 43, 44, 76, 77, 81, 93, 160, 161, 164, 165, 166, 167, 169, 170, 171, 172, 173, 174, 178, 179, 181, 182, 183, 184, 185, 188, 189, 190, 192, 196, 197, 202, 203, 204, 205, 207, 208, 209, 210, 213, 214, 215, 221, 222, 225, 226, 227, 228, 230 }

C grade: { }

F grade: { 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 268, 269, 270, 271, 272, 275, 276, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306 }

2.1.8 Mupad

A grade: { 263, 264, 265, 266, 267, 273, 274, 277, 278, 279, 280 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 148, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 270 }

C grade: { }

F grade: { 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 268, 269, 271, 272, 275, 276, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	106	107	113	115	0	293	111
N.S.	1	1.00	0.70	0.70	0.74	0.76	0.00	1.93	0.73
time (sec)	N/A	0.077	0.143	0.124	0.254	2.615	0.000	0.518	0.131

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	86	87	91	93	0	247	89
N.S.	1	1.00	0.72	0.73	0.76	0.78	0.00	2.08	0.75
time (sec)	N/A	0.072	0.092	0.149	0.266	2.835	0.000	0.478	0.079

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	83	67	69	71	0	201	67
N.S.	1	1.00	0.95	0.77	0.79	0.82	0.00	2.31	0.77
time (sec)	N/A	0.070	0.062	0.141	0.263	2.826	0.000	0.430	0.056

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	57	47	47	49	0	66	45
N.S.	1	1.00	0.98	0.81	0.81	0.84	0.00	1.14	0.78
time (sec)	N/A	0.061	0.037	0.112	0.268	2.105	0.000	0.436	0.063

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	37	24	23	25	0	32	20
N.S.	1	1.00	1.42	0.92	0.88	0.96	0.00	1.23	0.77
time (sec)	N/A	0.024	0.015	0.040	0.263	2.878	0.000	0.417	0.038

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	63	15	26	31	0	58	17
N.S.	1	1.00	2.10	0.50	0.87	1.03	0.00	1.93	0.57
time (sec)	N/A	0.046	0.026	0.065	0.270	2.660	0.000	0.434	0.124

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	114	41	52	93	0	102	53
N.S.	1	1.00	1.56	0.56	0.71	1.27	0.00	1.40	0.73
time (sec)	N/A	0.076	0.551	0.098	0.276	2.890	0.000	0.447	0.963

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	164	66	95	193	0	149	99
N.S.	1	1.00	1.39	0.56	0.81	1.64	0.00	1.26	0.84
time (sec)	N/A	0.095	0.243	0.115	0.260	3.140	0.000	0.457	0.099

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	165	89	136	307	0	196	142
N.S.	1	1.00	1.01	0.55	0.83	1.88	0.00	1.20	0.87
time (sec)	N/A	0.115	0.275	0.136	0.271	5.720	0.000	0.490	0.994

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	106	116	127	123	0	174	150
N.S.	1	1.00	0.64	0.70	0.77	0.75	0.00	1.05	0.91
time (sec)	N/A	0.114	0.222	0.167	0.271	3.141	0.000	0.467	1.144

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	86	96	106	102	0	146	120
N.S.	1	1.00	0.68	0.76	0.83	0.80	0.00	1.15	0.94
time (sec)	N/A	0.103	0.130	0.144	0.266	3.105	0.000	0.463	1.047

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	86	76	81	79	0	118	90
N.S.	1	1.00	0.97	0.85	0.91	0.89	0.00	1.33	1.01
time (sec)	N/A	0.086	0.085	0.126	0.265	4.525	0.000	0.472	1.025

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	54	55	59	55	0	88	80
N.S.	1	1.00	1.06	1.08	1.16	1.08	0.00	1.73	1.57
time (sec)	N/A	0.063	0.040	0.070	0.269	2.639	0.000	0.436	1.069

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	41	42	50	63	0	50	29
N.S.	1	1.00	1.11	1.14	1.35	1.70	0.00	1.35	0.78
time (sec)	N/A	0.075	0.025	0.097	0.268	3.928	0.000	0.624	0.961

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	63	76	108	0	79	65
N.S.	1	1.00	1.00	0.91	1.10	1.57	0.00	1.14	0.94
time (sec)	N/A	0.085	0.024	0.108	0.286	3.172	0.000	0.495	0.983

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	91	83	96	190	0	107	97
N.S.	1	1.00	0.90	0.82	0.95	1.88	0.00	1.06	0.96
time (sec)	N/A	0.089	0.024	0.119	0.279	3.296	0.000	0.558	1.005

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	113	103	116	281	0	136	128
N.S.	1	1.00	0.86	0.79	0.89	2.15	0.00	1.04	0.98
time (sec)	N/A	0.093	0.035	0.146	0.281	3.867	0.000	0.521	1.166

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	135	123	136	366	0	164	159
N.S.	1	1.00	0.82	0.75	0.82	2.22	0.00	0.99	0.96
time (sec)	N/A	0.098	0.041	0.142	0.274	4.394	0.000	0.597	1.643

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	127	181	146	167	0	370	146
N.S.	1	1.00	0.69	0.99	0.80	0.91	0.00	2.02	0.80
time (sec)	N/A	0.137	0.582	0.089	0.285	3.448	0.000	0.734	0.993

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	107	151	107	128	0	320	105
N.S.	1	1.00	0.82	1.15	0.82	0.98	0.00	2.44	0.80
time (sec)	N/A	0.119	0.384	0.140	0.286	3.015	0.000	0.578	0.950

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	87	121	94	115	0	270	91
N.S.	1	1.00	0.78	1.08	0.84	1.03	0.00	2.41	0.81
time (sec)	N/A	0.115	0.211	0.115	0.271	4.039	0.000	0.683	0.886

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	65	91	56	76	0	74	54
N.S.	1	1.00	1.05	1.47	0.90	1.23	0.00	1.19	0.87
time (sec)	N/A	0.091	0.142	0.091	0.266	3.079	0.000	0.666	0.058

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	31	34	41	51	0	51	41
N.S.	1	1.00	0.72	0.79	0.95	1.19	0.00	1.19	0.95
time (sec)	N/A	0.056	0.077	0.056	0.265	3.167	0.000	0.533	0.059

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	36	29	43	61	0	115	35
N.S.	1	1.00	0.75	0.60	0.90	1.27	0.00	2.40	0.73
time (sec)	N/A	0.082	0.054	0.075	0.272	2.137	0.000	0.546	0.082

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	75	38	68	112	0	135	61
N.S.	1	1.00	1.09	0.55	0.99	1.62	0.00	1.96	0.88
time (sec)	N/A	0.110	0.372	0.118	0.273	2.283	0.000	0.525	0.901

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	103	64	104	209	0	191	109
N.S.	1	1.00	0.90	0.56	0.90	1.82	0.00	1.66	0.95
time (sec)	N/A	0.128	1.032	0.115	0.287	2.791	0.000	0.536	0.095

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	136	85	143	289	0	238	147
N.S.	1	1.00	0.85	0.53	0.89	1.81	0.00	1.49	0.92
time (sec)	N/A	0.147	0.883	0.133	0.272	3.710	0.000	0.691	0.110

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	164	112	197	461	0	291	203
N.S.	1	1.00	0.80	0.55	0.96	2.25	0.00	1.42	0.99
time (sec)	N/A	0.177	2.284	0.162	0.274	2.817	0.000	0.757	0.171

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	144	194	215	185	0	225	293
N.S.	1	1.00	0.72	0.97	1.08	0.93	0.00	1.13	1.47
time (sec)	N/A	0.271	0.628	0.086	0.492	2.461	0.000	0.559	2.540

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	124	164	174	158	0	193	235
N.S.	1	1.00	0.79	1.04	1.11	1.01	0.00	1.23	1.50
time (sec)	N/A	0.204	0.349	0.121	0.478	2.562	0.000	0.609	2.157

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	94	134	126	133	0	161	177
N.S.	1	1.00	0.82	1.17	1.10	1.16	0.00	1.40	1.54
time (sec)	N/A	0.203	0.177	0.098	0.490	2.554	0.000	0.700	1.825

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	243	78	81	104	0	128	117
N.S.	1	1.00	3.33	1.07	1.11	1.42	0.00	1.75	1.60
time (sec)	N/A	0.107	0.780	0.072	0.490	3.032	0.000	0.536	1.160

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	401	77	74	101	0	90	70
N.S.	1	1.00	7.04	1.35	1.30	1.77	0.00	1.58	1.23
time (sec)	N/A	0.181	6.116	0.068	0.270	2.494	0.000	0.478	1.126

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	228	117	113	159	0	104	91
N.S.	1	1.00	2.62	1.34	1.30	1.83	0.00	1.20	1.05
time (sec)	N/A	0.218	1.274	0.086	0.274	2.737	0.000	0.470	2.485

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	317	155	144	206	0	136	124
N.S.	1	1.00	2.46	1.20	1.12	1.60	0.00	1.05	0.96
time (sec)	N/A	0.171	0.633	0.102	0.280	2.615	0.000	0.495	1.284

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	428	193	175	272	0	168	159
N.S.	1	1.00	2.63	1.18	1.07	1.67	0.00	1.03	0.98
time (sec)	N/A	0.180	0.817	0.116	0.266	2.668	0.000	0.534	0.988

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	1050	231	204	406	0	200	194
N.S.	1	1.00	5.22	1.15	1.01	2.02	0.00	1.00	0.97
time (sec)	N/A	0.196	6.559	0.120	0.281	2.756	0.000	0.537	0.972

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	148	252	158	182	0	396	157
N.S.	1	1.00	0.73	1.24	0.78	0.90	0.00	1.95	0.77
time (sec)	N/A	0.143	1.160	0.101	0.273	2.924	0.000	0.646	0.965

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	106	214	107	121	0	239	107
N.S.	1	1.00	0.81	1.63	0.82	0.92	0.00	1.82	0.82
time (sec)	N/A	0.123	0.583	0.092	0.260	3.504	0.000	0.588	0.897

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	108	172	106	130	0	297	107
N.S.	1	1.00	0.81	1.28	0.79	0.97	0.00	2.22	0.80
time (sec)	N/A	0.123	0.409	0.143	0.264	3.224	0.000	0.527	0.887

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	86	114	80	104	0	102	80
N.S.	1	1.00	0.88	1.16	0.82	1.06	0.00	1.04	0.82
time (sec)	N/A	0.073	0.134	0.119	0.262	2.972	0.000	0.489	0.882

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	65	46	55	65	0	64	52
N.S.	1	1.00	1.05	0.74	0.89	1.05	0.00	1.03	0.84
time (sec)	N/A	0.069	0.154	0.061	0.269	2.876	0.000	0.456	0.055

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	81	39	56	76	0	142	49
N.S.	1	1.00	1.21	0.58	0.84	1.13	0.00	2.12	0.73
time (sec)	N/A	0.092	0.090	0.080	0.269	3.126	0.000	0.476	0.074

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	88	50	84	132	0	189	75
N.S.	1	1.00	1.00	0.57	0.95	1.50	0.00	2.15	0.85
time (sec)	N/A	0.121	0.581	0.119	0.267	3.539	0.000	0.540	0.095

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	100	63	103	177	0	186	96
N.S.	1	1.00	0.90	0.57	0.93	1.59	0.00	1.68	0.86
time (sec)	N/A	0.123	0.634	0.125	0.269	4.172	0.000	0.542	0.931

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	129	85	145	297	0	243	151
N.S.	1	1.00	0.82	0.54	0.92	1.89	0.00	1.55	0.96
time (sec)	N/A	0.146	0.709	0.148	0.272	4.607	0.000	0.557	0.965

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	159	110	189	419	0	292	195
N.S.	1	1.00	0.79	0.54	0.94	2.07	0.00	1.45	0.97
time (sec)	N/A	0.172	0.806	0.168	0.272	2.911	0.000	0.570	1.013

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	156	272	291	204	0	244	320
N.S.	1	1.00	0.74	1.30	1.39	0.97	0.00	1.16	1.52
time (sec)	N/A	0.285	1.405	0.102	0.473	3.448	0.000	0.612	2.454

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	136	232	240	177	0	212	261
N.S.	1	1.00	0.75	1.27	1.32	0.97	0.00	1.16	1.43
time (sec)	N/A	0.215	0.605	0.141	0.487	3.721	0.000	0.594	2.225

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	114	192	182	152	0	180	204
N.S.	1	1.00	0.83	1.39	1.32	1.10	0.00	1.30	1.48
time (sec)	N/A	0.174	0.262	0.116	0.482	3.051	0.000	0.576	1.966

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	300	126	127	125	0	102	90
N.S.	1	1.00	3.06	1.29	1.30	1.28	0.00	1.04	0.92
time (sec)	N/A	0.143	1.587	0.086	0.484	3.875	0.000	0.546	1.278

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	244	127	137	122	0	106	98
N.S.	1	1.00	3.05	1.59	1.71	1.52	0.00	1.32	1.22
time (sec)	N/A	0.139	0.809	0.083	0.275	4.376	0.000	0.510	2.460

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	290	185	188	178	0	123	116
N.S.	1	1.00	2.64	1.68	1.71	1.62	0.00	1.12	1.05
time (sec)	N/A	0.173	5.376	0.112	0.272	2.696	0.000	0.528	5.348

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	353	241	228	225	0	141	136
N.S.	1	1.00	2.14	1.46	1.38	1.36	0.00	0.85	0.82
time (sec)	N/A	0.307	0.797	0.119	0.282	2.763	0.000	0.549	4.913

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	430	297	268	278	0	169	169
N.S.	1	1.00	2.24	1.55	1.40	1.45	0.00	0.88	0.88
time (sec)	N/A	0.232	0.811	0.132	0.280	3.580	0.000	0.555	2.896

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	1000	353	308	375	0	202	204
N.S.	1	1.00	4.31	1.52	1.33	1.62	0.00	0.87	0.88
time (sec)	N/A	0.239	6.453	0.135	0.281	3.153	0.000	0.573	1.039

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	62	89	89	89	0	141	110
N.S.	1	1.00	0.68	0.98	0.98	0.98	0.00	1.55	1.21
time (sec)	N/A	0.116	2.659	0.151	0.269	4.182	0.000	0.434	0.087

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	52	70	69	69	0	119	84
N.S.	1	1.00	0.71	0.96	0.95	0.95	0.00	1.63	1.15
time (sec)	N/A	0.115	1.004	0.122	0.271	2.832	0.000	0.456	0.060

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	42	49	49	49	0	97	58
N.S.	1	1.00	0.76	0.89	0.89	0.89	0.00	1.76	1.05
time (sec)	N/A	0.109	0.231	0.095	0.278	2.443	0.000	0.433	0.074

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	32	30	29	29	0	32	26
N.S.	1	1.00	0.86	0.81	0.78	0.78	0.00	0.86	0.70
time (sec)	N/A	0.093	0.075	0.078	0.273	2.678	0.000	0.460	0.882

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	28	37	30	28	0	34	25
N.S.	1	1.00	0.90	1.19	0.97	0.90	0.00	1.10	0.81
time (sec)	N/A	0.052	0.060	0.033	0.276	3.025	0.000	0.412	0.054

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	67	43	47	60	0	56	33
N.S.	1	1.00	1.16	0.74	0.81	1.03	0.00	0.97	0.57
time (sec)	N/A	0.073	0.080	0.074	0.266	4.218	0.000	0.461	0.919

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	91	55	86	138	0	129	75
N.S.	1	1.00	1.11	0.67	1.05	1.68	0.00	1.57	0.91
time (sec)	N/A	0.120	0.271	0.106	0.275	2.682	0.000	0.481	0.935

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	122	79	130	217	0	182	115
N.S.	1	1.00	1.15	0.75	1.23	2.05	0.00	1.72	1.08
time (sec)	N/A	0.130	0.323	0.118	0.273	2.665	0.000	0.476	1.008

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	132	142	360	91	0	139	132
N.S.	1	1.00	1.06	1.14	2.88	0.73	0.00	1.11	1.06
time (sec)	N/A	0.156	0.820	0.155	0.488	3.395	0.000	0.452	3.896

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	112	116	278	70	0	113	106
N.S.	1	1.00	1.13	1.17	2.81	0.71	0.00	1.14	1.07
time (sec)	N/A	0.138	0.474	0.128	0.482	3.063	0.000	0.426	3.664

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	83	90	196	51	0	87	55
N.S.	1	1.00	1.14	1.23	2.68	0.70	0.00	1.19	0.75
time (sec)	N/A	0.111	0.420	0.090	0.479	2.776	0.000	0.433	0.984

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	68	64	112	27	0	58	30
N.S.	1	1.00	1.55	1.45	2.55	0.61	0.00	1.32	0.68
time (sec)	N/A	0.079	0.206	0.076	0.481	2.465	0.000	0.421	0.963

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	66	36	49	41	0	37	32
N.S.	1	1.00	1.78	0.97	1.32	1.11	0.00	1.00	0.86
time (sec)	N/A	0.092	0.162	0.076	0.266	3.496	0.000	0.458	0.927

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	116	62	96	89	0	74	60
N.S.	1	1.00	2.11	1.13	1.75	1.62	0.00	1.35	1.09
time (sec)	N/A	0.105	0.354	0.096	0.269	3.207	0.000	0.452	1.087

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	158	88	136	131	0	103	153
N.S.	1	1.00	2.16	1.21	1.86	1.79	0.00	1.41	2.10
time (sec)	N/A	0.108	0.424	0.105	0.277	3.667	0.000	0.462	1.161

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	200	114	176	177	0	132	201
N.S.	1	1.00	2.20	1.25	1.93	1.95	0.00	1.45	2.21
time (sec)	N/A	0.117	0.649	0.114	0.270	2.824	0.000	0.479	1.454

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	242	140	216	219	0	161	139
N.S.	1	1.00	2.22	1.28	1.98	2.01	0.00	1.48	1.28
time (sec)	N/A	0.119	0.967	0.118	0.278	3.164	0.000	0.503	3.108

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	72	88	89	89	0	185	109
N.S.	1	1.00	0.53	0.64	0.65	0.65	0.00	1.35	0.80
time (sec)	N/A	0.135	3.131	0.141	0.269	3.281	0.000	0.530	0.087

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	62	79	79	79	0	141	96
N.S.	1	1.00	0.54	0.69	0.69	0.69	0.00	1.24	0.84
time (sec)	N/A	0.128	2.279	0.166	0.270	3.346	0.000	0.500	0.910

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	53	50	49	49	0	141	58
N.S.	1	1.00	0.73	0.68	0.67	0.67	0.00	1.93	0.79
time (sec)	N/A	0.115	1.188	0.144	0.265	3.776	0.000	0.456	0.918

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	42	39	39	39	0	119	36
N.S.	1	1.00	0.76	0.71	0.71	0.71	0.00	2.16	0.65
time (sec)	N/A	0.108	0.435	0.112	0.272	3.846	0.000	0.482	0.061

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	51	58	51	48	0	75	56
N.S.	1	1.00	0.77	0.88	0.77	0.73	0.00	1.14	0.85
time (sec)	N/A	0.114	0.138	0.117	0.276	4.597	0.000	0.501	0.064

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	64	51	46	58	0	52	46
N.S.	1	1.00	1.23	0.98	0.88	1.12	0.00	1.00	0.88
time (sec)	N/A	0.073	0.103	0.043	0.276	4.073	0.000	0.484	0.923

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	83	55	74	106	0	87	60
N.S.	1	1.00	1.38	0.92	1.23	1.77	0.00	1.45	1.00
time (sec)	N/A	0.092	0.122	0.100	0.275	2.872	0.000	0.465	0.096

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	38	57	59	60	0	82	58
N.S.	1	1.00	0.90	1.36	1.40	1.43	0.00	1.95	1.38
time (sec)	N/A	0.089	0.064	0.101	0.270	3.540	0.000	0.471	0.942

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	152	103	167	283	0	207	152
N.S.	1	1.00	1.04	0.71	1.14	1.94	0.00	1.42	1.04
time (sec)	N/A	0.158	0.519	0.146	0.271	3.488	0.000	0.510	1.053

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	131	141	378	90	0	139	133
N.S.	1	1.00	0.78	0.84	2.26	0.54	0.00	0.83	0.80
time (sec)	N/A	0.320	1.772	0.153	0.488	3.555	0.000	0.492	3.934

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	111	115	292	71	0	113	107
N.S.	1	1.00	1.07	1.11	2.81	0.68	0.00	1.09	1.03
time (sec)	N/A	0.223	0.589	0.139	0.485	3.752	0.000	0.506	3.750

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	91	89	206	50	0	87	81
N.S.	1	1.00	1.05	1.02	2.37	0.57	0.00	1.00	0.93
time (sec)	N/A	0.170	0.373	0.125	0.483	4.130	0.000	0.441	4.505

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	121	73	140	61	0	75	91
N.S.	1	1.00	1.75	1.06	2.03	0.88	0.00	1.09	1.32
time (sec)	N/A	0.226	0.220	0.116	0.475	2.563	0.000	0.459	1.030

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	105	60	90	71	0	74	71
N.S.	1	1.00	1.44	0.82	1.23	0.97	0.00	1.01	0.97
time (sec)	N/A	0.148	0.299	0.084	0.273	3.064	0.000	0.477	0.982

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	149	86	134	108	0	105	121
N.S.	1	1.00	1.64	0.95	1.47	1.19	0.00	1.15	1.33
time (sec)	N/A	0.240	0.447	0.105	0.271	3.106	0.000	0.500	1.068

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	191	112	174	169	0	134	106
N.S.	1	1.00	1.75	1.03	1.60	1.55	0.00	1.23	0.97
time (sec)	N/A	0.258	0.652	0.118	0.282	2.820	0.000	0.544	2.185

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	233	112	174	204	0	134	201
N.S.	1	1.00	1.86	0.90	1.39	1.63	0.00	1.07	1.61
time (sec)	N/A	0.266	0.924	0.121	0.272	4.595	0.000	0.495	1.687

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	120	90	89	89	0	207	110
N.S.	1	1.00	0.86	0.65	0.64	0.64	0.00	1.49	0.79
time (sec)	N/A	0.142	2.694	0.147	0.274	4.212	0.000	0.551	0.089

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	100	69	69	69	0	185	84
N.S.	1	1.00	0.92	0.63	0.63	0.63	0.00	1.70	0.77
time (sec)	N/A	0.126	1.850	0.188	0.271	2.750	0.000	0.577	0.921

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	80	50	49	49	0	163	58
N.S.	1	1.00	1.10	0.68	0.67	0.67	0.00	2.23	0.79
time (sec)	N/A	0.118	1.096	0.135	0.277	2.508	0.000	0.551	0.070

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	73	79	73	70	0	172	82
N.S.	1	1.00	0.72	0.77	0.72	0.69	0.00	1.69	0.80
time (sec)	N/A	0.130	0.544	0.151	0.270	2.762	0.000	0.544	0.895

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	99	72	72	82	0	94	75
N.S.	1	1.00	1.11	0.81	0.81	0.92	0.00	1.06	0.84
time (sec)	N/A	0.132	0.274	0.134	0.269	2.617	0.000	0.513	0.888

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	103	63	71	96	0	63	59
N.S.	1	1.00	1.37	0.84	0.95	1.28	0.00	0.84	0.79
time (sec)	N/A	0.089	0.206	0.057	0.264	3.055	0.000	0.491	0.079

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	97	67	98	151	0	113	83
N.S.	1	1.00	1.18	0.82	1.20	1.84	0.00	1.38	1.01
time (sec)	N/A	0.111	0.214	0.115	0.278	2.768	0.000	0.545	0.111

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	138	91	146	240	0	182	130
N.S.	1	1.00	1.10	0.72	1.16	1.90	0.00	1.44	1.03
time (sec)	N/A	0.100	0.394	0.135	0.272	2.689	0.000	0.564	0.167

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	137	91	188	317	0	232	173
N.S.	1	1.00	1.07	0.71	1.47	2.48	0.00	1.81	1.35
time (sec)	N/A	0.152	3.005	0.148	0.277	2.733	0.000	0.606	1.090

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	131	142	378	91	0	139	132
N.S.	1	1.00	0.83	0.90	2.41	0.58	0.00	0.89	0.84
time (sec)	N/A	0.327	3.070	0.156	0.486	2.946	0.000	0.538	3.865

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	111	116	292	70	0	113	106
N.S.	1	1.00	0.86	0.90	2.26	0.54	0.00	0.88	0.82
time (sec)	N/A	0.217	1.147	0.148	0.505	2.995	0.000	0.509	3.669

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	173	100	227	83	0	101	98
N.S.	1	1.00	1.60	0.93	2.10	0.77	0.00	0.94	0.91
time (sec)	N/A	0.230	0.477	0.145	0.488	3.092	0.000	0.524	2.659

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	177	87	164	99	0	96	115
N.S.	1	1.00	1.82	0.90	1.69	1.02	0.00	0.99	1.19
time (sec)	N/A	0.224	0.291	0.120	0.476	2.776	0.000	0.496	1.057

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	137	60	90	95	0	73	84
N.S.	1	1.00	1.54	0.67	1.01	1.07	0.00	0.82	0.94
time (sec)	N/A	0.262	0.398	0.103	0.276	3.692	0.000	0.529	1.009

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	175	60	92	146	0	73	105
N.S.	1	1.00	1.70	0.58	0.89	1.42	0.00	0.71	1.02
time (sec)	N/A	0.263	0.493	0.125	0.278	3.137	0.000	0.528	1.112

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	223	112	174	191	0	134	201
N.S.	1	1.00	1.76	0.88	1.37	1.50	0.00	1.06	1.58
time (sec)	N/A	0.287	0.810	0.134	0.269	3.453	0.000	0.577	1.435

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	265	138	214	214	0	163	249
N.S.	1	1.00	1.83	0.95	1.48	1.48	0.00	1.12	1.72
time (sec)	N/A	0.296	1.154	0.136	0.279	3.881	0.000	0.525	2.128

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	106	210	0	228	0	0	-1
N.S.	1	1.00	0.68	1.34	0.00	1.45	0.00	0.00	-0.01
time (sec)	N/A	0.151	0.206	0.216	0.000	1.433	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	170	154	0	221	0	0	-1
N.S.	1	1.00	1.10	1.00	0.00	1.44	0.00	0.00	-0.01
time (sec)	N/A	0.137	0.383	0.200	0.000	10.856	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	69	142	0	200	0	0	-1
N.S.	1	1.00	0.66	1.37	0.00	1.92	0.00	0.00	-0.01
time (sec)	N/A	0.111	0.076	0.179	0.000	1.929	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	193	117	0	144	0	0	-1
N.S.	1	1.00	1.87	1.14	0.00	1.40	0.00	0.00	-0.01
time (sec)	N/A	0.114	1.972	0.174	0.000	4.056	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	143	201	0	202	0	0	-1
N.S.	1	1.00	0.92	1.30	0.00	1.30	0.00	0.00	-0.01
time (sec)	N/A	0.143	0.259	0.183	0.000	1.412	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	120	164	0	219	0	0	-1
N.S.	1	1.00	0.75	1.02	0.00	1.37	0.00	0.00	-0.01
time (sec)	N/A	0.142	0.238	0.204	0.000	4.330	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	248	265	0	287	0	0	-1
N.S.	1	1.00	1.28	1.37	0.00	1.48	0.00	0.00	-0.01
time (sec)	N/A	0.266	21.942	0.296	0.000	1.928	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	220	201	0	279	0	0	-1
N.S.	1	1.00	1.15	1.05	0.00	1.45	0.00	0.00	-0.01
time (sec)	N/A	0.258	30.399	0.225	0.000	9.159	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	206	219	0	255	0	0	-1
N.S.	1	1.00	1.49	1.59	0.00	1.85	0.00	0.00	-0.01
time (sec)	N/A	0.219	11.601	0.243	0.000	1.475	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	179	163	0	197	0	0	-1
N.S.	1	1.00	1.29	1.17	0.00	1.42	0.00	0.00	-0.01
time (sec)	N/A	0.218	72.923	0.201	0.000	2.942	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	260	238	0	262	0	0	-1
N.S.	1	1.00	1.16	1.06	0.00	1.17	0.00	0.00	-0.00
time (sec)	N/A	0.290	17.675	0.244	0.000	1.370	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	C	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	241	301	0	291	0	0	-1
N.S.	1	1.00	1.03	1.29	0.00	1.24	0.00	0.00	-0.00
time (sec)	N/A	0.291	40.434	0.283	0.000	3.654	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	122	128	0	114	0	0	-1
N.S.	1	1.00	0.88	0.92	0.00	0.82	0.00	0.00	-0.01
time (sec)	N/A	0.187	0.467	0.187	0.000	0.949	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	232	173	0	95	0	0	-1
N.S.	1	1.00	2.23	1.66	0.00	0.91	0.00	0.00	-0.01
time (sec)	N/A	0.156	3.144	0.309	0.000	0.959	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	69	112	0	86	0	0	-1
N.S.	1	1.00	0.68	1.10	0.00	0.84	0.00	0.00	-0.01
time (sec)	N/A	0.154	13.520	0.183	0.000	0.823	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	249	148	0	121	0	0	-1
N.S.	1	1.00	2.62	1.56	0.00	1.27	0.00	0.00	-0.01
time (sec)	N/A	0.147	0.419	0.171	0.000	1.278	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	77	121	0	104	0	0	-1
N.S.	1	1.00	0.76	1.20	0.00	1.03	0.00	0.00	-0.01
time (sec)	N/A	0.157	0.365	0.175	0.000	0.688	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	124	187	0	155	0	0	-1
N.S.	1	1.00	0.92	1.39	0.00	1.15	0.00	0.00	-0.01
time (sec)	N/A	0.179	0.749	0.203	0.000	0.619	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	91	136	0	205	0	0	-1
N.S.	1	1.00	0.67	1.01	0.00	1.52	0.00	0.00	-0.01
time (sec)	N/A	0.180	0.848	0.188	0.000	0.788	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	94	145	0	113	0	0	-1
N.S.	1	1.00	0.58	0.90	0.00	0.70	0.00	0.00	-0.01
time (sec)	N/A	0.380	1.035	0.230	0.000	0.916	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	249	173	0	152	0	0	-1
N.S.	1	1.00	1.33	0.93	0.00	0.81	0.00	0.00	-0.01
time (sec)	N/A	0.415	1.956	0.222	0.000	1.118	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	119	153	0	141	0	0	-1
N.S.	1	1.00	0.63	0.81	0.00	0.75	0.00	0.00	-0.01
time (sec)	N/A	0.417	1.237	0.225	0.000	0.794	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	222	205	0	184	0	0	-1
N.S.	1	1.00	1.18	1.09	0.00	0.98	0.00	0.00	-0.01
time (sec)	N/A	0.405	0.905	0.238	0.000	0.717	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	82	148	0	162	0	0	-1
N.S.	1	1.00	0.43	0.78	0.00	0.85	0.00	0.00	-0.01
time (sec)	N/A	0.429	0.938	0.217	0.000	0.967	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	163	213	0	213	0	0	-1
N.S.	1	1.00	0.73	0.95	0.00	0.95	0.00	0.00	-0.00
time (sec)	N/A	0.462	0.956	0.237	0.000	1.081	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	113	160	0	226	0	0	-1
N.S.	1	1.00	0.50	0.71	0.00	1.01	0.00	0.00	-0.00
time (sec)	N/A	0.457	0.654	0.257	0.000	0.744	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	287	0	0	0	0	0	-1
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.249	4.447	0.149	0.000	0.000	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	230	0	0	0	0	0	-1
N.S.	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.202	2.676	0.236	0.000	0.000	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	97	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.102	0.100	0.109	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.153	19.793	0.076	0.000	0.000	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	2272	0	0	0	0	0	-1
N.S.	1	1.00	10.98	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.379	17.277	0.105	0.000	0.000	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.450	6.409	0.112	0.000	0.000	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	1243	0	0	0	0	0	-1
N.S.	1	1.00	11.73	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.245	8.429	0.106	0.000	0.000	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	433	0	0	0	0	0	-1
N.S.	1	1.00	4.05	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.210	1.910	0.102	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	277	0	0	0	0	0	-1
N.S.	1	1.00	2.41	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.224	1.373	0.099	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	484	0	0	0	0	0	-1
N.S.	1	1.00	4.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.262	1.953	0.092	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	276	0	0	0	0	0	-1
N.S.	1	1.00	2.12	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.196	1.321	0.100	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	113	0	0	0	0	0	-1
N.S.	1	1.00	0.63	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.124	0.990	0.319	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	84	0	0	0	0	0	-1
N.S.	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	0.345	0.288	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	67	0	0	0	0	0	-1
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.055	0.101	0.282	0.000	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	0	0	0	0	0	64
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.52
time (sec)	N/A	0.029	0.028	0.036	0.000	0.000	0.000	0.000	1.182

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	92	0	0	0	0	0	-1
N.S.	1	1.00	2.30	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.037	0.417	0.091	0.000	0.000	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	179	0	0	0	0	0	-1
N.S.	1	1.00	1.60	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.072	1.429	0.095	0.000	0.000	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	492	0	0	0	0	0	-1
N.S.	1	1.00	2.05	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.165	4.168	0.101	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	7069	0	0	0	0	0	-1
N.S.	1	1.00	30.73	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.471	20.636	0.323	0.000	0.000	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	4297	0	0	0	0	0	-1
N.S.	1	1.00	45.23	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.242	15.298	0.253	0.000	0.000	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	142	0	0	0	0	0	-1
N.S.	1	1.00	1.45	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.803	0.092	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	350	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.390	4.257	0.095	0.000	0.000	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	382	0	0	0	0	0	-1
N.S.	1	1.00	3.64	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.168	11.204	0.092	0.000	0.000	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	214	0	0	0	0	0	-1
N.S.	1	1.00	2.04	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.159	1.976	0.088	0.000	0.000	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	212	0	0	0	0	0	-1
N.S.	1	1.00	2.02	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.162	2.116	0.082	0.000	0.000	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	212	0	0	0	0	0	-1
N.S.	1	1.00	2.02	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.182	2.566	0.085	0.000	0.000	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	115	87	91	93	0	317	89
N.S.	1	1.00	0.97	0.73	0.76	0.78	0.00	2.66	0.75
time (sec)	N/A	0.076	0.096	0.122	0.271	3.233	0.000	0.464	0.934

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	83	67	69	71	0	248	67
N.S.	1	1.00	0.95	0.77	0.79	0.82	0.00	2.85	0.77
time (sec)	N/A	0.069	0.061	0.091	0.267	3.550	0.000	0.486	0.906

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	57	47	47	49	0	66	45
N.S.	1	1.00	0.98	0.81	0.81	0.84	0.00	1.14	0.78
time (sec)	N/A	0.060	0.035	0.069	0.272	3.660	0.000	0.463	0.057

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	37	26	23	25	0	32	23
N.S.	1	1.00	1.42	1.00	0.88	0.96	0.00	1.23	0.88
time (sec)	N/A	0.023	0.018	0.050	0.267	4.823	0.000	0.445	0.041

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	63	33	45	51	0	61	63
N.S.	1	1.00	2.42	1.27	1.73	1.96	0.00	2.35	2.42
time (sec)	N/A	0.050	0.025	0.074	0.277	3.327	0.000	0.448	0.108

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	114	61	71	123	0	169	76
N.S.	1	1.00	1.78	0.95	1.11	1.92	0.00	2.64	1.19
time (sec)	N/A	0.078	0.323	0.136	0.279	4.530	0.000	0.491	0.097

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	164	83	110	201	0	266	117
N.S.	1	1.00	1.64	0.83	1.10	2.01	0.00	2.66	1.17
time (sec)	N/A	0.090	0.384	0.163	0.270	2.740	0.000	0.477	0.984

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	216	103	143	284	0	357	148
N.S.	1	1.00	1.54	0.74	1.02	2.03	0.00	2.55	1.06
time (sec)	N/A	0.107	0.400	0.184	0.276	4.097	0.000	0.471	1.021

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	118	96	106	102	0	228	332
N.S.	1	1.00	0.93	0.76	0.83	0.80	0.00	1.80	2.61
time (sec)	N/A	0.097	0.138	0.104	0.270	4.981	0.000	0.449	2.169

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	86	76	81	79	0	172	267
N.S.	1	1.00	0.97	0.85	0.91	0.89	0.00	1.93	3.00
time (sec)	N/A	0.086	0.104	0.079	0.261	4.730	0.000	0.454	1.884

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	54	55	59	55	0	114	83
N.S.	1	1.00	1.06	1.08	1.16	1.08	0.00	2.24	1.63
time (sec)	N/A	0.064	0.045	0.072	0.270	3.127	0.000	0.477	1.094

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	41	42	50	63	0	77	60
N.S.	1	1.00	1.11	1.14	1.35	1.70	0.00	2.08	1.62
time (sec)	N/A	0.072	0.022	0.062	0.282	2.903	0.000	0.437	1.022

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	63	76	125	0	133	101
N.S.	1	1.00	1.00	0.91	1.10	1.81	0.00	1.93	1.46
time (sec)	N/A	0.077	0.021	0.087	0.261	3.794	0.000	0.466	0.999

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	91	83	96	174	0	194	142
N.S.	1	1.00	0.90	0.82	0.95	1.72	0.00	1.92	1.41
time (sec)	N/A	0.084	0.021	0.109	0.270	3.432	0.000	0.461	1.094

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	112	120	105	125	0	418	104
N.S.	1	1.00	0.90	0.97	0.85	1.01	0.00	3.37	0.84
time (sec)	N/A	0.143	0.236	0.114	0.264	3.046	0.000	0.510	0.951

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	72	90	71	92	0	100	69
N.S.	1	1.00	0.90	1.12	0.89	1.15	0.00	1.25	0.86
time (sec)	N/A	0.106	0.121	0.089	0.274	2.821	0.000	0.518	0.915

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	37	40	40	50	0	50	40
N.S.	1	1.00	0.88	0.95	0.95	1.19	0.00	1.19	0.95
time (sec)	N/A	0.058	0.039	0.047	0.281	2.919	0.000	0.465	0.052

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	91	66	73	97	0	124	62
N.S.	1	1.00	1.23	0.89	0.99	1.31	0.00	1.68	0.84
time (sec)	N/A	0.133	0.103	0.088	0.265	3.250	0.000	0.487	0.966

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	329	116	119	205	0	314	120
N.S.	1	1.00	2.89	1.02	1.04	1.80	0.00	2.75	1.05
time (sec)	N/A	0.211	0.409	0.120	0.265	3.058	0.000	0.509	0.121

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	193	163	173	176	0	379	231
N.S.	1	1.00	1.10	0.93	0.99	1.01	0.00	2.17	1.32
time (sec)	N/A	0.342	1.085	0.125	0.496	4.022	0.000	0.499	3.102

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	157	133	125	142	0	285	207
N.S.	1	1.00	0.88	0.75	0.70	0.80	0.00	1.60	1.16
time (sec)	N/A	0.390	0.682	0.112	0.487	2.877	0.000	0.495	1.227

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	121	77	80	108	0	159	143
N.S.	1	1.00	1.57	1.00	1.04	1.40	0.00	2.06	1.86
time (sec)	N/A	0.094	0.399	0.086	0.486	2.886	0.000	0.525	1.168

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	138	76	73	104	0	167	108
N.S.	1	1.00	2.34	1.29	1.24	1.76	0.00	2.83	1.83
time (sec)	N/A	0.298	0.323	0.071	0.263	3.592	0.000	0.512	1.071

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	259	116	112	178	0	226	182
N.S.	1	1.00	2.59	1.16	1.12	1.78	0.00	2.26	1.82
time (sec)	N/A	0.226	0.421	0.096	0.278	3.289	0.000	0.527	1.102

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	368	154	143	241	0	326	248
N.S.	1	1.00	2.57	1.08	1.00	1.69	0.00	2.28	1.73
time (sec)	N/A	0.289	0.474	0.122	0.270	3.899	0.000	0.542	1.045

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	154	174	142	175	0	695	143
N.S.	1	1.00	0.91	1.02	0.84	1.03	0.00	4.09	0.84
time (sec)	N/A	0.185	0.409	0.142	0.292	3.220	0.000	0.603	0.965

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	102	116	98	123	0	128	99
N.S.	1	1.00	0.88	1.00	0.84	1.06	0.00	1.10	0.85
time (sec)	N/A	0.093	0.216	0.110	0.269	3.734	0.000	0.536	0.072

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	56	57	57	67	0	66	57
N.S.	1	1.00	0.88	0.89	0.89	1.05	0.00	1.03	0.89
time (sec)	N/A	0.071	0.076	0.065	0.263	4.679	0.000	0.502	0.925

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	89	92	112	139	0	250	85
N.S.	1	1.00	0.87	0.90	1.10	1.36	0.00	2.45	0.83
time (sec)	N/A	0.155	0.196	0.102	0.266	7.581	0.000	0.518	0.130

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	260	162	171	290	0	482	159
N.S.	1	1.00	1.60	1.00	1.06	1.79	0.00	2.98	0.98
time (sec)	N/A	0.251	4.695	0.142	0.266	4.671	0.000	0.536	1.008

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	818	234	242	241	0	563	373
N.S.	1	1.00	2.74	0.78	0.81	0.81	0.00	1.88	1.25
time (sec)	N/A	0.253	6.179	0.162	0.488	4.576	0.000	0.573	1.685

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	696	194	183	196	0	431	281
N.S.	1	1.00	2.95	0.82	0.78	0.83	0.00	1.83	1.19
time (sec)	N/A	0.515	6.112	0.125	0.487	3.258	0.000	0.588	1.472

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	327	128	129	151	0	346	202
N.S.	1	1.00	2.37	0.93	0.93	1.09	0.00	2.51	1.46
time (sec)	N/A	0.347	0.576	0.098	0.488	3.899	0.000	0.520	1.269

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	406	129	139	151	0	225	181
N.S.	1	1.00	3.05	0.97	1.05	1.14	0.00	1.69	1.36
time (sec)	N/A	0.196	0.443	0.092	0.268	4.846	0.000	0.515	1.466

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	610	187	190	260	0	361	260
N.S.	1	1.00	2.98	0.91	0.93	1.27	0.00	1.76	1.27
time (sec)	N/A	0.206	0.603	0.138	0.273	3.407	0.000	0.542	1.165

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	812	243	230	354	0	498	363
N.S.	1	1.00	2.91	0.87	0.82	1.27	0.00	1.78	1.30
time (sec)	N/A	0.223	0.929	0.149	0.278	4.153	0.000	0.554	1.222

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	282	275	224	222	0	1559	249
N.S.	1	1.00	1.26	1.23	1.00	1.00	0.00	6.99	1.12
time (sec)	N/A	0.183	0.956	0.199	0.262	3.498	0.000	0.492	0.155

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	172	160	141	140	0	867	151
N.S.	1	1.00	1.13	1.05	0.93	0.92	0.00	5.70	0.99
time (sec)	N/A	0.143	0.256	0.133	0.269	2.575	0.000	0.518	0.091

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	89	82	80	78	0	102	79
N.S.	1	1.00	1.00	0.92	0.90	0.88	0.00	1.15	0.89
time (sec)	N/A	0.108	0.112	0.102	0.271	2.407	0.000	0.476	1.023

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	30	48	33	31	0	38	30
N.S.	1	1.00	0.88	1.41	0.97	0.91	0.00	1.12	0.88
time (sec)	N/A	0.055	0.015	0.046	0.272	2.109	0.000	0.501	0.058

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	63	70	64	64	0	100	68
N.S.	1	1.00	0.85	0.95	0.86	0.86	0.00	1.35	0.92
time (sec)	N/A	0.074	0.067	0.095	0.282	2.759	0.000	0.476	1.147

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	123	110	132	216	0	202	133
N.S.	1	1.00	1.06	0.95	1.14	1.86	0.00	1.74	1.15
time (sec)	N/A	0.151	0.404	0.154	0.272	2.496	0.000	0.482	0.303

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	207	172	268	469	0	419	297
N.S.	1	1.00	1.16	0.96	1.50	2.62	0.00	2.34	1.66
time (sec)	N/A	0.223	3.061	0.194	0.275	3.246	0.000	0.483	1.614

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	268	441	0	553	0	781	2500
N.S.	1	1.00	1.17	1.92	0.00	2.40	0.00	3.40	10.87
time (sec)	N/A	0.408	1.574	0.193	0.000	3.146	0.000	0.491	3.879

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	172	264	0	393	0	407	317
N.S.	1	1.00	1.07	1.64	0.00	2.44	0.00	2.53	1.97
time (sec)	N/A	0.271	0.473	0.149	0.000	3.217	0.000	0.441	2.293

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	96	142	0	258	0	185	147
N.S.	1	1.00	0.96	1.42	0.00	2.58	0.00	1.85	1.47
time (sec)	N/A	0.147	0.224	0.108	0.000	3.905	0.000	0.434	1.478

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	118	96	0	300	0	129	109
N.S.	1	1.00	1.40	1.14	0.00	3.57	0.00	1.54	1.30
time (sec)	N/A	0.109	0.137	0.119	0.000	2.681	0.000	0.443	1.304

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	162	165	0	558	0	269	219
N.S.	1	1.00	1.16	1.18	0.00	3.99	0.00	1.92	1.56
time (sec)	N/A	0.217	0.580	0.171	0.000	2.478	0.000	0.486	1.375

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	277	282	0	861	0	541	387
N.S.	1	1.00	1.38	1.40	0.00	4.28	0.00	2.69	1.93
time (sec)	N/A	0.363	0.843	0.233	0.000	3.212	0.000	0.518	1.687

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	417	321	271	344	0	1861	588
N.S.	1	1.00	1.56	1.20	1.01	1.29	0.00	6.97	2.20
time (sec)	N/A	0.257	2.337	0.290	0.271	2.701	0.000	0.521	0.188

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	280	198	184	240	0	1102	253
N.S.	1	1.00	1.44	1.02	0.95	1.24	0.00	5.68	1.30
time (sec)	N/A	0.210	0.795	0.195	0.283	2.612	0.000	0.478	0.122

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	167	112	112	150	0	139	113
N.S.	1	1.00	1.40	0.94	0.94	1.26	0.00	1.17	0.95
time (sec)	N/A	0.157	0.326	0.132	0.264	2.698	0.000	0.456	0.090

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	76	67	55	75	0	61	60
N.S.	1	1.00	1.33	1.18	0.96	1.32	0.00	1.07	1.05
time (sec)	N/A	0.076	0.109	0.060	0.273	2.687	0.000	0.470	1.023

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	165	98	123	210	0	213	103
N.S.	1	1.00	1.51	0.90	1.13	1.93	0.00	1.95	0.94
time (sec)	N/A	0.157	0.200	0.141	0.271	3.109	0.000	0.503	0.226

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	224	151	274	630	0	456	228
N.S.	1	1.00	1.33	0.90	1.63	3.75	0.00	2.71	1.36
time (sec)	N/A	0.330	0.819	0.199	0.286	3.475	0.000	0.540	1.469

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	320	221	511	1205	0	710	447
N.S.	1	1.00	1.24	0.85	1.97	4.65	0.00	2.74	1.73
time (sec)	N/A	0.592	0.884	0.255	0.278	4.584	0.000	0.571	1.911

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	473	473	402	509	0	793	0	870	2500
N.S.	1	1.00	0.85	1.08	0.00	1.68	0.00	1.84	5.29
time (sec)	N/A	1.170	4.576	0.264	0.000	5.238	0.000	0.493	4.865

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	282	325	0	581	0	482	2500
N.S.	1	1.00	1.08	1.25	0.00	2.23	0.00	1.85	9.58
time (sec)	N/A	0.572	2.134	0.198	0.000	5.009	0.000	0.467	3.861

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	178	199	0	551	0	240	1655
N.S.	1	1.00	1.17	1.31	0.00	3.62	0.00	1.58	10.89
time (sec)	N/A	0.391	0.549	0.151	0.000	4.615	0.000	0.467	3.569

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	128	162	0	526	0	289	245
N.S.	1	1.00	0.63	0.80	0.00	2.59	0.00	1.42	1.21
time (sec)	N/A	0.337	0.590	0.178	0.000	3.528	0.000	0.490	1.788

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	281	242	0	1040	0	457	403
N.S.	1	1.00	0.82	0.71	0.00	3.03	0.00	1.33	1.17
time (sec)	N/A	0.392	0.752	0.247	0.000	4.157	0.000	0.525	1.618

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	550	367	326	447	0	2150	762
N.S.	1	1.00	1.67	1.12	0.99	1.36	0.00	6.53	2.32
time (sec)	N/A	0.358	3.194	0.148	0.268	4.555	0.000	0.623	0.236

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	388	239	234	331	0	1337	315
N.S.	1	1.00	1.62	1.00	0.98	1.38	0.00	5.59	1.32
time (sec)	N/A	0.254	1.866	0.332	0.289	3.985	0.000	0.563	1.111

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	208	144	154	226	0	170	167
N.S.	1	1.00	1.32	0.91	0.97	1.43	0.00	1.08	1.06
time (sec)	N/A	0.196	0.519	0.224	0.264	4.481	0.000	0.553	0.105

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	111	85	87	126	0	77	93
N.S.	1	1.00	1.34	1.02	1.05	1.52	0.00	0.93	1.12
time (sec)	N/A	0.092	0.270	0.121	0.262	2.969	0.000	0.523	1.073

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	203	148	241	474	0	452	182
N.S.	1	1.00	1.25	0.91	1.48	2.91	0.00	2.77	1.12
time (sec)	N/A	0.240	0.380	0.178	0.279	3.535	0.000	0.546	1.394

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	332	196	435	1071	0	800	378
N.S.	1	1.00	1.45	0.86	1.90	4.68	0.00	3.49	1.65
time (sec)	N/A	0.434	6.213	0.310	0.289	4.098	0.000	0.606	1.702

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	496	255	707	1803	0	1551	673
N.S.	1	1.00	1.58	0.81	2.26	5.76	0.00	4.96	2.15
time (sec)	N/A	0.881	2.988	0.391	0.289	4.474	0.000	0.689	2.554

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	539	539	599	582	0	1057	0	1030	2500
N.S.	1	1.00	1.11	1.08	0.00	1.96	0.00	1.91	4.64
time (sec)	N/A	1.612	7.857	0.480	0.000	4.385	0.000	0.642	5.710

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	333	1178	392	0	1041	0	584	2500
N.S.	1	1.00	3.54	1.18	0.00	3.13	0.00	1.75	7.51
time (sec)	N/A	0.762	5.751	0.317	0.000	4.025	0.000	0.575	5.761

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	282	276	0	984	0	1193	2500
N.S.	1	1.00	1.06	1.03	0.00	3.69	0.00	4.47	9.36
time (sec)	N/A	0.652	2.678	0.246	0.000	6.006	0.000	0.642	9.095

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	376	231	234	0	841	0	386	423
N.S.	1	1.00	0.61	0.62	0.00	2.24	0.00	1.03	1.12
time (sec)	N/A	0.490	0.711	0.251	0.000	4.492	0.000	0.562	2.858

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	515	515	388	328	0	1550	0	709	588
N.S.	1	1.00	0.75	0.64	0.00	3.01	0.00	1.38	1.14
time (sec)	N/A	0.569	0.755	0.351	0.000	3.774	0.000	0.584	1.781

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	516	516	2049	1179	0	0	0	0	-1
N.S.	1	1.00	3.97	2.28	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.180	45.460	0.473	0.000	0.000	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	430	430	853	852	0	0	0	0	-1
N.S.	1	1.00	1.98	1.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.789	33.875	0.377	0.000	0.000	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	444	444	1959	860	0	0	0	0	-1
N.S.	1	1.00	4.41	1.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.737	43.398	0.375	0.000	0.000	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	351	460	0	0	0	0	-1
N.S.	1	1.00	0.99	1.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.576	50.447	0.240	0.000	0.000	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	370	370	546	530	0	0	0	0	-1
N.S.	1	1.00	1.48	1.43	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.550	14.003	0.254	0.000	0.000	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	430	430	711	689	0	0	0	0	-1
N.S.	1	1.00	1.65	1.60	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.742	21.985	0.265	0.000	0.000	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	452	452	1233	657	0	0	0	0	-1
N.S.	1	1.00	2.73	1.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.757	30.436	0.389	0.000	0.000	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	511	511	797	924	0	0	0	0	-1
N.S.	1	1.00	1.56	1.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.992	15.334	0.353	0.000	0.000	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1070	1070	974	2081	0	0	0	0	-1
N.S.	1	1.00	0.91	1.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.098	24.246	0.858	0.000	0.000	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1101	1101	2095	1943	0	0	0	0	-1
N.S.	1	1.00	1.90	1.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.134	45.358	0.968	0.000	0.000	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	850	850	886	1644	0	0	0	0	-1
N.S.	1	1.00	1.04	1.93	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.557	23.920	0.701	0.000	0.000	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	882	882	2012	1569	0	0	0	0	-1
N.S.	1	1.00	2.28	1.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.563	44.523	0.828	0.000	0.000	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	809	809	717	1499	0	0	0	0	-1
N.S.	1	1.00	0.89	1.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.351	23.268	0.609	0.000	0.000	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	838	838	1246	1476	0	0	0	0	-1
N.S.	1	1.00	1.49	1.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.409	30.722	0.749	0.000	0.000	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1054	1054	772	1623	0	0	0	0	-1
N.S.	1	1.00	0.73	1.54	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.982	15.474	0.634	0.000	0.000	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1089	1089	1320	1594	0	0	0	0	-1
N.S.	1	1.00	1.21	1.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.069	30.954	0.918	0.000	0.000	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	151	215	0	0	0	0	-1
N.S.	1	1.00	1.21	1.72	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.021	0.187	1.457	0.000	0.000	0.000	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	120	264	0	0	0	0	-1
N.S.	1	1.00	0.99	2.18	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.077	0.900	0.234	0.000	0.000	0.000	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	882	1199	0	0	0	0	-1
N.S.	1	1.00	2.85	3.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.160	6.079	0.569	0.000	0.000	0.000	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	276	849	0	0	0	0	-1
N.S.	1	1.00	1.21	3.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.167	9.005	0.235	0.000	0.000	0.000	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	138	178	0	0	0	0	-1
N.S.	1	1.00	1.30	1.68	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.017	0.154	0.228	0.000	0.000	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	259	852	0	0	0	0	-1
N.S.	1	1.00	1.02	3.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.227	4.560	0.243	0.000	0.000	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	347	347	1249	1209	0	0	0	0	-1
N.S.	1	1.00	3.60	3.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.234	6.110	0.308	0.000	0.000	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	259	1065	0	0	0	0	-1
N.S.	1	1.00	0.81	3.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.364	5.069	0.198	0.000	0.000	0.000	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	182	0	0	0	0	0	-1
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.264	0.213	0.148	0.000	0.000	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	134	0	0	0	0	0	-1
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.600	0.171	0.120	0.000	0.000	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	98	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.106	0.069	0.105	0.000	0.000	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	687	0	0	0	0	0	-1
N.S.	1	1.00	2.96	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.183	4.085	0.089	0.000	0.000	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	405	405	1433	0	0	0	0	0	-1
N.S.	1	1.00	3.54	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.307	9.850	0.112	0.000	0.000	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	580	580	2700	0	0	0	0	0	-1
N.S.	1	1.00	4.66	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.397	13.754	0.119	0.000	0.000	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.045	10.489	0.112	0.000	0.000	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.039	0.304	0.109	0.000	0.000	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.042	2.180	0.104	0.000	0.000	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.048	3.356	0.096	0.000	0.000	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.029	2.215	0.112	0.000	0.000	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	562	0	0	0	0	0	-1
N.S.	1	1.00	3.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.095	5.608	0.349	0.000	0.000	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	155	0	0	0	0	0	-1
N.S.	1	1.00	1.28	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.075	1.211	0.343	0.000	0.000	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	72	0	0	0	0	0	73
N.S.	1	1.00	1.50	0.00	0.00	0.00	0.00	0.00	1.52
time (sec)	N/A	0.028	0.362	0.033	0.000	0.000	0.000	0.000	1.386

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	132	0	0	0	0	0	-1
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	0.660	0.098	0.000	0.000	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	513	0	0	0	0	0	-1
N.S.	1	1.00	2.22	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.142	13.802	0.097	0.000	0.000	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.026	7.530	0.371	0.000	0.000	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.026	2.020	0.277	0.000	0.000	0.000	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	3614	0	0	0	0	0	-1
N.S.	1	1.00	26.57	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.115	16.127	0.092	0.000	0.000	0.000	0.000	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	B	F	F	F	F(-1)	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	424	0	5928	0	0	0	0	0	-1
N.S.	1	0.00	13.98	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.026	21.380	0.097	0.000	0.000	0.000	0.000	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.026	2.669	0.089	0.000	0.000	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.025	4.464	0.090	0.000	0.000	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.026	3.257	0.091	0.000	0.000	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.026	3.851	0.089	0.000	0.000	0.000	0.000	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	135	679	0	0	0	0	-1
N.S.	1	1.00	0.71	3.57	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.111	0.994	1.616	0.000	0.000	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	146	1522	0	186	0	0	-1
N.S.	1	1.00	0.86	9.01	0.00	1.10	0.00	0.00	-0.01
time (sec)	N/A	0.116	10.861	0.274	0.000	1.062	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	111	291	0	0	0	0	-1
N.S.	1	1.00	0.92	2.40	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.573	0.183	0.000	0.000	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	130	1535	0	206	0	0	-1
N.S.	1	1.00	1.07	12.58	0.00	1.69	0.00	0.00	-0.01
time (sec)	N/A	0.104	10.572	0.208	0.000	1.424	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	135	695	0	0	0	0	-1
N.S.	1	1.00	0.74	3.82	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.125	9.204	0.204	0.000	0.000	0.000	0.000	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	165	1529	0	252	0	0	-1
N.S.	1	1.00	0.84	7.76	0.00	1.28	0.00	0.00	-0.01
time (sec)	N/A	0.127	10.976	0.192	0.000	1.313	0.000	0.000	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	244	745	0	0	0	0	-1
N.S.	1	1.00	0.90	2.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.228	27.533	0.221	0.000	0.000	0.000	0.000	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	C	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	307	1559	0	247	0	0	-1
N.S.	1	1.00	1.28	6.50	0.00	1.03	0.00	0.00	-0.00
time (sec)	N/A	0.232	28.002	0.204	0.000	1.271	0.000	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	216	744	0	0	0	0	-1
N.S.	1	1.00	1.40	4.83	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.178	23.097	0.236	0.000	0.000	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	288	1605	0	272	0	0	-1
N.S.	1	1.00	1.88	10.49	0.00	1.78	0.00	0.00	-0.01
time (sec)	N/A	0.190	27.703	0.226	0.000	1.036	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	302	763	0	0	0	0	-1
N.S.	1	1.00	1.36	3.44	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.220	30.009	0.193	0.000	0.000	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	322	1636	0	313	0	0	-1
N.S.	1	1.00	1.36	6.93	0.00	1.33	0.00	0.00	-0.00
time (sec)	N/A	0.220	31.628	0.205	0.000	1.214	0.000	0.000	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	131	465	0	148	0	0	-1
N.S.	1	1.00	0.85	3.00	0.00	0.95	0.00	0.00	-0.01
time (sec)	N/A	0.163	0.683	0.187	0.000	1.166	0.000	0.000	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	230	799	0	126	0	0	-1
N.S.	1	1.00	1.59	5.51	0.00	0.87	0.00	0.00	-0.01
time (sec)	N/A	0.161	0.898	0.204	0.000	0.950	0.000	0.000	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	60	326	0	103	0	0	-1
N.S.	1	1.00	0.57	3.10	0.00	0.98	0.00	0.00	-0.01
time (sec)	N/A	0.142	0.255	0.180	0.000	0.798	0.000	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	95	536	0	78	0	0	-1
N.S.	1	1.00	0.96	5.41	0.00	0.79	0.00	0.00	-0.01
time (sec)	N/A	0.156	0.421	0.167	0.000	0.882	0.000	0.000	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	70	194	0	74	0	0	-1
N.S.	1	1.00	0.66	1.83	0.00	0.70	0.00	0.00	-0.01
time (sec)	N/A	0.161	0.262	0.166	0.000	0.729	0.000	0.000	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	100	551	0	103	0	0	-1
N.S.	1	1.00	0.83	4.59	0.00	0.86	0.00	0.00	-0.01
time (sec)	N/A	0.157	0.549	0.188	0.000	0.606	0.000	0.000	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	91	221	0	96	0	0	-1
N.S.	1	1.00	0.61	1.48	0.00	0.64	0.00	0.00	-0.01
time (sec)	N/A	0.181	0.376	0.168	0.000	0.766	0.000	0.000	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	115	621	0	202	0	0	-1
N.S.	1	1.00	0.43	2.32	0.00	0.75	0.00	0.00	-0.00
time (sec)	N/A	0.359	0.733	0.210	0.000	0.519	0.000	0.000	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	247	1044	0	184	0	0	-1
N.S.	1	1.00	0.99	4.18	0.00	0.74	0.00	0.00	-0.00
time (sec)	N/A	0.342	1.154	0.201	0.000	0.525	0.000	0.000	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	82	474	0	160	0	0	-1
N.S.	1	1.00	0.41	2.36	0.00	0.80	0.00	0.00	-0.00
time (sec)	N/A	0.314	0.454	0.204	0.000	0.470	0.000	0.000	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	252	811	0	123	0	0	-1
N.S.	1	1.00	1.27	4.08	0.00	0.62	0.00	0.00	-0.01
time (sec)	N/A	0.323	1.048	0.197	0.000	0.957	0.000	0.000	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	101	327	0	119	0	0	-1
N.S.	1	1.00	0.47	1.54	0.00	0.56	0.00	0.00	-0.00
time (sec)	N/A	0.323	0.382	0.183	0.000	0.792	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	125	551	0	103	0	0	-1
N.S.	1	1.00	0.58	2.56	0.00	0.48	0.00	0.00	-0.00
time (sec)	N/A	0.325	1.392	0.171	0.000	0.956	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	94	221	0	96	0	0	-1
N.S.	1	1.00	0.55	1.28	0.00	0.56	0.00	0.00	-0.01
time (sec)	N/A	0.313	1.426	0.171	0.000	0.729	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [114] had the largest ratio of [25]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	4	1.00	19	0.210
2	A	5	4	1.00	19	0.210
3	A	5	4	1.00	19	0.210
4	A	5	4	1.00	19	0.210
5	A	4	3	1.00	17	0.176
6	A	6	6	1.00	17	0.353
7	A	5	4	1.00	19	0.210
8	A	5	4	1.00	19	0.210
9	A	5	4	1.00	19	0.210
10	A	11	7	1.00	19	0.368
11	A	10	7	1.00	19	0.368
12	A	9	7	1.00	19	0.368
13	A	7	7	1.00	19	0.368
14	A	7	7	1.00	19	0.368
15	A	8	6	1.00	19	0.316
16	A	8	6	1.00	19	0.316
17	A	8	6	1.00	19	0.316
18	A	8	6	1.00	19	0.316
19	A	5	4	1.00	21	0.190
20	A	5	4	1.00	21	0.190
21	A	5	4	1.00	21	0.190
22	A	5	4	1.00	21	0.190
23	A	5	4	1.00	19	0.210
24	A	5	4	1.00	19	0.210
25	A	5	4	1.00	21	0.190

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	5	4	1.00	21	0.190
27	A	5	4	1.00	21	0.190
28	A	5	4	1.00	21	0.190
29	A	27	8	1.00	21	0.381
30	A	18	8	1.00	21	0.381
31	A	14	8	1.00	21	0.381
32	A	9	7	1.00	21	0.333
33	A	11	9	1.00	21	0.429
34	A	8	8	1.00	21	0.381
35	A	12	8	1.00	21	0.381
36	A	12	8	1.00	21	0.381
37	A	12	8	1.00	21	0.381
38	A	5	4	1.00	21	0.190
39	A	5	4	1.00	21	0.190
40	A	5	4	1.00	21	0.190
41	A	4	3	1.00	21	0.143
42	A	5	4	1.00	19	0.210
43	A	5	4	1.00	19	0.210
44	A	5	4	1.00	21	0.190
45	A	5	4	1.00	21	0.190
46	A	5	4	1.00	21	0.190
47	A	5	4	1.00	21	0.190
48	A	29	9	1.00	21	0.429
49	A	18	9	1.00	21	0.429
50	A	16	9	1.00	21	0.429
51	A	11	8	1.00	21	0.381
52	A	9	7	1.00	21	0.333
53	A	11	8	1.00	21	0.381
54	A	10	9	1.00	21	0.429
55	A	17	9	1.00	21	0.429
56	A	17	9	1.00	21	0.429
57	A	7	6	1.00	21	0.286
58	A	7	6	1.00	21	0.286
59	A	7	6	1.00	21	0.286
60	A	6	5	1.00	21	0.238

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	5	4	1.00	19	0.210
62	A	6	6	1.00	19	0.316
63	A	7	7	1.00	21	0.333
64	A	8	7	1.00	21	0.333
65	A	9	7	1.00	21	0.333
66	A	8	7	1.00	21	0.333
67	A	7	7	1.00	21	0.333
68	A	5	5	1.00	21	0.238
69	A	6	5	1.00	21	0.238
70	A	7	6	1.00	21	0.286
71	A	7	6	1.00	21	0.286
72	A	7	6	1.00	21	0.286
73	A	7	6	1.00	21	0.286
74	A	5	4	1.00	21	0.190
75	A	5	4	1.00	21	0.190
76	A	5	4	1.00	21	0.190
77	A	5	4	1.00	21	0.190
78	A	5	4	1.00	21	0.190
79	A	5	4	1.00	19	0.210
80	A	6	5	1.00	19	0.263
81	A	4	4	1.00	21	0.190
82	A	6	5	1.00	21	0.238
83	A	16	8	1.00	21	0.381
84	A	7	6	1.00	21	0.286
85	A	11	6	1.00	21	0.286
86	A	9	8	1.00	21	0.381
87	A	11	6	1.00	21	0.286
88	A	13	7	1.00	21	0.333
89	A	13	7	1.00	21	0.333
90	A	13	7	1.00	21	0.333
91	A	5	4	1.00	21	0.190
92	A	5	4	1.00	21	0.190
93	A	5	4	1.00	21	0.190
94	A	5	4	1.00	21	0.190
95	A	5	4	1.00	21	0.190

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	5	4	1.00	19	0.210
97	A	6	5	1.00	19	0.263
98	A	5	4	1.00	21	0.190
99	A	6	5	1.00	21	0.238
100	A	19	9	1.00	21	0.429
101	A	15	6	1.00	21	0.286
102	A	13	8	1.00	21	0.381
103	A	10	8	1.00	21	0.381
104	A	15	8	1.00	21	0.381
105	A	16	7	1.00	21	0.333
106	A	16	7	1.00	21	0.333
107	A	16	7	1.00	21	0.333
108	A	11	11	1.00	23	0.478
109	A	11	11	1.00	23	0.478
110	A	9	9	1.00	23	0.391
111	A	9	9	1.00	23	0.391
112	A	11	11	1.00	23	0.478
113	A	11	11	1.00	23	0.478
114	A	15	12	1.00	25	0.480
115	A	15	12	1.00	25	0.480
116	A	13	10	1.00	25	0.400
117	A	13	10	1.00	25	0.400
118	A	16	13	1.00	25	0.520
119	A	16	13	1.00	25	0.520
120	A	8	8	1.00	25	0.320
121	A	7	7	1.00	25	0.280
122	A	7	7	1.00	25	0.280
123	A	7	7	1.00	25	0.280
124	A	7	7	1.00	25	0.280
125	A	8	8	1.00	25	0.320
126	A	8	8	1.00	25	0.320
127	A	14	8	1.00	25	0.320
128	A	14	9	1.00	25	0.360
129	A	14	9	1.00	25	0.360
130	A	15	9	1.00	25	0.360

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	15	9	1.00	25	0.360
132	A	17	9	1.00	25	0.360
133	A	17	9	1.00	25	0.360
134	A	9	6	1.00	23	0.261
135	A	7	6	1.00	23	0.261
136	A	5	5	1.00	21	0.238
137	A	5	5	1.00	23	0.217
138	A	9	6	1.00	23	0.261
139	A	12	7	1.00	23	0.304
140	A	5	4	1.00	25	0.160
141	A	5	4	1.00	25	0.160
142	A	5	4	1.00	25	0.160
143	A	5	4	1.00	25	0.160
144	A	5	4	1.00	23	0.174
145	A	4	4	1.00	21	0.190
146	A	4	4	1.00	21	0.190
147	A	3	3	1.00	21	0.143
148	A	2	2	1.00	19	0.105
149	A	2	2	1.00	19	0.105
150	A	4	4	1.00	21	0.190
151	A	5	5	1.00	21	0.238
152	A	11	9	1.00	21	0.429
153	A	6	5	1.00	21	0.238
154	A	4	4	1.00	21	0.190
155	A	7	6	1.00	21	0.286
156	A	5	4	1.00	23	0.174
157	A	5	4	1.00	23	0.174
158	A	5	4	1.00	23	0.174
159	A	5	4	1.00	23	0.174
160	A	5	4	1.00	19	0.210
161	A	5	4	1.00	19	0.210
162	A	5	4	1.00	19	0.210
163	A	4	3	1.00	17	0.176
164	A	5	5	1.00	17	0.294
165	A	7	6	1.00	19	0.316

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	9	7	1.00	19	0.368
167	A	10	7	1.00	19	0.368
168	A	10	7	1.00	19	0.368
169	A	9	7	1.00	19	0.368
170	A	7	7	1.00	19	0.368
171	A	7	7	1.00	19	0.368
172	A	8	6	1.00	19	0.316
173	A	8	6	1.00	19	0.316
174	A	5	4	1.00	21	0.190
175	A	5	4	1.00	21	0.190
176	A	5	4	1.00	19	0.210
177	A	5	4	1.00	19	0.210
178	A	6	5	1.00	21	0.238
179	A	12	10	1.00	21	0.476
180	A	7	7	1.00	21	0.333
181	A	10	8	1.00	21	0.381
182	A	8	6	1.00	21	0.286
183	A	9	6	1.00	21	0.286
184	A	9	6	1.00	21	0.286
185	A	5	4	1.00	21	0.190
186	A	4	3	1.00	21	0.143
187	A	5	4	1.00	19	0.210
188	A	5	4	1.00	19	0.210
189	A	6	5	1.00	21	0.238
190	A	21	11	1.00	21	0.524
191	A	8	7	1.00	21	0.333
192	A	8	8	1.00	21	0.381
193	A	15	10	1.00	21	0.476
194	A	17	9	1.00	21	0.429
195	A	17	9	1.00	21	0.429
196	A	5	4	1.00	21	0.190
197	A	5	4	1.00	21	0.190
198	A	5	4	1.00	21	0.190
199	A	5	4	1.00	19	0.210
200	A	4	3	1.00	19	0.158

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	6	5	1.00	21	0.238
202	A	7	5	1.00	21	0.238
203	A	7	5	1.00	21	0.238
204	A	6	5	1.00	21	0.238
205	A	5	5	1.00	21	0.238
206	A	5	5	1.00	21	0.238
207	A	6	5	1.00	21	0.238
208	A	7	5	1.00	21	0.238
209	A	5	4	1.00	21	0.190
210	A	5	4	1.00	21	0.190
211	A	5	4	1.00	21	0.190
212	A	5	4	1.00	19	0.210
213	A	5	4	1.00	19	0.210
214	A	6	5	1.00	21	0.238
215	A	7	5	1.00	21	0.238
216	A	10	8	1.00	21	0.381
217	A	8	7	1.00	21	0.333
218	A	8	8	1.00	21	0.381
219	A	11	7	1.00	21	0.333
220	A	15	8	1.00	21	0.381
221	A	5	4	1.00	21	0.190
222	A	5	4	1.00	21	0.190
223	A	5	4	1.00	21	0.190
224	A	5	4	1.00	19	0.210
225	A	5	4	1.00	19	0.210
226	A	5	4	1.00	21	0.190
227	A	7	5	1.00	21	0.238
228	A	11	8	1.00	21	0.381
229	A	9	7	1.00	21	0.333
230	A	9	8	1.00	21	0.381
231	A	16	8	1.00	21	0.381
232	A	20	9	1.00	21	0.429
233	A	15	12	1.00	25	0.480
234	A	14	12	1.00	25	0.480
235	A	14	12	1.00	25	0.480

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	13	11	1.00	25	0.440
237	A	13	11	1.00	25	0.440
238	A	14	12	1.00	25	0.480
239	A	14	12	1.00	25	0.480
240	A	15	12	1.00	25	0.480
241	A	35	16	1.00	25	0.640
242	A	35	16	1.00	25	0.640
243	A	32	15	1.00	25	0.600
244	A	32	15	1.00	25	0.600
245	A	27	13	1.00	25	0.520
246	A	27	13	1.00	25	0.520
247	A	33	16	1.00	25	0.640
248	A	33	16	1.00	25	0.640
249	A	1	1	1.00	14	0.071
250	A	2	2	1.00	23	0.087
251	A	5	5	1.00	14	0.357
252	A	4	4	1.00	23	0.174
253	A	1	1	1.00	14	0.071
254	A	6	6	1.00	23	0.261
255	A	6	6	1.00	14	0.429
256	A	6	6	1.00	23	0.261
257	A	9	6	1.00	23	0.261
258	A	9	8	1.00	23	0.348
259	A	5	5	1.00	21	0.238
260	A	4	4	1.00	23	0.174
261	A	6	4	1.00	23	0.174
262	A	7	4	1.00	23	0.174
263	A	0	0	0.00	0	0.000
264	A	0	0	0.00	0	0.000
265	A	0	0	0.00	0	0.000
266	A	0	0	0.00	0	0.000
267	A	0	0	0.00	0	0.000
268	A	6	3	1.00	21	0.143
269	A	3	3	1.00	21	0.143
270	A	2	2	1.00	19	0.105

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	4	3	1.00	19	0.158
272	A	9	4	1.00	21	0.190
273	A	0	0	0.00	0	0.000
274	A	0	0	0.00	0	0.000
275	A	4	4	1.00	21	0.190
276	F	0	0	N/A	0.	N/A
277	A	0	0	0.00	0	0.000
278	A	0	0	0.00	0	0.000
279	A	0	0	0.00	0	0.000
280	A	0	0	0.00	0	0.000
281	A	11	11	1.00	23	0.478
282	A	11	11	1.00	23	0.478
283	A	9	9	1.00	23	0.391
284	A	9	9	1.00	23	0.391
285	A	11	11	1.00	23	0.478
286	A	11	11	1.00	23	0.478
287	A	15	13	1.00	25	0.520
288	A	15	13	1.00	25	0.520
289	A	12	10	1.00	25	0.400
290	A	12	10	1.00	25	0.400
291	A	14	12	1.00	25	0.480
292	A	14	12	1.00	25	0.480
293	A	8	8	1.00	25	0.320
294	A	8	8	1.00	25	0.320
295	A	7	7	1.00	25	0.280
296	A	7	7	1.00	25	0.280
297	A	7	7	1.00	25	0.280
298	A	7	7	1.00	25	0.280
299	A	8	8	1.00	25	0.320
300	A	16	9	1.00	25	0.360
301	A	16	9	1.00	25	0.360
302	A	14	9	1.00	25	0.360
303	A	14	9	1.00	25	0.360
304	A	13	9	1.00	25	0.360
305	A	13	9	1.00	25	0.360

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
306	A	13	8	1.00	25	0.320

Chapter 3

Listing of integrals

Local contents

3.1 $\int (a + a\sec(c + dx))\sin^9(c + dx)\,dx$

Optimal. Leaf size=152

$$-\frac{a\cos(c+dx)}{d}+\frac{2a\cos^2(c+dx)}{d}+\frac{4a\cos^3(c+dx)}{3d}-\frac{3a\cos^4(c+dx)}{2d}-\frac{6a\cos^5(c+dx)}{5d}+\frac{2a\cos^6(c+dx)}{3d}+4a\,c$$

```
[Out] -a*cos(d*x+c)/d+2*a*cos(d*x+c)^2/d+4/3*a*cos(d*x+c)^3/d-3/2*a*cos(d*x+c)^4/
d-6/5*a*cos(d*x+c)^5/d+2/3*a*cos(d*x+c)^6/d+4/7*a*cos(d*x+c)^7/d-1/8*a*cos(
d*x+c)^8/d-1/9*a*cos(d*x+c)^9/d-a*ln(cos(d*x+c))/d
```

Rubi [A]
time = 0.08, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3957, 2915, 12, 90}

$$-\frac{a\cos^9(c+dx)}{9d}-\frac{a\cos^8(c+dx)}{8d}+\frac{4a\cos^7(c+dx)}{7d}+\frac{2a\cos^6(c+dx)}{3d}-\frac{6a\cos^5(c+dx)}{5d}-\frac{3a\cos^4(c+dx)}{2d}+\frac{4a\cos^3(c+dx)}{3d}+\frac{2a\cos^2(c+dx)}{d}-\frac{a\cos(c+dx)}{d}-\frac{a\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Sin[c + d*x]^9,x]
```

```
[Out] -((a*Cos[c + d*x])/d) + (2*a*Cos[c + d*x]^2)/d + (4*a*Cos[c + d*x]^3)/(3*d)
 - (3*a*Cos[c + d*x]^4)/(2*d) - (6*a*Cos[c + d*x]^5)/(5*d) + (2*a*Cos[c + d
*x]^6)/(3*d) + (4*a*Cos[c + d*x]^7)/(7*d) - (a*Cos[c + d*x]^8)/(8*d) - (a*C
os[c + d*x]^9)/(9*d) - (a*Log[Cos[c + d*x]])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+a\sec(c+dx))\sin^9(c+dx)\,dx = -\int (-a-a\cos(c+dx))\sin^8(c+dx)\tan(c+dx)\,dx$$

$$= \frac{\text{Subst}\left(\int \frac{a(-a-x)^4(-a+x)^5}{x}\,dx, x, -a\cos(c+dx)\right)}{a^9 d}$$

$$= \frac{\text{Subst}\left(\int \frac{(-a-x)^4(-a+x)^5}{x}\,dx, x, -a\cos(c+dx)\right)}{a^8 d}$$

$$= \frac{\text{Subst}\left(\int \left(a^8 - \frac{a^9}{x} + 4a^7x - 4a^6x^2 - 6a^5x^3 + 6a^4x^4 + 4a^3x^5 - 4a^2x\right.\right.}{a^8 d}$$

$$= -\frac{a\cos(c+dx)}{d} + \frac{2a\cos^2(c+dx)}{d} + \frac{4a\cos^3(c+dx)}{3d} - \frac{3a\cos^4(c+}{2d}$$

Mathematica [A]
time = 0.14, size = 106, normalized size = 0.70

$$-\frac{a(39690\cos(c+dx)-161280\cos^2(c+dx)+120960\cos^4(c+dx)-53760\cos^6(c+dx)+10080\cos^8(c+dx)-8820\cos(3(c+dx))+2268\cos(5(c+dx))-405\cos(7(c+dx))+35\cos(9(c+dx))+80640\log(\cos(c+dx)))}{80640d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Sin[c + d*x]^9,x]
```

```
[Out] -1/80640*(a*(39690*Cos[c + d*x] - 161280*Cos[c + d*x]^2 + 120960*Cos[c + d*
x]^4 - 53760*Cos[c + d*x]^6 + 10080*Cos[c + d*x]^8 - 8820*Cos[3*(c + d*x)]
+ 2268*Cos[5*(c + d*x)] - 405*Cos[7*(c + d*x)] + 35*Cos[9*(c + d*x)] + 8064
0*Log[Cos[c + d*x]]))/d
```

Maple [A]
time = 0.12, size = 107, normalized size = 0.70

method	result
derivativedivides	$\frac{a\left(-\frac{\left(\sin^8(dx+c)\right)}{8}-\frac{\left(\sin^6(dx+c)\right)}{6}-\frac{\left(\sin^4(dx+c)\right)}{4}-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)-\frac{a\left(\frac{128}{35}+\sin^8(dx+c)+\frac{8\left(\sin^6(dx+c)\right)}{7}+\frac{4}{}\right.}{}}{d}$
default	$\frac{a\left(-\frac{\left(\sin^8(dx+c)\right)}{8}-\frac{\left(\sin^6(dx+c)\right)}{6}-\frac{\left(\sin^4(dx+c)\right)}{4}-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)-\frac{a\left(\frac{128}{35}+\sin^8(dx+c)+\frac{8\left(\sin^6(dx+c)\right)}{7}+\frac{4}{}\right.}{}}{d}$

risch	$iax+\frac{2iac}{d}+\frac{65a\,\mathrm{e}^{2i(dx+c)}}{256d}+\frac{65a\,\mathrm{e}^{-2i(dx+c)}}{256d}-\frac{a\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{d}-\frac{63a\cos(dx+c)}{128d}-\frac{a\cos(9dx+9c)}{2304d}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*sin(d*x+c)^9,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/8*sin(d*x+c)^8-1/6*sin(d*x+c)^6-1/4*sin(d*x+c)^4-1/2*sin(d*x+c)^
2-ln(cos(d*x+c)))-1/9*a*(128/35+sin(d*x+c)^8+8/7*sin(d*x+c)^6+48/35*sin(d*x
+c)^4+64/35*sin(d*x+c)^2)*cos(d*x+c))
```

Maxima [A]
time = 0.25, size = 113, normalized size = 0.74

$$-\frac{280\,a\cos\left(dx+c\right)^{9}+315\,a\cos\left(dx+c\right)^{8}-1440\,a\cos\left(dx+c\right)^{7}-1680\,a\cos\left(dx+c\right)^{6}+3024\,a\cos\left(dx+c\right)^{5}+3780\,a\cos\left(dx+c\right)^{4}-3360\,a\cos\left(dx+c\right)^{3}-5040\,a\cos\left(dx+c\right)^{2}+2520\,a\cos\left(dx+c\right)+2520\,a\log\left(\cos\left(dx+c\right)\right)}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^9,x, algorithm="maxima")
```

```
[Out] -1/2520*(280*a*cos(d*x + c)^9 + 315*a*cos(d*x + c)^8 - 1440*a*cos(d*x + c)^
7 - 1680*a*cos(d*x + c)^6 + 3024*a*cos(d*x + c)^5 + 3780*a*cos(d*x + c)^4 -
 3360*a*cos(d*x + c)^3 - 5040*a*cos(d*x + c)^2 + 2520*a*cos(d*x + c) + 2520
*a*log(cos(d*x + c)))/d
```

Fricas [A]
time = 2.62, size = 115, normalized size = 0.76

$$-\frac{280\,a\cos\left(dx+c\right)^{9}+315\,a\cos\left(dx+c\right)^{8}-1440\,a\cos\left(dx+c\right)^{7}-1680\,a\cos\left(dx+c\right)^{6}+3024\,a\cos\left(dx+c\right)^{5}+3780\,a\cos\left(dx+c\right)^{4}-3360\,a\cos\left(dx+c\right)^{3}-5040\,a\cos\left(dx+c\right)^{2}+2520\,a\cos\left(dx+c\right)+2520\,a\log\left(-\cos\left(dx+c\right)\right)}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^9,x, algorithm="fricas")
```

```
[Out] -1/2520*(280*a*cos(d*x + c)^9 + 315*a*cos(d*x + c)^8 - 1440*a*cos(d*x + c)^
7 - 1680*a*cos(d*x + c)^6 + 3024*a*cos(d*x + c)^5 + 3780*a*cos(d*x + c)^4 -
 3360*a*cos(d*x + c)^3 - 5040*a*cos(d*x + c)^2 + 2520*a*cos(d*x + c) + 2520
*a*log(-cos(d*x + c)))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)**9,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 293 vs. 2(138) = 276.
time = 0.52, size = 293, normalized size = 1.93

$$\frac{2520\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-2520\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{9177\,a-\frac{87633\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{375732\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{953988\,a(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{1594782\,a(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}-\frac{1336734\,a(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5}+\frac{781956\,a(\cos(dx+c)-1)^6}{(\cos(dx+c)+1)^6}-\frac{302004\,a(\cos(dx+c)-1)^7}{(\cos(dx+c)+1)^7}+\frac{69201\,a(\cos(dx+c)-1)^8}{(\cos(dx+c)+1)^8}-\frac{7129\,a(\cos(dx+c)-1)^9}{(\cos(dx+c)+1)^9}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^9}}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^9,x, algorithm="giac")
```

```
[Out] 1/2520*(2520*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 2520*
a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (9177*a - 87633*a*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 375732*a*(cos(d*x + c) - 1)^2/(cos(
d*x + c) + 1)^2 - 953988*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 1594
782*a*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 1336734*a*(cos(d*x + c) -
 1)^5/(cos(d*x + c) + 1)^5 + 781956*a*(cos(d*x + c) - 1)^6/(cos(d*x + c) +
1)^6 - 302004*a*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 + 69201*a*(cos(d*
x + c) - 1)^8/(cos(d*x + c) + 1)^8 - 7129*a*(cos(d*x + c) - 1)^9/(cos(d*x +
 c) + 1)^9)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^9)/d
```

Mupad [B]
time = 0.13, size = 111, normalized size = 0.73

$$-\frac{a\,\cos\left(c+d\,x\right)-2\,a\,{\cos\left(c+d\,x\right)}^2-\frac{4\,a\,{\cos\left(c+d\,x\right)}^3}{3}+\frac{3\,a\,{\cos\left(c+d\,x\right)}^4}{2}+\frac{6\,a\,{\cos\left(c+d\,x\right)}^5}{5}-\frac{2\,a\,{\cos\left(c+d\,x\right)}^6}{3}-\frac{4\,a\,{\cos\left(c+d\,x\right)}^7}{7}+\frac{a\,{\cos\left(c+d\,x\right)}^8}{8}+\frac{a\,{\cos\left(c+d\,x\right)}^9}{9}+a\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^9*(a + a/cos(c + d*x)),x)
```

```
[Out] -(a*cos(c + d*x) - 2*a*cos(c + d*x)^2 - (4*a*cos(c + d*x)^3)/3 + (3*a*cos(c
 + d*x)^4)/2 + (6*a*cos(c + d*x)^5)/5 - (2*a*cos(c + d*x)^6)/3 - (4*a*cos(c
 + d*x)^7)/7 + (a*cos(c + d*x)^8)/8 + (a*cos(c + d*x)^9)/9 + a*log(cos(c +
d*x)))/d
```

3.2 $\int (a + a\sec(c + dx))\sin^7(c + dx)\,dx$

Optimal. Leaf size=119

$$-\frac{a\cos(c+dx)}{d}+\frac{3a\cos^2(c+dx)}{2d}+\frac{a\cos^3(c+dx)}{d}-\frac{3a\cos^4(c+dx)}{4d}-\frac{3a\cos^5(c+dx)}{5d}+\frac{a\cos^6(c+dx)}{6d}+\frac{a\cos^7}{}$$

```
[Out] -a*cos(d*x+c)/d+3/2*a*cos(d*x+c)^2/d+a*cos(d*x+c)^3/d-3/4*a*cos(d*x+c)^4/d-
3/5*a*cos(d*x+c)^5/d+1/6*a*cos(d*x+c)^6/d+1/7*a*cos(d*x+c)^7/d-a*ln(cos(d*x
+c))/d
```

Rubi [A]
time = 0.07, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3957, 2915, 12, 90}

$$\frac{a\cos^7(c+dx)}{7d}+\frac{a\cos^6(c+dx)}{6d}-\frac{3a\cos^5(c+dx)}{5d}-\frac{3a\cos^4(c+dx)}{4d}+\frac{a\cos^3(c+dx)}{d}+\frac{3a\cos^2(c+dx)}{2d}-\frac{a\cos(c+dx)}{d}-\frac{a\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Sin[c + d*x]^7,x]
```

```
[Out] -((a*Cos[c + d*x])/d) + (3*a*Cos[c + d*x]^2)/(2*d) + (a*Cos[c + d*x]^3)/d -
 (3*a*Cos[c + d*x]^4)/(4*d) - (3*a*Cos[c + d*x]^5)/(5*d) + (a*Cos[c + d*x]^
6)/(6*d) + (a*Cos[c + d*x]^7)/(7*d) - (a*Log[Cos[c + d*x]])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+a\sec(c+dx))\sin^7(c+dx)\,dx = -\int (-a-a\cos(c+dx))\sin^6(c+dx)\tan(c+dx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{a(-a-x)^3(-a+x)^4}{x}\,dx, x, -a\cos(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(-a-x)^3(-a+x)^4}{x}\,dx, x, -a\cos(c+dx)\right)}{a^6 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^6-\frac{a^7}{x}+3a^5x-3a^4x^2-3a^3x^3+3a^2x^4+ax^5-x^6\right) dx$$
$$a^6 d$$
$$= -\frac{a\cos(c+dx)}{d}+\frac{3a\cos^2(c+dx)}{2d}+\frac{a\cos^3(c+dx)}{d}-\frac{3a\cos^4(c+d}{4d}$$

Mathematica [A]
time = 0.09, size = 86, normalized size = 0.72

$$\frac{a(-3675\cos(c+dx)+10080\cos^2(c+dx)-5040\cos^4(c+dx)+1120\cos^6(c+dx)+735\cos(3(c+dx))-147\cos(5(c+dx))+15\cos(7(c+dx))-6720\log(\cos(c+dx)))}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Sin[c + d*x]^7,x]
```

```
[Out] (a*(-3675*Cos[c + d*x] + 10080*Cos[c + d*x]^2 - 5040*Cos[c + d*x]^4 + 1120*
Cos[c + d*x]^6 + 735*Cos[3*(c + d*x)] - 147*Cos[5*(c + d*x)] + 15*Cos[7*(c
+ d*x)] - 6720*Log[Cos[c + d*x]]))/(6720*d)
```

Maple [A]
time = 0.15, size = 87, normalized size = 0.73

method	result
derivativedivides	$\frac{a\left(-\frac{\left(\sin^6(dx+c)\right)}{6}-\frac{\left(\sin^4(dx+c)\right)}{4}-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)-\frac{a\left(\frac{16}{5}+\sin^6(dx+c)+\frac{6\left(\sin^4(dx+c)\right)}{5}+\frac{8\left(\sin^2(dx+c)\right)}{5}\right)\text{co}}{7}}{d}$
default	$\frac{a\left(-\frac{\left(\sin^6(dx+c)\right)}{6}-\frac{\left(\sin^4(dx+c)\right)}{4}-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)-\frac{a\left(\frac{16}{5}+\sin^6(dx+c)+\frac{6\left(\sin^4(dx+c)\right)}{5}+\frac{8\left(\sin^2(dx+c)\right)}{5}\right)\text{co}}{7}}{d}$

risch	$iax+\frac{2iac}{d}+\frac{29a\,\mathrm{e}^{2i(dx+c)}}{128d}+\frac{29a\,\mathrm{e}^{-2i(dx+c)}}{128d}-\frac{a\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{d}-\frac{35a\cos(dx+c)}{64d}+\frac{a\cos(7dx+7c)}{448d}+\frac{a\cos}{}$
norman	$\frac{-\frac{32a}{35d}-\frac{128a\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{166a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}-\frac{224a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{2a\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{42a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}-\frac{14a\left(\tan\right.}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*sin(d*x+c)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/6*sin(d*x+c)^6-1/4*sin(d*x+c)^4-1/2*sin(d*x+c)^2-ln(cos(d*x+c)))
-1/7*a*(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c))
```

Maxima [A]
time = 0.27, size = 91, normalized size = 0.76

$$\frac{60\,a\cos\left(dx+c\right)^7+70\,a\cos\left(dx+c\right)^6-252\,a\cos\left(dx+c\right)^5-315\,a\cos\left(dx+c\right)^4+420\,a\cos\left(dx+c\right)^3+630\,a\cos\left(dx+c\right)^2-420\,a\cos\left(dx+c\right)-420\,a\log\left(\cos\left(dx+c\right)\right)}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/420*(60*a*cos(d*x + c)^7 + 70*a*cos(d*x + c)^6 - 252*a*cos(d*x + c)^5 - 3
15*a*cos(d*x + c)^4 + 420*a*cos(d*x + c)^3 + 630*a*cos(d*x + c)^2 - 420*a*c
os(d*x + c) - 420*a*log(cos(d*x + c)))/d
```

Fricas [A]
time = 2.83, size = 93, normalized size = 0.78

$$\frac{60\,a\cos\left(dx+c\right)^7+70\,a\cos\left(dx+c\right)^6-252\,a\cos\left(dx+c\right)^5-315\,a\cos\left(dx+c\right)^4+420\,a\cos\left(dx+c\right)^3+630\,a\cos\left(dx+c\right)^2-420\,a\cos\left(dx+c\right)-420\,a\log\left(-\cos\left(dx+c\right)\right)}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/420*(60*a*cos(d*x + c)^7 + 70*a*cos(d*x + c)^6 - 252*a*cos(d*x + c)^5 - 3
15*a*cos(d*x + c)^4 + 420*a*cos(d*x + c)^3 + 630*a*cos(d*x + c)^2 - 420*a*c
os(d*x + c) - 420*a*log(-cos(d*x + c)))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)**7,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(109) = 218$.

time = 0.48, size = 247, normalized size = 2.08

$$\frac{420\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-420\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{1473\,a-\frac{11151\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{36813\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{69475\,a(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{56035\,a(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}-\frac{28749\,a(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5}+\frac{8463\,a(\cos(dx+c)-1)^6}{(\cos(dx+c)+1)^6}-\frac{1089\,a(\cos(dx+c)-1)^7}{(\cos(dx+c)+1)^7}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^7}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/420*(420*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 420*a*l
og(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (1473*a - 11151*a*(co
s(d*x + c) - 1)/(cos(d*x + c) + 1) + 36813*a*(cos(d*x + c) - 1)^2/(cos(d*x
+ c) + 1)^2 - 69475*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 56035*a*(
cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 28749*a*(cos(d*x + c) - 1)^5/(co
s(d*x + c) + 1)^5 + 8463*a*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 1089
*a*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7)/((cos(d*x + c) - 1)/(cos(d*x
+ c) + 1) - 1)^7)/d
```

Mupad [B]

time = 0.08, size = 89, normalized size = 0.75

$$-\frac{a\,\cos\left(c+d\,x\right)-\frac{3\,a\,{\cos\left(c+d\,x\right)}^2}{2}-a\,{\cos\left(c+d\,x\right)}^3+\frac{3\,a\,{\cos\left(c+d\,x\right)}^4}{4}+\frac{3\,a\,{\cos\left(c+d\,x\right)}^5}{5}-\frac{a\,{\cos\left(c+d\,x\right)}^6}{6}-\frac{a\,{\cos\left(c+d\,x\right)}^7}{7}+a\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^7*(a + a/cos(c + d*x)),x)
```

```
[Out] -(a*cos(c + d*x) - (3*a*cos(c + d*x)^2)/2 - a*cos(c + d*x)^3 + (3*a*cos(c +
 d*x)^4)/4 + (3*a*cos(c + d*x)^5)/5 - (a*cos(c + d*x)^6)/6 - (a*cos(c + d*x
)^7)/7 + a*log(cos(c + d*x)))/d
```

3.3 $\int (a+a\sec(c+dx))\sin^5(c+dx)\,dx$

Optimal. Leaf size=87

$$-\frac{a\cos(c+dx)}{d}+\frac{a\cos^2(c+dx)}{d}+\frac{2a\cos^3(c+dx)}{3d}-\frac{a\cos^4(c+dx)}{4d}-\frac{a\cos^5(c+dx)}{5d}-\frac{a\log(\cos(c+dx))}{d}$$

```
[Out] -a*cos(d*x+c)/d+a*cos(d*x+c)^2/d+2/3*a*cos(d*x+c)^3/d-1/4*a*cos(d*x+c)^4/d-
1/5*a*cos(d*x+c)^5/d-a*ln(cos(d*x+c))/d
```

Rubi [A]
time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3957, 2915, 12, 90}

$$-\frac{a\cos^5(c+dx)}{5d}-\frac{a\cos^4(c+dx)}{4d}+\frac{2a\cos^3(c+dx)}{3d}+\frac{a\cos^2(c+dx)}{d}-\frac{a\cos(c+dx)}{d}-\frac{a\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Sin[c + d*x]^5,x]
```

```
[Out] -((a*Cos[c + d*x])/d) + (a*Cos[c + d*x]^2)/d + (2*a*Cos[c + d*x]^3)/(3*d) -
 (a*Cos[c + d*x]^4)/(4*d) - (a*Cos[c + d*x]^5)/(5*d) - (a*Log[Cos[c + d*x]]
)/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int (a+a\sec(c+dx))\sin^5(c+dx)\,dx &= -\int (-a-a\cos(c+dx))\sin^4(c+dx)\tan(c+dx)\,dx \\
&= \frac{\text{Subst}\left(\int \frac{a(-a-x)^2(-a+x)^3}{x}\,dx, x, -a\cos(c+dx)\right)}{a^5 d} \\
&= \frac{\text{Subst}\left(\int \frac{(-a-x)^2(-a+x)^3}{x}\,dx, x, -a\cos(c+dx)\right)}{a^4 d} \\
&= \frac{\text{Subst}\left(\int \left(a^4-\frac{a^5}{x}+2a^3x-2a^2x^2-ax^3+x^4\right)dx, x, -a\cos(c+d\right.}{a^4 d} \\
&= -\frac{a\cos(c+dx)}{d}+\frac{a\cos^2(c+dx)}{d}+\frac{2a\cos^3(c+dx)}{3d}-\frac{a\cos^4(c+dx}{4d}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 83, normalized size = 0.95

$$-\frac{5a\cos(c+dx)}{8d}+\frac{5a\cos(3(c+dx))}{48d}-\frac{a\cos(5(c+dx))}{80d}-\frac{a\left(-\cos^2(c+dx)+\frac{1}{4}\cos^4(c+dx)+\log(\cos(c+dx))\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Sin[c + d*x]^5,x]
```

```
[Out] (-5*a*Cos[c + d*x])/(8*d) + (5*a*Cos[3*(c + d*x)])/(48*d) - (a*Cos[5*(c + d
*x)])/(80*d) - (a*(-Cos[c + d*x]^2 + Cos[c + d*x]^4/4 + Log[Cos[c + d*x]]))
/d
```

Maple [A]
time = 0.14, size = 67, normalized size = 0.77

method	result
derivativedivides	$\frac{a\left(-\frac{\left(\sin^4(dx+c)\right)}{4}-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)-\frac{a\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)}{5}}{d}$
default	$\frac{a\left(-\frac{\left(\sin^4(dx+c)\right)}{4}-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)-\frac{a\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)}{5}}{d}$

risch	$iax+\frac{3a\,e^{2i(dx+c)}}{16d}+\frac{3a\,e^{-2i(dx+c)}}{16d}+\frac{2iac}{d}-\frac{a\ln\left(e^{2i(dx+c)}+1\right)}{d}-\frac{5a\cos(dx+c)}{8d}-\frac{a\cos(5dx+5c)}{80d}-\frac{a\cos(4d}{32d}$
norman	$\frac{-\frac{16a}{15d}-\frac{22a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{62a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{2a\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{10a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}+\frac{a\ln\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*sin(d*x+c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/4*sin(d*x+c)^4-1/2*sin(d*x+c)^2-ln(cos(d*x+c)))-1/5*a*(8/3+sin(d
*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))
```

Maxima [A]
time = 0.26, size = 69, normalized size = 0.79

$$-\frac{12\,a\cos\left(dx+c\right)^5+15\,a\cos\left(dx+c\right)^4-40\,a\cos\left(dx+c\right)^3-60\,a\cos\left(dx+c\right)^2+60\,a\cos\left(dx+c\right)+60\,a\log\left(\cos\left(dx+c\right)\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/60*(12*a*cos(d*x + c)^5 + 15*a*cos(d*x + c)^4 - 40*a*cos(d*x + c)^3 - 60
*a*cos(d*x + c)^2 + 60*a*cos(d*x + c) + 60*a*log(cos(d*x + c)))/d
```

Fricas [A]
time = 2.83, size = 71, normalized size = 0.82

$$-\frac{12\,a\cos\left(dx+c\right)^5+15\,a\cos\left(dx+c\right)^4-40\,a\cos\left(dx+c\right)^3-60\,a\cos\left(dx+c\right)^2+60\,a\cos\left(dx+c\right)+60\,a\log\left(-\cos\left(dx+c\right)\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/60*(12*a*cos(d*x + c)^5 + 15*a*cos(d*x + c)^4 - 40*a*cos(d*x + c)^3 - 60
*a*cos(d*x + c)^2 + 60*a*cos(d*x + c) + 60*a*log(-cos(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sin^5\left(c+dx\right)\sec\left(c+dx\right)\,dx+\int\sin^5\left(c+dx\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)**5,x)
```

```
[Out] a*(Integral(sin(c + d*x)**5*sec(c + d*x), x) + Integral(sin(c + d*x)**5, x)
)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(81) = 162.
time = 0.43, size = 201, normalized size = 2.31

$$\frac{60\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-60\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{201\,a-\frac{1125\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{2610\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{1970\,a(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{805\,a(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}-\frac{137\,a(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/60*(60*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 60*a*log(
abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (201*a - 1125*a*(cos(d*x
 + c) - 1)/(cos(d*x + c) + 1) + 2610*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) +
 1)^2 - 1970*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 805*a*(cos(d*x +
 c) - 1)^4/(cos(d*x + c) + 1)^4 - 137*a*(cos(d*x + c) - 1)^5/(cos(d*x + c)
+ 1)^5)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^5)/d
```

Mupad [B]
time = 0.06, size = 67, normalized size = 0.77

$$-\frac{a\,\cos\left(c+d\,x\right)-a\cos\left(c+d\,x\right)^2-\frac{2\,a\cos\left(c+d\,x\right)^3}{3}+\frac{a\cos\left(c+d\,x\right)^4}{4}+\frac{a\cos\left(c+d\,x\right)^5}{5}+a\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5*(a + a/cos(c + d*x)),x)
```

```
[Out] -(a*cos(c + d*x) - a*cos(c + d*x)^2 - (2*a*cos(c + d*x)^3)/3 + (a*cos(c + d
*x)^4)/4 + (a*cos(c + d*x)^5)/5 + a*log(cos(c + d*x)))/d
```

3.4 $\int (a+a\sec(c+dx))\sin^3(c+dx)\,dx$

Optimal. Leaf size=58

$$-\frac{a\cos(c+dx)}{d}+\frac{a\cos^2(c+dx)}{2d}+\frac{a\cos^3(c+dx)}{3d}-\frac{a\log(\cos(c+dx))}{d}$$

```
[Out] -a*cos(d*x+c)/d+1/2*a*cos(d*x+c)^2/d+1/3*a*cos(d*x+c)^3/d-a*ln(cos(d*x+c))/
d
```

Rubi [A]
time = 0.06, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3957, 2915, 12, 76}

$$\frac{a\cos^3(c+dx)}{3d}+\frac{a\cos^2(c+dx)}{2d}-\frac{a\cos(c+dx)}{d}-\frac{a\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Sin[c + d*x]^3,x]
```

```
[Out] -((a*Cos[c + d*x])/d) + (a*Cos[c + d*x]^2)/(2*d) + (a*Cos[c + d*x]^3)/(3*d)
 - (a*Log[Cos[c + d*x]])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
```

```
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+a\sec(c+dx))\sin^3(c+dx)\,dx = -\int (-a-a\cos(c+dx))\sin^2(c+dx)\tan(c+dx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{a(-a-x)(-a+x)^2}{x}\,dx, x, -a\cos(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \frac{(-a-x)(-a+x)^2}{x}\,dx, x, -a\cos(c+dx)\right)}{a^2d}$$
$$= \frac{\text{Subst}\left(\int \left(a^2-\frac{a^3}{x}+ax-x^2\right)\,dx, x, -a\cos(c+dx)\right)}{a^2d}$$
$$= -\frac{a\cos(c+dx)}{d}+\frac{a\cos^2(c+dx)}{2d}+\frac{a\cos^3(c+dx)}{3d}-\frac{a\log(\cos(c+$$

Mathematica [A]
time = 0.04, size = 57, normalized size = 0.98

$$-\frac{3a\cos(c+dx)}{4d}+\frac{a\cos(3(c+dx))}{12d}-\frac{a\left(-\frac{1}{2}\cos^2(c+dx)+\log(\cos(c+dx))\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Sin[c + d*x]^3,x]
[Out] (-3*a*Cos[c + d*x])/(4*d) + (a*Cos[3*(c + d*x)])/(12*d) - (a*(-1/2*Cos[c +
d*x]^2 + Log[Cos[c + d*x]]))/d
```

Maple [A]
time = 0.11, size = 47, normalized size = 0.81

method	result
derivativedivides	$\frac{a\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)-\frac{a\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}}{d}$
default	$\frac{a\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)-\frac{a\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}}{d}$
risch	$iax+\frac{a\,e^{2i(dx+c)}}{8d}+\frac{a\,e^{-2i(dx+c)}}{8d}+\frac{2iac}{d}-\frac{a\ln\left(e^{2i(dx+c)}+1\right)}{d}-\frac{3a\cos(dx+c)}{4d}+\frac{a\cos(3dx+3c)}{12d}$

norman	$\frac{-\frac{4a}{3d}-\frac{2a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{6a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{a\ln\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*sin(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/2*sin(d*x+c)^2-ln(cos(d*x+c)))-1/3*a*(2+sin(d*x+c)^2)*cos(d*x+c)
)
```

Maxima [A]
time = 0.27, size = 47, normalized size = 0.81

$$\frac{2\,a\cos\left(dx+c\right)^3+3\,a\cos\left(dx+c\right)^2-6\,a\cos\left(dx+c\right)-6\,a\log\left(\cos\left(dx+c\right)\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(2*a*cos(d*x + c)^3 + 3*a*cos(d*x + c)^2 - 6*a*cos(d*x + c) - 6*a*log(c
os(d*x + c)))/d
```

Fricas [A]
time = 2.11, size = 49, normalized size = 0.84

$$\frac{2\,a\cos\left(dx+c\right)^3+3\,a\cos\left(dx+c\right)^2-6\,a\cos\left(dx+c\right)-6\,a\log\left(-\cos\left(dx+c\right)\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*cos(d*x + c)^3 + 3*a*cos(d*x + c)^2 - 6*a*cos(d*x + c) - 6*a*log(-
cos(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sin^3\left(c+dx\right)\sec\left(c+dx\right)\,dx+\int\sin^3\left(c+dx\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)**3,x)
```

```
[Out] a*(Integral(sin(c + d*x)**3*sec(c + d*x), x) + Integral(sin(c + d*x)**3, x)
)
```

Giac [A]
time = 0.44, size = 66, normalized size = 1.14

$$-\frac{a\log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d}+\frac{2\,ad^2\cos\left(dx+c\right)^3+3\,ad^2\cos\left(dx+c\right)^2-6\,ad^2\cos\left(dx+c\right)}{6\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -a*log(abs(cos(d*x + c))/abs(d))/d + 1/6*(2*a*d^2*cos(d*x + c)^3 + 3*a*d^2*
cos(d*x + c)^2 - 6*a*d^2*cos(d*x + c))/d^3
```

Mupad [B]
time = 0.06, size = 45, normalized size = 0.78

$$-\frac{a\,\cos\left(c+d\,x\right)-\frac{a\cos(c+d\,x)^2}{2}-\frac{a\cos(c+d\,x)^3}{3}+a\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3*(a + a/cos(c + d*x)),x)
```

```
[Out] -(a*cos(c + d*x) - (a*cos(c + d*x)^2)/2 - (a*cos(c + d*x)^3)/3 + a*log(cos(
c + d*x)))/d
```

3.5 $\int (a+a\sec(c+dx))\sin(c+dx)\,dx$

Optimal. Leaf size=26

$$-\frac{a\cos(c+dx)}{d}-\frac{a\log(\cos(c+dx))}{d}$$

```
[Out] -a*cos(d*x+c)/d-a*ln(cos(d*x+c))/d
```

Rubi [A]
time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3957, 2786, 45}

$$-\frac{a\cos(c+dx)}{d}-\frac{a\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Sin[c + d*x],x]
```

```
[Out] -((a*Cos[c + d*x])/d) - (a*Log[Cos[c + d*x]])/d
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))\sin(c+dx)\,dx &= -\int (-a-a\cos(c+dx))\tan(c+dx)\,dx\\ &= \frac{\text{Subst}\left(\int \frac{-a+x}{x}\,dx, x, -a\cos(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(1-\frac{a}{x}\right)\,dx, x, -a\cos(c+dx)\right)}{d}\\ &= -\frac{a\cos(c+dx)}{d}-\frac{a\log(\cos(c+dx))}{d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 37, normalized size = 1.42

$$-\frac{a\cos(c)\cos(dx)}{d}-\frac{a\log(\cos(c+dx))}{d}+\frac{a\sin(c)\sin(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Sin[c + d*x],x]
[Out] -((a*Cos[c]*Cos[d*x])/d) - (a*Log[Cos[c + d*x]])/d + (a*Sin[c]*Sin[d*x])/d
```

Maple [A]
time = 0.04, size = 24, normalized size = 0.92

method	result	size
derivativedivides	$\frac{a\left(-\frac{1}{\sec(dx+c)}+\ln(\sec(dx+c))\right)}{d}$	24
default	$\frac{a\left(-\frac{1}{\sec(dx+c)}+\ln(\sec(dx+c))\right)}{d}$	24
risch	$iax+\frac{2iac}{d}-\frac{a\ln\left(e^{2i(dx+c)}+1\right)}{d}-\frac{a\cos(dx+c)}{d}$	45
norman	$-\frac{2a}{d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{a\ln\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*sin(d*x+c),x,method=_RETURNVERBOSE)
[Out] a/d*(-1/sec(d*x+c)+ln(sec(d*x+c)))
```

Maxima [A]
time = 0.26, size = 23, normalized size = 0.88

$$-\frac{a\cos\left(dx+c\right)+a\log\left(\cos\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -(a*cos(d*x + c) + a*log(cos(d*x + c)))/d
```

Fricas [A]
time = 2.88, size = 25, normalized size = 0.96

$$-\frac{a\cos(dx+c)+a\log(-\cos(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -(a*cos(d*x + c) + a*log(-cos(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sin(c+dx)\sec(c+dx)\,dx+\int\sin(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c),x)
```

```
[Out] a*(Integral(sin(c + d*x)*sec(c + d*x), x) + Integral(sin(c + d*x), x))
```

Giac [A]
time = 0.42, size = 32, normalized size = 1.23

$$-\frac{a\cos(dx+c)}{d}-\frac{a\log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -a*cos(d*x + c)/d - a*log(abs(cos(d*x + c))/abs(d))/d
```

Mupad [B]
time = 0.04, size = 20, normalized size = 0.77

$$-\frac{a\left(\cos(c+d\,x)+\ln(\cos(c+d\,x))\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a + a/cos(c + d*x)),x)
```

```
[Out] -(a*(cos(c + d*x) + log(cos(c + d*x))))/d
```

3.6 $\int \csc(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=30

$$\frac{a\log(1-\cos(c+dx))}{d}-\frac{a\log(\cos(c+dx))}{d}$$

```
[Out] a*ln(1-cos(d*x+c))/d-a*ln(cos(d*x+c))/d
```

Rubi [A]
time = 0.05, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.353$, Rules used = {3957, 2915, 12, 36, 31, 29}

$$\frac{a\log(1-\cos(c+dx))}{d}-\frac{a\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*Log[1 - Cos[c + d*x]])/d - (a*Log[Cos[c + d*x]])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
```

```
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \csc(c+dx)(a+a\sec(c+dx))\,dx &= -\int(-a-a\cos(c+dx))\csc(c+dx)\sec(c+dx)\,dx\\ &= \frac{a\text{Subst}\left(\int \frac{a}{(-a-x)x}\,dx,x,-a\cos(c+dx)\right)}{d}\\ &= \frac{a^2\text{Subst}\left(\int \frac{1}{(-a-x)x}\,dx,x,-a\cos(c+dx)\right)}{d}\\ &= -\frac{a\text{Subst}\left(\int \frac{1}{-a-x}\,dx,x,-a\cos(c+dx)\right)}{d}-\frac{a\text{Subst}\left(\int \frac{1}{x}\,dx,x,-a\cos(c\right.}{d}\\ &= \frac{a\log(1-\cos(c+dx))}{d}-\frac{a\log(\cos(c+dx))}{d}\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 63 vs. 2(30) = 60.
time = 0.03, size = 63, normalized size = 2.10

$$-\frac{a\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{a\log\left(\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{a(\log(\cos(c+dx))-\log(\sin(c+dx)))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*(a + a*Sec[c + d*x]),x]
[Out] -((a*Log[Cos[c/2 + (d*x)/2]])/d) + (a*Log[Sin[c/2 + (d*x)/2]])/d - (a*(Log[
Cos[c + d*x]] - Log[Sin[c + d*x]]))/d
```

Maple [**A**]
time = 0.06, size = 15, normalized size = 0.50

method	result	size
derivativedivides	$\frac{a\ln(-1+\sec(dx+c))}{d}$	15
default	$\frac{a\ln(-1+\sec(dx+c))}{d}$	15

risch	$\frac{2a\ln\left(e^{i(dx+c)}-1\right)}{d}-\frac{a\ln\left(e^{2i(dx+c)}+1\right)}{d}$	38
norman	$\frac{2a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*a*ln(-1+sec(d*x+c))
```

Maxima [A]
time = 0.27, size = 26, normalized size = 0.87

$$\frac{a\log\left(\cos\left(dx+c\right)-1\right)-a\log\left(\cos\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] (a*log(cos(d*x + c) - 1) - a*log(cos(d*x + c)))/d
```

Fricas [A]
time = 2.66, size = 31, normalized size = 1.03

$$-\frac{a\log\left(-\cos\left(dx+c\right)\right)-a\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] -(a*log(-cos(d*x + c)) - a*log(-1/2*cos(d*x + c) + 1/2))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\csc\left(c+dx\right)\sec\left(c+dx\right)dx+\int\csc\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c)),x)
[Out] a*(Integral(csc(c + d*x)*sec(c + d*x), x) + Integral(csc(c + d*x), x))
```

Giac [A]
time = 0.43, size = 58, normalized size = 1.93

$$\frac{a\log\left(\frac{\left|-\cos\left(dx+c\right)+1\right|}{\left|\cos\left(dx+c\right)+1\right|}\right)-a\log\left(\left|-\frac{\cos\left(dx+c\right)-1}{\cos\left(dx+c\right)+1}-1\right|\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] (a*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - a*log(abs(-(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) - 1)))/d
```

Mupad [B]
time = 0.12, size = 17, normalized size = 0.57

$$\frac{2\,a\,\mathrm{atanh}\left(1-2\,\cos\left(c+d\,x\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/sin(c + d*x),x)
```

```
[Out] (2*a*atanh(1 - 2*cos(c + d*x)))/d
```

3.7 $\int \csc^3(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=73

$$-\frac{a^2}{2d(a-a\cos(c+dx))}+\frac{3a\log(1-\cos(c+dx))}{4d}-\frac{a\log(\cos(c+dx))}{d}+\frac{a\log(1+\cos(c+dx))}{4d}$$

```
[Out] -1/2*a^2/d/(a-a*cos(d*x+c))+3/4*a*ln(1-cos(d*x+c))/d-a*ln(cos(d*x+c))/d+1/4
*a*ln(1+cos(d*x+c))/d
```

Rubi [A]
time = 0.08, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3957, 2915, 12, 84}

$$-\frac{a^2}{2d(a-a\cos(c+dx))}+\frac{3a\log(1-\cos(c+dx))}{4d}-\frac{a\log(\cos(c+dx))}{d}+\frac{a\log(\cos(c+dx)+1)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/2*a^2/(d*(a - a*Cos[c + d*x])) + (3*a*Log[1 - Cos[c + d*x]])/(4*d) - (a*
Log[Cos[c + d*x]])/d + (a*Log[1 + Cos[c + d*x]])/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
```

```
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc^3(c+dx)(a+a\sec(c+dx))\,dx = -\int(-a-a\cos(c+dx))\csc^3(c+dx)\sec(c+dx)\,dx$$
$$= \frac{a^3\text{Subst}\left(\int \frac{a}{(-a-x)^2x(-a+x)}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^4\text{Subst}\left(\int \frac{1}{(-a-x)^2x(-a+x)}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^4\text{Subst}\left(\int \left(-\frac{1}{4a^3(a-x)}-\frac{1}{a^3x}+\frac{1}{2a^2(a+x)^2}+\frac{3}{4a^3(a+x)}\right)dx, x, -a\cos(c-\right.}{d}$$
$$= -\frac{a^2}{2d(a-a\cos(c+dx))}+\frac{3a\log(1-\cos(c+dx))}{4d}-\frac{a\log(\cos(c+d}{d}$$

Mathematica [A]
time = 0.55, size = 114, normalized size = 1.56

$$-\frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+\frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{a(\csc^2(c+dx)+2\log(\cos(c+dx))-2\log(\sin(c+dx)))}{2d}+\frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/8*(a*Csc[(c + d*x)/2]^2)/d - (a*Log[Cos[(c + d*x)/2]])/(2*d) + (a*Log[Si
n[(c + d*x)/2]])/(2*d) - (a*(Csc[c + d*x]^2 + 2*Log[Cos[c + d*x]] - 2*Log[S
in[c + d*x]]))/(2*d) + (a*Sec[(c + d*x)/2]^2)/(8*d)
```

Maple [A]
time = 0.10, size = 41, normalized size = 0.56

method	result	size
derivativedivides	$\frac{a\left(-\frac{1}{2(-1+\sec(dx+c))}+\frac{3\ln(-1+\sec(dx+c))}{4}+\frac{\ln(1+\sec(dx+c))}{4}\right)}{d}$	41
default	$\frac{a\left(-\frac{1}{2(-1+\sec(dx+c))}+\frac{3\ln(-1+\sec(dx+c))}{4}+\frac{\ln(1+\sec(dx+c))}{4}\right)}{d}$	41
norman	$-\frac{a}{4d\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{3a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}$	71
risch	$\frac{a\,e^{i(dx+c)}}{d(e^{i(dx+c)}-1)^2}+\frac{3a\ln(e^{i(dx+c)}-1)}{2d}+\frac{a\ln(e^{i(dx+c)}+1)}{2d}-\frac{a\ln(e^{2i(dx+c)}+1)}{d}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*a*(-1/2/(-1+sec(d*x+c))+3/4*ln(-1+sec(d*x+c))+1/4*ln(1+sec(d*x+c)))
```

Maxima [A]
time = 0.28, size = 52, normalized size = 0.71

$$\frac{a\log\left(\cos\left(dx+c\right)+1\right)+3\,a\log\left(\cos\left(dx+c\right)-1\right)-4\,a\log\left(\cos\left(dx+c\right)\right)+\frac{2\,a}{\cos(dx+c)-1}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(a*log(cos(d*x + c) + 1) + 3*a*log(cos(d*x + c) - 1) - 4*a*log(cos(d*x
+ c)) + 2*a/(cos(d*x + c) - 1))/d
```

Fricas [A]
time = 2.89, size = 93, normalized size = 1.27

$$-\frac{4\left(a\cos\left(dx+c\right)-a\right)\log\left(-\cos\left(dx+c\right)\right)-\left(a\cos\left(dx+c\right)-a\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-3\left(a\cos\left(dx+c\right)-a\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-2\,a}{4\left(d\cos\left(dx+c\right)-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(4*(a*cos(d*x + c) - a)*log(-cos(d*x + c)) - (a*cos(d*x + c) - a)*log(
1/2*cos(d*x + c) + 1/2) - 3*(a*cos(d*x + c) - a)*log(-1/2*cos(d*x + c) + 1/
2) - 2*a)/(d*cos(d*x + c) - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\csc^3\left(c+dx\right)\sec\left(c+dx\right)dx+\int\csc^3\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral(csc(c + d*x)**3*sec(c + d*x), x) + Integral(csc(c + d*x)**3, x)
)
```

Giac [A]
time = 0.45, size = 102, normalized size = 1.40

$$\frac{3\,a\log\left(\frac{\left|-\cos(dx+c)+1\right|}{\left|\cos(dx+c)+1\right|}\right)-4\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{\left(a-\frac{3\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{\cos(dx+c)-1}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(3*a*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 4*a*log(abs(-(
cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (a - 3*a*(cos(d*x + c) - 1)/(c
os(d*x + c) + 1))*(cos(d*x + c) + 1)/(cos(d*x + c) - 1))/d
```

Mupad [B]
time = 0.96, size = 53, normalized size = 0.73

$$\frac{\frac{a}{2\,\left(\cos(c+d\,x)-1\right)}-a\,\ln\left(\cos\left(c+d\,x\right)\right)+\frac{3\,a\,\ln(\cos(c+d\,x)-1)}{4}+\frac{a\,\ln(\cos(c+d\,x)+1)}{4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/sin(c + d*x)^3,x)
```

```
[Out] (a/(2*(cos(c + d*x) - 1)) - a*log(cos(c + d*x)) + (3*a*log(cos(c + d*x) - 1
))/4 + (a*log(cos(c + d*x) + 1))/4)/d
```

3.8 $\int \csc^5(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=118

$$-\frac{a^3}{8d(a-a\cos(c+dx))^2}-\frac{a^2}{2d(a-a\cos(c+dx))}-\frac{a^2}{8d(a+a\cos(c+dx))}+\frac{11a\log(1-\cos(c+dx))}{16d}-\frac{a\log(\cdots}{\cdots}$$

```
[Out] -1/8*a^3/d/(a-a*cos(d*x+c))^2-1/2*a^2/d/(a-a*cos(d*x+c))-1/8*a^2/d/(a+a*cos
(d*x+c))+11/16*a*ln(1-cos(d*x+c))/d-a*ln(cos(d*x+c))/d+5/16*a*ln(1+cos(d*x+
c))/d
```

Rubi [A]
time = 0.09, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3957, 2915, 12, 90}

$$-\frac{a^3}{8d(a-a\cos(c+dx))^2}-\frac{a^2}{2d(a-a\cos(c+dx))}-\frac{a^2}{8d(a\cos(c+dx)+a)}+\frac{11a\log(1-\cos(c+dx))}{16d}-\frac{a\log(\cos(c+dx))}{d}+\frac{5a\log(\cos(c+dx)+1)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5*(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/8*a^3/(d*(a - a*Cos[c + d*x])^2) - a^2/(2*d*(a - a*Cos[c + d*x])) - a^2/
(8*d*(a + a*Cos[c + d*x])) + (11*a*Log[1 - Cos[c + d*x]])/(16*d) - (a*Log[C
os[c + d*x]])/d + (5*a*Log[1 + Cos[c + d*x]])/(16*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \csc^5(c+dx)(a+a\sec(c+dx))\,dx &= -\int (-a-a\cos(c+dx))\csc^5(c+dx)\sec(c+dx)\,dx \\
&= \frac{a^5\text{Subst}\left(\int \frac{a}{(-a-x)^3x(-a+x)^2}\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= \frac{a^6\text{Subst}\left(\int \frac{1}{(-a-x)^3x(-a+x)^2}\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= \frac{a^6\text{Subst}\left(\int \left(-\frac{1}{8a^4(a-x)^2} - \frac{5}{16a^5(a-x)} - \frac{1}{a^5x} + \frac{1}{4a^3(a+x)^3} + \frac{1}{2a^4(a+x)^2} + \frac{1}{16a}\right.\right.}{d} \\
&= -\frac{a^3}{8d(a-a\cos(c+dx))^2} - \frac{a^2}{2d(a-a\cos(c+dx))} - \frac{a^2}{8d(a+a\cos(c+}
\end{aligned}$$

Mathematica [A]
time = 0.24, size = 164, normalized size = 1.39

$$-\frac{3a\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d} - \frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{3a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{8d} + \frac{3a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d} - \frac{a(2\csc^2(c+dx)+\csc^4(c+dx)+4\log(\cos(c+dx))-4\log(\sin(c+dx)))}{4d} + \frac{3a\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^5*(a + a*Sec[c + d*x]),x]
```

```
[Out] (-3*a*Csc[(c + d*x)/2]^2)/(32*d) - (a*Csc[(c + d*x)/2]^4)/(64*d) - (3*a*Log
[Cos[(c + d*x)/2]])/(8*d) + (3*a*Log[Sin[(c + d*x)/2]])/(8*d) - (a*(2*Csc[c
 + d*x]^2 + Csc[c + d*x]^4 + 4*Log[Cos[c + d*x]] - 4*Log[Sin[c + d*x]]))/(4
*d) + (3*a*Sec[(c + d*x)/2]^2)/(32*d) + (a*Sec[(c + d*x)/2]^4)/(64*d)
```

Maple [A]
time = 0.12, size = 66, normalized size = 0.56

method	result
derivativedivides	$-\frac{a\left(\frac{1}{8(-1+\sec(dx+c))^2}+\frac{3}{4(-1+\sec(dx+c))}-\frac{11\ln(-1+\sec(dx+c))}{16}-\frac{1}{8(1+\sec(dx+c))}-\frac{5\ln(1+\sec(dx+c))}{16}\right)}{d}$
default	$-\frac{a\left(\frac{1}{8(-1+\sec(dx+c))^2}+\frac{3}{4(-1+\sec(dx+c))}-\frac{11\ln(-1+\sec(dx+c))}{16}-\frac{1}{8(1+\sec(dx+c))}-\frac{5\ln(1+\sec(dx+c))}{16}\right)}{d}$

norman	$\frac{-\frac{a}{32d}-\frac{5a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16d}-\frac{a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}+\frac{11a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{d}$
risch	$\frac{a(3\,\mathrm{e}^{5i(dx+c)}+2\,\mathrm{e}^{4i(dx+c)}-18\,\mathrm{e}^{3i(dx+c)}+2\,\mathrm{e}^{2i(dx+c)}+3\,\mathrm{e}^{i(dx+c)})}{4d(\mathrm{e}^{i(dx+c)}-1)^4(\mathrm{e}^{i(dx+c)}+1)^2}+\frac{5a\ln(\mathrm{e}^{i(dx+c)}+1)}{8d}+\frac{11a\ln(\mathrm{e}^{i(dx+c)}-1)}{8d}-\frac{a\,\mathrm{l}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*a*(1/8/(-1+sec(d*x+c))^2+3/4/(-1+sec(d*x+c))-11/16*ln(-1+sec(d*x+c))-1
/8/(1+sec(d*x+c))-5/16*ln(1+sec(d*x+c)))
```

Maxima [A]
time = 0.26, size = 95, normalized size = 0.81

$$\frac{5\,a\log\left(\cos\left(dx+c\right)+1\right)+11\,a\log\left(\cos\left(dx+c\right)-1\right)-16\,a\log\left(\cos\left(dx+c\right)\right)+\frac{2\left(3\,a\cos\left(dx+c\right)^2+a\cos\left(dx+c\right)-6\,a\right)}{\cos\left(dx+c\right)^3-\cos\left(dx+c\right)^2-\cos\left(dx+c\right)+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(5*a*log(cos(d*x + c) + 1) + 11*a*log(cos(d*x + c) - 1) - 16*a*log(cos
(d*x + c)) + 2*(3*a*cos(d*x + c)^2 + a*cos(d*x + c) - 6*a)/(cos(d*x + c)^3
- cos(d*x + c)^2 - cos(d*x + c) + 1))/d
```

Fricas [A]
time = 3.14, size = 193, normalized size = 1.64

$$\frac{6\,a\cos\left(dx+c\right)^2+2\,a\cos\left(dx+c\right)-16\,\left(a\cos\left(dx+c\right)^3-a\cos\left(dx+c\right)^2-a\cos\left(dx+c\right)+a\right)\log\left(-\cos\left(dx+c\right)\right)+5\,\left(a\cos\left(dx+c\right)^3-a\cos\left(dx+c\right)^2-a\cos\left(dx+c\right)+a\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+11\,\left(a\cos\left(dx+c\right)^3-a\cos\left(dx+c\right)^2-a\cos\left(dx+c\right)+a\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-12\,a}{16\,\left(d\cos\left(dx+c\right)^3-d\cos\left(dx+c\right)^2-d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(6*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - 16*(a*cos(d*x + c)^3 - a*cos(
d*x + c)^2 - a*cos(d*x + c) + a)*log(-cos(d*x + c)) + 5*(a*cos(d*x + c)^3 -
 a*cos(d*x + c)^2 - a*cos(d*x + c) + a)*log(1/2*cos(d*x + c) + 1/2) + 11*(a
*cos(d*x + c)^3 - a*cos(d*x + c)^2 - a*cos(d*x + c) + a)*log(-1/2*cos(d*x +
 c) + 1/2) - 12*a)/(d*cos(d*x + c)^3 - d*cos(d*x + c)^2 - d*cos(d*x + c) +
d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\csc^5\left(c+dx\right)\sec\left(c+dx\right)dx+\int\csc^5\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)**5*(a+a*sec(d*x+c)),x)

[Out] a*(Integral(csc(c + d*x)**5*sec(c + d*x), x) + Integral(csc(c + d*x)**5, x)
)

Giac [A]
time = 0.46, size = 149, normalized size = 1.26

$$\frac{22\,a\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-32\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)-\frac{\left(a-\frac{10\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{33\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)^2}{(\cos(dx+c)-1)^2}+\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^5*(a+a*sec(d*x+c)),x, algorithm="giac")

[Out] 1/32*(22*a*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 32*a*log(abs
(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - (a - 10*a*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) + 33*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)*(cos
(d*x + c) + 1)^2/(cos(d*x + c) - 1)^2 + 2*a*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1))/d

Mupad [B]
time = 0.10, size = 99, normalized size = 0.84

$$\frac{11\,a\,\ln\left(\cos\left(c+d\,x\right)-1\right)}{16\,d}-\frac{a\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}+\frac{5\,a\,\ln\left(\cos\left(c+d\,x\right)+1\right)}{16\,d}+\frac{\frac{3\,a\cos(c+d\,x)^2}{8}+\frac{a\,\cos(c+d\,x)}{8}-\frac{3\,a}{4}}{d\,\left({\cos\left(c+d\,x\right)}^3-\cos\left(c+d\,x\right)+{\sin\left(c+d\,x\right)}^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(c + d*x))/sin(c + d*x)^5,x)

[Out] (11*a*log(cos(c + d*x) - 1))/(16*d) - (a*log(cos(c + d*x)))/d + (5*a*log(co
s(c + d*x) + 1))/(16*d) + ((a*cos(c + d*x))/8 - (3*a)/4 + (3*a*cos(c + d*x)
^2)/8)/(d*(cos(c + d*x)^3 - cos(c + d*x) + sin(c + d*x)^2))

3.9 $\int \csc^7(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=163

$$-\frac{a^4}{24d(a-a\cos(c+dx))^3}-\frac{5a^3}{32d(a-a\cos(c+dx))^2}-\frac{a^2}{2d(a-a\cos(c+dx))}-\frac{a^3}{32d(a+a\cos(c+dx))^2}-16d$$

```
[Out] -1/24*a^4/d/(a-a*cos(d*x+c))^3-5/32*a^3/d/(a-a*cos(d*x+c))^2-1/2*a^2/d/(a-a
*cos(d*x+c))-1/32*a^3/d/(a+a*cos(d*x+c))^2-3/16*a^2/d/(a+a*cos(d*x+c))+21/3
2*a*ln(1-cos(d*x+c))/d-a*ln(cos(d*x+c))/d+11/32*a*ln(1+cos(d*x+c))/d
```

Rubi [A]
time = 0.12, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3957, 2915, 12, 90}

$$-\frac{a^4}{24d(a-a\cos(c+dx))^3}-\frac{5a^3}{32d(a-a\cos(c+dx))^2}-\frac{a^3}{32d(a\cos(c+dx)+a)^2}-\frac{a^2}{2d(a-a\cos(c+dx))}-\frac{3a^2}{16d(a\cos(c+dx)+a)}+\frac{21a\log(1-\cos(c+dx))}{32d}-\frac{a\log(\cos(c+dx))}{d}+\frac{11a\log(\cos(c+dx)+1)}{32d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^7*(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/24*a^4/(d*(a - a*Cos[c + d*x])^3) - (5*a^3)/(32*d*(a - a*Cos[c + d*x])^2
) - a^2/(2*d*(a - a*Cos[c + d*x])) - a^3/(32*d*(a + a*Cos[c + d*x])^2) - (3
*a^2)/(16*d*(a + a*Cos[c + d*x])) + (21*a*Log[1 - Cos[c + d*x]])/(32*d) - (
a*Log[Cos[c + d*x]])/d + (11*a*Log[1 + Cos[c + d*x]])/(32*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \csc^7(c+dx)(a+a\sec(c+dx))\,dx &= -\int (-a-a\cos(c+dx))\csc^7(c+dx)\sec(c+dx)\,dx\\ &= \frac{a^7\text{Subst}\left(\int \frac{a}{(-a-x)^4x(-a+x)^3}\,dx,x,-a\cos(c+dx)\right)}{d}\\ &= \frac{a^8\text{Subst}\left(\int \frac{1}{(-a-x)^4x(-a+x)^3}\,dx,x,-a\cos(c+dx)\right)}{d}\\ &= \frac{a^8\text{Subst}\left(\int \left(-\frac{1}{16a^5(a-x)^3}-\frac{3}{16a^6(a-x)^2}-\frac{11}{32a^7(a-x)}-\frac{1}{a^7x}+\frac{1}{8a^4(a+x)^4}+\right.\right.}{d}\\ &= -\frac{a^4}{24d(a-a\cos(c+dx))^3}-\frac{5a^3}{32d(a-a\cos(c+dx))^2}-\frac{a^2}{2d(a-a\cos}\end{aligned}$$

Mathematica [A]
time = 0.27, size = 165, normalized size = 1.01

$$-\frac{a\left(30\csc^2\left(\frac{1}{2}(c+dx)\right)+6\csc^4\left(\frac{1}{2}(c+dx)\right)+\csc^6\left(\frac{1}{2}(c+dx)\right)+192\csc^2(c+dx)+96\csc^4(c+dx)+64\csc^6(c+dx)+120\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+384\log(\cos(c+dx))-120\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-384\log(\sin(c+dx))-30\sec^2\left(\frac{1}{2}(c+dx)\right)-6\sec^4\left(\frac{1}{2}(c+dx)\right)-\sec^6\left(\frac{1}{2}(c+dx)\right)\right)}{384d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^7*(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/384*(a*(30*Csc[(c + d*x)/2]^2 + 6*Csc[(c + d*x)/2]^4 + Csc[(c + d*x)/2]^
6 + 192*Csc[c + d*x]^2 + 96*Csc[c + d*x]^4 + 64*Csc[c + d*x]^6 + 120*Log[Co
s[(c + d*x)/2]] + 384*Log[Cos[c + d*x]] - 120*Log[Sin[(c + d*x)/2]] - 384*L
og[Sin[c + d*x]] - 30*Sec[(c + d*x)/2]^2 - 6*Sec[(c + d*x)/2]^4 - Sec[(c +
d*x)/2]^6))/d
```

Maple [A]
time = 0.14, size = 89, normalized size = 0.55

method	result
derivativedivides	$\frac{a\left(-\frac{1}{24(-1+\sec(dx+c))^3}-\frac{9}{32(-1+\sec(dx+c))^2}-\frac{15}{16(-1+\sec(dx+c))}+\frac{21\ln(-1+\sec(dx+c))}{32}-\frac{1}{32(1+\sec(dx+c))^2}+\frac{1}{4+4\sec(dx+c)}+\right.}{d}$
default	$\frac{a\left(-\frac{1}{24(-1+\sec(dx+c))^3}-\frac{9}{32(-1+\sec(dx+c))^2}-\frac{15}{16(-1+\sec(dx+c))}+\frac{21\ln(-1+\sec(dx+c))}{32}-\frac{1}{32(1+\sec(dx+c))^2}+\frac{1}{4+4\sec(dx+c)}+\right.}{d}$

norman	$\frac{-\frac{a}{192d}-\frac{7a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128d}-\frac{11a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32d}-\frac{7a\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}-\frac{a\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}+\frac{21a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16d}-\frac{a\,\mathrm{l}}{}$
risch	$\frac{a\left(15\,\mathrm{e}^{9i(dx+c)}+18\,\mathrm{e}^{8i(dx+c)}-136\,\mathrm{e}^{7i(dx+c)}-34\,\mathrm{e}^{6i(dx+c)}+402\,\mathrm{e}^{5i(dx+c)}-34\,\mathrm{e}^{4i(dx+c)}-136\,\mathrm{e}^{3i(dx+c)}+18\,\mathrm{e}^{2i(dx+c)}+15\right.}{24d\left(\mathrm{e}^{i(dx+c)}-1\right)^6\left(\mathrm{e}^{i(dx+c)}+1\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^7*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*a*(-1/24/(-1+sec(d*x+c))^3-9/32/(-1+sec(d*x+c))^2-15/16/(-1+sec(d*x+c))
+21/32*ln(-1+sec(d*x+c))-1/32/(1+sec(d*x+c))^2+1/4/(1+sec(d*x+c))+11/32*ln(
1+sec(d*x+c)))
```

Maxima [A]
time = 0.27, size = 136, normalized size = 0.83

$$\frac{33\,a\log\left(\cos\left(dx+c\right)+1\right)+63\,a\log\left(\cos\left(dx+c\right)-1\right)-96\,a\log\left(\cos\left(dx+c\right)\right)+\frac{2\left(15\,a\cos\left(dx+c\right)^4+9\,a\cos\left(dx+c\right)^3-49\,a\cos\left(dx+c\right)^2-11\,a\cos\left(dx+c\right)+44\,a\right)}{\cos\left(dx+c\right)^5-\cos\left(dx+c\right)^4-2\cos\left(dx+c\right)^3+2\cos\left(dx+c\right)^2+\cos\left(dx+c\right)-1}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^7*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/96*(33*a*log(cos(d*x + c) + 1) + 63*a*log(cos(d*x + c) - 1) - 96*a*log(co
s(d*x + c)) + 2*(15*a*cos(d*x + c)^4 + 9*a*cos(d*x + c)^3 - 49*a*cos(d*x +
c)^2 - 11*a*cos(d*x + c) + 44*a)/(cos(d*x + c)^5 - cos(d*x + c)^4 - 2*cos(d
*x + c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c) - 1))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(152) = 304$.
time = 5.72, size = 307, normalized size = 1.88

$$\frac{30\,a\cos\left(dx+c\right)^4+18\,a\cos\left(dx+c\right)^3-98\,a\cos\left(dx+c\right)^2-22\,a\cos\left(dx+c\right)-96\left(a\cos\left(dx+c\right)^5-a\cos\left(dx+c\right)^4-2\,a\cos\left(dx+c\right)^3+2\,a\cos\left(dx+c\right)^2+a\cos\left(dx+c\right)-a\right)\log\left(-\cos\left(dx+c\right)\right)+33\left(a\cos\left(dx+c\right)^5-a\cos\left(dx+c\right)^4-2\,a\cos\left(dx+c\right)^3+2\,a\cos\left(dx+c\right)^2+a\cos\left(dx+c\right)-a\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+63\left(a\cos\left(dx+c\right)^5-a\cos\left(dx+c\right)^4-2\,a\cos\left(dx+c\right)^3+2\,a\cos\left(dx+c\right)^2+a\cos\left(dx+c\right)-a\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+88\,a}{96\left(d\cos\left(dx+c\right)^5-d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^3+2\,d\cos\left(dx+c\right)^2+d\cos\left(dx+c\right)-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^7*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/96*(30*a*cos(d*x + c)^4 + 18*a*cos(d*x + c)^3 - 98*a*cos(d*x + c)^2 - 22*
a*cos(d*x + c) - 96*(a*cos(d*x + c)^5 - a*cos(d*x + c)^4 - 2*a*cos(d*x + c)
^3 + 2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a)*log(-cos(d*x + c)) + 33*(a*co
s(d*x + c)^5 - a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^3 + 2*a*cos(d*x + c)^2 +
 a*cos(d*x + c) - a)*log(1/2*cos(d*x + c) + 1/2) + 63*(a*cos(d*x + c)^5 - a
*cos(d*x + c)^4 - 2*a*cos(d*x + c)^3 + 2*a*cos(d*x + c)^2 + a*cos(d*x + c)
- a)*log(-1/2*cos(d*x + c) + 1/2) + 88*a)/(d*cos(d*x + c)^5 - d*cos(d*x + c
)^4 - 2*d*cos(d*x + c)^3 + 2*d*cos(d*x + c)^2 + d*cos(d*x + c) - d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**7*(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.49, size = 196, normalized size = 1.20

$$\frac{252\,a\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-384\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{\left(2\,a-\frac{21\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{132\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{462\,a(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}\right)(\cos(dx+c)+1)^3}{(\cos(dx+c)-1)^3}+\frac{42\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{3\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^7*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/384*(252*a*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 384*a*log(
abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (2*a - 21*a*(cos(d*x + c
) - 1)/(cos(d*x + c) + 1) + 132*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2
 - 462*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)*(cos(d*x + c) + 1)^3/(c
os(d*x + c) - 1)^3 + 42*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 3*a*(cos(
d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/d
```

Mupad [B]
time = 0.99, size = 142, normalized size = 0.87

$$\frac{\frac{5\,a\cos(c+d\,x)^4}{16}+\frac{3\,a\cos(c+d\,x)^3}{16}-\frac{49\,a\cos(c+d\,x)^2}{48}-\frac{11\,a\,\cos(c+d\,x)}{48}+\frac{11\,a}{12}}{d\,\left(\cos(c+d\,x)^5-\cos(c+d\,x)^4-2\cos(c+d\,x)^3+2\cos(c+d\,x)^2+\cos(c+d\,x)-1\right)}-\frac{a\,\ln(\cos(c+d\,x))}{d}+\frac{21\,a\,\ln(\cos(c+d\,x)-1)}{32\,d}+\frac{11\,a\,\ln(\cos(c+d\,x)+1)}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/sin(c + d*x)^7,x)
```

```
[Out] ((11*a)/12 - (11*a*cos(c + d*x))/48 - (49*a*cos(c + d*x)^2)/48 + (3*a*cos(c
 + d*x)^3)/16 + (5*a*cos(c + d*x)^4)/16)/(d*(cos(c + d*x) + 2*cos(c + d*x)^
2 - 2*cos(c + d*x)^3 - cos(c + d*x)^4 + cos(c + d*x)^5 - 1)) - (a*log(cos(c
 + d*x)))/d + (21*a*log(cos(c + d*x) - 1))/(32*d) + (11*a*log(cos(c + d*x)
+ 1))/(32*d)
```

3.10 $\int (a+a\sec(c+dx))\sin^8(c+dx)\,dx$

Optimal. Leaf size=165

$$\frac{35ax}{128}+\frac{a\tanh^{-1}(\sin(c+dx))}{d}-\frac{a\sin(c+dx)}{d}-\frac{35a\cos(c+dx)\sin(c+dx)}{128d}-\frac{a\sin^3(c+dx)}{3d}-\frac{35a\cos(c+dx)}{192}$$

```
[Out] 35/128*a*x+a*arctanh(sin(d*x+c))/d-a*sin(d*x+c)/d-35/128*a*cos(d*x+c)*sin(d
*x+c)/d-1/3*a*sin(d*x+c)^3/d-35/192*a*cos(d*x+c)*sin(d*x+c)^3/d-1/5*a*sin(d
*x+c)^5/d-7/48*a*cos(d*x+c)*sin(d*x+c)^5/d-1/7*a*sin(d*x+c)^7/d-1/8*a*cos(d
*x+c)*sin(d*x+c)^7/d
```

Rubi [A]
time = 0.11, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3957, 2917, 2672, 308, 212, 2715, 8}

$$-\frac{a\sin^7(c+dx)}{7d}-\frac{a\sin^5(c+dx)}{5d}-\frac{a\sin^3(c+dx)}{3d}-\frac{a\sin(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{d}-\frac{a\sin^7(c+dx)\cos(c+dx)}{8d}-\frac{7a\sin^5(c+dx)\cos(c+dx)}{48d}-\frac{35a\sin^3(c+dx)\cos(c+dx)}{192d}-\frac{35a\sin(c+dx)\cos(c+dx)}{128d}+\frac{35ax}{128}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Sin[c + d*x]^8,x]
```

```
[Out] (35*a*x)/128 + (a*ArcTanh[Sin[c + d*x]])/d - (a*Sin[c + d*x])/d - (35*a*Cos
[c + d*x]*Sin[c + d*x])/(128*d) - (a*Sin[c + d*x]^3)/(3*d) - (35*a*Cos[c +
d*x]*Sin[c + d*x]^3)/(192*d) - (a*Sin[c + d*x]^5)/(5*d) - (7*a*Cos[c + d*x]
*Sin[c + d*x]^5)/(48*d) - (a*Sin[c + d*x]^7)/(7*d) - (a*Cos[c + d*x]*Sin[c
+ d*x]^7)/(8*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
```

```
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int (a+a\sec(c+dx))\sin^8(c+dx)\,dx &= -\int (-a-a\cos(c+dx))\sin^7(c+dx)\tan(c+dx)\,dx \\
&= a\int \sin^8(c+dx)\,dx + a\int \sin^7(c+dx)\tan(c+dx)\,dx \\
&= -\frac{a\cos(c+dx)\sin^7(c+dx)}{8d} + \frac{1}{8}(7a)\int \sin^6(c+dx)\,dx + \frac{a\text{Subst}\Big(\int \\
&= -\frac{7a\cos(c+dx)\sin^5(c+dx)}{48d} - \frac{a\cos(c+dx)\sin^7(c+dx)}{8d} + \frac{1}{48}(35a \\
&= -\frac{a\sin(c+dx)}{d} - \frac{a\sin^3(c+dx)}{3d} - \frac{35a\cos(c+dx)\sin^3(c+dx)}{192d} - \frac{a \\
&= \frac{a\tanh^{-1}(\sin(c+dx))}{d} - \frac{a\sin(c+dx)}{d} - \frac{35a\cos(c+dx)\sin(c+dx)}{128d} \\
&= \frac{35ax}{128} + \frac{a\tanh^{-1}(\sin(c+dx))}{d} - \frac{a\sin(c+dx)}{d} - \frac{35a\cos(c+dx)\sin}{128d}
\end{aligned}$$

Mathematica [A]

time = 0.22, size = 106, normalized size = 0.64

$$\frac{a(107520\tanh^{-1}(\sin(c+dx)) - 107520\sin(c+dx) - 35840\sin^3(c+dx) - 21504\sin^5(c+dx) - 15360\sin^7(c+dx) + 35(840c + 840dx - 672\sin(2(c+dx)) + 168\sin(4(c+dx)) - 32\sin(6(c+dx)) + 3\sin(8(c+dx))))}{107520d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Sin[c + d*x]^8,x]
```

```
[Out] (a*(107520*ArcTanh[Sin[c + d*x]] - 107520*Sin[c + d*x] - 35840*Sin[c + d*x]
^3 - 21504*Sin[c + d*x]^5 - 15360*Sin[c + d*x]^7 + 35*(840*c + 840*d*x - 67
2*Sin[2*(c + d*x)] + 168*Sin[4*(c + d*x)] - 32*Sin[6*(c + d*x)] + 3*Sin[8*(
c + d*x)])))/(107520*d)
```

Maple [A]
time = 0.17, size = 116, normalized size = 0.70

method	result
derivativedivides	$\frac{a\left(-\frac{\left(\sin^7(dx+c)\right)}{7}-\frac{\left(\sin^5(dx+c)\right)}{5}-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(-\frac{\left(\sin^7(dx+c)+\frac{7\left(\sin^5(dx\right.}{6}\right.}{}\right.}{d}$
default	$\frac{a\left(-\frac{\left(\sin^7(dx+c)\right)}{7}-\frac{\left(\sin^5(dx+c)\right)}{5}-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(-\frac{\left(\sin^7(dx+c)+\frac{7\left(\sin^5(dx\right.}{6}\right.}{}\right.}{d}$
risch	$\frac{35ax}{128}+\frac{93ia\,e^{i(dx+c)}}{128d}-\frac{93ia\,e^{-i(dx+c)}}{128d}+\frac{a\ln(e^{i(dx+c)}+i)}{d}-\frac{a\ln(e^{i(dx+c)}-i)}{d}+\frac{a\sin(8dx+8c)}{1024d}+\frac{a\sin(7dx+}{448d}$
norman	$\frac{35ax}{128}-\frac{163a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64d}-\frac{1335a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}-\frac{24223a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{320d}-\frac{359453a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2240d}-\frac{724649a\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6720d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*sin(d*x+c)^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/7*sin(d*x+c)^7-1/5*sin(d*x+c)^5-1/3*sin(d*x+c)^3-sin(d*x+c)+ln(s
ec(d*x+c)+tan(d*x+c)))+a*(-1/8*(sin(d*x+c)^7+7/6*sin(d*x+c)^5+35/24*sin(d*x
+c)^3+35/16*sin(d*x+c))*cos(d*x+c)+35/128*d*x+35/128*c))
```

Maxima [A]
time = 0.27, size = 127, normalized size = 0.77

$$-\frac{512\,(30\,\sin(dx+c)^7 + 42\,\sin(dx+c)^5 + 70\,\sin(dx+c)^3 - 105\,\log(\sin(dx+c)+1) + 105\,\log(\sin(dx+c)-1) + 210\,\sin(dx+c))a - 35\,(128\,\sin(2\,dx+2\,c)^3 + 840\,dx + 840\,c + 3\,\sin(8\,dx+8\,c) + 168\,\sin(4\,dx+4\,c) - 768\,\sin(2\,dx+2\,c))a}{107520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] -1/107520*(512*(30*sin(d*x + c)^7 + 42*sin(d*x + c)^5 + 70*sin(d*x + c)^3 -
 105*log(sin(d*x + c) + 1) + 105*log(sin(d*x + c) - 1) + 210*sin(d*x + c))*
a - 35*(128*sin(2*d*x + 2*c)^3 + 840*d*x + 840*c + 3*sin(8*d*x + 8*c) + 168
*sin(4*d*x + 4*c) - 768*sin(2*d*x + 2*c))*a)/d
```

Fricas [A]
time = 3.14, size = 123, normalized size = 0.75

$$\frac{3675\,adx + 6720\,a\log\left(\sin\left(dx+c\right)+1\right) - 6720\,a\log\left(-\sin\left(dx+c\right)+1\right) + \left(1680\,a\cos\left(dx+c\right)^{7} + 1920\,a\cos\left(dx+c\right)^{6} - 7000\,a\cos\left(dx+c\right)^{5} - 8448\,a\cos\left(dx+c\right)^{4} + 11410\,a\cos\left(dx+c\right)^{3} + 15616\,a\cos\left(dx+c\right)^{2} - 9765\,a\cos\left(dx+c\right) - 22528\,a\right)\sin\left(dx+c\right)}{13440\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] 1/13440*(3675*a*d*x + 6720*a*log(sin(d*x + c) + 1) - 6720*a*log(-sin(d*x +
c) + 1) + (1680*a*cos(d*x + c)^7 + 1920*a*cos(d*x + c)^6 - 7000*a*cos(d*x +
 c)^5 - 8448*a*cos(d*x + c)^4 + 11410*a*cos(d*x + c)^3 + 15616*a*cos(d*x +
c)^2 - 9765*a*cos(d*x + c) - 22528*a)*sin(d*x + c))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)**8,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [A]
time = 0.47, size = 174, normalized size = 1.05

$$\frac{3675\,(dx+c)a + 13440\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 13440\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\left(9765\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}+83825\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+321013\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+724649\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+1078359\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+508683\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+140175\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+17115\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{8}}}{13440\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^8,x, algorithm="giac")
```

```
[Out] 1/13440*(3675*(d*x + c)*a + 13440*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 13
440*a*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(9765*a*tan(1/2*d*x + 1/2*c)^1
5 + 83825*a*tan(1/2*d*x + 1/2*c)^13 + 321013*a*tan(1/2*d*x + 1/2*c)^11 + 72
4649*a*tan(1/2*d*x + 1/2*c)^9 + 1078359*a*tan(1/2*d*x + 1/2*c)^7 + 508683*a
*tan(1/2*d*x + 1/2*c)^5 + 140175*a*tan(1/2*d*x + 1/2*c)^3 + 17115*a*tan(1/2
*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^8)/d
```

Mupad [B]
time = 1.14, size = 150, normalized size = 0.91

$$\frac{35\,a\,x}{128}+\frac{2\,a\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{7\,a\,\sin\left(2\,c+2\,d\,x\right)}{32\,d}+\frac{37\,a\,\sin\left(3\,c+3\,d\,x\right)}{192\,d}+\frac{7\,a\,\sin\left(4\,c+4\,d\,x\right)}{128\,d}-\frac{9\,a\,\sin\left(5\,c+5\,d\,x\right)}{320\,d}-\frac{a\,\sin\left(6\,c+6\,d\,x\right)}{96\,d}+\frac{a\,\sin\left(7\,c+7\,d\,x\right)}{448\,d}+\frac{a\,\sin\left(8\,c+8\,d\,x\right)}{1024\,d}-\frac{93\,a\,\sin\left(c+d\,x\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^8*(a + a/cos(c + d*x)),x)
```

```
[Out] (35*a*x)/128 + (2*a*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (7*a*
sin(2*c + 2*d*x))/(32*d) + (37*a*sin(3*c + 3*d*x))/(192*d) + (7*a*sin(4*c +
 4*d*x))/(128*d) - (9*a*sin(5*c + 5*d*x))/(320*d) - (a*sin(6*c + 6*d*x))/(9
6*d) + (a*sin(7*c + 7*d*x))/(448*d) + (a*sin(8*c + 8*d*x))/(1024*d) - (93*a
*sin(c + d*x))/(64*d)
```

3.11 $\int (a+a\sec(c+dx))\sin^6(c+dx)\,dx$

Optimal. Leaf size=127

$$\frac{5ax}{16}+\frac{a\tanh^{-1}(\sin(c+dx))}{d}-\frac{a\sin(c+dx)}{d}-\frac{5a\cos(c+dx)\sin(c+dx)}{16d}-\frac{a\sin^3(c+dx)}{3d}-\frac{5a\cos(c+dx)\sin^3}{24d}$$

```
[Out] 5/16*a*x+a*arctanh(sin(d*x+c))/d-a*sin(d*x+c)/d-5/16*a*cos(d*x+c)*sin(d*x+c
)/d-1/3*a*sin(d*x+c)^3/d-5/24*a*cos(d*x+c)*sin(d*x+c)^3/d-1/5*a*sin(d*x+c)^
5/d-1/6*a*cos(d*x+c)*sin(d*x+c)^5/d
```

Rubi [A]
time = 0.10, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3957, 2917, 2672, 308, 212, 2715, 8}

$$-\frac{a\sin^5(c+dx)}{5d}-\frac{a\sin^3(c+dx)}{3d}-\frac{a\sin(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{d}-\frac{a\sin^5(c+dx)\cos(c+dx)}{6d}-\frac{5a\sin^3(c+dx)\cos(c+dx)}{24d}-\frac{5a\sin(c+dx)\cos(c+dx)}{16d}+\frac{5ax}{16}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Sin[c + d*x]^6,x]
```

```
[Out] (5*a*x)/16 + (a*ArcTanh[Sin[c + d*x]])/d - (a*Sin[c + d*x])/d - (5*a*Cos[c
+ d*x]*Sin[c + d*x])/(16*d) - (a*Sin[c + d*x]^3)/(3*d) - (5*a*Cos[c + d*x]*
Sin[c + d*x]^3)/(24*d) - (a*Sin[c + d*x]^5)/(5*d) - (a*Cos[c + d*x]*Sin[c +
 d*x]^5)/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
```

```
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))\sin^6(c+dx)\,dx &= -\int (-a-a\cos(c+dx))\sin^5(c+dx)\tan(c+dx)\,dx\\ &= a\int \sin^6(c+dx)\,dx + a\int \sin^5(c+dx)\tan(c+dx)\,dx\\ &= -\frac{a\cos(c+dx)\sin^5(c+dx)}{6d} + \frac{1}{6}(5a)\int \sin^4(c+dx)\,dx + \frac{a\mathrm{Subst}\Big(}{}\\ &= -\frac{5a\cos(c+dx)\sin^3(c+dx)}{24d} - \frac{a\cos(c+dx)\sin^5(c+dx)}{6d} + \frac{1}{8}(5a)\\ &= -\frac{a\sin(c+dx)}{d} - \frac{5a\cos(c+dx)\sin(c+dx)}{16d} - \frac{a\sin^3(c+dx)}{3d} - 5a\\ &= \frac{5ax}{16} + \frac{a\tanh^{-1}(\sin(c+dx))}{d} - \frac{a\sin(c+dx)}{d} - \frac{5a\cos(c+dx)\sin(}{16d}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 86, normalized size = 0.68

$$\frac{a(960\tanh^{-1}(\sin(c+dx)) - 960\sin(c+dx) - 320\sin^3(c+dx) - 192\sin^5(c+dx) + 5(60c+60dx-45\sin(2(c+dx))+9\sin(4(c+dx))-\sin(6(c+dx))))}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Sin[c + d*x]^6,x]
```

```
[Out] (a*(960*ArcTanh[Sin[c + d*x]] - 960*Sin[c + d*x] - 320*Sin[c + d*x]^3 - 192
*Sin[c + d*x]^5 + 5*(60*c + 60*d*x - 45*Sin[2*(c + d*x)] + 9*Sin[4*(c + d*x
)] - Sin[6*(c + d*x)])))/(960*d)
```

Maple [A]
time = 0.14, size = 96, normalized size = 0.76

method	result
derivativedivides	$\frac{a\left(-\frac{\left(\sin^5(dx+c)\right)}{5}-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(-\frac{\left(\sin^5(dx+c)+\frac{5\left(\sin^3(dx+c)\right)}{4}+\frac{15\sin(dx+c}{8}\right.}{6}\right.}{d}$
default	$\frac{a\left(-\frac{\left(\sin^5(dx+c)\right)}{5}-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(-\frac{\left(\sin^5(dx+c)+\frac{5\left(\sin^3(dx+c)\right)}{4}+\frac{15\sin(dx+c}{8}\right.}{6}\right.}{d}$
risch	$\frac{5ax}{16}+\frac{11ia\,e^{i(dx+c)}}{16d}-\frac{11ia\,e^{-i(dx+c)}}{16d}+\frac{a\ln\left(e^{i(dx+c)}+i\right)}{d}-\frac{a\ln\left(e^{i(dx+c)}-i\right)}{d}-\frac{a\sin(6dx+6c)}{192d}-\frac{a\sin(5dx+5c)}{80d}$
norman	$\frac{\frac{5ax}{16}-\frac{21a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}-\frac{389a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}-\frac{853a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20d}-\frac{523a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20d}-\frac{73a\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{11a\left(\tan^{11}\left(\frac{dx}{2}\right.\right.}{8d}}{\left(1\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*sin(d*x+c)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/5*sin(d*x+c)^5-1/3*sin(d*x+c)^3-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x
+c)))+a*(-1/6*(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/
16*d*x+5/16*c))
```

Maxima [A]
time = 0.27, size = 106, normalized size = 0.83

$$-\frac{32\left(6\sin\left(dx+c\right)^5+10\sin\left(dx+c\right)^3-15\log\left(\sin\left(dx+c\right)+1\right)+15\log\left(\sin\left(dx+c\right)-1\right)+30\sin\left(dx+c\right)\right)a-5\left(4\sin\left(2\,dx+2\,c\right)^3+60\,dx+60\,c+9\sin\left(4\,dx+4\,c\right)-48\sin\left(2\,dx+2\,c\right)\right)a}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] -1/960*(32*(6*sin(d*x + c)^5 + 10*sin(d*x + c)^3 - 15*log(sin(d*x + c) + 1)
 + 15*log(sin(d*x + c) - 1) + 30*sin(d*x + c))*a - 5*(4*sin(2*d*x + 2*c)^3
+ 60*d*x + 60*c + 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a)/d
```

Fricas [A]
time = 3.11, size = 102, normalized size = 0.80

$$\frac{75\,adx+120\,a\log\left(\sin\left(dx+c\right)+1\right)-120\,a\log\left(-\sin\left(dx+c\right)+1\right)-\left(40\,a\cos\left(dx+c\right)^5+48\,a\cos\left(dx+c\right)^4-130\,a\cos\left(dx+c\right)^3-176\,a\cos\left(dx+c\right)^2+165\,a\cos\left(dx+c\right)+368\,a\right)\sin\left(dx+c\right)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] 1/240*(75*a*d*x + 120*a*log(sin(d*x + c) + 1) - 120*a*log(-sin(d*x + c) + 1
) - (40*a*cos(d*x + c)^5 + 48*a*cos(d*x + c)^4 - 130*a*cos(d*x + c)^3 - 176
*a*cos(d*x + c)^2 + 165*a*cos(d*x + c) + 368*a)*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \sin^6\left(c+dx\right)\sec\left(c+dx\right)dx + \int \sin^6\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)**6,x)
```

```
[Out] a*(Integral(sin(c + d*x)**6*sec(c + d*x), x) + Integral(sin(c + d*x)**6, x)
)
```

Giac [A]
time = 0.46, size = 146, normalized size = 1.15

$$\frac{75\left(dx+c\right)a + 240\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 240\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\left(165\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+1095\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+3138\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+5118\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+1945\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+315\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{6}}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^6,x, algorithm="giac")
```

```
[Out] 1/240*(75*(d*x + c)*a + 240*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 240*a*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(165*a*tan(1/2*d*x + 1/2*c)^11 + 1095*
a*tan(1/2*d*x + 1/2*c)^9 + 3138*a*tan(1/2*d*x + 1/2*c)^7 + 5118*a*tan(1/2*d
*x + 1/2*c)^5 + 1945*a*tan(1/2*d*x + 1/2*c)^3 + 315*a*tan(1/2*d*x + 1/2*c))
/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

Mupad [B]
time = 1.05, size = 120, normalized size = 0.94

$$\frac{5\,a\,x}{16} + \frac{2\,a\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} - \frac{15\,a\,\sin\left(2\,c+2\,d\,x\right)}{64\,d} + \frac{7\,a\,\sin\left(3\,c+3\,d\,x\right)}{48\,d} + \frac{3\,a\,\sin\left(4\,c+4\,d\,x\right)}{64\,d} - \frac{a\,\sin\left(5\,c+5\,d\,x\right)}{80\,d} - \frac{a\,\sin\left(6\,c+6\,d\,x\right)}{192\,d} - \frac{11\,a\,\sin\left(c+d\,x\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6*(a + a/cos(c + d*x)),x)
```

```
[Out] (5*a*x)/16 + (2*a*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (15*a*s
in(2*c + 2*d*x))/(64*d) + (7*a*sin(3*c + 3*d*x))/(48*d) + (3*a*sin(4*c + 4*
d*x))/(64*d) - (a*sin(5*c + 5*d*x))/(80*d) - (a*sin(6*c + 6*d*x))/(192*d) -
 (11*a*sin(c + d*x))/(8*d)
```

3.12 $\int (a + a\sec(c + dx))\sin^4(c + dx)\,dx$

Optimal. Leaf size=89

$$\frac{3ax}{8}+\frac{a\tanh^{-1}(\sin(c+dx))}{d}-\frac{a\sin(c+dx)}{d}-\frac{3a\cos(c+dx)\sin(c+dx)}{8d}-\frac{a\sin^3(c+dx)}{3d}-\frac{a\cos(c+dx)\sin^3(c+dx)}{4d}$$

```
[Out] 3/8*a*x+a*arctanh(sin(d*x+c))/d-a*sin(d*x+c)/d-3/8*a*cos(d*x+c)*sin(d*x+c)/
d-1/3*a*sin(d*x+c)^3/d-1/4*a*cos(d*x+c)*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3957, 2917, 2672, 308, 212, 2715, 8}

$$-\frac{a\sin^3(c+dx)}{3d}-\frac{a\sin(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{d}-\frac{a\sin^3(c+dx)\cos(c+dx)}{4d}-\frac{3a\sin(c+dx)\cos(c+dx)}{8d}+\frac{3ax}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Sin[c + d*x]^4,x]
```

```
[Out] (3*a*x)/8 + (a*ArcTanh[Sin[c + d*x]])/d - (a*Sin[c + d*x])/d - (3*a*Cos[c +
 d*x]*Sin[c + d*x])/(8*d) - (a*Sin[c + d*x]^3)/(3*d) - (a*Cos[c + d*x]*Sin[
c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))\sin^4(c+dx)\,dx &= -\int (-a-a\cos(c+dx))\sin^3(c+dx)\tan(c+dx)\,dx\\ &= a\int \sin^4(c+dx)\,dx + a\int \sin^3(c+dx)\tan(c+dx)\,dx\\ &= -\frac{a\cos(c+dx)\sin^3(c+dx)}{4d}+\frac{1}{4}(3a)\int \sin^2(c+dx)\,dx+\frac{a\text{Subst}\Big(\dots}{\dots}\\ &= -\frac{3a\cos(c+dx)\sin(c+dx)}{8d}-\frac{a\cos(c+dx)\sin^3(c+dx)}{4d}+\frac{1}{8}(3a)\dots\\ &= \frac{3ax}{8}-\frac{a\sin(c+dx)}{d}-\frac{3a\cos(c+dx)\sin(c+dx)}{8d}-\frac{a\sin^3(c+dx)}{3d}\dots\\ &= \frac{3ax}{8}+\frac{a\tanh^{-1}(\sin(c+dx))}{d}-\frac{a\sin(c+dx)}{d}-\frac{3a\cos(c+dx)\sin(\dots}{8d}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 86, normalized size = 0.97

$$\frac{3a(c+dx)}{8d}+\frac{a\tanh^{-1}(\sin(c+dx))}{d}-\frac{a\sin(c+dx)}{d}-\frac{a\sin^3(c+dx)}{3d}-\frac{a\sin(2(c+dx))}{4d}+\frac{a\sin(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Sin[c + d*x]^4,x]
```

```
[Out] (3*a*(c + d*x))/(8*d) + (a*ArcTanh[Sin[c + d*x]])/d - (a*Sin[c + d*x])/d -
(a*Sin[c + d*x]^3)/(3*d) - (a*Sin[2*(c + d*x)])/(4*d) + (a*Sin[4*(c + d*x)]
)/(32*d)
```

Maple [A]
time = 0.13, size = 76, normalized size = 0.85

method	result
derivativedivides	$\frac{a\left(-\frac{\left(\sin^{3}(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(-\frac{\left(\sin^{3}(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
default	$\frac{a\left(-\frac{\left(\sin^{3}(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(-\frac{\left(\sin^{3}(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
risch	$\frac{3ax}{8}+\frac{5ia\,e^{i(dx+c)}}{8d}-\frac{5ia\,e^{-i(dx+c)}}{8d}+\frac{a\ln\left(e^{i(dx+c)}+i\right)}{d}-\frac{a\ln\left(e^{i(dx+c)}-i\right)}{d}+\frac{a\sin(4dx+4c)}{32d}+\frac{a\sin(3dx+3c)}{12d}-$
norman	$\frac{\frac{3ax}{8}-\frac{11a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{137a\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}-\frac{71a\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}-\frac{5a\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{3ax\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{9ax\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\ldots\right.}{4}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*sin(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/3*sin(d*x+c)^3-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+a*(-1/4*(si
n(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A]
time = 0.27, size = 81, normalized size = 0.91

$$-\frac{16\left(2\sin\left(dx+c\right)^{3}-3\log\left(\sin\left(dx+c\right)+1\right)+3\log\left(\sin\left(dx+c\right)-1\right)+6\sin\left(dx+c\right)\right)a-3\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)-8\sin\left(2\,dx+2\,c\right)\right)a}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/96*(16*(2*sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c)
- 1) + 6*sin(d*x + c))*a - 3*(12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*d*
x + 2*c))*a)/d
```

Fricas [A]
time = 4.53, size = 79, normalized size = 0.89

$$\frac{9\,adx+12\,a\log\left(\sin\left(dx+c\right)+1\right)-12\,a\log\left(-\sin\left(dx+c\right)+1\right)+\left(6\,a\cos\left(dx+c\right)^{3}+8\,a\cos\left(dx+c\right)^{2}-15\,a\cos\left(dx+c\right)-32\,a\right)\sin\left(dx+c\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/24*(9*a*d*x + 12*a*log(sin(d*x + c) + 1) - 12*a*log(-sin(d*x + c) + 1) +
(6*a*cos(d*x + c)^3 + 8*a*cos(d*x + c)^2 - 15*a*cos(d*x + c) - 32*a)*sin(d*
x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \sin^4\left(c+dx\right)\sec\left(c+dx\right)dx+\int \sin^4\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)**4,x)
```

```
[Out] a*(Integral(sin(c + d*x)**4*sec(c + d*x), x) + Integral(sin(c + d*x)**4, x)
)
```

Giac [A]
time = 0.47, size = 118, normalized size = 1.33

$$\frac{9\left(dx+c\right)a+24\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-24\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+71\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+137\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+33\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/24*(9*(d*x + c)*a + 24*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 24*a*log(ab
s(tan(1/2*d*x + 1/2*c) - 1)) - 2*(15*a*tan(1/2*d*x + 1/2*c)^7 + 71*a*tan(1/
2*d*x + 1/2*c)^5 + 137*a*tan(1/2*d*x + 1/2*c)^3 + 33*a*tan(1/2*d*x + 1/2*c)
)/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

Mupad [B]
time = 1.03, size = 90, normalized size = 1.01

$$\frac{3\,a\,x}{8}+\frac{2\,a\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{a\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{a\,\sin\left(3\,c+3\,d\,x\right)}{12\,d}+\frac{a\,\sin\left(4\,c+4\,d\,x\right)}{32\,d}-\frac{5\,a\,\sin\left(c+d\,x\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4*(a + a/cos(c + d*x)),x)
```

```
[Out] (3*a*x)/8 + (2*a*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (a*sin(2
*c + 2*d*x))/(4*d) + (a*sin(3*c + 3*d*x))/(12*d) + (a*sin(4*c + 4*d*x))/(32
*d) - (5*a*sin(c + d*x))/(4*d)
```

3.13 $\int (a + a\sec(c + dx))\sin^2(c + dx)\,dx$

Optimal. Leaf size=51

$$\frac{ax}{2} + \frac{a\tanh^{-1}(\sin(c + dx))}{d} - \frac{a\sin(c + dx)}{d} - \frac{a\cos(c + dx)\sin(c + dx)}{2d}$$

```
[Out] 1/2*a*x+a*arctanh(sin(d*x+c))/d-a*sin(d*x+c)/d-1/2*a*cos(d*x+c)*sin(d*x+c)/
d
```

Rubi [A]
time = 0.06, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3957, 2917, 2672, 327, 212, 2715, 8}

$$-\frac{a\sin(c + dx)}{d} + \frac{a\tanh^{-1}(\sin(c + dx))}{d} - \frac{a\sin(c + dx)\cos(c + dx)}{2d} + \frac{ax}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Sin[c + d*x]^2,x]
```

```
[Out] (a*x)/2 + (a*ArcTanh[Sin[c + d*x]])/d - (a*Sin[c + d*x])/d - (a*Cos[c + d*x
]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
```

```
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int (a+a\sec(c+dx))\sin^2(c+dx)\,dx &= -\int (-a-a\cos(c+dx))\sin(c+dx)\tan(c+dx)\,dx \\
&= a\int \sin^2(c+dx)\,dx + a\int \sin(c+dx)\tan(c+dx)\,dx \\
&= -\frac{a\cos(c+dx)\sin(c+dx)}{2d} + \frac{1}{2}a\int 1\,dx + \frac{a\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \sin \right.}{d} \\
&= \frac{ax}{2} - \frac{a\sin(c+dx)}{d} - \frac{a\cos(c+dx)\sin(c+dx)}{2d} + \frac{a\text{Subst}\left(\int \frac{1}{1-x^2}\,d\right.}{d} \\
&= \frac{ax}{2} + \frac{a\tanh^{-1}(\sin(c+dx))}{d} - \frac{a\sin(c+dx)}{d} - \frac{a\cos(c+dx)\sin(c-}{2d}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 54, normalized size = 1.06

$$
\frac{a(c+dx)}{2d} + \frac{a\tanh^{-1}(\sin(c+dx))}{d} - \frac{a\sin(c+dx)}{d} - \frac{a\sin(2(c+dx))}{4d}
$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[c + d*x])*Sin[c + d*x]^2,x]

[Out] (a*(c + d*x))/(2*d) + (a*ArcTanh[Sin[c + d*x]])/d - (a*Sin[c + d*x])/d - (a*Sin[2*(c + d*x)])/(4*d)

Maple [A]
time = 0.07, size = 55, normalized size = 1.08

method	result
derivativedivides	$\frac{a(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+a\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
default	$\frac{a(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+a\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
risch	$\frac{ax}{2}+\frac{ia\,e^{i(dx+c)}}{2d}-\frac{ia\,e^{-i(dx+c)}}{2d}-\frac{a\ln\left(e^{i(dx+c)}-i\right)}{d}+\frac{a\ln\left(e^{i(dx+c)}+i\right)}{d}-\frac{a\sin(2dx+2c)}{4d}$
norman	$\frac{ax\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{ax}{2}-\frac{3a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{ax\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{a\ln\left(\tan\left(\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))*sin(d*x+c)^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(a*(-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+a*(-1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))

Maxima [A]
time = 0.27, size = 59, normalized size = 1.16

$$\frac{(2\,dx+2\,c-\sin{(2\,dx+2\,c)})a+2\,a(\log{(\sin{(dx+c)}+1)}-\log{(\sin{(dx+c)}-1)}-2\,\sin{(dx+c)})}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^2,x, algorithm="maxima")

[Out] 1/4*((2*d*x + 2*c - sin(2*d*x + 2*c))*a + 2*a*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x + c)))/d

Fricas [A]
time = 2.64, size = 55, normalized size = 1.08

$$\frac{adx+a\log{(\sin{(dx+c)}+1)}-a\log{(-\sin{(dx+c)}+1)}-(a\cos{(dx+c)}+2\,a)\sin{(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^2,x, algorithm="fricas")

[Out] 1/2*(a*d*x + a*log(sin(d*x + c) + 1) - a*log(-sin(d*x + c) + 1) - (a*cos(d*x + c) + 2*a)*sin(d*x + c))/d

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \sin^{2}\left(c+dx\right)\sec\left(c+dx\right)dx+\int \sin^{2}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)**2,x)
```

```
[Out] a*(Integral(sin(c + d*x)**2*sec(c + d*x), x) + Integral(sin(c + d*x)**2, x)
)
```

Giac [A]
time = 0.44, size = 88, normalized size = 1.73

$$\frac{(dx+c)a+2\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-2\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*((d*x + c)*a + 2*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*a*log(abs(tan
(1/2*d*x + 1/2*c) - 1)) - 2*(a*tan(1/2*d*x + 1/2*c)^3 + 3*a*tan(1/2*d*x + 1
/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

Mupad [B]
time = 1.07, size = 80, normalized size = 1.57

$$\frac{a\,x}{2}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}+3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)}+\frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2*(a + a/cos(c + d*x)),x)
```

```
[Out] (a*x)/2 - (3*a*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^3)/(d*(2*tan(c/2 +
 (d*x)/2)^2 + tan(c/2 + (d*x)/2)^4 + 1)) + (2*a*atanh(tan(c/2 + (d*x)/2)))/
d
```

3.14 $\int \csc^2(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=37

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d}-\frac{a\cot(c+dx)}{d}-\frac{a\csc(c+dx)}{d}$$

[Out] a*arctanh(sin(d*x+c))/d-a*cot(d*x+c)/d-a*csc(d*x+c)/d

Rubi [A]
time = 0.08, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.368$, Rules used = {3957, 2917, 2701, 327, 213, 3852, 8}

$$-\frac{a\cot(c+dx)}{d}-\frac{a\csc(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[Csc[c + d*x]^2*(a + a*Sec[c + d*x]),x]

[Out] (a*ArcTanh[Sin[c + d*x]])/d - (a*Cot[c + d*x])/d - (a*Csc[c + d*x])/d

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc^2(c+dx)(a+a\sec(c+dx))\,dx = -\int (-a-a\cos(c+dx))\csc^2(c+dx)\sec(c+dx)\,dx$$
$$= a\int \csc^2(c+dx)\,dx + a\int \csc^2(c+dx)\sec(c+dx)\,dx$$
$$= -\frac{a\text{Subst}\left(\int 1\,dx, x, \cot(c+dx)\right)}{d} - \frac{a\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \csc(c+d\right)}{d}$$
$$= -\frac{a\cot(c+dx)}{d} - \frac{a\csc(c+dx)}{d} - \frac{a\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \csc(c+dx\right)}{d}$$
$$= \frac{a\tanh^{-1}(\sin(c+dx))}{d} - \frac{a\cot(c+dx)}{d} - \frac{a\csc(c+dx)}{d}$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.02, size = 41, normalized size = 1.11

$$-\frac{a\cot(c+dx)}{d} - \frac{a\csc(c+dx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\sin^2(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*(a + a*Sec[c + d*x]),x]
```

[Out] -((a*Cot[c + d*x])/d) - (a*Csc[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, Sin[c + d*x]^2])/d

Maple [A]
time = 0.10, size = 42, normalized size = 1.14

method	result	size
derivativedivides	$\frac{a\left(-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)-a\cot(dx+c)}{d}$	42
default	$\frac{a\left(-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)-a\cot(dx+c)}{d}$	42
norman	$-\frac{a}{d\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}$	54
risch	$-\frac{2ia}{d\left(e^{i(dx+c)}-1\right)}-\frac{a\ln\left(e^{i(dx+c)}-i\right)}{d}+\frac{a\ln\left(e^{i(dx+c)}+i\right)}{d}$	59

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^2*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d*(a*(-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))-a*cot(d*x+c))

Maxima [A]
time = 0.27, size = 50, normalized size = 1.35

$$-\frac{a\left(\frac{2}{\sin(dx+c)}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+\frac{2\,a}{\tan(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="maxima")

[Out] -1/2*(a*(2/sin(d*x + c) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 2*a/tan(d*x + c))/d

Fricas [A]
time = 3.93, size = 63, normalized size = 1.70

$$\frac{a\log\left(\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)-a\log\left(-\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)-2\,a\cos\left(dx+c\right)-2\,a}{2\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/2*(a*log(sin(d*x + c) + 1)*sin(d*x + c) - a*log(-sin(d*x + c) + 1)*sin(d*x + c) - 2*a*cos(d*x + c) - 2*a)/(d*sin(d*x + c))

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \csc^2(c+dx)\sec(c+dx)\,dx + \int \csc^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+a*sec(d*x+c)),x)
[Out] a*(Integral(csc(c + d*x)**2*sec(c + d*x), x) + Integral(csc(c + d*x)**2, x)
)
```

Giac [A]
time = 0.62, size = 50, normalized size = 1.35

$$\frac{a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] (a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan(1/2*d*x + 1/2*c) - 1)
) - a/tan(1/2*d*x + 1/2*c))/d
```

Mupad [B]
time = 0.96, size = 29, normalized size = 0.78

$$\frac{a\left(2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/sin(c + d*x)^2,x)
[Out] (a*(2*atanh(tan(c/2 + (d*x)/2)) - cot(c/2 + (d*x)/2)))/d
```

3.15 $\int \csc^4(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=69

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d}-\frac{a\cot(c+dx)}{d}-\frac{a\cot^3(c+dx)}{3d}-\frac{a\csc(c+dx)}{d}-\frac{a\csc^3(c+dx)}{3d}$$

```
[Out] a*arctanh(sin(d*x+c))/d-a*cot(d*x+c)/d-1/3*a*cot(d*x+c)^3/d-a*csc(d*x+c)/d-
1/3*a*csc(d*x+c)^3/d
```

Rubi [A]
time = 0.09, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.316$, Rules used = {3957, 2917, 2701, 308, 213, 3852}

$$-\frac{a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}-\frac{a\csc^3(c+dx)}{3d}-\frac{a\csc(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d - (a*Cot[c + d*x])/d - (a*Cot[c + d*x]^3)/(3*d)
 - (a*Csc[c + d*x])/d - (a*Csc[c + d*x]^3)/(3*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
```

```
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \csc^4(c+dx)(a+a\sec(c+dx))\,dx &= -\int (-a-a\cos(c+dx))\csc^4(c+dx)\sec(c+dx)\,dx \\
&= a\int \csc^4(c+dx)\,dx + a\int \csc^4(c+dx)\sec(c+dx)\,dx \\
&= -\frac{a\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{d} - \frac{a\text{Subst}\left(\int \left(1+x^2\right)\,dx, x, c\right.}{d} \\
&= -\frac{a\cot(c+dx)}{d} - \frac{a\cot^3(c+dx)}{3d} - \frac{a\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)\,dx\right.}{d} \\
&= -\frac{a\cot(c+dx)}{d} - \frac{a\cot^3(c+dx)}{3d} - \frac{a\csc(c+dx)}{d} - \frac{a\csc^3(c+dx)}{3d} \\
&= \frac{a\tanh^{-1}(\sin(c+dx))}{d} - \frac{a\cot(c+dx)}{d} - \frac{a\cot^3(c+dx)}{3d} - \frac{a\csc(c}{d}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.02, size = 69, normalized size = 1.00

$$-\frac{2a\cot(c+dx)}{3d} - \frac{a\cot(c+dx)\csc^2(c+dx)}{3d} - \frac{a\csc^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\sin^2(c+dx)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*(a + a*Sec[c + d*x]),x]
```

```
[Out] (-2*a*Cot[c + d*x])/(3*d) - (a*Cot[c + d*x]*Csc[c + d*x]^2)/(3*d) - (a*Csc[
c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, Sin[c + d*x]^2])/(3*d)
```

Maple [A]
time = 0.11, size = 63, normalized size = 0.91

method	result	size
derivativedivides	$\frac{a\left(-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(-\frac{2}{3}-\frac{\left(\csc^2(dx+c)\right)}{3}\right)\cot(dx+c)}{d}$	63
default	$\frac{a\left(-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(-\frac{2}{3}-\frac{\left(\csc^2(dx+c)\right)}{3}\right)\cot(dx+c)}{d}$	63
norman	$\frac{-\frac{a}{12d}-\frac{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}$	90
risch	$-\frac{2ia\left(3\,\mathrm{e}^{3i(dx+c)}-6\,\mathrm{e}^{2i(dx+c)}-\mathrm{e}^{i(dx+c)}+2\right)}{3d\left(\mathrm{e}^{i(dx+c)}-1\right)^3\left(\mathrm{e}^{i(dx+c)}+1\right)}+\frac{a\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{d}-\frac{a\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{d}$	107

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/3/sin(d*x+c)^3-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+a*(-2/3-1
/3*csc(d*x+c)^2)*cot(d*x+c))
```

Maxima [A]
time = 0.29, size = 76, normalized size = 1.10

$$-\frac{a\left(\frac{2\left(3\sin\left(dx+c\right)^2+1\right)}{\sin\left(dx+c\right)^3}-3\log\left(\sin\left(dx+c\right)+1\right)+3\log\left(\sin\left(dx+c\right)-1\right)\right)+\frac{2\left(3\tan\left(dx+c\right)^2+1\right)a}{\tan\left(dx+c\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6*(a*(2*(3*sin(d*x + c)^2 + 1)/sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1)
+ 3*log(sin(d*x + c) - 1)) + 2*(3*tan(d*x + c)^2 + 1)*a/tan(d*x + c)^3)/d
```

Fricas [A]
time = 3.17, size = 108, normalized size = 1.57

$$-\frac{4\,a\cos\left(dx+c\right)^2-3\,\left(a\cos\left(dx+c\right)-a\right)\log\left(\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)+3\,\left(a\cos\left(dx+c\right)-a\right)\log\left(-\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)+2\,a\cos\left(dx+c\right)-8\,a}{6\,\left(d\cos\left(dx+c\right)-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(4*a*cos(d*x + c)^2 - 3*(a*cos(d*x + c) - a)*log(sin(d*x + c) + 1)*sin
(d*x + c) + 3*(a*cos(d*x + c) - a)*log(-sin(d*x + c) + 1)*sin(d*x + c) + 2*
a*cos(d*x + c) - 8*a)/((d*cos(d*x + c) - d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \csc^4(c+dx)\sec(c+dx)\,dx+\int \csc^4(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral(csc(c + d*x)**4*sec(c + d*x), x) + Integral(csc(c + d*x)**4, x)
)
```

Giac [A]
time = 0.49, size = 79, normalized size = 1.14

$$\frac{12\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-12\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{12\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(12*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 12*a*log(abs(tan(1/2*d*x +
1/2*c) - 1)) - 3*a*tan(1/2*d*x + 1/2*c) - (12*a*tan(1/2*d*x + 1/2*c)^2 + a)
/tan(1/2*d*x + 1/2*c)^3)/d
```

Mupad [B]
time = 0.98, size = 65, normalized size = 0.94

$$\frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{a}{12}}{d\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/sin(c + d*x)^4,x)
```

```
[Out] (2*a*atanh(tan(c/2 + (d*x)/2)))/d - (a/12 + a*tan(c/2 + (d*x)/2)^2)/(d*tan(
c/2 + (d*x)/2)^3) - (a*tan(c/2 + (d*x)/2))/(4*d)
```

3.16 $\int \csc^6(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=101

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d}-\frac{a\cot(c+dx)}{d}-\frac{2a\cot^3(c+dx)}{3d}-\frac{a\cot^5(c+dx)}{5d}-\frac{a\csc(c+dx)}{d}-\frac{a\csc^3(c+dx)}{3d}-\frac{a\,\text{cs}}{}$$

```
[Out] a*arctanh(sin(d*x+c))/d-a*cot(d*x+c)/d-2/3*a*cot(d*x+c)^3/d-1/5*a*cot(d*x+c
)^5/d-a*csc(d*x+c)/d-1/3*a*csc(d*x+c)^3/d-1/5*a*csc(d*x+c)^5/d
```

Rubi [A]
time = 0.09, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3957, 2917, 2701, 308, 213, 3852}

$$-\frac{a\cot^5(c+dx)}{5d}-\frac{2a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}-\frac{a\csc^5(c+dx)}{5d}-\frac{a\csc^3(c+dx)}{3d}-\frac{a\csc(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d - (a*Cot[c + d*x])/d - (2*a*Cot[c + d*x]^3)/(3*
d) - (a*Cot[c + d*x]^5)/(5*d) - (a*Csc[c + d*x])/d - (a*Csc[c + d*x]^3)/(3*
d) - (a*Csc[c + d*x]^5)/(5*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
```

```
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \csc^6(c+dx)(a+a\sec(c+dx))\,dx &= -\int (-a-a\cos(c+dx))\csc^6(c+dx)\sec(c+dx)\,dx \\
&= a\int \csc^6(c+dx)\,dx + a\int \csc^6(c+dx)\sec(c+dx)\,dx \\
&= -\frac{a\text{Subst}\left(\int \frac{x^6}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{d} - \frac{a\text{Subst}\left(\int \left(1+2x^2+x^4\right)\right.}{d} \\
&= -\frac{a\cot(c+dx)}{d} - \frac{2a\cot^3(c+dx)}{3d} - \frac{a\cot^5(c+dx)}{5d} - \frac{a\text{Subst}\left(\int (1\right.}{} \\
&= -\frac{a\cot(c+dx)}{d} - \frac{2a\cot^3(c+dx)}{3d} - \frac{a\cot^5(c+dx)}{5d} - \frac{a\csc(c+dx}{d} \\
&= \frac{a\tanh^{-1}(\sin(c+dx))}{d} - \frac{a\cot(c+dx)}{d} - \frac{2a\cot^3(c+dx)}{3d} - \frac{a\cot^5(}{5}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.02, size = 91, normalized size = 0.90

$$-\frac{8a\cot(c+dx)}{15d} - \frac{4a\cot(c+dx)\csc^2(c+dx)}{15d} - \frac{a\cot(c+dx)\csc^4(c+dx)}{5d} - \frac{a\csc^5(c+dx)\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};\sin^2(c+dx)\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6*(a + a*Sec[c + d*x]),x]
```

```
[Out] (-8*a*Cot[c + d*x])/(15*d) - (4*a*Cot[c + d*x]*Csc[c + d*x]^2)/(15*d) - (a*
Cot[c + d*x]*Csc[c + d*x]^4)/(5*d) - (a*Csc[c + d*x]^5*Hypergeometric2F1[-5
/2, 1, -3/2, Sin[c + d*x]^2])/(5*d)
```

Maple [A]
time = 0.12, size = 83, normalized size = 0.82

method	result
derivativedivides	$\frac{a\left(-\frac{1}{5\sin(dx+c)^5}-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(-\frac{8}{15}-\frac{\left(\csc^4(dx+c)\right)}{5}-\frac{4\left(\csc^2(dx+c)\right)}{15}\right)\cot(dx\cdots}{d}$
default	$\frac{a\left(-\frac{1}{5\sin(dx+c)^5}-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(-\frac{8}{15}-\frac{\left(\csc^4(dx+c)\right)}{5}-\frac{4\left(\csc^2(dx+c)\right)}{15}\right)\cot(dx\cdots}{d}$
norman	$\frac{-\frac{a}{80d}-\frac{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{3a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{a\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{48d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{a\ln\left(\tan\left(\cdots\right.\right.}{}$
risch	$-\frac{2ia\left(15\,\mathrm{e}^{7i(dx+c)}-30\,\mathrm{e}^{6i(dx+c)}-35\,\mathrm{e}^{5i(dx+c)}+100\,\mathrm{e}^{4i(dx+c)}+13\,\mathrm{e}^{3i(dx+c)}-46\,\mathrm{e}^{2i(dx+c)}-\mathrm{e}^{i(dx+c)}+8\right)}{15d\left(\mathrm{e}^{i(dx+c)}-1\right)^5\left(\mathrm{e}^{i(dx+c)}+1\right)^3}-\frac{a\ln\left(\mathrm{e}^{i(dx+c)}\cdots\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/5/sin(d*x+c)^5-1/3/sin(d*x+c)^3-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d
*x+c)))+a*(-8/15-1/5*csc(d*x+c)^4-4/15*csc(d*x+c)^2)*cot(d*x+c))
```

Maxima [A]
time = 0.28, size = 96, normalized size = 0.95

$$-\frac{a\left(\frac{2\left(15\sin(dx+c)^4+5\sin(dx+c)^2+3\right)}{\sin(dx+c)^5}-15\log\left(\sin\left(dx+c\right)+1\right)+15\log\left(\sin\left(dx+c\right)-1\right)\right)+\frac{2\left(15\tan(dx+c)^4+10\tan(dx+c)^2+3\right)a}{\tan(dx+c)^5}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/30*(a*(2*(15*sin(d*x + c)^4 + 5*sin(d*x + c)^2 + 3)/sin(d*x + c)^5 - 15*
log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) + 2*(15*tan(d*x + c)^4 +
10*tan(d*x + c)^2 + 3)*a/tan(d*x + c)^5)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(93) = 186$.
time = 3.30, size = 190, normalized size = 1.88

$$-\frac{16\,a\cos\left(dx+c\right)^4+14\,a\cos\left(dx+c\right)^3-54\,a\cos\left(dx+c\right)^2-15\left(a\cos\left(dx+c\right)^3-a\cos\left(dx+c\right)^2-a\cos\left(dx+c\right)+a\right)\log\left(\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)+15\left(a\cos\left(dx+c\right)^3-a\cos\left(dx+c\right)^2-a\cos\left(dx+c\right)+a\right)\log\left(-\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)-16\,a\cos\left(dx+c\right)+46\,a}{30\left(d\cos\left(dx+c\right)^3-d\cos\left(dx+c\right)^2-d\cos\left(dx+c\right)+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/30*(16*a*cos(d*x + c)^4 + 14*a*cos(d*x + c)^3 - 54*a*cos(d*x + c)^2 - 15
*(a*cos(d*x + c)^3 - a*cos(d*x + c)^2 - a*cos(d*x + c) + a)*log(sin(d*x + c
```

```
) + 1)*sin(d*x + c) + 15*(a*cos(d*x + c)^3 - a*cos(d*x + c)^2 - a*cos(d*x +
 c) + a)*log(-sin(d*x + c) + 1)*sin(d*x + c) - 16*a*cos(d*x + c) + 46*a)/((
d*cos(d*x + c)^3 - d*cos(d*x + c)^2 - d*cos(d*x + c) + d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \csc^{6}\left(c+dx\right)\sec\left(c+dx\right)dx+\int \csc^{6}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral(csc(c + d*x)**6*sec(c + d*x), x) + Integral(csc(c + d*x)**6, x)
)
```

Giac [A]
time = 0.56, size = 107, normalized size = 1.06

$$-\frac{5\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-240\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)+240\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+90\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{3\left(80\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+10\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/240*(5*a*tan(1/2*d*x + 1/2*c)^3 - 240*a*log(abs(tan(1/2*d*x + 1/2*c) + 1
)) + 240*a*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 90*a*tan(1/2*d*x + 1/2*c) +
 3*(80*a*tan(1/2*d*x + 1/2*c)^4 + 10*a*tan(1/2*d*x + 1/2*c)^2 + a)/tan(1/2*
d*x + 1/2*c)^5)/d
```

Mupad [B]
time = 1.00, size = 97, normalized size = 0.96

$$\frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{3\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}-\frac{a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{48\,d}-\frac{{\mathrm{cot}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(16\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+2\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+\frac{a}{5}\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/sin(c + d*x)^6,x)
```

```
[Out] (2*a*atanh(tan(c/2 + (d*x)/2)))/d - (3*a*tan(c/2 + (d*x)/2))/(8*d) - (a*tan
(c/2 + (d*x)/2)^3)/(48*d) - (cot(c/2 + (d*x)/2)^5*(a/5 + 2*a*tan(c/2 + (d*x
)/2)^2 + 16*a*tan(c/2 + (d*x)/2)^4))/(16*d)
```

3.17 $\int \csc^8(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=131

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d}-\frac{a\cot(c+dx)}{d}-\frac{a\cot^3(c+dx)}{d}-\frac{3a\cot^5(c+dx)}{5d}-\frac{a\cot^7(c+dx)}{7d}-\frac{a\csc(c+dx)}{d}-\frac{a\,\mathrm{cs}}{}$$

```
[Out] a*arctanh(sin(d*x+c))/d-a*cot(d*x+c)/d-a*cot(d*x+c)^3/d-3/5*a*cot(d*x+c)^5/
d-1/7*a*cot(d*x+c)^7/d-a*csc(d*x+c)/d-1/3*a*csc(d*x+c)^3/d-1/5*a*csc(d*x+c)
^5/d-1/7*a*csc(d*x+c)^7/d
```

Rubi [A]
time = 0.09, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3957, 2917, 2701, 308, 213, 3852}

$$-\frac{a\cot^7(c+dx)}{7d}-\frac{3a\cot^5(c+dx)}{5d}-\frac{a\cot^3(c+dx)}{d}-\frac{a\cot(c+dx)}{d}-\frac{a\csc^7(c+dx)}{7d}-\frac{a\csc^5(c+dx)}{5d}-\frac{a\csc^3(c+dx)}{3d}-\frac{a\csc(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^8*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d - (a*Cot[c + d*x])/d - (a*Cot[c + d*x]^3)/d - (
3*a*Cot[c + d*x]^5)/(5*d) - (a*Cot[c + d*x]^7)/(7*d) - (a*Csc[c + d*x])/d -
 (a*Csc[c + d*x]^3)/(3*d) - (a*Csc[c + d*x]^5)/(5*d) - (a*Csc[c + d*x]^7)/(
7*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc^8(c+dx)(a+a\sec(c+dx))\,dx = -\int (-a-a\cos(c+dx))\csc^8(c+dx)\sec(c+dx)\,dx$$
$$= a\int \csc^8(c+dx)\,dx + a\int \csc^8(c+dx)\sec(c+dx)\,dx$$
$$= -\frac{a\text{Subst}\left(\int \frac{x^8}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{d} - \frac{a\text{Subst}\left(\int (1+3x^2+3x^4 -\right.}{d}$$
$$= -\frac{a\cot(c+dx)}{d} - \frac{a\cot^3(c+dx)}{d} - \frac{3a\cot^5(c+dx)}{5d} - \frac{a\cot^7(c+dx}{7d}$$
$$= -\frac{a\cot(c+dx)}{d} - \frac{a\cot^3(c+dx)}{d} - \frac{3a\cot^5(c+dx)}{5d} - \frac{a\cot^7(c+dx}{7d}$$
$$= \frac{a\tanh^{-1}(\sin(c+dx))}{d} - \frac{a\cot(c+dx)}{d} - \frac{a\cot^3(c+dx)}{d} - \frac{3a\cot^5(}{5}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.04, size = 113, normalized size = 0.86

$$-\frac{16a\cot(c+dx)}{35d} - \frac{8a\cot(c+dx)\csc^2(c+dx)}{35d} - \frac{6a\cot(c+dx)\csc^4(c+dx)}{35d} - \frac{a\cot(c+dx)\csc^6(c+dx)}{7d} - \frac{a\csc^7(c+dx)\,{}_2F_1\left(-\frac{7}{2},1;-\frac{5}{2};\sin^2(c+dx)\right)}{7d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^8*(a + a*Sec[c + d*x]),x]
```

```
[Out] (-16*a*Cot[c + d*x])/(35*d) - (8*a*Cot[c + d*x]*Csc[c + d*x]^2)/(35*d) - (6
*a*Cot[c + d*x]*Csc[c + d*x]^4)/(35*d) - (a*Cot[c + d*x]*Csc[c + d*x]^6)/(7
*d) - (a*Csc[c + d*x]^7*Hypergeometric2F1[-7/2, 1, -5/2, Sin[c + d*x]^2])/(
7*d)
```

Maple [A]
time = 0.15, size = 103, normalized size = 0.79

method	result
derivativedivides	$\frac{a\left(-\frac{1}{7\sin(dx+c)^7}-\frac{1}{5\sin(dx+c)^5}-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(-\frac{16}{35}-\frac{\left(\csc^6(dx+c)\right)}{7}-\frac{6\left(\csc^4(dx\right.}{35}\right.}{d}$
default	$\frac{a\left(-\frac{1}{7\sin(dx+c)^7}-\frac{1}{5\sin(dx+c)^5}-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(-\frac{16}{35}-\frac{\left(\csc^6(dx+c)\right)}{7}-\frac{6\left(\csc^4(dx\right.}{35}\right.}{d}$
norman	$\frac{-\frac{a}{448d}-\frac{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{40d}-\frac{29a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192d}-\frac{a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{29a\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}-\frac{a\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}-\frac{a\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{320d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}$
risch	$-\frac{2ia\left(105\,e^{11i(dx+c)}-210\,e^{10i(dx+c)}-455\,e^{9i(dx+c)}+1120\,e^{8i(dx+c)}+686\,e^{7i(dx+c)}-2492\,e^{6i(dx+c)}-274\,e^{5i(dx+c)}+1360\right.}{105d\left(e^{i(dx+c)}-1\right)^7\left(e^{i(dx+c)}+1\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^8*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/7/sin(d*x+c)^7-1/5/sin(d*x+c)^5-1/3/sin(d*x+c)^3-1/sin(d*x+c)+ln
(sec(d*x+c)+tan(d*x+c)))+a*(-16/35-1/7*csc(d*x+c)^6-6/35*csc(d*x+c)^4-8/35*
csc(d*x+c)^2)*cot(d*x+c))
```

Maxima [A]
time = 0.28, size = 116, normalized size = 0.89

$$-\frac{a\left(\frac{2\left(105\sin(dx+c)^6+35\sin(dx+c)^4+21\sin(dx+c)^2+15\right)}{\sin(dx+c)^7}-105\log\left(\sin\left(dx+c\right)+1\right)+105\log\left(\sin\left(dx+c\right)-1\right)\right)+\frac{6\left(35\tan(dx+c)^6+35\tan(dx+c)^4+21\tan(dx+c)^2+5\right)a}{\tan(dx+c)^7}}{210\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/210*(a*(2*(105*sin(d*x + c)^6 + 35*sin(d*x + c)^4 + 21*sin(d*x + c)^2 +
15)/sin(d*x + c)^7 - 105*log(sin(d*x + c) + 1) + 105*log(sin(d*x + c) - 1))
 + 6*(35*tan(d*x + c)^6 + 35*tan(d*x + c)^4 + 21*tan(d*x + c)^2 + 5)*a/tan(
d*x + c)^7)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 281 vs. $2(121) = 242$.
time = 3.87, size = 281, normalized size = 2.15

$$-\frac{96\,a\cos\left(dx+c\right)^6+114\,a\cos\left(dx+c\right)^5-450\,a\cos\left(dx+c\right)^4-250\,a\cos\left(dx+c\right)^3+670\,a\cos\left(dx+c\right)^2-105\left(a\cos\left(dx+c\right)^5-a\cos\left(dx+c\right)^4-2\,a\cos\left(dx+c\right)^3+2\,a\cos\left(dx+c\right)^2+a\cos\left(dx+c\right)-a\right)\log\left(\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)+105\left(a\cos\left(dx+c\right)^5-a\cos\left(dx+c\right)^4-2\,a\cos\left(dx+c\right)^3+2\,a\cos\left(dx+c\right)^2+a\cos\left(dx+c\right)-a\right)\log\left(-\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)+142\,a\cos\left(dx+c\right)-352\,a}{210\left(d\cos\left(dx+c\right)^5-d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^3+2\,d\cos\left(dx+c\right)^2+d\cos\left(dx+c\right)-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/210*(96*a*cos(d*x + c)^6 + 114*a*cos(d*x + c)^5 - 450*a*cos(d*x + c)^4 -
 250*a*cos(d*x + c)^3 + 670*a*cos(d*x + c)^2 - 105*(a*cos(d*x + c)^5 - a*co
s(d*x + c)^4 - 2*a*cos(d*x + c)^3 + 2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a
)*log(sin(d*x + c) + 1)*sin(d*x + c) + 105*(a*cos(d*x + c)^5 - a*cos(d*x +
c)^4 - 2*a*cos(d*x + c)^3 + 2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a)*log(-s
in(d*x + c) + 1)*sin(d*x + c) + 142*a*cos(d*x + c) - 352*a)/((d*cos(d*x + c
)^5 - d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^3 + 2*d*cos(d*x + c)^2 + d*cos(d*
x + c) - d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**8*(a+a*sec(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [A]
time = 0.52, size = 136, normalized size = 1.04

$$-\frac{21\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+280\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-6720\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)+6720\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+3045\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{6720\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+1015\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+168\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+15\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}}}{6720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6720*(21*a*tan(1/2*d*x + 1/2*c)^5 + 280*a*tan(1/2*d*x + 1/2*c)^3 - 6720*
a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) + 6720*a*log(abs(tan(1/2*d*x + 1/2*c)
- 1)) + 3045*a*tan(1/2*d*x + 1/2*c) + (6720*a*tan(1/2*d*x + 1/2*c)^6 + 1015
*a*tan(1/2*d*x + 1/2*c)^4 + 168*a*tan(1/2*d*x + 1/2*c)^2 + 15*a)/tan(1/2*d*
x + 1/2*c)^7)/d
```

Mupad [B]
time = 1.17, size = 128, normalized size = 0.98

$$\frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{29\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{24\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{320\,d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}\left(64\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+\frac{29\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}}{3}+\frac{8\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{5}+\frac{a}{7}\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/sin(c + d*x)^8,x)
```

```
[Out] (2*a*atanh(tan(c/2 + (d*x)/2)))/d - (29*a*tan(c/2 + (d*x)/2))/(64*d) - (a*t
an(c/2 + (d*x)/2)^3)/(24*d) - (a*tan(c/2 + (d*x)/2)^5)/(320*d) - (cot(c/2 +
 (d*x)/2)^7*(a/7 + (8*a*tan(c/2 + (d*x)/2)^2)/5 + (29*a*tan(c/2 + (d*x)/2)^
4)/3 + 64*a*tan(c/2 + (d*x)/2)^6))/(64*d)
```

3.18 $\int \csc^{10}(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=165

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d}-\frac{a\cot(c+dx)}{d}-\frac{4a\cot^3(c+dx)}{3d}-\frac{6a\cot^5(c+dx)}{5d}-\frac{4a\cot^7(c+dx)}{7d}-\frac{a\cot^9(c+dx)}{9d}-$$

```
[Out] a*arctanh(sin(d*x+c))/d-a*cot(d*x+c)/d-4/3*a*cot(d*x+c)^3/d-6/5*a*cot(d*x+c
)^5/d-4/7*a*cot(d*x+c)^7/d-1/9*a*cot(d*x+c)^9/d-a*csc(d*x+c)/d-1/3*a*csc(d*
x+c)^3/d-1/5*a*csc(d*x+c)^5/d-1/7*a*csc(d*x+c)^7/d-1/9*a*csc(d*x+c)^9/d
```

Rubi [A]
time = 0.10, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3957, 2917, 2701, 308, 213, 3852}

$$-\frac{a\cot^9(c+dx)}{9d}-\frac{4a\cot^7(c+dx)}{7d}-\frac{6a\cot^5(c+dx)}{5d}-\frac{4a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}-\frac{a\csc^9(c+dx)}{9d}-\frac{a\csc^7(c+dx)}{7d}-\frac{a\csc^5(c+dx)}{5d}-\frac{a\csc^3(c+dx)}{3d}-\frac{a\csc(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^10*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d - (a*Cot[c + d*x])/d - (4*a*Cot[c + d*x]^3)/(3*
d) - (6*a*Cot[c + d*x]^5)/(5*d) - (4*a*Cot[c + d*x]^7)/(7*d) - (a*Cot[c + d
*x]^9)/(9*d) - (a*Csc[c + d*x])/d - (a*Csc[c + d*x]^3)/(3*d) - (a*Csc[c + d
*x]^5)/(5*d) - (a*Csc[c + d*x]^7)/(7*d) - (a*Csc[c + d*x]^9)/(9*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \csc^{10}(c+dx)(a+a\sec(c+dx))\,dx &= -\int (-a-a\cos(c+dx))\csc^{10}(c+dx)\sec(c+dx)\,dx \\
&= a\int \csc^{10}(c+dx)\,dx + a\int \csc^{10}(c+dx)\sec(c+dx)\,dx \\
&= -\frac{a\text{Subst}\left(\int \frac{x^{10}}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{d} - \frac{a\text{Subst}\left(\int (1+4x^2+6x^4\right.}{} \\
&= -\frac{a\cot(c+dx)}{d} - \frac{4a\cot^3(c+dx)}{3d} - \frac{6a\cot^5(c+dx)}{5d} - \frac{4a\cot^7(c+}{7d} \\
&= -\frac{a\cot(c+dx)}{d} - \frac{4a\cot^3(c+dx)}{3d} - \frac{6a\cot^5(c+dx)}{5d} - \frac{4a\cot^7(c+}{7d} \\
&= \frac{a\tanh^{-1}(\sin(c+dx))}{d} - \frac{a\cot(c+dx)}{d} - \frac{4a\cot^3(c+dx)}{3d} - \frac{6a\cot}{}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.04, size = 135, normalized size = 0.82

$$-\frac{128a\cot(c+dx)}{315d} - \frac{64a\cot(c+dx)\csc^2(c+dx)}{315d} - \frac{16a\cot(c+dx)\csc^4(c+dx)}{105d} - \frac{8a\cot(c+dx)\csc^6(c+dx)}{63d} - \frac{a\cot(c+dx)\csc^8(c+dx)}{9d} - \frac{a\csc^9(c+dx)\,{}_2F_1\left(-\frac{9}{2},1;-\frac{7}{2};\sin^2(c+dx)\right)}{9d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^10*(a + a*Sec[c + d*x]),x]
```

[Out] (-128*a*Cot[c + d*x])/(315*d) - (64*a*Cot[c + d*x]*Csc[c + d*x]^2)/(315*d) - (16*a*Cot[c + d*x]*Csc[c + d*x]^4)/(105*d) - (8*a*Cot[c + d*x]*Csc[c + d*x]^6)/(63*d) - (a*Cot[c + d*x]*Csc[c + d*x]^8)/(9*d) - (a*Csc[c + d*x]^9*Hypergeometric2F1[-9/2, 1, -7/2, Sin[c + d*x]^2])/(9*d)

Maple [A]
time = 0.14, size = 123, normalized size = 0.75

method	result
derivativedivides	$\frac{a\left(-\frac{1}{9\sin(dx+c)^9}-\frac{1}{7\sin(dx+c)^7}-\frac{1}{5\sin(dx+c)^5}-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(-\frac{128}{315}-\frac{\left(\csc^8(dx+\right.}{9}\right.}{d}$
default	$\frac{a\left(-\frac{1}{9\sin(dx+c)^9}-\frac{1}{7\sin(dx+c)^7}-\frac{1}{5\sin(dx+c)^5}-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(-\frac{128}{315}-\frac{\left(\csc^8(dx+\right.}{9}\right.}{d}$
risch	$-\frac{2ia\left(315\,e^{15i(dx+c)}-630\,e^{14i(dx+c)}-1995\,e^{13i(dx+c)}+4620\,e^{12i(dx+c)}+5103\,e^{11i(dx+c)}-14826\,e^{10i(dx+c)}-6303\,e^{9i(dx+\right.}{315d\left(e^{i(\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^10*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d*(a*(-1/9/sin(d*x+c)^9-1/7/sin(d*x+c)^7-1/5/sin(d*x+c)^5-1/3/sin(d*x+c)^3-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+a*(-128/315-1/9*csc(d*x+c)^8-8/63*csc(d*x+c)^6-16/105*csc(d*x+c)^4-64/315*csc(d*x+c)^2)*cot(d*x+c))

Maxima [A]
time = 0.27, size = 136, normalized size = 0.82

$$-\frac{a\left(\frac{2\left(315\sin(dx+c)^8+105\sin(dx+c)^6+63\sin(dx+c)^4+45\sin(dx+c)^2+35\right)}{\sin(dx+c)^9}-315\log\left(\sin\left(dx+c\right)+1\right)+315\log\left(\sin\left(dx+c\right)-1\right)\right)+\frac{2\left(315\tan(dx+c)^8+420\tan(dx+c)^6+378\tan(dx+c)^4+180\tan(dx+c)^2+35\right)a}{\tan(dx+c)^9}}{630\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^10*(a+a*sec(d*x+c)),x, algorithm="maxima")

[Out] -1/630*(a*(2*(315*sin(d*x + c)^8 + 105*sin(d*x + c)^6 + 63*sin(d*x + c)^4 + 45*sin(d*x + c)^2 + 35)/sin(d*x + c)^9 - 315*log(sin(d*x + c) + 1) + 315*log(sin(d*x + c) - 1)) + 2*(315*tan(d*x + c)^8 + 420*tan(d*x + c)^6 + 378*tan(d*x + c)^4 + 180*tan(d*x + c)^2 + 35)*a/tan(d*x + c)^9)/d

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 366 vs. 2(149) = 298.
time = 4.39, size = 366, normalized size = 2.22

$$-\frac{256\,a\cos\left(dx+c\right)^8+374\,a\cos\left(dx+c\right)^7-1526\,a\cos\left(dx+c\right)^6-1204\,a\cos\left(dx+c\right)^5+3220\,a\cos\left(dx+c\right)^4+1316\,a\cos\left(dx+c\right)^3-2996\,a\cos\left(dx+c\right)^2-315\left(a\cos\left(dx+c\right)^7-a\cos\left(dx+c\right)^6-3\,a\cos\left(dx+c\right)^5+3\,a\cos\left(dx+c\right)^4+3\,a\cos\left(dx+c\right)^3-3\,a\cos\left(dx+c\right)^2-a\cos\left(dx+c\right)+a\right)\log\left(\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)+315\left(a\cos\left(dx+c\right)^7-a\cos\left(dx+c\right)^6-3\,a\cos\left(dx+c\right)^5+3\,a\cos\left(dx+c\right)^4+3\,a\cos\left(dx+c\right)^3-3\,a\cos\left(dx+c\right)^2-a\cos\left(dx+c\right)+a\right)\log\left(-\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)-496\,a\cos\left(dx+c\right)+1126\,a}{630\left(d\cos\left(dx+c\right)^7-d\cos\left(dx+c\right)^6-3\,d\cos\left(dx+c\right)^5+3\,d\cos\left(dx+c\right)^4+3\,d\cos\left(dx+c\right)^3-3\,d\cos\left(dx+c\right)^2-d\cos\left(dx+c\right)+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^10*(a+a*sec(d*x+c)),x, algorithm="fricas")

```
[Out] -1/630*(256*a*cos(d*x + c)^8 + 374*a*cos(d*x + c)^7 - 1526*a*cos(d*x + c)^6
 - 1204*a*cos(d*x + c)^5 + 3220*a*cos(d*x + c)^4 + 1316*a*cos(d*x + c)^3 -
2996*a*cos(d*x + c)^2 - 315*(a*cos(d*x + c)^7 - a*cos(d*x + c)^6 - 3*a*cos(
d*x + c)^5 + 3*a*cos(d*x + c)^4 + 3*a*cos(d*x + c)^3 - 3*a*cos(d*x + c)^2 -
 a*cos(d*x + c) + a)*log(sin(d*x + c) + 1)*sin(d*x + c) + 315*(a*cos(d*x +
c)^7 - a*cos(d*x + c)^6 - 3*a*cos(d*x + c)^5 + 3*a*cos(d*x + c)^4 + 3*a*cos
(d*x + c)^3 - 3*a*cos(d*x + c)^2 - a*cos(d*x + c) + a)*log(-sin(d*x + c) +
1)*sin(d*x + c) - 496*a*cos(d*x + c) + 1126*a)/((d*cos(d*x + c)^7 - d*cos(d
*x + c)^6 - 3*d*cos(d*x + c)^5 + 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^3 -
3*d*cos(d*x + c)^2 - d*cos(d*x + c) + d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**10*(a+a*sec(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [A]
time = 0.60, size = 164, normalized size = 0.99

$$-\frac{45\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+630\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+4830\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-80640\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)+80640\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+40950\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{80640\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+13650\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+2898\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+450\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+35\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}}}{80640\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^10*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/80640*(45*a*tan(1/2*d*x + 1/2*c)^7 + 630*a*tan(1/2*d*x + 1/2*c)^5 + 4830
*a*tan(1/2*d*x + 1/2*c)^3 - 80640*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) + 80
640*a*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 40950*a*tan(1/2*d*x + 1/2*c) + (
80640*a*tan(1/2*d*x + 1/2*c)^8 + 13650*a*tan(1/2*d*x + 1/2*c)^6 + 2898*a*ta
n(1/2*d*x + 1/2*c)^4 + 450*a*tan(1/2*d*x + 1/2*c)^2 + 35*a)/tan(1/2*d*x + 1
/2*c)^9)/d
```

Mupad [B]
time = 1.64, size = 159, normalized size = 0.96

$$\frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{65\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{128\,d}-\frac{23\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{384\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{128\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{1792\,d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}\left(256\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+\frac{130\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}{3}+\frac{46\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}}{5}+\frac{10\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{7}+\frac{a}{9}\right)}{256\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/sin(c + d*x)^10,x)
```

```
[Out] (2*a*atanh(tan(c/2 + (d*x)/2)))/d - (65*a*tan(c/2 + (d*x)/2))/(128*d) - (23
*a*tan(c/2 + (d*x)/2)^3)/(384*d) - (a*tan(c/2 + (d*x)/2)^5)/(128*d) - (a*ta
n(c/2 + (d*x)/2)^7)/(1792*d) - (cot(c/2 + (d*x)/2)^9*(a/9 + (10*a*tan(c/2 +
 (d*x)/2)^2)/7 + (46*a*tan(c/2 + (d*x)/2)^4)/5 + (130*a*tan(c/2 + (d*x)/2)^
6)/3 + 256*a*tan(c/2 + (d*x)/2)^8))/(256*d)
```

3.19 $\int (a+a\sec(c+dx))^2\sin^9(c+dx)\,dx$

Optimal. Leaf size=183

$$\frac{3a^2\cos(c+dx)}{d}+\frac{4a^2\cos^2(c+dx)}{d}-\frac{2a^2\cos^3(c+dx)}{3d}-\frac{3a^2\cos^4(c+dx)}{d}-\frac{2a^2\cos^5(c+dx)}{5d}+\frac{4a^2\cos^6(c+dx)}{3d}$$

```
[Out] 3*a^2*cos(d*x+c)/d+4*a^2*cos(d*x+c)^2/d-2/3*a^2*cos(d*x+c)^3/d-3*a^2*cos(d*
x+c)^4/d-2/5*a^2*cos(d*x+c)^5/d+4/3*a^2*cos(d*x+c)^6/d+3/7*a^2*cos(d*x+c)^7
/d-1/4*a^2*cos(d*x+c)^8/d-1/9*a^2*cos(d*x+c)^9/d-2*a^2*ln(cos(d*x+c))/d+a^2
*sec(d*x+c)/d
```

Rubi [A]
time = 0.14, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3957, 2915, 12, 90}

$$-\frac{a^2\cos^9(c+dx)}{9d}-\frac{a^2\cos^8(c+dx)}{4d}+\frac{3a^2\cos^7(c+dx)}{7d}+\frac{4a^2\cos^6(c+dx)}{3d}-\frac{2a^2\cos^5(c+dx)}{5d}-\frac{3a^2\cos^4(c+dx)}{d}-\frac{2a^2\cos^3(c+dx)}{3d}+\frac{4a^2\cos^2(c+dx)}{d}+\frac{3a^2\cos(c+dx)}{d}+\frac{a^2\sec(c+dx)}{d}-\frac{2a^2\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^9,x]
```

```
[Out] (3*a^2*Cos[c + d*x])/d + (4*a^2*Cos[c + d*x]^2)/d - (2*a^2*Cos[c + d*x]^3)/
(3*d) - (3*a^2*Cos[c + d*x]^4)/d - (2*a^2*Cos[c + d*x]^5)/(5*d) + (4*a^2*Co
s[c + d*x]^6)/(3*d) + (3*a^2*Cos[c + d*x]^7)/(7*d) - (a^2*Cos[c + d*x]^8)/(
4*d) - (a^2*Cos[c + d*x]^9)/(9*d) - (2*a^2*Log[Cos[c + d*x]])/d + (a^2*Sec[
c + d*x])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
```

```
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int (a+a\sec(c+dx))^2\sin^9(c+dx)\,dx &= \int (-a-a\cos(c+dx))^2\sin^7(c+dx)\tan^2(c+dx)\,dx \\
&= \frac{\text{Subst}\left(\int \frac{a^2(-a-x)^4(-a+x)^6}{x^2}\,dx, x, -a\cos(c+dx)\right)}{a^9 d} \\
&= \frac{\text{Subst}\left(\int \frac{(-a-x)^4(-a+x)^6}{x^2}\,dx, x, -a\cos(c+dx)\right)}{a^7 d} \\
&= \frac{\text{Subst}\left(\int \left(-3a^8+\frac{a^{10}}{x^2}-\frac{2a^9}{x}+8a^7x+2a^6x^2-12a^5x^3+2a^4x^4+8\right.\right.}{a^7 d} \\
&= \frac{3a^2\cos(c+dx)}{d}+\frac{4a^2\cos^2(c+dx)}{d}-\frac{2a^2\cos^3(c+dx)}{3d}-\frac{3a^2\cos^4(}{d}
\end{aligned}$$

Mathematica [**A**]
time = 0.58, size = 127, normalized size = 0.69

$$-\frac{a^2(-714420-361620\cos(2(c+dx))-134820\cos(3(c+dx))+29232\cos(4(c+dx))+24780\cos(5(c+dx))-1458\cos(6(c+dx))-3885\cos(7(c+dx))-380\cos(8(c+dx))+315\cos(9(c+dx))+70\cos(10(c+dx))+210\cos(c+dx)(205+3072\log(\cos(c+dx))))\sec(c+dx)}{322560d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^9,x]
[Out] -1/322560*(a^2*(-714420 - 361620*Cos[2*(c + d*x)] - 134820*Cos[3*(c + d*x)]
 + 29232*Cos[4*(c + d*x)] + 24780*Cos[5*(c + d*x)] - 1458*Cos[6*(c + d*x)]
- 3885*Cos[7*(c + d*x)] - 380*Cos[8*(c + d*x)] + 315*Cos[9*(c + d*x)] + 70*
Cos[10*(c + d*x)] + 210*Cos[c + d*x]*(205 + 3072*Log[Cos[c + d*x]]))*Sec[c
+ d*x])/d
```

Maple [**A**]
time = 0.09, size = 181, normalized size = 0.99

method	result

derivativedivides	$\frac{a^2\left(\frac{\sin^{10}(dx+c)}{\cos(dx+c)}+\left(\frac{128}{35}+\sin^8(dx+c)+\frac{8\left(\sin^6(dx+c)\right)}{7}+\frac{48\left(\sin^4(dx+c)\right)}{35}+\frac{64\left(\sin^2(dx+c)\right)}{35}\right)\cos(dx+c)\right)+2a^2\left(-\frac{\left(\sin^8(dx+c\right.}{8}\right.}{}$
default	$\frac{a^2\left(\frac{\sin^{10}(dx+c)}{\cos(dx+c)}+\left(\frac{128}{35}+\sin^8(dx+c)+\frac{8\left(\sin^6(dx+c)\right)}{7}+\frac{48\left(\sin^4(dx+c)\right)}{35}+\frac{64\left(\sin^2(dx+c)\right)}{35}\right)\cos(dx+c)\right)+2a^2\left(-\frac{\left(\sin^8(dx+c\right.}{8}\right.}{}$
risch	$\frac{65a^2e^{-2i(dx+c)}}{128d}+2ia^2x+\frac{4ia^2c}{d}-\frac{a^2\cos(9dx+9c)}{2304d}-\frac{2a^2\ln\left(e^{2i(dx+c)}+1\right)}{d}+\frac{2a^2e^{i(dx+c)}}{d\left(e^{2i(dx+c)}+1\right)}+\frac{311a^2e^{-i(dx+}}{256d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*sin(d*x+c)^9,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(sin(d*x+c)^10/cos(d*x+c)+(128/35+sin(d*x+c)^8+8/7*sin(d*x+c)^6+48
/35*sin(d*x+c)^4+64/35*sin(d*x+c)^2)*cos(d*x+c))+2*a^2*(-1/8*sin(d*x+c)^8-1
/6*sin(d*x+c)^6-1/4*sin(d*x+c)^4-1/2*sin(d*x+c)^2-ln(cos(d*x+c)))-1/9*a^2*(
128/35+sin(d*x+c)^8+8/7*sin(d*x+c)^6+48/35*sin(d*x+c)^4+64/35*sin(d*x+c)^2)
*cos(d*x+c))
```

Maxima [A]
time = 0.28, size = 146, normalized size = 0.80

$$-\frac{140\,a^2\cos(dx+c)^9+315\,a^2\cos(dx+c)^8-540\,a^2\cos(dx+c)^7-1680\,a^2\cos(dx+c)^6+504\,a^2\cos(dx+c)^5+3780\,a^2\cos(dx+c)^4+840\,a^2\cos(dx+c)^3-5040\,a^2\cos(dx+c)^2-3780\,a^2\cos(dx+c)+2520\,a^2\log(\cos(dx+c))-\frac{1260\,a^2}{\cos(dx+c)}}{1260\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^9,x, algorithm="maxima")
```

```
[Out] -1/1260*(140*a^2*cos(d*x + c)^9 + 315*a^2*cos(d*x + c)^8 - 540*a^2*cos(d*x
+ c)^7 - 1680*a^2*cos(d*x + c)^6 + 504*a^2*cos(d*x + c)^5 + 3780*a^2*cos(d*
x + c)^4 + 840*a^2*cos(d*x + c)^3 - 5040*a^2*cos(d*x + c)^2 - 3780*a^2*cos(
d*x + c) + 2520*a^2*log(cos(d*x + c)) - 1260*a^2/cos(d*x + c))/d
```

Fricas [A]
time = 3.45, size = 167, normalized size = 0.91

$$-\frac{17920\,a^2\cos(dx+c)^{10}+40320\,a^2\cos(dx+c)^9-69120\,a^2\cos(dx+c)^8-215040\,a^2\cos(dx+c)^7+64512\,a^2\cos(dx+c)^6+483840\,a^2\cos(dx+c)^5+107520\,a^2\cos(dx+c)^4-645120\,a^2\cos(dx+c)^3-483840\,a^2\cos(dx+c)^2+322560\,a^2\cos(dx+c)\log(-\cos(dx+c))+197295\,a^2\cos(dx+c)-161280\,a^2}{161280\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^9,x, algorithm="fricas")
```

```
[Out] -1/161280*(17920*a^2*cos(d*x + c)^10 + 40320*a^2*cos(d*x + c)^9 - 69120*a^2
*cos(d*x + c)^8 - 215040*a^2*cos(d*x + c)^7 + 64512*a^2*cos(d*x + c)^6 + 48
3840*a^2*cos(d*x + c)^5 + 107520*a^2*cos(d*x + c)^4 - 645120*a^2*cos(d*x +
c)^3 - 483840*a^2*cos(d*x + c)^2 + 322560*a^2*cos(d*x + c)*log(-cos(d*x + c
)) + 197295*a^2*cos(d*x + c) - 161280*a^2)/(d*cos(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*sin(d*x+c)**9,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 370 vs. 2(171) = 342.
time = 0.73, size = 370, normalized size = 2.02

$$\frac{2520\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-2520\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{2520\left(2\,a^2+\frac{a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1}+\frac{1457\,a^2-\frac{20673\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{123012\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{421428\,a^2(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{949662\,a^2(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}-\frac{1009134\,a^2(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5}+\frac{666036\,a^2(\cos(dx+c)-1)^6}{(\cos(dx+c)+1)^6}-\frac{276804\,a^2(\cos(dx+c)-1)^7}{(\cos(dx+c)+1)^7}+\frac{66681\,a^2(\cos(dx+c)-1)^8}{(\cos(dx+c)+1)^8}-\frac{7129\,a^2(\cos(dx+c)-1)^9}{(\cos(dx+c)+1)^9}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^9}}{1260\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^9,x, algorithm="giac")
```

```
[Out] 1/1260*(2520*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 252
0*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + 2520*(2*a^2 +
a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c
) + 1) + 1) + (1457*a^2 - 20673*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) +
 123012*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 421428*a^2*(cos(d*x
 + c) - 1)^3/(cos(d*x + c) + 1)^3 + 949662*a^2*(cos(d*x + c) - 1)^4/(cos(d*
x + c) + 1)^4 - 1009134*a^2*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 666
036*a^2*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 276804*a^2*(cos(d*x + c
) - 1)^7/(cos(d*x + c) + 1)^7 + 66681*a^2*(cos(d*x + c) - 1)^8/(cos(d*x + c
) + 1)^8 - 7129*a^2*(cos(d*x + c) - 1)^9/(cos(d*x + c) + 1)^9)/((cos(d*x +
c) - 1)/(cos(d*x + c) + 1) - 1)^9)/d
```

Mupad [B]
time = 0.99, size = 146, normalized size = 0.80

$$-\frac{\frac{2\,a^2\cos(c+d\,x)^3}{3}-\frac{a^2}{\cos(c+d\,x)}-4\,a^2\cos\left(c+d\,x\right)^2-3\,a^2\,\cos\left(c+d\,x\right)+3\,a^2\cos\left(c+d\,x\right)^4+\frac{2\,a^2\cos(c+d\,x)^5}{5}-\frac{4\,a^2\cos(c+d\,x)^6}{3}-\frac{3\,a^2\cos(c+d\,x)^7}{7}+\frac{a^2\cos(c+d\,x)^8}{4}+\frac{a^2\cos(c+d\,x)^9}{9}+2\,a^2\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^9*(a + a/cos(c + d*x))^2,x)
```

```
[Out] -((2*a^2*cos(c + d*x)^3)/3 - a^2/cos(c + d*x) - 4*a^2*cos(c + d*x)^2 - 3*a^
2*cos(c + d*x) + 3*a^2*cos(c + d*x)^4 + (2*a^2*cos(c + d*x)^5)/5 - (4*a^2*c
os(c + d*x)^6)/3 - (3*a^2*cos(c + d*x)^7)/7 + (a^2*cos(c + d*x)^8)/4 + (a^2
*cos(c + d*x)^9)/9 + 2*a^2*log(cos(c + d*x)))/d
```

3.20 $\int (a+a\sec(c+dx))^2\sin^7(c+dx)\,dx$

Optimal. Leaf size=131

$$\frac{2a^2\cos(c+dx)}{d}+\frac{3a^2\cos^2(c+dx)}{d}-\frac{3a^2\cos^4(c+dx)}{2d}-\frac{2a^2\cos^5(c+dx)}{5d}+\frac{a^2\cos^6(c+dx)}{3d}+\frac{a^2\cos^7(c+dx)}{7d}-2$$

```
[Out] 2*a^2*cos(d*x+c)/d+3*a^2*cos(d*x+c)^2/d-3/2*a^2*cos(d*x+c)^4/d-2/5*a^2*cos(
d*x+c)^5/d+1/3*a^2*cos(d*x+c)^6/d+1/7*a^2*cos(d*x+c)^7/d-2*a^2*ln(cos(d*x+c
))/d+a^2*sec(d*x+c)/d
```

Rubi [A]

time = 0.12, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3957, 2915, 12, 90}

$$\frac{a^2\cos^7(c+dx)}{7d}+\frac{a^2\cos^6(c+dx)}{3d}-\frac{2a^2\cos^5(c+dx)}{5d}-\frac{3a^2\cos^4(c+dx)}{2d}+\frac{3a^2\cos^2(c+dx)}{d}+\frac{2a^2\cos(c+dx)}{d}+\frac{a^2\sec(c+dx)}{d}-\frac{2a^2\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^7,x]
```

```
[Out] (2*a^2*Cos[c + d*x])/d + (3*a^2*Cos[c + d*x]^2)/d - (3*a^2*Cos[c + d*x]^4)/
(2*d) - (2*a^2*Cos[c + d*x]^5)/(5*d) + (a^2*Cos[c + d*x]^6)/(3*d) + (a^2*Co
s[c + d*x]^7)/(7*d) - (2*a^2*Log[Cos[c + d*x]])/d + (a^2*Sec[c + d*x])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+a\sec(c+dx))^2\sin^7(c+dx)\,dx=\int(-a-a\cos(c+dx))^2\sin^5(c+dx)\tan^2(c+dx)\,dx$$
$$=\frac{\text{Subst}\left(\int\frac{a^2(-a-x)^3(-a+x)^5}{x^2}\,dx,x,-a\cos(c+dx)\right)}{a^7d}$$
$$=\frac{\text{Subst}\left(\int\frac{(-a-x)^3(-a+x)^5}{x^2}\,dx,x,-a\cos(c+dx)\right)}{a^5d}$$
$$=\frac{\text{Subst}\left(\int\left(-2a^6+\frac{a^8}{x^2}-\frac{2a^7}{x}+6a^5x-6a^3x^3+2a^2x^4+2ax^5-x^6\right)\right.}{a^5d}$$
$$=\frac{2a^2\cos(c+dx)}{d}+\frac{3a^2\cos^2(c+dx)}{d}-\frac{3a^2\cos^4(c+dx)}{2d}-\frac{2a^2\cos^5(}{5}$$

Mathematica [A]
time = 0.38, size = 107, normalized size = 0.82

$$\frac{a^2(25725+11760\cos(2(c+dx))+5250\cos(3(c+dx))-588\cos(4(c+dx))-770\cos(5(c+dx))-48\cos(6(c+dx))+70\cos(7(c+dx))+15\cos(8(c+dx))-70\cos(c+dx)(5+384\log(\cos(c+dx))))\sec(c+dx)}{13440d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^7,x]
[Out] (a^2*(25725 + 11760*Cos[2*(c + d*x)] + 5250*Cos[3*(c + d*x)] - 588*Cos[4*(c
 + d*x)] - 770*Cos[5*(c + d*x)] - 48*Cos[6*(c + d*x)] + 70*Cos[7*(c + d*x)]
 + 15*Cos[8*(c + d*x)] - 70*Cos[c + d*x]*(5 + 384*Log[Cos[c + d*x]]))*Sec[c
 + d*x])/(13440*d)
```

Maple [A]
time = 0.14, size = 151, normalized size = 1.15

method	result
derivativedivides	$\frac{a^2\left(\frac{\sin^8(dx+c)}{\cos(dx+c)}+\left(\frac{16}{5}+\sin^6(dx+c)+\frac{6\left(\sin^4(dx+c)\right)}{5}+\frac{8\left(\sin^2(dx+c)\right)}{5}\right)\cos(dx+c)\right)+2a^2\left(-\frac{\left(\sin^6(dx+c)\right)}{6}-\frac{\left(\sin^4(dx+c)\right)}{4}-\right.}{d}$
default	$\frac{a^2\left(\frac{\sin^8(dx+c)}{\cos(dx+c)}+\left(\frac{16}{5}+\sin^6(dx+c)+\frac{6\left(\sin^4(dx+c)\right)}{5}+\frac{8\left(\sin^2(dx+c)\right)}{5}\right)\cos(dx+c)\right)+2a^2\left(-\frac{\left(\sin^6(dx+c)\right)}{6}-\frac{\left(\sin^4(dx+c)\right)}{4}-\right.}{d}$

risch	$2ia^2x+\frac{29a^2e^{2i(dx+c)}}{64d}+\frac{117a^2e^{i(dx+c)}}{128d}+\frac{117a^2e^{-i(dx+c)}}{128d}+\frac{29a^2e^{-2i(dx+c)}}{64d}-\frac{2a^2\ln\left(e^{2i(dx+c)}+1\right)}{d}+\frac{2a^2e}{d(e^{2i(}}$
norman	$\frac{-\frac{192a^2}{35d}-\frac{64a^2\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{4a^2\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{24a^2\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{172a^2\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{264a^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}-}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*sin(d*x+c)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(sin(d*x+c)^8/cos(d*x+c)+(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*s
in(d*x+c)^2)*cos(d*x+c))+2*a^2*(-1/6*sin(d*x+c)^6-1/4*sin(d*x+c)^4-1/2*sin(
d*x+c)^2-ln(cos(d*x+c)))-1/7*a^2*(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*si
n(d*x+c)^2)*cos(d*x+c))
```

Maxima [A]
time = 0.29, size = 107, normalized size = 0.82

$$\frac{30\,a^2\cos(dx+c)^7+70\,a^2\cos(dx+c)^6-84\,a^2\cos(dx+c)^5-315\,a^2\cos(dx+c)^4+630\,a^2\cos(dx+c)^2+420\,a^2\cos(dx+c)-420\,a^2\log(\cos(dx+c))+\frac{210\,a^2}{\cos(dx+c)}}{210\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/210*(30*a^2*cos(d*x + c)^7 + 70*a^2*cos(d*x + c)^6 - 84*a^2*cos(d*x + c)^
5 - 315*a^2*cos(d*x + c)^4 + 630*a^2*cos(d*x + c)^2 + 420*a^2*cos(d*x + c)
- 420*a^2*log(cos(d*x + c)) + 210*a^2/cos(d*x + c))/d
```

Fricas [A]
time = 3.01, size = 128, normalized size = 0.98

$$\frac{120\,a^2\cos(dx+c)^8+280\,a^2\cos(dx+c)^7-336\,a^2\cos(dx+c)^6-1260\,a^2\cos(dx+c)^5+2520\,a^2\cos(dx+c)^3+1680\,a^2\cos(dx+c)^2-1680\,a^2\cos(dx+c)\log(-\cos(dx+c))-875\,a^2\cos(dx+c)+840\,a^2}{840\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/840*(120*a^2*cos(d*x + c)^8 + 280*a^2*cos(d*x + c)^7 - 336*a^2*cos(d*x +
c)^6 - 1260*a^2*cos(d*x + c)^5 + 2520*a^2*cos(d*x + c)^3 + 1680*a^2*cos(d*x
 + c)^2 - 1680*a^2*cos(d*x + c)*log(-cos(d*x + c)) - 875*a^2*cos(d*x + c) +
 840*a^2)/(d*cos(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*sin(d*x+c)**7,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 320 vs. 2(123) = 246.
time = 0.58, size = 320, normalized size = 2.44

$$\frac{420\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-420\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{420\left(2\,a^2+\frac{a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1}+\frac{357\,a^2-\frac{3759\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{16737\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{42595\,a^2(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{43855\,a^2(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}-\frac{25389\,a^2(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5}+\frac{8043\,a^2(\cos(dx+c)-1)^6}{(\cos(dx+c)+1)^6}-\frac{1089\,a^2(\cos(dx+c)-1)^7}{(\cos(dx+c)+1)^7}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^7}}{210\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/210*(420*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 420*a
^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + 420*(2*a^2 + a^2*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c) +
1) + 1) + (357*a^2 - 3759*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 16737
*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 42595*a^2*(cos(d*x + c) -
1)^3/(cos(d*x + c) + 1)^3 + 43855*a^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) +
1)^4 - 25389*a^2*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 8043*a^2*(cos(
d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 1089*a^2*(cos(d*x + c) - 1)^7/(cos(d
*x + c) + 1)^7)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^7)/d
```

Mupad [B]
time = 0.95, size = 105, normalized size = 0.80

$$\frac{2\,a^2\,\cos\left(c+d\,x\right)+\frac{a^2}{\cos(c+d\,x)}+3\,a^2\,\cos\left(c+d\,x\right)^2-\frac{3\,a^2\,\cos(c+d\,x)^4}{2}-\frac{2\,a^2\,\cos(c+d\,x)^5}{5}+\frac{a^2\,\cos(c+d\,x)^6}{3}+\frac{a^2\,\cos(c+d\,x)^7}{7}-2\,a^2\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^7*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (2*a^2*cos(c + d*x) + a^2/cos(c + d*x) + 3*a^2*cos(c + d*x)^2 - (3*a^2*cos(
c + d*x)^4)/2 - (2*a^2*cos(c + d*x)^5)/5 + (a^2*cos(c + d*x)^6)/3 + (a^2*co
s(c + d*x)^7)/7 - 2*a^2*log(cos(c + d*x)))/d
```

3.21 $\int (a+a\sec(c+dx))^2\sin^5(c+dx)\,dx$

Optimal. Leaf size=112

$$\frac{a^2\cos(c+dx)}{d}+\frac{2a^2\cos^2(c+dx)}{d}+\frac{a^2\cos^3(c+dx)}{3d}-\frac{a^2\cos^4(c+dx)}{2d}-\frac{a^2\cos^5(c+dx)}{5d}-\frac{2a^2\log(\cos(c+dx))}{d}+$$

```
[Out] a^2*cos(d*x+c)/d+2*a^2*cos(d*x+c)^2/d+1/3*a^2*cos(d*x+c)^3/d-1/2*a^2*cos(d*
x+c)^4/d-1/5*a^2*cos(d*x+c)^5/d-2*a^2*ln(cos(d*x+c))/d+a^2*sec(d*x+c)/d
```

Rubi [A]

time = 0.11, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3957, 2915, 12, 90}

$$-\frac{a^2\cos^5(c+dx)}{5d}-\frac{a^2\cos^4(c+dx)}{2d}+\frac{a^2\cos^3(c+dx)}{3d}+\frac{2a^2\cos^2(c+dx)}{d}+\frac{a^2\cos(c+dx)}{d}+\frac{a^2\sec(c+dx)}{d}-\frac{2a^2\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^5,x]
```

```
[Out] (a^2*Cos[c + d*x])/d + (2*a^2*Cos[c + d*x]^2)/d + (a^2*Cos[c + d*x]^3)/(3*d
) - (a^2*Cos[c + d*x]^4)/(2*d) - (a^2*Cos[c + d*x]^5)/(5*d) - (2*a^2*Log[Co
s[c + d*x]])/d + (a^2*Sec[c + d*x])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
```

```
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+a\sec(c+dx))^2\sin^5(c+dx)\,dx = \int (-a-a\cos(c+dx))^2\sin^3(c+dx)\tan^2(c+dx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{a^2(-a-x)^2(-a+x)^4}{x^2}\,dx, x, -a\cos(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int \frac{(-a-x)^2(-a+x)^4}{x^2}\,dx, x, -a\cos(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \left(-a^4+\frac{a^6}{x^2}-\frac{2a^5}{x}+4a^3x-a^2x^2-2ax^3+x^4\right)\,dx, x, -a\cos\right.}{a^3d}$$
$$= \frac{a^2\cos(c+dx)}{d}+\frac{2a^2\cos^2(c+dx)}{d}+\frac{a^2\cos^3(c+dx)}{3d}-\frac{a^2\cos^4(c+}{2d}$$

Mathematica [A]
time = 0.21, size = 87, normalized size = 0.78

$$-\frac{a^2(-750-275\cos(2(c+dx))-165\cos(3(c+dx))-2\cos(4(c+dx))+15\cos(5(c+dx))+3\cos(6(c+dx))+30\cos(c+dx)(-3+32\log(\cos(c+dx))))\sec(c+dx)}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^5,x]
```

```
[Out] -1/480*(a^2*(-750 - 275*Cos[2*(c + d*x)] - 165*Cos[3*(c + d*x)] - 2*Cos[4*(
c + d*x)] + 15*Cos[5*(c + d*x)] + 3*Cos[6*(c + d*x)] + 30*Cos[c + d*x]*(-3
+ 32*Log[Cos[c + d*x]]))*Sec[c + d*x])/d
```

Maple [A]
time = 0.12, size = 121, normalized size = 1.08

method	result
derivativedivides	$\frac{a^2\left(\frac{\sin^6(dx+c)}{\cos(dx+c)}+\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)\right)+2a^2\left(-\frac{\left(\sin^4(dx+c)\right)}{4}-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)}{d}$
default	$\frac{a^2\left(\frac{\sin^6(dx+c)}{\cos(dx+c)}+\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)\right)+2a^2\left(-\frac{\left(\sin^4(dx+c)\right)}{4}-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)}{d}$
risch	$2ia^2x+\frac{3a^2e^{2i(dx+c)}}{8d}+\frac{9a^2e^{i(dx+c)}}{16d}+\frac{9a^2e^{-i(dx+c)}}{16d}+\frac{3a^2e^{-2i(dx+c)}}{8d}+\frac{4ia^2c}{d}+\frac{2a^2e^{i(dx+c)}}{d\left(e^{2i(dx+c)}+1\right)}-\frac{2a^2\ln}{}$

norman	$\frac{-\frac{64a^2}{15d}-\frac{64a^2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{4a^2\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{16a^2\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{16a^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{196a^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5} -$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*sin(d*x+c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d
*x+c))+2*a^2*(-1/4*sin(d*x+c)^4-1/2*sin(d*x+c)^2-ln(cos(d*x+c)))-1/5*a^2*(8
/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))
```

Maxima [A]
time = 0.27, size = 94, normalized size = 0.84

$$-\frac{6\,a^2\cos\left(dx+c\right)^5+15\,a^2\cos\left(dx+c\right)^4-10\,a^2\cos\left(dx+c\right)^3-60\,a^2\cos\left(dx+c\right)^2-30\,a^2\cos\left(dx+c\right)+60\,a^2\log\left(\cos\left(dx+c\right)\right)-\frac{30\,a^2}{\cos(dx+c)}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/30*(6*a^2*cos(d*x + c)^5 + 15*a^2*cos(d*x + c)^4 - 10*a^2*cos(d*x + c)^3
 - 60*a^2*cos(d*x + c)^2 - 30*a^2*cos(d*x + c) + 60*a^2*log(cos(d*x + c)) -
 30*a^2/cos(d*x + c))/d
```

Fricas [A]
time = 4.04, size = 115, normalized size = 1.03

$$-\frac{48\,a^2\cos\left(dx+c\right)^6+120\,a^2\cos\left(dx+c\right)^5-80\,a^2\cos\left(dx+c\right)^4-480\,a^2\cos\left(dx+c\right)^3-240\,a^2\cos\left(dx+c\right)^2+480\,a^2\cos\left(dx+c\right)\log\left(-\cos\left(dx+c\right)\right)+195\,a^2\cos\left(dx+c\right)-240\,a^2}{240\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/240*(48*a^2*cos(d*x + c)^6 + 120*a^2*cos(d*x + c)^5 - 80*a^2*cos(d*x + c
)^4 - 480*a^2*cos(d*x + c)^3 - 240*a^2*cos(d*x + c)^2 + 480*a^2*cos(d*x + c
)*log(-cos(d*x + c)) + 195*a^2*cos(d*x + c) - 240*a^2)/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\sin^5\left(c+dx\right)\sec\left(c+dx\right)dx+\int\sin^5\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int\sin^5\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*sin(d*x+c)**5,x)
```

```
[Out] a**2*(Integral(2*sin(c + d*x)**5*sec(c + d*x), x) + Integral(sin(c + d*x)**
5*sec(c + d*x)**2, x) + Integral(sin(c + d*x)**5, x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(106) = 212$.
time = 0.68, size = 270, normalized size = 2.41

$$\frac{60\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-60\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{60\left(2\,a^2+\frac{a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1}+\frac{69\,a^2-\frac{525\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{1650\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{1610\,a^2(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{745\,a^2(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}-\frac{137\,a^2(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^5}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/30*(60*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 60*a^2*
log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + 60*(2*a^2 + a^2*(cos
(d*x + c) - 1)/(cos(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) +
 1) + (69*a^2 - 525*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1650*a^2*(c
os(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 1610*a^2*(cos(d*x + c) - 1)^3/(co
s(d*x + c) + 1)^3 + 745*a^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 137
*a^2*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5)/((cos(d*x + c) - 1)/(cos(d*
x + c) + 1) - 1)^5)/d
```

Mupad [B]
time = 0.89, size = 91, normalized size = 0.81

$$\frac{a^2\,\cos\left(c+d\,x\right)+\frac{a^2}{\cos\left(c+d\,x\right)}+2\,a^2\,{\cos\left(c+d\,x\right)}^2+\frac{a^2\,{\cos\left(c+d\,x\right)}^3}{3}-\frac{a^2\,{\cos\left(c+d\,x\right)}^4}{2}-\frac{a^2\,{\cos\left(c+d\,x\right)}^5}{5}-2\,a^2\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (a^2*cos(c + d*x) + a^2/cos(c + d*x) + 2*a^2*cos(c + d*x)^2 + (a^2*cos(c +
d*x)^3)/3 - (a^2*cos(c + d*x)^4)/2 - (a^2*cos(c + d*x)^5)/5 - 2*a^2*log(cos
(c + d*x)))/d
```

3.22 $\int (a+a\sec (c+dx))^2 \sin ^3(c+dx) \, dx$

Optimal. Leaf size=62

$$\frac{a^2\cos^2(c+dx)}{d}+\frac{a^2\cos^3(c+dx)}{3d}-\frac{2a^2\log(\cos(c+dx))}{d}+\frac{a^2\sec(c+dx)}{d}$$

```
[Out] a^2*cos(d*x+c)^2/d+1/3*a^2*cos(d*x+c)^3/d-2*a^2*ln(cos(d*x+c))/d+a^2*sec(d*
x+c)/d
```

Rubi [A]
time = 0.09, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3957, 2915, 12, 76}

$$\frac{a^2\cos^3(c+dx)}{3d}+\frac{a^2\cos^2(c+dx)}{d}+\frac{a^2\sec(c+dx)}{d}-\frac{2a^2\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^3,x]
```

```
[Out] (a^2*Cos[c + d*x]^2)/d + (a^2*Cos[c + d*x]^3)/(3*d) - (2*a^2*Log[Cos[c + d*
x]])/d + (a^2*Sec[c + d*x])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
```

```
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+a\sec(c+dx))^2\sin^3(c+dx)\,dx = \int (-a-a\cos(c+dx))^2\sin(c+dx)\tan^2(c+dx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{a^2(-a-x)(-a+x)^3}{x^2}\,dx, x, -a\cos(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \frac{(-a-x)(-a+x)^3}{x^2}\,dx, x, -a\cos(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^4}{x^2}-\frac{2a^3}{x}+2ax-x^2\right)\,dx, x, -a\cos(c+dx)\right)}{ad}$$
$$= \frac{a^2\cos^2(c+dx)}{d}+\frac{a^2\cos^3(c+dx)}{3d}-\frac{2a^2\log(\cos(c+dx))}{d}+\frac{a^2\sec$$

Mathematica [A]
time = 0.14, size = 65, normalized size = 1.05

$$\frac{a^2(27+4\cos(2(c+dx))+6\cos(3(c+dx))+\cos(4(c+dx))-6\cos(c+dx)(1+8\log(\cos(c+dx))))\sec(c+dx)}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^3,x]
[Out] (a^2*(27 + 4*Cos[2*(c + d*x)] + 6*Cos[3*(c + d*x)] + Cos[4*(c + d*x)] - 6*C
os[c + d*x]*(1 + 8*Log[Cos[c + d*x]]))*Sec[c + d*x])/(24*d)
```

Maple [A]
time = 0.09, size = 91, normalized size = 1.47

method	result
derivativedivides	$\frac{a^2\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)+2a^2\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)-\frac{a^2\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}}{d}$
default	$\frac{a^2\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)+2a^2\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)-\frac{a^2\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}}{d}$
risch	$2ia^2x+\frac{a^2e^{2i(dx+c)}}{4d}+\frac{a^2e^{i(dx+c)}}{8d}+\frac{a^2e^{-i(dx+c)}}{8d}+\frac{a^2e^{-2i(dx+c)}}{4d}+\frac{4ia^2c}{d}+\frac{2a^2e^{i(dx+c)}}{d(e^{2i(dx+c)}+1)}-\frac{2a^2\ln(e^{2i(}}{d}$
norman	$\frac{-\frac{8a^2}{3d}-\frac{8a^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{4a^2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{4a^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{2a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{2a^2\ln\left(\tan\left(\frac{dx}{2}+\right.\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*sin(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+2*a^2*(-1/2*
sin(d*x+c)^2-ln(cos(d*x+c)))-1/3*a^2*(2+sin(d*x+c)^2)*cos(d*x+c))
```

Maxima [A]
time = 0.27, size = 56, normalized size = 0.90

$$\frac{a^2 \cos{(dx+c)}^3 + 3\,a^2 \cos{(dx+c)}^2 - 6\,a^2 \log{(\cos{(dx+c)})} + \frac{3\,a^2}{\cos(dx+c)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/3*(a^2*cos(d*x + c)^3 + 3*a^2*cos(d*x + c)^2 - 6*a^2*log(cos(d*x + c)) +
3*a^2/cos(d*x + c))/d
```

Fricas [A]
time = 3.08, size = 76, normalized size = 1.23

$$\frac{2\,a^2 \cos{(dx+c)}^4 + 6\,a^2 \cos{(dx+c)}^3 - 12\,a^2 \cos{(dx+c)} \log{(-\cos{(dx+c)})} - 3\,a^2 \cos{(dx+c)} + 6\,a^2}{6\,d \cos{(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a^2*cos(d*x + c)^4 + 6*a^2*cos(d*x + c)^3 - 12*a^2*cos(d*x + c)*log(
-cos(d*x + c)) - 3*a^2*cos(d*x + c) + 6*a^2)/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\sin^3{(c+dx)}\sec{(c+dx)}\,dx + \int \sin^3{(c+dx)}\sec^2{(c+dx)}\,dx + \int \sin^3{(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*sin(d*x+c)**3,x)
```

```
[Out] a**2*(Integral(2*sin(c + d*x)**3*sec(c + d*x), x) + Integral(sin(c + d*x)**
3*sec(c + d*x)**2, x) + Integral(sin(c + d*x)**3, x))
```

Giac [A]
time = 0.67, size = 74, normalized size = 1.19

$$-\frac{2\,a^2 \log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d} + \frac{a^2}{d\cos{(dx+c)}} + \frac{a^2 d^5 \cos{(dx+c)}^3 + 3\,a^2 d^5 \cos{(dx+c)}^2}{3\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -2*a^2*log(abs(cos(d*x + c))/abs(d))/d + a^2/(d*cos(d*x + c)) + 1/3*(a^2*d^
5*cos(d*x + c)^3 + 3*a^2*d^5*cos(d*x + c)^2)/d^6
```

Mupad [B]
time = 0.06, size = 54, normalized size = 0.87

$$\frac{\frac{a^2}{\cos(c+d\,x)}+a^2\cos\left(c+d\,x\right)^2+\frac{a^2\cos(c+d\,x)^3}{3}-2\,a^2\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (a^2/cos(c + d*x) + a^2*cos(c + d*x)^2 + (a^2*cos(c + d*x)^3)/3 - 2*a^2*log
(cos(c + d*x)))/d
```

3.23 $\int (a+a\sec(c+dx))^2\sin(c+dx)\,dx$

Optimal. Leaf size=43

$$-\frac{a^2\cos(c+dx)}{d}-\frac{2a^2\log(\cos(c+dx))}{d}+\frac{a^2\sec(c+dx)}{d}$$

```
[Out] -a^2*cos(d*x+c)/d-2*a^2*ln(cos(d*x+c))/d+a^2*sec(d*x+c)/d
```

Rubi [A]
time = 0.06, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3957, 2912, 12, 45}

$$-\frac{a^2\cos(c+dx)}{d}+\frac{a^2\sec(c+dx)}{d}-\frac{2a^2\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Sin[c + d*x],x]
```

```
[Out] -((a^2*Cos[c + d*x])/d) - (2*a^2*Log[Cos[c + d*x]])/d + (a^2*Sec[c + d*x])/
d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int (a+a\sec(c+dx))^2\sin(c+dx)\,dx &= \int (-a-a\cos(c+dx))^2\sec(c+dx)\tan(c+dx)\,dx\\
&= \frac{\text{Subst}\left(\int \frac{a^2(-a+x)^2}{x^2}\,dx, x, -a\cos(c+dx)\right)}{ad}\\
&= \frac{a\text{Subst}\left(\int \frac{(-a+x)^2}{x^2}\,dx, x, -a\cos(c+dx)\right)}{d}\\
&= \frac{a\text{Subst}\left(\int \left(1+\frac{a^2}{x^2}-\frac{2a}{x}\right)\,dx, x, -a\cos(c+dx)\right)}{d}\\
&= -\frac{a^2\cos(c+dx)}{d}-\frac{2a^2\log(\cos(c+dx))}{d}+\frac{a^2\sec(c+dx)}{d}
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 31, normalized size = 0.72

$$\frac{a^2(1-2\log(\cos(c+dx))+\sin(c+dx)\tan(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Sin[c + d*x],x]
[Out] (a^2*(1 - 2*Log[Cos[c + d*x]] + Sin[c + d*x]*Tan[c + d*x]))/d
```

Maple [A]
time = 0.06, size = 34, normalized size = 0.79

method	result
derivativedivides	$\frac{a^2\left(\sec(dx+c)-\frac{1}{\sec(dx+c)}+2\ln(\sec(dx+c))\right)}{d}$
default	$\frac{a^2\left(\sec(dx+c)-\frac{1}{\sec(dx+c)}+2\ln(\sec(dx+c))\right)}{d}$
risch	$2ia^2x-\frac{a^2e^{i(dx+c)}}{2d}-\frac{a^2e^{-i(dx+c)}}{2d}+\frac{4ia^2c}{d}+\frac{2a^2e^{i(dx+c)}}{d\left(e^{2i(dx+c)}+1\right)}-\frac{2a^2\ln\left(e^{2i(dx+c)}+1\right)}{d}$
norman	$-\frac{4a^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{2a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{2a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}+\frac{2a^2\ln\left(1+\tan\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*sin(d*x+c),x,method=_RETURNVERBOSE)
[Out] a^2/d*(sec(d*x+c)-1/sec(d*x+c)+2*ln(sec(d*x+c)))
```

Maxima [A]
time = 0.27, size = 41, normalized size = 0.95

$$-\frac{a^2\cos(dx+c)+2a^2\log(\cos(dx+c))-\frac{a^2}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c),x, algorithm="maxima")
[Out] -(a^2*cos(d*x + c) + 2*a^2*log(cos(d*x + c)) - a^2/cos(d*x + c))/d
```

Fricas [A]
time = 3.17, size = 51, normalized size = 1.19

$$-\frac{a^2\cos(dx+c)^2+2a^2\cos(dx+c)\log(-\cos(dx+c))-a^2}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c),x, algorithm="fricas")
[Out] -(a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c)*log(-cos(d*x + c)) - a^2)/(d*cos(
d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\sin(c+dx)\sec(c+dx)\,dx+\int\sin(c+dx)\sec^2(c+dx)\,dx+\int\sin(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*sin(d*x+c),x)
[Out] a**2*(Integral(2*sin(c + d*x)*sec(c + d*x), x) + Integral(sin(c + d*x)*sec(
c + d*x)**2, x) + Integral(sin(c + d*x), x))
```

Giac [A]
time = 0.53, size = 51, normalized size = 1.19

$$-\frac{a^2\cos(dx+c)}{d}-\frac{2a^2\log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d}+\frac{a^2}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c),x, algorithm="giac")
[Out] -a^2*cos(d*x + c)/d - 2*a^2*log(abs(cos(d*x + c))/abs(d))/d + a^2/(d*cos(d*
x + c))
```

Mupad [B]
time = 0.06, size = 41, normalized size = 0.95

$$-\frac{a^2 \left(2 \cos \left(c+d x\right) \ln \left(\cos \left(c+d x\right)\right)+\cos \left(c+d x\right)^2-1\right)}{d \cos \left(c+d x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a + a/cos(c + d*x))^2,x)
```

```
[Out] -(a^2*(2*cos(c + d*x)*log(cos(c + d*x)) + cos(c + d*x)^2 - 1))/(d*cos(c + d
*x))
```

3.24 $\int \csc(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=48

$$\frac{2a^2\log(1-\cos(c+dx))}{d}-\frac{2a^2\log(\cos(c+dx))}{d}+\frac{a^2\sec(c+dx)}{d}$$

```
[Out] 2*a^2*ln(1-cos(d*x+c))/d-2*a^2*ln(cos(d*x+c))/d+a^2*sec(d*x+c)/d
```

Rubi [A]
time = 0.08, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3957, 2915, 12, 78}

$$\frac{a^2\sec(c+dx)}{d}+\frac{2a^2\log(1-\cos(c+dx))}{d}-\frac{2a^2\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*a^2*Log[1 - Cos[c + d*x]])/d - (2*a^2*Log[Cos[c + d*x]])/d + (a^2*Sec[c
+ d*x])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \csc(c+dx)(a+a\sec(c+dx))^2\,dx &= \int (-a-a\cos(c+dx))^2\csc(c+dx)\sec^2(c+dx)\,dx\\
&= \frac{a\text{Subst}\left(\int \frac{a^2(-a+x)}{(-a-x)x^2}\,dx, x, -a\cos(c+dx)\right)}{d}\\
&= \frac{a^3\text{Subst}\left(\int \frac{-a+x}{(-a-x)x^2}\,dx, x, -a\cos(c+dx)\right)}{d}\\
&= \frac{a^3\text{Subst}\left(\int \left(\frac{1}{x^2}-\frac{2}{ax}+\frac{2}{a(a+x)}\right)\,dx, x, -a\cos(c+dx)\right)}{d}\\
&= \frac{2a^2\log(1-\cos(c+dx))}{d}-\frac{2a^2\log(\cos(c+dx))}{d}+\frac{a^2\sec(c+dx)}{d}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 36, normalized size = 0.75

$$\frac{a^2\left(-2\log(\cos(c+dx))+4\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\sec(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*(a + a*Sec[c + d*x])^2,x]
[Out] (a^2*(-2*Log[Cos[c + d*x]] + 4*Log[Sin[(c + d*x)/2]] + Sec[c + d*x]))/d
```

Maple [A]
time = 0.08, size = 29, normalized size = 0.60

method	result	size
derivativedivides	$-\frac{a^2(-\sec(dx+c)-2\ln(-1+\sec(dx+c)))}{d}$	29
default	$-\frac{a^2(-\sec(dx+c)-2\ln(-1+\sec(dx+c)))}{d}$	29
risch	$\frac{2a^2 e^{i(dx+c)}}{d(e^{2i(dx+c)}+1)}+\frac{4a^2\ln(e^{i(dx+c)}-1)}{d}-\frac{2a^2\ln(e^{2i(dx+c)}+1)}{d}$	72
norman	$-\frac{2a^2}{d\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{4a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{2a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*a^2*(-sec(d*x+c)-2*ln(-1+sec(d*x+c)))
```

Maxima [A]
time = 0.27, size = 43, normalized size = 0.90

$$\frac{2\,a^2\log\left(\cos\left(dx+c\right)-1\right)-2\,a^2\log\left(\cos\left(dx+c\right)\right)+\frac{a^2}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (2*a^2*log(cos(d*x + c) - 1) - 2*a^2*log(cos(d*x + c)) + a^2/cos(d*x + c))/
d
```

Fricas [A]
time = 2.14, size = 61, normalized size = 1.27

$$-\frac{2\,a^2\cos\left(dx+c\right)\log\left(-\cos\left(dx+c\right)\right)-2\,a^2\cos\left(dx+c\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-a^2}{d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(2*a^2*cos(d*x + c)*log(-cos(d*x + c)) - 2*a^2*cos(d*x + c)*log(-1/2*cos(d
*x + c) + 1/2) - a^2)/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\csc\left(c+dx\right)\sec\left(c+dx\right)dx+\int\csc\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int\csc\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*csc(c + d*x)*sec(c + d*x), x) + Integral(csc(c + d*x)*sec(
c + d*x)**2, x) + Integral(csc(c + d*x), x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(48) = 96.
time = 0.55, size = 115, normalized size = 2.40

$$\frac{2\left(a^2\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{2\,a^2+\frac{a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2*(a^2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - a^2*log(abs(-(co
s(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (2*a^2 + a^2*(cos(d*x + c) - 1)/
(cos(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/d
```

Mupad [B]
time = 0.08, size = 35, normalized size = 0.73

$$\frac{a^2}{d\cos(c+dx)}-\frac{4a^2\,\mathrm{atanh}(2\cos(c+dx)-1)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/sin(c + d*x),x)
```

```
[Out] a^2/(d*cos(c + d*x)) - (4*a^2*atanh(2*cos(c + d*x) - 1))/d
```

3.25 $\int \csc^3(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=69

$$-\frac{a^3}{d(a-a\cos(c+dx))}+\frac{2a^2\log(1-\cos(c+dx))}{d}-\frac{2a^2\log(\cos(c+dx))}{d}+\frac{a^2\sec(c+dx)}{d}$$

```
[Out] -a^3/d/(a-a*cos(d*x+c))+2*a^2*ln(1-cos(d*x+c))/d-2*a^2*ln(cos(d*x+c))/d+a^2
*sec(d*x+c)/d
```

Rubi [A]
time = 0.11, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3957, 2915, 12, 46}

$$-\frac{a^3}{d(a-a\cos(c+dx))}+\frac{a^2\sec(c+dx)}{d}+\frac{2a^2\log(1-\cos(c+dx))}{d}-\frac{2a^2\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(a^3/(d*(a - a*Cos[c + d*x]))) + (2*a^2*Log[1 - Cos[c + d*x]])/d - (2*a^2*
Log[Cos[c + d*x]])/d + (a^2*Sec[c + d*x])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
```

```
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc^3(c+dx)(a+a\sec(c+dx))^2\,dx = \int (-a-a\cos(c+dx))^2\csc^3(c+dx)\sec^2(c+dx)\,dx$$
$$= \frac{a^3\text{Subst}\left(\int \frac{a^2}{(-a-x)^2x^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^5\text{Subst}\left(\int \frac{1}{(-a-x)^2x^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^5\text{Subst}\left(\int \left(\frac{1}{a^2x^2}-\frac{2}{a^3x}+\frac{1}{a^2(a+x)^2}+\frac{2}{a^3(a+x)}\right)\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= -\frac{a^3}{d(a-a\cos(c+dx))}+\frac{2a^2\log(1-\cos(c+dx))}{d}-\frac{2a^2\log(\cos(c}{d}$$

Mathematica [A]
time = 0.37, size = 75, normalized size = 1.09

$$-\frac{a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\csc^2\left(\frac{1}{2}(c+dx)\right)+4\log(\cos(c+dx))-8\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-2\sec(c+dx)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*(a + a*Sec[c + d*x])^2,x]
[Out] -1/8*(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(Csc[(c + d*x)/2]^2 + 4*L
og[Cos[c + d*x]] - 8*Log[Sin[(c + d*x)/2]] - 2*Sec[c + d*x]))/d
```

Maple [A]
time = 0.12, size = 38, normalized size = 0.55

method	result
derivativedivides	$\frac{a^2\left(\sec(dx+c)-\frac{1}{-1+\sec(dx+c)}+2\ln(-1+\sec(dx+c))\right)}{d}$
default	$\frac{a^2\left(\sec(dx+c)-\frac{1}{-1+\sec(dx+c)}+2\ln(-1+\sec(dx+c))\right)}{d}$
risch	$\frac{4a^2\left(e^{3i(dx+c)}-e^{2i(dx+c)}+e^{i(dx+c)}\right)}{d\left(e^{i(dx+c)}-1\right)^2\left(e^{2i(dx+c)}+1\right)}+\frac{4a^2\ln\left(e^{i(dx+c)}-1\right)}{d}-\frac{2a^2\ln\left(e^{2i(dx+c)}+1\right)}{d}$
norman	$\frac{\frac{a^2}{2d}-\frac{5a^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{4a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{2a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*a^2*(sec(d*x+c)-1/(-1+sec(d*x+c))+2*ln(-1+sec(d*x+c)))
```

Maxima [A]
time = 0.27, size = 68, normalized size = 0.99

$$\frac{2\,a^2\log\left(\cos\left(dx+c\right)-1\right)-2\,a^2\log\left(\cos\left(dx+c\right)\right)+\frac{2\,a^2\cos(dx+c)-a^2}{\cos(dx+c)^2-\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (2*a^2*log(cos(d*x + c) - 1) - 2*a^2*log(cos(d*x + c)) + (2*a^2*cos(d*x + c
) - a^2)/(cos(d*x + c)^2 - cos(d*x + c)))/d
```

Fricas [A]
time = 2.28, size = 112, normalized size = 1.62

$$\frac{2\,a^2\cos\left(dx+c\right)-a^2-2\left(a^2\cos\left(dx+c\right)^2-a^2\cos\left(dx+c\right)\right)\log\left(-\cos\left(dx+c\right)\right)+2\left(a^2\cos\left(dx+c\right)^2-a^2\cos\left(dx+c\right)\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{d\cos\left(dx+c\right)^2-d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (2*a^2*cos(d*x + c) - a^2 - 2*(a^2*cos(d*x + c)^2 - a^2*cos(d*x + c))*log(-
cos(d*x + c)) + 2*(a^2*cos(d*x + c)^2 - a^2*cos(d*x + c))*log(-1/2*cos(d*x
+ c) + 1/2))/(d*cos(d*x + c)^2 - d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\csc^3\left(c+dx\right)\sec\left(c+dx\right)dx+\int\csc^3\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int\csc^3\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*csc(c + d*x)**3*sec(c + d*x), x) + Integral(csc(c + d*x)**
3*sec(c + d*x)**2, x) + Integral(csc(c + d*x)**3, x))
```

Giac [A]
time = 0.53, size = 135, normalized size = 1.96

$$\frac{4\,a^2\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-4\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{a^2+\frac{5\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+\frac{(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*a^2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 4*a^2*log(ab
s(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (a^2 + 5*a^2*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + (cos(d*x
 + c) - 1)^2/(cos(d*x + c) + 1)^2))/d
```

Mupad [B]
time = 0.90, size = 61, normalized size = 0.88

$$-\frac{2\,a^2\,\cos\left(c+d\,x\right)-a^2}{d\,\left(\cos\left(c+d\,x\right)-{\cos\left(c+d\,x\right)}^2\right)}-\frac{4\,a^2\,\mathrm{atanh}\left(2\,\cos\left(c+d\,x\right)-1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/sin(c + d*x)^3,x)
```

```
[Out] - (2*a^2*cos(c + d*x) - a^2)/(d*(cos(c + d*x) - cos(c + d*x)^2)) - (4*a^2*a
tanh(2*cos(c + d*x) - 1))/d
```

3.26 $\int \csc^5(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=115

$$-\frac{a^4}{4d(a-a\cos(c+dx))^2}-\frac{5a^3}{4d(a-a\cos(c+dx))}+\frac{17a^2\log(1-\cos(c+dx))}{8d}-\frac{2a^2\log(\cos(c+dx))}{d}-\frac{a^2\log(1$$

```
[Out] -1/4*a^4/d/(a-a*cos(d*x+c))^2-5/4*a^3/d/(a-a*cos(d*x+c))+17/8*a^2*ln(1-cos(
d*x+c))/d-2*a^2*ln(cos(d*x+c))/d-1/8*a^2*ln(1+cos(d*x+c))/d+a^2*sec(d*x+c)/
d
```

Rubi [A]
time = 0.13, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3957, 2915, 12, 90}

$$-\frac{a^4}{4d(a-a\cos(c+dx))^2}-\frac{5a^3}{4d(a-a\cos(c+dx))}+\frac{a^2\sec(c+dx)}{d}+\frac{17a^2\log(1-\cos(c+dx))}{8d}-\frac{2a^2\log(\cos(c+dx))}{d}-\frac{a^2\log(\cos(c+dx)+1)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/4*a^4/(d*(a - a*Cos[c + d*x])^2) - (5*a^3)/(4*d*(a - a*Cos[c + d*x])) +
(17*a^2*Log[1 - Cos[c + d*x]])/(8*d) - (2*a^2*Log[Cos[c + d*x]])/d - (a^2*L
og[1 + Cos[c + d*x]])/(8*d) + (a^2*Sec[c + d*x])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc^5(c+dx)(a+a\sec(c+dx))^2\,dx = \int (-a-a\cos(c+dx))^2\csc^5(c+dx)\sec^2(c+dx)\,dx$$
$$= \frac{a^5\text{Subst}\left(\int \frac{a^2}{(-a-x)^3x^2(-a+x)}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^7\text{Subst}\left(\int \frac{1}{(-a-x)^3x^2(-a+x)}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^7\text{Subst}\left(\int \left(\frac{1}{8a^5(a-x)} + \frac{1}{a^4x^2} - \frac{2}{a^5x} + \frac{1}{2a^3(a+x)^3} + \frac{5}{4a^4(a+x)^2} + \frac{17}{8a^5(a+x)}\right.\right.}{d}$$
$$= -\frac{a^4}{4d(a-a\cos(c+dx))^2} - \frac{5a^3}{4d(a-a\cos(c+dx))} + \frac{17a^2\log(1-}{8a}$$

Mathematica [A]
time = 1.03, size = 103, normalized size = 0.90

$$-\frac{a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(10\csc^2\left(\frac{1}{2}(c+dx)\right)+\csc^4\left(\frac{1}{2}(c+dx)\right)+4\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+8\log(\cos(c+dx))-17\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-4\sec(c+dx)\right)\right)}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^5*(a + a*Sec[c + d*x])^2,x]
[Out] -1/64*(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(10*Csc[(c + d*x)/2]^2 +
 Csc[(c + d*x)/2]^4 + 4*(Log[Cos[(c + d*x)/2]] + 8*Log[Cos[c + d*x]] - 17*L
og[Sin[(c + d*x)/2]] - 4*Sec[c + d*x])))/d
```

Maple [A]
time = 0.12, size = 64, normalized size = 0.56

method	result
derivativedivides	$-\frac{a^2\left(-\sec(dx+c)+\frac{1}{4(-1+\sec(dx+c))^2}+\frac{7}{4(-1+\sec(dx+c))}-\frac{17\ln(-1+\sec(dx+c))}{8}+\frac{\ln(1+\sec(dx+c))}{8}\right)}{d}$
default	$-\frac{a^2\left(-\sec(dx+c)+\frac{1}{4(-1+\sec(dx+c))^2}+\frac{7}{4(-1+\sec(dx+c))}-\frac{17\ln(-1+\sec(dx+c))}{8}+\frac{\ln(1+\sec(dx+c))}{8}\right)}{d}$
norman	$\frac{\frac{a^2}{16d}+\frac{11a^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16d}-\frac{11a^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{17a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{2a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{2a^2\ln\left(\tan\right.}{}$

risch	$\frac{a^2(9\,\mathrm{e}^{5i(dx+c)}-28\,\mathrm{e}^{4i(dx+c)}+34\,\mathrm{e}^{3i(dx+c)}-28\,\mathrm{e}^{2i(dx+c)}+9\,\mathrm{e}^{i(dx+c)})}{2d(\mathrm{e}^{i(dx+c)}-1)^4(\mathrm{e}^{2i(dx+c)}+1)}+\frac{17a^2\ln(\mathrm{e}^{i(dx+c)}-1)}{4d}-\frac{a^2\ln(\mathrm{e}^{i(dx+c)}+1)}{4d}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*a^2*(-sec(d*x+c)+1/4/(-1+sec(d*x+c))^2+7/4/(-1+sec(d*x+c))-17/8*ln(-1+
sec(d*x+c))+1/8*ln(1+sec(d*x+c)))
```

Maxima [A]
time = 0.29, size = 104, normalized size = 0.90

$$-\frac{a^2\log\left(\cos\left(dx+c\right)+1\right)-17\,a^2\log\left(\cos\left(dx+c\right)-1\right)+16\,a^2\log\left(\cos\left(dx+c\right)\right)-\frac{2\left(9\,a^2\cos(dx+c)^2-14\,a^2\cos(dx+c)+4\,a^2\right)}{\cos(dx+c)^3-2\,\cos(dx+c)^2+\cos(dx+c)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/8*(a^2*log(cos(d*x + c) + 1) - 17*a^2*log(cos(d*x + c) - 1) + 16*a^2*log
(cos(d*x + c)) - 2*(9*a^2*cos(d*x + c)^2 - 14*a^2*cos(d*x + c) + 4*a^2)/(co
s(d*x + c)^3 - 2*cos(d*x + c)^2 + cos(d*x + c)))/d
```

Fricas [A]
time = 2.79, size = 209, normalized size = 1.82

$$\frac{18\,a^2\cos\left(dx+c\right)^2-28\,a^2\cos\left(dx+c\right)+8\,a^2-16\left(a^2\cos\left(dx+c\right)^3-2\,a^2\cos\left(dx+c\right)^2+a^2\cos\left(dx+c\right)\right)\log\left(-\cos\left(dx+c\right)\right)-\left(a^2\cos\left(dx+c\right)^3-2\,a^2\cos\left(dx+c\right)^2+a^2\cos\left(dx+c\right)\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+17\left(a^2\cos\left(dx+c\right)^3-2\,a^2\cos\left(dx+c\right)^2+a^2\cos\left(dx+c\right)\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{8\left(d\cos\left(dx+c\right)^3-2\,d\cos\left(dx+c\right)^2+d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(18*a^2*cos(d*x + c)^2 - 28*a^2*cos(d*x + c) + 8*a^2 - 16*(a^2*cos(d*x
+ c)^3 - 2*a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))*log(-cos(d*x + c)) - (a^2
*cos(d*x + c)^3 - 2*a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))*log(1/2*cos(d*x
+ c) + 1/2) + 17*(a^2*cos(d*x + c)^3 - 2*a^2*cos(d*x + c)^2 + a^2*cos(d*x +
 c))*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^3 - 2*d*cos(d*x + c)^2 +
 d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\csc^5\left(c+dx\right)\sec\left(c+dx\right)dx+\int\csc^5\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int\csc^5\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*csc(c + d*x)**5*sec(c + d*x), x) + Integral(csc(c + d*x)**
5*sec(c + d*x)**2, x) + Integral(csc(c + d*x)**5, x))
```

Giac [A]
time = 0.54, size = 191, normalized size = 1.66

$$\frac{34\,a^2\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-32\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)-\frac{\left(a^2-\frac{12\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{51\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)^2}{(\cos(dx+c)-1)^2}+\frac{32\left(2\,a^2+\frac{a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/16*(34*a^2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 32*a^2*log
(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - (a^2 - 12*a^2*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) + 51*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) +
1)^2)*(cos(d*x + c) + 1)^2/(cos(d*x + c) - 1)^2 + 32*(2*a^2 + a^2*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/
d
```

Mupad [B]
time = 0.10, size = 109, normalized size = 0.95

$$\frac{17\,a^2\,\ln\left(\cos\left(c+d\,x\right)-1\right)}{8\,d}-\frac{a^2\,\ln\left(\cos\left(c+d\,x\right)+1\right)}{8\,d}+\frac{\frac{9\,a^2\,\cos(c+d\,x)^2}{4}-\frac{7\,a^2\,\cos(c+d\,x)}{2}+a^2}{d\,\left({\cos\left(c+d\,x\right)}^3-2\,{\cos\left(c+d\,x\right)}^2+\cos\left(c+d\,x\right)\right)}-\frac{2\,a^2\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/sin(c + d*x)^5,x)
```

```
[Out] (17*a^2*log(cos(c + d*x) - 1))/(8*d) - (a^2*log(cos(c + d*x) + 1))/(8*d) +
(a^2 - (7*a^2*cos(c + d*x))/2 + (9*a^2*cos(c + d*x)^2)/4)/(d*(cos(c + d*x)
- 2*cos(c + d*x)^2 + cos(c + d*x)^3)) - (2*a^2*log(cos(c + d*x)))/d
```

3.27 $\int \csc^7(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=160

$$-\frac{a^5}{12d(a-a\cos(c+dx))^3}-\frac{3a^4}{8d(a-a\cos(c+dx))^2}-\frac{23a^3}{16d(a-a\cos(c+dx))}+\frac{a^3}{16d(a+a\cos(c+dx))}+\frac{9a^2\,\mathrm{lo}}{}$$

```
[Out] -1/12*a^5/d/(a-a*cos(d*x+c))^3-3/8*a^4/d/(a-a*cos(d*x+c))^2-23/16*a^3/d/(a-
a*cos(d*x+c))+1/16*a^3/d/(a+a*cos(d*x+c))+9/4*a^2*ln(1-cos(d*x+c))/d-2*a^2*
ln(cos(d*x+c))/d-1/4*a^2*ln(1+cos(d*x+c))/d+a^2*sec(d*x+c)/d
```

Rubi [A]
time = 0.15, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3957, 2915, 12, 90}

$$-\frac{a^5}{12d(a-a\cos(c+dx))^3}-\frac{3a^4}{8d(a-a\cos(c+dx))^2}-\frac{23a^3}{16d(a-a\cos(c+dx))}+\frac{a^3}{16d(a\cos(c+dx)+a)}+\frac{a^2\sec(c+dx)}{d}+\frac{9a^2\log(1-\cos(c+dx))}{4d}-\frac{2a^2\log(\cos(c+dx))}{d}-\frac{a^2\log(\cos(c+dx)+1)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^7*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/12*a^5/(d*(a - a*Cos[c + d*x])^3) - (3*a^4)/(8*d*(a - a*Cos[c + d*x])^2)
 - (23*a^3)/(16*d*(a - a*Cos[c + d*x])) + a^3/(16*d*(a + a*Cos[c + d*x])) +
 (9*a^2*Log[1 - Cos[c + d*x]])/(4*d) - (2*a^2*Log[Cos[c + d*x]])/d - (a^2*L
og[1 + Cos[c + d*x]])/(4*d) + (a^2*Sec[c + d*x])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \csc^7(c+dx)(a+a\sec(c+dx))^2\,dx &= \int (-a-a\cos(c+dx))^2\csc^7(c+dx)\sec^2(c+dx)\,dx \\
&= \frac{a^7\text{Subst}\left(\int \frac{a^2}{(-a-x)^4x^2(-a+x)^2}\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= \frac{a^9\text{Subst}\left(\int \frac{1}{(-a-x)^4x^2(-a+x)^2}\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= \frac{a^9\text{Subst}\left(\int \left(\frac{1}{16a^6(a-x)^2}+\frac{1}{4a^7(a-x)}+\frac{1}{a^6x^2}-\frac{2}{a^7x}+\frac{1}{4a^4(a+x)^4}+\frac{3}{4a^5(a+}\right.\right.}{d} \\
&= -\frac{a^5}{12d(a-a\cos(c+dx))^3}-\frac{3a^4}{8d(a-a\cos(c+dx))^2}-\frac{23}{16d(a-a\,\mathrm{c}}
\end{aligned}
$$

Mathematica [A]
time = 0.88, size = 136, normalized size = 0.85

$$
-\frac{a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(120\csc^2\left(\frac{1}{2}(c+dx)\right)+36\csc^4\left(\frac{1}{2}(c+dx)\right)+48\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+4\log(\cos(c+dx))-9\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+\csc^6\left(\frac{1}{2}(c+dx)\right)\left(16-3\sec^2\left(\frac{1}{2}(c+dx)\right)(3+2\sec(c+dx))\right)\right)}{384d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^7*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/384*(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(120*Csc[(c + d*x)/2]^2
 + 36*Csc[(c + d*x)/2]^4 + 48*(Log[Cos[(c + d*x)/2]] + 4*Log[Cos[c + d*x]]
- 9*Log[Sin[(c + d*x)/2]]) + Csc[(c + d*x)/2]^6*(16 - 3*Sec[(c + d*x)/2]^2*
(3 + 2*Sec[c + d*x]))))/d
```

Maple [A]
time = 0.13, size = 85, normalized size = 0.53

method	result
derivativedivides	$\frac{a^2\left(\sec(dx+c)-\frac{1}{12(-1+\sec(dx+c))^3}-\frac{5}{8(-1+\sec(dx+c))^2}-\frac{39}{16(-1+\sec(dx+c))}+\frac{9\ln(-1+\sec(dx+c))}{4}-\frac{1}{16(1+\sec(dx+c))}-\frac{\ln(1+\mathrm{s}}{}\right.}{d}$
default	$\frac{a^2\left(\sec(dx+c)-\frac{1}{12(-1+\sec(dx+c))^3}-\frac{5}{8(-1+\sec(dx+c))^2}-\frac{39}{16(-1+\sec(dx+c))}+\frac{9\ln(-1+\sec(dx+c))}{4}-\frac{1}{16(1+\sec(dx+c))}-\frac{\ln(1+\mathrm{s}}{}\right.}{d}$

norman	$\frac{\frac{a^2}{96d}+\frac{11a^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96d}+\frac{13a^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16d}+\frac{a^2\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32d}-\frac{95a^2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{9a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-$
risch	$\frac{a^2\left(15\,e^{9i(dx+c)}-48\,e^{8i(dx+c)}+32\,e^{7i(dx+c)}+40\,e^{6i(dx+c)}-62\,e^{5i(dx+c)}+40\,e^{4i(dx+c)}+32\,e^{3i(dx+c)}-48\,e^{2i(dx+c)}+15\,e^{i(dx+c)}\right.}{3d\left(e^{i(dx+c)}-1\right)^6\left(e^{i(dx+c)}+1\right)^2\left(e^{2i(dx+c)}+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^7*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*a^2*(sec(d*x+c)-1/12/(-1+sec(d*x+c))^3-5/8/(-1+sec(d*x+c))^2-39/16/(-1+
sec(d*x+c))+9/4*ln(-1+sec(d*x+c))-1/16/(1+sec(d*x+c))-1/4*ln(1+sec(d*x+c)))
```

Maxima [A]
time = 0.27, size = 143, normalized size = 0.89

$$-\frac{3\,a^2\log\left(\cos\left(dx+c\right)+1\right)-27\,a^2\log\left(\cos\left(dx+c\right)-1\right)+24\,a^2\log\left(\cos\left(dx+c\right)\right)-\frac{2\left(15\,a^2\cos\left(dx+c\right)^4-24\,a^2\cos\left(dx+c\right)^3-7\,a^2\cos\left(dx+c\right)^2+23\,a^2\cos\left(dx+c\right)-6\,a^2\right)}{\cos\left(dx+c\right)^5-2\,\cos\left(dx+c\right)^4+2\,\cos\left(dx+c\right)^2-\cos\left(dx+c\right)}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^7*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/12*(3*a^2*log(cos(d*x + c) + 1) - 27*a^2*log(cos(d*x + c) - 1) + 24*a^2*
log(cos(d*x + c)) - 2*(15*a^2*cos(d*x + c)^4 - 24*a^2*cos(d*x + c)^3 - 7*a^
2*cos(d*x + c)^2 + 23*a^2*cos(d*x + c) - 6*a^2)/(cos(d*x + c)^5 - 2*cos(d*x
 + c)^4 + 2*cos(d*x + c)^2 - cos(d*x + c)))/d
```

Fricas [A]
time = 3.71, size = 289, normalized size = 1.81

$$\frac{30\,a^2\cos\left(dx+c\right)^4-48\,a^2\cos\left(dx+c\right)^3-14\,a^2\cos\left(dx+c\right)^2+46\,a^2\cos\left(dx+c\right)-12\,a^2-24\left(a^2\cos\left(dx+c\right)^5-2\,a^2\cos\left(dx+c\right)^4+2\,a^2\cos\left(dx+c\right)^2-a^2\cos\left(dx+c\right)\right)\log\left(-\cos\left(dx+c\right)\right)-3\left(a^2\cos\left(dx+c\right)^5-2\,a^2\cos\left(dx+c\right)^4+2\,a^2\cos\left(dx+c\right)^2-a^2\cos\left(dx+c\right)\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+27\left(a^2\cos\left(dx+c\right)^5-2\,a^2\cos\left(dx+c\right)^4+2\,a^2\cos\left(dx+c\right)^2-a^2\cos\left(dx+c\right)\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{12\left(d\cos\left(dx+c\right)^5-2\,d\cos\left(dx+c\right)^4+2\,d\cos\left(dx+c\right)^2-d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^7*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(30*a^2*cos(d*x + c)^4 - 48*a^2*cos(d*x + c)^3 - 14*a^2*cos(d*x + c)^2
 + 46*a^2*cos(d*x + c) - 12*a^2 - 24*(a^2*cos(d*x + c)^5 - 2*a^2*cos(d*x +
c)^4 + 2*a^2*cos(d*x + c)^2 - a^2*cos(d*x + c))*log(-cos(d*x + c)) - 3*(a^2
*cos(d*x + c)^5 - 2*a^2*cos(d*x + c)^4 + 2*a^2*cos(d*x + c)^2 - a^2*cos(d*x
 + c))*log(1/2*cos(d*x + c) + 1/2) + 27*(a^2*cos(d*x + c)^5 - 2*a^2*cos(d*x
 + c)^4 + 2*a^2*cos(d*x + c)^2 - a^2*cos(d*x + c))*log(-1/2*cos(d*x + c) +
1/2))/(d*cos(d*x + c)^5 - 2*d*cos(d*x + c)^4 + 2*d*cos(d*x + c)^2 - d*cos(d
*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**7*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [A]
time = 0.69, size = 238, normalized size = 1.49

$$\frac{216 a^2 \log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - 192 a^2 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) - \frac{3a^2(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{\left(a^2 - \frac{12a^2(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{90a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} - \frac{396a^2(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}\right)(\cos(dx+c)+1)^3}{(\cos(dx+c)-1)^3} + \frac{192\left(2a^2+\frac{a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1}}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^7*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/96*(216*a^2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 192*a^2*l
og(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - 3*a^2*(cos(d*x + c) -
 1)/(cos(d*x + c) + 1) + (a^2 - 12*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1
) + 90*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 396*a^2*(cos(d*x + c
) - 1)^3/(cos(d*x + c) + 1)^3)*(cos(d*x + c) + 1)^3/(cos(d*x + c) - 1)^3 +
192*(2*a^2 + a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((cos(d*x + c) - 1)
/(cos(d*x + c) + 1) + 1))/d
```

Mupad [B]
time = 0.11, size = 147, normalized size = 0.92

$$\frac{9a^2\ln(\cos(c+d\,x)-1)}{4\,d} - \frac{a^2\ln(\cos(c+d\,x)+1)}{4\,d} - \frac{2\,a^2\ln(\cos(c+d\,x))}{d} + \frac{-\frac{5\,a^2\cos(c+d\,x)^4}{2}+4\,a^2\cos(c+d\,x)^3+\frac{7\,a^2\cos(c+d\,x)^2}{6}-\frac{23\,a^2\cos(c+d\,x)}{6}+a^2}{d\left(-\cos(c+d\,x)^5+2\cos(c+d\,x)^4-2\cos(c+d\,x)^2+\cos(c+d\,x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/sin(c + d*x)^7,x)
```

```
[Out] (9*a^2*log(cos(c + d*x) - 1))/(4*d) - (a^2*log(cos(c + d*x) + 1))/(4*d) - (
2*a^2*log(cos(c + d*x)))/d + (a^2 - (23*a^2*cos(c + d*x))/6 + (7*a^2*cos(c
+ d*x)^2)/6 + 4*a^2*cos(c + d*x)^3 - (5*a^2*cos(c + d*x)^4)/2)/(d*(cos(c +
d*x) - 2*cos(c + d*x)^2 + 2*cos(c + d*x)^4 - cos(c + d*x)^5))
```

3.28 $\int \csc^9(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=205

$$-\frac{a^6}{32d(a-a\cos(c+dx))^4}-\frac{7a^5}{48d(a-a\cos(c+dx))^3}-\frac{15a^4}{32d(a-a\cos(c+dx))^2}-\frac{51a^3}{32d(a-a\cos(c+dx))}+\frac{}{64d}$$

```
[Out] -1/32*a^6/d/(a-a*cos(d*x+c))^4-7/48*a^5/d/(a-a*cos(d*x+c))^3-15/32*a^4/d/(a
-a*cos(d*x+c))^2-51/32*a^3/d/(a-a*cos(d*x+c))+1/64*a^4/d/(a+a*cos(d*x+c))^2
+9/64*a^3/d/(a+a*cos(d*x+c))+303/128*a^2*ln(1-cos(d*x+c))/d-2*a^2*ln(cos(d*
x+c))/d-47/128*a^2*ln(1+cos(d*x+c))/d+a^2*sec(d*x+c)/d
```

Rubi [A]
time = 0.18, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3957, 2915, 12, 90}

$$-\frac{a^6}{32d(a-a\cos(c+dx))^4}-\frac{7a^5}{48d(a-a\cos(c+dx))^3}-\frac{15a^4}{32d(a-a\cos(c+dx))^2}+\frac{a^4}{64d(a\cos(c+dx)+a)^2}-\frac{51a^3}{32d(a-a\cos(c+dx))}+\frac{9a^3}{64d(a\cos(c+dx)+a)}+\frac{a^2\sec(c+dx)}{d}+\frac{303a^2\log(1-\cos(c+dx))}{128d}-\frac{2a^2\log(\cos(c+dx))}{d}-\frac{47a^2\log(\cos(c+dx)+1)}{128d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^9*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/32*a^6/(d*(a - a*Cos[c + d*x])^4) - (7*a^5)/(48*d*(a - a*Cos[c + d*x])^3
) - (15*a^4)/(32*d*(a - a*Cos[c + d*x])^2) - (51*a^3)/(32*d*(a - a*Cos[c +
d*x])) + a^4/(64*d*(a + a*Cos[c + d*x])^2) + (9*a^3)/(64*d*(a + a*Cos[c + d
*x])) + (303*a^2*Log[1 - Cos[c + d*x]])/(128*d) - (2*a^2*Log[Cos[c + d*x]])
/d - (47*a^2*Log[1 + Cos[c + d*x]])/(128*d) + (a^2*Sec[c + d*x])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
```

```
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \csc^9(c+dx)(a+a\sec(c+dx))^2\,dx &= \int(-a-a\cos(c+dx))^2\csc^9(c+dx)\sec^2(c+dx)\,dx\\
&= \frac{a^9\text{Subst}\left(\int\frac{a^2}{(-a-x)^5x^2(-a+x)^3}\,dx,x,-a\cos(c+dx)\right)}{d}\\
&= \frac{a^{11}\text{Subst}\left(\int\frac{1}{(-a-x)^5x^2(-a+x)^3}\,dx,x,-a\cos(c+dx)\right)}{d}\\
&= \frac{a^{11}\text{Subst}\left(\int\left(\frac{1}{32a^7(a-x)^3}+\frac{9}{64a^8(a-x)^2}+\frac{47}{128a^9(a-x)}+\frac{1}{a^8x^2}-\frac{2}{a^9x}+\frac{}{8a^5}\right.\right.}{}\\
&= -\frac{a^6}{32d(a-a\cos(c+dx))^4}-\frac{7a^5}{48d(a-a\cos(c+dx))^3}-\frac{}{32d(a-a}\end{aligned}$$

Mathematica [A]
time = 2.28, size = 164, normalized size = 0.80

$$-\frac{a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(1224\csc^2\left(\frac{1}{2}(c+dx)\right)+180\csc^4\left(\frac{1}{2}(c+dx)\right)+28\csc^6\left(\frac{1}{2}(c+dx)\right)+3\csc^8\left(\frac{1}{2}(c+dx)\right)-6\left(18\sec^2\left(\frac{1}{2}(c+dx)\right)+\sec^4\left(\frac{1}{2}(c+dx)\right)+4\left(-47\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-128\log(\cos(c+dx))+303\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+64\sec(c+dx)\right)\right)\right)}{6144d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^9*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/6144*(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(1224*Csc[(c + d*x)/2]
^2 + 180*Csc[(c + d*x)/2]^4 + 28*Csc[(c + d*x)/2]^6 + 3*Csc[(c + d*x)/2]^8
- 6*(18*Sec[(c + d*x)/2]^2 + Sec[(c + d*x)/2]^4 + 4*(-47*Log[Cos[(c + d*x)/
2]] - 128*Log[Cos[c + d*x]] + 303*Log[Sin[(c + d*x)/2]] + 64*Sec[c + d*x]))
))/d
```

Maple [A]
time = 0.16, size = 112, normalized size = 0.55

method	result

derivativedivides	$-\frac{a^2\left(-\sec(dx+c)+\frac{1}{32(-1+\sec(dx+c))^4}+\frac{13}{48(-1+\sec(dx+c))^3}+\frac{35}{32(-1+\sec(dx+c))^2}+\frac{99}{32(-1+\sec(dx+c))}-\frac{303\ln(-1+\sec(dx+c))}{128}\right.}{d}$
default	$-\frac{a^2\left(-\sec(dx+c)+\frac{1}{32(-1+\sec(dx+c))^4}+\frac{13}{48(-1+\sec(dx+c))^3}+\frac{35}{32(-1+\sec(dx+c))^2}+\frac{99}{32(-1+\sec(dx+c))}-\frac{303\ln(-1+\sec(dx+c))}{128}\right.}{d}$
norman	$\frac{\frac{a^2}{512d}+\frac{37a^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1536d}+\frac{121a^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{768d}+\frac{233a^2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{256d}+\frac{19a^2\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{256d}+\frac{a^2\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{256d}-\frac{203a}{}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$
risch	$\frac{a^2\left(525\,e^{13i(dx+c)}-1716\,e^{12i(dx+c)}+214\,e^{11i(dx+c)}+4652\,e^{10i(dx+c)}-4173\,e^{9i(dx+c)}-2552\,e^{8i(dx+c)}+4564\,e^{7i(dx+c)}-25\right.}{96d\left(e^{i(dx+c)}-1\right)^8\left(e^{i(dx+c)}+1\right)^4\left(e^{2i(dx}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^9*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*a^2*(-sec(d*x+c)+1/32/(-1+sec(d*x+c))^4+13/48/(-1+sec(d*x+c))^3+35/32/
(-1+sec(d*x+c))^2+99/32/(-1+sec(d*x+c))-303/128*ln(-1+sec(d*x+c))-1/64/(1+s
ec(d*x+c))^2+11/64/(1+sec(d*x+c))+47/128*ln(1+sec(d*x+c)))
```

Maxima [A]
time = 0.27, size = 197, normalized size = 0.96

$$-\frac{141\,a^2\log\left(\cos\left(dx+c\right)+1\right)-909\,a^2\log\left(\cos\left(dx+c\right)-1\right)+768\,a^2\log\left(\cos\left(dx+c\right)\right)-\frac{2\left(525\,a^2\cos\left(dx+c\right)^6-858\,a^2\cos\left(dx+c\right)^5-734\,a^2\cos\left(dx+c\right)^4+1654\,a^2\cos\left(dx+c\right)^3-19\,a^2\cos\left(dx+c\right)^2-784\,a^2\cos\left(dx+c\right)+192\,a^2\right)}{\cos\left(dx+c\right)^7-2\cos\left(dx+c\right)^6-\cos\left(dx+c\right)^5+4\cos\left(dx+c\right)^4-\cos\left(dx+c\right)^3-2\cos\left(dx+c\right)^2+\cos\left(dx+c\right)}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^9*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/384*(141*a^2*log(cos(d*x + c) + 1) - 909*a^2*log(cos(d*x + c) - 1) + 768
*a^2*log(cos(d*x + c)) - 2*(525*a^2*cos(d*x + c)^6 - 858*a^2*cos(d*x + c)^5
 - 734*a^2*cos(d*x + c)^4 + 1654*a^2*cos(d*x + c)^3 - 19*a^2*cos(d*x + c)^2
 - 784*a^2*cos(d*x + c) + 192*a^2)/(cos(d*x + c)^7 - 2*cos(d*x + c)^6 - cos
(d*x + c)^5 + 4*cos(d*x + c)^4 - cos(d*x + c)^3 - 2*cos(d*x + c)^2 + cos(d*
x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(193) = 386$.
time = 2.82, size = 461, normalized size = 2.25

$$\frac{1050\,a^2\cos\left(dx+c\right)^6-1716\,a^2\cos\left(dx+c\right)^5-1468\,a^2\cos\left(dx+c\right)^4+3308\,a^2\cos\left(dx+c\right)^3-38\,a^2\cos\left(dx+c\right)^2-1568\,a^2\cos\left(dx+c\right)+384\,a^2-768\left(a^2\cos\left(dx+c\right)^7-2\,a^2\cos\left(dx+c\right)^6-a^2\cos\left(dx+c\right)^5+4\,a^2\cos\left(dx+c\right)^4-a^2\cos\left(dx+c\right)^3-2\,a^2\cos\left(dx+c\right)^2+a^2\cos\left(dx+c\right)\right)\log\left(-\cos\left(dx+c\right)\right)-141\left(a^2\cos\left(dx+c\right)^7-2\,a^2\cos\left(dx+c\right)^6-a^2\cos\left(dx+c\right)^5+4\,a^2\cos\left(dx+c\right)^4-a^2\cos\left(dx+c\right)^3-2\,a^2\cos\left(dx+c\right)^2+a^2\cos\left(dx+c\right)\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+909\left(a^2\cos\left(dx+c\right)^7-2\,a^2\cos\left(dx+c\right)^6-a^2\cos\left(dx+c\right)^5+4\,a^2\cos\left(dx+c\right)^4-a^2\cos\left(dx+c\right)^3-2\,a^2\cos\left(dx+c\right)^2+a^2\cos\left(dx+c\right)\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{384\left(d\cos\left(dx+c\right)^7-2\,d\cos\left(dx+c\right)^6-d\cos\left(dx+c\right)^5+4\,d\cos\left(dx+c\right)^4-d\cos\left(dx+c\right)^3-2\,d\cos\left(dx+c\right)^2+d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^9*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/384*(1050*a^2*cos(d*x + c)^6 - 1716*a^2*cos(d*x + c)^5 - 1468*a^2*cos(d*x
 + c)^4 + 3308*a^2*cos(d*x + c)^3 - 38*a^2*cos(d*x + c)^2 - 1568*a^2*cos(d*
x + c) + 384*a^2 - 768*(a^2*cos(d*x + c)^7 - 2*a^2*cos(d*x + c)^6 - a^2*cos
```

```
(d*x + c)^5 + 4*a^2*cos(d*x + c)^4 - a^2*cos(d*x + c)^3 - 2*a^2*cos(d*x + c
)^2 + a^2*cos(d*x + c))*log(-cos(d*x + c)) - 141*(a^2*cos(d*x + c)^7 - 2*a^
2*cos(d*x + c)^6 - a^2*cos(d*x + c)^5 + 4*a^2*cos(d*x + c)^4 - a^2*cos(d*x
+ c)^3 - 2*a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/
2) + 909*(a^2*cos(d*x + c)^7 - 2*a^2*cos(d*x + c)^6 - a^2*cos(d*x + c)^5 +
4*a^2*cos(d*x + c)^4 - a^2*cos(d*x + c)^3 - 2*a^2*cos(d*x + c)^2 + a^2*cos(
d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^7 - 2*d*cos(d*x + c
)^6 - d*cos(d*x + c)^5 + 4*d*cos(d*x + c)^4 - d*cos(d*x + c)^3 - 2*d*cos(d*
x + c)^2 + d*cos(d*x + c))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**9*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [**A**]
time = 0.76, size = 291, normalized size = 1.42

$$\frac{3636\,a^2\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-3072\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)-\frac{120\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{6\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{\left(3\,a^2-\frac{40\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{282\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{1680\,a^2(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{7575\,a^2(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}\right)(\cos(dx+c)+1)^4}{(\cos(dx+c)-1)^4}+\frac{3072\left(2\,a^2+\frac{a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1}}{1536\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^9*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/1536*(3636*a^2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 3072*a
^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - 120*a^2*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) + 6*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1
)^2 - (3*a^2 - 40*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 282*a^2*(cos(
d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 1680*a^2*(cos(d*x + c) - 1)^3/(cos(d
*x + c) + 1)^3 + 7575*a^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4)*(cos(d
*x + c) + 1)^4/(cos(d*x + c) - 1)^4 + 3072*(2*a^2 + a^2*(cos(d*x + c) - 1)/
(cos(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/d
```

Mupad [**B**]
time = 0.17, size = 203, normalized size = 0.99

$$\frac{-\frac{175\,a^2\cos(c+d\,x)^6}{64}+\frac{143\,a^2\cos(c+d\,x)^5}{32}+\frac{367\,a^2\cos(c+d\,x)^4}{96}-\frac{827\,a^2\cos(c+d\,x)^3}{96}+\frac{19\,a^2\cos(c+d\,x)^2}{192}+\frac{49\,a^2\,\cos(c+d\,x)}{12}-a^2}{d\,\left(-\cos\left(c+d\,x\right)^7+2\cos\left(c+d\,x\right)^6+\cos\left(c+d\,x\right)^5-4\cos\left(c+d\,x\right)^4+\cos\left(c+d\,x\right)^3+2\cos\left(c+d\,x\right)^2-\cos\left(c+d\,x\right)\right)}+\frac{303\,a^2\,\ln\left(\cos\left(c+d\,x\right)-1\right)}{128\,d}-\frac{47\,a^2\,\ln\left(\cos\left(c+d\,x\right)+1\right)}{128\,d}-\frac{2\,a^2\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/sin(c + d*x)^9,x)
```

```
[Out] ((49*a^2*cos(c + d*x))/12 - a^2 + (19*a^2*cos(c + d*x)^2)/192 - (827*a^2*co
s(c + d*x)^3)/96 + (367*a^2*cos(c + d*x)^4)/96 + (143*a^2*cos(c + d*x)^5)/3
2 - (175*a^2*cos(c + d*x)^6)/64)/(d*(2*cos(c + d*x)^2 - cos(c + d*x) + cos(
c + d*x)^3 - 4*cos(c + d*x)^4 + cos(c + d*x)^5 + 2*cos(c + d*x)^6 - cos(c +
 d*x)^7)) + (303*a^2*log(cos(c + d*x) - 1))/(128*d) - (47*a^2*log(cos(c + d
*x) + 1))/(128*d) - (2*a^2*log(cos(c + d*x)))/d
```

3.29 $\int (a+a\sec(c+dx))^2\sin^8(c+dx)\,dx$

Optimal. Leaf size=199

$$-\frac{245a^2x}{128}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{2a^2\sin(c+dx)}{d}+\frac{139a^2\cos(c+dx)\sin(c+dx)}{128d}+\frac{11a^2\cos^3(c+dx)\sin}{192d}$$

```
[Out] -245/128*a^2*x+2*a^2*arctanh(sin(d*x+c))/d-2*a^2*sin(d*x+c)/d+139/128*a^2*c
os(d*x+c)*sin(d*x+c)/d+11/192*a^2*cos(d*x+c)^3*sin(d*x+c)/d-17/48*a^2*cos(d
*x+c)^5*sin(d*x+c)/d+1/8*a^2*cos(d*x+c)^7*sin(d*x+c)/d-2/3*a^2*sin(d*x+c)^3
/d-2/5*a^2*sin(d*x+c)^5/d-2/7*a^2*sin(d*x+c)^7/d+a^2*tan(d*x+c)/d
```

Rubi [A]
time = 0.27, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3957, 2951, 2717, 2715, 8, 2713, 3855, 3852}

$$-\frac{2a^2\sin^7(c+dx)}{7d}-\frac{2a^2\sin^5(c+dx)}{5d}-\frac{2a^2\sin^3(c+dx)}{3d}-\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\tan(c+dx)}{d}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2\sin(c+dx)\cos^7(c+dx)}{8d}-\frac{17a^2\sin(c+dx)\cos^5(c+dx)}{48d}+\frac{11a^2\sin(c+dx)\cos^3(c+dx)}{192d}+\frac{139a^2\sin(c+dx)\cos(c+dx)}{128d}-\frac{245a^2x}{128}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^8,x]
```

```
[Out] (-245*a^2*x)/128 + (2*a^2*ArcTanh[Sin[c + d*x]])/d - (2*a^2*Sin[c + d*x])/d
 + (139*a^2*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (11*a^2*Cos[c + d*x]^3*Sin
[c + d*x])/(192*d) - (17*a^2*Cos[c + d*x]^5*Sin[c + d*x])/(48*d) + (a^2*Cos
[c + d*x]^7*Sin[c + d*x])/(8*d) - (2*a^2*Sin[c + d*x]^3)/(3*d) - (2*a^2*Sin
[c + d*x]^5)/(5*d) - (2*a^2*Sin[c + d*x]^7)/(7*d) + (a^2*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+a\sec(c+dx))^2\sin^8(c+dx)\,dx = \int (-a-a\cos(c+dx))^2\sin^6(c+dx)\tan^2(c+dx)\,dx$$
$$= \frac{\int \left(-3a^{10}-8a^{10}\cos(c+dx)+2a^{10}\cos^2(c+dx)+12a^{10}\cos^3(c+\right.}{}$$
$$= -3a^2x+a^2\int \cos^8(c+dx)\,dx+a^2\int \sec^2(c+dx)\,dx+\left(2a^2\right)\int \mathrm{c}$$
$$= -3a^2x+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{8a^2\sin(c+dx)}{d}+\frac{a^2\cos(c+}{}$$
$$= -2a^2x+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{2a^2\sin(c+dx)}{d}+\frac{7a^2\cos(c+}{}$$
$$= -\frac{5a^2x}{4}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{2a^2\sin(c+dx)}{d}+\frac{13a^2\cos(c}{}$$
$$= -\frac{35a^2x}{16}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{2a^2\sin(c+dx)}{d}+\frac{139a^2\cos}{}$$
$$= -\frac{245a^2x}{128}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{2a^2\sin(c+dx)}{d}+\frac{139a^2\mathrm{co}}{}$$

Mathematica [A]
time = 0.63, size = 144, normalized size = 0.72

$$-\frac{a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(168000c+168000dx+37800\text{ArcTan}(\tan(c+dx))-215040\tanh^{-1}(\sin(c+dx))+215040\sin(c+dx)+71680\sin^3(c+dx)+43008\sin^5(c+dx)+30720\sin^7(c+dx)-55440\sin(2(c+dx))+2520\sin(4(c+dx))+560\sin(6(c+dx))-105\sin(8(c+dx))-107520\tan(c+dx)\right)}{430080d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^8,x]
```

```
[Out] -1/430080*(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(168000*c + 168000*d
*x + 37800*ArcTan[Tan[c + d*x]] - 215040*ArcTanh[Sin[c + d*x]] + 215040*Sin
[c + d*x] + 71680*Sin[c + d*x]^3 + 43008*Sin[c + d*x]^5 + 30720*Sin[c + d*x
]^7 - 55440*Sin[2*(c + d*x)] + 2520*Sin[4*(c + d*x)] + 560*Sin[6*(c + d*x)]
 - 105*Sin[8*(c + d*x)] - 107520*Tan[c + d*x]))/d
```

Maple [A]
time = 0.09, size = 194, normalized size = 0.97

method	result
derivativedivides	$\frac{a^2\left(\frac{\sin^9(dx+c)}{\cos(dx+c)}+\left(\sin^7(dx+c)+\frac{7\left(\sin^5(dx+c)\right)}{6}+\frac{35\left(\sin^3(dx+c)\right)}{24}+\frac{35\sin(dx+c)}{16}\right)\cos(dx+c)-\frac{35dx}{16}-\frac{35c}{16}\right)+2a^2\left(-\frac{\left(\sin^7(d\right.}{7}\right.}{}$

default	$\frac{a^2\left(\frac{\sin^9(dx+c)}{\cos(dx+c)}+\left(\sin^7(dx+c)+\frac{7\left(\sin^5(dx+c)\right)}{6}+\frac{35\left(\sin^3(dx+c)\right)}{24}+\frac{35\sin(dx+c)}{16}\right)\cos(dx+c)-\frac{35dx}{16}-\frac{35c}{16}\right)+2a^2\left(-\frac{\left(\sin^7(dx-\right.}{7}\right.}{}$
risch	$-\frac{245a^2x}{128}+\frac{93ia^2\mathrm{e}^{i(dx+c)}}{64d}-\frac{33ia^2\mathrm{e}^{2i(dx+c)}}{128d}-\frac{93ia^2\mathrm{e}^{-i(dx+c)}}{64d}+\frac{33ia^2\mathrm{e}^{-2i(dx+c)}}{128d}+\frac{2ia^2}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)}-\frac{2a^2\ln(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*sin(d*x+c)^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(sin(d*x+c)^9/cos(d*x+c)+(sin(d*x+c)^7+7/6*sin(d*x+c)^5+35/24*sin(
d*x+c)^3+35/16*sin(d*x+c))*cos(d*x+c)-35/16*d*x-35/16*c)+2*a^2*(-1/7*sin(d*
x+c)^7-1/5*sin(d*x+c)^5-1/3*sin(d*x+c)^3-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c
)))+a^2*(-1/8*(sin(d*x+c)^7+7/6*sin(d*x+c)^5+35/24*sin(d*x+c)^3+35/16*sin(d
*x+c))*cos(d*x+c)+35/128*d*x+35/128*c))
```

Maxima [A]
time = 0.49, size = 215, normalized size = 1.08

$$-\frac{1024\left(30\sin\left(dx+c\right)^7+42\sin\left(dx+c\right)^5+70\sin\left(dx+c\right)^3-105\log\left(\sin\left(dx+c\right)+1\right)+105\log\left(\sin\left(dx+c\right)-1\right)+210\sin\left(dx+c\right)\right)a^2-35\left(128\sin\left(2dx+2c\right)^3+840dx+840c+3\sin\left(8dx+8c\right)+168\sin\left(4dx+4c\right)-768\sin\left(2dx+2c\right)\right)a^2+2240\left(105dx+105c-\frac{87\tan(dx+c)^5+136\tan(dx+c)^3+57\tan(dx+c)}{\tan(dx+c)^6+3\tan(dx+c)^4+3\tan(dx+c)^2+1}-48\tan\left(dx+c\right)\right)a^2}{107520d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] -1/107520*(1024*(30*sin(d*x + c)^7 + 42*sin(d*x + c)^5 + 70*sin(d*x + c)^3
- 105*log(sin(d*x + c) + 1) + 105*log(sin(d*x + c) - 1) + 210*sin(d*x + c))
*a^2 - 35*(128*sin(2*d*x + 2*c)^3 + 840*d*x + 840*c + 3*sin(8*d*x + 8*c) +
168*sin(4*d*x + 4*c) - 768*sin(2*d*x + 2*c))*a^2 + 2240*(105*d*x + 105*c -
(87*tan(d*x + c)^5 + 136*tan(d*x + c)^3 + 57*tan(d*x + c))/(tan(d*x + c)^6
+ 3*tan(d*x + c)^4 + 3*tan(d*x + c)^2 + 1) - 48*tan(d*x + c))*a^2)/d
```

Fricas [A]
time = 2.46, size = 185, normalized size = 0.93

$$-\frac{25725a^2dx\cos\left(dx+c\right)-13440a^2\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)+13440a^2\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right)-\left(1680a^2\cos\left(dx+c\right)^8+3840a^2\cos\left(dx+c\right)^7-4760a^2\cos\left(dx+c\right)^6-16896a^2\cos\left(dx+c\right)^5+770a^2\cos\left(dx+c\right)^4+31232a^2\cos\left(dx+c\right)^3+14595a^2\cos\left(dx+c\right)^2-45056a^2\cos\left(dx+c\right)+13440a^2\right)\sin\left(dx+c\right)}{13440d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] -1/13440*(25725*a^2*d*x*cos(d*x + c) - 13440*a^2*cos(d*x + c)*log(sin(d*x +
 c) + 1) + 13440*a^2*cos(d*x + c)*log(-sin(d*x + c) + 1) - (1680*a^2*cos(d*
x + c)^8 + 3840*a^2*cos(d*x + c)^7 - 4760*a^2*cos(d*x + c)^6 - 16896*a^2*co
s(d*x + c)^5 + 770*a^2*cos(d*x + c)^4 + 31232*a^2*cos(d*x + c)^3 + 14595*a^
2*cos(d*x + c)^2 - 45056*a^2*cos(d*x + c) + 13440*a^2)*sin(d*x + c))/(d*cos
(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*sin(d*x+c)**8,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [A]
time = 0.56, size = 225, normalized size = 1.13

$$-\frac{25725\,(dx+c)a^2 - 26880\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) + 26880\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{26880\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1} + \frac{2\left(39165\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}+300265\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+989261\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+1791073\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+1814943\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+670131\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+147735\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+14595\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^8}}{13440\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^8,x, algorithm="giac")
```

```
[Out] -1/13440*(25725*(d*x + c)*a^2 - 26880*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)
) + 26880*a^2*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 26880*a^2*tan(1/2*d*x +
1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) + 2*(39165*a^2*tan(1/2*d*x + 1/2*c)^15
+ 300265*a^2*tan(1/2*d*x + 1/2*c)^13 + 989261*a^2*tan(1/2*d*x + 1/2*c)^11 +
 1791073*a^2*tan(1/2*d*x + 1/2*c)^9 + 1814943*a^2*tan(1/2*d*x + 1/2*c)^7 +
670131*a^2*tan(1/2*d*x + 1/2*c)^5 + 147735*a^2*tan(1/2*d*x + 1/2*c)^3 + 145
95*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^8)/d
```

Mupad [B]
time = 2.54, size = 293, normalized size = 1.47

$$\frac{4\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{245\,a^2\,x}{128}+\frac{\frac{501\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}}{64}+\frac{2633\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{48}+\frac{38047\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{240}+\frac{388613\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{1680}+\frac{13781\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{96}-\frac{32681\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{560}-\frac{1739\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{80}-\frac{61\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{16}-\frac{11\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64}}{d\,\left(-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{18}-7\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{16}-20\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{14}-28\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}-14\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+14\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{8}+28\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{6}+20\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{4}+7\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^8*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (4*a^2*atanh(tan(c/2 + (d*x)/2)))/d - (245*a^2*x)/128 + ((13781*a^2*tan(c/2
 + (d*x)/2)^9)/96 - (1739*a^2*tan(c/2 + (d*x)/2)^5)/80 - (32681*a^2*tan(c/2
 + (d*x)/2)^7)/560 - (61*a^2*tan(c/2 + (d*x)/2)^3)/16 + (388613*a^2*tan(c/2
 + (d*x)/2)^11)/1680 + (38047*a^2*tan(c/2 + (d*x)/2)^13)/240 + (2633*a^2*ta
n(c/2 + (d*x)/2)^15)/48 + (501*a^2*tan(c/2 + (d*x)/2)^17)/64 - (11*a^2*tan(
c/2 + (d*x)/2))/64)/(d*(7*tan(c/2 + (d*x)/2)^2 + 20*tan(c/2 + (d*x)/2)^4 +
28*tan(c/2 + (d*x)/2)^6 + 14*tan(c/2 + (d*x)/2)^8 - 14*tan(c/2 + (d*x)/2)^1
0 - 28*tan(c/2 + (d*x)/2)^12 - 20*tan(c/2 + (d*x)/2)^14 - 7*tan(c/2 + (d*x)
/2)^16 - tan(c/2 + (d*x)/2)^18 + 1))
```

3.30 $\int (a+a\sec(c+dx))^2\sin^6(c+dx)\,dx$

Optimal. Leaf size=157

$$-\frac{25a^2x}{16}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{2a^2\sin(c+dx)}{d}+\frac{7a^2\cos(c+dx)\sin(c+dx)}{16d}+\frac{7a^2\cos^3(c+dx)\sin(c+d$$

```
[Out] -25/16*a^2*x+2*a^2*arctanh(sin(d*x+c))/d-2*a^2*sin(d*x+c)/d+7/16*a^2*cos(d*
x+c)*sin(d*x+c)/d+7/24*a^2*cos(d*x+c)^3*sin(d*x+c)/d-1/6*a^2*cos(d*x+c)^5*s
in(d*x+c)/d-2/3*a^2*sin(d*x+c)^3/d-2/5*a^2*sin(d*x+c)^5/d+a^2*tan(d*x+c)/d
```

Rubi [A]
time = 0.20, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3957, 2951, 2717, 2713, 2715, 8, 3855, 3852}

$$-\frac{2a^2\sin^5(c+dx)}{5d}-\frac{2a^2\sin^3(c+dx)}{3d}-\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\tan(c+dx)}{d}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^2\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{7a^2\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{7a^2\sin(c+dx)\cos(c+dx)}{16d}-\frac{25a^2x}{16}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^6,x]
```

```
[Out] (-25*a^2*x)/16 + (2*a^2*ArcTanh[Sin[c + d*x]])/d - (2*a^2*Sin[c + d*x])/d +
 (7*a^2*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (7*a^2*Cos[c + d*x]^3*Sin[c + d
*x])/(24*d) - (a^2*Cos[c + d*x]^5*Sin[c + d*x])/(6*d) - (2*a^2*Sin[c + d*x]
^3)/(3*d) - (2*a^2*Sin[c + d*x]^5)/(5*d) + (a^2*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+a\sec(c+dx))^2\sin^6(c+dx)\,dx=\int(-a-a\cos(c+dx))^2\sin^4(c+dx)\tan^2(c+dx)\,dx$$

$$=\frac{\int\left(-2a^8-6a^8\cos(c+dx)+6a^8\cos^3(c+dx)+2a^8\cos^4(c+dx)\right.}{}$$

$$=-2a^2x-a^2\int\cos^6(c+dx)\,dx+a^2\int\sec^2(c+dx)\,dx+\left(2a^2\right)\int c$$

$$=-2a^2x+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{6a^2\sin(c+dx)}{d}+\frac{a^2\cos^3(c+}{}$$

$$=-2a^2x+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{2a^2\sin(c+dx)}{d}+\frac{3a^2\cos(c+}{}$$

$$=-\frac{5a^2x}{4}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{2a^2\sin(c+dx)}{d}+\frac{7a^2\cos(c+}{}$$

$$=-\frac{25a^2x}{16}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{2a^2\sin(c+dx)}{d}+\frac{7a^2\cos(c}{}$$

Mathematica [A]

time = 0.35, size = 124, normalized size = 0.79

$$-\frac{a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(1080c+1080dx+420\text{ArcTan}(\tan(c+dx))-1920\tanh^{-1}(\sin(c+dx))+1920\sin(c+dx)+640\sin^3(c+dx)+384\sin^5(c+dx)-255\sin(2(c+dx))-15\sin(4(c+dx))+5\sin(6(c+dx))-960\tan(c+dx)\right)}{3840d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^6,x]
```

```
[Out] -1/3840*(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(1080*c + 1080*d*x + 4
20*ArcTan[Tan[c + d*x]] - 1920*ArcTanh[Sin[c + d*x]] + 1920*Sin[c + d*x] +
640*Sin[c + d*x]^3 + 384*Sin[c + d*x]^5 - 255*Sin[2*(c + d*x)] - 15*Sin[4*(
c + d*x)] + 5*Sin[6*(c + d*x)] - 960*Tan[c + d*x]))/d
```

Maple [A]

time = 0.12, size = 164, normalized size = 1.04

method	result
derivativedivides	$\frac{a^2\left(\frac{\sin^7(dx+c)}{\cos(dx+c)}+\left(\sin^5(dx+c)+\frac{5\left(\sin^3(dx+c)\right)}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)-\frac{15dx}{8}-\frac{15c}{8}\right)+2a^2\left(-\frac{\left(\sin^5(dx+c)\right)}{5}-\frac{\left(\sin^3(dx+c\right.}{3}\right.}{d}$
default	$\frac{a^2\left(\frac{\sin^7(dx+c)}{\cos(dx+c)}+\left(\sin^5(dx+c)+\frac{5\left(\sin^3(dx+c)\right)}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)-\frac{15dx}{8}-\frac{15c}{8}\right)+2a^2\left(-\frac{\left(\sin^5(dx+c)\right)}{5}-\frac{\left(\sin^3(dx+c\right.}{3}\right.}{d}$
risch	$-\frac{25a^2x}{16}+\frac{11ia^2e^{i(dx+c)}}{8d}-\frac{17ia^2e^{2i(dx+c)}}{128d}-\frac{11ia^2e^{-i(dx+c)}}{8d}+\frac{2ia^2}{d\left(e^{2i(dx+c)}+1\right)}+\frac{17ia^2e^{-2i(dx+c)}}{128d}-\frac{2a^2\ln\left(e\right.}{}$
norman	$\frac{\frac{25a^2x}{16}+\frac{7a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}+\frac{27a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{797a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{40d}-\frac{91a^2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{8041a^2\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{120d}-\frac{431a^2}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*sin(d*x+c)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(sin(d*x+c)^7/cos(d*x+c)+(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d
*x+c))*cos(d*x+c)-15/8*d*x-15/8*c)+2*a^2*(-1/5*sin(d*x+c)^5-1/3*sin(d*x+c)^
3-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+a^2*(-1/6*(sin(d*x+c)^5+5/4*sin(d*x
+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/16*d*x+5/16*c))
```

Maxima [A]

time = 0.48, size = 174, normalized size = 1.11

$$-\frac{64\left(6\sin(dx+c)^5+10\sin(dx+c)^3-15\log(\sin(dx+c)+1)+15\log(\sin(dx+c)-1)+30\sin(dx+c)\right)a^2-5\left(4\sin(2dx+2c)^3+60dx+60c+9\sin(4dx+4c)-48\sin(2dx+2c)\right)a^2+120\left(15dx+15c-\frac{9\tan(dx+c)^3+7\tan(dx+c)}{\tan(dx+c)^4+2\tan(dx+c)^2+1}-8\tan(dx+c)\right)a^2}{960d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] -1/960*(64*(6*sin(d*x + c)^5 + 10*sin(d*x + c)^3 - 15*log(sin(d*x + c) + 1)
 + 15*log(sin(d*x + c) - 1) + 30*sin(d*x + c))*a^2 - 5*(4*sin(2*d*x + 2*c)^
3 + 60*d*x + 60*c + 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^2 + 120*(15
*d*x + 15*c - (9*tan(d*x + c)^3 + 7*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d
*x + c)^2 + 1) - 8*tan(d*x + c))*a^2)/d
```

Fricas [A]
time = 2.56, size = 158, normalized size = 1.01

$$-\frac{375\,a^2dx\cos\left(dx+c\right)-240\,a^2\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)+240\,a^2\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right)+\left(40\,a^2\cos\left(dx+c\right)^6+96\,a^2\cos\left(dx+c\right)^5-70\,a^2\cos\left(dx+c\right)^4-352\,a^2\cos\left(dx+c\right)^3-105\,a^2\cos\left(dx+c\right)^2+736\,a^2\cos\left(dx+c\right)-240\,a^2\right)\sin\left(dx+c\right)}{240\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] -1/240*(375*a^2*d*x*cos(d*x + c) - 240*a^2*cos(d*x + c)*log(sin(d*x + c) +
1) + 240*a^2*cos(d*x + c)*log(-sin(d*x + c) + 1) + (40*a^2*cos(d*x + c)^6 +
 96*a^2*cos(d*x + c)^5 - 70*a^2*cos(d*x + c)^4 - 352*a^2*cos(d*x + c)^3 - 1
05*a^2*cos(d*x + c)^2 + 736*a^2*cos(d*x + c) - 240*a^2)*sin(d*x + c))/(d*co
s(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*sin(d*x+c)**6,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.61, size = 193, normalized size = 1.23

$$-\frac{375\,(dx+c)a^2-480\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)+480\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{480\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}+\frac{2\left(615\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+3485\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+7926\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+8586\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+2595\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+345\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^6}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^6,x, algorithm="giac")
```

```
[Out] -1/240*(375*(d*x + c)*a^2 - 480*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) + 48
0*a^2*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 480*a^2*tan(1/2*d*x + 1/2*c)/(ta
n(1/2*d*x + 1/2*c)^2 - 1) + 2*(615*a^2*tan(1/2*d*x + 1/2*c)^11 + 3485*a^2*t
an(1/2*d*x + 1/2*c)^9 + 7926*a^2*tan(1/2*d*x + 1/2*c)^7 + 8586*a^2*tan(1/2*
d*x + 1/2*c)^5 + 2595*a^2*tan(1/2*d*x + 1/2*c)^3 + 345*a^2*tan(1/2*d*x + 1/
2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

Mupad [B]
time = 2.16, size = 235, normalized size = 1.50

$$\frac{\frac{57\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{8}+\frac{431\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{12}+\frac{8041\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{120}+\frac{91\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{2}-\frac{797\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{40}-\frac{27\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{4}-\frac{7\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}}{d\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)}-\frac{25\,a^2\,x}{16}+\frac{4\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6*(a + a/cos(c + d*x))^2,x)
```

```
[Out] ((91*a^2*tan(c/2 + (d*x)/2)^7)/2 - (797*a^2*tan(c/2 + (d*x)/2)^5)/40 - (27*
a^2*tan(c/2 + (d*x)/2)^3)/4 + (8041*a^2*tan(c/2 + (d*x)/2)^9)/120 + (431*a^
2*tan(c/2 + (d*x)/2)^11)/12 + (57*a^2*tan(c/2 + (d*x)/2)^13)/8 - (7*a^2*tan
(c/2 + (d*x)/2))/8)/(d*(5*tan(c/2 + (d*x)/2)^2 + 9*tan(c/2 + (d*x)/2)^4 + 5
*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 - 9*tan(c/2 + (d*x)/2)^10 -
5*tan(c/2 + (d*x)/2)^12 - tan(c/2 + (d*x)/2)^14 + 1)) - (25*a^2*x)/16 + (4*
a^2*atanh(tan(c/2 + (d*x)/2)))/d
```

3.31 $\int (a+a\sec(c+dx))^2 \sin^4(c+dx)\,dx$

Optimal. Leaf size=115

$$-\frac{9a^2x}{8}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{2a^2\sin(c+dx)}{d}-\frac{a^2\cos(c+dx)\sin(c+dx)}{8d}+\frac{a^2\cos^3(c+dx)\sin(c+dx)}{4d}$$

```
[Out] -9/8*a^2*x+2*a^2*arctanh(sin(d*x+c))/d-2*a^2*sin(d*x+c)/d-1/8*a^2*cos(d*x+c
)*sin(d*x+c)/d+1/4*a^2*cos(d*x+c)^3*sin(d*x+c)/d-2/3*a^2*sin(d*x+c)^3/d+a^2
*tan(d*x+c)/d
```

Rubi [A]
time = 0.20, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3957, 2951, 2717, 2715, 8, 2713, 3855, 3852}

$$-\frac{2a^2\sin^3(c+dx)}{3d}-\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\tan(c+dx)}{d}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{4d}-\frac{a^2\sin(c+dx)\cos(c+dx)}{8d}-\frac{9a^2x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^4,x]
```

```
[Out] (-9*a^2*x)/8 + (2*a^2*ArcTanh[Sin[c + d*x]])/d - (2*a^2*Sin[c + d*x])/d - (
a^2*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^2*Cos[c + d*x]^3*Sin[c + d*x])/(4
*d) - (2*a^2*Sin[c + d*x]^3)/(3*d) + (a^2*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+a\sec(c+dx))^2\sin^4(c+dx)\,dx = \int (-a-a\cos(c+dx))^2\sin^2(c+dx)\tan^2(c+dx)\,dx$$
$$= \frac{\int \left(-a^6-4a^6\cos(c+dx)-a^6\cos^2(c+dx)+2a^6\cos^3(c+dx)+a^6$$
$$a^4$$
$$= -a^2x-a^2\int \cos^2(c+dx)\,dx+a^2\int \cos^4(c+dx)\,dx+a^2\int \sec^2(c$$
$$= -a^2x+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{4a^2\sin(c+dx)}{d}-\frac{a^2\cos(c+dx)}{2d}$$
$$= -\frac{3a^2x}{2}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{2a^2\sin(c+dx)}{d}-\frac{a^2\cos(c+d}{8}$$
$$= -\frac{9a^2x}{8}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{2a^2\sin(c+dx)}{d}-\frac{a^2\cos(c+d}{8}$$

Mathematica [A]
time = 0.18, size = 94, normalized size = 0.82

$$-\frac{a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(48c+48dx+60\text{ArcTan}(\tan(c+dx))-192\tanh^{-1}(\sin(c+dx))+192\sin(c+dx)+64\sin^3(c+dx)-3\sin(4(c+dx))-96\tan(c+dx)\right)}{384d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^4,x]
```

```
[Out] -1/384*(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(48*c + 48*d*x + 60*Arc
Tan[Tan[c + d*x]] - 192*ArcTanh[Sin[c + d*x]] + 192*Sin[c + d*x] + 64*Sin[c
 + d*x]^3 - 3*Sin[4*(c + d*x)] - 96*Tan[c + d*x]))/d
```

Maple [A]
time = 0.10, size = 134, normalized size = 1.17

method	result
derivativedivides	$\frac{a^2\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)+2a^2\left(-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\mathrm{ta}\right.}{d}$
default	$\frac{a^2\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)+2a^2\left(-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\mathrm{ta}\right.}{d}$
risch	$-\frac{9a^2x}{8}+\frac{5ia^2e^{i(dx+c)}}{4d}-\frac{5ia^2e^{-i(dx+c)}}{4d}+\frac{2ia^2}{d\left(e^{2i(dx+c)}+1\right)}-\frac{2a^2\ln\left(e^{i(dx+c)}-i\right)}{d}+\frac{2a^2\ln\left(e^{i(dx+c)}+i\right)}{d}+\frac{a^2}{}$
norman	$\frac{\frac{9a^2x}{8}+\frac{7a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{22a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{31a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{58a^2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{25a^2\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{27a^2x(\mathrm{t}}{}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(1+\tan^2\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*sin(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-
3/2*d*x-3/2*c)+2*a^2*(-1/3*sin(d*x+c)^3-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)
))+a^2*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A]
time = 0.49, size = 126, normalized size = 1.10

$$-\frac{32\,{\left(2\,\sin\left(dx+c\right)^{3}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)+6\,\sin\left(dx+c\right)\right)}a^{2}-3\,{\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)-8\,\sin\left(2\,dx+2\,c\right)\right)}a^{2}+48\,{\left(3\,dx+3\,c-\frac{\tan\left(dx+c\right)}{\tan\left(dx+c\right)^{2}+1}-2\,\tan\left(dx+c\right)\right)}a^{2}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/96*(32*(2*sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c)
- 1) + 6*sin(d*x + c))*a^2 - 3*(12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*
d*x + 2*c))*a^2 + 48*(3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*t
an(d*x + c))*a^2)/d
```

Fricas [A]
time = 2.55, size = 133, normalized size = 1.16

$$-\frac{27\,a^{2}dx\cos\left(dx+c\right)-24\,a^{2}\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)+24\,a^{2}\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right)-{\left(6\,a^{2}\cos\left(dx+c\right)^{4}+16\,a^{2}\cos\left(dx+c\right)^{3}-3\,a^{2}\cos\left(dx+c\right)^{2}-64\,a^{2}\cos\left(dx+c\right)+24\,a^{2}\right)}\sin\left(dx+c\right)}{24\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/24*(27*a^2*d*x*cos(d*x + c) - 24*a^2*cos(d*x + c)*log(sin(d*x + c) + 1)
+ 24*a^2*cos(d*x + c)*log(-sin(d*x + c) + 1) - (6*a^2*cos(d*x + c)^4 + 16*a
^2*cos(d*x + c)^3 - 3*a^2*cos(d*x + c)^2 - 64*a^2*cos(d*x + c) + 24*a^2)*si
n(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int 2\sin^{4}\left(c+dx\right)\sec\left(c+dx\right)dx+\int\sin^{4}\left(c+dx\right)\sec^{2}\left(c+dx\right)dx+\int\sin^{4}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*sin(d*x+c)**4,x)
```

```
[Out] a**2*(Integral(2*sin(c + d*x)**4*sec(c + d*x), x) + Integral(sin(c + d*x)**
4*sec(c + d*x)**2, x) + Integral(sin(c + d*x)**4, x))
```

Giac [A]
time = 0.70, size = 161, normalized size = 1.40

$$-\frac{27\,(dx+c)a^{2}-48\,a^{2}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)+48\,a^{2}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{48\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1}+\frac{2\left(51\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+187\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+229\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+45\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{4}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^4,x, algorithm="giac")
```

```
[Out] -1/24*(27*(d*x + c)*a^2 - 48*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) + 48*a^
2*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 48*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2
*d*x + 1/2*c)^2 - 1) + 2*(51*a^2*tan(1/2*d*x + 1/2*c)^7 + 187*a^2*tan(1/2*d
*x + 1/2*c)^5 + 229*a^2*tan(1/2*d*x + 1/2*c)^3 + 45*a^2*tan(1/2*d*x + 1/2*c
))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

Mupad [B]
time = 1.83, size = 177, normalized size = 1.54

$$\frac{4\,a^{2}\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{9\,a^{2}\,x}{8}+\frac{\frac{25\,a^{2}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{4}+\frac{58\,a^{2}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{3}+\frac{31\,a^{2}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{2}-\frac{22\,a^{2}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{3}-\frac{7\,a^{2}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (4*a^2*atanh(tan(c/2 + (d*x)/2)))/d - (9*a^2*x)/8 + ((31*a^2*tan(c/2 + (d*x
)/2)^5)/2 - (22*a^2*tan(c/2 + (d*x)/2)^3)/3 + (58*a^2*tan(c/2 + (d*x)/2)^7)
/3 + (25*a^2*tan(c/2 + (d*x)/2)^9)/4 - (7*a^2*tan(c/2 + (d*x)/2))/4)/(d*(3*
tan(c/2 + (d*x)/2)^2 + 2*tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^6 - 3*
tan(c/2 + (d*x)/2)^8 - tan(c/2 + (d*x)/2)^10 + 1))
```

3.32 $\int (a + a\sec(c + dx))^2 \sin^2(c + dx)\,dx$

Optimal. Leaf size=73

$$-\frac{a^2x}{2}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{2a^2\sin(c+dx)}{d}-\frac{a^2\cos(c+dx)\sin(c+dx)}{2d}+\frac{a^2\tan(c+dx)}{d}$$

```
[Out] -1/2*a^2*x+2*a^2*arctanh(sin(d*x+c))/d-2*a^2*sin(d*x+c)/d-1/2*a^2*cos(d*x+c
)*sin(d*x+c)/d+a^2*tan(d*x+c)/d
```

Rubi [A]
time = 0.11, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3957, 2788, 2717, 2715, 8, 3855, 3852}

$$-\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\tan(c+dx)}{d}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}-\frac{a^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^2,x]
```

```
[Out] -1/2*(a^2*x) + (2*a^2*ArcTanh[Sin[c + d*x]])/d - (2*a^2*Sin[c + d*x])/d - (
a^2*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (a^2*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
+ f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))^2\sin^2(c+dx)\,dx &= \int (-a-a\cos(c+dx))^2\tan^2(c+dx)\,dx\\ &= \frac{\int \left(-2a^4\cos(c+dx)-a^4\cos^2(c+dx)+2a^4\sec(c+dx)+a^4\sec^2(c\right.}{a^2}\\ &= -\left(a^2\int\cos^2(c+dx)\,dx\right)+a^2\int\sec^2(c+dx)\,dx-(2a^2)\int\cos(c\\ &= \frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{2a^2\sin(c+dx)}{d}-\frac{a^2\cos(c+dx)\sin(c+}{2d}\\ &= -\frac{a^2x}{2}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{2a^2\sin(c+dx)}{d}-\frac{a^2\cos(c+dx}{2}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 243 vs. 2(73) = 146.
time = 0.78, size = 243, normalized size = 3.33

$$\frac{1}{16}a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(-2x-\frac{8\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{8\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}-\frac{8\cos(dx)\sin(c)}{d}-\frac{\cos(2dx)\sin(2c)}{d}-\frac{8\cos(c)\sin(dx)}{d}-\frac{\cos(2c)\sin(2dx)}{d}+\frac{4\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{4\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^2,x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(-2*x - (8*Log[Cos[(c + d*x)/2
] - Sin[(c + d*x)/2]])/d + (8*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d -
 (8*Cos[d*x]*Sin[c])/d - (Cos[2*d*x]*Sin[2*c])/d - (8*Cos[c]*Sin[d*x])/d -
```

```
(Cos[2*c]*Sin[2*d*x])/d + (4*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c
 + d*x)/2] - Sin[(c + d*x)/2])) + (4*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])
*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/16
```

Maple [A]
time = 0.07, size = 78, normalized size = 1.07

method	result
derivativedivides	$\frac{a^2(\tan(dx+c)-dx-c)+2a^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+a^2\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
default	$\frac{a^2(\tan(dx+c)-dx-c)+2a^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+a^2\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
risch	$-\frac{a^2x}{2}+\frac{ia^2e^{2i(dx+c)}}{8d}+\frac{ia^2e^{i(dx+c)}}{d}-\frac{ia^2e^{-i(dx+c)}}{d}-\frac{ia^2e^{-2i(dx+c)}}{8d}+\frac{2ia^2}{d\left(e^{2i(dx+c)}+1\right)}-\cdots$
norman	$\frac{\frac{a^2x}{2}+\frac{3a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{6a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{5a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{a^2x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{a^2x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\cdots}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*sin(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(tan(d*x+c)-d*x-c)+2*a^2*(-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+a
^2*(-1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.49, size = 81, normalized size = 1.11

$$\frac{(2\,dx+2\,c-\sin(2\,dx+2\,c))a^2-4\,(dx+c-\tan(dx+c))a^2+4\,a^2(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)-2\,\sin(dx+c))}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c - sin(2*d*x + 2*c))*a^2 - 4*(d*x + c - tan(d*x + c))*a^2
+ 4*a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x + c)))/d
```

Fricas [A]
time = 3.03, size = 104, normalized size = 1.42

$$-\frac{a^2dx\cos(dx+c)-2\,a^2\cos(dx+c)\log(\sin(dx+c)+1)+2\,a^2\cos(dx+c)\log(-\sin(dx+c)+1)+\left(a^2\cos(dx+c)^2+4\,a^2\cos(dx+c)-2\,a^2\right)\sin(dx+c)}{2\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*d*x*cos(d*x + c) - 2*a^2*cos(d*x + c)*log(sin(d*x + c) + 1) + 2*a
^2*cos(d*x + c)*log(-sin(d*x + c) + 1) + (a^2*cos(d*x + c)^2 + 4*a^2*cos(d*
x + c) - 2*a^2)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\sin^2(c+dx)\sec(c+dx)\,dx+\int \sin^2(c+dx)\sec^2(c+dx)\,dx+\int \sin^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*sin(d*x+c)**2,x)
```

```
[Out] a**2*(Integral(2*sin(c + d*x)**2*sec(c + d*x), x) + Integral(sin(c + d*x)**
2*sec(c + d*x)**2, x) + Integral(sin(c + d*x)**2, x))
```

Giac [A]
time = 0.54, size = 128, normalized size = 1.75

$$-\frac{(dx+c)a^2-4\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)+4\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{4\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}+\frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+5\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*((d*x + c)*a^2 - 4*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) + 4*a^2*log(
abs(tan(1/2*d*x + 1/2*c) - 1)) + 4*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x +
1/2*c)^2 - 1) + 2*(3*a^2*tan(1/2*d*x + 1/2*c)^3 + 5*a^2*tan(1/2*d*x + 1/2*c
))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

Mupad [B]
time = 1.16, size = 117, normalized size = 1.60

$$\frac{4\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{a^2\,x}{2}+\frac{5\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+6\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-3\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (4*a^2*atanh(tan(c/2 + (d*x)/2)))/d - (a^2*x)/2 + (6*a^2*tan(c/2 + (d*x)/2)
^3 + 5*a^2*tan(c/2 + (d*x)/2)^5 - 3*a^2*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (
d*x)/2)^2 - tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2)^6 + 1))
```

3.33 $\int \csc^2(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=57

$$\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{2a^2\cot(c+dx)}{d}-\frac{2a^2\csc(c+dx)}{d}+\frac{a^2\tan(c+dx)}{d}$$

```
[Out] 2*a^2*arctanh(sin(d*x+c))/d-2*a^2*cot(d*x+c)/d-2*a^2*csc(d*x+c)/d+a^2*tan(d
*x+c)/d
```

Rubi [A]
time = 0.18, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.429$, Rules used = {3957, 2952, 3852, 8, 2701, 327, 213, 2700, 14}

$$\frac{a^2\tan(c+dx)}{d}-\frac{2a^2\cot(c+dx)}{d}-\frac{2a^2\csc(c+dx)}{d}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*a^2*ArcTanh[Sin[c + d*x]])/d - (2*a^2*Cot[c + d*x])/d - (2*a^2*Csc[c + d
*x])/d + (a^2*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \csc^2(c+dx)(a+a\sec(c+dx))^2\,dx &= \int (-a-a\cos(c+dx))^2\csc^2(c+dx)\sec^2(c+dx)\,dx \\
&= \int \left(a^2\csc^2(c+dx)+2a^2\csc^2(c+dx)\sec(c+dx)+a^2\csc^2(c+dx\right. \\
&= a^2\int \csc^2(c+dx)\,dx+a^2\int \csc^2(c+dx)\sec^2(c+dx)\,dx+\left(2a^2\right) \\
&= -\frac{a^2\text{Subst}\left(\int 1\,dx,x,\cot(c+dx)\right)}{d}+\frac{a^2\text{Subst}\left(\int \frac{1+x^2}{x^2}\,dx,x,\tan(c+\right.}{d} \\
&= -\frac{a^2\cot(c+dx)}{d}-\frac{2a^2\csc(c+dx)}{d}+\frac{a^2\text{Subst}\left(\int \left(1+\frac{1}{x^2}\right)\,dx,x,\text{ta}\right.}{d} \\
&= \frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{2a^2\cot(c+dx)}{d}-\frac{2a^2\csc(c+dx)}{d}+\frac{a^2}{}
\end{aligned}
$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 401 vs. $2(57)=114$.
time = 6.12, size = 401, normalized size = 7.04

$$
-\frac{\cos^2(c+dx)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2}{2d}+\frac{\cos^2(c+dx)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2}{2d}+\frac{\cos^2(c+dx)\csc\left(\frac{c}{2}\right)\csc\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2\sin\left(\frac{dx}{2}\right)}{2d}+\frac{\cos^2(c+dx)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2\sin\left(\frac{dx}{2}\right)}{4d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}+\frac{\cos^2(c+dx)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2\sin\left(\frac{dx}{2}\right)}{4d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/2*(Cos[c + d*x]^2*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c/2 +
 (d*x)/2]^4*(a + a*Sec[c + d*x])^2)/d + (Cos[c + d*x]^2*Log[Cos[c/2 + (d*x)
/2] + Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2)/(2*d
) + (Cos[c + d*x]^2*Csc[c/2]*Csc[c/2 + (d*x)/2]*Sec[c/2 + (d*x)/2]^4*(a + a
*Sec[c + d*x])^2*Sin[(d*x)/2])/(2*d) + (Cos[c + d*x]^2*Sec[c/2 + (d*x)/2]^4
*(a + a*Sec[c + d*x])^2*Sin[(d*x)/2])/(4*d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 +
 (d*x)/2] - Sin[c/2 + (d*x)/2])) + (Cos[c + d*x]^2*Sec[c/2 + (d*x)/2]^4*(a
+ a*Sec[c + d*x])^2*Sin[(d*x)/2])/(4*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*
x)/2] + Sin[c/2 + (d*x)/2]))
```

Maple [**A**]
time = 0.07, size = 77, normalized size = 1.35

method	result	size
derivativedivides	$\frac{a^2\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)+2a^2\left(-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)-a^2\cot(dx+c)}{d}$	77
default	$\frac{a^2\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)+2a^2\left(-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)-a^2\cot(dx+c)}{d}$	77

norman	$\frac{\frac{2a^2}{d}-\frac{4a^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}+\frac{2a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}$	97
risch	$-\frac{2ia^2\left(2\,e^{2i(dx+c)}-e^{i(dx+c)}+3\right)}{d\left(e^{i(dx+c)}-1\right)\left(e^{2i(dx+c)}+1\right)}-\frac{2a^2\ln\left(e^{i(dx+c)}-i\right)}{d}+\frac{2a^2\ln\left(e^{i(dx+c)}+i\right)}{d}$	103

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(1/sin(d*x+c)/cos(d*x+c)-2*cot(d*x+c))+2*a^2*(-1/sin(d*x+c)+ln(sec
(d*x+c)+tan(d*x+c)))-a^2*cot(d*x+c))
```

Maxima [A]
time = 0.27, size = 74, normalized size = 1.30

$$-\frac{a^2\left(\frac{2}{\sin(dx+c)}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+a^2\left(\frac{1}{\tan(dx+c)}-\tan\left(dx+c\right)\right)+\frac{a^2}{\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(a^2*(2/sin(d*x + c) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + a^
2*(1/tan(d*x + c) - tan(d*x + c)) + a^2/tan(d*x + c))/d
```

Fricas [A]
time = 2.49, size = 101, normalized size = 1.77

$$\frac{a^2\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)-a^2\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)-3\,a^2\cos\left(dx+c\right)^2-2\,a^2\cos\left(dx+c\right)+a^2}{d\cos\left(dx+c\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (a^2*cos(d*x + c)*log(sin(d*x + c) + 1)*sin(d*x + c) - a^2*cos(d*x + c)*log
(-sin(d*x + c) + 1)*sin(d*x + c) - 3*a^2*cos(d*x + c)^2 - 2*a^2*cos(d*x + c
) + a^2)/(d*cos(d*x + c)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\csc^2\left(c+dx\right)\sec\left(c+dx\right)dx+\int\csc^2\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int\csc^2\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*csc(c + d*x)**2*sec(c + d*x), x) + Integral(csc(c + d*x)**
2*sec(c + d*x)**2, x) + Integral(csc(c + d*x)**2, x))
```

Giac [A]
time = 0.48, size = 90, normalized size = 1.58

$$\frac{2\left(a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2*(a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a^2*log(abs(tan(1/2*d*x + 1/2*c
) - 1)) - (2*a^2*tan(1/2*d*x + 1/2*c)^2 - a^2)/(tan(1/2*d*x + 1/2*c)^3 - ta
n(1/2*d*x + 1/2*c)))/d
```

Mupad [B]
time = 1.13, size = 70, normalized size = 1.23

$$\frac{4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-2\,a^2}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right)}+\frac{4\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/sin(c + d*x)^2,x)
```

```
[Out] (4*a^2*tan(c/2 + (d*x)/2)^2 - 2*a^2)/(d*(tan(c/2 + (d*x)/2) - tan(c/2 + (d*
x)/2)^3)) + (4*a^2*atanh(tan(c/2 + (d*x)/2)))/d
```

3.34 $\int \csc^4(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=87

$$\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{10a^2\tan(c+dx)}{3d}-\frac{2a^2\tan(c+dx)}{d(1-\cos(c+dx))}-\frac{a^4\tan(c+dx)}{3d(a-a\cos(c+dx))^2}$$

```
[Out] 2*a^2*arctanh(sin(d*x+c))/d+10/3*a^2*tan(d*x+c)/d-2*a^2*tan(d*x+c)/d/(1-cos
(d*x+c))-1/3*a^4*tan(d*x+c)/d/(a-a*cos(d*x+c))^2
```

Rubi [A]
time = 0.22, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3957, 2948, 2845, 3057, 2827, 3852, 8, 3855}

$$-\frac{a^4\tan(c+dx)}{3d(a-a\cos(c+dx))^2}+\frac{10a^2\tan(c+dx)}{3d}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{2a^2\tan(c+dx)}{d(1-\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*a^2*ArcTanh[Sin[c + d*x]])/d + (10*a^2*Tan[c + d*x])/(3*d) - (2*a^2*Tan[
c + d*x])/(d*(1 - Cos[c + d*x])) - (a^4*Tan[c + d*x])/(3*d*(a - a*Cos[c + d
*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2948

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \csc^4(c+dx)(a+a\sec(c+dx))^2\,dx &= \int (-a-a\cos(c+dx))^2\csc^4(c+dx)\sec^2(c+dx)\,dx\\ &= a^4\int \frac{\sec^2(c+dx)}{(-a+a\cos(c+dx))^2}\,dx\\ &= -\frac{a^4\tan(c+dx)}{3d(a-a\cos(c+dx))^2}+\frac{1}{3}a^2\int \frac{(-4a-2a\cos(c+dx))\sec^2(c+\\ {-a+a\cos(c+dx)}\\ &= -\frac{2a^2\tan(c+dx)}{d(1-\cos(c+dx))}-\frac{a^4\tan(c+dx)}{3d(a-a\cos(c+dx))^2}+\frac{1}{3}\int (10a^2+6a^2\,\mathrm{c}\\ &= -\frac{2a^2\tan(c+dx)}{d(1-\cos(c+dx))}-\frac{a^4\tan(c+dx)}{3d(a-a\cos(c+dx))^2}+\left(2a^2\right)\int \sec(c+d\\ &= \frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{2a^2\tan(c+dx)}{d(1-\cos(c+dx))}-\frac{a^4\tan(c+dx)}{3d(a-a\cos(c+a}\\ &= \frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{10a^2\tan(c+dx)}{3d}-\frac{2a^2\tan(c+dx)}{d(1-\cos(c+dx))}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 228 vs. $2(87) = 174$.
time = 1.27, size = 228, normalized size = 2.62

$$\frac{a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(-\cot\left(\frac{c}{2}\right)\csc^2\left(\frac{1}{2}(c+dx)\right)-(-8+7\cos(c+dx))\csc\left(\frac{c}{2}\right)\csc^3\left(\frac{1}{2}(c+dx)\right)\sin\left(\frac{dx}{2}\right)+6\left(-2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{\sin(dx)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right)\right)}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(-(Cot[c/2]*Csc[(c + d*x)/2]^2
) - (-8 + 7*Cos[c + d*x])*Csc[c/2]*Csc[(c + d*x)/2]^3*Sin[(d*x)/2] + 6*(-2*
Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*Log[Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2]] + Sin[d*x]/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(c
+ d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))))/(24
*d)
```

Maple [A]
time = 0.09, size = 117, normalized size = 1.34

method	result
norman	$\frac{\frac{a^2}{6d}+\frac{7a^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{9a^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}+\frac{2a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}$
derivativedivides	$\frac{a^2\left(-\frac{1}{3\sin(dx+c)^3\cos(dx+c)}+\frac{4}{3\sin(dx+c)\cos(dx+c)}-\frac{8\cot(dx+c)}{3}\right)+2a^2\left(-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+\right.}{d}$

default	$\frac{a^2\left(-\frac{1}{3\sin(dx+c)^3\cos(dx+c)}+\frac{4}{3\sin(dx+c)\cos(dx+c)}-\frac{8\cot(dx+c)}{3}\right)+2a^2\left(-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx}{d}$
risch	$-\frac{4ia^2\left(3\,e^{4i(dx+c)}-9\,e^{3i(dx+c)}+11\,e^{2i(dx+c)}-12\,e^{i(dx+c)}+5\right)}{3d\left(e^{i(dx+c)}-1\right)^3\left(e^{2i(dx+c)}+1\right)}+\frac{2a^2\ln\left(e^{i(dx+c)}+i\right)}{d}-\frac{2a^2\ln\left(e^{i(dx+c)}-i\right)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/3/sin(d*x+c)^3/cos(d*x+c)+4/3/sin(d*x+c)/cos(d*x+c)-8/3*cot(d*
x+c))+2*a^2*(-1/3/sin(d*x+c)^3-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+a^2*
(-2/3-1/3*csc(d*x+c)^2)*cot(d*x+c))
```

Maxima [A]
time = 0.27, size = 113, normalized size = 1.30

$$-\frac{a^2\left(\frac{2\left(3\,\sin(dx+c)^2+1\right)}{\sin(dx+c)^3}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)+a^2\left(\frac{6\,\tan(dx+c)^2+1}{\tan(dx+c)^3}-3\,\tan\left(dx+c\right)\right)+\frac{\left(3\,\tan(dx+c)^2+1\right)a^2}{\tan(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3*(a^2*(2*(3*sin(d*x + c)^2 + 1)/sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1
) + 3*log(sin(d*x + c) - 1)) + a^2*((6*tan(d*x + c)^2 + 1)/tan(d*x + c)^3 -
 3*tan(d*x + c)) + (3*tan(d*x + c)^2 + 1)*a^2/tan(d*x + c)^3)/d
```

Fricas [A]
time = 2.74, size = 159, normalized size = 1.83

$$-\frac{10\,a^2\cos\left(dx+c\right)^3-4\,a^2\cos\left(dx+c\right)^2-11\,a^2\cos\left(dx+c\right)-3\,\left(a^2\cos\left(dx+c\right)^2-a^2\cos\left(dx+c\right)\right)\log\left(\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)+3\,\left(a^2\cos\left(dx+c\right)^2-a^2\cos\left(dx+c\right)\right)\log\left(-\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)+3\,a^2}{3\,\left(d\cos\left(dx+c\right)^2-d\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(10*a^2*cos(d*x + c)^3 - 4*a^2*cos(d*x + c)^2 - 11*a^2*cos(d*x + c) -
3*(a^2*cos(d*x + c)^2 - a^2*cos(d*x + c))*log(sin(d*x + c) + 1)*sin(d*x + c
) + 3*(a^2*cos(d*x + c)^2 - a^2*cos(d*x + c))*log(-sin(d*x + c) + 1)*sin(d*
x + c) + 3*a^2)/((d*cos(d*x + c)^2 - d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\csc^4\left(c+dx\right)\sec\left(c+dx\right)dx+\int\csc^4\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int\csc^4\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*csc(c + d*x)**4*sec(c + d*x), x) + Integral(csc(c + d*x)**
4*sec(c + d*x)**2, x) + Integral(csc(c + d*x)**4, x))
```

Giac [A]
time = 0.47, size = 104, normalized size = 1.20

$$\frac{12\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-12\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{12\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}-\frac{15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/6*(12*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 12*a^2*log(abs(tan(1/2*d*x
 + 1/2*c) - 1)) - 12*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1)
- (15*a^2*tan(1/2*d*x + 1/2*c)^2 + a^2)/tan(1/2*d*x + 1/2*c)^3)/d
```

Mupad [B]
time = 2.48, size = 91, normalized size = 1.05

$$\frac{4\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{-9\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{14\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+\frac{a^2}{3}}{d\,\left(2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/sin(c + d*x)^4,x)
```

```
[Out] (4*a^2*atanh(tan(c/2 + (d*x)/2)))/d - ((14*a^2*tan(c/2 + (d*x)/2)^2)/3 - 9*
a^2*tan(c/2 + (d*x)/2)^4 + a^2/3)/(d*(2*tan(c/2 + (d*x)/2)^3 - 2*tan(c/2 +
(d*x)/2)^5))
```

3.35 $\int \csc^6(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=129

$$\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{4a^2\cot(c+dx)}{d}-\frac{5a^2\cot^3(c+dx)}{3d}-\frac{2a^2\cot^5(c+dx)}{5d}-\frac{2a^2\csc(c+dx)}{d}-\frac{2a^2\csc^3(}{3c}$$

```
[Out] 2*a^2*arctanh(sin(d*x+c))/d-4*a^2*cot(d*x+c)/d-5/3*a^2*cot(d*x+c)^3/d-2/5*a
^2*cot(d*x+c)^5/d-2*a^2*csc(d*x+c)/d-2/3*a^2*csc(d*x+c)^3/d-2/5*a^2*csc(d*x
+c)^5/d+a^2*tan(d*x+c)/d
```

Rubi [A]
time = 0.17, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.381$, Rules used = {3957, 2952, 3852, 2701, 308, 213, 2700, 276}

$$\frac{a^2\tan(c+dx)}{d}-\frac{2a^2\cot^5(c+dx)}{5d}-\frac{5a^2\cot^3(c+dx)}{3d}-\frac{4a^2\cot(c+dx)}{d}-\frac{2a^2\csc^5(c+dx)}{5d}-\frac{2a^2\csc^3(c+dx)}{3d}-\frac{2a^2\csc(c+dx)}{d}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*a^2*ArcTanh[Sin[c + d*x]])/d - (4*a^2*Cot[c + d*x])/d - (5*a^2*Cot[c + d
*x]^3)/(3*d) - (2*a^2*Cot[c + d*x]^5)/(5*d) - (2*a^2*Csc[c + d*x])/d - (2*a
^2*Csc[c + d*x]^3)/(3*d) - (2*a^2*Csc[c + d*x]^5)/(5*d) + (a^2*Tan[c + d*x]
)/d
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
```

```
x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc^6(c+dx)(a+a\sec(c+dx))^2\,dx = \int (-a-a\cos(c+dx))^2\csc^6(c+dx)\sec^2(c+dx)\,dx$$
$$= \int \left(a^2\csc^6(c+dx)+2a^2\csc^6(c+dx)\sec(c+dx)+a^2\csc^6(c+dx\right.$$
$$= a^2\int \csc^6(c+dx)\,dx + a^2\int \csc^6(c+dx)\sec^2(c+dx)\,dx + \left(2a^2\right)$$
$$= \frac{a^2\text{Subst}\left(\int \frac{(1+x^2)^3}{x^6}\,dx, x, \tan(c+dx)\right)}{d} - \frac{a^2\text{Subst}\left(\int (1+2x^2+x^4\right.}{d}$$
$$= -\frac{a^2\cot(c+dx)}{d} - \frac{2a^2\cot^3(c+dx)}{3d} - \frac{a^2\cot^5(c+dx)}{5d} + \frac{a^2\text{Subst}(}{}$$
$$= -\frac{4a^2\cot(c+dx)}{d} - \frac{5a^2\cot^3(c+dx)}{3d} - \frac{2a^2\cot^5(c+dx)}{5d} - \frac{2a^2\csc}{}$$
$$= \frac{2a^2\tanh^{-1}(\sin(c+dx))}{d} - \frac{4a^2\cot(c+dx)}{d} - \frac{5a^2\cot^3(c+dx)}{3d} - $$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 317 vs. 2(129) = 258.
time = 0.63, size = 317, normalized size = 2.46

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*Cos[c + d*x]*Sec[(c + d*x)/2]^4*(1 + Sec[c + d*x])^2*(-3840*Cos[c + d*
x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 3840*Cos[c + d*x]*Log[Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2]] + Csc[2*c]*Csc[(c + d*x)/2]^4*Csc[c + d*x]*(3
20*Sin[2*c] - 596*Sin[d*x] + 864*Sin[2*d*x] + 216*Sin[c - d*x] - 416*Sin[c
+ d*x] + 624*Sin[2*(c + d*x)] - 416*Sin[3*(c + d*x)] + 104*Sin[4*(c + d*x)]
 - 596*Sin[2*c + d*x] - 680*Sin[3*c + d*x] + 894*Sin[c + 2*d*x] + 224*Sin[2
*(c + 2*d*x)] + 894*Sin[3*c + 2*d*x] + 480*Sin[4*c + 2*d*x] - 776*Sin[c + 3
*d*x] - 596*Sin[2*c + 3*d*x] - 596*Sin[4*c + 3*d*x] - 120*Sin[5*c + 3*d*x]
+ 149*Sin[3*c + 4*d*x] + 149*Sin[5*c + 4*d*x])))/(7680*d)
```

Maple [A]
time = 0.10, size = 155, normalized size = 1.20

method	result
norman	$\frac{\frac{a^2}{40d}+\frac{4a^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}+\frac{31a^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}-\frac{5a^2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{a^2\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)} - \frac{2a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d} +$

derivativedivides	$\frac{a^2\left(-\frac{1}{5\sin(dx+c)^5\cos(dx+c)}-\frac{2}{5\sin(dx+c)^3\cos(dx+c)}+\frac{8}{5\sin(dx+c)\cos(dx+c)}-\frac{16\cot(dx+c)}{5}\right)+2a^2\left(-\frac{1}{5\sin(dx+c)^5}-\frac{1}{3\sin(dx+c)}\right.}{d}$
default	$\frac{a^2\left(-\frac{1}{5\sin(dx+c)^5\cos(dx+c)}-\frac{2}{5\sin(dx+c)^3\cos(dx+c)}+\frac{8}{5\sin(dx+c)\cos(dx+c)}-\frac{16\cot(dx+c)}{5}\right)+2a^2\left(-\frac{1}{5\sin(dx+c)^5}-\frac{1}{3\sin(dx+c)}\right.}{d}$
risch	$-\frac{4ia^2\left(15\,\mathrm{e}^{7i(dx+c)}-60\,\mathrm{e}^{6i(dx+c)}+85\,\mathrm{e}^{5i(dx+c)}-40\,\mathrm{e}^{4i(dx+c)}-27\,\mathrm{e}^{3i(dx+c)}+108\,\mathrm{e}^{2i(dx+c)}-97\,\mathrm{e}^{i(dx+c)}+28\right)}{15d\left(\mathrm{e}^{i(dx+c)}-1\right)^5\left(\mathrm{e}^{2i(dx+c)}+1\right)\left(\mathrm{e}^{i(dx+c)}+1\right)}-\frac{2a^2\ln(\mathrm{e}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/5/sin(d*x+c)^5/cos(d*x+c)-2/5/sin(d*x+c)^3/cos(d*x+c)+8/5/sin(
d*x+c)/cos(d*x+c)-16/5*cot(d*x+c))+2*a^2*(-1/5/sin(d*x+c)^5-1/3/sin(d*x+c)^
3-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+a^2*(-8/15-1/5*csc(d*x+c)^4-4/15*
csc(d*x+c)^2)*cot(d*x+c))
```

Maxima [A]

time = 0.28, size = 144, normalized size = 1.12

$$-\frac{a^2\left(\frac{2\left(15\sin(dx+c)^4+5\sin(dx+c)^2+3\right)}{\sin(dx+c)^5}-15\log(\sin(dx+c)+1)+15\log(\sin(dx+c)-1)\right)+3a^2\left(\frac{15\tan(dx+c)^4+5\tan(dx+c)^2+1}{\tan(dx+c)^5}-5\tan(dx+c)\right)+\frac{\left(15\tan(dx+c)^4+10\tan(dx+c)^2+3\right)a^2}{\tan(dx+c)^5}}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/15*(a^2*(2*(15*sin(d*x + c)^4 + 5*sin(d*x + c)^2 + 3)/sin(d*x + c)^5 - 1
5*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) + 3*a^2*((15*tan(d*x +
c)^4 + 5*tan(d*x + c)^2 + 1)/tan(d*x + c)^5 - 5*tan(d*x + c)) + (15*tan(d*x
 + c)^4 + 10*tan(d*x + c)^2 + 3)*a^2/tan(d*x + c)^5)/d
```

Fricas [A]

time = 2.61, size = 206, normalized size = 1.60

$$-\frac{56a^2\cos(dx+c)^4-82a^2\cos(dx+c)^3-32a^2\cos(dx+c)^2+76a^2\cos(dx+c)-15\left(a^2\cos(dx+c)^3-2a^2\cos(dx+c)^2+a^2\cos(dx+c)\right)\log(\sin(dx+c)+1)\sin(dx+c)+15\left(a^2\cos(dx+c)^3-2a^2\cos(dx+c)^2+a^2\cos(dx+c)\right)\log(-\sin(dx+c)+1)\sin(dx+c)-15a^2}{15\left(d\cos(dx+c)^3-2d\cos(dx+c)^2+d\cos(dx+c)\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/15*(56*a^2*cos(d*x + c)^4 - 82*a^2*cos(d*x + c)^3 - 32*a^2*cos(d*x + c)^
2 + 76*a^2*cos(d*x + c) - 15*(a^2*cos(d*x + c)^3 - 2*a^2*cos(d*x + c)^2 + a
^2*cos(d*x + c))*log(sin(d*x + c) + 1)*sin(d*x + c) + 15*(a^2*cos(d*x + c)^
3 - 2*a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))*log(-sin(d*x + c) + 1)*sin(d*x
 + c) - 15*a^2)/((d*cos(d*x + c)^3 - 2*d*cos(d*x + c)^2 + d*cos(d*x + c))*s
in(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.49, size = 136, normalized size = 1.05

$$\frac{240\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-240\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{240\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}-\frac{345\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+35\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/120*(240*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 240*a^2*log(abs(tan(1/2
*d*x + 1/2*c) - 1)) + 15*a^2*tan(1/2*d*x + 1/2*c) - 240*a^2*tan(1/2*d*x + 1
/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - (345*a^2*tan(1/2*d*x + 1/2*c)^4 + 35*a
^2*tan(1/2*d*x + 1/2*c)^2 + 3*a^2)/tan(1/2*d*x + 1/2*c)^5)/d
```

Mupad [**B**]
time = 1.28, size = 124, normalized size = 0.96

$$\frac{4\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{-39\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\frac{62\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}+\frac{32\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{15}+\frac{a^2}{5}}{d\left(8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\right)}+\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/sin(c + d*x)^6,x)
```

```
[Out] (4*a^2*atanh(tan(c/2 + (d*x)/2)))/d - ((32*a^2*tan(c/2 + (d*x)/2)^2)/15 + (
62*a^2*tan(c/2 + (d*x)/2)^4)/3 - 39*a^2*tan(c/2 + (d*x)/2)^6 + a^2/5)/(d*(8
*tan(c/2 + (d*x)/2)^5 - 8*tan(c/2 + (d*x)/2)^7)) + (a^2*tan(c/2 + (d*x)/2))
/(8*d)
```

3.36 $\int \csc^8(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=163

$$\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{5a^2\cot(c+dx)}{d}-\frac{3a^2\cot^3(c+dx)}{d}-\frac{7a^2\cot^5(c+dx)}{5d}-\frac{2a^2\cot^7(c+dx)}{7d}-\frac{2a^2\csc(c}{d}$$

```
[Out] 2*a^2*arctanh(sin(d*x+c))/d-5*a^2*cot(d*x+c)/d-3*a^2*cot(d*x+c)^3/d-7/5*a^2
*cot(d*x+c)^5/d-2/7*a^2*cot(d*x+c)^7/d-2*a^2*csc(d*x+c)/d-2/3*a^2*csc(d*x+c
)^3/d-2/5*a^2*csc(d*x+c)^5/d-2/7*a^2*csc(d*x+c)^7/d+a^2*tan(d*x+c)/d
```

Rubi [A]
time = 0.18, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3957, 2952, 3852, 2701, 308, 213, 2700, 276}

$$\frac{a^2\tan(c+dx)}{d}-\frac{2a^2\cot^7(c+dx)}{7d}-\frac{7a^2\cot^5(c+dx)}{5d}-\frac{3a^2\cot^3(c+dx)}{d}-\frac{5a^2\cot(c+dx)}{d}-\frac{2a^2\csc^7(c+dx)}{7d}-\frac{2a^2\csc^5(c+dx)}{5d}-\frac{2a^2\csc^3(c+dx)}{3d}-\frac{2a^2\csc(c+dx)}{d}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^8*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*a^2*ArcTanh[Sin[c + d*x]])/d - (5*a^2*Cot[c + d*x])/d - (3*a^2*Cot[c + d
*x]^3)/d - (7*a^2*Cot[c + d*x]^5)/(5*d) - (2*a^2*Cot[c + d*x]^7)/(7*d) - (2
*a^2*Csc[c + d*x])/d - (2*a^2*Csc[c + d*x]^3)/(3*d) - (2*a^2*Csc[c + d*x]^5
)/(5*d) - (2*a^2*Csc[c + d*x]^7)/(7*d) + (a^2*Tan[c + d*x])/d
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
```

```
x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc^8(c+dx)(a+a\sec(c+dx))^2\,dx = \int (-a-a\cos(c+dx))^2\csc^8(c+dx)\sec^2(c+dx)\,dx$$
$$= \int \left(a^2\csc^8(c+dx)+2a^2\csc^8(c+dx)\sec(c+dx)+a^2\csc^8(c+dx)\right.$$
$$= a^2\int \csc^8(c+dx)\,dx + a^2\int \csc^8(c+dx)\sec^2(c+dx)\,dx + \left(2a^2\right)\int$$
$$= \frac{a^2\text{Subst}\left(\int \frac{(1+x^2)^4}{x^8}\,dx, x, \tan(c+dx)\right)}{d} - \frac{a^2\text{Subst}\left(\int (1+3x^2+3x^4\right.}{d}$$
$$= -\frac{a^2\cot(c+dx)}{d} - \frac{a^2\cot^3(c+dx)}{d} - \frac{3a^2\cot^5(c+dx)}{5d} - \frac{a^2\cot^7(c-}{7d}$$
$$= -\frac{5a^2\cot(c+dx)}{d} - \frac{3a^2\cot^3(c+dx)}{d} - \frac{7a^2\cot^5(c+dx)}{5d} - \frac{2a^2\cot^7}{7}$$
$$= \frac{2a^2\tanh^{-1}(\sin(c+dx))}{d} - \frac{5a^2\cot(c+dx)}{d} - \frac{3a^2\cot^3(c+dx)}{d} - \frac{7a}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 428 vs. 2(163) = 326.
time = 0.82, size = 428, normalized size = 2.63

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^8*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*Cos[c + d*x]*Sec[(c + d*x)/2]^4*(1 + Sec[c + d*x])^2*(-6881280*Cos[c +
 d*x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 6881280*Cos[c + d*x]*Log[C
os[(c + d*x)/2] + Sin[(c + d*x)/2]] - 32*Csc[2*c]*Csc[(c + d*x)/2]^4*Csc[c
+ d*x]^3*(-9856*Sin[2*c] + 17288*Sin[d*x] - 29056*Sin[2*d*x] - 7264*Sin[c -
 d*x] + 14208*Sin[c + d*x] - 19536*Sin[2*(c + d*x)] + 7104*Sin[3*(c + d*x)]
 + 7104*Sin[4*(c + d*x)] - 7104*Sin[5*(c + d*x)] + 1776*Sin[6*(c + d*x)] +
17288*Sin[2*c + d*x] + 20384*Sin[3*c + d*x] - 23771*Sin[c + 2*d*x] + 7104*S
in[2*(c + 2*d*x)] - 23771*Sin[3*c + 2*d*x] - 8960*Sin[4*c + 2*d*x] + 19984*
Sin[c + 3*d*x] + 8644*Sin[2*c + 3*d*x] + 8644*Sin[4*c + 3*d*x] - 6160*Sin[5
*c + 3*d*x] + 8644*Sin[3*c + 4*d*x] + 8644*Sin[5*c + 4*d*x] + 6720*Sin[6*c
+ 4*d*x] - 12144*Sin[3*c + 5*d*x] - 8644*Sin[4*c + 5*d*x] - 8644*Sin[6*c +
5*d*x] - 1680*Sin[7*c + 5*d*x] + 3456*Sin[4*c + 6*d*x] + 2161*Sin[5*c + 6*d
*x] + 2161*Sin[7*c + 6*d*x])))/(13762560*d)
```

Maple [A]
time = 0.12, size = 193, normalized size = 1.18

method	result
norman	$\frac{\frac{a^2}{224d}+\frac{29a^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{560d}+\frac{163a^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{480d}+\frac{67a^2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}-\frac{175a^2\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32d}+\frac{13a^2\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{48d}+}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$
derivativedivides	$a^2\left(-\frac{1}{7\sin(dx+c)^7\cos(dx+c)}-\frac{8}{35\sin(dx+c)^5\cos(dx+c)}-\frac{16}{35\sin(dx+c)^3\cos(dx+c)}+\frac{64}{35\sin(dx+c)\cos(dx+c)}-\frac{128\cot(dx+c)}{35}\right)+2a$
default	$a^2\left(-\frac{1}{7\sin(dx+c)^7\cos(dx+c)}-\frac{8}{35\sin(dx+c)^5\cos(dx+c)}-\frac{16}{35\sin(dx+c)^3\cos(dx+c)}+\frac{64}{35\sin(dx+c)\cos(dx+c)}-\frac{128\cot(dx+c)}{35}\right)+2a$
risch	$-\frac{4ia^2\left(105\,e^{11i(dx+c)}-420\,e^{10i(dx+c)}+385\,e^{9i(dx+c)}+560\,e^{8i(dx+c)}-1274\,e^{7i(dx+c)}+616\,e^{6i(dx+c)}+454\,e^{5i(dx+c)}-18\right.}{105d\left(e^{i(dx+c)}-1\right)^7\left(e^{i(dx+c)}+1\right)^3\left(e^{2i(dx+c)}+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^8*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/7/sin(d*x+c)^7/cos(d*x+c)-8/35/sin(d*x+c)^5/cos(d*x+c)-16/35/s
in(d*x+c)^3/cos(d*x+c)+64/35/sin(d*x+c)/cos(d*x+c)-128/35*cot(d*x+c))+2*a^2
*(-1/7/sin(d*x+c)^7-1/5/sin(d*x+c)^5-1/3/sin(d*x+c)^3-1/sin(d*x+c)+ln(sec(d
*x+c)+tan(d*x+c)))+a^2*(-16/35-1/7*csc(d*x+c)^6-6/35*csc(d*x+c)^4-8/35*csc(
d*x+c)^2)*cot(d*x+c))
```

Maxima [A]

time = 0.27, size = 175, normalized size = 1.07

$$-\frac{a^2\left(\frac{2\left(105\sin(dx+c)^6+35\sin(dx+c)^4+21\sin(dx+c)^2+15\right)}{\sin(dx+c)^7}-105\log\left(\sin\left(dx+c\right)+1\right)+105\log\left(\sin\left(dx+c\right)-1\right)\right)+3a^2\left(\frac{140\tan(dx+c)^6+70\tan(dx+c)^4+28\tan(dx+c)^2+5}{\tan(dx+c)^7}-35\tan\left(dx+c\right)\right)+\frac{3\left(35\tan(dx+c)^6+35\tan(dx+c)^4+21\tan(dx+c)^2+5\right)a^2}{\tan(dx+c)^7}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/105*(a^2*(2*(105*sin(d*x + c)^6 + 35*sin(d*x + c)^4 + 21*sin(d*x + c)^2
+ 15)/sin(d*x + c)^7 - 105*log(sin(d*x + c) + 1) + 105*log(sin(d*x + c) - 1
)) + 3*a^2*((140*tan(d*x + c)^6 + 70*tan(d*x + c)^4 + 28*tan(d*x + c)^2 + 5
)/tan(d*x + c)^7 - 35*tan(d*x + c)) + 3*(35*tan(d*x + c)^6 + 35*tan(d*x + c
)^4 + 21*tan(d*x + c)^2 + 5)*a^2/tan(d*x + c)^7)/d
```

Fricas [A]

time = 2.67, size = 272, normalized size = 1.67

$$-\frac{432\,a^2\cos\left(dx+c\right)^6-654\,a^2\cos\left(dx+c\right)^5-636\,a^2\cos\left(dx+c\right)^4+1226\,a^2\cos\left(dx+c\right)^3+74\,a^2\cos\left(dx+c\right)^2-562\,a^2\cos\left(dx+c\right)-105\left(a^2\cos\left(dx+c\right)^5-2\,a^2\cos\left(dx+c\right)^4+2\,a^2\cos\left(dx+c\right)^2-a^2\cos\left(dx+c\right)\right)\log\left(\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)+105\left(a^2\cos\left(dx+c\right)^5-2\,a^2\cos\left(dx+c\right)^4+2\,a^2\cos\left(dx+c\right)^2-a^2\cos\left(dx+c\right)\right)\log\left(-\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)+105\,a^2}{105\left(d\cos\left(dx+c\right)^5-2\,d\cos\left(dx+c\right)^4+2\,d\cos\left(dx+c\right)^2-d\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/105*(432*a^2*cos(d*x + c)^6 - 654*a^2*cos(d*x + c)^5 - 636*a^2*cos(d*x +
 c)^4 + 1226*a^2*cos(d*x + c)^3 + 74*a^2*cos(d*x + c)^2 - 562*a^2*cos(d*x +
```

```
c) - 105*(a^2*cos(d*x + c)^5 - 2*a^2*cos(d*x + c)^4 + 2*a^2*cos(d*x + c)^2
 - a^2*cos(d*x + c))*log(sin(d*x + c) + 1)*sin(d*x + c) + 105*(a^2*cos(d*x
+ c)^5 - 2*a^2*cos(d*x + c)^4 + 2*a^2*cos(d*x + c)^2 - a^2*cos(d*x + c))*lo
g(-sin(d*x + c) + 1)*sin(d*x + c) + 105*a^2)/((d*cos(d*x + c)^5 - 2*d*cos(d
*x + c)^4 + 2*d*cos(d*x + c)^2 - d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**8*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [A]
time = 0.53, size = 168, normalized size = 1.03

$$\frac{35\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6720\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-6720\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+945\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{6720\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}-\frac{10710\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+1330\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+189\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+15\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7}}{3360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3360*(35*a^2*tan(1/2*d*x + 1/2*c)^3 + 6720*a^2*log(abs(tan(1/2*d*x + 1/2*
c) + 1)) - 6720*a^2*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 945*a^2*tan(1/2*d*
x + 1/2*c) - 6720*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - (
10710*a^2*tan(1/2*d*x + 1/2*c)^6 + 1330*a^2*tan(1/2*d*x + 1/2*c)^4 + 189*a^
2*tan(1/2*d*x + 1/2*c)^2 + 15*a^2)/tan(1/2*d*x + 1/2*c)^7)/d
```

Mupad [B]
time = 0.99, size = 159, normalized size = 0.98

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{96\,d}+\frac{4\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{9\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{32\,d}-\frac{-166\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+\frac{268\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{3}+\frac{163\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{15}+\frac{58\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{35}+\frac{a^2}{7}}{d\,\left(32\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-32\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/sin(c + d*x)^8,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^3)/(96*d) + (4*a^2*atanh(tan(c/2 + (d*x)/2)))/d + (
9*a^2*tan(c/2 + (d*x)/2))/(32*d) - ((58*a^2*tan(c/2 + (d*x)/2)^2)/35 + (163
*a^2*tan(c/2 + (d*x)/2)^4)/15 + (268*a^2*tan(c/2 + (d*x)/2)^6)/3 - 166*a^2*
tan(c/2 + (d*x)/2)^8 + a^2/7)/(d*(32*tan(c/2 + (d*x)/2)^7 - 32*tan(c/2 + (d
*x)/2)^9))
```

3.37 $\int \csc^{10}(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=201

$$\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{6a^2\cot(c+dx)}{d}-\frac{14a^2\cot^3(c+dx)}{3d}-\frac{16a^2\cot^5(c+dx)}{5d}-\frac{9a^2\cot^7(c+dx)}{7d}-2a^2\,\mathrm{co}$$

```
[Out] 2*a^2*arctanh(sin(d*x+c))/d-6*a^2*cot(d*x+c)/d-14/3*a^2*cot(d*x+c)^3/d-16/5
*a^2*cot(d*x+c)^5/d-9/7*a^2*cot(d*x+c)^7/d-2/9*a^2*cot(d*x+c)^9/d-2*a^2*csc
(d*x+c)/d-2/3*a^2*csc(d*x+c)^3/d-2/5*a^2*csc(d*x+c)^5/d-2/7*a^2*csc(d*x+c)^
7/d-2/9*a^2*csc(d*x+c)^9/d+a^2*tan(d*x+c)/d
```

Rubi [A]
time = 0.20, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3957, 2952, 3852, 2701, 308, 213, 2700, 276}

$$\frac{a^2\tan(c+dx)}{d}-\frac{2a^2\cot^9(c+dx)}{9d}-\frac{9a^2\cot^7(c+dx)}{7d}-\frac{16a^2\cot^5(c+dx)}{5d}-\frac{14a^2\cot^3(c+dx)}{3d}-\frac{6a^2\cot(c+dx)}{d}-\frac{2a^2\csc^9(c+dx)}{9d}-\frac{2a^2\csc^7(c+dx)}{7d}-\frac{2a^2\csc^5(c+dx)}{5d}-\frac{2a^2\csc^3(c+dx)}{3d}-\frac{2a^2\csc(c+dx)}{d}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^10*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*a^2*ArcTanh[Sin[c + d*x]])/d - (6*a^2*Cot[c + d*x])/d - (14*a^2*Cot[c +
d*x]^3)/(3*d) - (16*a^2*Cot[c + d*x]^5)/(5*d) - (9*a^2*Cot[c + d*x]^7)/(7*d
) - (2*a^2*Cot[c + d*x]^9)/(9*d) - (2*a^2*Csc[c + d*x])/d - (2*a^2*Csc[c +
d*x]^3)/(3*d) - (2*a^2*Csc[c + d*x]^5)/(5*d) - (2*a^2*Csc[c + d*x]^7)/(7*d)
 - (2*a^2*Csc[c + d*x]^9)/(9*d) + (a^2*Tan[c + d*x])/d
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \csc^{10}(c+dx)(a+a\sec(c+dx))^2\,dx &= \int(-a-a\cos(c+dx))^2\csc^{10}(c+dx)\sec^2(c+dx)\,dx \\
&= \int \left(a^2\csc^{10}(c+dx)+2a^2\csc^{10}(c+dx)\sec(c+dx)+a^2\csc^{10}(c+\right. \\
&= a^2\int\csc^{10}(c+dx)\,dx+a^2\int\csc^{10}(c+dx)\sec^2(c+dx)\,dx+\left(2a\right. \\
&= \frac{a^2\text{Subst}\left(\int\frac{(1+x^2)^5}{x^{10}}\,dx,x,\tan(c+dx)\right)}{d}-\frac{a^2\text{Subst}\left(\int(1+4x^2+6\right.}{} \\
&= -\frac{a^2\cot(c+dx)}{d}-\frac{4a^2\cot^3(c+dx)}{3d}-\frac{6a^2\cot^5(c+dx)}{5d}-\frac{4a^2\cot}{} \\
&= -\frac{6a^2\cot(c+dx)}{d}-\frac{14a^2\cot^3(c+dx)}{3d}-\frac{16a^2\cot^5(c+dx)}{5d}-\frac{9a^2}{} \\
&= \frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{6a^2\cot(c+dx)}{d}-\frac{14a^2\cot^3(c+dx)}{3d}-
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1050 vs. $2(201) = 402$.
time = 6.56, size = 1050, normalized size = 5.22

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^10*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-6899*Cos[c + d*x]^2*Cot[c/2]*Csc[c/2 + (d*x)/2]^2*Sec[c/2 + (d*x)/2]^4*(a
 + a*Sec[c + d*x])^2)/(80640*d) - (193*Cos[c + d*x]^2*Cot[c/2]*Csc[c/2 + (d
*x)/2]^4*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2)/(13440*d) - (71*Cos[c
 + d*x]^2*Cot[c/2]*Csc[c/2 + (d*x)/2]^6*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c +
 d*x])^2)/(32256*d) - (Cos[c + d*x]^2*Cot[c/2]*Csc[c/2 + (d*x)/2]^8*Sec[c/2
 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2)/(4608*d) - (Cos[c + d*x]^2*Log[Cos[c/
2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x]
)^2)/(2*d) + (Cos[c + d*x]^2*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*S
ec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2)/(2*d) + (123041*Cos[c + d*x]^2*
Csc[c/2]*Csc[c/2 + (d*x)/2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*Sin
[(d*x)/2])/(161280*d) + (6899*Cos[c + d*x]^2*Csc[c/2]*Csc[c/2 + (d*x)/2]^3*
Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*Sin[(d*x)/2])/(80640*d) + (193*
Cos[c + d*x]^2*Csc[c/2]*Csc[c/2 + (d*x)/2]^5*Sec[c/2 + (d*x)/2]^4*(a + a*Se
c[c + d*x])^2*Sin[(d*x)/2])/(13440*d) + (71*Cos[c + d*x]^2*Csc[c/2]*Csc[c/2
 + (d*x)/2]^7*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*Sin[(d*x)/2])/(32
256*d) + (Cos[c + d*x]^2*Csc[c/2]*Csc[c/2 + (d*x)/2]^9*Sec[c/2 + (d*x)/2]^4
```

```
*(a + a*Sec[c + d*x])^2*Sin[(d*x)/2])/(4608*d) + (803*Cos[c + d*x]^2*Sec[c/
2]*Sec[c/2 + (d*x)/2]^5*(a + a*Sec[c + d*x])^2*Sin[(d*x)/2])/(7680*d) + (49
*Cos[c + d*x]^2*Sec[c/2]*Sec[c/2 + (d*x)/2]^7*(a + a*Sec[c + d*x])^2*Sin[(d
*x)/2])/(7680*d) + (Cos[c + d*x]^2*Sec[c/2]*Sec[c/2 + (d*x)/2]^9*(a + a*Sec
[c + d*x])^2*Sin[(d*x)/2])/(2560*d) + (Cos[c + d*x]*Sec[c]*Sec[c/2 + (d*x)/
2]^4*(a + a*Sec[c + d*x])^2*Sin[d*x])/(4*d) + (49*Cos[c + d*x]^2*Sec[c/2 +
(d*x)/2]^6*(a + a*Sec[c + d*x])^2*Tan[c/2])/(7680*d) + (Cos[c + d*x]^2*Sec[
c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^2*Tan[c/2])/(2560*d)
```

Maple [A]
time = 0.12, size = 231, normalized size = 1.15

method	result
derivativedivides	$\frac{a^2\left(-\frac{1}{9\sin(dx+c)^9\cos(dx+c)}-\frac{10}{63\sin(dx+c)^7\cos(dx+c)}-\frac{16}{63\sin(dx+c)^5\cos(dx+c)}-\frac{32}{63\sin(dx+c)^3\cos(dx+c)}+\frac{128}{63\sin(dx+c)\cos(dx+\right.}{}$
default	$\frac{a^2\left(-\frac{1}{9\sin(dx+c)^9\cos(dx+c)}-\frac{10}{63\sin(dx+c)^7\cos(dx+c)}-\frac{16}{63\sin(dx+c)^5\cos(dx+c)}-\frac{32}{63\sin(dx+c)^3\cos(dx+c)}+\frac{128}{63\sin(dx+c)\cos(dx+\right.}{}$
risch	$-\frac{4ia^2\left(315\,\mathrm{e}^{15i(dx+c)}-1260\,\mathrm{e}^{14i(dx+c)}+525\,\mathrm{e}^{13i(dx+c)}+4200\,\mathrm{e}^{12i(dx+c)}-5817\,\mathrm{e}^{11i(dx+c)}-2772\,\mathrm{e}^{10i(dx+c)}+10161\,\mathrm{e}^{9i(dx\right.}{315d(\mathrm{e}^{i(dx+c)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^10*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/9/sin(d*x+c)^9/cos(d*x+c)-10/63/sin(d*x+c)^7/cos(d*x+c)-16/63/
sin(d*x+c)^5/cos(d*x+c)-32/63/sin(d*x+c)^3/cos(d*x+c)+128/63/sin(d*x+c)/cos
(d*x+c)-256/63*cot(d*x+c))+2*a^2*(-1/9/sin(d*x+c)^9-1/7/sin(d*x+c)^7-1/5/si
n(d*x+c)^5-1/3/sin(d*x+c)^3-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+a^2*(-1
28/315-1/9*csc(d*x+c)^8-8/63*csc(d*x+c)^6-16/105*csc(d*x+c)^4-64/315*csc(d*
x+c)^2)*cot(d*x+c))
```

Maxima [A]
time = 0.28, size = 204, normalized size = 1.01

$$-\frac{a^2\left(\frac{2\left(315\,\sin(dx+c)^8+105\,\sin(dx+c)^6+63\,\sin(dx+c)^4+45\,\sin(dx+c)^2+35\right)}{\sin(dx+c)^9}-315\,\log\left(\sin\left(dx+c\right)+1\right)+315\,\log\left(\sin\left(dx+c\right)-1\right)\right)+5\,a^2\left(\frac{315\,\tan(dx+c)^8+210\,\tan(dx+c)^6+126\,\tan(dx+c)^4+45\,\tan(dx+c)^2+7}{\tan(dx+c)^9}-63\,\tan\left(dx+c\right)\right)+\frac{\left(315\,\tan(dx+c)^8+420\,\tan(dx+c)^6+378\,\tan(dx+c)^4+180\,\tan(dx+c)^2+35\right)a^2}{\tan(dx+c)^9}}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^10*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/315*(a^2*(2*(315*sin(d*x + c)^8 + 105*sin(d*x + c)^6 + 63*sin(d*x + c)^4
 + 45*sin(d*x + c)^2 + 35)/sin(d*x + c)^9 - 315*log(sin(d*x + c) + 1) + 315
*log(sin(d*x + c) - 1)) + 5*a^2*((315*tan(d*x + c)^8 + 210*tan(d*x + c)^6 +
 126*tan(d*x + c)^4 + 45*tan(d*x + c)^2 + 7)/tan(d*x + c)^9 - 63*tan(d*x +
c)) + (315*tan(d*x + c)^8 + 420*tan(d*x + c)^6 + 378*tan(d*x + c)^4 + 180*t
an(d*x + c)^2 + 35)*a^2/tan(d*x + c)^9)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 406 vs. $2(185) = 370$.
time = 2.76, size = 406, normalized size = 2.02

$$-\frac{1408 a^2 \cos (dx+c)^8 - 2186 a^2 \cos (dx+c)^7 - 3372 a^2 \cos (dx+c)^6 + 6200 a^2 \cos (dx+c)^5 + 2060 a^2 \cos (dx+c)^4 - 5784 a^2 \cos (dx+c)^3 + 268 a^2 \cos (dx+c)^2 + 1756 a^2 \cos (dx+c) - 315 \left(a^2 \cos (dx+c)^7 - 2 a^2 \cos (dx+c)^6 - a^2 \cos (dx+c)^5 + 4 a^2 \cos (dx+c)^4 - a^2 \cos (dx+c)^3 - 2 a^2 \cos (dx+c)^2 + a^2 \cos (dx+c)\right) \log (\sin (dx+c)+1) \sin (dx+c) + 315 \left(a^2 \cos (dx+c)^7 - 2 a^2 \cos (dx+c)^6 - a^2 \cos (dx+c)^5 + 4 a^2 \cos (dx+c)^4 - a^2 \cos (dx+c)^3 - 2 a^2 \cos (dx+c)^2 + a^2 \cos (dx+c)\right) \log (-\sin (dx+c)+1) \sin (dx+c) - 315 a^2}{315 \left(d \cos (dx+c)^7 - 2 d \cos (dx+c)^6 - d \cos (dx+c)^5 + 4 d \cos (dx+c)^4 - d \cos (dx+c)^3 - 2 d \cos (dx+c)^2 + d \cos (dx+c)\right) \sin (dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^10*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/315*(1408*a^2*cos(d*x + c)^8 - 2186*a^2*cos(d*x + c)^7 - 3372*a^2*cos(d*
x + c)^6 + 6200*a^2*cos(d*x + c)^5 + 2060*a^2*cos(d*x + c)^4 - 5784*a^2*cos
(d*x + c)^3 + 268*a^2*cos(d*x + c)^2 + 1756*a^2*cos(d*x + c) - 315*(a^2*cos
(d*x + c)^7 - 2*a^2*cos(d*x + c)^6 - a^2*cos(d*x + c)^5 + 4*a^2*cos(d*x + c
)^4 - a^2*cos(d*x + c)^3 - 2*a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))*log(sin
(d*x + c) + 1)*sin(d*x + c) + 315*(a^2*cos(d*x + c)^7 - 2*a^2*cos(d*x + c)^
6 - a^2*cos(d*x + c)^5 + 4*a^2*cos(d*x + c)^4 - a^2*cos(d*x + c)^3 - 2*a^2*
cos(d*x + c)^2 + a^2*cos(d*x + c))*log(-sin(d*x + c) + 1)*sin(d*x + c) - 31
5*a^2)/((d*cos(d*x + c)^7 - 2*d*cos(d*x + c)^6 - d*cos(d*x + c)^5 + 4*d*cos
(d*x + c)^4 - d*cos(d*x + c)^3 - 2*d*cos(d*x + c)^2 + d*cos(d*x + c))*sin(d
*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**10*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8570 deep
```

Giac [A]
time = 0.54, size = 200, normalized size = 1.00

$$\frac{63 a^2 \tan \left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 + 1155 a^2 \tan \left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 80640 a^2 \log \left(\left|\tan \left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right|\right) - 80640 a^2 \log \left(\left|\tan \left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1\right|\right) + 17955 a^2 \tan \left(\frac{1}{2} dx + \frac{1}{2} c\right) - \frac{80640 a^2 \tan \left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\tan \left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 1} - \frac{139545 a^2 \tan \left(\frac{1}{2} dx + \frac{1}{2} c\right)^8 + 19635 a^2 \tan \left(\frac{1}{2} dx + \frac{1}{2} c\right)^6 + 3591 a^2 \tan \left(\frac{1}{2} dx + \frac{1}{2} c\right)^4 + 495 a^2 \tan \left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 35 a^2}{\tan \left(\frac{1}{2} dx + \frac{1}{2} c\right)^9}}{40320 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^10*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/40320*(63*a^2*tan(1/2*d*x + 1/2*c)^5 + 1155*a^2*tan(1/2*d*x + 1/2*c)^3 +
80640*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 80640*a^2*log(abs(tan(1/2*d*
x + 1/2*c) - 1)) + 17955*a^2*tan(1/2*d*x + 1/2*c) - 80640*a^2*tan(1/2*d*x +
 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - (139545*a^2*tan(1/2*d*x + 1/2*c)^8 +
 19635*a^2*tan(1/2*d*x + 1/2*c)^6 + 3591*a^2*tan(1/2*d*x + 1/2*c)^4 + 495*a
^2*tan(1/2*d*x + 1/2*c)^2 + 35*a^2)/tan(1/2*d*x + 1/2*c)^9)/d
```

Mupad [B]
time = 0.97, size = 194, normalized size = 0.97

$$\frac{11\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{384\,d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{640\,d}+\frac{4\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{-699\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+\frac{1142\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{3}+\frac{764\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{15}+\frac{344\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{35}+\frac{92\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{63}+\frac{a^2}{9}}{d\left(128\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9-128\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\right)}+\frac{57\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/sin(c + d*x)^10,x)
```

```
[Out] (11*a^2*tan(c/2 + (d*x)/2)^3)/(384*d) + (a^2*tan(c/2 + (d*x)/2)^5)/(640*d)
+ (4*a^2*atanh(tan(c/2 + (d*x)/2)))/d - ((92*a^2*tan(c/2 + (d*x)/2)^2)/63 +
 (344*a^2*tan(c/2 + (d*x)/2)^4)/35 + (764*a^2*tan(c/2 + (d*x)/2)^6)/15 + (1
142*a^2*tan(c/2 + (d*x)/2)^8)/3 - 699*a^2*tan(c/2 + (d*x)/2)^10 + a^2/9)/(d
*(128*tan(c/2 + (d*x)/2)^9 - 128*tan(c/2 + (d*x)/2)^11)) + (57*a^2*tan(c/2
+ (d*x)/2))/(128*d)
```

3.38 $\int (a+a\sec(c+dx))^3\sin^9(c+dx)\,dx$

Optimal. Leaf size=203

$$\frac{11a^3\cos(c+dx)}{d}+\frac{3a^3\cos^2(c+dx)}{d}-\frac{14a^3\cos^3(c+dx)}{3d}-\frac{7a^3\cos^4(c+dx)}{2d}+\frac{6a^3\cos^5(c+dx)}{5d}+\frac{11a^3\cos^6(c+}{6d}$$

```
[Out] 11*a^3*cos(d*x+c)/d+3*a^3*cos(d*x+c)^2/d-14/3*a^3*cos(d*x+c)^3/d-7/2*a^3*co
s(d*x+c)^4/d+6/5*a^3*cos(d*x+c)^5/d+11/6*a^3*cos(d*x+c)^6/d+1/7*a^3*cos(d*x
+c)^7/d-3/8*a^3*cos(d*x+c)^8/d-1/9*a^3*cos(d*x+c)^9/d+a^3*ln(cos(d*x+c))/d+
3*a^3*sec(d*x+c)/d+1/2*a^3*sec(d*x+c)^2/d
```

Rubi [A]
time = 0.14, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3957, 2915, 12, 90}

$$-\frac{a^3\cos^9(c+dx)}{9d}-\frac{3a^3\cos^8(c+dx)}{8d}+\frac{a^3\cos^7(c+dx)}{7d}+\frac{11a^3\cos^6(c+dx)}{6d}+\frac{6a^3\cos^5(c+dx)}{5d}-\frac{7a^3\cos^4(c+dx)}{2d}-\frac{14a^3\cos^3(c+dx)}{3d}+\frac{3a^3\cos^2(c+dx)}{d}+\frac{11a^3\cos(c+dx)}{d}+\frac{a^3\sec^2(c+dx)}{2d}+\frac{3a^3\sec(c+dx)}{d}+\frac{a^3\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^9,x]
```

```
[Out] (11*a^3*Cos[c + d*x])/d + (3*a^3*Cos[c + d*x]^2)/d - (14*a^3*Cos[c + d*x]^3
)/(3*d) - (7*a^3*Cos[c + d*x]^4)/(2*d) + (6*a^3*Cos[c + d*x]^5)/(5*d) + (11
*a^3*Cos[c + d*x]^6)/(6*d) + (a^3*Cos[c + d*x]^7)/(7*d) - (3*a^3*Cos[c + d*
x]^8)/(8*d) - (a^3*Cos[c + d*x]^9)/(9*d) + (a^3*Log[Cos[c + d*x]])/d + (3*a
^3*Sec[c + d*x])/d + (a^3*Sec[c + d*x]^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
```

```
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))^3\sin^9(c+dx)\,dx &= -\int (-a-a\cos(c+dx))^3\sin^6(c+dx)\tan^3(c+dx)\,dx\\ &= \frac{\text{Subst}\left(\int \frac{a^3(-a-x)^4(-a+x)^7}{x^3}\,dx, x, -a\cos(c+dx)\right)}{a^9d}\\ &= \frac{\text{Subst}\left(\int \frac{(-a-x)^4(-a+x)^7}{x^3}\,dx, x, -a\cos(c+dx)\right)}{a^6d}\\ &= \frac{\text{Subst}\left(\int \left(-11a^8-\frac{a^{11}}{x^3}+\frac{3a^{10}}{x^2}+\frac{a^9}{x}+6a^7x+14a^6x^2-14a^5x^3-6a\right.\right.}{a^6d}\\ &= \frac{11a^3\cos(c+dx)}{d}+\frac{3a^3\cos^2(c+dx)}{d}-\frac{14a^3\cos^3(c+dx)}{3d}-\frac{7a^3\cos}{}\end{aligned}$$

Mathematica [A]
time = 1.16, size = 148, normalized size = 0.73

$$\frac{a^3(471450+11624760\cos(c+dx)+2188872\cos(3(c+dx))+41160\cos(4(c+dx))-204156\cos(5(c+dx))-35805\cos(6(c+dx))+22972\cos(7(c+dx))+9030\cos(8(c+dx))-820\cos(9(c+dx))-945\cos(10(c+dx))-140\cos(11(c+dx))+645120\log(\cos(c+dx))+210\cos(2(c+dx))(-413+3072\log(\cos(c+dx))))\sec^2(c+dx)}{1290240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^9,x]
```

```
[Out] (a^3*(471450 + 11624760*Cos[c + d*x] + 2188872*Cos[3*(c + d*x)] + 41160*Cos
[4*(c + d*x)] - 204156*Cos[5*(c + d*x)] - 35805*Cos[6*(c + d*x)] + 22972*Co
s[7*(c + d*x)] + 9030*Cos[8*(c + d*x)] - 820*Cos[9*(c + d*x)] - 945*Cos[10*
(c + d*x)] - 140*Cos[11*(c + d*x)] + 645120*Log[Cos[c + d*x]] + 210*Cos[2*(
c + d*x)]*(-413 + 3072*Log[Cos[c + d*x]]))*Sec[c + d*x]^2)/(1290240*d)
```

Maple [A]
time = 0.10, size = 252, normalized size = 1.24

method	result

derivativedivides	$\frac{a^3\left(\frac{\sin^{10}(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^8(dx+c)\right)}{2}+\frac{2\left(\sin^6(dx+c)\right)}{3}+\sin^4(dx+c)+2\left(\sin^2(dx+c)\right)+4\ln(\cos(dx+c))\right)+3a^3\left(\frac{\sin^{10}(dx+c)}{\cos(dx+c)}+\left(\right.\right.}{}$
default	$\frac{a^3\left(\frac{\sin^{10}(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^8(dx+c)\right)}{2}+\frac{2\left(\sin^6(dx+c)\right)}{3}+\sin^4(dx+c)+2\left(\sin^2(dx+c)\right)+4\ln(\cos(dx+c))\right)+3a^3\left(\frac{\sin^{10}(dx+c)}{\cos(dx+c)}+\left(\right.\right.}{}$
risch	$-\frac{25a^3e^{-3i(dx+c)}}{64d}+\frac{1059a^3e^{-i(dx+c)}}{256d}+\frac{57a^3e^{-2i(dx+c)}}{256d}+\frac{57a^3e^{2i(dx+c)}}{256d}+\frac{1059a^3e^{i(dx+c)}}{256d}-\frac{25a^3e^{3i(dx+c)}}{64d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*sin(d*x+c)^9,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/2*sin(d*x+c)^10/cos(d*x+c)^2+1/2*sin(d*x+c)^8+2/3*sin(d*x+c)^6+
sin(d*x+c)^4+2*sin(d*x+c)^2+4*ln(cos(d*x+c)))+3*a^3*(sin(d*x+c)^10/cos(d*x+
c)+(128/35+sin(d*x+c)^8+8/7*sin(d*x+c)^6+48/35*sin(d*x+c)^4+64/35*sin(d*x+c
)^2)*cos(d*x+c))+3*a^3*(-1/8*sin(d*x+c)^8-1/6*sin(d*x+c)^6-1/4*sin(d*x+c)^4
-1/2*sin(d*x+c)^2-ln(cos(d*x+c)))-1/9*a^3*(128/35+sin(d*x+c)^8+8/7*sin(d*x+
c)^6+48/35*sin(d*x+c)^4+64/35*sin(d*x+c)^2)*cos(d*x+c))
```

Maxima [A]
time = 0.27, size = 158, normalized size = 0.78

$$-\frac{280\,a^3\cos(dx+c)^9+945\,a^3\cos(dx+c)^8-360\,a^3\cos(dx+c)^7-4620\,a^3\cos(dx+c)^6-3024\,a^3\cos(dx+c)^5+8820\,a^3\cos(dx+c)^4+11760\,a^3\cos(dx+c)^3-7560\,a^3\cos(dx+c)^2-27720\,a^3\cos(dx+c)-2520\,a^3\log(\cos(dx+c))-\frac{1260\,(6\,a^3\cos(dx+c)+a^3)}{\cos(dx+c)^2}}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^9,x, algorithm="maxima")
```

```
[Out] -1/2520*(280*a^3*cos(d*x + c)^9 + 945*a^3*cos(d*x + c)^8 - 360*a^3*cos(d*x
+ c)^7 - 4620*a^3*cos(d*x + c)^6 - 3024*a^3*cos(d*x + c)^5 + 8820*a^3*cos(d
*x + c)^4 + 11760*a^3*cos(d*x + c)^3 - 7560*a^3*cos(d*x + c)^2 - 27720*a^3*
cos(d*x + c) - 2520*a^3*log(cos(d*x + c)) - 1260*(6*a^3*cos(d*x + c) + a^3)
/cos(d*x + c)^2)/d
```

Fricas [A]
time = 2.92, size = 182, normalized size = 0.90

$$-\frac{35840\,a^3\cos(dx+c)^{11}+120960\,a^3\cos(dx+c)^{10}-46080\,a^3\cos(dx+c)^9-591360\,a^3\cos(dx+c)^8-387072\,a^3\cos(dx+c)^7+1128960\,a^3\cos(dx+c)^6+1505280\,a^3\cos(dx+c)^5-967680\,a^3\cos(dx+c)^4-3548160\,a^3\cos(dx+c)^3-322560\,a^3\cos(dx+c)^2\log(-\cos(dx+c))+212205\,a^3\cos(dx+c)^2-967680\,a^3\cos(dx+c)-161280\,a^3}{322560\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^9,x, algorithm="fricas")
```

```
[Out] -1/322560*(35840*a^3*cos(d*x + c)^11 + 120960*a^3*cos(d*x + c)^10 - 46080*a
^3*cos(d*x + c)^9 - 591360*a^3*cos(d*x + c)^8 - 387072*a^3*cos(d*x + c)^7 +
 1128960*a^3*cos(d*x + c)^6 + 1505280*a^3*cos(d*x + c)^5 - 967680*a^3*cos(d
```

```
*x + c)^4 - 3548160*a^3*cos(d*x + c)^3 - 322560*a^3*cos(d*x + c)^2*log(-cos
(d*x + c)) + 212205*a^3*cos(d*x + c)^2 - 967680*a^3*cos(d*x + c) - 161280*a
^3)/(d*cos(d*x + c)^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*sin(d*x+c)**9,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8570 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 396 vs. 2(187) = 374.
time = 0.65, size = 396, normalized size = 1.95

$$-\frac{2520\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-2520\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)-\frac{1260\left(9\,a^3+\frac{2\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{3\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^2}+\frac{45257\,a^3-\frac{392193\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{1467972\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{3001908\,a^3(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{3232782\,a^3(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}-\frac{2359854\,a^3(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5}+\frac{1190196\,a^3(\cos(dx+c)-1)^6}{(\cos(dx+c)+1)^6}-\frac{397764\,a^3(\cos(dx+c)-1)^7}{(\cos(dx+c)+1)^7}+\frac{79281\,a^3(\cos(dx+c)-1)^8}{(\cos(dx+c)+1)^8}-\frac{7129\,a^3(\cos(dx+c)-1)^9}{(\cos(dx+c)+1)^9}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^9}}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^9,x, algorithm="giac")
```

```
[Out] -1/2520*(2520*a^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 25
20*a^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - 1260*(9*a^3 +
 2*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 3*a^3*(cos(d*x + c) - 1)^2/(
cos(d*x + c) + 1)^2)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^2 + (45257
*a^3 - 392193*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1467972*a^3*(cos(
d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 3001908*a^3*(cos(d*x + c) - 1)^3/(co
s(d*x + c) + 1)^3 + 3232782*a^3*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 -
 2359854*a^3*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 1190196*a^3*(cos(d
*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 397764*a^3*(cos(d*x + c) - 1)^7/(cos(
d*x + c) + 1)^7 + 79281*a^3*(cos(d*x + c) - 1)^8/(cos(d*x + c) + 1)^8 - 712
9*a^3*(cos(d*x + c) - 1)^9/(cos(d*x + c) + 1)^9)/((cos(d*x + c) - 1)/(cos(d
*x + c) + 1) - 1)^9)/d
```

Mupad [B]
time = 0.96, size = 157, normalized size = 0.77

$$\frac{\frac{3\,a^3\,\cos(c+d\,x)+\frac{a^3}{2}}{\cos(c+d\,x)^2}+11\,a^3\,\cos\left(c+d\,x\right)+3\,a^3\cos\left(c+d\,x\right)^2-\frac{14\,a^3\cos(c+d\,x)^3}{3}-\frac{7\,a^3\cos(c+d\,x)^4}{2}+\frac{6\,a^3\cos(c+d\,x)^5}{5}+\frac{11\,a^3\cos(c+d\,x)^6}{6}+\frac{a^3\cos(c+d\,x)^7}{7}-\frac{3\,a^3\cos(c+d\,x)^8}{8}-\frac{a^3\cos(c+d\,x)^9}{9}+a^3\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^9*(a + a/cos(c + d*x))^3,x)
```

```
[Out] ((3*a^3*cos(c + d*x) + a^3/2)/cos(c + d*x)^2 + 11*a^3*cos(c + d*x) + 3*a^3*
cos(c + d*x)^2 - (14*a^3*cos(c + d*x)^3)/3 - (7*a^3*cos(c + d*x)^4)/2 + (6*
a^3*cos(c + d*x)^5)/5 + (11*a^3*cos(c + d*x)^6)/6 + (a^3*cos(c + d*x)^7)/7
- (3*a^3*cos(c + d*x)^8)/8 - (a^3*cos(c + d*x)^9)/9 + a^3*log(cos(c + d*x))
)/d
```

3.39 $\int (a+a\sec(c+dx))^3\sin^7(c+dx)\,dx$

Optimal. Leaf size=131

$$\frac{8a^3\cos(c+dx)}{d}+\frac{3a^3\cos^2(c+dx)}{d}-\frac{2a^3\cos^3(c+dx)}{d}-\frac{2a^3\cos^4(c+dx)}{d}+\frac{a^3\cos^6(c+dx)}{2d}+\frac{a^3\cos^7(c+dx)}{7d}+$$

```
[Out] 8*a^3*cos(d*x+c)/d+3*a^3*cos(d*x+c)^2/d-2*a^3*cos(d*x+c)^3/d-2*a^3*cos(d*x+
c)^4/d+1/2*a^3*cos(d*x+c)^6/d+1/7*a^3*cos(d*x+c)^7/d+3*a^3*sec(d*x+c)/d+1/2
*a^3*sec(d*x+c)^2/d
```

Rubi [A]
time = 0.12, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3957, 2915, 12, 90}

$$\frac{a^3\cos^7(c+dx)}{7d}+\frac{a^3\cos^6(c+dx)}{2d}-\frac{2a^3\cos^4(c+dx)}{d}-\frac{2a^3\cos^3(c+dx)}{d}+\frac{3a^3\cos^2(c+dx)}{d}+\frac{8a^3\cos(c+dx)}{d}+\frac{a^3\sec^2(c+dx)}{2d}+\frac{3a^3\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^7,x]
```

```
[Out] (8*a^3*Cos[c + d*x])/d + (3*a^3*Cos[c + d*x]^2)/d - (2*a^3*Cos[c + d*x]^3)/
d - (2*a^3*Cos[c + d*x]^4)/d + (a^3*Cos[c + d*x]^6)/(2*d) + (a^3*Cos[c + d*
x]^7)/(7*d) + (3*a^3*Sec[c + d*x])/d + (a^3*Sec[c + d*x]^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int (a+a\sec(c+dx))^3\sin^7(c+dx)\,dx &= -\int (-a-a\cos(c+dx))^3\sin^4(c+dx)\tan^3(c+dx)\,dx\\
&= \frac{\text{Subst}\left(\int \frac{a^3(-a-x)^3(-a+x)^6}{x^3}\,dx, x, -a\cos(c+dx)\right)}{a^7 d}\\
&= \frac{\text{Subst}\left(\int \frac{(-a-x)^3(-a+x)^6}{x^3}\,dx, x, -a\cos(c+dx)\right)}{a^4 d}\\
&= \frac{\text{Subst}\left(\int \left(-8a^6-\frac{a^9}{x^3}+\frac{3a^8}{x^2}+6a^5x+6a^4x^2-8a^3x^3+3ax^5-x^6\right)\, d\right.}{a^4 d}\\
&= \frac{8a^3\cos(c+dx)}{d}+\frac{3a^3\cos^2(c+dx)}{d}-\frac{2a^3\cos^3(c+dx)}{d}-\frac{2a^3\cos^4(c}{d}
\end{aligned}$$

Mathematica [A]
time = 0.58, size = 106, normalized size = 0.81

$$\frac{a^3(427+14014\cos(c+dx)-210\cos(2(c+dx))+2548\cos(3(c+dx))+196\cos(4(c+dx))-188\cos(5(c+dx))-56\cos(6(c+dx))+9\cos(7(c+dx))+7\cos(8(c+dx))+\cos(9(c+dx)))\sec^2(c+dx)}{1792d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^7,x]
```

```
[Out] (a^3*(427 + 14014*Cos[c + d*x] - 210*Cos[2*(c + d*x)] + 2548*Cos[3*(c + d*x
)] + 196*Cos[4*(c + d*x)] - 188*Cos[5*(c + d*x)] - 56*Cos[6*(c + d*x)] + 9*
Cos[7*(c + d*x)] + 7*Cos[8*(c + d*x)] + Cos[9*(c + d*x)])*Sec[c + d*x]^2)/(
1792*d)
```

Maple [A]
time = 0.09, size = 214, normalized size = 1.63

method	result
derivativedivides	$a^3\left(\frac{\sin^8(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^6(dx+c)\right)}{2}+\frac{3\left(\sin^4(dx+c)\right)}{4}+\frac{3\left(\sin^2(dx+c)\right)}{2}+3\ln(\cos(dx+c))\right)+3a^3\left(\frac{\sin^8(dx+c)}{\cos(dx+c)}+\left(\frac{16}{5}+\sin^6(dx+c)+\right.\right.$
default	$a^3\left(\frac{\sin^8(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^6(dx+c)\right)}{2}+\frac{3\left(\sin^4(dx+c)\right)}{4}+\frac{3\left(\sin^2(dx+c)\right)}{2}+3\ln(\cos(dx+c))\right)+3a^3\left(\frac{\sin^8(dx+c)}{\cos(dx+c)}+\left(\frac{16}{5}+\sin^6(dx+c)+\right.\right.$

risch	$-\frac{29a^3 e^{3i(dx+c)}}{128d}+\frac{47a^3 e^{2i(dx+c)}}{128d}+\frac{421a^3 e^{i(dx+c)}}{128d}+\frac{421a^3 e^{-i(dx+c)}}{128d}+\frac{47a^3 e^{-2i(dx+c)}}{128d}-\frac{29a^3 e^{-3i(dx+c)}}{128d}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*sin(d*x+c)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/2*sin(d*x+c)^8/cos(d*x+c)^2+1/2*sin(d*x+c)^6+3/4*sin(d*x+c)^4+3
/2*sin(d*x+c)^2+3*ln(cos(d*x+c)))+3*a^3*(sin(d*x+c)^8/cos(d*x+c)+(16/5+sin(
d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c))+3*a^3*(-1/6*sin(d*x
+c)^6-1/4*sin(d*x+c)^4-1/2*sin(d*x+c)^2-ln(cos(d*x+c)))-1/7*a^3*(16/5+sin(d
*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c))
```

Maxima [A]
time = 0.26, size = 107, normalized size = 0.82

$$\frac{2\,a^3\cos\left(dx+c\right)^7+7\,a^3\cos\left(dx+c\right)^6-28\,a^3\cos\left(dx+c\right)^4-28\,a^3\cos\left(dx+c\right)^3+42\,a^3\cos\left(dx+c\right)^2+112\,a^3\cos\left(dx+c\right)+\frac{7\,(6\,a^3\cos(dx+c)+a^3)}{\cos(dx+c)^2}}{14\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/14*(2*a^3*cos(d*x + c)^7 + 7*a^3*cos(d*x + c)^6 - 28*a^3*cos(d*x + c)^4 -
 28*a^3*cos(d*x + c)^3 + 42*a^3*cos(d*x + c)^2 + 112*a^3*cos(d*x + c) + 7*(
6*a^3*cos(d*x + c) + a^3)/cos(d*x + c)^2)/d
```

Fricas [A]
time = 3.50, size = 121, normalized size = 0.92

$$\frac{32\,a^3\cos\left(dx+c\right)^9+112\,a^3\cos\left(dx+c\right)^8-448\,a^3\cos\left(dx+c\right)^6-448\,a^3\cos\left(dx+c\right)^5+672\,a^3\cos\left(dx+c\right)^4+1792\,a^3\cos\left(dx+c\right)^3-203\,a^3\cos\left(dx+c\right)^2+672\,a^3\cos\left(dx+c\right)+112\,a^3}{224\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/224*(32*a^3*cos(d*x + c)^9 + 112*a^3*cos(d*x + c)^8 - 448*a^3*cos(d*x + c
)^6 - 448*a^3*cos(d*x + c)^5 + 672*a^3*cos(d*x + c)^4 + 1792*a^3*cos(d*x +
c)^3 - 203*a^3*cos(d*x + c)^2 + 672*a^3*cos(d*x + c) + 112*a^3)/(d*cos(d*x
+ c)^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*sin(d*x+c)**7,x)
```

[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep

Giac [A]
time = 0.59, size = 239, normalized size = 1.82

$$\frac{2\left(\frac{7\left(3a^3+\frac{2a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^2}-\frac{43a^3-\frac{273a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{672a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{630a^3(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{343a^3(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}-\frac{105a^3(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5}+\frac{14a^3(\cos(dx+c)-1)^6}{(\cos(dx+c)+1)^6}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^7}\right)}{7d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^7,x, algorithm="giac")

[Out] 2/7*(7*(3*a^3 + 2*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^2 - (43*a^3 - 273*a^3*(cos(d*x + c) - 1)/(cos (d*x + c) + 1) + 672*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 630*a^ 3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 343*a^3*(cos(d*x + c) - 1)^4/ (cos(d*x + c) + 1)^4 - 105*a^3*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 14*a^3*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^7)/d

Mupad [B]
time = 0.90, size = 107, normalized size = 0.82

$$\frac{\frac{3a^3\cos(c+dx)+\frac{a^3}{2}}{\cos(c+dx)^2}+8a^3\cos(c+dx)+3a^3\cos(c+dx)^2-2a^3\cos(c+dx)^3-2a^3\cos(c+dx)^4+\frac{a^3\cos(c+dx)^6}{2}+\frac{a^3\cos(c+dx)^7}{7}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(c + d*x)^7*(a + a/cos(c + d*x))^3,x)

[Out] ((3*a^3*cos(c + d*x) + a^3/2)/cos(c + d*x)^2 + 8*a^3*cos(c + d*x) + 3*a^3*c os(c + d*x)^2 - 2*a^3*cos(c + d*x)^3 - 2*a^3*cos(c + d*x)^4 + (a^3*cos(c + d*x)^6)/2 + (a^3*cos(c + d*x)^7)/7)/d

3.40 $\int (a+a\sec(c+dx))^3 \sin^5(c+dx)\,dx$

Optimal. Leaf size=134

$$\frac{5a^3\cos(c+dx)}{d}+\frac{5a^3\cos^2(c+dx)}{2d}-\frac{a^3\cos^3(c+dx)}{3d}-\frac{3a^3\cos^4(c+dx)}{4d}-\frac{a^3\cos^5(c+dx)}{5d}-\frac{a^3\log(\cos(c+dx)}{d}$$

```
[Out] 5*a^3*cos(d*x+c)/d+5/2*a^3*cos(d*x+c)^2/d-1/3*a^3*cos(d*x+c)^3/d-3/4*a^3*co
s(d*x+c)^4/d-1/5*a^3*cos(d*x+c)^5/d-a^3*ln(cos(d*x+c))/d+3*a^3*sec(d*x+c)/d
+1/2*a^3*sec(d*x+c)^2/d
```

Rubi [A]
time = 0.12, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3957, 2915, 12, 90}

$$-\frac{a^3\cos^5(c+dx)}{5d}-\frac{3a^3\cos^4(c+dx)}{4d}-\frac{a^3\cos^3(c+dx)}{3d}+\frac{5a^3\cos^2(c+dx)}{2d}+\frac{5a^3\cos(c+dx)}{d}+\frac{a^3\sec^2(c+dx)}{2d}+\frac{3a^3\sec(c+dx)}{d}-\frac{a^3\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^5,x]
```

```
[Out] (5*a^3*Cos[c + d*x])/d + (5*a^3*Cos[c + d*x]^2)/(2*d) - (a^3*Cos[c + d*x]^3
)/(3*d) - (3*a^3*Cos[c + d*x]^4)/(4*d) - (a^3*Cos[c + d*x]^5)/(5*d) - (a^3*
Log[Cos[c + d*x]])/d + (3*a^3*Sec[c + d*x])/d + (a^3*Sec[c + d*x]^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int (a+a\sec(c+dx))^3\sin^5(c+dx)\,dx &= -\int (-a-a\cos(c+dx))^3\sin^2(c+dx)\tan^3(c+dx)\,dx \\
&= \frac{\text{Subst}\left(\int \frac{a^3(-a-x)^2(-a+x)^5}{x^3}\,dx, x, -a\cos(c+dx)\right)}{a^5d} \\
&= \frac{\text{Subst}\left(\int \frac{(-a-x)^2(-a+x)^5}{x^3}\,dx, x, -a\cos(c+dx)\right)}{a^2d} \\
&= \frac{\text{Subst}\left(\int \left(-5a^4-\frac{a^7}{x^3}+\frac{3a^6}{x^2}-\frac{a^5}{x}+5a^3x+a^2x^2-3ax^3+x^4\right)\,dx, x\right.}{a^2d} \\
&= \frac{5a^3\cos(c+dx)}{d}+\frac{5a^3\cos^2(c+dx)}{2d}-\frac{a^3\cos^3(c+dx)}{3d}-\frac{3a^3\cos^4(c}{4d}
\end{aligned}$$

Mathematica [A]

time = 0.41, size = 108, normalized size = 0.81

$$-\frac{a^3(-120-12350\cos(c+dx)-2074\cos(3(c+dx))-330\cos(4(c+dx))+82\cos(5(c+dx))+45\cos(6(c+dx))+6\cos(7(c+dx))+960\log(\cos(c+dx))+15\cos(2(c+dx))(31+64\log(\cos(c+dx))))\sec^2(c+dx)}{1920d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^5,x]
```

```
[Out] -1/1920*(a^3*(-120 - 12350*Cos[c + d*x] - 2074*Cos[3*(c + d*x)] - 330*Cos[4
*(c + d*x)] + 82*Cos[5*(c + d*x)] + 45*Cos[6*(c + d*x)] + 6*Cos[7*(c + d*x)
] + 960*Log[Cos[c + d*x]] + 15*Cos[2*(c + d*x)]*(31 + 64*Log[Cos[c + d*x]])
)*Sec[c + d*x]^2)/d
```

Maple [A]

time = 0.14, size = 172, normalized size = 1.28

method	result
derivativedivides	$\frac{a^3\left(\frac{\sin^6(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^4(dx+c)\right)}{2}+\sin^2(dx+c)+2\ln(\cos(dx+c))\right)+3a^3\left(\frac{\sin^6(dx+c)}{\cos(dx+c)}+\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos\right.}{d}$
default	$\frac{a^3\left(\frac{\sin^6(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^4(dx+c)\right)}{2}+\sin^2(dx+c)+2\ln(\cos(dx+c))\right)+3a^3\left(\frac{\sin^6(dx+c)}{\cos(dx+c)}+\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos\right.}{d}$

norman	$\frac{\frac{8a^3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{224a^3}{15d}-\frac{2a^3\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{6a^3\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{8a^3\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{134a^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}+\frac{214a}{}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}$
risch	$ia^3x-\frac{7a^3e^{3i(dx+c)}}{96d}+\frac{7a^3e^{2i(dx+c)}}{16d}+\frac{37a^3e^{i(dx+c)}}{16d}+\frac{37a^3e^{-i(dx+c)}}{16d}+\frac{7a^3e^{-2i(dx+c)}}{16d}-\frac{7a^3e^{-3i(dx+c)}}{96d}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*sin(d*x+c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/2*sin(d*x+c)^6/cos(d*x+c)^2+1/2*sin(d*x+c)^4+sin(d*x+c)^2+2*ln(
cos(d*x+c)))+3*a^3*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin(d*x+c)^4+4/3*sin(d*x+c
)^2)*cos(d*x+c))+3*a^3*(-1/4*sin(d*x+c)^4-1/2*sin(d*x+c)^2-ln(cos(d*x+c)))-
1/5*a^3*(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))
```

Maxima [A]
time = 0.26, size = 106, normalized size = 0.79

$$-\frac{12\,a^3\cos\left(dx+c\right)^5+45\,a^3\cos\left(dx+c\right)^4+20\,a^3\cos\left(dx+c\right)^3-150\,a^3\cos\left(dx+c\right)^2-300\,a^3\cos\left(dx+c\right)+60\,a^3\log\left(\cos\left(dx+c\right)\right)-\frac{30\,\left(6\,a^3\cos\left(dx+c\right)+a^3\right)}{\cos\left(dx+c\right)^2}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/60*(12*a^3*cos(d*x + c)^5 + 45*a^3*cos(d*x + c)^4 + 20*a^3*cos(d*x + c)^
3 - 150*a^3*cos(d*x + c)^2 - 300*a^3*cos(d*x + c) + 60*a^3*log(cos(d*x + c)
) - 30*(6*a^3*cos(d*x + c) + a^3)/cos(d*x + c)^2)/d
```

Fricas [A]
time = 3.22, size = 130, normalized size = 0.97

$$-\frac{96\,a^3\cos\left(dx+c\right)^7+360\,a^3\cos\left(dx+c\right)^6+160\,a^3\cos\left(dx+c\right)^5-1200\,a^3\cos\left(dx+c\right)^4-2400\,a^3\cos\left(dx+c\right)^3+480\,a^3\cos\left(dx+c\right)^2\log\left(-\cos\left(dx+c\right)\right)+465\,a^3\cos\left(dx+c\right)^2-1440\,a^3\cos\left(dx+c\right)-240\,a^3}{480\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/480*(96*a^3*cos(d*x + c)^7 + 360*a^3*cos(d*x + c)^6 + 160*a^3*cos(d*x +
c)^5 - 1200*a^3*cos(d*x + c)^4 - 2400*a^3*cos(d*x + c)^3 + 480*a^3*cos(d*x
+ c)^2*log(-cos(d*x + c)) + 465*a^3*cos(d*x + c)^2 - 1440*a^3*cos(d*x + c)
- 240*a^3)/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*sin(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 297 vs. $2(124) = 248$.
time = 0.53, size = 297, normalized size = 2.22

$$\frac{60\,a^3 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 60\,a^3 \log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{30\left(15\,a^3+\frac{14\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{3\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^2} - \frac{399\,a^3-\frac{1395\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{390\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{650\,a^3(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}-\frac{565\,a^3(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}+\frac{137\,a^3(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/60*(60*a^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 60*a^3*
log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + 30*(15*a^3 + 14*a^3*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 3*a^3*(cos(d*x + c) - 1)^2/(cos(d*x
 + c) + 1)^2)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^2 - (399*a^3 - 13
95*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 390*a^3*(cos(d*x + c) - 1)^2
/(cos(d*x + c) + 1)^2 + 650*a^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 -
 565*a^3*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 137*a^3*(cos(d*x + c)
- 1)^5/(cos(d*x + c) + 1)^5)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^5)
/d
```

Mupad [B]
time = 0.89, size = 107, normalized size = 0.80

$$-\frac{\frac{a^3\cos(c+d\,x)^3}{3}-5\,a^3\,\cos\left(c+d\,x\right)-\frac{5\,a^3\cos(c+d\,x)^2}{2}-\frac{3\,a^3\,\cos(c+d\,x)+\frac{a^3}{2}}{\cos(c+d\,x)^2}+\frac{3\,a^3\cos(c+d\,x)^4}{4}+\frac{a^3\cos(c+d\,x)^5}{5}+a^3\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5*(a + a/cos(c + d*x))^3,x)
```

```
[Out] -((a^3*cos(c + d*x)^3)/3 - 5*a^3*cos(c + d*x) - (5*a^3*cos(c + d*x)^2)/2 -
(3*a^3*cos(c + d*x) + a^3/2)/cos(c + d*x)^2 + (3*a^3*cos(c + d*x)^4)/4 + (a
^3*cos(c + d*x)^5)/5 + a^3*log(cos(c + d*x)))/d
```

3.41 $\int (a+a\sec(c+dx))^3\sin^3(c+dx)\,dx$

Optimal. Leaf size=98

$$\frac{2a^3\cos(c+dx)}{d}+\frac{3a^3\cos^2(c+dx)}{2d}+\frac{a^3\cos^3(c+dx)}{3d}-\frac{2a^3\log(\cos(c+dx))}{d}+\frac{3a^3\sec(c+dx)}{d}+\frac{a^3\sec^2(c+dx)}{2d}$$

```
[Out] 2*a^3*cos(d*x+c)/d+3/2*a^3*cos(d*x+c)^2/d+1/3*a^3*cos(d*x+c)^3/d-2*a^3*ln(c
os(d*x+c))/d+3*a^3*sec(d*x+c)/d+1/2*a^3*sec(d*x+c)^2/d
```

Rubi [A]
time = 0.07, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3957, 2786, 76}

$$\frac{a^3\cos^3(c+dx)}{3d}+\frac{3a^3\cos^2(c+dx)}{2d}+\frac{2a^3\cos(c+dx)}{d}+\frac{a^3\sec^2(c+dx)}{2d}+\frac{3a^3\sec(c+dx)}{d}-\frac{2a^3\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^3,x]
```

```
[Out] (2*a^3*Cos[c + d*x])/d + (3*a^3*Cos[c + d*x]^2)/(2*d) + (a^3*Cos[c + d*x]^3
)/(3*d) - (2*a^3*Log[Cos[c + d*x]])/d + (3*a^3*Sec[c + d*x])/d + (a^3*Sec[c
 + d*x]^2)/(2*d)
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+a\sec(c+dx))^3\sin^3(c+dx)\,dx = -\int (-a-a\cos(c+dx))^3\tan^3(c+dx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{(-a-x)(-a+x)^4}{x^3}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-2a^2-\frac{a^5}{x^3}+\frac{3a^4}{x^2}-\frac{2a^3}{x}+3ax-x^2\right)dx, x, -a\cos(c+dx\right.}{d}$$
$$= \frac{2a^3\cos(c+dx)}{d}+\frac{3a^3\cos^2(c+dx)}{2d}+\frac{a^3\cos^3(c+dx)}{3d}-\frac{2a^3\log(\cos}{d}$$

Mathematica [A]
time = 0.13, size = 86, normalized size = 0.88

$$\frac{a^3(-41+226\cos(c+dx)+29\cos(3(c+dx))+9\cos(4(c+dx))+\cos(5(c+dx))-48\log(\cos(c+dx))-8\cos(2(c+dx))(7+6\log(\cos(c+dx))))\sec^2(c+dx)}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^3,x]
```

```
[Out] (a^3*(-41 + 226*Cos[c + d*x] + 29*Cos[3*(c + d*x)] + 9*Cos[4*(c + d*x)] + C
os[5*(c + d*x)] - 48*Log[Cos[c + d*x]] - 8*Cos[2*(c + d*x)]*(7 + 6*Log[Cos[
c + d*x]]))*Sec[c + d*x]^2)/(48*d)
```

Maple [A]
time = 0.12, size = 114, normalized size = 1.16

method	result
derivativedivides	$\frac{a^3\left(\frac{\left(\tan^2(dx+c)\right)}{2}+\ln(\cos(dx+c))\right)+3a^3\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)+3a^3\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c)\right.}{d}$
default	$\frac{a^3\left(\frac{\left(\tan^2(dx+c)\right)}{2}+\ln(\cos(dx+c))\right)+3a^3\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)+3a^3\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c)\right.}{d}$
norman	$\frac{\frac{32a^3}{3d}-\frac{4a^3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{4a^3\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{20a^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{20a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{2a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-$
risch	$2ia^3x+\frac{a^3e^{3i(dx+c)}}{24d}+\frac{3a^3e^{2i(dx+c)}}{8d}+\frac{9a^3e^{i(dx+c)}}{8d}+\frac{9a^3e^{-i(dx+c)}}{8d}+\frac{3a^3e^{-2i(dx+c)}}{8d}+\frac{a^3e^{-3i(dx+c)}}{24d}+\frac{4ia^3}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*sin(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/2*tan(d*x+c)^2+ln(cos(d*x+c)))+3*a^3*(sin(d*x+c)^4/cos(d*x+c)+(
2+sin(d*x+c)^2)*cos(d*x+c))+3*a^3*(-1/2*sin(d*x+c)^2-ln(cos(d*x+c)))-1/3*a^
3*(2+sin(d*x+c)^2)*cos(d*x+c))
```

Maxima [A]
time = 0.26, size = 80, normalized size = 0.82

$$\frac{2\,a^{3}\cos\left(dx+c\right)^{3}+9\,a^{3}\cos\left(dx+c\right)^{2}+12\,a^{3}\cos\left(dx+c\right)-12\,a^{3}\log\left(\cos\left(dx+c\right)\right)+\frac{3\,\left(6\,a^{3}\cos(dx+c)+a^{3}\right)}{\cos(dx+c)^{2}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(2*a^3*cos(d*x + c)^3 + 9*a^3*cos(d*x + c)^2 + 12*a^3*cos(d*x + c) - 12
*a^3*log(cos(d*x + c)) + 3*(6*a^3*cos(d*x + c) + a^3)/cos(d*x + c)^2)/d
```

Fricas [A]
time = 2.97, size = 104, normalized size = 1.06

$$\frac{4\,a^{3}\cos\left(dx+c\right)^{5}+18\,a^{3}\cos\left(dx+c\right)^{4}+24\,a^{3}\cos\left(dx+c\right)^{3}-24\,a^{3}\cos\left(dx+c\right)^{2}\log\left(-\cos\left(dx+c\right)\right)-9\,a^{3}\cos\left(dx+c\right)^{2}+36\,a^{3}\cos\left(dx+c\right)+6\,a^{3}}{12\,d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(4*a^3*cos(d*x + c)^5 + 18*a^3*cos(d*x + c)^4 + 24*a^3*cos(d*x + c)^3
- 24*a^3*cos(d*x + c)^2*log(-cos(d*x + c)) - 9*a^3*cos(d*x + c)^2 + 36*a^3*
cos(d*x + c) + 6*a^3)/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{3}\left(\int 3\sin^{3}\left(c+dx\right)\sec\left(c+dx\right)dx+\int 3\sin^{3}\left(c+dx\right)\sec^{2}\left(c+dx\right)dx+\int \sin^{3}\left(c+dx\right)\sec^{3}\left(c+dx\right)dx+\int \sin^{3}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*sin(d*x+c)**3,x)
```

```
[Out] a**3*(Integral(3*sin(c + d*x)**3*sec(c + d*x), x) + Integral(3*sin(c + d*x)
**3*sec(c + d*x)**2, x) + Integral(sin(c + d*x)**3*sec(c + d*x)**3, x) + In
tegral(sin(c + d*x)**3, x))
```

Giac [A]
time = 0.49, size = 102, normalized size = 1.04

$$-\frac{2\,a^{3}\log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d}+\frac{6\,a^{3}\cos\left(dx+c\right)+a^{3}}{2\,d\cos\left(dx+c\right)^{2}}+\frac{2\,a^{3}d^{8}\cos\left(dx+c\right)^{3}+9\,a^{3}d^{8}\cos\left(dx+c\right)^{2}+12\,a^{3}d^{8}\cos\left(dx+c\right)}{6\,d^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^3,x, algorithm="giac")
```

[Out] -2*a^3*log(abs(cos(d*x + c))/abs(d))/d + 1/2*(6*a^3*cos(d*x + c) + a^3)/(d*cos(d*x + c)^2) + 1/6*(2*a^3*d^8*cos(d*x + c)^3 + 9*a^3*d^8*cos(d*x + c)^2 + 12*a^3*d^8*cos(d*x + c))/d^9

Mupad [B]
time = 0.88, size = 80, normalized size = 0.82

$$\frac{\frac{3\,a^3\,\cos(c+d\,x)+\frac{a^3}{2}}{\cos(c+d\,x)^2}+2\,a^3\,\cos\left(c+d\,x\right)+\frac{3\,a^3\,\cos(c+d\,x)^2}{2}+\frac{a^3\,\cos(c+d\,x)^3}{3}-2\,a^3\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(c + d*x)^3*(a + a/cos(c + d*x))^3,x)

[Out] ((3*a^3*cos(c + d*x) + a^3/2)/cos(c + d*x)^2 + 2*a^3*cos(c + d*x) + (3*a^3*cos(c + d*x)^2)/2 + (a^3*cos(c + d*x)^3)/3 - 2*a^3*log(cos(c + d*x)))/d

3.42 $\int (a+a\sec(c+dx))^3\sin(c+dx)\,dx$

Optimal. Leaf size=62

$$-\frac{a^3\cos(c+dx)}{d}-\frac{3a^3\log(\cos(c+dx))}{d}+\frac{3a^3\sec(c+dx)}{d}+\frac{a^3\sec^2(c+dx)}{2d}$$

```
[Out] -a^3*cos(d*x+c)/d-3*a^3*ln(cos(d*x+c))/d+3*a^3*sec(d*x+c)/d+1/2*a^3*sec(d*x
+c)^2/d
```

Rubi [A]
time = 0.07, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.210$, Rules used = {3957, 2912, 12, 45}

$$-\frac{a^3\cos(c+dx)}{d}+\frac{a^3\sec^2(c+dx)}{2d}+\frac{3a^3\sec(c+dx)}{d}-\frac{3a^3\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*Sin[c + d*x],x]
```

```
[Out] -((a^3*Cos[c + d*x])/d) - (3*a^3*Log[Cos[c + d*x]])/d + (3*a^3*Sec[c + d*x]
)/d + (a^3*Sec[c + d*x]^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
```

```
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+a\sec(c+dx))^3\sin(c+dx)\,dx = -\int (-a-a\cos(c+dx))^3\sec^2(c+dx)\tan(c+dx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{a^3(-a+x)^3}{x^3}\,dx, x, -a\cos(c+dx)\right)}{ad}$$
$$= \frac{a^2\text{Subst}\left(\int \frac{(-a+x)^3}{x^3}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^2\text{Subst}\left(\int \left(1-\frac{a^3}{x^3}+\frac{3a^2}{x^2}-\frac{3a}{x}\right)\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= -\frac{a^3\cos(c+dx)}{d}-\frac{3a^3\log(\cos(c+dx))}{d}+\frac{3a^3\sec(c+dx)}{d}+\frac{a^3\sec^2(}{2}$$

Mathematica [A]
time = 0.15, size = 65, normalized size = 1.05

$$-\frac{a^3(-4-9\cos(c+dx)+\cos(3(c+dx))+6\log(\cos(c+dx))+\cos(2(c+dx))(-2+6\log(\cos(c+dx))))\sec^2(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Sin[c + d*x],x]
[Out] -1/4*(a^3*(-4 - 9*Cos[c + d*x] + Cos[3*(c + d*x)] + 6*Log[Cos[c + d*x]] + C
os[2*(c + d*x)]*(-2 + 6*Log[Cos[c + d*x]]))*Sec[c + d*x]^2)/d
```

Maple [A]
time = 0.06, size = 46, normalized size = 0.74

method	result
derivativedivides	$\frac{a^3\left(\frac{\left(\sec^2(dx+c)\right)}{2}+3\sec(dx+c)+3\ln(\sec(dx+c))-\frac{1}{\sec(dx+c)}\right)}{d}$
default	$\frac{a^3\left(\frac{\left(\sec^2(dx+c)\right)}{2}+3\sec(dx+c)+3\ln(\sec(dx+c))-\frac{1}{\sec(dx+c)}\right)}{d}$
risch	$3ia^3x-\frac{a^3e^{i(dx+c)}}{2d}-\frac{a^3e^{-i(dx+c)}}{2d}+\frac{6ia^3c}{d}+\frac{2a^3\left(3e^{3i(dx+c)}+e^{2i(dx+c)}+3e^{i(dx+c)}\right)}{d\left(e^{2i(dx+c)}+1\right)^2}-\frac{3a^3\ln\left(e^{2i(dx+c)}+1\right)}{d}$
norman	$\frac{\frac{4a^3}{d}+\frac{6a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{6a^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{3a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{3a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}+\frac{3a^3\ln\left(1+\tan\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*sin(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] a^3/d*(1/2*sec(d*x+c)^2+3*sec(d*x+c)+3*ln(sec(d*x+c))-1/sec(d*x+c))
```

Maxima [A]
time = 0.27, size = 55, normalized size = 0.89

$$-\frac{2\,a^3\cos\left(dx+c\right)+6\,a^3\log\left(\cos\left(dx+c\right)\right)-\frac{6\,a^3}{\cos(dx+c)}-\frac{a^3}{\cos(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*(2*a^3*cos(d*x + c) + 6*a^3*log(cos(d*x + c)) - 6*a^3/cos(d*x + c) - a
^3/cos(d*x + c)^2)/d
```

Fricas [A]
time = 2.88, size = 65, normalized size = 1.05

$$-\frac{2\,a^3\cos\left(dx+c\right)^3+6\,a^3\cos\left(dx+c\right)^2\log\left(-\cos\left(dx+c\right)\right)-6\,a^3\cos\left(dx+c\right)-a^3}{2\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^3*cos(d*x + c)^3 + 6*a^3*cos(d*x + c)^2*log(-cos(d*x + c)) - 6*a^
3*cos(d*x + c) - a^3)/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int 3\sin\left(c+dx\right)\sec\left(c+dx\right)dx+\int 3\sin\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int\sin\left(c+dx\right)\sec^3\left(c+dx\right)dx+\int\sin\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*sin(d*x+c),x)
```

```
[Out] a**3*(Integral(3*sin(c + d*x)*sec(c + d*x), x) + Integral(3*sin(c + d*x)*se
c(c + d*x)**2, x) + Integral(sin(c + d*x)*sec(c + d*x)**3, x) + Integral(si
n(c + d*x), x))
```

Giac [A]
time = 0.46, size = 64, normalized size = 1.03

$$-\frac{a^3\cos\left(dx+c\right)}{d}-\frac{3\,a^3\log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d}+\frac{6\,a^3\cos\left(dx+c\right)+a^3}{2\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -a^3*cos(d*x + c)/d - 3*a^3*log(abs(cos(d*x + c))/abs(d))/d + 1/2*(6*a^3*co
s(d*x + c) + a^3)/(d*cos(d*x + c)^2)
```

Mupad [B]
time = 0.06, size = 52, normalized size = 0.84

$$\frac{a^3\left(3\,\cos\left(c+d\,x\right)-\cos\left(c+d\,x\right)^3-3\cos\left(c+d\,x\right)^2\,\ln\left(\cos\left(c+d\,x\right)\right)+\frac{1}{2}\right)}{d\cos\left(c+d\,x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (a^3*(3*cos(c + d*x) - cos(c + d*x)^3 - 3*cos(c + d*x)^2*log(cos(c + d*x))
+ 1/2))/(d*cos(c + d*x)^2)
```

3.43 $\int \csc(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=67

$$\frac{4a^3\log(1-\cos(c+dx))}{d}-\frac{4a^3\log(\cos(c+dx))}{d}+\frac{3a^3\sec(c+dx)}{d}+\frac{a^3\sec^2(c+dx)}{2d}$$

```
[Out] 4*a^3*ln(1-cos(d*x+c))/d-4*a^3*ln(cos(d*x+c))/d+3*a^3*sec(d*x+c)/d+1/2*a^3*
sec(d*x+c)^2/d
```

Rubi [A]
time = 0.09, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3957, 2915, 12, 90}

$$\frac{a^3\sec^2(c+dx)}{2d}+\frac{3a^3\sec(c+dx)}{d}+\frac{4a^3\log(1-\cos(c+dx))}{d}-\frac{4a^3\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (4*a^3*Log[1 - Cos[c + d*x]])/d - (4*a^3*Log[Cos[c + d*x]])/d + (3*a^3*Sec[
c + d*x])/d + (a^3*Sec[c + d*x]^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
```

```
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc(c+dx)(a+a\sec(c+dx))^3\,dx = -\int (-a-a\cos(c+dx))^3\csc(c+dx)\sec^3(c+dx)\,dx$$
$$= \frac{a\text{Subst}\left(\int \frac{a^3(-a+x)^2}{(-a-x)x^3}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^4\text{Subst}\left(\int \frac{(-a+x)^2}{(-a-x)x^3}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^4\text{Subst}\left(\int \left(-\frac{a}{x^3}+\frac{3}{x^2}-\frac{4}{ax}+\frac{4}{a(a+x)}\right)dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{4a^3\log(1-\cos(c+dx))}{d} - \frac{4a^3\log(\cos(c+dx))}{d} + \frac{3a^3\sec(c+dx)}{d} +$$

Mathematica [A]
time = 0.09, size = 81, normalized size = 1.21

$$\frac{a^3\left(1+6\cos(c+dx)-4\log(\cos(c+dx))-4\cos(2(c+dx))\left(\log(\cos(c+dx))-2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+8\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sec^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + 6*Cos[c + d*x] - 4*Log[Cos[c + d*x]] - 4*Cos[2*(c + d*x)]*(Log[Co
s[c + d*x]] - 2*Log[Sin[(c + d*x)/2]]) + 8*Log[Sin[(c + d*x)/2]])*Sec[c + d
*x]^2)/(2*d)
```

Maple [A]
time = 0.08, size = 39, normalized size = 0.58

method	result	size
derivativedivides	$-\frac{a^3\left(-\frac{\left(\sec^2(dx+c)\right)}{2}-3\sec(dx+c)-4\ln(-1+\sec(dx+c))\right)}{d}$	39
default	$-\frac{a^3\left(-\frac{\left(\sec^2(dx+c)\right)}{2}-3\sec(dx+c)-4\ln(-1+\sec(dx+c))\right)}{d}$	39
risch	$\frac{2a^3\left(3\,e^{3i(dx+c)}+e^{2i(dx+c)}+3\,e^{i(dx+c)}\right)}{d\left(e^{2i(dx+c)}+1\right)^2}+\frac{8a^3\ln\left(e^{i(dx+c)}-1\right)}{d}-\frac{4a^3\ln\left(e^{2i(dx+c)}+1\right)}{d}$	95

norman	$\frac{\frac{6a^3}{d}-\frac{4a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{8a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{4a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{4a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}$	104

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*a^3*(-1/2*sec(d*x+c)^2-3*sec(d*x+c)-4*ln(-1+sec(d*x+c)))
```

Maxima [A]
time = 0.27, size = 56, normalized size = 0.84

$$\frac{8\,a^3\log\left(\cos\left(dx+c\right)-1\right)-8\,a^3\log\left(\cos\left(dx+c\right)\right)+\frac{6\,a^3\cos(dx+c)+a^3}{\cos(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(8*a^3*log(cos(d*x + c) - 1) - 8*a^3*log(cos(d*x + c)) + (6*a^3*cos(d*x
 + c) + a^3)/cos(d*x + c)^2)/d
```

Fricas [A]
time = 3.13, size = 76, normalized size = 1.13

$$-\frac{8\,a^3\cos\left(dx+c\right)^2\log\left(-\cos\left(dx+c\right)\right)-8\,a^3\cos\left(dx+c\right)^2\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-6\,a^3\cos\left(dx+c\right)-a^3}{2\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(8*a^3*cos(d*x + c)^2*log(-cos(d*x + c)) - 8*a^3*cos(d*x + c)^2*log(-1
/2*cos(d*x + c) + 1/2) - 6*a^3*cos(d*x + c) - a^3)/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int 3\csc\left(c+dx\right)\sec\left(c+dx\right)dx+\int 3\csc\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int \csc\left(c+dx\right)\sec^3\left(c+dx\right)dx+\int \csc\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(3*csc(c + d*x)*sec(c + d*x), x) + Integral(3*csc(c + d*x)*se
c(c + d*x)**2, x) + Integral(csc(c + d*x)*sec(c + d*x)**3, x) + Integral(cs
c(c + d*x), x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(65) = 130.
time = 0.48, size = 142, normalized size = 2.12

$$\frac{2\left(2\,a^{3}\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-2\,a^{3}\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{6\,a^{3}+\frac{8\,a^{3}(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{3\,a^{3}(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^{2}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 2*(2*a^3*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 2*a^3*log(abs(
-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (6*a^3 + 8*a^3*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1) + 3*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)
/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^2)/d
```

Mupad [B]
time = 0.07, size = 49, normalized size = 0.73

$$\frac{3\,a^{3}\,\cos\left(c+d\,x\right)+\frac{a^{3}}{2}}{d\cos\left(c+d\,x\right)^{2}}-\frac{8\,a^{3}\,\mathrm{atanh}(2\,\cos\left(c+d\,x\right)-1)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^3/sin(c + d*x),x)
```

```
[Out] (3*a^3*cos(c + d*x) + a^3/2)/(d*cos(c + d*x)^2) - (8*a^3*atanh(2*cos(c + d*
x) - 1))/d
```

3.44 $\int \csc^3(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=88

$$-\frac{2a^4}{d(a-a\cos(c+dx))}+\frac{5a^3\log(1-\cos(c+dx))}{d}-\frac{5a^3\log(\cos(c+dx))}{d}+\frac{3a^3\sec(c+dx)}{d}+\frac{a^3\sec^2(c+dx)}{2d}$$

```
[Out] -2*a^4/d/(a-a*cos(d*x+c))+5*a^3*ln(1-cos(d*x+c))/d-5*a^3*ln(cos(d*x+c))/d+3
*a^3*sec(d*x+c)/d+1/2*a^3*sec(d*x+c)^2/d
```

Rubi [A]
time = 0.12, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3957, 2915, 12, 78}

$$-\frac{2a^4}{d(a-a\cos(c+dx))}+\frac{a^3\sec^2(c+dx)}{2d}+\frac{3a^3\sec(c+dx)}{d}+\frac{5a^3\log(1-\cos(c+dx))}{d}-\frac{5a^3\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-2*a^4)/(d*(a - a*Cos[c + d*x])) + (5*a^3*Log[1 - Cos[c + d*x]])/d - (5*a^
3*Log[Cos[c + d*x]])/d + (3*a^3*Sec[c + d*x])/d + (a^3*Sec[c + d*x]^2)/(2*d
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc^3(c+dx)(a+a\sec(c+dx))^3\,dx = -\int (-a-a\cos(c+dx))^3\csc^3(c+dx)\sec^3(c+dx)\,dx$$
$$= \frac{a^3\text{Subst}\left(\int \frac{a^3(-a+x)}{(-a-x)^2x^3}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^6\text{Subst}\left(\int \frac{-a+x}{(-a-x)^2x^3}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^6\text{Subst}\left(\int \left(-\frac{1}{ax^3}+\frac{3}{a^2x^2}-\frac{5}{a^3x}+\frac{2}{a^2(a+x)^2}+\frac{5}{a^3(a+x)}\right)dx, x, -a\cos(c\right.}{d}$$
$$= -\frac{2a^4}{d(a-a\cos(c+dx))}+\frac{5a^3\log(1-\cos(c+dx))}{d}-\frac{5a^3\log(\cos(c-}{d}$$

Mathematica [A]
time = 0.58, size = 88, normalized size = 1.00

$$-\frac{a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(2\csc^2\left(\frac{1}{2}(c+dx)\right)+10\left(\log(\cos(c+dx))-2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-6\sec(c+dx)-\sec^2(c+dx)\right)}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*(a + a*Sec[c + d*x])^3,x]
[Out] -1/16*(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(2*Csc[(c + d*x)/2]^2 +
10*(Log[Cos[c + d*x]] - 2*Log[Sin[(c + d*x)/2]]) - 6*Sec[c + d*x] - Sec[c +
 d*x]^2))/d
```

Maple [A]
time = 0.12, size = 50, normalized size = 0.57

method	result
derivativedivides	$\frac{a^3\left(\frac{\left(\sec^2(dx+c)\right)}{2}+3\sec(dx+c)-\frac{2}{-1+\sec(dx+c)}+5\ln(-1+\sec(dx+c))\right)}{d}$
default	$\frac{a^3\left(\frac{\left(\sec^2(dx+c)\right)}{2}+3\sec(dx+c)-\frac{2}{-1+\sec(dx+c)}+5\ln(-1+\sec(dx+c))\right)}{d}$

risch	$\frac{2a^3\left(5\,e^{5i(dx+c)}-5\,e^{4i(dx+c)}+8\,e^{3i(dx+c)}-5\,e^{2i(dx+c)}+5\,e^{i(dx+c)}\right)}{d\left(e^{2i(dx+c)}+1\right)^2\left(e^{i(dx+c)}-1\right)^2}+\frac{10a^3\ln\left(e^{i(dx+c)}-1\right)}{d}-\frac{5a^3\ln\left(e^{2i(dx+c)}+1\right)}{d}$
norman	$\frac{\frac{8a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a^3}{d}-\frac{5a^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{10a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{5a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{5a^3\ln\left(\tan\left(\frac{d}{2}\right.\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^3*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 1/d*a^3*(1/2*sec(d*x+c)^2+3*sec(d*x+c)-2/(-1+sec(d*x+c))+5*ln(-1+sec(d*x+c)
))

Maxima [A]
time = 0.27, size = 84, normalized size = 0.95

$$\frac{10\,a^3\log\left(\cos\left(dx+c\right)-1\right)-10\,a^3\log\left(\cos\left(dx+c\right)\right)+\frac{10\,a^3\cos(dx+c)^2-5\,a^3\cos(dx+c)-a^3}{\cos(dx+c)^3-\cos(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="maxima")

[Out] 1/2*(10*a^3*log(cos(d*x + c) - 1) - 10*a^3*log(cos(d*x + c)) + (10*a^3*cos(
d*x + c)^2 - 5*a^3*cos(d*x + c) - a^3)/(cos(d*x + c)^3 - cos(d*x + c)^2))/d

Fricas [A]
time = 3.54, size = 132, normalized size = 1.50

$$\frac{10\,a^3\cos\left(dx+c\right)^2-5\,a^3\cos\left(dx+c\right)-a^3-10\,\left(a^3\cos\left(dx+c\right)^3-a^3\cos\left(dx+c\right)^2\right)\log\left(-\cos\left(dx+c\right)\right)+10\,\left(a^3\cos\left(dx+c\right)^3-a^3\cos\left(dx+c\right)^2\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\,\left(d\cos\left(dx+c\right)^3-d\cos\left(dx+c\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="fricas")

[Out] 1/2*(10*a^3*cos(d*x + c)^2 - 5*a^3*cos(d*x + c) - a^3 - 10*(a^3*cos(d*x + c
)^3 - a^3*cos(d*x + c)^2)*log(-cos(d*x + c)) + 10*(a^3*cos(d*x + c)^3 - a^3
*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^3 - d*cos(d*
x + c)^2)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int 3\csc^3\left(c+dx\right)\sec\left(c+dx\right)\,dx+\int 3\csc^3\left(c+dx\right)\sec^2\left(c+dx\right)\,dx+\int \csc^3\left(c+dx\right)\sec^3\left(c+dx\right)\,dx+\int \csc^3\left(c+dx\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)**3*(a+a*sec(d*x+c))**3,x)

```
[Out] a**3*(Integral(3*csc(c + d*x)**3*sec(c + d*x), x) + Integral(3*csc(c + d*x)
**3*sec(c + d*x)**2, x) + Integral(csc(c + d*x)**3*sec(c + d*x)**3, x) + In
tegral(csc(c + d*x)**3, x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(87) = 174.
time = 0.54, size = 189, normalized size = 2.15

$$\frac{10\,a^3\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-10\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{2\left(a^3-\frac{5\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{\cos(dx+c)-1}+\frac{27\,a^3+\frac{38\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{15\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(10*a^3*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 10*a^3*log(
abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + 2*(a^3 - 5*a^3*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1))*(cos(d*x + c) + 1)/(cos(d*x + c) - 1) + (27*a
^3 + 38*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 15*a^3*(cos(d*x + c) -
1)^2/(cos(d*x + c) + 1)^2)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^2)/d
```

Mupad [B]
time = 0.10, size = 75, normalized size = 0.85

$$\frac{-5\,a^3\cos\left(c+d\,x\right)^2+\frac{5\,a^3\,\cos(c+d\,x)}{2}+\frac{a^3}{2}}{d\,\left(\cos\left(c+d\,x\right)^2-\cos\left(c+d\,x\right)^3\right)}-\frac{10\,a^3\,\mathrm{atanh}(2\,\cos\left(c+d\,x\right)-1)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^3/sin(c + d*x)^3,x)
```

```
[Out] ((5*a^3*cos(c + d*x))/2 + a^3/2 - 5*a^3*cos(c + d*x)^2)/(d*(cos(c + d*x)^2
- cos(c + d*x)^3)) - (10*a^3*atanh(2*cos(c + d*x) - 1))/d
```

3.45 $\int \csc^5(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=111

$$-\frac{a^5}{2d(a-a\cos(c+dx))^2}-\frac{3a^4}{d(a-a\cos(c+dx))}+\frac{6a^3\log(1-\cos(c+dx))}{d}-\frac{6a^3\log(\cos(c+dx))}{d}+\frac{3a^3\sec(c$$

```
[Out] -1/2*a^5/d/(a-a*cos(d*x+c))^2-3*a^4/d/(a-a*cos(d*x+c))+6*a^3*ln(1-cos(d*x+c
))/d-6*a^3*ln(cos(d*x+c))/d+3*a^3*sec(d*x+c)/d+1/2*a^3*sec(d*x+c)^2/d
```

Rubi [A]
time = 0.12, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3957, 2915, 12, 46}

$$-\frac{a^5}{2d(a-a\cos(c+dx))^2}-\frac{3a^4}{d(a-a\cos(c+dx))}+\frac{a^3\sec^2(c+dx)}{2d}+\frac{3a^3\sec(c+dx)}{d}+\frac{6a^3\log(1-\cos(c+dx))}{d}-\frac{6a^3\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/2*a^5/(d*(a - a*Cos[c + d*x])^2) - (3*a^4)/(d*(a - a*Cos[c + d*x])) + (6
*a^3*Log[1 - Cos[c + d*x]])/d - (6*a^3*Log[Cos[c + d*x]])/d + (3*a^3*Sec[c
+ d*x])/d + (a^3*Sec[c + d*x]^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \csc^5(c+dx)(a+a\sec(c+dx))^3\,dx &= -\int (-a-a\cos(c+dx))^3\csc^5(c+dx)\sec^3(c+dx)\,dx\\ &= \frac{a^5\text{Subst}\left(\int \frac{a^3}{(-a-x)^3x^3}\,dx,x,-a\cos(c+dx)\right)}{d}\\ &= \frac{a^8\text{Subst}\left(\int \frac{1}{(-a-x)^3x^3}\,dx,x,-a\cos(c+dx)\right)}{d}\\ &= \frac{a^8\text{Subst}\left(\int \left(-\frac{1}{a^3x^3}+\frac{3}{a^4x^2}-\frac{6}{a^5x}+\frac{1}{a^3(a+x)^3}+\frac{3}{a^4(a+x)^2}+\frac{6}{a^5(a+x)}\right)dx\right.}{d}\\ &= -\frac{a^5}{2d(a-a\cos(c+dx))^2}-\frac{3a^4}{d(a-a\cos(c+dx))}+\frac{6a^3\log(1-\cos(}{d}\end{aligned}$$

Mathematica [A]
time = 0.63, size = 100, normalized size = 0.90

$$-\frac{a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(12\csc^2\left(\frac{1}{2}(c+dx)\right)+\csc^4\left(\frac{1}{2}(c+dx)\right)+48\left(\log(\cos(c+dx))-2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-24\sec(c+dx)-4\sec^2(c+dx)\right)}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^5*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/64*(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(12*Csc[(c + d*x)/2]^2 +
 Csc[(c + d*x)/2]^4 + 48*(Log[Cos[c + d*x]] - 2*Log[Sin[(c + d*x)/2]]) - 24
*Sec[c + d*x] - 4*Sec[c + d*x]^2))/d
```

Maple [A]
time = 0.12, size = 63, normalized size = 0.57

method	result
derivativedivides	$-\frac{a^3\left(-\frac{\left(\sec^2(dx+c)\right)}{2}-3\sec(dx+c)+\frac{4}{-1+\sec(dx+c)}-6\ln(-1+\sec(dx+c))+\frac{1}{2(-1+\sec(dx+c))^2}\right)}{d}$
default	$-\frac{a^3\left(-\frac{\left(\sec^2(dx+c)\right)}{2}-3\sec(dx+c)+\frac{4}{-1+\sec(dx+c)}-6\ln(-1+\sec(dx+c))+\frac{1}{2(-1+\sec(dx+c))^2}\right)}{d}$

norman	$\frac{-\frac{a^{3}}{8d}-\frac{3a^{3}\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{23a^{3}\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{75a^{3}\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{12a^{3}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{6a^{3}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{d}$
risch	$\frac{4a^{3}\left(3\,e^{7i(dx+c)}-9\,e^{6i(dx+c)}+13\,e^{5i(dx+c)}-16\,e^{4i(dx+c)}+13\,e^{3i(dx+c)}-9\,e^{2i(dx+c)}+3\,e^{i(dx+c)}\right)}{d\left(e^{i(dx+c)}-1\right)^{4}\left(e^{2i(dx+c)}+1\right)^{2}}+\frac{12a^{3}\ln\left(e^{i(dx+c)}-1\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*a^3*(-1/2*sec(d*x+c)^2-3*sec(d*x+c)+4/(-1+sec(d*x+c))-6*ln(-1+sec(d*x+
c))+1/2/(-1+sec(d*x+c))^2)
```

Maxima [A]
time = 0.27, size = 103, normalized size = 0.93

$$\frac{12\,a^{3}\log\left(\cos\left(dx+c\right)-1\right)-12\,a^{3}\log\left(\cos\left(dx+c\right)\right)+\frac{12\,a^{3}\cos\left(dx+c\right)^{3}-18\,a^{3}\cos\left(dx+c\right)^{2}+4\,a^{3}\cos\left(dx+c\right)+a^{3}}{\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(12*a^3*log(cos(d*x + c) - 1) - 12*a^3*log(cos(d*x + c)) + (12*a^3*cos(
d*x + c)^3 - 18*a^3*cos(d*x + c)^2 + 4*a^3*cos(d*x + c) + a^3)/(cos(d*x + c
)^4 - 2*cos(d*x + c)^3 + cos(d*x + c)^2))/d
```

Fricas [A]
time = 4.17, size = 177, normalized size = 1.59

$$\frac{12\,a^{3}\cos\left(dx+c\right)^{3}-18\,a^{3}\cos\left(dx+c\right)^{2}+4\,a^{3}\cos\left(dx+c\right)+a^{3}-12\,\left(a^{3}\cos\left(dx+c\right)^{4}-2\,a^{3}\cos\left(dx+c\right)^{3}+a^{3}\cos\left(dx+c\right)^{2}\right)\log\left(-\cos\left(dx+c\right)\right)+12\,\left(a^{3}\cos\left(dx+c\right)^{4}-2\,a^{3}\cos\left(dx+c\right)^{3}+a^{3}\cos\left(dx+c\right)^{2}\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\,\left(d\cos\left(dx+c\right)^{4}-2\,d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(12*a^3*cos(d*x + c)^3 - 18*a^3*cos(d*x + c)^2 + 4*a^3*cos(d*x + c) + a
^3 - 12*(a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x + c)^3 + a^3*cos(d*x + c)^2)*lo
g(-cos(d*x + c)) + 12*(a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x + c)^3 + a^3*cos(
d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^4 - 2*d*cos(d*x +
 c)^3 + d*cos(d*x + c)^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.54, size = 186, normalized size = 1.68

$$\frac{48\,a^3\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-48\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)-\frac{a^3-\frac{12\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{75\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{46\,a^3(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+\frac{(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(48*a^3*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 48*a^3*log(
abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - (a^3 - 12*a^3*(cos(d*x +
 c) - 1)/(cos(d*x + c) + 1) - 75*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1
)^2 - 46*a^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)/((cos(d*x + c) - 1)
/(cos(d*x + c) + 1) + (cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)^2)/d
```

Mupad [B]
time = 0.93, size = 96, normalized size = 0.86

$$\frac{6\,a^3\cos\left(c+d\,x\right)^3-9\,a^3\cos\left(c+d\,x\right)^2+2\,a^3\,\cos\left(c+d\,x\right)+\frac{a^3}{2}}{d\,\left({\cos\left(c+d\,x\right)}^4-2\cos\left(c+d\,x\right)^3+\cos\left(c+d\,x\right)^2\right)}-\frac{12\,a^3\,\mathrm{atanh}(2\,\cos\left(c+d\,x\right)-1)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^3/sin(c + d*x)^5,x)
```

```
[Out] (2*a^3*cos(c + d*x) + a^3/2 - 9*a^3*cos(c + d*x)^2 + 6*a^3*cos(c + d*x)^3)/
(d*(cos(c + d*x)^2 - 2*cos(c + d*x)^3 + cos(c + d*x)^4)) - (12*a^3*atanh(2*
cos(c + d*x) - 1))/d
```

3.46 $\int \csc^7(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=157

$$-\frac{a^6}{6d(a-a\cos(c+dx))^3}-\frac{7a^5}{8d(a-a\cos(c+dx))^2}-\frac{31a^4}{8d(a-a\cos(c+dx))}+\frac{111a^3\log(1-\cos(c+dx))}{16d}-\frac{7a^3}{}$$

```
[Out] -1/6*a^6/d/(a-a*cos(d*x+c))^3-7/8*a^5/d/(a-a*cos(d*x+c))^2-31/8*a^4/d/(a-a*
cos(d*x+c))+111/16*a^3*ln(1-cos(d*x+c))/d-7*a^3*ln(cos(d*x+c))/d+1/16*a^3*l
n(1+cos(d*x+c))/d+3*a^3*sec(d*x+c)/d+1/2*a^3*sec(d*x+c)^2/d
```

Rubi [A]
time = 0.15, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3957, 2915, 12, 90}

$$-\frac{a^6}{6d(a-a\cos(c+dx))^3}-\frac{7a^5}{8d(a-a\cos(c+dx))^2}-\frac{31a^4}{8d(a-a\cos(c+dx))}+\frac{a^3\sec^2(c+dx)}{2d}+\frac{3a^3\sec(c+dx)}{d}+\frac{111a^3\log(1-\cos(c+dx))}{16d}-\frac{7a^3\log(\cos(c+dx))}{d}+\frac{a^3\log(\cos(c+dx)+1)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^7*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/6*a^6/(d*(a - a*Cos[c + d*x])^3) - (7*a^5)/(8*d*(a - a*Cos[c + d*x])^2)
- (31*a^4)/(8*d*(a - a*Cos[c + d*x])) + (111*a^3*Log[1 - Cos[c + d*x]])/(16
*d) - (7*a^3*Log[Cos[c + d*x]])/d + (a^3*Log[1 + Cos[c + d*x]])/(16*d) + (3
*a^3*Sec[c + d*x])/d + (a^3*Sec[c + d*x]^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \csc^7(c+dx)(a+a\sec(c+dx))^3\,dx &= -\int (-a-a\cos(c+dx))^3\csc^7(c+dx)\sec^3(c+dx)\,dx\\ &= \frac{a^7\text{Subst}\left(\int \frac{a^3}{(-a-x)^4x^3(-a+x)}\,dx,x,-a\cos(c+dx)\right)}{d}\\ &= \frac{a^{10}\text{Subst}\left(\int \frac{1}{(-a-x)^4x^3(-a+x)}\,dx,x,-a\cos(c+dx)\right)}{d}\\ &= \frac{a^{10}\text{Subst}\left(\int \left(-\frac{1}{16a^7(a-x)}-\frac{1}{a^5x^3}+\frac{3}{a^6x^2}-\frac{7}{a^7x}+\frac{1}{2a^4(a+x)^4}+\frac{7}{4a^5(a+x)^3}\right.\right.}{d}\\ &= -\frac{a^6}{6d(a-a\cos(c+dx))^3}-\frac{7a^5}{8d(a-a\cos(c+dx))^2}-\frac{31a^4}{8d(a-a\cos(c}\end{aligned}$$

Mathematica [A]
time = 0.71, size = 129, normalized size = 0.82

$$-\frac{a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(186\csc^2\left(\frac{1}{2}(c+dx)\right)+21\csc^4\left(\frac{1}{2}(c+dx)\right)+2\csc^6\left(\frac{1}{2}(c+dx)\right)-12\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-56\log(\cos(c+dx))+111\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+24\sec(c+dx)+4\sec^2(c+dx)\right)\right)}{768d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^7*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/768*(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(186*Csc[(c + d*x)/2]^2
 + 21*Csc[(c + d*x)/2]^4 + 2*Csc[(c + d*x)/2]^6 - 12*(Log[Cos[(c + d*x)/2]]
 - 56*Log[Cos[c + d*x]] + 111*Log[Sin[(c + d*x)/2]] + 24*Sec[c + d*x] + 4*S
ec[c + d*x]^2)))/d
```

Maple [A]
time = 0.15, size = 85, normalized size = 0.54

method	result
derivativedivides	$\frac{a^3\left(\frac{\left(\sec^2(dx+c)\right)}{2}+3\sec(dx+c)-\frac{1}{6(-1+\sec(dx+c))^3}-\frac{11}{8(-1+\sec(dx+c))^2}-\frac{49}{8(-1+\sec(dx+c))}+\frac{111\ln(-1+\sec(dx+c))}{16}+\frac{\ln(1+\sec(}{16}\right.}{d}$

default	$\frac{a^3\left(\frac{\left(\sec^2(dx+c)\right)}{2}+3\sec(dx+c)-\frac{1}{6(-1+\sec(dx+c))^3}-\frac{11}{8(-1+\sec(dx+c))^2}-\frac{49}{8(-1+\sec(dx+c))}+\frac{111\ln(-1+\sec(dx+c))}{16}+\frac{\ln(1+\text{se}}{}\right.}{d}$
norman	$\frac{-\frac{a^3}{48d}-\frac{23a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96d}-\frac{91a^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{48d}-\frac{103a^3\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16d}+\frac{339a^3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{111a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{8d}$
risch	$\frac{a^3\left(165\,e^{9i(dx+c)}-822\,e^{8i(dx+c)}+1852\,e^{7i(dx+c)}-2754\,e^{6i(dx+c)}+3182\,e^{5i(dx+c)}-2754\,e^{4i(dx+c)}+1852\,e^{3i(dx+c)}-822\,e\right.}{12d\left(e^{i(dx+c)}-1\right)^6\left(e^{2i(dx+c)}+1\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^7*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*a^3*(1/2*sec(d*x+c)^2+3*sec(d*x+c)-1/6/(-1+sec(d*x+c))^3-11/8/(-1+sec(d
*x+c))^2-49/8/(-1+sec(d*x+c))+111/16*ln(-1+sec(d*x+c))+1/16*ln(1+sec(d*x+c)
))
```

Maxima [A]
time = 0.27, size = 145, normalized size = 0.92

$$\frac{3\,a^3\log\left(\cos\left(dx+c\right)+1\right)+333\,a^3\log\left(\cos\left(dx+c\right)-1\right)-336\,a^3\log\left(\cos\left(dx+c\right)\right)+\frac{2\left(165\,a^3\cos\left(dx+c\right)^4-411\,a^3\cos\left(dx+c\right)^3+298\,a^3\cos\left(dx+c\right)^2-36\,a^3\cos\left(dx+c\right)-12\,a^3\right)}{\cos\left(dx+c\right)^5-3\cos\left(dx+c\right)^4+3\cos\left(dx+c\right)^3-\cos\left(dx+c\right)^2}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^7*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/48*(3*a^3*log(cos(d*x + c) + 1) + 333*a^3*log(cos(d*x + c) - 1) - 336*a^3
*log(cos(d*x + c)) + 2*(165*a^3*cos(d*x + c)^4 - 411*a^3*cos(d*x + c)^3 + 2
98*a^3*cos(d*x + c)^2 - 36*a^3*cos(d*x + c) - 12*a^3)/(cos(d*x + c)^5 - 3*c
os(d*x + c)^4 + 3*cos(d*x + c)^3 - cos(d*x + c)^2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 297 vs. $2(148) = 296$.
time = 4.61, size = 297, normalized size = 1.89

$$\frac{330\,a^3\cos\left(dx+c\right)^4-822\,a^3\cos\left(dx+c\right)^3+596\,a^3\cos\left(dx+c\right)^2-72\,a^3\cos\left(dx+c\right)-24\,a^3-336\left(a^3\cos\left(dx+c\right)^5-3\,a^3\cos\left(dx+c\right)^4+3\,a^3\cos\left(dx+c\right)^3-a^3\cos\left(dx+c\right)^2\right)\log\left(-\cos\left(dx+c\right)\right)+3\left(a^3\cos\left(dx+c\right)^5-3\,a^3\cos\left(dx+c\right)^4+3\,a^3\cos\left(dx+c\right)^3-a^3\cos\left(dx+c\right)^2\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+333\left(a^3\cos\left(dx+c\right)^5-3\,a^3\cos\left(dx+c\right)^4+3\,a^3\cos\left(dx+c\right)^3-a^3\cos\left(dx+c\right)^2\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{48\left(d\cos\left(dx+c\right)^5-3\,d\cos\left(dx+c\right)^4+3\,d\cos\left(dx+c\right)^3-d\cos\left(dx+c\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^7*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/48*(330*a^3*cos(d*x + c)^4 - 822*a^3*cos(d*x + c)^3 + 596*a^3*cos(d*x + c
)^2 - 72*a^3*cos(d*x + c) - 24*a^3 - 336*(a^3*cos(d*x + c)^5 - 3*a^3*cos(d*
x + c)^4 + 3*a^3*cos(d*x + c)^3 - a^3*cos(d*x + c)^2)*log(-cos(d*x + c)) +
3*(a^3*cos(d*x + c)^5 - 3*a^3*cos(d*x + c)^4 + 3*a^3*cos(d*x + c)^3 - a^3*c
os(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) + 333*(a^3*cos(d*x + c)^5 - 3*a^
3*cos(d*x + c)^4 + 3*a^3*cos(d*x + c)^3 - a^3*cos(d*x + c)^2)*log(-1/2*cos(
d*x + c) + 1/2))/(d*cos(d*x + c)^5 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^
3 - d*cos(d*x + c)^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**7*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [A]
time = 0.56, size = 243, normalized size = 1.55

$$\frac{666\,a^3\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-672\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{\left(2\,a^3-\frac{27\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{234\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{1221\,a^3(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}\right)(\cos(dx+c)+1)^3}{(\cos(dx+c)-1)^3}+\frac{48\left(33\,a^3+\frac{50\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{21\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^2}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^7*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/96*(666*a^3*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 672*a^3*l
og(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (2*a^3 - 27*a^3*(cos(
d*x + c) - 1)/(cos(d*x + c) + 1) + 234*a^3*(cos(d*x + c) - 1)^2/(cos(d*x +
c) + 1)^2 - 1221*a^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)*(cos(d*x +
c) + 1)^3/(cos(d*x + c) - 1)^3 + 48*(33*a^3 + 50*a^3*(cos(d*x + c) - 1)/(co
s(d*x + c) + 1) + 21*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/((cos(d
*x + c) - 1)/(cos(d*x + c) + 1) + 1)^2)/d
```

Mupad [B]
time = 0.96, size = 151, normalized size = 0.96

$$\frac{111\,a^3\,\ln\left(\cos\left(c+d\,x\right)-1\right)}{16\,d}+\frac{a^3\,\ln\left(\cos\left(c+d\,x\right)+1\right)}{16\,d}+\frac{-\frac{55\,a^3\,{\cos\left(c+d\,x\right)}^4}{8}+\frac{137\,a^3\,{\cos\left(c+d\,x\right)}^3}{8}-\frac{149\,a^3\,{\cos\left(c+d\,x\right)}^2}{12}+\frac{3\,a^3\,\cos\left(c+d\,x\right)}{2}+\frac{a^3}{2}}{d\,\left(-{\cos\left(c+d\,x\right)}^5+3\,{\cos\left(c+d\,x\right)}^4-3\,{\cos\left(c+d\,x\right)}^3+{\cos\left(c+d\,x\right)}^2\right)}-\frac{7\,a^3\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^3/sin(c + d*x)^7,x)
```

```
[Out] (111*a^3*log(cos(c + d*x) - 1))/(16*d) + (a^3*log(cos(c + d*x) + 1))/(16*d)
 + ((3*a^3*cos(c + d*x))/2 + a^3/2 - (149*a^3*cos(c + d*x)^2)/12 + (137*a^3
*cos(c + d*x)^3)/8 - (55*a^3*cos(c + d*x)^4)/8)/(d*(cos(c + d*x)^2 - 3*cos(
c + d*x)^3 + 3*cos(c + d*x)^4 - cos(c + d*x)^5)) - (7*a^3*log(cos(c + d*x))
)/d
```

3.47 $\int \csc^9(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=202

$$-\frac{a^7}{16d(a-a\cos(c+dx))^4}-\frac{a^6}{3d(a-a\cos(c+dx))^3}-\frac{39a^5}{32d(a-a\cos(c+dx))^2}-\frac{75a^4}{16d(a-a\cos(c+dx))}-\frac{32a}{}$$

```
[Out] -1/16*a^7/d/(a-a*cos(d*x+c))^4-1/3*a^6/d/(a-a*cos(d*x+c))^3-39/32*a^5/d/(a-
a*cos(d*x+c))^2-75/16*a^4/d/(a-a*cos(d*x+c))-1/32*a^4/d/(a+a*cos(d*x+c))+50
1/64*a^3*ln(1-cos(d*x+c))/d-8*a^3*ln(cos(d*x+c))/d+11/64*a^3*ln(1+cos(d*x+c
))/d+3*a^3*sec(d*x+c)/d+1/2*a^3*sec(d*x+c)^2/d
```

Rubi [A]
time = 0.17, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$,
Rules used = {3957, 2915, 12, 90}

$$-\frac{a^7}{16d(a-a\cos(c+dx))^4}-\frac{a^6}{3d(a-a\cos(c+dx))^3}-\frac{39a^5}{32d(a-a\cos(c+dx))^2}-\frac{75a^4}{16d(a-a\cos(c+dx))}-\frac{a^4}{32d(a\cos(c+dx)+a)}+\frac{a^3\sec^2(c+dx)}{2d}+\frac{3a^3\sec(c+dx)}{d}+\frac{501a^3\log(1-\cos(c+dx))}{64d}-\frac{8a^3\log(\cos(c+dx))}{d}+\frac{11a^3\log(\cos(c+dx)+1)}{64d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^9*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/16*a^7/(d*(a - a*Cos[c + d*x])^4) - a^6/(3*d*(a - a*Cos[c + d*x])^3) - (
39*a^5)/(32*d*(a - a*Cos[c + d*x])^2) - (75*a^4)/(16*d*(a - a*Cos[c + d*x])
) - a^4/(32*d*(a + a*Cos[c + d*x])) + (501*a^3*Log[1 - Cos[c + d*x]])/(64*d
) - (8*a^3*Log[Cos[c + d*x]])/d + (11*a^3*Log[1 + Cos[c + d*x]])/(64*d) + (
3*a^3*Sec[c + d*x])/d + (a^3*Sec[c + d*x]^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
```

```
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc^9(c+dx)(a+a\sec(c+dx))^3\,dx = -\int(-a-a\cos(c+dx))^3\csc^9(c+dx)\sec^3(c+dx)\,dx$$

$$= \frac{a^9\text{Subst}\left(\int \frac{a^3}{(-a-x)^5x^3(-a+x)^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$

$$= \frac{a^{12}\text{Subst}\left(\int \frac{1}{(-a-x)^5x^3(-a+x)^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$

$$= \frac{a^{12}\text{Subst}\left(\int \left(-\frac{1}{32a^8(a-x)^2} - \frac{11}{64a^9(a-x)} - \frac{1}{a^7x^3} + \frac{3}{a^8x^2} - \frac{8}{a^9x} + \frac{1}{4a^5(a+x)^5}\right.\right.}{}$$

$$= -\frac{a^7}{16d(a-a\cos(c+dx))^4} - \frac{a^6}{3d(a-a\cos(c+dx))^3} - \frac{39a}{32d(a-a\cos}$$

Mathematica [A]
time = 0.81, size = 159, normalized size = 0.79

$$-\frac{a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(1800\csc^2\left(\frac{1}{2}(c+dx)\right)+234\csc^4\left(\frac{1}{2}(c+dx)\right)+32\csc^6\left(\frac{1}{2}(c+dx)\right)+3\csc^8\left(\frac{1}{2}(c+dx)\right)-12\left(22\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-512\log(\cos(c+dx))+1002\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-\sec^2\left(\frac{1}{2}(c+dx)\right)+192\sec(c+dx)+32\sec^2(c+dx)\right)\right)}{6144d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^9*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/6144*(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(1800*Csc[(c + d*x)/2]
^2 + 234*Csc[(c + d*x)/2]^4 + 32*Csc[(c + d*x)/2]^6 + 3*Csc[(c + d*x)/2]^8
- 12*(22*Log[Cos[(c + d*x)/2]] - 512*Log[Cos[c + d*x]] + 1002*Log[Sin[(c +
d*x)/2]] - Sec[(c + d*x)/2]^2 + 192*Sec[c + d*x] + 32*Sec[c + d*x]^2)))/d
```

Maple [A]
time = 0.17, size = 110, normalized size = 0.54

method	result
derivativedivides	$-\frac{a^3\left(-\frac{\left(\sec^2(dx+c)\right)}{2}-3\sec(dx+c)+\frac{1}{16(-1+\sec(dx+c))^4}+\frac{7}{12(-1+\sec(dx+c))^3}+\frac{83}{32(-1+\sec(dx+c))^2}+\frac{67}{8(-1+\sec(dx+c))}-\frac{501\,\mathrm{l}}{}\right.}{d}$

default	$-\frac{a^3\left(-\frac{\left(\sec^2(dx+c)\right)}{2}-3\sec(dx+c)+\frac{1}{16(-1+\sec(dx+c))^4}+\frac{7}{12(-1+\sec(dx+c))^3}+\frac{83}{32(-1+\sec(dx+c))^2}+\frac{67}{8(-1+\sec(dx+c))}-\frac{50}{}\right.}{d}$
norman	$\frac{-\frac{a^3}{256d}-\frac{19a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{384d}-\frac{263a^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{768d}-\frac{431a^3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192d}-\frac{a^3\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}-\frac{451a^3\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}+}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}$
risch	$\frac{a^3\left(735\,\mathrm{e}^{13i(dx+c)}-3642\,\mathrm{e}^{12i(dx+c)}+6662\,\mathrm{e}^{11i(dx+c)}-4650\,\mathrm{e}^{10i(dx+c)}-1983\,\mathrm{e}^{9i(dx+c)}+8868\,\mathrm{e}^{8i(dx+c)}-12748\,\mathrm{e}^{7i(dx+c)}\right.}{48d\left(\mathrm{e}^{i(dx+c)}-1\right)^8\left(\mathrm{e}^{i(dx+c)}+1\right)^2\left(\mathrm{e}^{2}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^9*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*a^3*(-1/2*sec(d*x+c)^2-3*sec(d*x+c)+1/16/(-1+sec(d*x+c))^4+7/12/(-1+se
c(d*x+c))^3+83/32/(-1+sec(d*x+c))^2+67/8/(-1+sec(d*x+c))-501/64*ln(-1+sec(d
*x+c))-1/32/(1+sec(d*x+c))-11/64*ln(1+sec(d*x+c)))
```

Maxima [A]
time = 0.27, size = 189, normalized size = 0.94

$$\frac{33\,a^3\log\left(\cos\left(dx+c\right)+1\right)+1503\,a^3\log\left(\cos\left(dx+c\right)-1\right)-1536\,a^3\log\left(\cos\left(dx+c\right)\right)+\frac{2\left(735\,a^3\cos(dx+c)^6-1821\,a^3\cos(dx+c)^5+563\,a^3\cos(dx+c)^4+1695\,a^3\cos(dx+c)^3-1376\,a^3\cos(dx+c)^2+144\,a^3\cos(dx+c)+48\,a^3\right)}{\cos(dx+c)^7-3\cos(dx+c)^6+2\cos(dx+c)^5+2\cos(dx+c)^4-3\cos(dx+c)^3+\cos(dx+c)^2}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^9*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/192*(33*a^3*log(cos(d*x + c) + 1) + 1503*a^3*log(cos(d*x + c) - 1) - 1536
*a^3*log(cos(d*x + c)) + 2*(735*a^3*cos(d*x + c)^6 - 1821*a^3*cos(d*x + c)^
5 + 563*a^3*cos(d*x + c)^4 + 1695*a^3*cos(d*x + c)^3 - 1376*a^3*cos(d*x + c
)^2 + 144*a^3*cos(d*x + c) + 48*a^3)/(cos(d*x + c)^7 - 3*cos(d*x + c)^6 + 2
*cos(d*x + c)^5 + 2*cos(d*x + c)^4 - 3*cos(d*x + c)^3 + cos(d*x + c)^2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 419 vs. 2(190) = 380.
time = 2.91, size = 419, normalized size = 2.07

$$\frac{1470\,a^3\cos\left(dx+c\right)^6-3642\,a^3\cos\left(dx+c\right)^5+1126\,a^3\cos\left(dx+c\right)^4+3390\,a^3\cos\left(dx+c\right)^3-2752\,a^3\cos\left(dx+c\right)^2+288\,a^3\cos\left(dx+c\right)+96\,a^3-1536\left(a^3\cos\left(dx+c\right)^7-3\,a^3\cos\left(dx+c\right)^6+2\,a^3\cos\left(dx+c\right)^5+2\,a^3\cos\left(dx+c\right)^4-3\,a^3\cos\left(dx+c\right)^3+a^3\cos\left(dx+c\right)^2\right)\log\left(-\cos\left(dx+c\right)\right)+33\left(a^3\cos\left(dx+c\right)^7-3\,a^3\cos\left(dx+c\right)^6+2\,a^3\cos\left(dx+c\right)^5+2\,a^3\cos\left(dx+c\right)^4-3\,a^3\cos\left(dx+c\right)^3+a^3\cos\left(dx+c\right)^2\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+1503\left(a^3\cos\left(dx+c\right)^7-3\,a^3\cos\left(dx+c\right)^6+2\,a^3\cos\left(dx+c\right)^5+2\,a^3\cos\left(dx+c\right)^4-3\,a^3\cos\left(dx+c\right)^3+a^3\cos\left(dx+c\right)^2\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{192\left(d\cos\left(dx+c\right)^7-3\,d\cos\left(dx+c\right)^6+2\,d\cos\left(dx+c\right)^5+2\,d\cos\left(dx+c\right)^4-3\,d\cos\left(dx+c\right)^3+d\cos\left(dx+c\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^9*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/192*(1470*a^3*cos(d*x + c)^6 - 3642*a^3*cos(d*x + c)^5 + 1126*a^3*cos(d*x
 + c)^4 + 3390*a^3*cos(d*x + c)^3 - 2752*a^3*cos(d*x + c)^2 + 288*a^3*cos(d
*x + c) + 96*a^3 - 1536*(a^3*cos(d*x + c)^7 - 3*a^3*cos(d*x + c)^6 + 2*a^3*
cos(d*x + c)^5 + 2*a^3*cos(d*x + c)^4 - 3*a^3*cos(d*x + c)^3 + a^3*cos(d*x
+ c)^2)*log(-cos(d*x + c)) + 33*(a^3*cos(d*x + c)^7 - 3*a^3*cos(d*x + c)^6
+ 2*a^3*cos(d*x + c)^5 + 2*a^3*cos(d*x + c)^4 - 3*a^3*cos(d*x + c)^3 + a^3*
```

```
cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) + 1503*(a^3*cos(d*x + c)^7 - 3*
a^3*cos(d*x + c)^6 + 2*a^3*cos(d*x + c)^5 + 2*a^3*cos(d*x + c)^4 - 3*a^3*co
s(d*x + c)^3 + a^3*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x
 + c)^7 - 3*d*cos(d*x + c)^6 + 2*d*cos(d*x + c)^5 + 2*d*cos(d*x + c)^4 - 3*
d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**9*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8570 deep
```

Giac [A]
time = 0.57, size = 292, normalized size = 1.45

$$\frac{6012\,a^3\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - 6144\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{12\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{\left(3\,a^3 - \frac{44\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{348\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} - \frac{2376\,a^3(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3} + \frac{12525\,a^3(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}\right)(\cos(dx+c)+1)^4}{(\cos(dx+c)-1)^4} + \frac{1536\left(9\,a^3 + \frac{14\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{6\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^2}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^9*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/768*(6012*a^3*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 6144*a^
3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + 12*a^3*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) - (3*a^3 - 44*a^3*(cos(d*x + c) - 1)/(cos(d*x +
c) + 1) + 348*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 2376*a^3*(cos
(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 12525*a^3*(cos(d*x + c) - 1)^4/(cos
(d*x + c) + 1)^4)*(cos(d*x + c) + 1)^4/(cos(d*x + c) - 1)^4 + 1536*(9*a^3 +
 14*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 6*a^3*(cos(d*x + c) - 1)^2/
(cos(d*x + c) + 1)^2)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^2)/d
```

Mupad [B]
time = 1.01, size = 195, normalized size = 0.97

$$\frac{501\,a^3\,\ln\left(\cos\left(c+d\,x\right)-1\right)}{64\,d} + \frac{11\,a^3\,\ln\left(\cos\left(c+d\,x\right)+1\right)}{64\,d} + \frac{\frac{245\,a^3\cos(c+d\,x)^6}{32} - \frac{607\,a^3\cos(c+d\,x)^5}{32} + \frac{563\,a^3\cos(c+d\,x)^4}{96} + \frac{565\,a^3\cos(c+d\,x)^3}{32} - \frac{43\,a^3\cos(c+d\,x)^2}{3} + \frac{3\,a^3\cos(c+d\,x)}{2} + \frac{a^3}{2}}{d\,\left(\cos\left(c+d\,x\right)^7 - 3\cos\left(c+d\,x\right)^6 + 2\cos\left(c+d\,x\right)^5 + 2\cos\left(c+d\,x\right)^4 - 3\cos\left(c+d\,x\right)^3 + \cos\left(c+d\,x\right)^2\right)} - \frac{8\,a^3\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^3/sin(c + d*x)^9,x)
```

```
[Out] (501*a^3*log(cos(c + d*x) - 1))/(64*d) + (11*a^3*log(cos(c + d*x) + 1))/(64
*d) + ((3*a^3*cos(c + d*x))/2 + a^3/2 - (43*a^3*cos(c + d*x)^2)/3 + (565*a^
3*cos(c + d*x)^3)/32 + (563*a^3*cos(c + d*x)^4)/96 - (607*a^3*cos(c + d*x)^
5)/32 + (245*a^3*cos(c + d*x)^6)/32)/(d*(cos(c + d*x)^2 - 3*cos(c + d*x)^3
+ 2*cos(c + d*x)^4 + 2*cos(c + d*x)^5 - 3*cos(c + d*x)^6 + cos(c + d*x)^7))
 - (8*a^3*log(cos(c + d*x)))/d
```

3.48 $\int (a+a\sec(c+dx))^3\sin^8(c+dx)\,dx$

Optimal. Leaf size=210

$$-\frac{805a^3x}{128}-\frac{a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{603a^3\cos(c+dx)\sin(c+dx)}{128d}-\frac{293a^3\cos^3(c+dx)\sin(c+dx)}{192d}-\frac{a^3\cos^5}{}$$

```
[Out] -805/128*a^3*x-1/2*a^3*arctanh(sin(d*x+c))/d+603/128*a^3*cos(d*x+c)*sin(d*x
+c)/d-293/192*a^3*cos(d*x+c)^3*sin(d*x+c)/d-1/48*a^3*cos(d*x+c)^5*sin(d*x+c
)/d+1/8*a^3*cos(d*x+c)^7*sin(d*x+c)/d-1/3*a^3*sin(d*x+c)^3/d-2/5*a^3*sin(d*
x+c)^5/d-3/7*a^3*sin(d*x+c)^7/d+3*a^3*tan(d*x+c)/d+1/2*a^3*sec(d*x+c)*tan(d
*x+c)/d
```

Rubi [A]
time = 0.29, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3957, 2951, 2717, 2715, 8, 2713, 3855, 3852, 3853}

$$-\frac{3a^3\sin^7(c+dx)}{7d}-\frac{2a^3\sin^5(c+dx)}{5d}-\frac{a^3\sin^3(c+dx)}{3d}+\frac{3a^3\tan(c+dx)}{d}-\frac{a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^3\sin(c+dx)\cos^7(c+dx)}{8d}-\frac{a^3\sin(c+dx)\cos^5(c+dx)}{48d}-\frac{293a^3\sin(c+dx)\cos^3(c+dx)}{192d}+\frac{603a^3\sin(c+dx)\cos(c+dx)}{128d}+\frac{a^3\tan(c+dx)\sec(c+dx)}{2d}-\frac{805a^3x}{128}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^8,x]
```

```
[Out] (-805*a^3*x)/128 - (a^3*ArcTanh[Sin[c + d*x]])/(2*d) + (603*a^3*Cos[c + d*x
]*Sin[c + d*x])/(128*d) - (293*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(192*d) - (
a^3*Cos[c + d*x]^5*Sin[c + d*x])/(48*d) + (a^3*Cos[c + d*x]^7*Sin[c + d*x])
/(8*d) - (a^3*Sin[c + d*x]^3)/(3*d) - (2*a^3*Sin[c + d*x]^5)/(5*d) - (3*a^3
*Sin[c + d*x]^7)/(7*d) + (3*a^3*Tan[c + d*x])/d + (a^3*Sec[c + d*x]*Tan[c +
 d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))^3\sin^8(c+dx)\,dx &= -\int (-a-a\cos(c+dx))^3\sin^5(c+dx)\tan^3(c+dx)\,dx\\ &= -\frac{\int (11a^{11}+6a^{11}\cos(c+dx)-14a^{11}\cos^2(c+dx)-14a^{11}\cos^3(c+\\ \\ &= -11a^3x-a^3\int \cos^6(c+dx)\,dx+a^3\int \cos^8(c+dx)\,dx-a^3\int \sec\\ &= -11a^3x-\frac{a^3\tanh^{-1}(\sin(c+dx))}{d}-\frac{6a^3\sin(c+dx)}{d}+\frac{7a^3\cos(c+\\ \\ &= -4a^3x-\frac{a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{19a^3\cos(c+dx)\sin(c+dx)}{4d}-\\ &= -\frac{25a^3x}{4}-\frac{a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{71a^3\cos(c+dx)\sin(c+dx)}{16d}\\ &= -\frac{105a^3x}{16}-\frac{a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{603a^3\cos(c+dx)\sin(c+dx}{128d}\\ &= -\frac{805a^3x}{128}-\frac{a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{603a^3\cos(c+dx)\sin(c+dx}{128d}\end{aligned}$$

Mathematica [A]
time = 1.41, size = 156, normalized size = 0.74

$$\frac{a^3\sec^2(c+dx)\left(-1352400c-1352400dx-215040\tanh^{-1}(\sin(c+dx))\cos^2(c+dx)-1352400(c+dx)\cos(2(c+dx))+173600\sin(c+dx)+1052520\sin(2(c+dx))-11648\sin(3(c+dx))+175280\sin(4(c+dx))+22784\sin(5(c+dx))-18095\sin(6(c+dx))-6288\sin(7(c+dx))+770\sin(8(c+dx))+720\sin(9(c+dx))+105\sin(10(c+dx))\right)}{430080d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^8,x]
```

```
[Out] (a^3*Sec[c + d*x]^2*(-1352400*c - 1352400*d*x - 215040*ArcTanh[Sin[c + d*x]
]*Cos[c + d*x]^2 - 1352400*(c + d*x)*Cos[2*(c + d*x)] + 173600*Sin[c + d*x]
 + 1052520*Sin[2*(c + d*x)] - 11648*Sin[3*(c + d*x)] + 175280*Sin[4*(c + d*
x)] + 22784*Sin[5*(c + d*x)] - 18095*Sin[6*(c + d*x)] - 6288*Sin[7*(c + d*x
)] + 770*Sin[8*(c + d*x)] + 720*Sin[9*(c + d*x)] + 105*Sin[10*(c + d*x)]))/
(430080*d)
```

Maple [A]
time = 0.10, size = 272, normalized size = 1.30

method	result
derivativedivides	$\frac{a^3\left(\frac{\sin^9(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^7(dx+c)\right)}{2}+\frac{7\left(\sin^5(dx+c)\right)}{10}+\frac{7\left(\sin^3(dx+c)\right)}{6}+\frac{7\sin(dx+c)}{2}-\frac{7\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a^3\left(\frac{\sin^9(dx\cdot}{\cos(dx\cdot}\right.}{}$

default	$\frac{a^3\left(\frac{\sin^9(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^7(dx+c)\right)}{2}+\frac{7\left(\sin^5(dx+c)\right)}{10}+\frac{7\left(\sin^3(dx+c)\right)}{6}+\frac{7\sin(dx+c)}{2}-\frac{7\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a^3\left(\frac{\sin^9(dx+c}{\cos(dx+c}\right.}{}$
risch	$-\frac{805a^3x}{128}-\frac{47ia^3\mathrm{e}^{-i(dx+c)}}{128d}+\frac{127ia^3\mathrm{e}^{-2i(dx+c)}}{128d}-\frac{67ia^3\mathrm{e}^{3i(dx+c)}}{384d}-\frac{127ia^3\mathrm{e}^{2i(dx+c)}}{128d}+\frac{67ia^3\mathrm{e}^{-3i(dx+c)}}{384d}-ia^3$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*sin(d*x+c)^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/2*sin(d*x+c)^9/cos(d*x+c)^2+1/2*sin(d*x+c)^7+7/10*sin(d*x+c)^5+
7/6*sin(d*x+c)^3+7/2*sin(d*x+c)-7/2*ln(sec(d*x+c)+tan(d*x+c)))+3*a^3*(sin(d
*x+c)^9/cos(d*x+c)+(sin(d*x+c)^7+7/6*sin(d*x+c)^5+35/24*sin(d*x+c)^3+35/16*
sin(d*x+c))*cos(d*x+c)-35/16*d*x-35/16*c)+3*a^3*(-1/7*sin(d*x+c)^7-1/5*sin(
d*x+c)^5-1/3*sin(d*x+c)^3-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+a^3*(-1/8*(
sin(d*x+c)^7+7/6*sin(d*x+c)^5+35/24*sin(d*x+c)^3+35/16*sin(d*x+c))*cos(d*x+
c)+35/128*d*x+35/128*c))
```

Maxima [A]
time = 0.47, size = 291, normalized size = 1.39

$$-\frac{1536\left(30\sin(dx+c)^7+42\sin(dx+c)^5+70\sin(dx+c)^3-105\log(\sin(dx+c)+1)+105\log(\sin(dx+c)-1)+210\sin(dx+c)\right)a^3-1792\left(12\sin(dx+c)^5+40\sin(dx+c)^3-\frac{30\sin(dx+c)}{\sin(dx+c)^2-1}-105\log(\sin(dx+c)+1)+105\log(\sin(dx+c)-1)+180\sin(dx+c)\right)a^3-35\left(128\sin(2dx+2c)^3+840dx+840c+3\sin(8dx+8c)+168\sin(4dx+4c)-768\sin(2dx+2c)\right)a^3+6720\left(105dx+105c-\frac{87\tan(dx+c)^5+136\tan(dx+c)^3+57\tan(dx+c)}{\tan(dx+c)^6+3\tan(dx+c)^4+3\tan(dx+c)^2+1}-48\tan(dx+c)\right)a^3}{107520d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] -1/107520*(1536*(30*sin(d*x + c)^7 + 42*sin(d*x + c)^5 + 70*sin(d*x + c)^3
- 105*log(sin(d*x + c) + 1) + 105*log(sin(d*x + c) - 1) + 210*sin(d*x + c))
*a^3 - 1792*(12*sin(d*x + c)^5 + 40*sin(d*x + c)^3 - 30*sin(d*x + c)/(sin(d
*x + c)^2 - 1) - 105*log(sin(d*x + c) + 1) + 105*log(sin(d*x + c) - 1) + 18
0*sin(d*x + c))*a^3 - 35*(128*sin(2*d*x + 2*c)^3 + 840*d*x + 840*c + 3*sin(
8*d*x + 8*c) + 168*sin(4*d*x + 4*c) - 768*sin(2*d*x + 2*c))*a^3 + 6720*(105
*d*x + 105*c - (87*tan(d*x + c)^5 + 136*tan(d*x + c)^3 + 57*tan(d*x + c))/(
tan(d*x + c)^6 + 3*tan(d*x + c)^4 + 3*tan(d*x + c)^2 + 1) - 48*tan(d*x + c)
)*a^3)/d
```

Fricas [A]
time = 3.45, size = 204, normalized size = 0.97

$$-\frac{84525a^3dx\cos(dx+c)^2+3360a^3\cos(dx+c)^2\log(\sin(dx+c)+1)-3360a^3\cos(dx+c)^2\log(-\sin(dx+c)+1)-\left(1680a^3\cos(dx+c)^9+5760a^3\cos(dx+c)^8-280a^3\cos(dx+c)^7-22656a^3\cos(dx+c)^6-20510a^3\cos(dx+c)^5+32512a^3\cos(dx+c)^4+63315a^3\cos(dx+c)^3-15616a^3\cos(dx+c)^2+40320a^3\cos(dx+c)+6720a^3\right)\sin(dx+c)}{13440d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] -1/13440*(84525*a^3*d*x*cos(d*x + c)^2 + 3360*a^3*cos(d*x + c)^2*log(sin(d*
x + c) + 1) - 3360*a^3*cos(d*x + c)^2*log(-sin(d*x + c) + 1) - (1680*a^3*co
```

```
s(d*x + c)^9 + 5760*a^3*cos(d*x + c)^8 - 280*a^3*cos(d*x + c)^7 - 22656*a^3
*cos(d*x + c)^6 - 20510*a^3*cos(d*x + c)^5 + 32512*a^3*cos(d*x + c)^4 + 633
15*a^3*cos(d*x + c)^3 - 15616*a^3*cos(d*x + c)^2 + 40320*a^3*cos(d*x + c) +
 6720*a^3)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*sin(d*x+c)**8,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [A]

time = 0.61, size = 244, normalized size = 1.16

$$-\frac{84525\,(dx+c)a^3 + 6720\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 6720\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{13440\left(5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-7\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2} + \frac{2\left(44205\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}+303065\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+841981\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+1123793\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+487983\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-490749\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-267225\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-44205\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^8}}{13440\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^8,x, algorithm="giac")
[Out] -1/13440*(84525*(d*x + c)*a^3 + 6720*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1))
 - 6720*a^3*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 13440*(5*a^3*tan(1/2*d*x +
 1/2*c)^3 - 7*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2 + 2*
(44205*a^3*tan(1/2*d*x + 1/2*c)^15 + 303065*a^3*tan(1/2*d*x + 1/2*c)^13 + 8
41981*a^3*tan(1/2*d*x + 1/2*c)^11 + 1123793*a^3*tan(1/2*d*x + 1/2*c)^9 + 48
7983*a^3*tan(1/2*d*x + 1/2*c)^7 - 490749*a^3*tan(1/2*d*x + 1/2*c)^5 - 26722
5*a^3*tan(1/2*d*x + 1/2*c)^3 - 44205*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x
 + 1/2*c)^2 + 1)^8)/d
```

Mupad [B]

time = 2.45, size = 320, normalized size = 1.52

$$\frac{-\frac{741\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{19}}{64}-\frac{12469\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}}{192}-\frac{5027\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{40}-\frac{19211\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{420}+\frac{199977\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{1120}+\frac{877061\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{3360}+\frac{10233\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{140}+\frac{6243\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{40}+\frac{4967\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{64}+\frac{869\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{20}+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{18}+13\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}+8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}-14\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-28\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-14\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+13\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)}-\frac{a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{805\,a^3\,x}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^8*(a + a/cos(c + d*x))^3,x)
[Out] ((4967*a^3*tan(c/2 + (d*x)/2)^3)/64 + (6243*a^3*tan(c/2 + (d*x)/2)^5)/40 +
(10233*a^3*tan(c/2 + (d*x)/2)^7)/140 + (877061*a^3*tan(c/2 + (d*x)/2)^9)/33
60 + (199977*a^3*tan(c/2 + (d*x)/2)^11)/1120 - (19211*a^3*tan(c/2 + (d*x)/2
)^13)/420 - (5027*a^3*tan(c/2 + (d*x)/2)^15)/40 - (12469*a^3*tan(c/2 + (d*x
)/2)^17)/192 - (741*a^3*tan(c/2 + (d*x)/2)^19)/64 + (869*a^3*tan(c/2 + (d*x
)/2))/64)/(d*(6*tan(c/2 + (d*x)/2)^2 + 13*tan(c/2 + (d*x)/2)^4 + 8*tan(c/2
+ (d*x)/2)^6 - 14*tan(c/2 + (d*x)/2)^8 - 28*tan(c/2 + (d*x)/2)^10 - 14*tan(
c/2 + (d*x)/2)^12 + 8*tan(c/2 + (d*x)/2)^14 + 13*tan(c/2 + (d*x)/2)^16 + 6*
tan(c/2 + (d*x)/2)^18 + tan(c/2 + (d*x)/2)^20 + 1)) - (a^3*atanh(tan(c/2 +
(d*x)/2)))/d - (805*a^3*x)/128
```

3.49 $\int (a+a\sec(c+dx))^3\sin^6(c+dx)\,dx$

Optimal. Leaf size=182

$$-\frac{85a^3x}{16}+\frac{a^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{a^3\sin(c+dx)}{d}+\frac{43a^3\cos(c+dx)\sin(c+dx)}{16d}-\frac{5a^3\cos^3(c+dx)\sin(c+dx}{24d}$$

```
[Out] -85/16*a^3*x+1/2*a^3*arctanh(sin(d*x+c))/d-a^3*sin(d*x+c)/d+43/16*a^3*cos(d
*x+c)*sin(d*x+c)/d-5/24*a^3*cos(d*x+c)^3*sin(d*x+c)/d-1/6*a^3*cos(d*x+c)^5*
sin(d*x+c)/d-2/3*a^3*sin(d*x+c)^3/d-3/5*a^3*sin(d*x+c)^5/d+3*a^3*tan(d*x+c)
/d+1/2*a^3*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A]
time = 0.22, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3957, 2951, 2717, 2715, 8, 2713, 3852, 3853, 3855}

$$-\frac{3a^3\sin^5(c+dx)}{5d}-\frac{2a^3\sin^3(c+dx)}{3d}-\frac{a^3\sin(c+dx)}{d}+\frac{3a^3\tan(c+dx)}{d}+\frac{a^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{a^3\sin(c+dx)\cos^5(c+dx)}{6d}-\frac{5a^3\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{43a^3\sin(c+dx)\cos(c+dx)}{16d}+\frac{a^3\tan(c+dx)\sec(c+dx)}{2d}-\frac{85a^3x}{16}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^6,x]
```

```
[Out] (-85*a^3*x)/16 + (a^3*ArcTanh[Sin[c + d*x]])/(2*d) - (a^3*Sin[c + d*x])/d +
 (43*a^3*Cos[c + d*x]*Sin[c + d*x])/(16*d) - (5*a^3*Cos[c + d*x]^3*Sin[c +
d*x])/(24*d) - (a^3*Cos[c + d*x]^5*Sin[c + d*x])/(6*d) - (2*a^3*Sin[c + d*x
]^3)/(3*d) - (3*a^3*Sin[c + d*x]^5)/(5*d) + (3*a^3*Tan[c + d*x])/d + (a^3*S
ec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+a\sec(c+dx))^3\sin^6(c+dx)\,dx = -\int (-a-a\cos(c+dx))^3\sin^3(c+dx)\tan^3(c+dx)\,dx$$

$$= -\frac{\int (8a^9+6a^9\cos(c+dx)-6a^9\cos^2(c+dx)-8a^9\cos^3(c+dx)+$$

$$= -8a^3x - a^3\int \cos^6(c+dx)\,dx + a^3\int \sec^3(c+dx)\,dx - \left(3a^3\right)\int \mathrm{co}$$

$$= -8a^3x - \frac{6a^3\sin(c+dx)}{d} + \frac{3a^3\cos(c+dx)\sin(c+dx)}{d} - \frac{a^3\cos^5(c$$

$$= -5a^3x + \frac{a^3\tanh^{-1}(\sin(c+dx))}{2d} - \frac{a^3\sin(c+dx)}{d} + \frac{3a^3\cos(c+dx)}{d}$$

$$= -5a^3x + \frac{a^3\tanh^{-1}(\sin(c+dx))}{2d} - \frac{a^3\sin(c+dx)}{d} + \frac{43a^3\cos(c+dx}{16}$$

$$= -\frac{85a^3x}{16} + \frac{a^3\tanh^{-1}(\sin(c+dx))}{2d} - \frac{a^3\sin(c+dx)}{d} + \frac{43a^3\cos(c+}{1}$$

Mathematica [A]
time = 0.60, size = 136, normalized size = 0.75

$$-\frac{a^3\sec^2(c+dx)\left(10200c+10200dx-1920\tanh^{-1}(\sin(c+dx))\cos^2(c+dx)+10200(c+dx)\cos(2(c+dx))-460\sin(c+dx)-8145\sin(2(c+dx))+1156\sin(3(c+dx))-1120\sin(4(c+dx))-268\sin(5(c+dx))+55\sin(6(c+dx))+36\sin(7(c+dx))+5\sin(8(c+dx))\right)}{3840d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^6,x]
```

```
[Out] -1/3840*(a^3*Sec[c + d*x]^2*(10200*c + 10200*d*x - 1920*ArcTanh[Sin[c + d*x
]]*Cos[c + d*x]^2 + 10200*(c + d*x)*Cos[2*(c + d*x)] - 460*Sin[c + d*x] - 8
145*Sin[2*(c + d*x)] + 1156*Sin[3*(c + d*x)] - 1120*Sin[4*(c + d*x)] - 268*
Sin[5*(c + d*x)] + 55*Sin[6*(c + d*x)] + 36*Sin[7*(c + d*x)] + 5*Sin[8*(c +
 d*x)]))/d
```

Maple [A]
time = 0.14, size = 232, normalized size = 1.27

method	result
derivativedivides	$\frac{a^3\left(\frac{\sin^7(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^5(dx+c)\right)}{2}+\frac{5\left(\sin^3(dx+c)\right)}{6}+\frac{5\sin(dx+c)}{2}-\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a^3\left(\frac{\sin^7(dx+c)}{\cos(dx+c)}+\left(\sin^5(dx+c)\right.\right.}{}$
default	$\frac{a^3\left(\frac{\sin^7(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^5(dx+c)\right)}{2}+\frac{5\left(\sin^3(dx+c)\right)}{6}+\frac{5\sin(dx+c)}{2}-\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a^3\left(\frac{\sin^7(dx+c)}{\cos(dx+c)}+\left(\sin^5(dx+c)\right.\right.}{}$

risch	$-\frac{85a^3x}{16}-\frac{15ia^3\mathrm{e}^{-i(dx+c)}}{16d}+\frac{17ia^3\mathrm{e}^{-3i(dx+c)}}{96d}+\frac{81ia^3\mathrm{e}^{-2i(dx+c)}}{128d}-\frac{ia^3\left(\mathrm{e}^{3i(dx+c)}-6\,\mathrm{e}^{2i(dx+c)}-\mathrm{e}^{i(dx+c)}-6\right)}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)^2}+$
norman	$-\frac{85a^3x}{16}+\frac{77a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}+\frac{277a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}+\frac{997a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{40d}+\frac{3933a^3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{40d}+\frac{6169a^3\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{120d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*sin(d*x+c)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/2*sin(d*x+c)^7/cos(d*x+c)^2+1/2*sin(d*x+c)^5+5/6*sin(d*x+c)^3+5
/2*sin(d*x+c)-5/2*ln(sec(d*x+c)+tan(d*x+c)))+3*a^3*(sin(d*x+c)^7/cos(d*x+c)
+(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)-15/8*d*x-15/8*c
)+3*a^3*(-1/5*sin(d*x+c)^5-1/3*sin(d*x+c)^3-sin(d*x+c)+ln(sec(d*x+c)+tan(d*
x+c)))+a^3*(-1/6*(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)
+5/16*d*x+5/16*c))
```

Maxima [A]
time = 0.49, size = 240, normalized size = 1.32

$$-\frac{96\left(6\sin\left(dx+c\right)^5+10\sin\left(dx+c\right)^3-15\log\left(\sin\left(dx+c\right)+1\right)+15\log\left(\sin\left(dx+c\right)-1\right)+30\sin\left(dx+c\right)\right)a^3-5\left(4\sin\left(2\,dx+2\,c\right)^3+60\,dx+60\,c+9\sin\left(4\,dx+4\,c\right)-48\sin\left(2\,dx+2\,c\right)\right)a^3-80\left(4\sin\left(dx+c\right)^3-\frac{6\sin(dx+c)}{\sin(dx+c)^2-1}-15\log\left(\sin\left(dx+c\right)+1\right)+15\log\left(\sin\left(dx+c\right)-1\right)+24\sin\left(dx+c\right)\right)a^3+360\left(15\,dx+15\,c-\frac{9\tan(dx+c)^3+7\tan(dx+c)}{\tan(dx+c)^4+2\tan(dx+c)^2+1}-8\tan\left(dx+c\right)\right)a^3}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] -1/960*(96*(6*sin(d*x + c)^5 + 10*sin(d*x + c)^3 - 15*log(sin(d*x + c) + 1)
 + 15*log(sin(d*x + c) - 1) + 30*sin(d*x + c))*a^3 - 5*(4*sin(2*d*x + 2*c)^
3 + 60*d*x + 60*c + 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^3 - 80*(4*s
in(d*x + c)^3 - 6*sin(d*x + c)/(sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) +
 1) + 15*log(sin(d*x + c) - 1) + 24*sin(d*x + c))*a^3 + 360*(15*d*x + 15*c
- (9*tan(d*x + c)^3 + 7*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d*x + c)^2 +
1) - 8*tan(d*x + c))*a^3)/d
```

Fricas [A]
time = 3.72, size = 177, normalized size = 0.97

$$-\frac{1275\,a^3dx\cos\left(dx+c\right)^2-60\,a^3\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)+60\,a^3\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)+\left(40\,a^3\cos\left(dx+c\right)^7+144\,a^3\cos\left(dx+c\right)^6+50\,a^3\cos\left(dx+c\right)^5-448\,a^3\cos\left(dx+c\right)^4-645\,a^3\cos\left(dx+c\right)^3+544\,a^3\cos\left(dx+c\right)^2-720\,a^3\cos\left(dx+c\right)-120\,a^3\right)\sin\left(dx+c\right)}{240\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] -1/240*(1275*a^3*d*x*cos(d*x + c)^2 - 60*a^3*cos(d*x + c)^2*log(sin(d*x + c
) + 1) + 60*a^3*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + (40*a^3*cos(d*x + c
)^7 + 144*a^3*cos(d*x + c)^6 + 50*a^3*cos(d*x + c)^5 - 448*a^3*cos(d*x + c)
^4 - 645*a^3*cos(d*x + c)^3 + 544*a^3*cos(d*x + c)^2 - 720*a^3*cos(d*x + c)
 - 120*a^3)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*sin(d*x+c)**6,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [A]
time = 0.59, size = 212, normalized size = 1.16

$$-\frac{1275\,(dx+c)a^3-120\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)+120\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{240\left(5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-7\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}+\frac{2\left(795\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+4025\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+7614\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+5634\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-345\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-315\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^6}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^6,x, algorithm="giac")
```

```
[Out] -1/240*(1275*(d*x + c)*a^3 - 120*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) + 1
20*a^3*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 240*(5*a^3*tan(1/2*d*x + 1/2*c)
^3 - 7*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2 + 2*(795*a^
3*tan(1/2*d*x + 1/2*c)^11 + 4025*a^3*tan(1/2*d*x + 1/2*c)^9 + 7614*a^3*tan(
1/2*d*x + 1/2*c)^7 + 5634*a^3*tan(1/2*d*x + 1/2*c)^5 - 345*a^3*tan(1/2*d*x
+ 1/2*c)^3 - 315*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/
d
```

Mupad [B]
time = 2.22, size = 261, normalized size = 1.43

$$\frac{a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{85\,a^3\,x}{16}+\frac{-\frac{93\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{15}}{8}-\frac{1039\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{13}}{24}-\frac{4319\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}}{120}+\frac{6169\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{9}}{120}+\frac{3933\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{7}}{40}+\frac{997\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{5}}{40}+\frac{277\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{3}}{8}+\frac{77\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{16}+4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{14}+4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}-10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{8}-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{6}+4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{4}+4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (a^3*atanh(tan(c/2 + (d*x)/2)))/d - (85*a^3*x)/16 + ((277*a^3*tan(c/2 + (d*
x)/2)^3)/8 + (997*a^3*tan(c/2 + (d*x)/2)^5)/40 + (3933*a^3*tan(c/2 + (d*x)/
2)^7)/40 + (6169*a^3*tan(c/2 + (d*x)/2)^9)/120 - (4319*a^3*tan(c/2 + (d*x)/
2)^11)/120 - (1039*a^3*tan(c/2 + (d*x)/2)^13)/24 - (93*a^3*tan(c/2 + (d*x)/
2)^15)/8 + (77*a^3*tan(c/2 + (d*x)/2))/8)/(d*(4*tan(c/2 + (d*x)/2)^2 + 4*ta
n(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^6 - 10*tan(c/2 + (d*x)/2)^8 - 4*t
an(c/2 + (d*x)/2)^10 + 4*tan(c/2 + (d*x)/2)^12 + 4*tan(c/2 + (d*x)/2)^14 +
tan(c/2 + (d*x)/2)^16 + 1))
```

3.50 $\int (a+a\sec(c+dx))^3 \sin^4(c+dx)\,dx$

Optimal. Leaf size=138

$$-\frac{33a^3x}{8}+\frac{3a^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{2a^3\sin(c+dx)}{d}+\frac{7a^3\cos(c+dx)\sin(c+dx)}{8d}+\frac{a^3\cos^3(c+dx)\sin(c+d}{4d}$$

```
[Out] -33/8*a^3*x+3/2*a^3*arctanh(sin(d*x+c))/d-2*a^3*sin(d*x+c)/d+7/8*a^3*cos(d*
x+c)*sin(d*x+c)/d+1/4*a^3*cos(d*x+c)^3*sin(d*x+c)/d-a^3*sin(d*x+c)^3/d+3*a^
3*tan(d*x+c)/d+1/2*a^3*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A]
time = 0.17, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3957, 2951, 2717, 2715, 8, 2713, 3855, 3852, 3853}

$$-\frac{a^3\sin^3(c+dx)}{d}-\frac{2a^3\sin(c+dx)}{d}+\frac{3a^3\tan(c+dx)}{d}+\frac{3a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^3\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{7a^3\sin(c+dx)\cos(c+dx)}{8d}+\frac{a^3\tan(c+dx)\sec(c+dx)}{2d}-\frac{33a^3x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^4,x]
```

```
[Out] (-33*a^3*x)/8 + (3*a^3*ArcTanh[Sin[c + d*x]])/(2*d) - (2*a^3*Sin[c + d*x])/
d + (7*a^3*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^3*Cos[c + d*x]^3*Sin[c + d
*x])/(4*d) - (a^3*Sin[c + d*x]^3)/d + (3*a^3*Tan[c + d*x])/d + (a^3*Sec[c +
 d*x]*Tan[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+a\sec(c+dx))^3\sin^4(c+dx)\,dx = -\int (-a-a\cos(c+dx))^3\sin(c+dx)\tan^3(c+dx)\,dx$$
$$= -\frac{\int (5a^7+5a^7\cos(c+dx)-a^7\cos^2(c+dx)-3a^7\cos^3(c+dx)-$$
$$= -5a^3x+a^3\int\cos^2(c+dx)\,dx+a^3\int\cos^4(c+dx)\,dx+a^3\int\sec($$
$$= -5a^3x+\frac{a^3\tanh^{-1}(\sin(c+dx))}{d}-\frac{5a^3\sin(c+dx)}{d}+\frac{a^3\cos(c+d$$
$$= -\frac{9a^3x}{2}+\frac{3a^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{2a^3\sin(c+dx)}{d}+\frac{7a^3\cos(c+$$
$$= -\frac{33a^3x}{8}+\frac{3a^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{2a^3\sin(c+dx)}{d}+\frac{7a^3\cos(c$$

Mathematica [A]

time = 0.26, size = 114, normalized size = 0.83

$$\frac{a^3\sec^2(c+dx)\left(-264c-264dx+192\tanh^{-1}(\sin(c+dx))\cos^2(c+dx)-264(c+dx)\cos(2(c+dx))-16\sin(c+dx)+225\sin(2(c+dx))-72\sin(3(c+dx))+18\sin(4(c+dx))+8\sin(5(c+dx))+\sin(6(c+dx))\right)}{128d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^4,x]
```

```
[Out] (a^3*Sec[c + d*x]^2*(-264*c - 264*d*x + 192*ArcTanh[Sin[c + d*x]]*Cos[c + d
*x]^2 - 264*(c + d*x)*Cos[2*(c + d*x)] - 16*Sin[c + d*x] + 225*Sin[2*(c + d
*x)] - 72*Sin[3*(c + d*x)] + 18*Sin[4*(c + d*x)] + 8*Sin[5*(c + d*x)] + Sin
[6*(c + d*x)]))/(128*d)
```

Maple [A]

time = 0.12, size = 192, normalized size = 1.39

method	result
derivativedivides	$\frac{a^3\left(\frac{\sin^5(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^3(dx+c)\right)}{2}+\frac{3\sin(dx+c)}{2}-\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a^3\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\right.}{}$
default	$\frac{a^3\left(\frac{\sin^5(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^3(dx+c)\right)}{2}+\frac{3\sin(dx+c)}{2}-\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a^3\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\right.}{}$
risch	$-\frac{33a^3x}{8}-\frac{ia^3e^{3i(dx+c)}}{8d}-\frac{ia^3e^{2i(dx+c)}}{4d}+\frac{11ia^3e^{i(dx+c)}}{8d}-\frac{11ia^3e^{-i(dx+c)}}{8d}+\frac{ia^3e^{-2i(dx+c)}}{4d}+\frac{ia^3e^{-3i(dx+c)}}{8d}$
norman	$\frac{-\frac{33a^3x}{8}+\frac{21a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{27a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{79a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{25a^3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{83a^3\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{45a^3}{}}{\left(\mathrm{t}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*sin(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/2*sin(d*x+c)^5/cos(d*x+c)^2+1/2*sin(d*x+c)^3+3/2*sin(d*x+c)-3/2
*ln(sec(d*x+c)+tan(d*x+c)))+3*a^3*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/
2*sin(d*x+c))*cos(d*x+c)-3/2*d*x-3/2*c)+3*a^3*(-1/3*sin(d*x+c)^3-sin(d*x+c)
+ln(sec(d*x+c)+tan(d*x+c)))+a^3*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x
+c)+3/8*d*x+3/8*c))
```

Maxima [A]
time = 0.48, size = 182, normalized size = 1.32

$$-\frac{16\left(2\sin\left(dx+c\right)^{3}-3\log\left(\sin\left(dx+c\right)+1\right)+3\log\left(\sin\left(dx+c\right)-1\right)+6\sin\left(dx+c\right)\right)a^{3}-\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)-8\sin\left(2\,dx+2\,c\right)\right)a^{3}+48\left(3\,dx+3\,c-\frac{\tan\left(dx+c\right)}{\tan\left(dx+c\right)^{2}+1}-2\tan\left(dx+c\right)\right)a^{3}+8\,a^{3}\left(\frac{2\sin\left(dx+c\right)}{\sin\left(dx+c\right)^{2}-1}+3\log\left(\sin\left(dx+c\right)+1\right)-3\log\left(\sin\left(dx+c\right)-1\right)-4\sin\left(dx+c\right)\right)}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/32*(16*(2*sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c)
- 1) + 6*sin(d*x + c))*a^3 - (12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*d*
x + 2*c))*a^3 + 48*(3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*tan
(d*x + c))*a^3 + 8*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + 3*log(sin(d*x
 + c) + 1) - 3*log(sin(d*x + c) - 1) - 4*sin(d*x + c)))/d
```

Fricas [A]
time = 3.05, size = 152, normalized size = 1.10

$$-\frac{33\,a^{3}dx\cos\left(dx+c\right)^{2}-6\,a^{3}\cos\left(dx+c\right)^{2}\log\left(\sin\left(dx+c\right)+1\right)+6\,a^{3}\cos\left(dx+c\right)^{2}\log\left(-\sin\left(dx+c\right)+1\right)-\left(2\,a^{3}\cos\left(dx+c\right)^{5}+8\,a^{3}\cos\left(dx+c\right)^{4}+7\,a^{3}\cos\left(dx+c\right)^{3}-24\,a^{3}\cos\left(dx+c\right)^{2}+24\,a^{3}\cos\left(dx+c\right)+4\,a^{3}\right)\sin\left(dx+c\right)}{8\,d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/8*(33*a^3*d*x*cos(d*x + c)^2 - 6*a^3*cos(d*x + c)^2*log(sin(d*x + c) + 1
) + 6*a^3*cos(d*x + c)^2*log(-sin(d*x + c) + 1) - (2*a^3*cos(d*x + c)^5 + 8
*a^3*cos(d*x + c)^4 + 7*a^3*cos(d*x + c)^3 - 24*a^3*cos(d*x + c)^2 + 24*a^3
*cos(d*x + c) + 4*a^3)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{3}\left(\int 3\sin^{4}\left(c+dx\right)\sec\left(c+dx\right)dx+\int 3\sin^{4}\left(c+dx\right)\sec^{2}\left(c+dx\right)dx+\int \sin^{4}\left(c+dx\right)\sec^{3}\left(c+dx\right)dx+\int \sin^{4}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*sin(d*x+c)**4,x)
```

```
[Out] a**3*(Integral(3*sin(c + d*x)**4*sec(c + d*x), x) + Integral(3*sin(c + d*x)
**4*sec(c + d*x)**2, x) + Integral(sin(c + d*x)**4*sec(c + d*x)**3, x) + In
tegral(sin(c + d*x)**4, x))
```

Giac [A]
time = 0.58, size = 180, normalized size = 1.30

$$-\frac{33\,(dx+c)a^3-12\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)+12\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{8\left(5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-7\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}+\frac{2\left(25\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+81\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+79\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+7\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^4,x, algorithm="giac")
```

```
[Out] -1/8*(33*(d*x + c)*a^3 - 12*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) + 12*a^3
*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 8*(5*a^3*tan(1/2*d*x + 1/2*c)^3 - 7*a
^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2 + 2*(25*a^3*tan(1/2
*d*x + 1/2*c)^7 + 81*a^3*tan(1/2*d*x + 1/2*c)^5 + 79*a^3*tan(1/2*d*x + 1/2*
c)^3 + 7*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

Mupad [B]
time = 1.97, size = 204, normalized size = 1.48

$$\frac{3\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{33\,a^3\,x}{8}+\frac{-\frac{45\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{4}-\frac{83\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4}+\frac{25\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{2}+\frac{79\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{2}+\frac{27\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}+\frac{21\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}+2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (3*a^3*atanh(tan(c/2 + (d*x)/2)))/d - (33*a^3*x)/8 + ((27*a^3*tan(c/2 + (d*
x)/2)^3)/4 + (79*a^3*tan(c/2 + (d*x)/2)^5)/2 + (25*a^3*tan(c/2 + (d*x)/2)^7
)/2 - (83*a^3*tan(c/2 + (d*x)/2)^9)/4 - (45*a^3*tan(c/2 + (d*x)/2)^11)/4 +
(21*a^3*tan(c/2 + (d*x)/2))/4)/(d*(2*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)
/2)^4 - 4*tan(c/2 + (d*x)/2)^6 - tan(c/2 + (d*x)/2)^8 + 2*tan(c/2 + (d*x)/2
)^10 + tan(c/2 + (d*x)/2)^12 + 1))
```

3.51 $\int (a+a\sec(c+dx))^3\sin^2(c+dx)\,dx$

Optimal. Leaf size=98

$$-\frac{5a^3x}{2}+\frac{5a^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{3a^3\sin(c+dx)}{d}-\frac{a^3\cos(c+dx)\sin(c+dx)}{2d}+\frac{3a^3\tan(c+dx)}{d}+\frac{a^3\sec(c+$$

```
[Out] -5/2*a^3*x+5/2*a^3*arctanh(sin(d*x+c))/d-3*a^3*sin(d*x+c)/d-1/2*a^3*cos(d*x
+c)*sin(d*x+c)/d+3*a^3*tan(d*x+c)/d+1/2*a^3*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A]
time = 0.14, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$,
Rules used = {3957, 2951, 2717, 2715, 8, 3855, 3852, 3853}

$$-\frac{3a^3\sin(c+dx)}{d}+\frac{3a^3\tan(c+dx)}{d}+\frac{5a^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{a^3\tan(c+dx)\sec(c+dx)}{2d}-\frac{5a^3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^2,x]
```

```
[Out] (-5*a^3*x)/2 + (5*a^3*ArcTanh[Sin[c + d*x]])/(2*d) - (3*a^3*Sin[c + d*x])/d
 - (a^3*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (3*a^3*Tan[c + d*x])/d + (a^3*Se
c[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
```

```
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))^3\sin^2(c+dx)\,dx &= -\int (-a-a\cos(c+dx))^3\sec(c+dx)\tan^2(c+dx)\,dx\\ &= -\frac{\int \left(2a^5+3a^5\cos(c+dx)+a^5\cos^2(c+dx)-2a^5\sec(c+dx)-3\right.}{a^2}\\ &= -2a^3x-a^3\int\cos^2(c+dx)\,dx+a^3\int\sec^3(c+dx)\,dx+\left(2a^3\right)\int \mathrm{s}\\ &= -2a^3x+\frac{2a^3\tanh^{-1}(\sin(c+dx))}{d}-\frac{3a^3\sin(c+dx)}{d}-\frac{a^3\cos(c+}{}\\ &= -\frac{5a^3x}{2}+\frac{5a^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{3a^3\sin(c+dx)}{d}-\frac{a^3\cos(c+}{}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 300 vs. 2(98) = 196.
time = 1.59, size = 300, normalized size = 3.06

$$\frac{1}{32}a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(-10x-\frac{10\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{10\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}-\frac{12\cos(dx)\sin(c)}{d}-\frac{\cos(2dx)\sin(2c)}{d}-\frac{12\cos(c)\sin(dx)}{d}-\frac{\cos(2c)\sin(2dx)}{d}+\frac{1}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{12\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}-\frac{1}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{12\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^2,x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(-10*x - (10*Log[Cos[(c + d*x)
/2] - Sin[(c + d*x)/2]])/d + (10*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/
d - (12*Cos[d*x]*Sin[c])/d - (Cos[2*d*x]*Sin[2*c])/d - (12*Cos[c]*Sin[d*x])
/d - (Cos[2*c]*Sin[2*d*x])/d + 1/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2
) + (12*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c +
 d*x)/2])) - 1/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (12*Sin[(d*x)/
2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/32
```

Maple [A]

time = 0.09, size = 126, normalized size = 1.29

method	result
derivativedivides	$\frac{a^3\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a^3(\tan(dx+c)-dx-c)+3a^3(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(d}{d}$
default	$\frac{a^3\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a^3(\tan(dx+c)-dx-c)+3a^3(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(d}{d}$
norman	$\frac{-\frac{5a^3x}{2}+\frac{18a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{10a^3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+5a^3x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{5a^3x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{5a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}{2d}$
risch	$-\frac{5a^3x}{2}+\frac{ia^3e^{2i(dx+c)}}{8d}+\frac{3ia^3e^{i(dx+c)}}{2d}-\frac{3ia^3e^{-i(dx+c)}}{2d}-\frac{ia^3e^{-2i(dx+c)}}{8d}-\frac{ia^3\left(e^{3i(dx+c)}-6e^{2i(dx+c)}-e^{i(dx+c)}\right.}{d\left(e^{2i(dx+c)}+1\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*sin(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*sin(d*x+c)-1/2*ln(sec(d*x+c)+ta
n(d*x+c)))+3*a^3*(tan(d*x+c)-d*x-c)+3*a^3*(-sin(d*x+c)+ln(sec(d*x+c)+tan(d*
x+c)))+a^3*(-1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A]

time = 0.48, size = 127, normalized size = 1.30

$$\frac{(2\,dx+2\,c-\sin(2\,dx+2\,c))a^3-12\,(dx+c-\tan(dx+c))a^3-a^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)\right)+6\,a^3(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)-2\,\sin(dx+c))}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c - sin(2*d*x + 2*c))*a^3 - 12*(d*x + c - tan(d*x + c))*a^3
 - a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - log(s
in(d*x + c) - 1)) + 6*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) -
2*sin(d*x + c)))/d
```

Fricas [A]
time = 3.87, size = 125, normalized size = 1.28

$$-\frac{10\,a^3dx\cos\left(dx+c\right)^2-5\,a^3\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)+5\,a^3\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)+2\,\left(a^3\cos\left(dx+c\right)^3+6\,a^3\cos\left(dx+c\right)^2-6\,a^3\cos\left(dx+c\right)-a^3\right)\sin\left(dx+c\right)}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(10*a^3*d*x*cos(d*x + c)^2 - 5*a^3*cos(d*x + c)^2*log(sin(d*x + c) + 1
) + 5*a^3*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(a^3*cos(d*x + c)^3 + 6
*a^3*cos(d*x + c)^2 - 6*a^3*cos(d*x + c) - a^3)*sin(d*x + c))/(d*cos(d*x +
c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int 3\sin^2\left(c+dx\right)\sec\left(c+dx\right)\,dx+\int 3\sin^2\left(c+dx\right)\sec^2\left(c+dx\right)\,dx+\int \sin^2\left(c+dx\right)\sec^3\left(c+dx\right)\,dx+\int \sin^2\left(c+dx\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*sin(d*x+c)**2,x)
```

```
[Out] a**3*(Integral(3*sin(c + d*x)**2*sec(c + d*x), x) + Integral(3*sin(c + d*x)
**2*sec(c + d*x)**2, x) + Integral(sin(c + d*x)**2*sec(c + d*x)**3, x) + In
tegral(sin(c + d*x)**2, x))
```

Giac [A]
time = 0.55, size = 102, normalized size = 1.04

$$-\frac{5\left(dx+c\right)a^3-5\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)+5\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{4\left(5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-9\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(5*(d*x + c)*a^3 - 5*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) + 5*a^3*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) + 4*(5*a^3*tan(1/2*d*x + 1/2*c)^7 - 9*a^3*
tan(1/2*d*x + 1/2*c)^3)/(tan(1/2*d*x + 1/2*c)^4 - 1)^2)/d
```

Mupad [B]
time = 1.28, size = 90, normalized size = 0.92

$$\frac{5\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{5\,a^3\,x}{2}+\frac{18\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-10\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (5*a^3*atanh(tan(c/2 + (d*x)/2)))/d - (5*a^3*x)/2 + (18*a^3*tan(c/2 + (d*x)
/2)^3 - 10*a^3*tan(c/2 + (d*x)/2)^7)/(d*(tan(c/2 + (d*x)/2)^8 - 2*tan(c/2 +
 (d*x)/2)^4 + 1))
```

3.52 $\int \csc^2(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=80

$$\frac{9a^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{4a^3\sin(c+dx)}{d(1-\cos(c+dx))}+\frac{3a^3\tan(c+dx)}{d}+\frac{a^3\sec(c+dx)\tan(c+dx)}{2d}$$

```
[Out] 9/2*a^3*arctanh(sin(d*x+c))/d-4*a^3*sin(d*x+c)/d/(1-cos(d*x+c))+3*a^3*tan(d
*x+c)/d+1/2*a^3*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A]
time = 0.14, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3957, 2951, 2727, 3855, 3852, 8, 3853}

$$\frac{3a^3\tan(c+dx)}{d}+\frac{9a^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{4a^3\sin(c+dx)}{d(1-\cos(c+dx))}+\frac{a^3\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (9*a^3*ArcTanh[Sin[c + d*x]])/(2*d) - (4*a^3*Sin[c + d*x])/(d*(1 - Cos[c +
d*x])) + (3*a^3*Tan[c + d*x])/d + (a^3*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \csc^2(c+dx)(a+a\sec(c+dx))^3\,dx &= -\int (-a-a\cos(c+dx))^3\csc^2(c+dx)\sec^3(c+dx)\,dx \\ &= a^2\int\left(\frac{4a}{1-\cos(c+dx)}+4a\sec(c+dx)+3a\sec^2(c+dx)+a\sec^3(\right. \\ &= a^3\int\sec^3(c+dx)\,dx+\left(3a^3\right)\int\sec^2(c+dx)\,dx+\left(4a^3\right)\int\frac{1}{1-\cos(} \\ &= \frac{4a^3\tanh^{-1}(\sin(c+dx))}{d}-\frac{4a^3\sin(c+dx)}{d(1-\cos(c+dx))}+\frac{a^3\sec(c+dx)\tan(}{2d} \\ &= \frac{9a^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{4a^3\sin(c+dx)}{d(1-\cos(c+dx))}+\frac{3a^3\tan(c+dx)}{d}+\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 244 vs. 2(80) = 160.
time = 0.81, size = 244, normalized size = 3.05

$$\frac{a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(-18\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+18\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+16\csc\left(\frac{c}{2}\right)\csc\left(\frac{1}{2}(c+dx)\right)\sin\left(\frac{dx}{2}\right)+\frac{1}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}-\frac{1}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{12\sin(dx)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right)}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*(a + a*Sec[c + d*x])^3,x]
```

[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(-18*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 18*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 16*Csc[c/2]*Csc[(c + d*x)/2]*Sin[(d*x)/2] + (Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^(-2) - (Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^(-2) + (12*Sin[d*x])/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/(32*d)

Maple [A]

time = 0.08, size = 127, normalized size = 1.59

method	result
norman	$\frac{-\frac{4a^3}{d}+\frac{15a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{9a^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{9a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}+\frac{9a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2d}$
risch	$-\frac{ia^3\left(9\,e^{4i(dx+c)}-7\,e^{3i(dx+c)}+21\,e^{2i(dx+c)}-5\,e^{i(dx+c)}+14\right)}{d\left(e^{i(dx+c)}-1\right)\left(e^{2i(dx+c)}+1\right)^2}-\frac{9a^3\ln\left(e^{i(dx+c)}-i\right)}{2d}+\frac{9a^3\ln\left(e^{i(dx+c)}+i\right)}{2d}$
derivativedivides	$\frac{a^3\left(\frac{1}{2\sin(dx+c)\cos(dx+c)^2}-\frac{3}{2\sin(dx+c)}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a^3\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)+3a^3\left(-\right.}{d}$
default	$\frac{a^3\left(\frac{1}{2\sin(dx+c)\cos(dx+c)^2}-\frac{3}{2\sin(dx+c)}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a^3\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)+3a^3\left(-\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^2*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(a^3*(1/2/sin(d*x+c)/cos(d*x+c)^2-3/2/sin(d*x+c)+3/2*ln(sec(d*x+c)+tan(d*x+c)))+3*a^3*(1/sin(d*x+c)/cos(d*x+c)-2*cot(d*x+c))+3*a^3*(-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))-a^3*cot(d*x+c))

Maxima [A]

time = 0.28, size = 137, normalized size = 1.71

$$-\frac{a^3\left(\frac{2\left(3\sin(dx+c)^2-2\right)}{\sin(dx+c)^3-\sin(dx+c)}-3\log\left(\sin\left(dx+c\right)+1\right)+3\log\left(\sin\left(dx+c\right)-1\right)\right)+6\,a^3\left(\frac{2}{\sin(dx+c)}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+12\,a^3\left(\frac{1}{\tan(dx+c)}-\tan\left(dx+c\right)\right)+\frac{4\,a^3}{\tan(dx+c)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="maxima")

[Out] -1/4*(a^3*(2*(3*sin(d*x + c)^2 - 2)/(sin(d*x + c)^3 - sin(d*x + c)) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) + 6*a^3*(2/sin(d*x + c) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 12*a^3*(1/tan(d*x + c) - tan(d*x + c)) + 4*a^3/tan(d*x + c))/d

Fricas [A]

time = 4.38, size = 122, normalized size = 1.52

$$\frac{9\,a^3\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)-9\,a^3\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)-28\,a^3\cos\left(dx+c\right)^3-18\,a^3\cos\left(dx+c\right)^2+12\,a^3\cos\left(dx+c\right)+2\,a^3}{4\,d\cos\left(dx+c\right)^2\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(9*a^3*cos(d*x + c)^2*log(sin(d*x + c) + 1)*sin(d*x + c) - 9*a^3*cos(d*
x + c)^2*log(-sin(d*x + c) + 1)*sin(d*x + c) - 28*a^3*cos(d*x + c)^3 - 18*a
^3*cos(d*x + c)^2 + 12*a^3*cos(d*x + c) + 2*a^3)/(d*cos(d*x + c)^2*sin(d*x
+ c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{3}\left(\int 3\csc^{2}(c+dx)\sec(c+dx)\,dx+\int 3\csc^{2}(c+dx)\sec^{2}(c+dx)\,dx+\int \csc^{2}(c+dx)\sec^{3}(c+dx)\,dx+\int \csc^{2}(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+a*sec(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(3*csc(c + d*x)**2*sec(c + d*x), x) + Integral(3*csc(c + d*x)
**2*sec(c + d*x)**2, x) + Integral(csc(c + d*x)**2*sec(c + d*x)**3, x) + In
tegral(csc(c + d*x)**2, x))
```

Giac [A]
time = 0.51, size = 106, normalized size = 1.32

$$\frac{9\,a^{3}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-9\,a^{3}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{8\,a^{3}}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{2\left(5\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-7\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(9*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 9*a^3*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) - 8*a^3/tan(1/2*d*x + 1/2*c) - 2*(5*a^3*tan(1/2*d*x + 1/2*c)^
3 - 7*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

Mupad [B]
time = 2.46, size = 98, normalized size = 1.22

$$\frac{9\,a^{3}\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{9\,a^{3}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}-15\,a^{3}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+4\,a^{3}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^3/sin(c + d*x)^2,x)
```

```
[Out] (9*a^3*atanh(tan(c/2 + (d*x)/2)))/d - (9*a^3*tan(c/2 + (d*x)/2)^4 - 15*a^3*
tan(c/2 + (d*x)/2)^2 + 4*a^3)/(d*(tan(c/2 + (d*x)/2) - 2*tan(c/2 + (d*x)/2)
^3 + tan(c/2 + (d*x)/2)^5))
```

3.53 $\int \csc^4(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=110

$$\frac{11a^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{2a^3\sin(c+dx)}{3d(1-\cos(c+dx))^2}-\frac{17a^3\sin(c+dx)}{3d(1-\cos(c+dx))}+\frac{3a^3\tan(c+dx)}{d}+\frac{a^3\sec(c+dx)\,\mathrm{ta}}{2d}$$

```
[Out] 11/2*a^3*arctanh(sin(d*x+c))/d-2/3*a^3*sin(d*x+c)/d/(1-cos(d*x+c))^2-17/3*a
^3*sin(d*x+c)/d/(1-cos(d*x+c))+3*a^3*tan(d*x+c)/d+1/2*a^3*sec(d*x+c)*tan(d*
x+c)/d
```

Rubi [A]
time = 0.17, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3957, 2951, 2729, 2727, 3855, 3852, 8, 3853}

$$\frac{3a^3\tan(c+dx)}{d}+\frac{11a^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{17a^3\sin(c+dx)}{3d(1-\cos(c+dx))}-\frac{2a^3\sin(c+dx)}{3d(1-\cos(c+dx))^2}+\frac{a^3\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (11*a^3*ArcTanh[Sin[c + d*x]])/(2*d) - (2*a^3*Sin[c + d*x])/(3*d*(1 - Cos[c
 + d*x])^2) - (17*a^3*Sin[c + d*x])/(3*d*(1 - Cos[c + d*x])) + (3*a^3*Tan[c
 + d*x])/d + (a^3*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
```

```
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc^4(c+dx)(a+a\sec(c+dx))^3\,dx = -\int (-a-a\cos(c+dx))^3\csc^4(c+dx)\sec^3(c+dx)\,dx$$

$$= a^4\int\left(\frac{2}{a(1-\cos(c+dx))^2}+\frac{5}{a(1-\cos(c+dx))}+\frac{5\sec(c+dx)}{a}+\right.$$

$$= a^3\int\sec^3(c+dx)\,dx+\left(2a^3\right)\int\frac{1}{(1-\cos(c+dx))^2}\,dx+\left(3a^3\right)\int \mathrm{se}$$

$$= \frac{5a^3\tanh^{-1}(\sin(c+dx))}{d}-\frac{2a^3\sin(c+dx)}{3d(1-\cos(c+dx))^2}-\frac{5a^3\sin(c+dx}{d(1-\cos(c+d}$$

$$= \frac{11a^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{2a^3\sin(c+dx)}{3d(1-\cos(c+dx))^2}-\frac{17a^3\sin(c+}{3d(1-\cos(c+}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 290 vs. 2(110) = 220.

time = 5.38, size = 290, normalized size = 2.64

$$\frac{a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(-4\cot\left(\frac{c}{2}\right)\csc^2\left(\frac{1}{2}(c+dx)\right)-66\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+66\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-2(-19+17\cos(c+dx))\csc\left(\frac{c}{2}\right)\csc^3\left(\frac{1}{2}(c+dx)\right)\sin\left(\frac{dx}{2}\right)+\frac{3}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{36\sin\left(\frac{dx}{2}\right)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}-\frac{3}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{36\sin\left(\frac{dx}{2}\right)}{\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right)}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(-4*Cot[c/2]*Csc[(c + d*x)/2]^
2 - 66*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 66*Log[Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]] - 2*(-19 + 17*Cos[c + d*x])*Csc[c/2]*Csc[(c + d*x)/2]^3*
Sin[(d*x)/2] + 3/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (36*Sin[(d*x)/2]
)/((Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) - 3/(Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2])^2 + (36*Sin[(d*x)/2])/((Cos[c/2] + Sin[c/2])
*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/(96*d)
```

Maple [A]

time = 0.11, size = 185, normalized size = 1.68

method	result
norman	$\frac{-\frac{a^3}{3d}-\frac{16a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{56a^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{11a^3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{11a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}+\frac{11a^3\ln\left(\tan\left(\right.\right.}{2d}$
risch	$-\frac{ia^3\left(33\,e^{6i(dx+c)}-99\,e^{5i(dx+c)}+154\,e^{4i(dx+c)}-210\,e^{3i(dx+c)}+161\,e^{2i(dx+c)}-123\,e^{i(dx+c)}+52\right)}{3d\left(e^{i(dx+c)}-1\right)^3\left(e^{2i(dx+c)}+1\right)^2}-\frac{11a^3\ln\left(e^{i(dx+c)}-\right.}{2d}$
derivativedivides	$a^3\left(-\frac{1}{3\sin(dx+c)^3\cos(dx+c)^2}+\frac{5}{6\sin(dx+c)\cos(dx+c)^2}-\frac{5}{2\sin(dx+c)}+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a^3\left(-\frac{1}{3\sin(dx+c)^3\cos(dx\right.$
default	$a^3\left(-\frac{1}{3\sin(dx+c)^3\cos(dx+c)^2}+\frac{5}{6\sin(dx+c)\cos(dx+c)^2}-\frac{5}{2\sin(dx+c)}+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a^3\left(-\frac{1}{3\sin(dx+c)^3\cos(dx\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/3/sin(d*x+c)^3/cos(d*x+c)^2+5/6/sin(d*x+c)/cos(d*x+c)^2-5/2/si
n(d*x+c)+5/2*ln(sec(d*x+c)+tan(d*x+c)))+3*a^3*(-1/3/sin(d*x+c)^3/cos(d*x+c)
+4/3/sin(d*x+c)/cos(d*x+c)-8/3*cot(d*x+c))+3*a^3*(-1/3/sin(d*x+c)^3-1/sin(d
*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+a^3*(-2/3-1/3*csc(d*x+c)^2)*cot(d*x+c))
```

Maxima [A]

time = 0.27, size = 188, normalized size = 1.71

$$-\frac{a^3\left(\frac{2\left(15\sin(dx+c)^4-10\sin(dx+c)^2-2\right)}{\sin(dx+c)^5-\sin(dx+c)^3}-15\log\left(\sin\left(dx+c\right)+1\right)+15\log\left(\sin\left(dx+c\right)-1\right)\right)+6a^3\left(\frac{2\left(3\sin(dx+c)^2+1\right)}{\sin(dx+c)^3}-3\log\left(\sin\left(dx+c\right)+1\right)+3\log\left(\sin\left(dx+c\right)-1\right)\right)+12a^3\left(\frac{6\tan(dx+c)^2+1}{\tan(dx+c)^3}-3\tan\left(dx+c\right)\right)+\frac{4\left(3\tan(dx+c)^2+1\right)a^3}{\tan(dx+c)^3}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/12*(a^3*(2*(15*sin(d*x + c)^4 - 10*sin(d*x + c)^2 - 2)/(sin(d*x + c)^5 -
 sin(d*x + c)^3) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) + 6
*a^3*(2*(3*sin(d*x + c)^2 + 1)/sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3
*log(sin(d*x + c) - 1)) + 12*a^3*((6*tan(d*x + c)^2 + 1)/tan(d*x + c)^3 - 3
*tan(d*x + c)) + 4*(3*tan(d*x + c)^2 + 1)*a^3/tan(d*x + c)^3)/d
```

Fricas [A]
time = 2.70, size = 178, normalized size = 1.62

$$-\frac{104\,a^{3}\cos\left(dx+c\right)^{4}-38\,a^{3}\cos\left(dx+c\right)^{3}-118\,a^{3}\cos\left(dx+c\right)^{2}+30\,a^{3}\cos\left(dx+c\right)+6\,a^{3}-33\,\left(a^{3}\cos\left(dx+c\right)^{3}-a^{3}\cos\left(dx+c\right)^{2}\right)\log\left(\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)+33\,\left(a^{3}\cos\left(dx+c\right)^{3}-a^{3}\cos\left(dx+c\right)^{2}\right)\log\left(-\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)}{12\,\left(d\cos\left(dx+c\right)^{3}-d\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/12*(104*a^3*cos(d*x + c)^4 - 38*a^3*cos(d*x + c)^3 - 118*a^3*cos(d*x + c
)^2 + 30*a^3*cos(d*x + c) + 6*a^3 - 33*(a^3*cos(d*x + c)^3 - a^3*cos(d*x +
c)^2)*log(sin(d*x + c) + 1)*sin(d*x + c) + 33*(a^3*cos(d*x + c)^3 - a^3*cos
(d*x + c)^2)*log(-sin(d*x + c) + 1)*sin(d*x + c))/((d*cos(d*x + c)^3 - d*co
s(d*x + c)^2)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{3}\left(\int 3\csc^{4}\left(c+dx\right)\sec\left(c+dx\right)\,dx+\int 3\csc^{4}\left(c+dx\right)\sec^{2}\left(c+dx\right)\,dx+\int\csc^{4}\left(c+dx\right)\sec^{3}\left(c+dx\right)\,dx+\int\csc^{4}\left(c+dx\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(a+a*sec(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(3*csc(c + d*x)**4*sec(c + d*x), x) + Integral(3*csc(c + d*x)
**4*sec(c + d*x)**2, x) + Integral(csc(c + d*x)**4*sec(c + d*x)**3, x) + In
tegral(csc(c + d*x)**4, x))
```

Giac [A]
time = 0.53, size = 123, normalized size = 1.12

$$\frac{33\,a^{3}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-33\,a^{3}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{6\,\left(5\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-7\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{2}}-\frac{2\,\left(18\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a^{3}\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(33*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 33*a^3*log(abs(tan(1/2*d*x
 + 1/2*c) - 1)) - 6*(5*a^3*tan(1/2*d*x + 1/2*c)^3 - 7*a^3*tan(1/2*d*x + 1/2
*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2 - 2*(18*a^3*tan(1/2*d*x + 1/2*c)^2 + a^
3)/tan(1/2*d*x + 1/2*c)^3)/d
```

Mupad [B]
time = 5.35, size = 116, normalized size = 1.05

$$\frac{11\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{11\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-\frac{56\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}+\frac{16\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+\frac{a^3}{3}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^3/sin(c + d*x)^4,x)
```

```
[Out] (11*a^3*atanh(tan(c/2 + (d*x)/2)))/d - ((16*a^3*tan(c/2 + (d*x)/2)^2)/3 - (
56*a^3*tan(c/2 + (d*x)/2)^4)/3 + 11*a^3*tan(c/2 + (d*x)/2)^6 + a^3/3)/(d*(t
an(c/2 + (d*x)/2)^3 - 2*tan(c/2 + (d*x)/2)^5 + tan(c/2 + (d*x)/2)^7))
```

3.54 $\int \csc^6(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=165

$$\frac{13a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{152a^3\tan(c+dx)}{15d}+\frac{13a^3\sec(c+dx)\tan(c+dx)}{2d}-\frac{a^6\sec(c+dx)\tan(c+dx)}{5d(a-a\cos(c+dx))^3}-11$$

```
[Out] 13/2*a^3*arctanh(sin(d*x+c))/d+152/15*a^3*tan(d*x+c)/d+13/2*a^3*sec(d*x+c)*
tan(d*x+c)/d-1/5*a^6*sec(d*x+c)*tan(d*x+c)/d/(a-a*cos(d*x+c))^3-11/15*a^5*s
ec(d*x+c)*tan(d*x+c)/d/(a-a*cos(d*x+c))^2-76/15*a^6*sec(d*x+c)*tan(d*x+c)/d
/(a^3-a^3*cos(d*x+c))
```

Rubi [A]
time = 0.31, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3957, 2948, 2845, 3057, 2827, 3853, 3855, 3852, 8}

$$-\frac{a^6\tan(c+dx)\sec(c+dx)}{5d(a-a\cos(c+dx))^3}-\frac{11a^5\tan(c+dx)\sec(c+dx)}{15d(a-a\cos(c+dx))^2}+\frac{152a^3\tan(c+dx)}{15d}+\frac{13a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{13a^3\tan(c+dx)\sec(c+dx)}{2d}-\frac{76a^6\tan(c+dx)\sec(c+dx)}{15d\,(a^3-a^3\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (13*a^3*ArcTanh[Sin[c + d*x]])/(2*d) + (152*a^3*Tan[c + d*x])/(15*d) + (13*
a^3*Sec[c + d*x]*Tan[c + d*x])/(2*d) - (a^6*Sec[c + d*x]*Tan[c + d*x])/(5*d
*(a - a*Cos[c + d*x])^3) - (11*a^5*Sec[c + d*x]*Tan[c + d*x])/(15*d*(a - a*
Cos[c + d*x])^2) - (76*a^6*Sec[c + d*x]*Tan[c + d*x])/(15*d*(a^3 - a^3*Cos[
c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
```

```
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2948

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc^6(c+dx)(a+a\sec(c+dx))^3\,dx = -\int(-a-a\cos(c+dx))^3\csc^6(c+dx)\sec^3(c+dx)\,dx$$
$$= -\left(a^6\int\frac{\sec^3(c+dx)}{(-a+a\cos(c+dx))^3}\,dx\right)$$
$$= -\frac{a^6\sec(c+dx)\tan(c+dx)}{5d(a-a\cos(c+dx))^3} - \frac{1}{5}a^4\int\frac{(-7a-4a\cos(c+dx))\sec^3(}{(-a+a\cos(c+dx))^2}$$
$$= -\frac{a^6\sec(c+dx)\tan(c+dx)}{5d(a-a\cos(c+dx))^3} - \frac{11a^5\sec(c+dx)\tan(c+dx)}{15d(a-a\cos(c+dx))^2} - \frac{1}{15}a$$
$$= -\frac{a^6\sec(c+dx)\tan(c+dx)}{5d(a-a\cos(c+dx))^3} - \frac{11a^5\sec(c+dx)\tan(c+dx)}{15d(a-a\cos(c+dx))^2} - \frac{76a^4}{1}$$
$$= -\frac{a^6\sec(c+dx)\tan(c+dx)}{5d(a-a\cos(c+dx))^3} - \frac{11a^5\sec(c+dx)\tan(c+dx)}{15d(a-a\cos(c+dx))^2} - \frac{76a^4}{1}$$
$$= \frac{13a^3\sec(c+dx)\tan(c+dx)}{2d} - \frac{a^6\sec(c+dx)\tan(c+dx)}{5d(a-a\cos(c+dx))^3} - \frac{11a^5 s}{15d}$$
$$= \frac{13a^3\tanh^{-1}(\sin(c+dx))}{2d} + \frac{152a^3\tan(c+dx)}{15d} + \frac{13a^3\sec(c+dx)\,t}{2d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 353 vs. 2(165) = 330.
time = 0.80, size = 353, normalized size = 2.14

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/30720*(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*Sec[c + d*x]^2*(24960
*Cos[c + d*x]^2*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*
x)/2] + Sin[(c + d*x)/2]]) + Csc[c/2]*Csc[(c + d*x)/2]^5*Sec[c]*(-1235*Sin[
(d*x)/2] + 3805*Sin[(3*d*x)/2] + 4329*Sin[c - (d*x)/2] - 1989*Sin[c + (d*x)
/2] - 3575*Sin[2*c + (d*x)/2] + 475*Sin[c + (3*d*x)/2] + 2005*Sin[2*c + (3*
d*x)/2] + 2275*Sin[3*c + (3*d*x)/2] - 2673*Sin[c + (5*d*x)/2] + 105*Sin[2*c
 + (5*d*x)/2] - 1593*Sin[3*c + (5*d*x)/2] - 975*Sin[4*c + (5*d*x)/2] + 1325
*Sin[2*c + (7*d*x)/2] - 255*Sin[3*c + (7*d*x)/2] + 875*Sin[4*c + (7*d*x)/2]
 + 195*Sin[5*c + (7*d*x)/2] - 304*Sin[3*c + (9*d*x)/2] + 90*Sin[4*c + (9*d*
x)/2] - 214*Sin[5*c + (9*d*x)/2])))/d
```

Maple [A]
time = 0.12, size = 241, normalized size = 1.46

method	result
norman	$\frac{-\frac{a^3}{20d}-\frac{17a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{30d}-\frac{97a^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}+\frac{131a^3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}-\frac{51a^3\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{13a^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{2d}$
risch	$-\frac{ia^3\left(195\,e^{8i(dx+c)}-975\,e^{7i(dx+c)}+2275\,e^{6i(dx+c)}-3575\,e^{5i(dx+c)}+4329\,e^{4i(dx+c)}-3805\,e^{3i(dx+c)}+2673\,e^{2i(dx+c)}-1\right.}{15d\left(e^{i(dx+c)}-1\right)^5\left(e^{2i(dx+c)}+1\right)^2}$
derivativedivides	$\frac{a^3\left(-\frac{1}{5\sin(dx+c)^5\cos(dx+c)^2}-\frac{7}{15\sin(dx+c)^3\cos(dx+c)^2}+\frac{7}{6\sin(dx+c)\cos(dx+c)^2}-\frac{7}{2\sin(dx+c)}+\frac{7\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+}{}$
default	$\frac{a^3\left(-\frac{1}{5\sin(dx+c)^5\cos(dx+c)^2}-\frac{7}{15\sin(dx+c)^3\cos(dx+c)^2}+\frac{7}{6\sin(dx+c)\cos(dx+c)^2}-\frac{7}{2\sin(dx+c)}+\frac{7\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/5/sin(d*x+c)^5/cos(d*x+c)^2-7/15/sin(d*x+c)^3/cos(d*x+c)^2+7/6
/sin(d*x+c)/cos(d*x+c)^2-7/2/sin(d*x+c)+7/2*ln(sec(d*x+c)+tan(d*x+c)))+3*a^
3*(-1/5/sin(d*x+c)^5/cos(d*x+c)-2/5/sin(d*x+c)^3/cos(d*x+c)+8/5/sin(d*x+c)/
cos(d*x+c)-16/5*cot(d*x+c))+3*a^3*(-1/5/sin(d*x+c)^5-1/3/sin(d*x+c)^3-1/sin
(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+a^3*(-8/15-1/5*csc(d*x+c)^4-4/15*csc(d*x
+c)^2)*cot(d*x+c))
```

Maxima [A]
time = 0.28, size = 228, normalized size = 1.38

$$-\frac{a^3\left(\frac{2\left(105\,\sin(dx+c)^6-70\,\sin(dx+c)^4-14\,\sin(dx+c)^2-6\right)}{\sin(dx+c)^7-\sin(dx+c)^5}-105\,\log\left(\sin\left(dx+c\right)+1\right)+105\,\log\left(\sin\left(dx+c\right)-1\right)\right)+6\,a^3\left(\frac{2\left(15\,\sin(dx+c)^4+5\,\sin(dx+c)^2+3\right)}{\sin(dx+c)^5}-15\,\log\left(\sin\left(dx+c\right)+1\right)+15\,\log\left(\sin\left(dx+c\right)-1\right)\right)+36\,a^3\left(\frac{15\,\tan(dx+c)^4+5\,\tan(dx+c)^2+1}{\tan(dx+c)^5}-5\,\tan\left(dx+c\right)\right)+\frac{4\left(15\,\tan(dx+c)^4+10\,\tan(dx+c)^2+3\right)a^3}{\tan(dx+c)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/60*(a^3*(2*(105*sin(d*x + c)^6 - 70*sin(d*x + c)^4 - 14*sin(d*x + c)^2 -
 6)/(sin(d*x + c)^7 - sin(d*x + c)^5) - 105*log(sin(d*x + c) + 1) + 105*log
(sin(d*x + c) - 1)) + 6*a^3*(2*(15*sin(d*x + c)^4 + 5*sin(d*x + c)^2 + 3)/s
in(d*x + c)^5 - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) + 36*a
^3*((15*tan(d*x + c)^4 + 5*tan(d*x + c)^2 + 1)/tan(d*x + c)^5 - 5*tan(d*x +
 c)) + 4*(15*tan(d*x + c)^4 + 10*tan(d*x + c)^2 + 3)*a^3/tan(d*x + c)^5)/d
```

Fricas [A]
time = 2.76, size = 225, normalized size = 1.36

$$-\frac{608\,a^3\cos\left(dx+c\right)^5-826\,a^3\cos\left(dx+c\right)^4-476\,a^3\cos\left(dx+c\right)^3+868\,a^3\cos\left(dx+c\right)^2-120\,a^3\cos\left(dx+c\right)-30\,a^3-195\left(a^3\cos\left(dx+c\right)^4-2\,a^3\cos\left(dx+c\right)^3+a^3\cos\left(dx+c\right)^2\right)\log\left(\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)+195\left(a^3\cos\left(dx+c\right)^4-2\,a^3\cos\left(dx+c\right)^3+a^3\cos\left(dx+c\right)^2\right)\log\left(-\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)}{60\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^3+d\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/60*(608*a^3*cos(d*x + c)^5 - 826*a^3*cos(d*x + c)^4 - 476*a^3*cos(d*x +
c)^3 + 868*a^3*cos(d*x + c)^2 - 120*a^3*cos(d*x + c) - 30*a^3 - 195*(a^3*co
s(d*x + c)^4 - 2*a^3*cos(d*x + c)^3 + a^3*cos(d*x + c)^2)*log(sin(d*x + c)
+ 1)*sin(d*x + c) + 195*(a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x + c)^3 + a^3*co
s(d*x + c)^2)*log(-sin(d*x + c) + 1)*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d
*cos(d*x + c)^3 + d*cos(d*x + c)^2)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [A]
time = 0.55, size = 141, normalized size = 0.85

$$\frac{390\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-390\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{60\,\left(5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-7\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}-\frac{465\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+40\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(390*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 390*a^3*log(abs(tan(1/2*
d*x + 1/2*c) - 1)) - 60*(5*a^3*tan(1/2*d*x + 1/2*c)^3 - 7*a^3*tan(1/2*d*x +
 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2 - (465*a^3*tan(1/2*d*x + 1/2*c)^4 +
 40*a^3*tan(1/2*d*x + 1/2*c)^2 + 3*a^3)/tan(1/2*d*x + 1/2*c)^5)/d
```

Mupad [B]
time = 4.91, size = 136, normalized size = 0.82

$$\frac{13\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{51\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-\frac{262\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{3}+\frac{388\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{15}+\frac{34\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{15}+\frac{a^3}{5}}{d\,\left(4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9-8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^3/sin(c + d*x)^6,x)
```

```
[Out] (13*a^3*atanh(tan(c/2 + (d*x)/2)))/d - ((34*a^3*tan(c/2 + (d*x)/2)^2)/15 +
(388*a^3*tan(c/2 + (d*x)/2)^4)/15 - (262*a^3*tan(c/2 + (d*x)/2)^6)/3 + 51*a
^3*tan(c/2 + (d*x)/2)^8 + a^3/5)/(d*(4*tan(c/2 + (d*x)/2)^5 - 8*tan(c/2 + (
d*x)/2)^7 + 4*tan(c/2 + (d*x)/2)^9))
```

3.55 $\int \csc^8(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=192

$$\frac{15a^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{13a^3\cot(c+dx)}{d}-\frac{7a^3\cot^3(c+dx)}{d}-\frac{3a^3\cot^5(c+dx)}{d}-\frac{4a^3\cot^7(c+dx)}{7d}-\frac{15a^3 c}{}$$

```
[Out] 15/2*a^3*arctanh(sin(d*x+c))/d-13*a^3*cot(d*x+c)/d-7*a^3*cot(d*x+c)^3/d-3*a
^3*cot(d*x+c)^5/d-4/7*a^3*cot(d*x+c)^7/d-15/2*a^3*csc(d*x+c)/d-5/2*a^3*csc(
d*x+c)^3/d-3/2*a^3*csc(d*x+c)^5/d-15/14*a^3*csc(d*x+c)^7/d+1/2*a^3*csc(d*x+
c)^7*sec(d*x+c)^2/d+3*a^3*tan(d*x+c)/d
```

Rubi [A]
time = 0.23, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3957, 2952, 3852, 2701, 308, 213, 2700, 276, 294}

$$\frac{3a^3\tan(c+dx)}{d}-\frac{4a^3\cot^7(c+dx)}{7d}-\frac{3a^3\cot^5(c+dx)}{d}-\frac{7a^3\cot^3(c+dx)}{d}-\frac{13a^3\cot(c+dx)}{d}-\frac{15a^3\csc^7(c+dx)}{14d}-\frac{3a^3\csc^5(c+dx)}{2d}-\frac{5a^3\csc^3(c+dx)}{2d}-\frac{15a^3\csc(c+dx)}{2d}+\frac{15a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^3\csc^7(c+dx)\sec^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^8*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (15*a^3*ArcTanh[Sin[c + d*x]])/(2*d) - (13*a^3*Cot[c + d*x])/d - (7*a^3*Cot
[c + d*x]^3)/d - (3*a^3*Cot[c + d*x]^5)/d - (4*a^3*Cot[c + d*x]^7)/(7*d) -
(15*a^3*Csc[c + d*x])/(2*d) - (5*a^3*Csc[c + d*x]^3)/(2*d) - (3*a^3*Csc[c +
 d*x]^5)/(2*d) - (15*a^3*Csc[c + d*x]^7)/(14*d) + (a^3*Csc[c + d*x]^7*Sec[c
 + d*x]^2)/(2*d) + (3*a^3*Tan[c + d*x])/d
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc^8(c+dx)(a+a\sec(c+dx))^3\,dx = -\int (-a-a\cos(c+dx))^3\csc^8(c+dx)\sec^3(c+dx)\,dx$$

$$= \int \left(a^3\csc^8(c+dx)+3a^3\csc^8(c+dx)\sec(c+dx)+3a^3\csc^8(c+d\right.$$

$$= a^3\int \csc^8(c+dx)\,dx + a^3\int \csc^8(c+dx)\sec^3(c+dx)\,dx + \left(3a^3\right)$$

$$= -\frac{a^3\text{Subst}\left(\int \frac{x^{10}}{(-1+x^2)^2}\,dx, x, \csc(c+dx)\right)}{d} - \frac{a^3\text{Subst}\left(\int (1+3x^2+\right.}{}$$

$$= -\frac{a^3\cot(c+dx)}{d} - \frac{a^3\cot^3(c+dx)}{d} - \frac{3a^3\cot^5(c+dx)}{5d} - \frac{a^3\cot^7(c}{7d}$$

$$= -\frac{13a^3\cot(c+dx)}{d} - \frac{7a^3\cot^3(c+dx)}{d} - \frac{3a^3\cot^5(c+dx)}{d} - \frac{4a^3\,c}{}$$

$$= \frac{3a^3\tanh^{-1}(\sin(c+dx))}{d} - \frac{13a^3\cot(c+dx)}{d} - \frac{7a^3\cot^3(c+dx)}{d} -$$

$$= \frac{15a^3\tanh^{-1}(\sin(c+dx))}{2d} - \frac{13a^3\cot(c+dx)}{d} - \frac{7a^3\cot^3(c+dx)}{d} -$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 430 vs. $2(192) = 384$.
time = 0.81, size = 430, normalized size = 2.24

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^8*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*Cos[c + d*x]*Sec[(c + d*x)/2]^6*(1 + Sec[c + d*x])^3*(-860160*Cos[c +
d*x]^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 860160*Cos[c + d*x]^2*Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 8*Csc[2*c]*Csc[(c + d*x)/2]^6*Csc[c
 + d*x]*(5264*Sin[2*c] - 9580*Sin[d*x] + 8480*Sin[2*d*x] + 2776*Sin[c - d*x
] - 6080*Sin[c + d*x] + 8816*Sin[2*(c + d*x)] - 7904*Sin[3*(c + d*x)] + 486
4*Sin[4*(c + d*x)] - 1824*Sin[5*(c + d*x)] + 304*Sin[6*(c + d*x)] - 9580*Si
n[2*c + d*x] - 10024*Sin[3*c + d*x] + 13891*Sin[c + 2*d*x] + 7720*Sin[2*(c
+ 2*d*x)] + 13891*Sin[3*c + 2*d*x] + 10080*Sin[4*c + 2*d*x] - 10060*Sin[c +
 3*d*x] - 12454*Sin[2*c + 3*d*x] - 12454*Sin[4*c + 3*d*x] - 6580*Sin[5*c +
3*d*x] + 7664*Sin[3*c + 4*d*x] + 7664*Sin[5*c + 4*d*x] + 2520*Sin[6*c + 4*d
*x] - 3420*Sin[3*c + 5*d*x] - 2874*Sin[4*c + 5*d*x] - 2874*Sin[6*c + 5*d*x]
 - 420*Sin[7*c + 5*d*x] + 640*Sin[4*c + 6*d*x] + 479*Sin[5*c + 6*d*x] + 479
*Sin[7*c + 6*d*x])))/(917504*d)
```

Maple [A]

time = 0.13, size = 297, normalized size = 1.55

method	result
norman	$\frac{-\frac{a^3}{112d}-\frac{3a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{28d}-\frac{85a^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{112d}-\frac{15a^3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{395a^3\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16d}-\frac{57a^3\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{a^3(}{}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}$
risch	$-\frac{ia^3\left(105\,e^{11i(dx+c)}-630\,e^{10i(dx+c)}+1645\,e^{9i(dx+c)}-2520\,e^{8i(dx+c)}+2506\,e^{7i(dx+c)}-1316\,e^{6i(dx+c)}-694\,e^{5i(dx+c)}+21\right.}{7d\left(e^{i(dx+c)}-1\right)^7\left(e^{i(dx+c)}+1\right)\left(e^{2i(dx+c)}+1\right)^2}$
derivativedivides	$\frac{a^3\left(-\frac{1}{7\sin(dx+c)^7\cos(dx+c)^2}-\frac{9}{35\sin(dx+c)^5\cos(dx+c)^2}-\frac{3}{5\sin(dx+c)^3\cos(dx+c)^2}+\frac{3}{2\sin(dx+c)\cos(dx+c)^2}-\frac{9}{2\sin(dx+c)}+\frac{9\ln(s}{}\right.}{}$
default	$\frac{a^3\left(-\frac{1}{7\sin(dx+c)^7\cos(dx+c)^2}-\frac{9}{35\sin(dx+c)^5\cos(dx+c)^2}-\frac{3}{5\sin(dx+c)^3\cos(dx+c)^2}+\frac{3}{2\sin(dx+c)\cos(dx+c)^2}-\frac{9}{2\sin(dx+c)}+\frac{9\ln(s}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^8*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/7/sin(d*x+c)^7/cos(d*x+c)^2-9/35/sin(d*x+c)^5/cos(d*x+c)^2-3/5
/sin(d*x+c)^3/cos(d*x+c)^2+3/2/sin(d*x+c)/cos(d*x+c)^2-9/2/sin(d*x+c)+9/2*l
n(sec(d*x+c)+tan(d*x+c)))+3*a^3*(-1/7/sin(d*x+c)^7/cos(d*x+c)-8/35/sin(d*x+
c)^5/cos(d*x+c)-16/35/sin(d*x+c)^3/cos(d*x+c)+64/35/sin(d*x+c)/cos(d*x+c)-1
28/35*cot(d*x+c))+3*a^3*(-1/7/sin(d*x+c)^7-1/5/sin(d*x+c)^5-1/3/sin(d*x+c)^
3-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+a^3*(-16/35-1/7*csc(d*x+c)^6-6/35
*csc(d*x+c)^4-8/35*csc(d*x+c)^2)*cot(d*x+c))
```

Maxima [A]

time = 0.28, size = 268, normalized size = 1.40

$$-\frac{a^3\left(\frac{2\left(315\sin(dx+c)^8-210\sin(dx+c)^6-42\sin(dx+c)^4-18\sin(dx+c)^2-10\right)}{\sin(dx+c)^9-\sin(dx+c)^7}-315\log(\sin(dx+c)+1)+315\log(\sin(dx+c)-1)\right)+2a^3\left(\frac{2\left(105\sin(dx+c)^6+35\sin(dx+c)^4+21\sin(dx+c)^2+15\right)}{\sin(dx+c)^7}-105\log(\sin(dx+c)+1)+105\log(\sin(dx+c)-1)\right)+12a^3\left(\frac{140\tan(dx+c)^6+70\tan(dx+c)^4+28\tan(dx+c)^2+5}{\tan(dx+c)^7}-35\tan(dx+c)\right)+\frac{4\left(35\tan(dx+c)^6+35\tan(dx+c)^4+21\tan(dx+c)^2+5\right)a^3}{\tan(dx+c)^7}}{140d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/140*(a^3*(2*(315*sin(d*x + c)^8 - 210*sin(d*x + c)^6 - 42*sin(d*x + c)^4
 - 18*sin(d*x + c)^2 - 10)/(sin(d*x + c)^9 - sin(d*x + c)^7) - 315*log(sin(
d*x + c) + 1) + 315*log(sin(d*x + c) - 1)) + 2*a^3*(2*(105*sin(d*x + c)^6 +
 35*sin(d*x + c)^4 + 21*sin(d*x + c)^2 + 15)/sin(d*x + c)^7 - 105*log(sin(d
*x + c) + 1) + 105*log(sin(d*x + c) - 1)) + 12*a^3*((140*tan(d*x + c)^6 + 7
0*tan(d*x + c)^4 + 28*tan(d*x + c)^2 + 5)/tan(d*x + c)^7 - 35*tan(d*x + c))
 + 4*(35*tan(d*x + c)^6 + 35*tan(d*x + c)^4 + 21*tan(d*x + c)^2 + 5)*a^3/ta
n(d*x + c)^7)/d
```

Fricas [A]

time = 3.58, size = 278, normalized size = 1.45

$$-\frac{320\,a^3\cos(dx+c)^6-750\,a^3\cos(dx+c)^5+170\,a^3\cos(dx+c)^4+720\,a^3\cos(dx+c)^3-520\,a^3\cos(dx+c)^2+42\,a^3\cos(dx+c)+14\,a^3-105\left(a^3\cos(dx+c)^5-3\,a^3\cos(dx+c)^4+3\,a^3\cos(dx+c)^3-a^3\cos(dx+c)^2\right)\log(\sin(dx+c)+1)\sin(dx+c)+105\left(a^3\cos(dx+c)^5-3\,a^3\cos(dx+c)^4+3\,a^3\cos(dx+c)^3-a^3\cos(dx+c)^2\right)\log(-\sin(dx+c)+1)\sin(dx+c)}{28\left(d\cos(dx+c)^5-3\,d\cos(dx+c)^4+3\,d\cos(dx+c)^3-d\cos(dx+c)^2\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/28*(320*a^3*cos(d*x + c)^6 - 750*a^3*cos(d*x + c)^5 + 170*a^3*cos(d*x +
c)^4 + 720*a^3*cos(d*x + c)^3 - 520*a^3*cos(d*x + c)^2 + 42*a^3*cos(d*x + c
) + 14*a^3 - 105*(a^3*cos(d*x + c)^5 - 3*a^3*cos(d*x + c)^4 + 3*a^3*cos(d*x
 + c)^3 - a^3*cos(d*x + c)^2)*log(sin(d*x + c) + 1)*sin(d*x + c) + 105*(a^3
*cos(d*x + c)^5 - 3*a^3*cos(d*x + c)^4 + 3*a^3*cos(d*x + c)^3 - a^3*cos(d*x
 + c)^2)*log(-sin(d*x + c) + 1)*sin(d*x + c))/((d*cos(d*x + c)^5 - 3*d*cos(
d*x + c)^4 + 3*d*cos(d*x + c)^3 - d*cos(d*x + c)^2)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**8*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [A]
time = 0.56, size = 169, normalized size = 0.88

$$\frac{840\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-840\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-7\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{112\left(5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-7\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}-\frac{1050\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+112\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+14\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7}}{112\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/112*(840*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 840*a^3*log(abs(tan(1/2
*d*x + 1/2*c) - 1)) - 7*a^3*tan(1/2*d*x + 1/2*c) - 112*(5*a^3*tan(1/2*d*x +
 1/2*c)^3 - 7*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2 - (1
050*a^3*tan(1/2*d*x + 1/2*c)^6 + 112*a^3*tan(1/2*d*x + 1/2*c)^4 + 14*a^3*ta
n(1/2*d*x + 1/2*c)^2 + a^3)/tan(1/2*d*x + 1/2*c)^7)/d
```

Mupad [B]
time = 2.90, size = 169, normalized size = 0.88

$$\frac{15\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{230\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-396\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+120\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\frac{85\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{7}+\frac{12\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{7}+\frac{a^3}{7}}{d\left(16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}-32\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\right)}-\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^3/sin(c + d*x)^8,x)
```

```
[Out] (15*a^3*atanh(tan(c/2 + (d*x)/2)))/d - ((12*a^3*tan(c/2 + (d*x)/2)^2)/7 + (
85*a^3*tan(c/2 + (d*x)/2)^4)/7 + 120*a^3*tan(c/2 + (d*x)/2)^6 - 396*a^3*tan
(c/2 + (d*x)/2)^8 + 230*a^3*tan(c/2 + (d*x)/2)^10 + a^3/7)/(d*(16*tan(c/2 +
 (d*x)/2)^7 - 32*tan(c/2 + (d*x)/2)^9 + 16*tan(c/2 + (d*x)/2)^11)) - (a^3*t
an(c/2 + (d*x)/2))/(16*d)
```

3.56 $\int \csc^{10}(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=232

$$\frac{17a^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{16a^3\cot(c+dx)}{d}-\frac{34a^3\cot^3(c+dx)}{3d}-\frac{36a^3\cot^5(c+dx)}{5d}-\frac{19a^3\cot^7(c+dx)}{7d}-\frac{4a^3}{}$$

```
[Out] 17/2*a^3*arctanh(sin(d*x+c))/d-16*a^3*cot(d*x+c)/d-34/3*a^3*cot(d*x+c)^3/d-
36/5*a^3*cot(d*x+c)^5/d-19/7*a^3*cot(d*x+c)^7/d-4/9*a^3*cot(d*x+c)^9/d-17/2
*a^3*csc(d*x+c)/d-17/6*a^3*csc(d*x+c)^3/d-17/10*a^3*csc(d*x+c)^5/d-17/14*a^
3*csc(d*x+c)^7/d-17/18*a^3*csc(d*x+c)^9/d+1/2*a^3*csc(d*x+c)^9*sec(d*x+c)^2
/d+3*a^3*tan(d*x+c)/d
```

Rubi [A]

time = 0.24, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3957, 2952, 3852, 2701, 308, 213, 2700, 276, 294}

$$\frac{3a^3\tan(c+dx)}{d}-\frac{4a^3\cot^9(c+dx)}{9d}-\frac{19a^3\cot^7(c+dx)}{7d}-\frac{36a^3\cot^5(c+dx)}{5d}-\frac{34a^3\cot^3(c+dx)}{3d}-\frac{16a^3\cot(c+dx)}{d}-\frac{17a^3\csc^9(c+dx)}{18d}-\frac{17a^3\csc^7(c+dx)}{14d}-\frac{17a^3\csc^5(c+dx)}{10d}-\frac{17a^3\csc^3(c+dx)}{6d}-\frac{17a^3\csc(c+dx)}{2d}+\frac{17a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^3\csc^9(c+dx)\sec^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^10*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (17*a^3*ArcTanh[Sin[c + d*x]])/(2*d) - (16*a^3*Cot[c + d*x])/d - (34*a^3*Co
t[c + d*x]^3)/(3*d) - (36*a^3*Cot[c + d*x]^5)/(5*d) - (19*a^3*Cot[c + d*x]^
7)/(7*d) - (4*a^3*Cot[c + d*x]^9)/(9*d) - (17*a^3*Csc[c + d*x])/(2*d) - (17
*a^3*Csc[c + d*x]^3)/(6*d) - (17*a^3*Csc[c + d*x]^5)/(10*d) - (17*a^3*Csc[c
 + d*x]^7)/(14*d) - (17*a^3*Csc[c + d*x]^9)/(18*d) + (a^3*Csc[c + d*x]^9*Se
c[c + d*x]^2)/(2*d) + (3*a^3*Tan[c + d*x])/d
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc^{10}(c+dx)(a+a\sec(c+dx))^3\,dx = -\int (-a-a\cos(c+dx))^3\csc^{10}(c+dx)\sec^3(c+dx)\,dx$$
$$= \int \left(a^3\csc^{10}(c+dx)+3a^3\csc^{10}(c+dx)\sec(c+dx)+3a^3\csc^{10}(c+\right.$$
$$= a^3\int \csc^{10}(c+dx)\,dx + a^3\int \csc^{10}(c+dx)\sec^3(c+dx)\,dx + \left(3a^3\right)$$
$$= -\frac{a^3\text{Subst}\left(\int \frac{x^{12}}{(-1+x^2)^2}\,dx, x, \csc(c+dx)\right)}{d} - \frac{a^3\text{Subst}\left(\int (1+4x^2+\right.}{}$$
$$= -\frac{a^3\cot(c+dx)}{d} - \frac{4a^3\cot^3(c+dx)}{3d} - \frac{6a^3\cot^5(c+dx)}{5d} - \frac{4a^3\cot^7}{7}$$
$$= -\frac{16a^3\cot(c+dx)}{d} - \frac{34a^3\cot^3(c+dx)}{3d} - \frac{36a^3\cot^5(c+dx)}{5d} - \frac{19a}{}$$
$$= \frac{3a^3\tanh^{-1}(\sin(c+dx))}{d} - \frac{16a^3\cot(c+dx)}{d} - \frac{34a^3\cot^3(c+dx)}{3d} -$$
$$= \frac{17a^3\tanh^{-1}(\sin(c+dx))}{2d} - \frac{16a^3\cot(c+dx)}{d} - \frac{34a^3\cot^3(c+dx)}{3d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1000 vs. $2(232) = 464$.
time = 6.45, size = 1000, normalized size = 4.31

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^10*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-9833*Cos[c + d*x]^3*Cot[c/2]*Csc[c/2 + (d*x)/2]^2*Sec[c/2 + (d*x)/2]^6*(a
 + a*Sec[c + d*x])^3)/(80640*d) - (979*Cos[c + d*x]^3*Cot[c/2]*Csc[c/2 + (d
*x)/2]^4*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3)/(53760*d) - (5*Cos[c
+ d*x]^3*Cot[c/2]*Csc[c/2 + (d*x)/2]^6*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c +
d*x])^3)/(2016*d) - (Cos[c + d*x]^3*Cot[c/2]*Csc[c/2 + (d*x)/2]^8*Sec[c/2 +
 (d*x)/2]^6*(a + a*Sec[c + d*x])^3)/(4608*d) - (17*Cos[c + d*x]^3*Log[Cos[c
/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x
])^3)/(16*d) + (17*Cos[c + d*x]^3*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/
2]]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3)/(16*d) + (197147*Cos[c + d
*x]^3*Csc[c/2]*Csc[c/2 + (d*x)/2]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])
^3*Sin[(d*x)/2])/(161280*d) + (9833*Cos[c + d*x]^3*Csc[c/2]*Csc[c/2 + (d*x)
/2]^3*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*Sin[(d*x)/2])/(80640*d) +
 (979*Cos[c + d*x]^3*Csc[c/2]*Csc[c/2 + (d*x)/2]^5*Sec[c/2 + (d*x)/2]^6*(a
+ a*Sec[c + d*x])^3*Sin[(d*x)/2])/(53760*d) + (5*Cos[c + d*x]^3*Csc[c/2]*Cs
```

```
c[c/2 + (d*x)/2]^7*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*Sin[(d*x)/2]
)/(2016*d) + (Cos[c + d*x]^3*Csc[c/2]*Csc[c/2 + (d*x)/2]^9*Sec[c/2 + (d*x)/
2]^6*(a + a*Sec[c + d*x])^3*Sin[(d*x)/2])/(4608*d) - (35*Cos[c + d*x]^3*Sec
[c/2]*Sec[c/2 + (d*x)/2]^7*(a + a*Sec[c + d*x])^3*Sin[(d*x)/2])/(1536*d) -
(Cos[c + d*x]^3*Sec[c/2]*Sec[c/2 + (d*x)/2]^9*(a + a*Sec[c + d*x])^3*Sin[(d
*x)/2])/(1536*d) + (Cos[c + d*x]*Sec[c]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c +
 d*x])^3*Sin[d*x])/(16*d) + (Cos[c + d*x]^2*Sec[c]*Sec[c/2 + (d*x)/2]^6*(a
+ a*Sec[c + d*x])^3*(Sin[c] + 6*Sin[d*x]))/(16*d) - (Cos[c + d*x]^3*Sec[c/2
 + (d*x)/2]^8*(a + a*Sec[c + d*x])^3*Tan[c/2])/(1536*d)
```

Maple [A]
time = 0.14, size = 353, normalized size = 1.52

method	result
risch	$-\frac{i a^{3}\left(5355\,\mathrm{e}^{15 i (d x+c)}-32130\,\mathrm{e}^{14 i (d x+c)}+73185\,\mathrm{e}^{13 i (d x+c)}-64260\,\mathrm{e}^{12 i (d x+c)}-34629\,\mathrm{e}^{11 i (d x+c)}+157794\,\mathrm{e}^{10 i (d x+c)}-20\right.}{3}$
derivativedivides	$\frac{a^{3}\left(-\frac{1}{9 \sin(d x+c)^{9} \cos(d x+c)^{2}}-\frac{11}{63 \sin(d x+c)^{7} \cos(d x+c)^{2}}-\frac{11}{35 \sin(d x+c)^{5} \cos(d x+c)^{2}}-\frac{11}{15 \sin(d x+c)^{3} \cos(d x+c)^{2}}+\frac{11}{6 \sin(d x+c) \cos}\right.}{}$
default	$\frac{a^{3}\left(-\frac{1}{9 \sin(d x+c)^{9} \cos(d x+c)^{2}}-\frac{11}{63 \sin(d x+c)^{7} \cos(d x+c)^{2}}-\frac{11}{35 \sin(d x+c)^{5} \cos(d x+c)^{2}}-\frac{11}{15 \sin(d x+c)^{3} \cos(d x+c)^{2}}+\frac{11}{6 \sin(d x+c) \cos}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^10*(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/9/sin(d*x+c)^9/cos(d*x+c)^2-11/63/sin(d*x+c)^7/cos(d*x+c)^2-11
/35/sin(d*x+c)^5/cos(d*x+c)^2-11/15/sin(d*x+c)^3/cos(d*x+c)^2+11/6/sin(d*x+
c)/cos(d*x+c)^2-11/2/sin(d*x+c)+11/2*ln(sec(d*x+c)+tan(d*x+c)))+3*a^3*(-1/9
/sin(d*x+c)^9/cos(d*x+c)-10/63/sin(d*x+c)^7/cos(d*x+c)-16/63/sin(d*x+c)^5/c
os(d*x+c)-32/63/sin(d*x+c)^3/cos(d*x+c)+128/63/sin(d*x+c)/cos(d*x+c)-256/63
*cot(d*x+c))+3*a^3*(-1/9/sin(d*x+c)^9-1/7/sin(d*x+c)^7-1/5/sin(d*x+c)^5-1/3
/sin(d*x+c)^3-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+a^3*(-128/315-1/9*csc
(d*x+c)^8-8/63*csc(d*x+c)^6-16/105*csc(d*x+c)^4-64/315*csc(d*x+c)^2)*cot(d*
x+c))
```

Maxima [A]
time = 0.28, size = 308, normalized size = 1.33

$$-\frac{a^{3}\left(\frac{2\left(3465 \sin(dx+c)^{10}-2310 \sin(dx+c)^{8}-462 \sin(dx+c)^{6}-198 \sin(dx+c)^{4}-110 \sin(dx+c)^{2}-70\right)}{\sin(dx+c)^{11}-\sin(dx+c)^{9}}-3465 \log\left(\sin\left(dx+c\right)+1\right)+3465 \log\left(\sin\left(dx+c\right)-1\right)\right)+6 a^{3}\left(\frac{2\left(315 \sin(dx+c)^{8}+105 \sin(dx+c)^{6}+63 \sin(dx+c)^{4}+45 \sin(dx+c)^{2}+35\right)}{\sin(dx+c)^{9}}-315 \log\left(\sin\left(dx+c\right)+1\right)+315 \log\left(\sin\left(dx+c\right)-1\right)\right)+60 a^{3}\left(\frac{315 \tan(dx+c)^{8}+210 \tan(dx+c)^{6}+126 \tan(dx+c)^{4}+45 \tan(dx+c)^{2}+7}{\tan(dx+c)^{9}}-63 \tan\left(dx+c\right)\right)+\frac{4\left(315 \tan(dx+c)^{8}+420 \tan(dx+c)^{6}+378 \tan(dx+c)^{4}+180 \tan(dx+c)^{2}+35\right)a^{3}}{\tan(dx+c)^{9}}}{1260\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^10*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/1260*(a^3*(2*(3465*sin(d*x + c)^10 - 2310*sin(d*x + c)^8 - 462*sin(d*x +
 c)^6 - 198*sin(d*x + c)^4 - 110*sin(d*x + c)^2 - 70)/(sin(d*x + c)^11 - si
```

```
n(d*x + c)^9) - 3465*log(sin(d*x + c) + 1) + 3465*log(sin(d*x + c) - 1)) +
6*a^3*(2*(315*sin(d*x + c)^8 + 105*sin(d*x + c)^6 + 63*sin(d*x + c)^4 + 45*
sin(d*x + c)^2 + 35)/sin(d*x + c)^9 - 315*log(sin(d*x + c) + 1) + 315*log(s
in(d*x + c) - 1)) + 60*a^3*((315*tan(d*x + c)^8 + 210*tan(d*x + c)^6 + 126*
tan(d*x + c)^4 + 45*tan(d*x + c)^2 + 7)/tan(d*x + c)^9 - 63*tan(d*x + c)) +
 4*(315*tan(d*x + c)^8 + 420*tan(d*x + c)^6 + 378*tan(d*x + c)^4 + 180*tan(
d*x + c)^2 + 35)*a^3/tan(d*x + c)^9)/d
```

Fricas [A]
time = 3.15, size = 375, normalized size = 1.62

$$-\frac{15872\,a^3\cos\left(dx+c\right)^8-36906\,a^3\cos\left(dx+c\right)^7-8322\,a^3\cos\left(dx+c\right)^6+73402\,a^3\cos\left(dx+c\right)^5-33342\,a^3\cos\left(dx+c\right)^4-34746\,a^3\cos\left(dx+c\right)^3+26702\,a^3\cos\left(dx+c\right)^2-1890\,a^3\cos\left(dx+c\right)-630\,a^3-5355\,\left(a^3\cos\left(dx+c\right)^7-3\,a^3\cos\left(dx+c\right)^6+2\,a^3\cos\left(dx+c\right)^5+2\,a^3\cos\left(dx+c\right)^4-3\,a^3\cos\left(dx+c\right)^3+a^3\cos\left(dx+c\right)^2\right)\log\left(\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)+5355\,\left(a^3\cos\left(dx+c\right)^7-3\,a^3\cos\left(dx+c\right)^6+2\,a^3\cos\left(dx+c\right)^5+2\,a^3\cos\left(dx+c\right)^4-3\,a^3\cos\left(dx+c\right)^3+a^3\cos\left(dx+c\right)^2\right)\log\left(-\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)}{1260\,\left(d\cos\left(dx+c\right)^7-3\,d\cos\left(dx+c\right)^6+2\,d\cos\left(dx+c\right)^5+2\,d\cos\left(dx+c\right)^4-3\,d\cos\left(dx+c\right)^3+d\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^10*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/1260*(15872*a^3*cos(d*x + c)^8 - 36906*a^3*cos(d*x + c)^7 - 8322*a^3*cos
(d*x + c)^6 + 73402*a^3*cos(d*x + c)^5 - 33342*a^3*cos(d*x + c)^4 - 34746*a
^3*cos(d*x + c)^3 + 26702*a^3*cos(d*x + c)^2 - 1890*a^3*cos(d*x + c) - 630*
a^3 - 5355*(a^3*cos(d*x + c)^7 - 3*a^3*cos(d*x + c)^6 + 2*a^3*cos(d*x + c)^
5 + 2*a^3*cos(d*x + c)^4 - 3*a^3*cos(d*x + c)^3 + a^3*cos(d*x + c)^2)*log(s
in(d*x + c) + 1)*sin(d*x + c) + 5355*(a^3*cos(d*x + c)^7 - 3*a^3*cos(d*x +
c)^6 + 2*a^3*cos(d*x + c)^5 + 2*a^3*cos(d*x + c)^4 - 3*a^3*cos(d*x + c)^3 +
 a^3*cos(d*x + c)^2)*log(-sin(d*x + c) + 1)*sin(d*x + c))/((d*cos(d*x + c)^
7 - 3*d*cos(d*x + c)^6 + 2*d*cos(d*x + c)^5 + 2*d*cos(d*x + c)^4 - 3*d*cos(
d*x + c)^3 + d*cos(d*x + c)^2)*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**10*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.57, size = 202, normalized size = 0.87

$$-\frac{105\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-171360\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)+171360\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+3780\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{20160\,\left(5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-7\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}+\frac{220185\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+26880\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+4347\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+540\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+35\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9}}{20160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^10*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/20160*(105*a^3*tan(1/2*d*x + 1/2*c)^3 - 171360*a^3*log(abs(tan(1/2*d*x +
 1/2*c) + 1)) + 171360*a^3*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 3780*a^3*ta
```

```
n(1/2*d*x + 1/2*c) + 20160*(5*a^3*tan(1/2*d*x + 1/2*c)^3 - 7*a^3*tan(1/2*d*
x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2 + (220185*a^3*tan(1/2*d*x + 1/2*
c)^8 + 26880*a^3*tan(1/2*d*x + 1/2*c)^6 + 4347*a^3*tan(1/2*d*x + 1/2*c)^4 +
 540*a^3*tan(1/2*d*x + 1/2*c)^2 + 35*a^3)/tan(1/2*d*x + 1/2*c)^9)/d
```

Mupad [B]
time = 1.04, size = 204, normalized size = 0.88

$$\frac{17\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{192\,d}-\frac{3\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,d}-\frac{1019\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-\frac{5282\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{3}+\frac{8132\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{15}+\frac{6242\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{105}+\frac{3302\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{315}+\frac{94\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{63}+\frac{a^3}{9}}{d\,\left(64\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}-128\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+64\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^3/sin(c + d*x)^10,x)
```

```
[Out] (17*a^3*atanh(tan(c/2 + (d*x)/2)))/d - (a^3*tan(c/2 + (d*x)/2)^3)/(192*d) -
 (3*a^3*tan(c/2 + (d*x)/2))/(16*d) - ((94*a^3*tan(c/2 + (d*x)/2)^2)/63 + (3
302*a^3*tan(c/2 + (d*x)/2)^4)/315 + (6242*a^3*tan(c/2 + (d*x)/2)^6)/105 + (
8132*a^3*tan(c/2 + (d*x)/2)^8)/15 - (5282*a^3*tan(c/2 + (d*x)/2)^10)/3 + 10
19*a^3*tan(c/2 + (d*x)/2)^12 + a^3/9)/(d*(64*tan(c/2 + (d*x)/2)^9 - 128*tan
(c/2 + (d*x)/2)^11 + 64*tan(c/2 + (d*x)/2)^13))
```

3.57 $\int \frac{\sin^9(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=91

$$\frac{\cos^3(c+dx)}{3ad}-\frac{3\cos^5(c+dx)}{5ad}+\frac{3\cos^7(c+dx)}{7ad}-\frac{\cos^9(c+dx)}{9ad}+\frac{\sin^8(c+dx)}{8ad}$$

```
[Out] 1/3*cos(d*x+c)^3/a/d-3/5*cos(d*x+c)^5/a/d+3/7*cos(d*x+c)^7/a/d-1/9*cos(d*x+
c)^9/a/d+1/8*sin(d*x+c)^8/a/d
```

Rubi [A]
time = 0.12, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3957, 2914, 2644, 30, 2645, 276}

$$\frac{\sin^8(c+dx)}{8ad}-\frac{\cos^9(c+dx)}{9ad}+\frac{3\cos^7(c+dx)}{7ad}-\frac{3\cos^5(c+dx)}{5ad}+\frac{\cos^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^9/(a + a*Sec[c + d*x]),x]
```

```
[Out] Cos[c + d*x]^3/(3*a*d) - (3*Cos[c + d*x]^5)/(5*a*d) + (3*Cos[c + d*x]^7)/(7
*a*d) - Cos[c + d*x]^9/(9*a*d) + Sin[c + d*x]^8/(8*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2914

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^9(c+dx)}{a+a\sec(c+dx)}\,dx &= -\int \frac{\cos(c+dx)\sin^9(c+dx)}{-a-a\cos(c+dx)}\,dx\\
&= \frac{\int \cos(c+dx)\sin^7(c+dx)\,dx}{a} - \frac{\int \cos^2(c+dx)\sin^7(c+dx)\,dx}{a}\\
&= \frac{\operatorname{Subst}\left(\int x^7\,dx, x, \sin(c+dx)\right)}{ad} + \frac{\operatorname{Subst}\left(\int x^2(1-x^2)^3\,dx, x, \cos(c+dx)\right)}{ad}\\
&= \frac{\sin^8(c+dx)}{8ad} + \frac{\operatorname{Subst}\left(\int (x^2-3x^4+3x^6-x^8)\,dx, x, \cos(c+dx)\right)}{ad}\\
&= \frac{\cos^3(c+dx)}{3ad} - \frac{3\cos^5(c+dx)}{5ad} + \frac{3\cos^7(c+dx)}{7ad} - \frac{\cos^9(c+dx)}{9ad} + \frac{\sin^8(c+dx)}{8ad}
\end{aligned}$$

Mathematica [A]
time = 2.66, size = 62, normalized size = 0.68

$$\frac{(4258+6995\cos(c+dx)+3650\cos(2(c+dx))+1085\cos(3(c+dx))+140\cos(4(c+dx)))\sin^{10}\left(\frac{1}{2}(c+dx)\right)}{315ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^9/(a + a*Sec[c + d*x]),x]
```

```
[Out] ((4258 + 6995*Cos[c + d*x] + 3650*Cos[2*(c + d*x)] + 1085*Cos[3*(c + d*x)]
+ 140*Cos[4*(c + d*x)])*Sin[(c + d*x)/2]^10)/(315*a*d)
```

Maple [A]
time = 0.15, size = 89, normalized size = 0.98

method	result
derivativedivides	$\frac{-\frac{1}{2\sec(dx+c)^6}-\frac{3}{5\sec(dx+c)^5}+\frac{1}{8\sec(dx+c)^8}+\frac{3}{4\sec(dx+c)^4}+\frac{3}{7\sec(dx+c)^7}-\frac{1}{9\sec(dx+c)^9}+\frac{1}{3\sec(dx+c)^3}-\frac{1}{2\sec(dx+c)^2}}{da}$
default	$\frac{-\frac{1}{2\sec(dx+c)^6}-\frac{3}{5\sec(dx+c)^5}+\frac{1}{8\sec(dx+c)^8}+\frac{3}{4\sec(dx+c)^4}+\frac{3}{7\sec(dx+c)^7}-\frac{1}{9\sec(dx+c)^9}+\frac{1}{3\sec(dx+c)^3}-\frac{1}{2\sec(dx+c)^2}}{da}$
norman	$\frac{\frac{32}{315ad}+\frac{64\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{32\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{35ad}+\frac{128\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{35ad}+\frac{128\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15ad}+\frac{64\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5ad}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9}$
risch	$\frac{7\cos(dx+c)}{128ad}-\frac{\cos(9dx+9c)}{2304ad}+\frac{\cos(8dx+8c)}{1024ad}+\frac{5\cos(7dx+7c)}{1792ad}-\frac{\cos(6dx+6c)}{128ad}-\frac{\cos(5dx+5c)}{160ad}+\frac{7\cos(4dx+4c)}{256ad}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^9/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-1/2/sec(d*x+c)^6-3/5/sec(d*x+c)^5+1/8/sec(d*x+c)^8+3/4/sec(d*x+c)^4
+3/7/sec(d*x+c)^7-1/9/sec(d*x+c)^9+1/3/sec(d*x+c)^3-1/2/sec(d*x+c)^2)
```

Maxima [A]
time = 0.27, size = 89, normalized size = 0.98

$$-\frac{280\cos(dx+c)^9-315\cos(dx+c)^8-1080\cos(dx+c)^7+1260\cos(dx+c)^6+1512\cos(dx+c)^5-1890\cos(dx+c)^4-840\cos(dx+c)^3+1260\cos(dx+c)^2}{2520\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^9/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2520*(280*cos(d*x + c)^9 - 315*cos(d*x + c)^8 - 1080*cos(d*x + c)^7 + 12
60*cos(d*x + c)^6 + 1512*cos(d*x + c)^5 - 1890*cos(d*x + c)^4 - 840*cos(d*x
 + c)^3 + 1260*cos(d*x + c)^2)/(a*d)
```

Fricas [A]
time = 4.18, size = 89, normalized size = 0.98

$$-\frac{280\cos(dx+c)^9-315\cos(dx+c)^8-1080\cos(dx+c)^7+1260\cos(dx+c)^6+1512\cos(dx+c)^5-1890\cos(dx+c)^4-840\cos(dx+c)^3+1260\cos(dx+c)^2}{2520\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^9/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2520*(280*cos(d*x + c)^9 - 315*cos(d*x + c)^8 - 1080*cos(d*x + c)^7 + 12
60*cos(d*x + c)^6 + 1512*cos(d*x + c)^5 - 1890*cos(d*x + c)^4 - 840*cos(d*x
 + c)^3 + 1260*cos(d*x + c)^2)/(a*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**9/(a+a*sec(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [A]
time = 0.43, size = 141, normalized size = 1.55

$$\frac{32\left(\frac{9\left(\cos\left(dx+c\right)-1\right)}{\cos\left(dx+c\right)+1}-\frac{36\left(\cos\left(dx+c\right)-1\right)^{2}}{\left(\cos\left(dx+c\right)+1\right)^{2}}+\frac{84\left(\cos\left(dx+c\right)-1\right)^{3}}{\left(\cos\left(dx+c\right)+1\right)^{3}}-\frac{126\left(\cos\left(dx+c\right)-1\right)^{4}}{\left(\cos\left(dx+c\right)+1\right)^{4}}+\frac{630\left(\cos\left(dx+c\right)-1\right)^{5}}{\left(\cos\left(dx+c\right)+1\right)^{5}}-1\right)}{315\,ad\left(\frac{\cos\left(dx+c\right)-1}{\cos\left(dx+c\right)+1}-1\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^9/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 32/315*(9*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 36*(cos(d*x + c) - 1)^2/(
cos(d*x + c) + 1)^2 + 84*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 126*(c
os(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 630*(cos(d*x + c) - 1)^5/(cos(d*x
 + c) + 1)^5 - 1)/(a*d*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^9)
```

Mupad [B]
time = 0.09, size = 110, normalized size = 1.21

$$-\frac{\frac{\cos\left(c+d\,x\right)^2}{2\,a}-\frac{\cos\left(c+d\,x\right)^3}{3\,a}-\frac{3\,\cos\left(c+d\,x\right)^4}{4\,a}+\frac{3\,\cos\left(c+d\,x\right)^5}{5\,a}+\frac{\cos\left(c+d\,x\right)^6}{2\,a}-\frac{3\,\cos\left(c+d\,x\right)^7}{7\,a}-\frac{\cos\left(c+d\,x\right)^8}{8\,a}+\frac{\cos\left(c+d\,x\right)^9}{9\,a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^9/(a + a/cos(c + d*x)),x)
```

```
[Out] -(cos(c + d*x)^2/(2*a) - cos(c + d*x)^3/(3*a) - (3*cos(c + d*x)^4)/(4*a) +
(3*cos(c + d*x)^5)/(5*a) + cos(c + d*x)^6/(2*a) - (3*cos(c + d*x)^7)/(7*a)
- cos(c + d*x)^8/(8*a) + cos(c + d*x)^9/(9*a))/d
```

3.58 $\int \frac{\sin^7(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=73

$$\frac{\cos^3(c+dx)}{3ad}-\frac{2\cos^5(c+dx)}{5ad}+\frac{\cos^7(c+dx)}{7ad}+\frac{\sin^6(c+dx)}{6ad}$$

```
[Out] 1/3*cos(d*x+c)^3/a/d-2/5*cos(d*x+c)^5/a/d+1/7*cos(d*x+c)^7/a/d+1/6*sin(d*x+
c)^6/a/d
```

Rubi [A]
time = 0.12, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3957, 2914, 2644, 30, 2645, 276}

$$\frac{\sin^6(c+dx)}{6ad}+\frac{\cos^7(c+dx)}{7ad}-\frac{2\cos^5(c+dx)}{5ad}+\frac{\cos^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^7/(a + a*Sec[c + d*x]),x]
```

```
[Out] Cos[c + d*x]^3/(3*a*d) - (2*Cos[c + d*x]^5)/(5*a*d) + Cos[c + d*x]^7/(7*a*d
) + Sin[c + d*x]^6/(6*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2914

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^7(c+dx)}{a+a\sec(c+dx)}\,dx &= -\int \frac{\cos(c+dx)\sin^7(c+dx)}{-a-a\cos(c+dx)}\,dx\\
&= \frac{\int \cos(c+dx)\sin^5(c+dx)\,dx}{a}-\frac{\int \cos^2(c+dx)\sin^5(c+dx)\,dx}{a}\\
&= \frac{\operatorname{Subst}\left(\int x^5\,dx,x,\sin(c+dx)\right)}{ad}+\frac{\operatorname{Subst}\left(\int x^2\left(1-x^2\right)^2\,dx,x,\cos(c+dx)\right)}{ad}\\
&= \frac{\sin^6(c+dx)}{6ad}+\frac{\operatorname{Subst}\left(\int \left(x^2-2x^4+x^6\right)\,dx,x,\cos(c+dx)\right)}{ad}\\
&= \frac{\cos^3(c+dx)}{3ad}-\frac{2\cos^5(c+dx)}{5ad}+\frac{\cos^7(c+dx)}{7ad}+\frac{\sin^6(c+dx)}{6ad}
\end{aligned}$$

Mathematica [A]
time = 1.00, size = 52, normalized size = 0.71

$$\frac{4(123+197\cos(c+dx)+85\cos(2(c+dx))+15\cos(3(c+dx)))\sin^8\left(\frac{1}{2}(c+dx)\right)}{105ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^7/(a + a*Sec[c + d*x]),x]
```

```
[Out] (4*(123 + 197*Cos[c + d*x] + 85*Cos[2*(c + d*x)] + 15*Cos[3*(c + d*x)])*Sin
[(c + d*x)/2]^8)/(105*a*d)
```

Maple [A]
time = 0.12, size = 70, normalized size = 0.96

method	result
derivativedivides	$-\frac{\frac{1}{2\sec(dx+c)^2}+\frac{2}{5\sec(dx+c)^5}-\frac{1}{3\sec(dx+c)^3}+\frac{1}{6\sec(dx+c)^6}-\frac{1}{7\sec(dx+c)^7}-\frac{1}{2\sec(dx+c)^4}}{da}$
default	$-\frac{\frac{1}{2\sec(dx+c)^2}+\frac{2}{5\sec(dx+c)^5}-\frac{1}{3\sec(dx+c)^3}+\frac{1}{6\sec(dx+c)^6}-\frac{1}{7\sec(dx+c)^7}-\frac{1}{2\sec(dx+c)^4}}{da}$
norman	$\frac{\frac{16}{105ad}+\frac{16\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15ad}+\frac{16\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5ad}+\frac{16\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{64\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}$
risch	$\frac{5\cos(dx+c)}{64ad}+\frac{\cos(7dx+7c)}{448ad}-\frac{\cos(6dx+6c)}{192ad}-\frac{3\cos(5dx+5c)}{320ad}+\frac{\cos(4dx+4c)}{32ad}+\frac{\cos(3dx+3c)}{192ad}-\frac{5\cos(2dx+2c)}{64ad}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^7/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d/a*(1/2/sec(d*x+c)^2+2/5/sec(d*x+c)^5-1/3/sec(d*x+c)^3+1/6/sec(d*x+c)^6
-1/7/sec(d*x+c)^7-1/2/sec(d*x+c)^4)
```

Maxima [A]
time = 0.27, size = 69, normalized size = 0.95

$$\frac{30\cos(dx+c)^7-35\cos(dx+c)^6-84\cos(dx+c)^5+105\cos(dx+c)^4+70\cos(dx+c)^3-105\cos(dx+c)^2}{210\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/210*(30*cos(d*x + c)^7 - 35*cos(d*x + c)^6 - 84*cos(d*x + c)^5 + 105*cos(
d*x + c)^4 + 70*cos(d*x + c)^3 - 105*cos(d*x + c)^2)/(a*d)
```

Fricas [A]
time = 2.83, size = 69, normalized size = 0.95

$$\frac{30\cos(dx+c)^7-35\cos(dx+c)^6-84\cos(dx+c)^5+105\cos(dx+c)^4+70\cos(dx+c)^3-105\cos(dx+c)^2}{210\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/210*(30*cos(d*x + c)^7 - 35*cos(d*x + c)^6 - 84*cos(d*x + c)^5 + 105*cos(
d*x + c)^4 + 70*cos(d*x + c)^3 - 105*cos(d*x + c)^2)/(a*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**7/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.46, size = 119, normalized size = 1.63

$$\frac{16\left(\frac{7(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{21(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{35(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}-\frac{140(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}-1\right)}{105\,ad\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 16/105*(7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 21*(cos(d*x + c) - 1)^2/(
cos(d*x + c) + 1)^2 + 35*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 140*(c
os(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 1)/(a*d*((cos(d*x + c) - 1)/(cos(
d*x + c) + 1) - 1)^7)
```

Mupad [B]
time = 0.06, size = 84, normalized size = 1.15

$$-\frac{\frac{\cos(c+d\,x)^2}{2\,a}-\frac{\cos(c+d\,x)^3}{3\,a}-\frac{\cos(c+d\,x)^4}{2\,a}+\frac{2\cos(c+d\,x)^5}{5\,a}+\frac{\cos(c+d\,x)^6}{6\,a}-\frac{\cos(c+d\,x)^7}{7\,a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^7/(a + a/cos(c + d*x)),x)
```

```
[Out] -(cos(c + d*x)^2/(2*a) - cos(c + d*x)^3/(3*a) - cos(c + d*x)^4/(2*a) + (2*c
os(c + d*x)^5)/(5*a) + cos(c + d*x)^6/(6*a) - cos(c + d*x)^7/(7*a))/d
```

3.59 $\int \frac{\sin^5(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=55

$$\frac{\cos^3(c+dx)}{3ad}-\frac{\cos^5(c+dx)}{5ad}+\frac{\sin^4(c+dx)}{4ad}$$

```
[Out] 1/3*cos(d*x+c)^3/a/d-1/5*cos(d*x+c)^5/a/d+1/4*sin(d*x+c)^4/a/d
```

Rubi [A]
time = 0.11, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3957, 2914, 2644, 30, 2645, 14}

$$\frac{\sin^4(c+dx)}{4ad}-\frac{\cos^5(c+dx)}{5ad}+\frac{\cos^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^5/(a + a*Sec[c + d*x]),x]
```

```
[Out] Cos[c + d*x]^3/(3*a*d) - Cos[c + d*x]^5/(5*a*d) + Sin[c + d*x]^4/(4*a*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2914

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^5(c+dx)}{a+a\sec(c+dx)}\,dx &= -\int \frac{\cos(c+dx)\sin^5(c+dx)}{-a-a\cos(c+dx)}\,dx\\
&= \frac{\int \cos(c+dx)\sin^3(c+dx)\,dx}{a}-\frac{\int \cos^2(c+dx)\sin^3(c+dx)\,dx}{a}\\
&= \frac{\text{Subst}\left(\int x^3\,dx,x,\sin(c+dx)\right)}{ad}+\frac{\text{Subst}\left(\int x^2\left(1-x^2\right)\,dx,x,\cos(c+dx)\right)}{ad}\\
&= \frac{\sin^4(c+dx)}{4ad}+\frac{\text{Subst}\left(\int \left(x^2-x^4\right)\,dx,x,\cos(c+dx)\right)}{ad}\\
&= \frac{\cos^3(c+dx)}{3ad}-\frac{\cos^5(c+dx)}{5ad}+\frac{\sin^4(c+dx)}{4ad}
\end{aligned}$$

Mathematica [A]
time = 0.23, size = 42, normalized size = 0.76

$$\frac{2(13+21\cos(c+dx)+6\cos(2(c+dx)))\sin^6\left(\frac{1}{2}(c+dx)\right)}{15ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^5/(a + a*Sec[c + d*x]),x]
[Out] (2*(13 + 21*Cos[c + d*x] + 6*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]^6)/(15*a*d)
```

Maple [A]
time = 0.10, size = 49, normalized size = 0.89

method	result	size
derivativedivides	$\frac{-\frac{1}{5\sec(dx+c)^5}+\frac{1}{4\sec(dx+c)^4}+\frac{1}{3\sec(dx+c)^3}-\frac{1}{2\sec(dx+c)^2}}{da}$	49
default	$\frac{-\frac{1}{5\sec(dx+c)^5}+\frac{1}{4\sec(dx+c)^4}+\frac{1}{3\sec(dx+c)^3}-\frac{1}{2\sec(dx+c)^2}}{da}$	49
norman	$\frac{\frac{4}{15ad}+\frac{8\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{8\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}$	83
risch	$\frac{\cos(dx+c)}{8ad}-\frac{\cos(5dx+5c)}{80ad}+\frac{\cos(4dx+4c)}{32ad}+\frac{\cos(3dx+3c)}{48ad}-\frac{\cos(2dx+2c)}{8ad}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^5/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-1/5/sec(d*x+c)^5+1/4/sec(d*x+c)^4+1/3/sec(d*x+c)^3-1/2/sec(d*x+c)^2
)
```

Maxima [A]
time = 0.28, size = 49, normalized size = 0.89

$$-\frac{12\cos(dx+c)^5-15\cos(dx+c)^4-20\cos(dx+c)^3+30\cos(dx+c)^2}{60\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/60*(12*cos(d*x + c)^5 - 15*cos(d*x + c)^4 - 20*cos(d*x + c)^3 + 30*cos(d
*x + c)^2)/(a*d)
```

Fricas [A]
time = 2.44, size = 49, normalized size = 0.89

$$-\frac{12\cos(dx+c)^5-15\cos(dx+c)^4-20\cos(dx+c)^3+30\cos(dx+c)^2}{60\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/60*(12*cos(d*x + c)^5 - 15*cos(d*x + c)^4 - 20*cos(d*x + c)^3 + 30*cos(d
*x + c)^2)/(a*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**5/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.43, size = 97, normalized size = 1.76

$$\frac{4\left(\frac{5(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{10(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{30(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}-1\right)}{15\,ad\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 4/15*(5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 10*(cos(d*x + c) - 1)^2/(co
s(d*x + c) + 1)^2 + 30*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 1)/(a*d*
((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^5)
```

Mupad [B]
time = 0.07, size = 58, normalized size = 1.05

$$-\frac{\frac{\cos(c+d\,x)^2}{2\,a}-\frac{\cos(c+d\,x)^3}{3\,a}-\frac{\cos(c+d\,x)^4}{4\,a}+\frac{\cos(c+d\,x)^5}{5\,a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5/(a + a/cos(c + d*x)),x)
```

```
[Out] -(cos(c + d*x)^2/(2*a) - cos(c + d*x)^3/(3*a) - cos(c + d*x)^4/(4*a) + cos(
c + d*x)^5/(5*a))/d
```

3.60 $\int \frac{\sin^3(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=37

$$\frac{\cos^3(c+dx)}{3ad}+\frac{\sin^2(c+dx)}{2ad}$$

```
[Out] 1/3*cos(d*x+c)^3/a/d+1/2*sin(d*x+c)^2/a/d
```

Rubi [A]
time = 0.09, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3957, 2914, 2644, 30, 2645}

$$\frac{\sin^2(c+dx)}{2ad}+\frac{\cos^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

```
[Out] Cos[c + d*x]^3/(3*a*d) + Sin[c + d*x]^2/(2*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2914

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
```

```
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^3(c+dx)}{a+a\sec(c+dx)}\,dx &= -\int \frac{\cos(c+dx)\sin^3(c+dx)}{-a-a\cos(c+dx)}\,dx \\
&= \frac{\int \cos(c+dx)\sin(c+dx)\,dx}{a} - \frac{\int \cos^2(c+dx)\sin(c+dx)\,dx}{a} \\
&= \frac{\text{Subst}(\int x\,dx, x, \sin(c+dx))}{ad} + \frac{\text{Subst}\left(\int x^2\,dx, x, \cos(c+dx)\right)}{ad} \\
&= \frac{\cos^3(c+dx)}{3ad} + \frac{\sin^2(c+dx)}{2ad}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 32, normalized size = 0.86

$$\frac{2(1+2\cos(c+dx))\sin^4\left(\frac{1}{2}(c+dx)\right)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/(a + a*Sec[c + d*x]),x]
[Out] (2*(1 + 2*Cos[c + d*x])*Sin[(c + d*x)/2]^4)/(3*a*d)
```

Maple [A]
time = 0.08, size = 30, normalized size = 0.81

method	result	size
derivativedivides	$-\frac{\frac{1}{2\sec(dx+c)^2}-\frac{1}{3\sec(dx+c)^3}}{da}$	30
default	$-\frac{\frac{1}{2\sec(dx+c)^2}-\frac{1}{3\sec(dx+c)^3}}{da}$	30
risch	$\frac{\cos(dx+c)}{4ad}+\frac{\cos(3dx+3c)}{12ad}-\frac{\cos(2dx+2c)}{4ad}$	50
norman	$\frac{\frac{2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{4\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{2}{3ad}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d/a*(1/2/sec(d*x+c)^2-1/3/sec(d*x+c)^3)
```

Maxima [A]
time = 0.27, size = 29, normalized size = 0.78

$$\frac{2\cos\left(dx+c\right)^{3}-3\cos\left(dx+c\right)^{2}}{6\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(2*cos(d*x + c)^3 - 3*cos(d*x + c)^2)/(a*d)
```

Fricas [A]
time = 2.68, size = 29, normalized size = 0.78

$$\frac{2\cos\left(dx+c\right)^{3}-3\cos\left(dx+c\right)^{2}}{6\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*cos(d*x + c)^3 - 3*cos(d*x + c)^2)/(a*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.46, size = 32, normalized size = 0.86

$$\frac{\frac{2\cos\left(dx+c\right)^{3}}{d}-\frac{3\cos\left(dx+c\right)^{2}}{d}}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(2*cos(d*x + c)^3/d - 3*cos(d*x + c)^2/d)/a
```

Mupad [B]
time = 0.88, size = 26, normalized size = 0.70

$$\frac{\cos\left(c+d\,x\right)^2\,\left(2\,\cos\left(c+d\,x\right)-3\right)}{6\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(a + a/cos(c + d*x)),x)
```

```
[Out] (cos(c + d*x)^2*(2*cos(c + d*x) - 3))/(6*a*d)
```

3.61 $\int \frac{\sin(c+dx)}{a+a\sec(c+dx)} \, dx$

Optimal. Leaf size=31

$$-\frac{\cos(c+dx)}{ad}+\frac{\log(1+\cos(c+dx))}{ad}$$

[Out] -cos(d*x+c)/a/d+ln(1+cos(d*x+c))/a/d

Rubi [A]
time = 0.05, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3957, 2912, 12, 45}

$$\frac{\log(\cos(c+dx)+1)}{ad}-\frac{\cos(c+dx)}{ad}$$

Antiderivative was successfully verified.

[In] Int[Sin[c + d*x]/(a + a*Sec[c + d*x]),x]

[Out] -(Cos[c + d*x]/(a*d)) + Log[1 + Cos[c + d*x]]/(a*d)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{a+a\sec(c+dx)}\,dx = -\int \frac{\cos(c+dx)\sin(c+dx)}{-a-a\cos(c+dx)}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{x}{a(-a+x)}\,dx, x, -a\cos(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x}{-a+x}\,dx, x, -a\cos(c+dx)\right)}{a^2d}$$
$$= \frac{\text{Subst}\left(\int \left(1-\frac{a}{a-x}\right)\,dx, x, -a\cos(c+dx)\right)}{a^2d}$$
$$= -\frac{\cos(c+dx)}{ad} + \frac{\log(1+\cos(c+dx))}{ad}$$

Mathematica [A]
time = 0.06, size = 28, normalized size = 0.90

$$-\frac{\cos(c+dx) - 2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + a*Sec[c + d*x]),x]
```

```
[Out] -((Cos[c + d*x] - 2*Log[Cos[(c + d*x)/2]])/(a*d))
```

Maple [A]
time = 0.03, size = 37, normalized size = 1.19

method	result	size
derivativedivides	$\frac{-\frac{1}{\sec(dx+c)}-\ln(\sec(dx+c))+\ln(1+\sec(dx+c))}{ad}$	37
default	$\frac{-\frac{1}{\sec(dx+c)}-\ln(\sec(dx+c))+\ln(1+\sec(dx+c))}{ad}$	37
norman	$\frac{2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)} - \frac{\ln\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}$	58
risch	$-\frac{ix}{a} - \frac{e^{i(dx+c)}}{2ad} - \frac{e^{-i(dx+c)}}{2ad} - \frac{2ic}{ad} + \frac{2\ln\left(e^{i(dx+c)}+1\right)}{ad}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a/d*(-1/sec(d*x+c)-ln(sec(d*x+c))+ln(1+sec(d*x+c)))
```

Maxima [A]
time = 0.28, size = 30, normalized size = 0.97

$$-\frac{\frac{\cos(dx+c)}{a} - \frac{\log(\cos(dx+c)+1)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] -(cos(d*x + c)/a - log(cos(d*x + c) + 1)/a)/d
```

Fricas [A]
time = 3.02, size = 28, normalized size = 0.90

$$-\frac{\cos(dx+c) - \log\left(\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] -(cos(d*x + c) - log(1/2*cos(d*x + c) + 1/2))/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sin(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sec(d*x+c)),x)
[Out] Integral(sin(c + d*x)/(sec(c + d*x) + 1), x)/a
```

Giac [A]
time = 0.41, size = 34, normalized size = 1.10

$$-\frac{\cos(dx+c)}{ad} + \frac{\log(|-\cos(dx+c) - 1|)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] -cos(d*x + c)/(a*d) + log(abs(-cos(d*x + c) - 1))/(a*d)
```

Mupad [B]
time = 0.05, size = 25, normalized size = 0.81

$$\frac{\ln(\cos(c+d\,x)+1) - \cos(c+d\,x)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(a + a/cos(c + d*x)),x)
[Out] (log(cos(c + d*x) + 1) - cos(c + d*x))/(a*d)
```

3.62 $\int \frac{\csc(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=58

$$-\frac{\tanh^{-1}(\cos(c+dx))}{2ad}+\frac{\cot(c+dx)\csc(c+dx)}{2ad}-\frac{\csc^2(c+dx)}{2ad}$$

```
[Out] -1/2*arctanh(cos(d*x+c))/a/d+1/2*cot(d*x+c)*csc(d*x+c)/a/d-1/2*csc(d*x+c)^2
/a/d
```

Rubi [A]
time = 0.07, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3957, 2785, 2686, 30, 2691, 3855}

$$-\frac{\csc^2(c+dx)}{2ad}-\frac{\tanh^{-1}(\cos(c+dx))}{2ad}+\frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/2*ArcTanh[Cos[c + d*x]]/(a*d) + (Cot[c + d*x]*Csc[c + d*x])/(2*a*d) - Cs
c[c + d*x]^2/(2*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc(c+dx)}{a+a\sec(c+dx)}\,dx &= -\int \frac{\cot(c+dx)}{-a-a\cos(c+dx)}\,dx\\ &= -\frac{\int \cot^2(c+dx)\csc(c+dx)\,dx}{a}+\frac{\int \cot(c+dx)\csc^2(c+dx)\,dx}{a}\\ &= \frac{\cot(c+dx)\csc(c+dx)}{2ad}+\frac{\int \csc(c+dx)\,dx}{2a}-\frac{\text{Subst}(\int x\,dx,x,\csc(c+dx))}{ad}\\ &= -\frac{\tanh^{-1}(\cos(c+dx))}{2ad}+\frac{\cot(c+dx)\csc(c+dx)}{2ad}-\frac{\csc^2(c+dx)}{2ad}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 67, normalized size = 1.16

$$-\frac{\left(1+2\cos^2\left(\frac{1}{2}(c+dx)\right)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sec(c+dx)}{2ad(1+\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/2*((1 + 2*Cos[(c + d*x)/2]^2*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/
2]]))*Sec[c + d*x])/(a*d*(1 + Sec[c + d*x]))
```

Maple [A]
time = 0.07, size = 43, normalized size = 0.74

method	result	size
norman	$-\frac{\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}$	39
derivativedivides	$\frac{\frac{\ln(-1+\cos(dx+c))}{4}-\frac{1}{2(1+\cos(dx+c))}-\frac{\ln(1+\cos(dx+c))}{4}}{da}$	43
default	$\frac{\frac{\ln(-1+\cos(dx+c))}{4}-\frac{1}{2(1+\cos(dx+c))}-\frac{\ln(1+\cos(dx+c))}{4}}{da}$	43
risch	$-\frac{e^{i(dx+c)}}{ad\left(e^{i(dx+c)}+1\right)^{2}}+\frac{\ln\left(e^{i(dx+c)}-1\right)}{2ad}-\frac{\ln\left(e^{i(dx+c)}+1\right)}{2ad}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(1/4*ln(-1+cos(d*x+c))-1/2/(1+cos(d*x+c))-1/4*ln(1+cos(d*x+c)))
```

Maxima [A]
time = 0.27, size = 47, normalized size = 0.81

$$-\frac{\frac{\log(\cos(dx+c)+1)}{a}-\frac{\log(\cos(dx+c)-1)}{a}+\frac{2}{a\cos(dx+c)+a}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(log(cos(d*x + c) + 1)/a - log(cos(d*x + c) - 1)/a + 2/(a*cos(d*x + c)
 + a))/d
```

Fricas [A]
time = 4.22, size = 60, normalized size = 1.03

$$-\frac{(\cos(dx+c)+1)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-(\cos(dx+c)+1)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+2}{4\,(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*((cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) - (cos(d*x + c) + 1)*l
og(-1/2*cos(d*x + c) + 1/2) + 2)/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\csc(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)/(sec(c + d*x) + 1), x)/a
```

Giac [A]
time = 0.46, size = 56, normalized size = 0.97

$$\frac{\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a} + \frac{\cos(dx+c)-1}{a(\cos(dx+c)+1)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a + (cos(d*x + c) -
1)/(a*(cos(d*x + c) + 1)))/d
```

Mupad [B]
time = 0.92, size = 33, normalized size = 0.57

$$-\frac{1}{2\,d\,\left(a+a\,\cos\left(c+d\,x\right)\right)}-\frac{\mathrm{atanh}(\cos\left(c+d\,x\right))}{2\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)*(a + a/cos(c + d*x))),x)
```

```
[Out] - 1/(2*d*(a + a*cos(c + d*x))) - atanh(cos(c + d*x))/(2*a*d)
```

3.63 $\int \frac{\csc^3(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=82

$$-\frac{\tanh^{-1}(\cos(c+dx))}{8ad}-\frac{\cot(c+dx)\csc(c+dx)}{8ad}+\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}-\frac{\csc^4(c+dx)}{4ad}$$

```
[Out] -1/8*arctanh(cos(d*x+c))/a/d-1/8*cot(d*x+c)*csc(d*x+c)/a/d+1/4*cot(d*x+c)*c
sc(d*x+c)^3/a/d-1/4*csc(d*x+c)^4/a/d
```

Rubi [A]
time = 0.12, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3957, 2914, 2686, 30, 2691, 3853, 3855}

$$-\frac{\csc^4(c+dx)}{4ad}-\frac{\tanh^{-1}(\cos(c+dx))}{8ad}+\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}-\frac{\cot(c+dx)\csc(c+dx)}{8ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/8*ArcTanh[Cos[c + d*x]]/(a*d) - (Cot[c + d*x]*Csc[c + d*x])/(8*a*d) + (C
ot[c + d*x]*Csc[c + d*x]^3)/(4*a*d) - Csc[c + d*x]^4/(4*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2914

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^3(c+dx)}{a+a\sec(c+dx)}\,dx &= -\int \frac{\cot(c+dx)\csc^2(c+dx)}{-a-a\cos(c+dx)}\,dx\\ &= -\frac{\int \cot^2(c+dx)\csc^3(c+dx)\,dx}{a}+\frac{\int \cot(c+dx)\csc^4(c+dx)\,dx}{a}\\ &= \frac{\cot(c+dx)\csc^3(c+dx)}{4ad}+\frac{\int \csc^3(c+dx)\,dx}{4a}-\frac{\operatorname{Subst}\left(\int x^3\,dx,x,\csc(c+dx)\right)}{ad}\\ &= -\frac{\cot(c+dx)\csc(c+dx)}{8ad}+\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}-\frac{\csc^4(c+dx)}{4ad}+\frac{\int \csc(c+ \\ &= -\frac{\tanh^{-1}(\cos(c+dx))}{8ad}-\frac{\cot(c+dx)\csc(c+dx)}{8ad}+\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}-\operatorname{cs}\end{aligned}$$

Mathematica [A]
time = 0.27, size = 91, normalized size = 1.11

$$-\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(2\csc^2\left(\frac{1}{2}(c+dx)\right)+4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-4\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\sec^4\left(\frac{1}{2}(c+dx)\right)\right)\sec(c+dx)}{16ad(1+\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/16*(Cos[(c + d*x)/2]^2*(2*Csc[(c + d*x)/2]^2 + 4*Log[Cos[(c + d*x)/2]] -
 4*Log[Sin[(c + d*x)/2]] + Sec[(c + d*x)/2]^4)*Sec[c + d*x])/(a*d*(1 + Sec[
c + d*x]))
```

Maple [A]
time = 0.11, size = 55, normalized size = 0.67

method	result	size
derivativedivides	$\frac{\frac{1}{-8+8\cos(dx+c)}+\frac{\ln(-1+\cos(dx+c))}{16}-\frac{1}{8(1+\cos(dx+c))^2}-\frac{\ln(1+\cos(dx+c))}{16}}{da}$	55
default	$\frac{\frac{1}{-8+8\cos(dx+c)}+\frac{\ln(-1+\cos(dx+c))}{16}-\frac{1}{8(1+\cos(dx+c))^2}-\frac{\ln(1+\cos(dx+c))}{16}}{da}$	55
norman	$\frac{-\frac{1}{16ad}-\frac{\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)}{16ad}-\frac{\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)}{32ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad}$	79
risch	$\frac{e^{5i(dx+c)}+2\,e^{4i(dx+c)}+10\,e^{3i(dx+c)}+2\,e^{2i(dx+c)}+e^{i(dx+c)}}{4ad\left(e^{i(dx+c)}+1\right)^4\left(e^{i(dx+c)}-1\right)^2}-\frac{\ln\left(e^{i(dx+c)}+1\right)}{8ad}+\frac{\ln\left(e^{i(dx+c)}-1\right)}{8ad}$	128

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(1/8/(-1+cos(d*x+c))+1/16*ln(-1+cos(d*x+c))-1/8/(1+cos(d*x+c))^2-1/16
*ln(1+cos(d*x+c)))
```

Maxima [A]
time = 0.28, size = 86, normalized size = 1.05

$$\frac{\frac{2\left(\cos(dx+c)^2+\cos(dx+c)+2\right)}{a\cos(dx+c)^3+a\cos(dx+c)^2-a\cos(dx+c)-a}-\frac{\log(\cos(dx+c)+1)}{a}+\frac{\log(\cos(dx+c)-1)}{a}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(2*(cos(d*x + c)^2 + cos(d*x + c) + 2)/(a*cos(d*x + c)^3 + a*cos(d*x +
 c)^2 - a*cos(d*x + c) - a) - log(cos(d*x + c) + 1)/a + log(cos(d*x + c) -
1)/a)/d
```

Fricas [A]
time = 2.68, size = 138, normalized size = 1.68

$$\frac{2\cos(dx+c)^2-\left(\cos(dx+c)^3+\cos(dx+c)^2-\cos(dx+c)-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+\left(\cos(dx+c)^3+\cos(dx+c)^2-\cos(dx+c)-1\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+2\cos(dx+c)+4}{16\left(ad\cos(dx+c)^3+ad\cos(dx+c)^2-ad\cos(dx+c)-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(2*cos(d*x + c)^2 - (cos(d*x + c)^3 + cos(d*x + c)^2 - cos(d*x + c) -
1)*log(1/2*cos(d*x + c) + 1/2) + (cos(d*x + c)^3 + cos(d*x + c)^2 - cos(d*x
 + c) - 1)*log(-1/2*cos(d*x + c) + 1/2) + 2*cos(d*x + c) + 4)/(a*d*cos(d*x
+ c)^3 + a*d*cos(d*x + c)^2 - a*d*cos(d*x + c) - a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^3(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**3/(sec(c + d*x) + 1), x)/a
```

Giac [A]
time = 0.48, size = 129, normalized size = 1.57

$$-\frac{\frac{2\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)(\cos(dx+c)+1)}{a(\cos(dx+c)-1)}-\frac{2\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a}-\frac{\frac{2a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{a^2}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/32*(2*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)*(cos(d*x + c) + 1)/(a*
(cos(d*x + c) - 1)) - 2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a
 - (2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - a*(cos(d*x + c) - 1)^2/(cos
(d*x + c) + 1)^2)/a^2)/d
```

Mupad [B]
time = 0.94, size = 75, normalized size = 0.91

$$-\frac{\operatorname{atanh}(\cos(c+d\,x))}{8\,a\,d}-\frac{\frac{\cos(c+d\,x)^2}{8}+\frac{\cos(c+d\,x)}{8}+\frac{1}{4}}{d\,\left(-a\cos(c+d\,x)^3-a\cos(c+d\,x)^2+a\,\cos(c+d\,x)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^3*(a + a/cos(c + d*x))),x)
```

```
[Out] - atanh(cos(c + d*x))/(8*a*d) - (cos(c + d*x)/8 + cos(c + d*x)^2/8 + 1/4)/(
d*(a + a*cos(c + d*x) - a*cos(c + d*x)^2 - a*cos(c + d*x)^3))
```

3.64 $\int \frac{\csc^5(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=106

$$-\frac{\tanh^{-1}(\cos(c+dx))}{16ad}-\frac{\cot(c+dx)\csc(c+dx)}{16ad}-\frac{\cot(c+dx)\csc^3(c+dx)}{24ad}+\frac{\cot(c+dx)\csc^5(c+dx)}{6ad}-\frac{\csc^6}{}$$

```
[Out] -1/16*arctanh(cos(d*x+c))/a/d-1/16*cot(d*x+c)*csc(d*x+c)/a/d-1/24*cot(d*x+c
)*csc(d*x+c)^3/a/d+1/6*cot(d*x+c)*csc(d*x+c)^5/a/d-1/6*csc(d*x+c)^6/a/d
```

Rubi [A]

time = 0.13, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3957, 2914, 2686, 30, 2691, 3853, 3855}

$$-\frac{\csc^6(c+dx)}{6ad}-\frac{\tanh^{-1}(\cos(c+dx))}{16ad}+\frac{\cot(c+dx)\csc^5(c+dx)}{6ad}-\frac{\cot(c+dx)\csc^3(c+dx)}{24ad}-\frac{\cot(c+dx)\csc(c+dx)}{16ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5/(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/16*ArcTanh[Cos[c + d*x]]/(a*d) - (Cot[c + d*x]*Csc[c + d*x])/(16*a*d) -
(Cot[c + d*x]*Csc[c + d*x]^3)/(24*a*d) + (Cot[c + d*x]*Csc[c + d*x]^5)/(6*a
*d) - Csc[c + d*x]^6/(6*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2914

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^5(c+dx)}{a+a\sec(c+dx)}\,dx &= -\int \frac{\cot(c+dx)\csc^4(c+dx)}{-a-a\cos(c+dx)}\,dx \\
&= -\frac{\int \cot^2(c+dx)\csc^5(c+dx)\,dx}{a}+\frac{\int \cot(c+dx)\csc^6(c+dx)\,dx}{a} \\
&= \frac{\cot(c+dx)\csc^5(c+dx)}{6ad}+\frac{\int \csc^5(c+dx)\,dx}{6a}-\frac{\operatorname{Subst}\left(\int x^5\,dx,x,\csc(c+dx)\right)}{ad} \\
&= -\frac{\cot(c+dx)\csc^3(c+dx)}{24ad}+\frac{\cot(c+dx)\csc^5(c+dx)}{6ad}-\frac{\csc^6(c+dx)}{6ad}+\frac{\int \csc^3(c
\end{aligned}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)}{16ad}-\frac{\cot(c+dx)\csc^3(c+dx)}{24ad}+\frac{\cot(c+dx)\csc^5(c+dx)}{6ad}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{16ad}-\frac{\cot(c+dx)\csc(c+dx)}{16ad}-\frac{\cot(c+dx)\csc^3(c+dx)}{24ad}+\text{co}$$

Mathematica [A]
time = 0.32, size = 122, normalized size = 1.15

$$-\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(12\csc^2\left(\frac{1}{2}(c+dx)\right)+3\csc^4\left(\frac{1}{2}(c+dx)\right)+24\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+3\sec^4\left(\frac{1}{2}(c+dx)\right)+2\sec^6\left(\frac{1}{2}(c+dx)\right)\right)\sec(c+dx)}{192ad(1+\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^5/(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/192*(Cos[(c + d*x)/2]^2*(12*Csc[(c + d*x)/2]^2 + 3*Csc[(c + d*x)/2]^4 +
24*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]]) + 3*Sec[(c + d*x)/2]^4 +
 2*Sec[(c + d*x)/2]^6)*Sec[c + d*x])/(a*d*(1 + Sec[c + d*x]))
```

Maple [A]
time = 0.12, size = 79, normalized size = 0.75

method	result
derivativedivides	$\frac{-\frac{1}{32(-1+\cos(dx+c))^2}+\frac{1}{-16+16\cos(dx+c)}+\frac{\ln(-1+\cos(dx+c))}{32}-\frac{1}{24(1+\cos(dx+c))^3}-\frac{1}{32(1+\cos(dx+c))^2}-\frac{\ln(1+\cos(dx+c))}{32}}{da}$
default	$\frac{-\frac{1}{32(-1+\cos(dx+c))^2}+\frac{1}{-16+16\cos(dx+c)}+\frac{\ln(-1+\cos(dx+c))}{32}-\frac{1}{24(1+\cos(dx+c))^3}-\frac{1}{32(1+\cos(dx+c))^2}-\frac{\ln(1+\cos(dx+c))}{32}}{da}$
norman	$\frac{-\frac{1}{128ad}-\frac{3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64ad}-\frac{\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)}{32ad}-\frac{3\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128ad}-\frac{\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)}{192ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16ad}$
risch	$\frac{3\,e^{9i(dx+c)}+6\,e^{8i(dx+c)}-8\,e^{7i(dx+c)}-22\,e^{6i(dx+c)}-150\,e^{5i(dx+c)}-22\,e^{4i(dx+c)}-8\,e^{3i(dx+c)}+6\,e^{2i(dx+c)}+3\,e^{i(dx+c)}}{24ad\left(e^{i(dx+c)}+1\right)^6\left(e^{i(dx+c)}-1\right)^4}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-1/32/(-1+cos(d*x+c))^2+1/16/(-1+cos(d*x+c))+1/32*ln(-1+cos(d*x+c))-
1/24/(1+cos(d*x+c))^3-1/32/(1+cos(d*x+c))^2-1/32*ln(1+cos(d*x+c)))
```

Maxima [A]
time = 0.27, size = 130, normalized size = 1.23

$$\frac{\frac{2\left(3\cos(dx+c)^4+3\cos(dx+c)^3-5\cos(dx+c)^2-5\cos(dx+c)-8\right)}{a\cos(dx+c)^5+a\cos(dx+c)^4-2a\cos(dx+c)^3-2a\cos(dx+c)^2+a\cos(dx+c)+a}-\frac{3\log(\cos(dx+c)+1)}{a}+\frac{3\log(\cos(dx+c)-1)}{a}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/96*(2*(3*cos(d*x + c)^4 + 3*cos(d*x + c)^3 - 5*cos(d*x + c)^2 - 5*cos(d*x
 + c) - 8)/(a*cos(d*x + c)^5 + a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^3 - 2*a*
cos(d*x + c)^2 + a*cos(d*x + c) + a) - 3*log(cos(d*x + c) + 1)/a + 3*log(co
s(d*x + c) - 1)/a)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(96) = 192.
time = 2.67, size = 217, normalized size = 2.05

$$\frac{6\cos(dx+c)^4+6\cos(dx+c)^3-10\cos(dx+c)^2-3\left(\cos(dx+c)^5+\cos(dx+c)^4-2\cos(dx+c)^3-2\cos(dx+c)^2+\cos(dx+c)+1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+3\left(\cos(dx+c)^5+\cos(dx+c)^4-2\cos(dx+c)^3-2\cos(dx+c)^2+\cos(dx+c)+1\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-10\cos(dx+c)-16}{96\left(ad\cos(dx+c)^5+ad\cos(dx+c)^4-2ad\cos(dx+c)^3-2ad\cos(dx+c)^2+ad\cos(dx+c)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/96*(6*cos(d*x + c)^4 + 6*cos(d*x + c)^3 - 10*cos(d*x + c)^2 - 3*(cos(d*x
+ c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 + cos(d*x + c
) + 1)*log(1/2*cos(d*x + c) + 1/2) + 3*(cos(d*x + c)^5 + cos(d*x + c)^4 - 2
*cos(d*x + c)^3 - 2*cos(d*x + c)^2 + cos(d*x + c) + 1)*log(-1/2*cos(d*x + c
) + 1/2) - 10*cos(d*x + c) - 16)/(a*d*cos(d*x + c)^5 + a*d*cos(d*x + c)^4 -
 2*a*d*cos(d*x + c)^3 - 2*a*d*cos(d*x + c)^2 + a*d*cos(d*x + c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^5(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**5/(sec(c + d*x) + 1), x)/a
```

Giac [A]
time = 0.48, size = 182, normalized size = 1.72

$$\frac{\frac{3\left(\frac{6(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{6(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-1\right)(\cos(dx+c)+1)^2}{a(\cos(dx+c)-1)^2}+\frac{12\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a}+\frac{\frac{12a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{9a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{2a^2(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}}{a^3}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/384*(3*(6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 6*(cos(d*x + c) - 1)^2/
(cos(d*x + c) + 1)^2 - 1)*(cos(d*x + c) + 1)^2/(a*(cos(d*x + c) - 1)^2) + 1
2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a + (12*a^2*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) - 9*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^
2 + 2*a^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)/a^3)/d
```

Mupad [B]
time = 1.01, size = 115, normalized size = 1.08

$$-\frac{\mathrm{atanh}(\cos(c+d\,x))}{16\,a\,d}-\frac{-\frac{\cos(c+d\,x)^4}{16}-\frac{\cos(c+d\,x)^3}{16}+\frac{5\cos(c+d\,x)^2}{48}+\frac{5\cos(c+d\,x)}{48}+\frac{1}{6}}{d\,\left(a\cos(c+d\,x)^5+a\cos(c+d\,x)^4-2\,a\cos(c+d\,x)^3-2\,a\cos(c+d\,x)^2+a\,\cos(c+d\,x)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^5*(a + a/cos(c + d*x))),x)
```

```
[Out] - atanh(cos(c + d*x))/(16*a*d) - ((5*cos(c + d*x))/48 + (5*cos(c + d*x)^2)/
48 - cos(c + d*x)^3/16 - cos(c + d*x)^4/16 + 1/6)/(d*(a + a*cos(c + d*x) -
2*a*cos(c + d*x)^2 - 2*a*cos(c + d*x)^3 + a*cos(c + d*x)^4 + a*cos(c + d*x)
^5))
```

3.65 $\int \frac{\sin^8(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=125

$$-\frac{5x}{128a}-\frac{5\cos(c+dx)\sin(c+dx)}{128ad}+\frac{5\cos^3(c+dx)\sin(c+dx)}{64ad}+\frac{5\cos^3(c+dx)\sin^3(c+dx)}{48ad}+\frac{\cos^3(c+dx)\,\mathrm{si}}{8ad}$$

```
[Out] -5/128*x/a-5/128*cos(d*x+c)*sin(d*x+c)/a/d+5/64*cos(d*x+c)^3*sin(d*x+c)/a/d
+5/48*cos(d*x+c)^3*sin(d*x+c)^3/a/d+1/8*cos(d*x+c)^3*sin(d*x+c)^5/a/d+1/7*s
in(d*x+c)^7/a/d
```

Rubi [A]
time = 0.16, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3957, 2918, 2644, 30, 2648, 2715, 8}

$$\frac{\sin^7(c+dx)}{7ad}+\frac{\sin^5(c+dx)\cos^3(c+dx)}{8ad}+\frac{5\sin^3(c+dx)\cos^3(c+dx)}{48ad}+\frac{5\sin(c+dx)\cos^3(c+dx)}{64ad}-\frac{5\sin(c+dx)\cos(c+dx)}{128ad}-\frac{5x}{128a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^8/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-5*x)/(128*a) - (5*Cos[c + d*x]*Sin[c + d*x])/(128*a*d) + (5*Cos[c + d*x]^
3*Sin[c + d*x])/(64*a*d) + (5*Cos[c + d*x]^3*Sin[c + d*x]^3)/(48*a*d) + (Co
s[c + d*x]^3*Sin[c + d*x]^5)/(8*a*d) + Sin[c + d*x]^7/(7*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
```

```
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^8(c+dx)}{a+a\sec(c+dx)}\,dx &= -\int \frac{\cos(c+dx)\sin^8(c+dx)}{-a-a\cos(c+dx)}\,dx \\
&= \frac{\int \cos(c+dx)\sin^6(c+dx)\,dx}{a} - \frac{\int \cos^2(c+dx)\sin^6(c+dx)\,dx}{a} \\
&= \frac{\cos^3(c+dx)\sin^5(c+dx)}{8ad} - \frac{5\int \cos^2(c+dx)\sin^4(c+dx)\,dx}{8a} + \frac{\text{Subst}\left(\int x^6\,dx, x, \text{s}\right.}{ad} \\
&= \frac{5\cos^3(c+dx)\sin^3(c+dx)}{48ad} + \frac{\cos^3(c+dx)\sin^5(c+dx)}{8ad} + \frac{\sin^7(c+dx)}{7ad} - \frac{5\int \cos^2(\right.}{} \\
&= \frac{5\cos^3(c+dx)\sin(c+dx)}{64ad} + \frac{5\cos^3(c+dx)\sin^3(c+dx)}{48ad} + \frac{\cos^3(c+dx)\sin^5(c+d}{8ad} \\
&= -\frac{5\cos(c+dx)\sin(c+dx)}{128ad} + \frac{5\cos^3(c+dx)\sin(c+dx)}{64ad} + \frac{5\cos^3(c+dx)\sin^3(c+}{48ad} \\
&= -\frac{5x}{128a} - \frac{5\cos(c+dx)\sin(c+dx)}{128ad} + \frac{5\cos^3(c+dx)\sin(c+dx)}{64ad} + \frac{5\cos^3(c+dx)\,\text{s}}{48a}
\end{aligned}$$

Mathematica [A]

time = 0.82, size = 132, normalized size = 1.06

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(1176c-840dx+1680\sin(c+dx)+336\sin(2(c+dx))-1008\sin(3(c+dx))+168\sin(4(c+dx))+336\sin(5(c+dx))-112\sin(6(c+dx))-48\sin(7(c+dx))+21\sin(8(c+dx))-1176\tan\left(\frac{c}{2}\right)\right)}{10752ad(1+\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^8/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*Sec[c + d*x]*(1176*c - 840*d*x + 1680*Sin[c + d*x] + 33
6*Sin[2*(c + d*x)] - 1008*Sin[3*(c + d*x)] + 168*Sin[4*(c + d*x)] + 336*Sin
[5*(c + d*x)] - 112*Sin[6*(c + d*x)] - 48*Sin[7*(c + d*x)] + 21*Sin[8*(c +
d*x)] - 1176*Tan[c/2]))/(10752*a*d*(1 + Sec[c + d*x]))
```

Maple [A]
time = 0.16, size = 142, normalized size = 1.14

method	result
risch	$-\frac{5x}{128a}+\frac{5\sin(dx+c)}{64ad}+\frac{\sin(8dx+8c)}{1024ad}-\frac{\sin(7dx+7c)}{448ad}-\frac{\sin(6dx+6c)}{192ad}+\frac{\sin(5dx+5c)}{64ad}+\frac{\sin(4dx+4c)}{128ad}-3\sin$
derivativedivides	$-\frac{\frac{256\left(-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16384}-\frac{115\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{49152}-\frac{383\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{49152}-\frac{5053\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{344064}-\frac{44099\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{344064}+\frac{383(\tan^{11}(}{49152}\right.}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}}{ad}$
default	$-\frac{\frac{256\left(-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16384}-\frac{115\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{49152}-\frac{383\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{49152}-\frac{5053\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{344064}-\frac{44099\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{344064}+\frac{383(\tan^{11}(}{49152}\right.}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}}{ad}$
norman	$\frac{-\frac{5x}{128a}+\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64ad}+\frac{115\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192ad}+\frac{383\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192ad}+\frac{5053\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1344ad}+\frac{44099\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1344ad}-\frac{383(\tan^{11}(}{192a}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^8/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 256/d/a*(-(-5/16384*tan(1/2*d*x+1/2*c)-115/49152*tan(1/2*d*x+1/2*c)^3-383/4
9152*tan(1/2*d*x+1/2*c)^5-5053/344064*tan(1/2*d*x+1/2*c)^7-44099/344064*tan
(1/2*d*x+1/2*c)^9+383/49152*tan(1/2*d*x+1/2*c)^11+115/49152*tan(1/2*d*x+1/2
*c)^13+5/16384*tan(1/2*d*x+1/2*c)^15)/(1+tan(1/2*d*x+1/2*c)^2)^8-5/16384*ar
ctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 360 vs. 2(113) = 226.
time = 0.49, size = 360, normalized size = 2.88

$$\frac{\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}+\frac{805\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{2681\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5053\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{44099\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{2681\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-\frac{805\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}}-\frac{105\sin(dx+c)^{15}}{(\cos(dx+c)+1)^{15}}}{a+\frac{8a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{28a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{56a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{70a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{56a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{28a\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}+\frac{8a\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}+\frac{a\sin(dx+c)^{16}}{(\cos(dx+c)+1)^{16}}}-\frac{105\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{1344\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^8/(a+a*sec(d*x+c)),x, algorithm="maxima")

[Out] 1/1344*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 805*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2681*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5053*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 44099*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 2681*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - 805*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 - 105*sin(d*x + c)^15/(cos(d*x + c) + 1)^15)/(a + 8*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 28*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 56*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 70*a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 56*a*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 28*a*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 + 8*a*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 + a*sin(d*x + c)^16/(cos(d*x + c) + 1)^16) - 105*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d

Fricas [A]
time = 3.40, size = 91, normalized size = 0.73

$$-\frac{105\,dx-\left(336\,\cos\left(dx+c\right)^{7}-384\,\cos\left(dx+c\right)^{6}-952\,\cos\left(dx+c\right)^{5}+1152\,\cos\left(dx+c\right)^{4}+826\,\cos\left(dx+c\right)^{3}-1152\,\cos\left(dx+c\right)^{2}-105\,\cos\left(dx+c\right)+384\right)\sin\left(dx+c\right)}{2688\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^8/(a+a*sec(d*x+c)),x, algorithm="fricas")

[Out] -1/2688*(105*d*x - (336*cos(d*x + c)^7 - 384*cos(d*x + c)^6 - 952*cos(d*x + c)^5 + 1152*cos(d*x + c)^4 + 826*cos(d*x + c)^3 - 1152*cos(d*x + c)^2 - 105*cos(d*x + c) + 384)*sin(d*x + c))/(a*d)

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)**8/(a+a*sec(d*x+c)),x)

[Out] Timed out

Giac [A]
time = 0.45, size = 139, normalized size = 1.11

$$-\frac{\frac{105\,(dx+c)}{a}+\frac{2\left(105\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}+805\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+2681\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-44099\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-5053\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-2681\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-805\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-105\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{8}a}}{2688\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^8/(a+a*sec(d*x+c)),x, algorithm="giac")

[Out] -1/2688*(105*(d*x + c)/a + 2*(105*tan(1/2*d*x + 1/2*c)^15 + 805*tan(1/2*d*x + 1/2*c)^13 + 2681*tan(1/2*d*x + 1/2*c)^11 - 44099*tan(1/2*d*x + 1/2*c)^9

```
- 5053*tan(1/2*d*x + 1/2*c)^7 - 2681*tan(1/2*d*x + 1/2*c)^5 - 805*tan(1/2*d
*x + 1/2*c)^3 - 105*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^8*a
))/d
```

Mupad [B]
time = 3.90, size = 132, normalized size = 1.06

$$\frac{-\frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}}{64}-\frac{115\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{192}-\frac{383\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{192}+\frac{44099\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}}{1344}+\frac{5053\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}{1344}+\frac{383\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}}{192}+\frac{115\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}}{192}+\frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{64}}{a\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^8}-\frac{5\,x}{128\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^8/(a + a/cos(c + d*x)),x)
```

```
[Out] ((5*tan(c/2 + (d*x)/2))/64 + (115*tan(c/2 + (d*x)/2)^3)/192 + (383*tan(c/2
+ (d*x)/2)^5)/192 + (5053*tan(c/2 + (d*x)/2)^7)/1344 + (44099*tan(c/2 + (d*
x)/2)^9)/1344 - (383*tan(c/2 + (d*x)/2)^11)/192 - (115*tan(c/2 + (d*x)/2)^1
3)/192 - (5*tan(c/2 + (d*x)/2)^15)/64)/(a*d*(tan(c/2 + (d*x)/2)^2 + 1)^8) -
 (5*x)/(128*a)
```

3.66 $\int \frac{\sin^6(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=99

$$-\frac{x}{16a}-\frac{\cos(c+dx)\sin(c+dx)}{16ad}+\frac{\cos^3(c+dx)\sin(c+dx)}{8ad}+\frac{\cos^3(c+dx)\sin^3(c+dx)}{6ad}+\frac{\sin^5(c+dx)}{5ad}$$

```
[Out] -1/16*x/a-1/16*cos(d*x+c)*sin(d*x+c)/a/d+1/8*cos(d*x+c)^3*sin(d*x+c)/a/d+1/
6*cos(d*x+c)^3*sin(d*x+c)^3/a/d+1/5*sin(d*x+c)^5/a/d
```

Rubi [A]
time = 0.14, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3957, 2918, 2644, 30, 2648, 2715, 8}

$$\frac{\sin^5(c+dx)}{5ad}+\frac{\sin^3(c+dx)\cos^3(c+dx)}{6ad}+\frac{\sin(c+dx)\cos^3(c+dx)}{8ad}-\frac{\sin(c+dx)\cos(c+dx)}{16ad}-\frac{x}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^6/(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/16*x/a - (Cos[c + d*x]*Sin[c + d*x])/(16*a*d) + (Cos[c + d*x]^3*Sin[c +
d*x])/(8*a*d) + (Cos[c + d*x]^3*Sin[c + d*x]^3)/(6*a*d) + Sin[c + d*x]^5/(5
*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
```

```
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^6(c+dx)}{a+a\sec(c+dx)}\,dx &= -\int \frac{\cos(c+dx)\sin^6(c+dx)}{-a-a\cos(c+dx)}\,dx \\
&= \frac{\int \cos(c+dx)\sin^4(c+dx)\,dx}{a} - \frac{\int \cos^2(c+dx)\sin^4(c+dx)\,dx}{a} \\
&= \frac{\cos^3(c+dx)\sin^3(c+dx)}{6ad} - \frac{\int \cos^2(c+dx)\sin^2(c+dx)\,dx}{2a} + \frac{\mathrm{Subst}\left(\int x^4\,dx, x, \mathrm{s}\right.}{ad} \\
&= \frac{\cos^3(c+dx)\sin(c+dx)}{8ad} + \frac{\cos^3(c+dx)\sin^3(c+dx)}{6ad} + \frac{\sin^5(c+dx)}{5ad} - \frac{\int \cos^2(c}{8a} \\
&= -\frac{\cos(c+dx)\sin(c+dx)}{16ad} + \frac{\cos^3(c+dx)\sin(c+dx)}{8ad} + \frac{\cos^3(c+dx)\sin^3(c+dx)}{6ad} \\
&= -\frac{x}{16a} - \frac{\cos(c+dx)\sin(c+dx)}{16ad} + \frac{\cos^3(c+dx)\sin(c+dx)}{8ad} + \frac{\cos^3(c+dx)\sin^3(}{6ad}
\end{aligned}$$

Mathematica [A]
time = 0.47, size = 112, normalized size = 1.13

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(75c-60dx+120\sin(c+dx)+15\sin(2(c+dx))-60\sin(3(c+dx))+15\sin(4(c+dx))+12\sin(5(c+dx))-5\sin(6(c+dx))-75\tan\left(\frac{c}{2}\right)\right)}{480ad(1+\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^6/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*Sec[c + d*x]*(75*c - 60*d*x + 120*Sin[c + d*x] + 15*Sin
[2*(c + d*x)] - 60*Sin[3*(c + d*x)] + 15*Sin[4*(c + d*x)] + 12*Sin[5*(c + d
*x)] - 5*Sin[6*(c + d*x)] - 75*Tan[c/2]))/(480*a*d*(1 + Sec[c + d*x]))
```

Maple [A]
time = 0.13, size = 116, normalized size = 1.17

method	result
risch	$-\frac{x}{16a}+\frac{\sin(dx+c)}{8ad}-\frac{\sin(6dx+6c)}{192ad}+\frac{\sin(5dx+5c)}{80ad}+\frac{\sin(4dx+4c)}{64ad}-\frac{\sin(3dx+3c)}{16ad}+\frac{\sin(2dx+2c)}{64ad}$
derivativedivides	$\frac{-\frac{64\left(\frac{\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{512}+\frac{17\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1536}-\frac{223\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1280}-\frac{33\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1280}-\frac{17\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1536}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{512}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\text{arct}\cdots}{ad}$
default	$\frac{-\frac{64\left(\frac{\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{512}+\frac{17\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1536}-\frac{223\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1280}-\frac{33\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1280}-\frac{17\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1536}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{512}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\text{arct}\cdots}{ad}$
norman	$\frac{-\frac{x}{16a}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{17\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24ad}+\frac{33\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20ad}+\frac{223\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20ad}-\frac{17\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24ad}-\frac{\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-3x\left(\text{ta}\cdots\right.}{\left(1+\tan^2\left(\frac{dx}{2}+\cdots\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^6/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 64/d/a*(-(1/512*tan(1/2*d*x+1/2*c)^11+17/1536*tan(1/2*d*x+1/2*c)^9-223/1280
*tan(1/2*d*x+1/2*c)^7-33/1280*tan(1/2*d*x+1/2*c)^5-17/1536*tan(1/2*d*x+1/2*
c)^3-1/512*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^6-1/512*arctan(tan(
1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 278 vs. 2(89) = 178.
time = 0.48, size = 278, normalized size = 2.81

$$\frac{\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}+\frac{85\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{198\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{1338\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{85\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{15\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}}{a+\frac{6a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{20a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{15a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{6a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{a\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}}-\frac{15\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{120d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/120*((15*sin(d*x + c)/(cos(d*x + c) + 1) + 85*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 + 198*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 1338*sin(d*x + c)^7/(c
os(d*x + c) + 1)^7 - 85*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 15*sin(d*x +
```

```
c)^11/(cos(d*x + c) + 1)^11)/(a + 6*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 +
 15*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 20*a*sin(d*x + c)^6/(cos(d*x +
c) + 1)^6 + 15*a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 6*a*sin(d*x + c)^10/
(cos(d*x + c) + 1)^10 + a*sin(d*x + c)^12/(cos(d*x + c) + 1)^12) - 15*arcta
n(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

Fricas [A]
time = 3.06, size = 70, normalized size = 0.71

$$-\frac{15\,dx+\left(40\,\cos\left(dx+c\right)^{5}-48\,\cos\left(dx+c\right)^{4}-70\,\cos\left(dx+c\right)^{3}+96\,\cos\left(dx+c\right)^{2}+15\,\cos\left(dx+c\right)-48\right)\sin\left(dx+c\right)}{240\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/240*(15*d*x + (40*cos(d*x + c)^5 - 48*cos(d*x + c)^4 - 70*cos(d*x + c)^3
 + 96*cos(d*x + c)^2 + 15*cos(d*x + c) - 48)*sin(d*x + c))/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sin^{6}\left(c+dx\right)}{\sec\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**6/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)**6/(sec(c + d*x) + 1), x)/a
```

Giac [A]
time = 0.43, size = 113, normalized size = 1.14

$$-\frac{\frac{15\,(dx+c)}{a}+\frac{2\left(15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+85\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-1338\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-198\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-85\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{6}a}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/240*(15*(d*x + c)/a + 2*(15*tan(1/2*d*x + 1/2*c)^11 + 85*tan(1/2*d*x + 1
/2*c)^9 - 1338*tan(1/2*d*x + 1/2*c)^7 - 198*tan(1/2*d*x + 1/2*c)^5 - 85*tan
(1/2*d*x + 1/2*c)^3 - 15*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1
)^6*a))/d
```

Mupad [B]
time = 3.66, size = 106, normalized size = 1.07

$$\frac{-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}-\frac{17\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}+\frac{223\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{20}+\frac{33\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{20}+\frac{17\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{24}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}}{a\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)^{6}}-\frac{x}{16\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6/(a + a/cos(c + d*x)),x)
```

```
[Out] (tan(c/2 + (d*x)/2)/8 + (17*tan(c/2 + (d*x)/2)^3)/24 + (33*tan(c/2 + (d*x)/
2)^5)/20 + (223*tan(c/2 + (d*x)/2)^7)/20 - (17*tan(c/2 + (d*x)/2)^9)/24 - t
an(c/2 + (d*x)/2)^11/8)/(a*d*(tan(c/2 + (d*x)/2)^2 + 1)^6) - x/(16*a)
```

3.67 $\int \frac{\sin^4(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=73

$$-\frac{x}{8a}-\frac{\cos(c+dx)\sin(c+dx)}{8ad}+\frac{\cos^3(c+dx)\sin(c+dx)}{4ad}+\frac{\sin^3(c+dx)}{3ad}$$

```
[Out] -1/8*x/a-1/8*cos(d*x+c)*sin(d*x+c)/a/d+1/4*cos(d*x+c)^3*sin(d*x+c)/a/d+1/3*
sin(d*x+c)^3/a/d
```

Rubi [A]
time = 0.11, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3957, 2918, 2644, 30, 2648, 2715, 8}

$$\frac{\sin^3(c+dx)}{3ad}+\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}-\frac{\sin(c+dx)\cos(c+dx)}{8ad}-\frac{x}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/8*x/a - (Cos[c + d*x]*Sin[c + d*x])/(8*a*d) + (Cos[c + d*x]^3*Sin[c + d*
x])/(4*a*d) + Sin[c + d*x]^3/(3*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
```

```
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^4(c+dx)}{a+a\sec(c+dx)}\,dx &= -\int \frac{\cos(c+dx)\sin^4(c+dx)}{-a-a\cos(c+dx)}\,dx\\
&= \frac{\int \cos(c+dx)\sin^2(c+dx)\,dx}{a}-\frac{\int \cos^2(c+dx)\sin^2(c+dx)\,dx}{a}\\
&= \frac{\cos^3(c+dx)\sin(c+dx)}{4ad}-\frac{\int \cos^2(c+dx)\,dx}{4a}+\frac{\operatorname{Subst}\left(\int x^2\,dx,x,\sin(c+dx)\right)}{ad}\\
&= -\frac{\cos(c+dx)\sin(c+dx)}{8ad}+\frac{\cos^3(c+dx)\sin(c+dx)}{4ad}+\frac{\sin^3(c+dx)}{3ad}-\frac{\int 1\,dx}{8a}\\
&= -\frac{x}{8a}-\frac{\cos(c+dx)\sin(c+dx)}{8ad}+\frac{\cos^3(c+dx)\sin(c+dx)}{4ad}+\frac{\sin^3(c+dx)}{3ad}
\end{aligned}
$$

Mathematica [**A**]
time = 0.42, size = 83, normalized size = 1.14

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(24\sin(c+dx)-8\sin(3(c+dx))+3\left(4c-4dx+\sin(4(c+dx))-4\tan\left(\frac{c}{2}\right)\right)\right)}{48ad(1+\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*Sec[c + d*x]*(24*Sin[c + d*x] - 8*Sin[3*(c + d*x)] + 3*
(4*c - 4*d*x + Sin[4*(c + d*x)] - 4*Tan[c/2])))/(48*a*d*(1 + Sec[c + d*x]))
```

Maple [A]
time = 0.09, size = 90, normalized size = 1.23

method	result
risch	$-\frac{x}{8a}+\frac{\sin(dx+c)}{4ad}+\frac{\sin(4dx+4c)}{32ad}-\frac{\sin(3dx+3c)}{12ad}$
derivativedivides	$\frac{-\frac{16\left(\frac{\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64}-\frac{53\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192}-\frac{11\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64}\right)}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}-\frac{\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}}{ad}$
default	$\frac{-\frac{16\left(\frac{\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64}-\frac{53\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192}-\frac{11\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64}\right)}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}-\frac{\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}}{ad}$
norman	$\frac{-\frac{x}{8a}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}+\frac{11\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12ad}+\frac{53\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12ad}-\frac{\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}-\frac{x\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{3x\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a}-\frac{x(\tan^{6}}{}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 16/d/a*(-(1/64*tan(1/2*d*x+1/2*c)^7-53/192*tan(1/2*d*x+1/2*c)^5-11/192*tan(
1/2*d*x+1/2*c)^3-1/64*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^4-1/64*a
rctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(65) = 130$.
time = 0.48, size = 196, normalized size = 2.68

$$\frac{\frac{\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{11\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{53\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{3\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}}{a+\frac{4a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{6a\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{4a\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{a\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}}-\frac{3\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*((3*sin(d*x + c)/(cos(d*x + c) + 1) + 11*sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3 + 53*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 3*sin(d*x + c)^7/(cos(d*x
 + c) + 1)^7)/(a + 4*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a*sin(d*x +
c)^4/(cos(d*x + c) + 1)^4 + 4*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a*sin
(d*x + c)^8/(cos(d*x + c) + 1)^8) - 3*arctan(sin(d*x + c)/(cos(d*x + c) + 1
))/a)/d
```

Fricas [A]
time = 2.78, size = 51, normalized size = 0.70

$$-\frac{3\,dx - \left(6\,\cos\left(dx+c\right)^{3} - 8\,\cos\left(dx+c\right)^{2} - 3\,\cos\left(dx+c\right) + 8\right)\sin\left(dx+c\right)}{24\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/24*(3*d*x - (6*cos(d*x + c)^3 - 8*cos(d*x + c)^2 - 3*cos(d*x + c) + 8)*s
in(d*x + c))/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sin^{4}\left(c+dx\right)}{\sec\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)**4/(sec(c + d*x) + 1), x)/a
```

Giac [A]
time = 0.43, size = 87, normalized size = 1.19

$$-\frac{\frac{3\,(dx+c)}{a} + \frac{2\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7} - 53\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5} - 11\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3} - 3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{4}a}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/24*(3*(d*x + c)/a + 2*(3*tan(1/2*d*x + 1/2*c)^7 - 53*tan(1/2*d*x + 1/2*c
)^5 - 11*tan(1/2*d*x + 1/2*c)^3 - 3*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1
/2*c)^2 + 1)^4*a))/d
```

Mupad [B]
time = 0.98, size = 55, normalized size = 0.75

$$\frac{\sin\left(c+d\,x\right)}{4\,a\,d} - \frac{x}{8\,a} - \frac{\sin\left(3\,c+3\,d\,x\right)}{12\,a\,d} + \frac{\sin\left(4\,c+4\,d\,x\right)}{32\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(a + a/cos(c + d*x)),x)
```

```
[Out] sin(c + d*x)/(4*a*d) - x/(8*a) - sin(3*c + 3*d*x)/(12*a*d) + sin(4*c + 4*d*
x)/(32*a*d)
```

3.68 $\int \frac{\sin^2(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=44

$$-\frac{x}{2a}+\frac{\sin(c+dx)}{ad}-\frac{\cos(c+dx)\sin(c+dx)}{2ad}$$

```
[Out] -1/2*x/a+sin(d*x+c)/a/d-1/2*cos(d*x+c)*sin(d*x+c)/a/d
```

Rubi [A]
time = 0.08, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.238$, Rules used = {3957, 2918, 2717, 2715, 8}

$$\frac{\sin(c+dx)}{ad}-\frac{\sin(c+dx)\cos(c+dx)}{2ad}-\frac{x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/2*x/a + Sin[c + d*x]/(a*d) - (Cos[c + d*x]*Sin[c + d*x])/(2*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^2(c+dx)}{a+a\sec(c+dx)}\,dx &= -\int \frac{\cos(c+dx)\sin^2(c+dx)}{-a-a\cos(c+dx)}\,dx\\
&= \frac{\int \cos(c+dx)\,dx}{a} - \frac{\int \cos^2(c+dx)\,dx}{a}\\
&= \frac{\sin(c+dx)}{ad} - \frac{\cos(c+dx)\sin(c+dx)}{2ad} - \frac{\int 1\,dx}{2a}\\
&= -\frac{x}{2a} + \frac{\sin(c+dx)}{ad} - \frac{\cos(c+dx)\sin(c+dx)}{2ad}
\end{aligned}
$$

Mathematica [A]
time = 0.21, size = 68, normalized size = 1.55

$$
-\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(-c+2dx-4\sin(c+dx)+\sin(2(c+dx))+\tan\left(\frac{c}{2}\right)\right)}{2ad(1+\sec(c+dx))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/2*(Cos[(c + d*x)/2]^2*Sec[c + d*x]*(-c + 2*d*x - 4*Sin[c + d*x] + Sin[2*
(c + d*x)] + Tan[c/2]))/(a*d*(1 + Sec[c + d*x]))
```

Maple [A]
time = 0.08, size = 64, normalized size = 1.45

method	result	size
risch	$-\frac{x}{2a}+\frac{\sin(dx+c)}{ad}-\frac{\sin(2dx+2c)}{4ad}$	38
derivativedivides	$\frac{-\frac{4\left(-\frac{3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}$	64
default	$\frac{-\frac{4\left(-\frac{3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}$	64

norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{x}{2a}+\frac{3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 4/d/a*(-(-3/4*tan(1/2*d*x+1/2*c)^3-1/4*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1
/2*c)^2)^2-1/4*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(40) = 80$.
time = 0.48, size = 112, normalized size = 2.55

$$\frac{\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a+\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] ((sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/
(a + 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(cos(d*x +
c) + 1)^4) - arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

Fricas [A]
time = 2.46, size = 27, normalized size = 0.61

$$-\frac{dx+(\cos(dx+c)-2)\sin(dx+c)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(d*x + (cos(d*x + c) - 2)*sin(d*x + c))/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sin^2(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)**2/(sec(c + d*x) + 1), x)/a
```

Giac [A]
time = 0.42, size = 58, normalized size = 1.32

$$-\frac{\frac{dx+c}{a}-\frac{2\left(3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{2}a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*((d*x + c)/a - 2*(3*tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))/((t
an(1/2*d*x + 1/2*c)^2 + 1)^2*a))/d
```

Mupad [B]
time = 0.96, size = 30, normalized size = 0.68

$$-\frac{\sin\left(2\,c+2\,d\,x\right)-4\,\sin\left(c+d\,x\right)+2\,d\,x}{4\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(a + a/cos(c + d*x)),x)
```

```
[Out] -(sin(2*c + 2*d*x) - 4*sin(c + d*x) + 2*d*x)/(4*a*d)
```

3.69 $\int \frac{\csc^2(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=37

$$\frac{\cot^3(c+dx)}{3ad}-\frac{\csc^3(c+dx)}{3ad}$$

```
[Out] 1/3*cot(d*x+c)^3/a/d-1/3*csc(d*x+c)^3/a/d
```

Rubi [A]
time = 0.09, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3957, 2918, 2686, 30, 2687}

$$\frac{\cot^3(c+dx)}{3ad}-\frac{\csc^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + a*Sec[c + d*x]),x]
```

```
[Out] Cot[c + d*x]^3/(3*a*d) - Csc[c + d*x]^3/(3*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
```

```
e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^2(c+dx)}{a+a\sec(c+dx)}\,dx &= -\int \frac{\cot(c+dx)\csc(c+dx)}{-a-a\cos(c+dx)}\,dx\\ &= -\frac{\int \cot^2(c+dx)\csc^2(c+dx)\,dx}{a}+\frac{\int \cot(c+dx)\csc^3(c+dx)\,dx}{a}\\ &= -\frac{\operatorname{Subst}\left(\int x^2\,dx,x,-\cot(c+dx)\right)}{ad}-\frac{\operatorname{Subst}\left(\int x^2\,dx,x,\csc(c+dx)\right)}{ad}\\ &= \frac{\cot^3(c+dx)}{3ad}-\frac{\csc^3(c+dx)}{3ad}\end{aligned}$$

Mathematica [A]
time = 0.16, size = 66, normalized size = 1.78

$$\frac{\csc(c)\csc(2(c+dx))(-6\sin(c)+4\sin(dx)+2\sin(c+dx)+\sin(2(c+dx))+2\sin(c+2dx))}{6ad(1+\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a + a*Sec[c + d*x]),x]
[Out] (Csc[c]*Csc[2*(c + d*x)]*(-6*Sin[c] + 4*Sin[d*x] + 2*Sin[c + d*x] + Sin[2*(
c + d*x)] + 2*Sin[c + 2*d*x]))/(6*a*d*(1 + Sec[c + d*x]))
```

Maple [A]
time = 0.08, size = 36, normalized size = 0.97

method	result	size
derivativedivides	$\frac{-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{4da}$	36
default	$\frac{-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{4da}$	36

norman	$\frac{-\frac{1}{4ad}-\frac{\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)}{12ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}$	41
risch	$-\frac{2i\left(3\,e^{2i(dx+c)}+2\,e^{i(dx+c)}+1\right)}{3ad\left(e^{i(dx+c)}+1\right)^3\left(e^{i(dx+c)}-1\right)}$	60

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^2/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/4/d/a*(-1/3*tan(1/2*d*x+1/2*c)^3-1/tan(1/2*d*x+1/2*c))

Maxima [A]
time = 0.27, size = 49, normalized size = 1.32

$$-\frac{\frac{3\,(\cos(dx+c)+1)}{a\sin(dx+c)}+\frac{\sin(dx+c)^3}{a(\cos(dx+c)+1)^3}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="maxima")

[Out] -1/12*(3*(cos(d*x + c) + 1)/(a*sin(d*x + c)) + sin(d*x + c)^3/(a*(cos(d*x + c) + 1)^3))/d

Fricas [A]
time = 3.50, size = 41, normalized size = 1.11

$$-\frac{\cos(dx+c)^2+\cos(dx+c)+1}{3\,(ad\cos(dx+c)+ad)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="fricas")

[Out] -1/3*(cos(d*x + c)^2 + cos(d*x + c) + 1)/((a*d*cos(d*x + c) + a*d)*sin(d*x + c))

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\csc^2(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)**2/(a+a*sec(d*x+c)),x)

[Out] Integral(csc(c + d*x)**2/(sec(c + d*x) + 1), x)/a

Giac [A]
time = 0.46, size = 37, normalized size = 1.00

$$-\frac{\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}}{a}+\frac{3}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*(tan(1/2*d*x + 1/2*c)^3/a + 3/(a*tan(1/2*d*x + 1/2*c)))/d
```

Mupad [B]
time = 0.93, size = 32, normalized size = 0.86

$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3}{12\,a\,d\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^2*(a + a/cos(c + d*x))),x)
```

```
[Out] -(tan(c/2 + (d*x)/2)^4 + 3)/(12*a*d*tan(c/2 + (d*x)/2))
```

3.70 $\int \frac{\csc^4(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=55

$$\frac{\cot^3(c+dx)}{3ad}+\frac{\cot^5(c+dx)}{5ad}-\frac{\csc^5(c+dx)}{5ad}$$

```
[Out] 1/3*cot(d*x+c)^3/a/d+1/5*cot(d*x+c)^5/a/d-1/5*csc(d*x+c)^5/a/d
```

Rubi [A]
time = 0.11, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3957, 2918, 2686, 30, 2687, 14}

$$\frac{\cot^5(c+dx)}{5ad}+\frac{\cot^3(c+dx)}{3ad}-\frac{\csc^5(c+dx)}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

```
[Out] Cot[c + d*x]^3/(3*a*d) + Cot[c + d*x]^5/(5*a*d) - Csc[c + d*x]^5/(5*a*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^4(c+dx)}{a+a\sec(c+dx)}\,dx &= -\int \frac{\cot(c+dx)\csc^3(c+dx)}{-a-a\cos(c+dx)}\,dx\\ &= -\frac{\int \cot^2(c+dx)\csc^4(c+dx)\,dx}{a}+\frac{\int \cot(c+dx)\csc^5(c+dx)\,dx}{a}\\ &= -\frac{\operatorname{Subst}\left(\int x^4\,dx,x,\csc(c+dx)\right)}{ad}-\frac{\operatorname{Subst}\left(\int x^2(1+x^2)\,dx,x,-\cot(c+dx)\right)}{ad}\\ &= -\frac{\csc^5(c+dx)}{5ad}-\frac{\operatorname{Subst}\left(\int (x^2+x^4)\,dx,x,-\cot(c+dx)\right)}{ad}\\ &= \frac{\cot^3(c+dx)}{3ad}+\frac{\cot^5(c+dx)}{5ad}-\frac{\csc^5(c+dx)}{5ad}\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 116 vs. 2(55) = 110.
time = 0.35, size = 116, normalized size = 2.11

$$-\frac{\csc(c)\csc^3(c+dx)\sec(c+dx)(240\sin(c)-96\sin(dx)-54\sin(c+dx)-18\sin(2(c+dx))+18\sin(3(c+dx))+9\sin(4(c+dx))-32\sin(c+2dx)+32\sin(2c+3dx)+16\sin(3c+4dx))}{960ad(1+\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/960*(Csc[c]*Csc[c + d*x]^3*Sec[c + d*x]*(240*Sin[c] - 96*Sin[d*x] - 54*S
in[c + d*x] - 18*Sin[2*(c + d*x)] + 18*Sin[3*(c + d*x)] + 9*Sin[4*(c + d*x)
] - 32*Sin[c + 2*d*x] + 32*Sin[2*c + 3*d*x] + 16*Sin[3*c + 4*d*x]))/(a*d*(1
 + Sec[c + d*x]))
```

Maple [**A**]
time = 0.10, size = 62, normalized size = 1.13

method	result	size
derivativedivides	$\frac{-\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}}{16da}$	62
default	$\frac{-\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}}{16da}$	62
norman	$\frac{-\frac{1}{48ad}-\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)}{24ad}-\frac{\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)}{80ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}$	79
risch	$\frac{4i\left(15\,e^{4i(dx+c)}+6\,e^{3i(dx+c)}+2\,e^{2i(dx+c)}-2\,e^{i(dx+c)}-1\right)}{15ad\left(e^{i(dx+c)}+1\right)^5\left(e^{i(dx+c)}-1\right)^3}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/d/a*(-1/5*tan(1/2*d*x+1/2*c)^5-2/3*tan(1/2*d*x+1/2*c)^3-2/tan(1/2*d*x+
1/2*c)-1/3/tan(1/2*d*x+1/2*c)^3)
```

Maxima [A]
time = 0.27, size = 96, normalized size = 1.75

$$-\frac{\frac{\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a}+\frac{5\left(\frac{6\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)(\cos(dx+c)+1)^3}{a\sin(dx+c)^3}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/240*((10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x
 + c) + 1)^5)/a + 5*(6*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)*(cos(d*x +
c) + 1)^3/(a*sin(d*x + c)^3))/d
```

Fricas [A]
time = 3.21, size = 89, normalized size = 1.62

$$-\frac{2\cos(dx+c)^4+2\cos(dx+c)^3-3\cos(dx+c)^2-3\cos(dx+c)-3}{15\left(ad\cos(dx+c)^3+ad\cos(dx+c)^2-ad\cos(dx+c)-ad\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/15*(2*cos(d*x + c)^4 + 2*cos(d*x + c)^3 - 3*cos(d*x + c)^2 - 3*cos(d*x +
 c) - 3)/((a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2 - a*d*cos(d*x + c) - a*d
)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^4(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**4/(sec(c + d*x) + 1), x)/a
```

Giac [A]
time = 0.45, size = 74, normalized size = 1.35

$$-\frac{\frac{5\left(6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}+\frac{3\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+10\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}{a^5}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/240*(5*(6*tan(1/2*d*x + 1/2*c)^2 + 1)/(a*tan(1/2*d*x + 1/2*c)^3) + (3*a^
4*tan(1/2*d*x + 1/2*c)^5 + 10*a^4*tan(1/2*d*x + 1/2*c)^3)/a^5)/d
```

Mupad [B]
time = 1.09, size = 60, normalized size = 1.09

$$-\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+30\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+5}{240\,a\,d\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^4*(a + a/cos(c + d*x))),x)
```

```
[Out] -(30*tan(c/2 + (d*x)/2)^2 + 10*tan(c/2 + (d*x)/2)^6 + 3*tan(c/2 + (d*x)/2)^
8 + 5)/(240*a*d*tan(c/2 + (d*x)/2)^3)
```

3.71 $\int \frac{\csc^6(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=73

$$\frac{\cot^3(c+dx)}{3ad}+\frac{2\cot^5(c+dx)}{5ad}+\frac{\cot^7(c+dx)}{7ad}-\frac{\csc^7(c+dx)}{7ad}$$

```
[Out] 1/3*cot(d*x+c)^3/a/d+2/5*cot(d*x+c)^5/a/d+1/7*cot(d*x+c)^7/a/d-1/7*csc(d*x+
c)^7/a/d
```

Rubi [A]
time = 0.11, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3957, 2918, 2686, 30, 2687, 276}

$$\frac{\cot^7(c+dx)}{7ad}+\frac{2\cot^5(c+dx)}{5ad}+\frac{\cot^3(c+dx)}{3ad}-\frac{\csc^7(c+dx)}{7ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6/(a + a*Sec[c + d*x]),x]
```

```
[Out] Cot[c + d*x]^3/(3*a*d) + (2*Cot[c + d*x]^5)/(5*a*d) + Cot[c + d*x]^7/(7*a*d
) - Csc[c + d*x]^7/(7*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc^6(c+dx)}{a+a\sec(c+dx)}\,dx &= -\int \frac{\cot(c+dx)\csc^5(c+dx)}{-a-a\cos(c+dx)}\,dx \\
&= -\frac{\int \cot^2(c+dx)\csc^6(c+dx)\,dx}{a}+\frac{\int \cot(c+dx)\csc^7(c+dx)\,dx}{a} \\
&= -\frac{\operatorname{Subst}\left(\int x^6\,dx,x,\csc(c+dx)\right)}{ad}-\frac{\operatorname{Subst}\left(\int x^2\left(1+x^2\right)^2\,dx,x,-\cot(c+dx)\right)}{ad} \\
&= -\frac{\csc^7(c+dx)}{7ad}-\frac{\operatorname{Subst}\left(\int \left(x^2+2x^4+x^6\right)\,dx,x,-\cot(c+dx)\right)}{ad} \\
&= \frac{\cot^3(c+dx)}{3ad}+\frac{2\cot^5(c+dx)}{5ad}+\frac{\cot^7(c+dx)}{7ad}-\frac{\csc^7(c+dx)}{7ad}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 158 vs. $2(73) = 146$.
time = 0.42, size = 158, normalized size = 2.16

$$
\frac{\csc(c)\csc^5(c+dx)\sec(c+dx)(-8960\sin(c)+2560\sin(dx)+1500\sin(c+dx)+375\sin(2(c+dx))-750\sin(3(c+dx))-300\sin(4(c+dx))+150\sin(5(c+dx))+75\sin(6(c+dx))+640\sin(c+2dx)-1280\sin(2c+3dx)-512\sin(3c+4dx)+256\sin(4c+5dx)+128\sin(5c+6dx))}{53760ad(1+\sec(c+dx))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Csc[c]*Csc[c + d*x]^5*Sec[c + d*x]*(-8960*Sin[c] + 2560*Sin[d*x] + 1500*Si
n[c + d*x] + 375*Sin[2*(c + d*x)] - 750*Sin[3*(c + d*x)] - 300*Sin[4*(c + d
*x)] + 150*Sin[5*(c + d*x)] + 75*Sin[6*(c + d*x)] + 640*Sin[c + 2*d*x] - 12
```

```
80*Sin[2*c + 3*d*x] - 512*Sin[3*c + 4*d*x] + 256*Sin[4*c + 5*d*x] + 128*Sin
[5*c + 6*d*x]))/(53760*a*d*(1 + Sec[c + d*x]))
```

Maple [A]
time = 0.10, size = 88, normalized size = 1.21

method	result	size
derivativedivides	$\frac{-\frac{\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{4\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{5\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{5}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{1}{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}-\frac{4}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}}{64da}$	88
default	$\frac{-\frac{\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{4\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{5\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{5}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{1}{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}-\frac{4}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}}{64da}$	88
risch	$-\frac{16i\left(70\,e^{6i(dx+c)}+20\,e^{5i(dx+c)}+5\,e^{4i(dx+c)}-10\,e^{3i(dx+c)}-4\,e^{2i(dx+c)}+2\,e^{i(dx+c)}+1\right)}{105ad\left(e^{i(dx+c)}+1\right)^{7}\left(e^{i(dx+c)}-1\right)^{5}}$	104
norman	$\frac{-\frac{1}{320ad}-\frac{\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}{48ad}-\frac{5\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64ad}-\frac{5\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192ad}-\frac{\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)}{80ad}-\frac{\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)}{448ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}$	117

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/64/d/a*(-1/7*tan(1/2*d*x+1/2*c)^7-4/5*tan(1/2*d*x+1/2*c)^5-5/3*tan(1/2*d*
x+1/2*c)^3-5/tan(1/2*d*x+1/2*c)-1/5/tan(1/2*d*x+1/2*c)^5-4/3/tan(1/2*d*x+1/
2*c)^3)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(65) = 130$.
time = 0.28, size = 136, normalized size = 1.86

$$-\frac{\frac{\frac{175\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{84\,\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{15\,\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}}{a}+\frac{7\left(\frac{20\,\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{75\,\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+3\right)(\cos(dx+c)+1)^{5}}{a\sin(dx+c)^{5}}}{6720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6720*((175*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 84*sin(d*x + c)^5/(cos(
d*x + c) + 1)^5 + 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a + 7*(20*sin(d*x
 + c)^2/(cos(d*x + c) + 1)^2 + 75*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 3)*
(cos(d*x + c) + 1)^5/(a*sin(d*x + c)^5))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(65) = 130$.
time = 3.67, size = 131, normalized size = 1.79

$$-\frac{8\cos(dx+c)^{6}+8\cos(dx+c)^{5}-20\cos(dx+c)^{4}-20\cos(dx+c)^{3}+15\cos(dx+c)^{2}+15\cos(dx+c)+15}{105\left(ad\cos(dx+c)^{5}+ad\cos(dx+c)^{4}-2\,ad\cos(dx+c)^{3}-2\,ad\cos(dx+c)^{2}+ad\cos(dx+c)+ad\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/105*(8*cos(d*x + c)^6 + 8*cos(d*x + c)^5 - 20*cos(d*x + c)^4 - 20*cos(d*
x + c)^3 + 15*cos(d*x + c)^2 + 15*cos(d*x + c) + 15)/((a*d*cos(d*x + c)^5 +
 a*d*cos(d*x + c)^4 - 2*a*d*cos(d*x + c)^3 - 2*a*d*cos(d*x + c)^2 + a*d*cos
(d*x + c) + a*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^6(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**6/(sec(c + d*x) + 1), x)/a
```

Giac [A]
time = 0.46, size = 103, normalized size = 1.41

$$-\frac{\frac{7\left(75\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+20\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+3\right)}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}}+\frac{15\,a^{6}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+84\,a^{6}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+175\,a^{6}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}}{a^{7}}}{6720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6720*(7*(75*tan(1/2*d*x + 1/2*c)^4 + 20*tan(1/2*d*x + 1/2*c)^2 + 3)/(a*t
an(1/2*d*x + 1/2*c)^5) + (15*a^6*tan(1/2*d*x + 1/2*c)^7 + 84*a^6*tan(1/2*d*
x + 1/2*c)^5 + 175*a^6*tan(1/2*d*x + 1/2*c)^3)/a^7)/d
```

Mupad [B]
time = 1.16, size = 153, normalized size = 2.10

$$-\frac{21\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+140\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+525\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}+175\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}+84\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+15\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}}{6720\,a\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^6*(a + a/cos(c + d*x))),x)
```

```
[Out] -(21*cos(c/2 + (d*x)/2)^12 + 15*sin(c/2 + (d*x)/2)^12 + 84*cos(c/2 + (d*x)/
2)^2*sin(c/2 + (d*x)/2)^10 + 175*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^8
+ 525*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^4 + 140*cos(c/2 + (d*x)/2)^10
*sin(c/2 + (d*x)/2)^2)/(6720*a*d*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^5)
```

3.72 $\int \frac{\csc^8(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=91

$$\frac{\cot^3(c+dx)}{3ad}+\frac{3\cot^5(c+dx)}{5ad}+\frac{3\cot^7(c+dx)}{7ad}+\frac{\cot^9(c+dx)}{9ad}-\frac{\csc^9(c+dx)}{9ad}$$

```
[Out] 1/3*cot(d*x+c)^3/a/d+3/5*cot(d*x+c)^5/a/d+3/7*cot(d*x+c)^7/a/d+1/9*cot(d*x+
c)^9/a/d-1/9*csc(d*x+c)^9/a/d
```

Rubi [A]
time = 0.12, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3957, 2918, 2686, 30, 2687, 276}

$$\frac{\cot^9(c+dx)}{9ad}+\frac{3\cot^7(c+dx)}{7ad}+\frac{3\cot^5(c+dx)}{5ad}+\frac{\cot^3(c+dx)}{3ad}-\frac{\csc^9(c+dx)}{9ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^8/(a + a*Sec[c + d*x]),x]
```

```
[Out] Cot[c + d*x]^3/(3*a*d) + (3*Cot[c + d*x]^5)/(5*a*d) + (3*Cot[c + d*x]^7)/(7
*a*d) + Cot[c + d*x]^9/(9*a*d) - Csc[c + d*x]^9/(9*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc^8(c+dx)}{a+a\sec(c+dx)}\,dx &= -\int \frac{\cot(c+dx)\csc^7(c+dx)}{-a-a\cos(c+dx)}\,dx \\
&= -\frac{\int \cot^2(c+dx)\csc^8(c+dx)\,dx}{a} + \frac{\int \cot(c+dx)\csc^9(c+dx)\,dx}{a} \\
&= -\frac{\operatorname{Subst}\left(\int x^8\,dx, x, \csc(c+dx)\right)}{ad} - \frac{\operatorname{Subst}\left(\int x^2(1+x^2)^3\,dx, x, -\cot(c+dx)\right)}{ad} \\
&= -\frac{\csc^9(c+dx)}{9ad} - \frac{\operatorname{Subst}\left(\int (x^2+3x^4+3x^6+x^8)\,dx, x, -\cot(c+dx)\right)}{ad} \\
&= \frac{\cot^3(c+dx)}{3ad} + \frac{3\cot^5(c+dx)}{5ad} + \frac{3\cot^7(c+dx)}{7ad} + \frac{\cot^9(c+dx)}{9ad} - \frac{\csc^9(c+dx)}{9ad}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 200 vs. 2(91) = 182.
time = 0.65, size = 200, normalized size = 2.20

$$
-\frac{\csc(c)\csc^7(c+dx)\sec(c+dx)(645120\sin(c)-143360\sin(dx)-85750\sin(c+dx)-17150\sin(2(c+dx))+51450\sin(3(c+dx))+17150\sin(4(c+dx))-17150\sin(5(c+dx))-7350\sin(6(c+dx))+2450\sin(7(c+dx))+1225\sin(8(c+dx))-28672\sin(c+2dx)+86016\sin(2c+3dx)+28672\sin(3c+4dx)-28672\sin(4c+5dx)-12288\sin(5c+6dx)+4096\sin(6c+7dx)+2048\sin(7c+8dx))}{5160960ad(1+\sec(c+dx))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^8/(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/5160960*(Csc[c]*Csc[c + d*x]^7*Sec[c + d*x]*(645120*Sin[c] - 143360*Sin[
d*x] - 85750*Sin[c + d*x] - 17150*Sin[2*(c + d*x)] + 51450*Sin[3*(c + d*x)]
 + 17150*Sin[4*(c + d*x)] - 17150*Sin[5*(c + d*x)] - 7350*Sin[6*(c + d*x)]
```

```
+ 2450*Sin[7*(c + d*x)] + 1225*Sin[8*(c + d*x)] - 28672*Sin[c + 2*d*x] + 86
016*Sin[2*c + 3*d*x] + 28672*Sin[3*c + 4*d*x] - 28672*Sin[4*c + 5*d*x] - 12
288*Sin[5*c + 6*d*x] + 4096*Sin[6*c + 7*d*x] + 2048*Sin[7*c + 8*d*x]))/(a*d
*(1 + Sec[c + d*x]))
```

Maple [A]
time = 0.11, size = 114, normalized size = 1.25

method	result
derivativedivides	$\frac{-\frac{\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}-\frac{6\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{14\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{14\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{14}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{6}{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}-\frac{14}{\tan\left(\frac{dx}{2}+\cdots\right.}\cdots}{256da}$
default	$\frac{-\frac{\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}-\frac{6\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{14\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{14\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{14}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{6}{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}-\frac{14}{\tan\left(\frac{dx}{2}+\cdots\right.}\cdots}{256da}$
risch	$\frac{32i\left(315\,\mathrm{e}^{8i(dx+c)}+70\,\mathrm{e}^{7i(dx+c)}+14\,\mathrm{e}^{6i(dx+c)}-42\,\mathrm{e}^{5i(dx+c)}-14\,\mathrm{e}^{4i(dx+c)}+14\,\mathrm{e}^{3i(dx+c)}+6\,\mathrm{e}^{2i(dx+c)}-2\,\mathrm{e}^{i(dx+c)}-1\right)}{315ad\left(\mathrm{e}^{i(dx+c)}+1\right)^9\left(\mathrm{e}^{i(dx+c)}-1\right)^7}$
norman	$\frac{-\frac{1}{1792ad}-\frac{3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{640ad}-\frac{7\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{384ad}-\frac{7\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128ad}-\frac{7\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{384ad}-\frac{7\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{640ad}-\frac{3\left(\tan^{14}\left(\frac{dx}{2}+\cdots\right)\right)}{896ad}\cdots}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^8/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/256/d/a*(-1/9*tan(1/2*d*x+1/2*c)^9-6/7*tan(1/2*d*x+1/2*c)^7-14/5*tan(1/2*
d*x+1/2*c)^5-14/3*tan(1/2*d*x+1/2*c)^3-14/3/tan(1/2*d*x+1/2*c)^3-6/5/tan(1/
2*d*x+1/2*c)^5-14/tan(1/2*d*x+1/2*c)-1/7/tan(1/2*d*x+1/2*c)^7)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(81) = 162$.
time = 0.27, size = 176, normalized size = 1.93

$$-\frac{\frac{\frac{1470\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{882\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{270\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{35\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a}+\frac{3\left(\frac{126\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{490\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{1470\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+15\right)(\cos(dx+c)+1)^7}{a\sin(dx+c)^7}}{80640\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/80640*((1470*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 882*sin(d*x + c)^5/(c
os(d*x + c) + 1)^5 + 270*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 35*sin(d*x +
 c)^9/(cos(d*x + c) + 1)^9)/a + 3*(126*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
+ 490*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1470*sin(d*x + c)^6/(cos(d*x +
c) + 1)^6 + 15)*(cos(d*x + c) + 1)^7/(a*sin(d*x + c)^7))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(81) = 162$.

time = 2.82, size = 177, normalized size = 1.95

$$-\frac{16 \cos \left(dx+c\right)^{8}+16 \cos \left(dx+c\right)^{7}-56 \cos \left(dx+c\right)^{6}-56 \cos \left(dx+c\right)^{5}+70 \cos \left(dx+c\right)^{4}+70 \cos \left(dx+c\right)^{3}-35 \cos \left(dx+c\right)^{2}-35 \cos \left(dx+c\right)-35}{315 \left(ad \cos \left(dx+c\right)^{7}+ad \cos \left(dx+c\right)^{6}-3 ad \cos \left(dx+c\right)^{5}-3 ad \cos \left(dx+c\right)^{4}+3 ad \cos \left(dx+c\right)^{3}+3 ad \cos \left(dx+c\right)^{2}-ad \cos \left(dx+c\right)-ad\right) \sin \left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/315*(16*cos(d*x + c)^8 + 16*cos(d*x + c)^7 - 56*cos(d*x + c)^6 - 56*cos(
d*x + c)^5 + 70*cos(d*x + c)^4 + 70*cos(d*x + c)^3 - 35*cos(d*x + c)^2 - 35
*cos(d*x + c) - 35)/((a*d*cos(d*x + c)^7 + a*d*cos(d*x + c)^6 - 3*a*d*cos(d
*x + c)^5 - 3*a*d*cos(d*x + c)^4 + 3*a*d*cos(d*x + c)^3 + 3*a*d*cos(d*x + c
)^2 - a*d*cos(d*x + c) - a*d)*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**8/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.48, size = 132, normalized size = 1.45

$$-\frac{\frac{3 \left(1470 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{6}+490 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{4}+126 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{2}+15\right)}{a \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{7}}+\frac{35 a^{8} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{9}+270 a^{8} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{7}+882 a^{8} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{5}+1470 a^{8} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{3}}{a^{9}}}{80640 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/80640*(3*(1470*tan(1/2*d*x + 1/2*c)^6 + 490*tan(1/2*d*x + 1/2*c)^4 + 126
*tan(1/2*d*x + 1/2*c)^2 + 15)/(a*tan(1/2*d*x + 1/2*c)^7) + (35*a^8*tan(1/2*
d*x + 1/2*c)^9 + 270*a^8*tan(1/2*d*x + 1/2*c)^7 + 882*a^8*tan(1/2*d*x + 1/2
*c)^5 + 1470*a^8*tan(1/2*d*x + 1/2*c)^3)/a^9)/d
```

Mupad [B]
time = 1.45, size = 201, normalized size = 2.21

$$-\frac{45 \cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}+378 \cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{14} \sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+1470 \cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12} \sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}+4410 \cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10} \sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+1470 \cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{6} \sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+882 \cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{4} \sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+270 \cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{2} \sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+35 \sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}}{80640\, a\, d \cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{9} \sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^8*(a + a/cos(c + d*x))),x)
```

```
[Out] -(45*cos(c/2 + (d*x)/2)^16 + 35*sin(c/2 + (d*x)/2)^16 + 270*cos(c/2 + (d*x)
/2)^2*sin(c/2 + (d*x)/2)^14 + 882*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^1
2 + 1470*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^10 + 4410*cos(c/2 + (d*x)/
2)^10*sin(c/2 + (d*x)/2)^6 + 1470*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^
4 + 378*cos(c/2 + (d*x)/2)^14*sin(c/2 + (d*x)/2)^2)/(80640*a*d*cos(c/2 + (d
*x)/2)^9*sin(c/2 + (d*x)/2)^7)
```

3.73 $\int \frac{\csc^{10}(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=109

$$\frac{\cot^3(c+dx)}{3ad}+\frac{4\cot^5(c+dx)}{5ad}+\frac{6\cot^7(c+dx)}{7ad}+\frac{4\cot^9(c+dx)}{9ad}+\frac{\cot^{11}(c+dx)}{11ad}-\frac{\csc^{11}(c+dx)}{11ad}$$

```
[Out] 1/3*cot(d*x+c)^3/a/d+4/5*cot(d*x+c)^5/a/d+6/7*cot(d*x+c)^7/a/d+4/9*cot(d*x+
c)^9/a/d+1/11*cot(d*x+c)^11/a/d-1/11*csc(d*x+c)^11/a/d
```

Rubi [A]
time = 0.12, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3957, 2918, 2686, 30, 2687, 276}

$$\frac{\cot^{11}(c+dx)}{11ad}+\frac{4\cot^9(c+dx)}{9ad}+\frac{6\cot^7(c+dx)}{7ad}+\frac{4\cot^5(c+dx)}{5ad}+\frac{\cot^3(c+dx)}{3ad}-\frac{\csc^{11}(c+dx)}{11ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^10/(a + a*Sec[c + d*x]),x]
```

```
[Out] Cot[c + d*x]^3/(3*a*d) + (4*Cot[c + d*x]^5)/(5*a*d) + (6*Cot[c + d*x]^7)/(7
*a*d) + (4*Cot[c + d*x]^9)/(9*a*d) + Cot[c + d*x]^11/(11*a*d) - Csc[c + d*x
]^11/(11*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^{10}(c+dx)}{a+a\sec(c+dx)}\,dx &= -\int \frac{\cot(c+dx)\csc^9(c+dx)}{-a-a\cos(c+dx)}\,dx \\
&= -\frac{\int \cot^2(c+dx)\csc^{10}(c+dx)\,dx}{a} + \frac{\int \cot(c+dx)\csc^{11}(c+dx)\,dx}{a} \\
&= -\frac{\text{Subst}\left(\int x^{10}\,dx, x, \csc(c+dx)\right)}{ad} - \frac{\text{Subst}\left(\int x^2\left(1+x^2\right)^4\,dx, x, -\cot(c+dx)\right)}{ad} \\
&= -\frac{\csc^{11}(c+dx)}{11ad} - \frac{\text{Subst}\left(\int \left(x^2+4x^4+6x^6+4x^8+x^{10}\right)dx, x, -\cot(c+dx)\right)}{ad} \\
&= \frac{\cot^3(c+dx)}{3ad} + \frac{4\cot^5(c+dx)}{5ad} + \frac{6\cot^7(c+dx)}{7ad} + \frac{4\cot^9(c+dx)}{9ad} + \frac{\cot^{11}(c+dx)}{11ad}
\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 242 vs. 2(109) = 218.
time = 0.97, size = 242, normalized size = 2.22

$$\frac{\csc(c)\csc^9(c+dx)\sec(c+dx)(-45416448\sin(c)+8257536\sin(dx)+5000940\sin(c+dx)+833490\sin(2(c+dx))-3333960\sin(3(c+dx))-952560\sin(4(c+dx))+1428840\sin(5(c+dx))+535815\sin(6(c+dx))-357210\sin(7(c+dx))-158760\sin(8(c+dx))+39690\sin(9(c+dx))+19845\sin(10(c+dx))+1376256\sin(c+2dx)-5505024\sin(2c+3dx)-1572864\sin(3c+4dx)+2359296\sin(4c+5dx)+884736\sin(5c+6dx)-589824\sin(6c+7dx)-262144\sin(7c+8dx)+65536\sin(8c+9dx)+32768\sin(9c+10dx))}{454164480ad(1+\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^10/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Csc[c]*Csc[c + d*x]^9*Sec[c + d*x]*(-45416448*Sin[c] + 8257536*Sin[d*x] +
5000940*Sin[c + d*x] + 833490*Sin[2*(c + d*x)] - 3333960*Sin[3*(c + d*x)] -
```

```
 952560*Sin[4*(c + d*x)] + 1428840*Sin[5*(c + d*x)] + 535815*Sin[6*(c + d*x
)] - 357210*Sin[7*(c + d*x)] - 158760*Sin[8*(c + d*x)] + 39690*Sin[9*(c + d
*x)] + 19845*Sin[10*(c + d*x)] + 1376256*Sin[c + 2*d*x] - 5505024*Sin[2*c +
 3*d*x] - 1572864*Sin[3*c + 4*d*x] + 2359296*Sin[4*c + 5*d*x] + 884736*Sin[
5*c + 6*d*x] - 589824*Sin[6*c + 7*d*x] - 262144*Sin[7*c + 8*d*x] + 65536*Si
n[8*c + 9*d*x] + 32768*Sin[9*c + 10*d*x]))/(454164480*a*d*(1 + Sec[c + d*x]
))
```

Maple [A]
time = 0.12, size = 140, normalized size = 1.28

method	result
derivativedivides	$\frac{-\frac{\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{11}-\frac{8\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}-\frac{27\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{48\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-14\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}-\frac{}{\tan(}}{1024da}$
default	$\frac{-\frac{\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{11}-\frac{8\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}-\frac{27\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{48\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-14\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}-\frac{}{\tan(}}{1024da}$
risch	$-\frac{256i\left(1386\,e^{10i(dx+c)}+252\,e^{9i(dx+c)}+42\,e^{8i(dx+c)}-168\,e^{7i(dx+c)}-48\,e^{6i(dx+c)}+72\,e^{5i(dx+c)}+27\,e^{4i(dx+c)}-18\,e^{3i(dx}\right.}{3465ad\left(e^{i(dx+c)}+1\right)^{11}\left(e^{i(dx+c)}-1\right)^{9}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^10/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/1024/d/a*(-1/11*tan(1/2*d*x+1/2*c)^11-8/9*tan(1/2*d*x+1/2*c)^9-27/7*tan(1
/2*d*x+1/2*c)^7-48/5*tan(1/2*d*x+1/2*c)^5-14*tan(1/2*d*x+1/2*c)^3-1/9/tan(1
/2*d*x+1/2*c)^9-16/tan(1/2*d*x+1/2*c)^3-42/tan(1/2*d*x+1/2*c)-8/7/tan(1/2*d
*x+1/2*c)^7-27/5/tan(1/2*d*x+1/2*c)^5)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(97) = 194$.
time = 0.28, size = 216, normalized size = 1.98

$$-\frac{\frac{\frac{48510\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{33264\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{13365\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{3080\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{315\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}}{a}+\frac{11\left(\frac{360\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{1701\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{5040\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{13230\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+35\right)(\cos(dx+c)+1)^9}{a\sin(dx+c)^9}}{3548160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^10/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/3548160*((48510*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 33264*sin(d*x + c)
^5/(cos(d*x + c) + 1)^5 + 13365*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 3080*
sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 315*sin(d*x + c)^11/(cos(d*x + c) + 1
)^11)/a + 11*(360*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1701*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4 + 5040*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 13230*si
n(d*x + c)^8/(cos(d*x + c) + 1)^8 + 35)*(cos(d*x + c) + 1)^9/(a*sin(d*x + c
)^9))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(97) = 194$.
time = 3.16, size = 219, normalized size = 2.01

$$-\frac{128 \cos\left(dx+c\right)^{10} + 128 \cos\left(dx+c\right)^{9} - 576 \cos\left(dx+c\right)^{8} - 576 \cos\left(dx+c\right)^{7} + 1008 \cos\left(dx+c\right)^{6} + 1008 \cos\left(dx+c\right)^{5} - 840 \cos\left(dx+c\right)^{4} - 840 \cos\left(dx+c\right)^{3} + 315 \cos\left(dx+c\right)^{2} + 315 \cos\left(dx+c\right) + 315}{3465 \left(ad \cos\left(dx+c\right)^{9} + ad \cos\left(dx+c\right)^{8} - 4\, ad \cos\left(dx+c\right)^{7} - 4\, ad \cos\left(dx+c\right)^{6} + 6\, ad \cos\left(dx+c\right)^{5} + 6\, ad \cos\left(dx+c\right)^{4} - 4\, ad \cos\left(dx+c\right)^{3} - 4\, ad \cos\left(dx+c\right)^{2} + ad \cos\left(dx+c\right) + ad\right) \sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^10/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/3465*(128*cos(d*x + c)^10 + 128*cos(d*x + c)^9 - 576*cos(d*x + c)^8 - 57
6*cos(d*x + c)^7 + 1008*cos(d*x + c)^6 + 1008*cos(d*x + c)^5 - 840*cos(d*x
+ c)^4 - 840*cos(d*x + c)^3 + 315*cos(d*x + c)^2 + 315*cos(d*x + c) + 315)/
((a*d*cos(d*x + c)^9 + a*d*cos(d*x + c)^8 - 4*a*d*cos(d*x + c)^7 - 4*a*d*co
s(d*x + c)^6 + 6*a*d*cos(d*x + c)^5 + 6*a*d*cos(d*x + c)^4 - 4*a*d*cos(d*x
+ c)^3 - 4*a*d*cos(d*x + c)^2 + a*d*cos(d*x + c) + a*d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**10/(a+a*sec(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4371 deep
```

Giac [A]
time = 0.50, size = 161, normalized size = 1.48

$$-\frac{\frac{11 \left(13230 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+5040 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+1701 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+360 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+35\right)}{a \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}} + \frac{315\, a^{10} \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+3080\, a^{10} \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+13365\, a^{10} \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+33264\, a^{10} \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+48510\, a^{10} \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}}{a^{11}}}{3548160\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^10/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/3548160*(11*(13230*tan(1/2*d*x + 1/2*c)^8 + 5040*tan(1/2*d*x + 1/2*c)^6
+ 1701*tan(1/2*d*x + 1/2*c)^4 + 360*tan(1/2*d*x + 1/2*c)^2 + 35)/(a*tan(1/2
*d*x + 1/2*c)^9) + (315*a^10*tan(1/2*d*x + 1/2*c)^11 + 3080*a^10*tan(1/2*d*
x + 1/2*c)^9 + 13365*a^10*tan(1/2*d*x + 1/2*c)^7 + 33264*a^10*tan(1/2*d*x +
 1/2*c)^5 + 48510*a^10*tan(1/2*d*x + 1/2*c)^3)/a^11)/d
```

Mupad [B]
time = 3.11, size = 139, normalized size = 1.28

$$-\frac{63 \cos\left(c+d\,x\right)+\frac{21 \cos\left(2\,c+2\,d\,x\right)}{2}-42 \cos\left(3\,c+3\,d\,x\right)-12 \cos\left(4\,c+4\,d\,x\right)+18 \cos\left(5\,c+5\,d\,x\right)+\frac{27 \cos\left(6\,c+6\,d\,x\right)}{4}-\frac{9 \cos\left(7\,c+7\,d\,x\right)}{2}-2 \cos\left(8\,c+8\,d\,x\right)+\frac{\cos\left(9\,c+9\,d\,x\right)}{2}+\frac{\cos\left(10\,c+10\,d\,x\right)}{4}+\frac{693}{2}}{3548160\, a\, d \cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11} \sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^10*(a + a/cos(c + d*x))),x)
```

```
[Out] -(63*cos(c + d*x) + (21*cos(2*c + 2*d*x))/2 - 42*cos(3*c + 3*d*x) - 12*cos(
4*c + 4*d*x) + 18*cos(5*c + 5*d*x) + (27*cos(6*c + 6*d*x))/4 - (9*cos(7*c +
 7*d*x))/2 - 2*cos(8*c + 8*d*x) + cos(9*c + 9*d*x)/2 + cos(10*c + 10*d*x)/4
 + 693/2)/(3548160*a*d*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2)^9)
```

3.74 $\int \frac{\sin^{11}(c+dx)}{(a+a\sec(c+dx))^2} dx$

Optimal. Leaf size=137

$$\frac{4(a-a\cos(c+dx))^6}{3a^8d}-\frac{4(a-a\cos(c+dx))^7}{a^9d}+\frac{19(a-a\cos(c+dx))^8}{4a^{10}d}-\frac{25(a-a\cos(c+dx))^9}{9a^{11}d}+\frac{4(a-a\cos(c+dx))^{10}}{5a^{12}d}$$

```
[Out] 4/3*(a-a*cos(d*x+c))^6/a^8/d-4*(a-a*cos(d*x+c))^7/a^9/d+19/4*(a-a*cos(d*x+c
))^8/a^10/d-25/9*(a-a*cos(d*x+c))^9/a^11/d+4/5*(a-a*cos(d*x+c))^10/a^12/d-1
/11*(a-a*cos(d*x+c))^11/a^13/d
```

Rubi [A]

time = 0.13, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3957, 2915, 12, 90}

$$-\frac{(a-a\cos(c+dx))^{11}}{11a^{13}d}+\frac{4(a-a\cos(c+dx))^{10}}{5a^{12}d}-\frac{25(a-a\cos(c+dx))^9}{9a^{11}d}+\frac{19(a-a\cos(c+dx))^8}{4a^{10}d}-\frac{4(a-a\cos(c+dx))^7}{a^9d}+\frac{4(a-a\cos(c+dx))^6}{3a^8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^11/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (4*(a - a*Cos[c + d*x])^6)/(3*a^8*d) - (4*(a - a*Cos[c + d*x])^7)/(a^9*d) +
 (19*(a - a*Cos[c + d*x])^8)/(4*a^10*d) - (25*(a - a*Cos[c + d*x])^9)/(9*a^
11*d) + (4*(a - a*Cos[c + d*x])^10)/(5*a^12*d) - (a - a*Cos[c + d*x])^11/(1
1*a^13*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^{11}(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= \int \frac{\cos^2(c+dx)\sin^{11}(c+dx)}{(-a-a\cos(c+dx))^2}\,dx\\ &= \frac{\text{Subst}\left(\int \frac{(-a-x)^5x^2(-a+x)^3}{a^2}\,dx, x, -a\cos(c+dx)\right)}{a^{11}d}\\ &= \frac{\text{Subst}\left(\int (-a-x)^5x^2(-a+x)^3\,dx, x, -a\cos(c+dx)\right)}{a^{13}d}\\ &= \frac{\text{Subst}\left(\int \left(-8a^5(-a-x)^5-28a^4(-a-x)^6-38a^3(-a-x)^7-25a^2(-a-x)^8\right.\right.}{a^{13}d}\\ &= \frac{4(a-a\cos(c+dx))^6}{3a^8d}-\frac{4(a-a\cos(c+dx))^7}{a^9d}+\frac{19(a-a\cos(c+dx))^8}{4a^{10}d}-\frac{25}{}\end{aligned}$$

Mathematica [A]
time = 3.13, size = 72, normalized size = 0.53

$$\frac{4(2360+4038\cos(c+dx)+2586\cos(2(c+dx))+1189\cos(3(c+dx))+342\cos(4(c+dx))+45\cos(5(c+dx)))\sin^{12}\left(\frac{1}{2}(c+dx)\right)}{495a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^11/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (4*(2360 + 4038*Cos[c + d*x] + 2586*Cos[2*(c + d*x)] + 1189*Cos[3*(c + d*x)
] + 342*Cos[4*(c + d*x)] + 45*Cos[5*(c + d*x)])*Sin[(c + d*x)/2]^12)/(495*a
^2*d)
```

Maple [A]
time = 0.14, size = 88, normalized size = 0.64

method	result
derivativedivides	$-\frac{\frac{1}{5\sec(dx+c)^{10}}-\frac{3}{4\sec(dx+c)^{8}}+\frac{2}{9\sec(dx+c)^{9}}+\frac{1}{3\sec(dx+c)^{3}}-\frac{1}{2\sec(dx+c)^{4}}-\frac{2}{5\sec(dx+c)^{5}}-\frac{1}{11\sec(dx+c)^{11}}+\frac{1}{\sec(dx+c)^{6}}}{d\,a^2}$
default	$-\frac{\frac{1}{5\sec(dx+c)^{10}}-\frac{3}{4\sec(dx+c)^{8}}+\frac{2}{9\sec(dx+c)^{9}}+\frac{1}{3\sec(dx+c)^{3}}-\frac{1}{2\sec(dx+c)^{4}}-\frac{2}{5\sec(dx+c)^{5}}-\frac{1}{11\sec(dx+c)^{11}}+\frac{1}{\sec(dx+c)^{6}}}{d\,a^2}$
risch	$-\frac{35\cos(dx+c)}{512a^2d}+\frac{\cos(11dx+11c)}{11264d\,a^2}-\frac{\cos(10dx+10c)}{2560d\,a^2}+\frac{\cos(9dx+9c)}{9216d\,a^2}+\frac{\cos(8dx+8c)}{512d\,a^2}-\frac{3\cos(7dx+7c)}{1024d\,a^2}-\frac{\cos(6}{51}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^11/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d/a^2*(1/5/sec(d*x+c)^10-3/4/sec(d*x+c)^8+2/9/sec(d*x+c)^9+1/3/sec(d*x+c
)^3-1/2/sec(d*x+c)^4-2/5/sec(d*x+c)^5-1/11/sec(d*x+c)^11+1/sec(d*x+c)^6)
```

Maxima [A]
time = 0.27, size = 89, normalized size = 0.65

$$\frac{180\cos(dx+c)^{11}-396\cos(dx+c)^{10}-440\cos(dx+c)^{9}+1485\cos(dx+c)^{8}-1980\cos(dx+c)^{6}+792\cos(dx+c)^{5}+990\cos(dx+c)^{4}-660\cos(dx+c)^{3}}{1980\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^11/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/1980*(180*cos(d*x + c)^11 - 396*cos(d*x + c)^10 - 440*cos(d*x + c)^9 + 14
85*cos(d*x + c)^8 - 1980*cos(d*x + c)^6 + 792*cos(d*x + c)^5 + 990*cos(d*x
+ c)^4 - 660*cos(d*x + c)^3)/(a^2*d)
```

Fricas [A]
time = 3.28, size = 89, normalized size = 0.65

$$\frac{180\cos(dx+c)^{11}-396\cos(dx+c)^{10}-440\cos(dx+c)^{9}+1485\cos(dx+c)^{8}-1980\cos(dx+c)^{6}+792\cos(dx+c)^{5}+990\cos(dx+c)^{4}-660\cos(dx+c)^{3}}{1980\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^11/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/1980*(180*cos(d*x + c)^11 - 396*cos(d*x + c)^10 - 440*cos(d*x + c)^9 + 14
85*cos(d*x + c)^8 - 1980*cos(d*x + c)^6 + 792*cos(d*x + c)^5 + 990*cos(d*x
+ c)^4 - 660*cos(d*x + c)^3)/(a^2*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**11/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6191 deep
```

Giac [A]
time = 0.53, size = 185, normalized size = 1.35

$$-\frac{64\left(\frac{11(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{55(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{165(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}-\frac{330(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}+\frac{462(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5}+\frac{198(\cos(dx+c)-1)^6}{(\cos(dx+c)+1)^6}+\frac{990(\cos(dx+c)-1)^7}{(\cos(dx+c)+1)^7}-1\right)}{495\,a^2d\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^11/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -64/495*(11*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 55*(cos(d*x + c) - 1)^2
/(cos(d*x + c) + 1)^2 + 165*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 330
*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 462*(cos(d*x + c) - 1)^5/(cos(
d*x + c) + 1)^5 + 198*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 990*(cos(
d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 - 1)/(a^2*d*((cos(d*x + c) - 1)/(cos(d
*x + c) + 1) - 1)^11)
```

Mupad [B]
time = 0.09, size = 109, normalized size = 0.80

$$-\frac{\frac{\cos(c+d\,x)^3}{3\,a^2}-\frac{\cos(c+d\,x)^4}{2\,a^2}-\frac{2\cos(c+d\,x)^5}{5\,a^2}+\frac{\cos(c+d\,x)^6}{a^2}-\frac{3\cos(c+d\,x)^8}{4\,a^2}+\frac{2\cos(c+d\,x)^9}{9\,a^2}+\frac{\cos(c+d\,x)^{10}}{5\,a^2}-\frac{\cos(c+d\,x)^{11}}{11\,a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^11/(a + a/cos(c + d*x))^2,x)
```

```
[Out] -(cos(c + d*x)^3/(3*a^2) - cos(c + d*x)^4/(2*a^2) - (2*cos(c + d*x)^5)/(5*a
^2) + cos(c + d*x)^6/a^2 - (3*cos(c + d*x)^8)/(4*a^2) + (2*cos(c + d*x)^9)/
(9*a^2) + cos(c + d*x)^10/(5*a^2) - cos(c + d*x)^11/(11*a^2))/d
```

3.75 $\int \frac{\sin^9(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=114

$$\frac{4(a-a\cos(c+dx))^5}{5a^7d}-\frac{2(a-a\cos(c+dx))^6}{a^8d}+\frac{13(a-a\cos(c+dx))^7}{7a^9d}-\frac{3(a-a\cos(c+dx))^8}{4a^{10}d}+\frac{(a-a\cos(c+dx))^9}{9a^{11}d}$$

```
[Out] 4/5*(a-a*cos(d*x+c))^5/a^7/d-2*(a-a*cos(d*x+c))^6/a^8/d+13/7*(a-a*cos(d*x+c
))^7/a^9/d-3/4*(a-a*cos(d*x+c))^8/a^10/d+1/9*(a-a*cos(d*x+c))^9/a^11/d
```

Rubi [A]
time = 0.13, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3957, 2915, 12, 90}

$$\frac{(a-a\cos(c+dx))^9}{9a^{11}d}-\frac{3(a-a\cos(c+dx))^8}{4a^{10}d}+\frac{13(a-a\cos(c+dx))^7}{7a^9d}-\frac{2(a-a\cos(c+dx))^6}{a^8d}+\frac{4(a-a\cos(c+dx))^5}{5a^7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^9/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (4*(a - a*Cos[c + d*x])^5)/(5*a^7*d) - (2*(a - a*Cos[c + d*x])^6)/(a^8*d) +
 (13*(a - a*Cos[c + d*x])^7)/(7*a^9*d) - (3*(a - a*Cos[c + d*x])^8)/(4*a^10
*d) + (a - a*Cos[c + d*x])^9/(9*a^11*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^9(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= \int \frac{\cos^2(c+dx)\sin^9(c+dx)}{(-a-a\cos(c+dx))^2}\,dx\\ &= \frac{\text{Subst}\left(\int \frac{(-a-x)^4x^2(-a+x)^2}{a^2}\,dx, x, -a\cos(c+dx)\right)}{a^9d}\\ &= \frac{\text{Subst}\left(\int (-a-x)^4x^2(-a+x)^2\,dx, x, -a\cos(c+dx)\right)}{a^{11}d}\\ &= \frac{\text{Subst}\left(\int \left(4a^4(-a-x)^4+12a^3(-a-x)^5+13a^2(-a-x)^6+6a(-a-x)^7+(\right.\right.}{a^{11}d}\\ &= \frac{4(a-a\cos(c+dx))^5}{5a^7d}-\frac{2(a-a\cos(c+dx))^6}{a^8d}+\frac{13(a-a\cos(c+dx))^7}{7a^9d}-\frac{3(a}{}\end{aligned}$$

Mathematica [A]
time = 2.28, size = 62, normalized size = 0.54

$$\frac{2(992+1615\cos(c+dx)+970\cos(2(c+dx))+385\cos(3(c+dx))+70\cos(4(c+dx)))\sin^{10}\left(\frac{1}{2}(c+dx)\right)}{315a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^9/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*(992 + 1615*Cos[c + d*x] + 970*Cos[2*(c + d*x)] + 385*Cos[3*(c + d*x)] +
 70*Cos[4*(c + d*x)])*Sin[(c + d*x)/2]^10)/(315*a^2*d)
```

Maple [A]
time = 0.17, size = 79, normalized size = 0.69

method	result
derivativedivides	$\frac{\frac{1}{2\sec(dx+c)^4}+\frac{1}{4\sec(dx+c)^8}-\frac{1}{3\sec(dx+c)^3}-\frac{1}{9\sec(dx+c)^9}-\frac{2}{3\sec(dx+c)^6}+\frac{1}{7\sec(dx+c)^7}+\frac{1}{5\sec(dx+c)^5}}{d\,a^2}$
default	$\frac{\frac{1}{2\sec(dx+c)^4}+\frac{1}{4\sec(dx+c)^8}-\frac{1}{3\sec(dx+c)^3}-\frac{1}{9\sec(dx+c)^9}-\frac{2}{3\sec(dx+c)^6}+\frac{1}{7\sec(dx+c)^7}+\frac{1}{5\sec(dx+c)^5}}{d\,a^2}$
norman	$\frac{-\frac{64}{315ad}-\frac{128\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{64\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{35ad}-\frac{256\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{35ad}-\frac{256\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15ad}-\frac{128\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5ad}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9a}$
risch	$-\frac{13\cos(dx+c)}{128a^2d}-\frac{\cos(9dx+9c)}{2304d\,a^2}+\frac{\cos(8dx+8c)}{512d\,a^2}-\frac{3\cos(7dx+7c)}{1792d\,a^2}-\frac{\cos(6dx+6c)}{192d\,a^2}+\frac{\cos(5dx+5c)}{80d\,a^2}-\frac{\cos(4dx+}{128d\,a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^9/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(1/2/sec(d*x+c)^4+1/4/sec(d*x+c)^8-1/3/sec(d*x+c)^3-1/9/sec(d*x+c)^
9-2/3/sec(d*x+c)^6+1/7/sec(d*x+c)^7+1/5/sec(d*x+c)^5)
```

Maxima [A]
time = 0.27, size = 79, normalized size = 0.69

$$-\frac{140 \cos(dx+c)^9 - 315 \cos(dx+c)^8 - 180 \cos(dx+c)^7 + 840 \cos(dx+c)^6 - 252 \cos(dx+c)^5 - 630 \cos(dx+c)^4 + 420 \cos(dx+c)^3}{1260\, a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^9/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/1260*(140*cos(d*x + c)^9 - 315*cos(d*x + c)^8 - 180*cos(d*x + c)^7 + 840
*cos(d*x + c)^6 - 252*cos(d*x + c)^5 - 630*cos(d*x + c)^4 + 420*cos(d*x + c
)^3)/(a^2*d)
```

Fricas [A]
time = 3.35, size = 79, normalized size = 0.69

$$-\frac{140 \cos(dx+c)^9 - 315 \cos(dx+c)^8 - 180 \cos(dx+c)^7 + 840 \cos(dx+c)^6 - 252 \cos(dx+c)^5 - 630 \cos(dx+c)^4 + 420 \cos(dx+c)^3}{1260\, a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^9/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/1260*(140*cos(d*x + c)^9 - 315*cos(d*x + c)^8 - 180*cos(d*x + c)^7 + 840
*cos(d*x + c)^6 - 252*cos(d*x + c)^5 - 630*cos(d*x + c)^4 + 420*cos(d*x + c
)^3)/(a^2*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**9/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [A]
time = 0.50, size = 141, normalized size = 1.24

$$-\frac{64 \left(\frac{9 (\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{36 (\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} + \frac{84 (\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3} - \frac{126 (\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4} - \frac{210 (\cos(dx+c)-1)^6}{(\cos(dx+c)+1)^6} - 1\right)}{315\, a^2 d \left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1} - 1\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^9/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -64/315*(9*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 36*(cos(d*x + c) - 1)^2/
(cos(d*x + c) + 1)^2 + 84*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 126*(
cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 210*(cos(d*x + c) - 1)^6/(cos(d*
x + c) + 1)^6 - 1)/(a^2*d*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^9)
```

Mupad [B]
time = 0.91, size = 96, normalized size = 0.84

$$\frac{\frac{\cos(c+d\,x)^4}{2\,a^2}-\frac{\cos(c+d\,x)^3}{3\,a^2}+\frac{\cos(c+d\,x)^5}{5\,a^2}-\frac{2\cos(c+d\,x)^6}{3\,a^2}+\frac{\cos(c+d\,x)^7}{7\,a^2}+\frac{\cos(c+d\,x)^8}{4\,a^2}-\frac{\cos(c+d\,x)^9}{9\,a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^9/(a + a/cos(c + d*x))^2,x)
```

```
[Out] (cos(c + d*x)^4/(2*a^2) - cos(c + d*x)^3/(3*a^2) + cos(c + d*x)^5/(5*a^2) -
 (2*cos(c + d*x)^6)/(3*a^2) + cos(c + d*x)^7/(7*a^2) + cos(c + d*x)^8/(4*a^
2) - cos(c + d*x)^9/(9*a^2))/d
```

3.76 $\int \frac{\sin^7(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=73

$$-\frac{\cos^3(c+dx)}{3a^2d}+\frac{\cos^4(c+dx)}{2a^2d}-\frac{\cos^6(c+dx)}{3a^2d}+\frac{\cos^7(c+dx)}{7a^2d}$$

```
[Out] -1/3*cos(d*x+c)^3/a^2/d+1/2*cos(d*x+c)^4/a^2/d-1/3*cos(d*x+c)^6/a^2/d+1/7*c
os(d*x+c)^7/a^2/d
```

Rubi [A]
time = 0.11, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.190$, Rules used = {3957, 2915, 12, 76}

$$\frac{\cos^7(c+dx)}{7a^2d}-\frac{\cos^6(c+dx)}{3a^2d}+\frac{\cos^4(c+dx)}{2a^2d}-\frac{\cos^3(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^7/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/3*Cos[c + d*x]^3/(a^2*d) + Cos[c + d*x]^4/(2*a^2*d) - Cos[c + d*x]^6/(3*
a^2*d) + Cos[c + d*x]^7/(7*a^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + 
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^7(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= \int \frac{\cos^2(c+dx)\sin^7(c+dx)}{(-a-a\cos(c+dx))^2}\,dx\\
&= \frac{\text{Subst}\left(\int \frac{(-a-x)^3x^2(-a+x)}{a^2}\,dx, x, -a\cos(c+dx)\right)}{a^7d}\\
&= \frac{\text{Subst}\left(\int(-a-x)^3x^2(-a+x)\,dx, x, -a\cos(c+dx)\right)}{a^9d}\\
&= \frac{\text{Subst}\left(\int\left(a^4x^2+2a^3x^3-2ax^5-x^6\right)\,dx, x, -a\cos(c+dx)\right)}{a^9d}\\
&= -\frac{\cos^3(c+dx)}{3a^2d}+\frac{\cos^4(c+dx)}{2a^2d}-\frac{\cos^6(c+dx)}{3a^2d}+\frac{\cos^7(c+dx)}{7a^2d}
\end{aligned}$$

Mathematica [A]
time = 1.19, size = 53, normalized size = 0.73

$$\frac{4(17\cos(c+dx)+10\cos(2(c+dx))+3(4+\cos(3(c+dx))))\sin^8\left(\frac{1}{2}(c+dx)\right)}{21a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^7/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (4*(17*Cos[c + d*x] + 10*Cos[2*(c + d*x)] + 3*(4 + Cos[3*(c + d*x)]))*Sin[(
c + d*x)/2]^8)/(21*a^2*d)
```

Maple [A]
time = 0.14, size = 50, normalized size = 0.68

method	result	size
derivativedivides	$-\frac{\frac{1}{3\sec(dx+c)^3}-\frac{1}{2\sec(dx+c)^4}-\frac{1}{7\sec(dx+c)^7}+\frac{1}{3\sec(dx+c)^6}}{d\,a^2}$	50
default	$-\frac{\frac{1}{3\sec(dx+c)^3}-\frac{1}{2\sec(dx+c)^4}-\frac{1}{7\sec(dx+c)^7}+\frac{1}{3\sec(dx+c)^6}}{d\,a^2}$	50
risch	$-\frac{11\cos(dx+c)}{64a^2d}+\frac{\cos(7dx+7c)}{448d\,a^2}-\frac{\cos(6dx+6c)}{96d\,a^2}+\frac{\cos(5dx+5c)}{64d\,a^2}-\frac{7\cos(3dx+3c)}{192d\,a^2}+\frac{3\cos(2dx+2c)}{32d\,a^2}$	101
norman	$\frac{-\frac{16\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{8\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{8}{21ad}-\frac{8\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{40\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{16\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{7}a}$	124

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^7/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d/a^2*(1/3/sec(d*x+c)^3-1/2/sec(d*x+c)^4-1/7/sec(d*x+c)^7+1/3/sec(d*x+c)
^6)
```

Maxima [A]
time = 0.26, size = 49, normalized size = 0.67

$$\frac{6 \cos(dx+c)^7 - 14 \cos(dx+c)^6 + 21 \cos(dx+c)^4 - 14 \cos(dx+c)^3}{42 a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/42*(6*cos(d*x + c)^7 - 14*cos(d*x + c)^6 + 21*cos(d*x + c)^4 - 14*cos(d*x
 + c)^3)/(a^2*d)
```

Fricas [A]
time = 3.78, size = 49, normalized size = 0.67

$$\frac{6 \cos(dx+c)^7 - 14 \cos(dx+c)^6 + 21 \cos(dx+c)^4 - 14 \cos(dx+c)^3}{42 a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/42*(6*cos(d*x + c)^7 - 14*cos(d*x + c)^6 + 21*cos(d*x + c)^4 - 14*cos(d*x
 + c)^3)/(a^2*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**7/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(65) = 130.
time = 0.46, size = 141, normalized size = 1.93

$$-\frac{8\left(\frac{7(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{21(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} + \frac{35(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3} - \frac{14(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4} + \frac{42(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5} - 1\right)}{21 a^2 d\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1} - 1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -8/21*(7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 21*(cos(d*x + c) - 1)^2/(c
os(d*x + c) + 1)^2 + 35*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 14*(cos
(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 42*(cos(d*x + c) - 1)^5/(cos(d*x +
c) + 1)^5 - 1)/(a^2*d*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^7)
```

Mupad [B]
time = 0.92, size = 58, normalized size = 0.79

$$-\frac{\frac{\cos(c+d\,x)^3}{3\,a^2}-\frac{\cos(c+d\,x)^4}{2\,a^2}+\frac{\cos(c+d\,x)^6}{3\,a^2}-\frac{\cos(c+d\,x)^7}{7\,a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^7/(a + a/cos(c + d*x))^2,x)
```

```
[Out] -(cos(c + d*x)^3/(3*a^2) - cos(c + d*x)^4/(2*a^2) + cos(c + d*x)^6/(3*a^2)
- cos(c + d*x)^7/(7*a^2))/d
```

3.77 $\int \frac{\sin^5(c+dx)}{(a+a\sec(c+dx))^2}\, dx$

Optimal. Leaf size=55

$$-\frac{\cos^3(c+dx)}{3a^2d}+\frac{\cos^4(c+dx)}{2a^2d}-\frac{\cos^5(c+dx)}{5a^2d}$$

```
[Out] -1/3*cos(d*x+c)^3/a^2/d+1/2*cos(d*x+c)^4/a^2/d-1/5*cos(d*x+c)^5/a^2/d
```

Rubi [A]
time = 0.11, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3957, 2915, 12, 45}

$$-\frac{\cos^5(c+dx)}{5a^2d}+\frac{\cos^4(c+dx)}{2a^2d}-\frac{\cos^3(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^5/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/3*Cos[c + d*x]^3/(a^2*d) + Cos[c + d*x]^4/(2*a^2*d) - Cos[c + d*x]^5/(5*
a^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
```

```
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)\sin^5(c+dx)}{(-a-a\cos(c+dx))^2}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{(-a-x)^2x^2}{a^2}\,dx, x, -a\cos(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int (-a-x)^2x^2\,dx, x, -a\cos(c+dx)\right)}{a^7d}$$
$$= \frac{\text{Subst}\left(\int \left(a^2x^2+2ax^3+x^4\right)\,dx, x, -a\cos(c+dx)\right)}{a^7d}$$
$$= -\frac{\cos^3(c+dx)}{3a^2d}+\frac{\cos^4(c+dx)}{2a^2d}-\frac{\cos^5(c+dx)}{5a^2d}$$

Mathematica [A]
time = 0.43, size = 42, normalized size = 0.76

$$\frac{4(4+3\cos(c+dx)+3\cos(2(c+dx)))\sin^6\left(\frac{1}{2}(c+dx)\right)}{15a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^5/(a + a*Sec[c + d*x])^2,x]
[Out] (4*(4 + 3*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]^6)/(15*a^2*d)
```

Maple [A]
time = 0.11, size = 39, normalized size = 0.71

method	result	size
derivativedivides	$\frac{\frac{1}{2\sec(dx+c)^4}-\frac{1}{5\sec(dx+c)^5}-\frac{1}{3\sec(dx+c)^3}}{d\,a^2}$	39
default	$\frac{\frac{1}{2\sec(dx+c)^4}-\frac{1}{5\sec(dx+c)^5}-\frac{1}{3\sec(dx+c)^3}}{d\,a^2}$	39
risch	$-\frac{3\cos(dx+c)}{8a^2d}-\frac{\cos(5dx+5c)}{80d\,a^2}+\frac{\cos(4dx+4c)}{16d\,a^2}-\frac{7\cos(3dx+3c)}{48d\,a^2}+\frac{\cos(2dx+2c)}{4d\,a^2}$	84
norman	$\frac{-\frac{8\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{16}{15ad}-\frac{8\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{16\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{32\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5a}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^5/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

[Out] 1/d/a^2*(1/2/sec(d*x+c)^4-1/5/sec(d*x+c)^5-1/3/sec(d*x+c)^3)

Maxima [A]
time = 0.27, size = 39, normalized size = 0.71

$$-\frac{6\cos(dx+c)^5-15\cos(dx+c)^4+10\cos(dx+c)^3}{30a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/30*(6*cos(d*x + c)^5 - 15*cos(d*x + c)^4 + 10*cos(d*x + c)^3)/(a^2*d)
```

Fricas [A]
time = 3.85, size = 39, normalized size = 0.71

$$-\frac{6\cos(dx+c)^5-15\cos(dx+c)^4+10\cos(dx+c)^3}{30a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/30*(6*cos(d*x + c)^5 - 15*cos(d*x + c)^4 + 10*cos(d*x + c)^3)/(a^2*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**5/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(49) = 98.
time = 0.48, size = 119, normalized size = 2.16

$$-\frac{8\left(\frac{10(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{20(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{15(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}-\frac{15(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}-2\right)}{15a^2d\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -8/15*(10*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 20*(cos(d*x + c) - 1)^2/(
cos(d*x + c) + 1)^2 + 15*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 15*(co
```

```
s(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 2)/(a^2*d*((cos(d*x + c) - 1)/(cos
(d*x + c) + 1) - 1)^5)
```

Mupad [B]
time = 0.06, size = 36, normalized size = 0.65

$$-\frac{\cos\left(c+d\,x\right)^3\,\left(6\cos\left(c+d\,x\right)^2-15\,\cos\left(c+d\,x\right)+10\right)}{30\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5/(a + a/cos(c + d*x))^2,x)
[Out] -(cos(c + d*x)^3*(6*cos(c + d*x)^2 - 15*cos(c + d*x) + 10))/(30*a^2*d)
```

3.78 $\int \frac{\sin^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=66

$$\frac{2\cos(c+dx)}{a^2d}-\frac{\cos^2(c+dx)}{a^2d}+\frac{\cos^3(c+dx)}{3a^2d}-\frac{2\log(1+\cos(c+dx))}{a^2d}$$

```
[Out] 2*cos(d*x+c)/a^2/d-cos(d*x+c)^2/a^2/d+1/3*cos(d*x+c)^3/a^2/d-2*ln(1+cos(d*x
+c))/a^2/d
```

Rubi [A]
time = 0.11, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.190$, Rules used = {3957, 2915, 12, 78}

$$\frac{\cos^3(c+dx)}{3a^2d}-\frac{\cos^2(c+dx)}{a^2d}+\frac{2\cos(c+dx)}{a^2d}-\frac{2\log(\cos(c+dx)+1)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*Cos[c + d*x])/(a^2*d) - Cos[c + d*x]^2/(a^2*d) + Cos[c + d*x]^3/(3*a^2*d
) - (2*Log[1 + Cos[c + d*x]])/(a^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= \int \frac{\cos^2(c+dx)\sin^3(c+dx)}{(-a-a\cos(c+dx))^2}\,dx \\
&= \frac{\text{Subst}\left(\int \frac{(-a-x)x^2}{a^2(-a+x)}\,dx, x, -a\cos(c+dx)\right)}{a^3d} \\
&= \frac{\text{Subst}\left(\int \frac{(-a-x)x^2}{-a+x}\,dx, x, -a\cos(c+dx)\right)}{a^5d} \\
&= \frac{\text{Subst}\left(\int \left(-2a^2+\frac{2a^3}{a-x}-2ax-x^2\right)\,dx, x, -a\cos(c+dx)\right)}{a^5d} \\
&= \frac{2\cos(c+dx)}{a^2d}-\frac{\cos^2(c+dx)}{a^2d}+\frac{\cos^3(c+dx)}{3a^2d}-\frac{2\log(1+\cos(c+dx))}{a^2d}
\end{aligned}$$

Mathematica [A]
time = 0.14, size = 51, normalized size = 0.77

$$\frac{-22+27\cos(c+dx)-6\cos(2(c+dx))+\cos(3(c+dx))-48\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{12a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
[Out] (-22 + 27*Cos[c + d*x] - 6*Cos[2*(c + d*x)] + Cos[3*(c + d*x)] - 48*Log[Cos
[(c + d*x)/2]])/(12*a^2*d)
```

Maple [A]
time = 0.12, size = 58, normalized size = 0.88

method	result	size
derivativedivides	$-\frac{-\frac{1}{3\sec(dx+c)^3}+\frac{1}{\sec(dx+c)^2}-\frac{2}{\sec(dx+c)}-2\ln(\sec(dx+c))+2\ln(1+\sec(dx+c))}{d\,a^2}$	58
default	$-\frac{-\frac{1}{3\sec(dx+c)^3}+\frac{1}{\sec(dx+c)^2}-\frac{2}{\sec(dx+c)}-2\ln(\sec(dx+c))+2\ln(1+\sec(dx+c))}{d\,a^2}$	58

norman	$\frac{\frac{10\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{14}{3ad}+\frac{12\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 a}+\frac{2\ln\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2 d}$	90
risch	$\frac{2ix}{a^2}+\frac{9\,e^{i(dx+c)}}{8a^2d}+\frac{9\,e^{-i(dx+c)}}{8a^2d}+\frac{4ic}{a^2d}-\frac{4\ln\left(e^{i(dx+c)}+1\right)}{a^2d}+\frac{\cos(3dx+3c)}{12d\,a^2}-\frac{\cos(2dx+2c)}{2d\,a^2}$	107

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d/a^2*(-1/3/sec(d*x+c)^3+1/sec(d*x+c)^2-2/sec(d*x+c)-2*ln(sec(d*x+c))+2*
ln(1+sec(d*x+c)))
```

Maxima [A]
time = 0.28, size = 51, normalized size = 0.77

$$\frac{\frac{\cos(dx+c)^3-3\cos(dx+c)^2+6\cos(dx+c)}{a^2}-\frac{6\log(\cos(dx+c)+1)}{a^2}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*((cos(d*x + c)^3 - 3*cos(d*x + c)^2 + 6*cos(d*x + c))/a^2 - 6*log(cos(d
*x + c) + 1)/a^2)/d
```

Fricas [A]
time = 4.60, size = 48, normalized size = 0.73

$$\frac{\cos(dx+c)^3-3\cos(dx+c)^2+6\cos(dx+c)-6\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)}{3\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(cos(d*x + c)^3 - 3*cos(d*x + c)^2 + 6*cos(d*x + c) - 6*log(1/2*cos(d*x
 + c) + 1/2))/(a^2*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.50, size = 75, normalized size = 1.14

$$-\frac{2\,\log\left(\left|-\cos\left(dx+c\right)-1\right|\right)}{a^2d}+\frac{a^4d^2\cos\left(dx+c\right)^3-3\,a^4d^2\cos\left(dx+c\right)^2+6\,a^4d^2\cos\left(dx+c\right)}{3\,a^6d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2*log(abs(-cos(d*x + c) - 1))/(a^2*d) + 1/3*(a^4*d^2*cos(d*x + c)^3 - 3*a^
4*d^2*cos(d*x + c)^2 + 6*a^4*d^2*cos(d*x + c))/(a^6*d^3)
```

Mupad [B]
time = 0.06, size = 56, normalized size = 0.85

$$-\frac{\frac{2\,\ln\left(\cos\left(c+d\,x\right)+1\right)}{a^2}-\frac{2\,\cos\left(c+d\,x\right)}{a^2}+\frac{{\cos\left(c+d\,x\right)}^2}{a^2}-\frac{{\cos\left(c+d\,x\right)}^3}{3\,a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(a + a/cos(c + d*x))^2,x)
```

```
[Out] -((2*log(cos(c + d*x) + 1))/a^2 - (2*cos(c + d*x))/a^2 + cos(c + d*x)^2/a^2
 - cos(c + d*x)^3/(3*a^2))/d
```

3.79 $\int \frac{\sin(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=52

$$-\frac{\cos(c+dx)}{a^2d}+\frac{1}{d\,(a^2+a^2\cos(c+dx))}+\frac{2\log(1+\cos(c+dx))}{a^2d}$$

```
[Out] -cos(d*x+c)/a^2/d+1/d/(a^2+a^2*cos(d*x+c))+2*ln(1+cos(d*x+c))/a^2/d
```

Rubi [A]
time = 0.07, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3957, 2912, 12, 45}

$$-\frac{\cos(c+dx)}{a^2d}+\frac{1}{d\,(a^2\cos(c+dx)+a^2)}+\frac{2\log(\cos(c+dx)+1)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(Cos[c + d*x]/(a^2*d)) + 1/(d*(a^2 + a^2*Cos[c + d*x])) + (2*Log[1 + Cos[c
 + d*x]])/(a^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
```

```
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= \int \frac{\cos^2(c+dx)\sin(c+dx)}{(-a-a\cos(c+dx))^2}\,dx\\ &= \frac{\text{Subst}\left(\int \frac{x^2}{a^2(-a+x)^2}\,dx, x, -a\cos(c+dx)\right)}{ad}\\ &= \frac{\text{Subst}\left(\int \frac{x^2}{(-a+x)^2}\,dx, x, -a\cos(c+dx)\right)}{a^3d}\\ &= \frac{\text{Subst}\left(\int \left(1+\frac{a^2}{(a-x)^2}-\frac{2a}{a-x}\right)\,dx, x, -a\cos(c+dx)\right)}{a^3d}\\ &= -\frac{\cos(c+dx)}{a^2d}+\frac{1}{d\,(a^2+a^2\cos(c+dx))}+\frac{2\log(1+\cos(c+dx))}{a^2d}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 64, normalized size = 1.23

$$-\frac{\left(-3+\cos(2(c+dx))-8\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-8\cos(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)\sec^2\left(\frac{1}{2}(c+dx)\right)}{4a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + a*Sec[c + d*x])^2,x]
[Out] -1/4*((-3 + Cos[2*(c + d*x)] - 8*Log[Cos[(c + d*x)/2]] - 8*Cos[c + d*x]*Log
[Cos[(c + d*x)/2]])*Sec[(c + d*x)/2]^2)/(a^2*d)
```

Maple [A]
time = 0.04, size = 51, normalized size = 0.98

method	result	size
derivativedivides	$\frac{-\frac{1}{\sec(dx+c)}-2\ln(\sec(dx+c))-\frac{1}{1+\sec(dx+c)}+2\ln(1+\sec(dx+c))}{a^2d}$	51
default	$\frac{-\frac{1}{\sec(dx+c)}-2\ln(\sec(dx+c))-\frac{1}{1+\sec(dx+c)}+2\ln(1+\sec(dx+c))}{a^2d}$	51
norman	$\frac{\frac{\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}-\frac{5}{2ad}}{a\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{2\ln\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2d}$	71
risch	$-\frac{2ix}{a^2}-\frac{e^{i(dx+c)}}{2a^2d}-\frac{e^{-i(dx+c)}}{2a^2d}-\frac{4ic}{a^2d}+\frac{2\,e^{i(dx+c)}}{a^2d\left(e^{i(dx+c)}+1\right)^2}+\frac{4\ln\left(e^{i(dx+c)}+1\right)}{a^2d}$	103

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2/d*(-1/sec(d*x+c)-2*ln(sec(d*x+c))-1/(1+sec(d*x+c))+2*ln(1+sec(d*x+c))
)
```

Maxima [A]
time = 0.28, size = 46, normalized size = 0.88

$$\frac{\frac{1}{a^2\cos(dx+c)+a^2} - \frac{\cos(dx+c)}{a^2} + \frac{2\log(\cos(dx+c)+1)}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (1/(a^2*cos(d*x + c) + a^2) - cos(d*x + c)/a^2 + 2*log(cos(d*x + c) + 1)/a^
2)/d
```

Fricas [A]
time = 4.07, size = 58, normalized size = 1.12

$$-\frac{\cos(dx+c)^2 - 2(\cos(dx+c)+1)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right) + \cos(dx+c) - 1}{a^2d\cos(dx+c)+a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(cos(d*x + c)^2 - 2*(cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) + cos(d
*x + c) - 1)/(a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sin(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(sin(c + d*x)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [A]
time = 0.48, size = 52, normalized size = 1.00

$$-\frac{\cos(dx+c)}{a^2d} + \frac{2\log(|-\cos(dx+c)-1|)}{a^2d} + \frac{1}{a^2d(\cos(dx+c)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -cos(d*x + c)/(a^2*d) + 2*log(abs(-cos(d*x + c) - 1))/(a^2*d) + 1/(a^2*d*(c
os(d*x + c) + 1))
```

Mupad [B]
time = 0.92, size = 46, normalized size = 0.88

$$\frac{2\,\ln\left(\cos\left(c+d\,x\right)+1\right)}{a^2\,d}-\frac{{\cos\left(c+d\,x\right)}^2-2}{a^2\,d\,\left(\cos\left(c+d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(a + a/cos(c + d*x))^2,x)
```

```
[Out] (2*log(cos(c + d*x) + 1))/(a^2*d) - (cos(c + d*x)^2 - 2)/(a^2*d*(cos(c + d*
x) + 1))
```

3.80 $\int \frac{\csc(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=60

$$-\frac{\tanh^{-1}(\cos(c+dx))}{4a^2d}+\frac{1}{4d(a+a\cos(c+dx))^2}-\frac{3}{4d\,(a^2+a^2\cos(c+dx))}$$

```
[Out] -1/4*arctanh(cos(d*x+c))/a^2/d+1/4/d/(a+a*cos(d*x+c))^2-3/4/d/(a^2+a^2*cos(
d*x+c))
```

Rubi [A]
time = 0.09, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3957, 2915, 12, 90, 212}

$$-\frac{3}{4d\,(a^2\cos(c+dx)+a^2)}-\frac{\tanh^{-1}(\cos(c+dx))}{4a^2d}+\frac{1}{4d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/4*ArcTanh[Cos[c + d*x]]/(a^2*d) + 1/(4*d*(a + a*Cos[c + d*x])^2) - 3/(4*
d*(a^2 + a^2*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
```

```
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= \int \frac{\cos(c+dx)\cot(c+dx)}{(-a-a\cos(c+dx))^2}\,dx\\ &= \frac{a\text{Subst}\left(\int \frac{x^2}{a^2(-a-x)(-a+x)^3}\,dx, x, -a\cos(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{x^2}{(-a-x)(-a+x)^3}\,dx, x, -a\cos(c+dx)\right)}{ad}\\ &= \frac{\text{Subst}\left(\int \left(\frac{a}{2(a-x)^3}-\frac{3}{4(a-x)^2}+\frac{1}{4(a^2-x^2)}\right)\,dx, x, -a\cos(c+dx)\right)}{ad}\\ &= \frac{1}{4d(a+a\cos(c+dx))^2}-\frac{3}{4d\,(a^2+a^2\cos(c+dx))}+\frac{\text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, -a\,\text{c}\right.}{4ad}\\ &= -\frac{\tanh^{-1}(\cos(c+dx))}{4a^2d}+\frac{1}{4d(a+a\cos(c+dx))^2}-\frac{3}{4d\,(a^2+a^2\cos(c+dx))}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 83, normalized size = 1.38

$$-\frac{\left(-1+6\cos^2\left(\frac{1}{2}(c+dx)\right)+4\cos^4\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\right)\sec^2(c+dx)}{4a^2d(1+\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/4*((-1 + 6*Cos[(c + d*x)/2]^2 + 4*Cos[(c + d*x)/2]^4*(Log[Cos[(c + d*x)/
2]] - Log[Sin[(c + d*x)/2]]))*Sec[c + d*x]^2)/(a^2*d*(1 + Sec[c + d*x])^2)
```

Maple [A]
time = 0.10, size = 55, normalized size = 0.92

method	result	size
derivativedivides	$\frac{\frac{\ln(-1+\cos(dx+c))}{8}+\frac{1}{4(1+\cos(dx+c))^2}-\frac{3}{4(1+\cos(dx+c))}-\frac{\ln(1+\cos(dx+c))}{8}}{d\,a^2}$	55
default	$\frac{\frac{\ln(-1+\cos(dx+c))}{8}+\frac{1}{4(1+\cos(dx+c))^2}-\frac{3}{4(1+\cos(dx+c))}-\frac{\ln(1+\cos(dx+c))}{8}}{d\,a^2}$	55
norman	$\frac{-\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}+\frac{\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)}{16ad}}{a}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^2d}$	63
risch	$-\frac{3\,\mathrm{e}^{3i(dx+c)}+4\,\mathrm{e}^{2i(dx+c)}+3\,\mathrm{e}^{i(dx+c)}}{2a^2d\left(\mathrm{e}^{i(dx+c)}+1\right)^4}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}-1\right)}{4a^2d}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}+1\right)}{4a^2d}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(1/8*ln(-1+cos(d*x+c))+1/4/(1+cos(d*x+c))^2-3/4/(1+cos(d*x+c))-1/8*
ln(1+cos(d*x+c)))
```

Maxima [A]
time = 0.28, size = 74, normalized size = 1.23

$$-\frac{\frac{2\,(3\,\cos(dx+c)+2)}{a^2\,\cos(dx+c)^2+2\,a^2\,\cos(dx+c)+a^2}+\frac{\log(\cos(dx+c)+1)}{a^2}-\frac{\log(\cos(dx+c)-1)}{a^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/8*(2*(3*cos(d*x + c) + 2)/(a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2
) + log(cos(d*x + c) + 1)/a^2 - log(cos(d*x + c) - 1)/a^2)/d
```

Fricas [A]
time = 2.87, size = 106, normalized size = 1.77

$$-\frac{\left(\cos\left(dx+c\right)^2+2\,\cos\left(dx+c\right)+1\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-\left(\cos\left(dx+c\right)^2+2\,\cos\left(dx+c\right)+1\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+6\,\cos\left(dx+c\right)+4}{8\,\left(a^2d\cos\left(dx+c\right)^2+2\,a^2d\cos\left(dx+c\right)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/8*((cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) - (
cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(-1/2*cos(d*x + c) + 1/2) + 6*cos(d
*x + c) + 4)/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc\left(c+dx\right)}{\sec^2\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(csc(c + d*x)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [A]
time = 0.46, size = 87, normalized size = 1.45

$$\frac{\frac{2\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^2}+\frac{\frac{4a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{a^4}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/16*(2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a^2 + (4*a^2*(cos
(d*x + c) - 1)/(cos(d*x + c) + 1) + a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c)
+ 1)^2)/a^4)/d
```

Mupad [B]
time = 0.10, size = 60, normalized size = 1.00

$$-\frac{\frac{3\cos(c+d\,x)}{4}+\frac{1}{2}}{d\,\left(a^2\cos\left(c+d\,x\right)^2+2\,a^2\,\cos\left(c+d\,x\right)+a^2\right)}-\frac{\mathrm{atanh}(\cos\left(c+d\,x\right))}{4\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)*(a + a/cos(c + d*x))^2),x)
```

```
[Out] - ((3*cos(c + d*x))/4 + 1/2)/(d*(2*a^2*cos(c + d*x) + a^2 + a^2*cos(c + d*x
)^2)) - atanh(cos(c + d*x))/(4*a^2*d)
```

3.81 $\int \frac{\csc^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=42

$$-\frac{a+2a\cos(c+dx)}{6d(1-\cos(c+dx))(a+a\cos(c+dx))^3}$$

```
[Out] 1/6*(-a-2*a*cos(d*x+c))/d/(1-cos(d*x+c))/(a+a*cos(d*x+c))^3
```

Rubi [A]
time = 0.09, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3957, 2915, 12, 82}

$$-\frac{2a\cos(c+dx)+a}{6d(1-\cos(c+dx))(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/6*(a + 2*a*Cos[c + d*x])/(d*(1 - Cos[c + d*x])*(a + a*Cos[c + d*x])^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 82

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*((2*a*d*f*(n
 + p + 3) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x)/(d^2*f^2*(
n + p + 2)*(n + p + 3))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n
 + p + 2, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p +
3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p +
 1))*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= \int \frac{\cot^2(c+dx)\csc(c+dx)}{(-a-a\cos(c+dx))^2}\,dx\\ &= \frac{a^3\text{Subst}\left(\int \frac{x^2}{a^2(-a-x)^2(-a+x)^4}\,dx, x, -a\cos(c+dx)\right)}{d}\\ &= \frac{a\text{Subst}\left(\int \frac{x^2}{(-a-x)^2(-a+x)^4}\,dx, x, -a\cos(c+dx)\right)}{d}\\ &= -\frac{a+2a\cos(c+dx)}{6d(1-\cos(c+dx))(a+a\cos(c+dx))^3}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 38, normalized size = 0.90

$$-\frac{(1+2\cos(c+dx))\csc^2(c+dx)}{6a^2d(1+\cos(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/6*((1 + 2*Cos[c + d*x])*Csc[c + d*x]^2)/(a^2*d*(1 + Cos[c + d*x])^2)
```

Maple [A]
time = 0.10, size = 57, normalized size = 1.36

method	result	size
derivativedivides	$\frac{\frac{1}{-16+16\cos(dx+c)}+\frac{1}{12(1+\cos(dx+c))^3}-\frac{1}{8(1+\cos(dx+c))^2}-\frac{1}{16(1+\cos(dx+c))}}{d\,a^2}$	57
default	$\frac{\frac{1}{-16+16\cos(dx+c)}+\frac{1}{12(1+\cos(dx+c))^3}-\frac{1}{8(1+\cos(dx+c))^2}-\frac{1}{16(1+\cos(dx+c))}}{d\,a^2}$	57
norman	$\frac{-\frac{1}{32ad}-\frac{\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)}{16ad}+\frac{\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)}{96ad}}{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}$	63
risch	$\frac{\frac{8\,e^{5i(dx+c)}}{3}+\frac{8\,e^{4i(dx+c)}}{3}+\frac{8\,e^{3i(dx+c)}}{3}}{a^2d\left(e^{i(dx+c)}+1\right)^6\left(e^{i(dx+c)}-1\right)^2}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(1/16/(-1+cos(d*x+c))+1/12/(1+cos(d*x+c))^3-1/8/(1+cos(d*x+c))^2-1/
16/(1+cos(d*x+c)))
```

Maxima [A]
time = 0.27, size = 59, normalized size = 1.40

$$\frac{2\cos\left(dx+c\right)+1}{6\left(a^2\cos\left(dx+c\right)^4+2\,a^2\cos\left(dx+c\right)^3-2\,a^2\cos\left(dx+c\right)-a^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(2*cos(d*x + c) + 1)/((a^2*cos(d*x + c)^4 + 2*a^2*cos(d*x + c)^3 - 2*a^
2*cos(d*x + c) - a^2)*d)
```

Fricas [A]
time = 3.54, size = 60, normalized size = 1.43

$$\frac{2\cos\left(dx+c\right)+1}{6\left(a^2d\cos\left(dx+c\right)^4+2\,a^2d\cos\left(dx+c\right)^3-2\,a^2d\cos\left(dx+c\right)-a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*cos(d*x + c) + 1)/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 - 2
*a^2*d*cos(d*x + c) - a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\csc^3\left(c+dx\right)}{\sec^2\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(csc(c + d*x)**3/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(38) = 76.
time = 0.47, size = 82, normalized size = 1.95

$$\frac{\frac{3\left(\cos(dx+c)+1\right)}{a^2\left(\cos(dx+c)-1\right)}+\frac{\frac{6\,a^4\left(\cos(dx+c)-1\right)}{\cos(dx+c)+1}-\frac{a^4\left(\cos(dx+c)-1\right)^3}{\left(\cos(dx+c)+1\right)^3}}{a^6}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/96*(3*(cos(d*x + c) + 1)/(a^2*(cos(d*x + c) - 1)) + (6*a^4*(cos(d*x + c)
- 1)/(cos(d*x + c) + 1) - a^4*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)/a^
6)/d
```

Mupad [B]
time = 0.94, size = 58, normalized size = 1.38

$$-\frac{\frac{\cos(c+d\,x)}{3}+\frac{1}{6}}{d\,\left(-a^2\cos\left(c+d\,x\right)^4-2\,a^2\cos\left(c+d\,x\right)^3+2\,a^2\,\cos\left(c+d\,x\right)+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^3*(a + a/cos(c + d*x))^2),x)
```

```
[Out] -(cos(c + d*x)/3 + 1/6)/(d*(2*a^2*cos(c + d*x) + a^2 - 2*a^2*cos(c + d*x)^3
 - a^2*cos(c + d*x)^4))
```

3.82 $\int \frac{\csc^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=146

$$\frac{\tanh^{-1}(\cos(c+dx))}{64a^2d}-\frac{1}{64d(a-a\cos(c+dx))^2}+\frac{a^2}{32d(a+a\cos(c+dx))^4}-\frac{a}{48d(a+a\cos(c+dx))^3}-\frac{}{32d(a+}$$

```
[Out] 1/64*arctanh(cos(d*x+c))/a^2/d-1/64/d/(a-a*cos(d*x+c))^2+1/32*a^2/d/(a+a*co
s(d*x+c))^4-1/48*a/d/(a+a*cos(d*x+c))^3-1/32/d/(a+a*cos(d*x+c))^2-1/64/d/(a
^2-a^2*cos(d*x+c))-1/32/d/(a^2+a^2*cos(d*x+c))
```

Rubi [A]
time = 0.16, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3957, 2915, 12, 90, 212}

$$\frac{a^2}{32d(a\cos(c+dx)+a)^4}-\frac{1}{64d\left(a^2-a^2\cos(c+dx)\right)}-\frac{1}{32d\left(a^2\cos(c+dx)+a^2\right)}+\frac{\tanh^{-1}(\cos(c+dx))}{64a^2d}-\frac{a}{48d(a\cos(c+dx)+a)^3}-\frac{1}{64d(a-a\cos(c+dx))^2}-\frac{1}{32d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] ArcTanh[Cos[c + d*x]]/(64*a^2*d) - 1/(64*d*(a - a*Cos[c + d*x])^2) + a^2/(3
2*d*(a + a*Cos[c + d*x])^4) - a/(48*d*(a + a*Cos[c + d*x])^3) - 1/(32*d*(a
+ a*Cos[c + d*x])^2) - 1/(64*d*(a^2 - a^2*Cos[c + d*x])) - 1/(32*d*(a^2 + a
^2*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\csc^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cot^2(c+dx)\csc^3(c+dx)}{(-a-a\cos(c+dx))^2}\,dx$$
$$= \frac{a^5\text{Subst}\left(\int \frac{x^2}{a^2(-a-x)^3(-a+x)^5}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^3\text{Subst}\left(\int \frac{x^2}{(-a-x)^3(-a+x)^5}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^3\text{Subst}\left(\int \left(\frac{1}{8a(a-x)^5} - \frac{1}{16a^2(a-x)^4} - \frac{1}{16a^3(a-x)^3} - \frac{1}{32a^4(a-x)^2} + \frac{1}{32a^3(a+x)^3} + \frac{1}{64a^4(a\cdots}\right.\right.}{d}$$
$$= -\frac{1}{64d(a-a\cos(c+dx))^2} + \frac{a^2}{32d(a+a\cos(c+dx))^4} - \frac{a}{48d(a+a\cos(c+dx)\cdots}$$
$$= \frac{\tanh^{-1}(\cos(c+dx))}{64a^2d} - \frac{1}{64d(a-a\cos(c+dx))^2} + \frac{a^2}{32d(a+a\cos(c+dx))^4} - \cdots$$

Mathematica [A]
time = 0.52, size = 152, normalized size = 1.04

$$-\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\left(12\csc^2\left(\frac{1}{2}(c+dx)\right)+6\csc^4\left(\frac{1}{2}(c+dx)\right)+24\left(-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+24\sec^2\left(\frac{1}{2}(c+dx)\right)+12\sec^4\left(\frac{1}{2}(c+dx)\right)+4\sec^6\left(\frac{1}{2}(c+dx)\right)-3\sec^8\left(\frac{1}{2}(c+dx)\right)\right)\sec^2(c+dx)}{384a^2d(1+\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^5/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/384*(Cos[(c + d*x)/2]^4*(12*Csc[(c + d*x)/2]^2 + 6*Csc[(c + d*x)/2]^4 +
24*(-Log[Cos[(c + d*x)/2]] + Log[Sin[(c + d*x)/2]]) + 24*Sec[(c + d*x)/2]^2
 + 12*Sec[(c + d*x)/2]^4 + 4*Sec[(c + d*x)/2]^6 - 3*Sec[(c + d*x)/2]^8)*Sec
[c + d*x]^2)/(a^2*d*(1 + Sec[c + d*x])^2)
```

Maple [A]

time = 0.15, size = 103, normalized size = 0.71

method	result
derivativedivides	$\frac{-\frac{1}{64(-1+\cos(dx+c))^2}+\frac{1}{-64+64\cos(dx+c)}-\frac{\ln(-1+\cos(dx+c))}{128}+\frac{1}{32(1+\cos(dx+c))^4}-\frac{1}{48(1+\cos(dx+c))^3}-\frac{1}{32(1+\cos(dx+c))^2}-32(1}{d\,a^2}$
default	$\frac{-\frac{1}{64(-1+\cos(dx+c))^2}+\frac{1}{-64+64\cos(dx+c)}-\frac{\ln(-1+\cos(dx+c))}{128}+\frac{1}{32(1+\cos(dx+c))^4}-\frac{1}{48(1+\cos(dx+c))^3}-\frac{1}{32(1+\cos(dx+c))^2}-32(1}{d\,a^2}$
norman	$\frac{-\frac{1}{256ad}-\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{64ad}-\frac{\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)}{32ad}-\frac{\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)}{256ad}+\frac{\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)}{192ad}+\frac{\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)}{512ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64a^2 d}$
risch	$-\frac{3\,e^{11i(dx+c)}+12\,e^{10i(dx+c)}+7\,e^{9i(dx+c)}-32\,e^{8i(dx+c)}+566\,e^{7i(dx+c)}+424\,e^{6i(dx+c)}+566\,e^{5i(dx+c)}-32\,e^{4i(dx+c)}+7\,e^{3i}}{96a^2 d\left(e^{i(dx+c)}+1\right)^8\left(e^{i(dx+c)}-1\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(-1/64/(-1+cos(d*x+c))^2+1/64/(-1+cos(d*x+c))-1/128*ln(-1+cos(d*x+c
))+1/32/(1+cos(d*x+c))^4-1/48/(1+cos(d*x+c))^3-1/32/(1+cos(d*x+c))^2-1/32/(
1+cos(d*x+c))+1/128*ln(1+cos(d*x+c)))
```

Maxima [A]

time = 0.27, size = 167, normalized size = 1.14

$$-\frac{\frac{2\left(3\cos(dx+c)^5+6\cos(dx+c)^4-2\cos(dx+c)^3-10\cos(dx+c)^2+35\cos(dx+c)+16\right)}{a^2\cos(dx+c)^6+2a^2\cos(dx+c)^5-a^2\cos(dx+c)^4-4a^2\cos(dx+c)^3-a^2\cos(dx+c)^2+2a^2\cos(dx+c)+a^2}-\frac{3\log(\cos(dx+c)+1)}{a^2}+\frac{3\log(\cos(dx+c)-1)}{a^2}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/384*(2*(3*cos(d*x + c)^5 + 6*cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 10*cos(
d*x + c)^2 + 35*cos(d*x + c) + 16)/(a^2*cos(d*x + c)^6 + 2*a^2*cos(d*x + c)
^5 - a^2*cos(d*x + c)^4 - 4*a^2*cos(d*x + c)^3 - a^2*cos(d*x + c)^2 + 2*a^2
*cos(d*x + c) + a^2) - 3*log(cos(d*x + c) + 1)/a^2 + 3*log(cos(d*x + c) - 1
)/a^2)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(134) = 268$.

time = 3.49, size = 283, normalized size = 1.94

$$-\frac{6\cos(dx+c)^5+12\cos(dx+c)^4-4\cos(dx+c)^3-20\cos(dx+c)^2-3\left(\cos(dx+c)^6+2\cos(dx+c)^5-\cos(dx+c)^4-4\cos(dx+c)^3-\cos(dx+c)^2+2\cos(dx+c)+1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+3\left(\cos(dx+c)^6+2\cos(dx+c)^5-\cos(dx+c)^4-4\cos(dx+c)^3-\cos(dx+c)^2+2\cos(dx+c)+1\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+70\cos(dx+c)+32}{384\left(a^2d\cos(dx+c)^6+2a^2d\cos(dx+c)^5-a^2d\cos(dx+c)^4-4a^2d\cos(dx+c)^3-a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/384*(6*cos(d*x + c)^5 + 12*cos(d*x + c)^4 - 4*cos(d*x + c)^3 - 20*cos(d*
x + c)^2 - 3*(cos(d*x + c)^6 + 2*cos(d*x + c)^5 - cos(d*x + c)^4 - 4*cos(d*
x + c)^3 - cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2)
 + 3*(cos(d*x + c)^6 + 2*cos(d*x + c)^5 - cos(d*x + c)^4 - 4*cos(d*x + c)^3
 - cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(-1/2*cos(d*x + c) + 1/2) + 70*c
os(d*x + c) + 32)/(a^2*d*cos(d*x + c)^6 + 2*a^2*d*cos(d*x + c)^5 - a^2*d*co
s(d*x + c)^4 - 4*a^2*d*cos(d*x + c)^3 - a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(
d*x + c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^5(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(csc(c + d*x)**5/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [A]
time = 0.51, size = 207, normalized size = 1.42

$$\frac{\frac{6\left(\frac{4(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-1\right)(\cos(dx+c)+1)^2}{a^2(\cos(dx+c)-1)^2}-\frac{12\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^2}+\frac{\frac{48a^6(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{6a^6(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{8a^6(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{3a^6(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}}{a^8}}{1536\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/1536*(6*(4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 3*(cos(d*x + c) - 1)^2
/(cos(d*x + c) + 1)^2 - 1)*(cos(d*x + c) + 1)^2/(a^2*(cos(d*x + c) - 1)^2)
- 12*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a^2 + (48*a^6*(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) - 6*a^6*(cos(d*x + c) - 1)^2/(cos(d*x + c)
+ 1)^2 - 8*a^6*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 3*a^6*(cos(d*x +
 c) - 1)^4/(cos(d*x + c) + 1)^4)/a^8)/d
```

Mupad [B]
time = 1.05, size = 152, normalized size = 1.04

$$\frac{\mathrm{atanh}(\cos(c+d\,x))}{64\,a^2\,d}-\frac{\frac{\cos(c+d\,x)^5}{64}+\frac{\cos(c+d\,x)^4}{32}-\frac{\cos(c+d\,x)^3}{96}-\frac{5\cos(c+d\,x)^2}{96}+\frac{35\,\cos(c+d\,x)}{192}+\frac{1}{12}}{d\,\left(a^2\cos(c+d\,x)^6+2\,a^2\cos(c+d\,x)^5-a^2\cos(c+d\,x)^4-4\,a^2\cos(c+d\,x)^3-a^2\cos(c+d\,x)^2+2\,a^2\,\cos(c+d\,x)+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^5*(a + a/cos(c + d*x))^2),x)
```

```
[Out] atanh(cos(c + d*x))/(64*a^2*d) - ((35*cos(c + d*x))/192 - (5*cos(c + d*x)^2
)/96 - cos(c + d*x)^3/96 + cos(c + d*x)^4/32 + cos(c + d*x)^5/64 + 1/12)/(d
*(2*a^2*cos(c + d*x) + a^2 - a^2*cos(c + d*x)^2 - 4*a^2*cos(c + d*x)^3 - a^
2*cos(c + d*x)^4 + 2*a^2*cos(c + d*x)^5 + a^2*cos(c + d*x)^6))
```

3.83 $\int \frac{\sin^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=167

$$\frac{11x}{128a^2}+\frac{11\cos(c+dx)\sin(c+dx)}{128a^2d}-\frac{7\cos^3(c+dx)\sin(c+dx)}{64a^2d}-\frac{\cos^5(c+dx)\sin(c+dx)}{16a^2d}-\frac{\cos^3(c+dx)\sin^3(c+dx)}{6a^2d}$$

```
[Out] 11/128*x/a^2+11/128*cos(d*x+c)*sin(d*x+c)/a^2/d-7/64*cos(d*x+c)^3*sin(d*x+c
)/a^2/d-1/16*cos(d*x+c)^5*sin(d*x+c)/a^2/d-1/6*cos(d*x+c)^3*sin(d*x+c)^3/a^
2/d-1/8*cos(d*x+c)^5*sin(d*x+c)^3/a^2/d-2/5*sin(d*x+c)^5/a^2/d+2/7*sin(d*x+
c)^7/a^2/d
```

Rubi [A]
time = 0.32, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3957, 2954, 2952, 2648, 2715, 8, 2644, 14}

$$\frac{2\sin^7(c+dx)}{7a^2d}-\frac{2\sin^5(c+dx)}{5a^2d}-\frac{\sin^3(c+dx)\cos^5(c+dx)}{8a^2d}-\frac{\sin(c+dx)\cos^5(c+dx)}{16a^2d}-\frac{\sin^3(c+dx)\cos^3(c+dx)}{6a^2d}-\frac{7\sin(c+dx)\cos^3(c+dx)}{64a^2d}+\frac{11\sin(c+dx)\cos(c+dx)}{128a^2d}+\frac{11x}{128a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^8/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (11*x)/(128*a^2) + (11*Cos[c + d*x]*Sin[c + d*x])/(128*a^2*d) - (7*Cos[c +
d*x]^3*Sin[c + d*x])/(64*a^2*d) - (Cos[c + d*x]^5*Sin[c + d*x])/(16*a^2*d)
- (Cos[c + d*x]^3*Sin[c + d*x]^3)/(6*a^2*d) - (Cos[c + d*x]^5*Sin[c + d*x]^
3)/(8*a^2*d) - (2*Sin[c + d*x]^5)/(5*a^2*d) + (2*Sin[c + d*x]^7)/(7*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= \int \frac{\cos^2(c+dx)\sin^8(c+dx)}{(-a-a\cos(c+dx))^2}\,dx \\
&= \frac{\int \cos^2(c+dx)(-a+a\cos(c+dx))^2\sin^4(c+dx)\,dx}{a^4} \\
&= \frac{\int \left(a^2\cos^2(c+dx)\sin^4(c+dx) - 2a^2\cos^3(c+dx)\sin^4(c+dx) + a^2\cos^4(c+dx\right.}{a^4} \\
&= \frac{\int \cos^2(c+dx)\sin^4(c+dx)\,dx}{a^2} + \frac{\int \cos^4(c+dx)\sin^4(c+dx)\,dx}{a^2} - \frac{2\int \cos^3(c+}{} \\
&= -\frac{\cos^3(c+dx)\sin^3(c+dx)}{6a^2d} - \frac{\cos^5(c+dx)\sin^3(c+dx)}{8a^2d} + \frac{3\int \cos^4(c+dx)\sin^2}{8a^2} \\
&= -\frac{\cos^3(c+dx)\sin(c+dx)}{8a^2d} - \frac{\cos^5(c+dx)\sin(c+dx)}{16a^2d} - \frac{\cos^3(c+dx)\sin^3(c+a}{6a^2d} \\
&= \frac{\cos(c+dx)\sin(c+dx)}{16a^2d} - \frac{7\cos^3(c+dx)\sin(c+dx)}{64a^2d} - \frac{\cos^5(c+dx)\sin(c+dx)}{16a^2d} \\
&= \frac{x}{16a^2} + \frac{11\cos(c+dx)\sin(c+dx)}{128a^2d} - \frac{7\cos^3(c+dx)\sin(c+dx)}{64a^2d} - \frac{\cos^5(c+dx)}{16a} \\
&= \frac{11x}{128a^2} + \frac{11\cos(c+dx)\sin(c+dx)}{128a^2d} - \frac{7\cos^3(c+dx)\sin(c+dx)}{64a^2d} - \frac{\cos^5(c+dx}{16}
\end{aligned}
$$

Mathematica [A]

time = 1.77, size = 131, normalized size = 0.78

$$
\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(9240dx - 10080\sin(c+dx) - 1680\sin(2(c+dx)) + 3360\sin(3(c+dx)) - 2520\sin(4(c+dx)) + 672\sin(5(c+dx)) + 560\sin(6(c+dx)) - 480\sin(7(c+dx)) + 105\sin(8(c+dx)) + 980\tan\left(\frac{c}{2}\right)\right)}{26880a^2d(1+\sec(c+dx))^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^8/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]^4*Sec[c + d*x]^2*(9240*d*x - 10080*Sin[c + d*x] - 1680*Si
n[2*(c + d*x)] + 3360*Sin[3*(c + d*x)] - 2520*Sin[4*(c + d*x)] + 672*Sin[5*
(c + d*x)] + 560*Sin[6*(c + d*x)] - 480*Sin[7*(c + d*x)] + 105*Sin[8*(c + d
*x)] + 980*Tan[c/2]))/(26880*a^2*d*(1 + Sec[c + d*x])^2)
```

Maple [A]

time = 0.15, size = 141, normalized size = 0.84

method	result
derivativedivides	$\frac{\frac{128\left(-\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8192} - \frac{253\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24576} - \frac{4213\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{122880} - \frac{55583\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{860160} + \frac{31007\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{860160} - \frac{20363\left(\tan^{11}\right.}{12288}\right.}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}}{a^2d}$

default	$\frac{\frac{128\left(-\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8192}-\frac{253\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24576}-\frac{4213\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{122880}-\frac{55583\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{860160}+\frac{31007\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{860160}-\frac{20363\left(\tan^1\right.}{122}\right.}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}}{a^2d}$
risch	$\frac{11x}{128a^2}-\frac{3\sin(dx+c)}{32a^2d}+\frac{\sin(8dx+8c)}{1024a^2d}-\frac{\sin(7dx+7c)}{224a^2d}+\frac{\sin(6dx+6c)}{192a^2d}+\frac{\sin(5dx+5c)}{160a^2d}-\frac{3\sin(4dx+4c)}{128a^2d}+\frac{\sin(3}{3}$
norman	$\frac{\frac{11x}{128a}-\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64ad}-\frac{253\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192ad}-\frac{4213\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{960ad}-\frac{55583\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6720ad}+\frac{31007\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6720ad}-\frac{20363\left(\tan\right.}{96}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^8/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 128/d/a^2*((-11/8192*tan(1/2*d*x+1/2*c)-253/24576*tan(1/2*d*x+1/2*c)^3-4213
/122880*tan(1/2*d*x+1/2*c)^5-55583/860160*tan(1/2*d*x+1/2*c)^7+31007/860160
*tan(1/2*d*x+1/2*c)^9-20363/122880*tan(1/2*d*x+1/2*c)^11+253/24576*tan(1/2*
d*x+1/2*c)^13+11/8192*tan(1/2*d*x+1/2*c)^15)/(1+tan(1/2*d*x+1/2*c)^2)^8+11/
8192*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 378 vs. $2(151) = 302$.
time = 0.49, size = 378, normalized size = 2.26

$$-\frac{\frac{\frac{1155\sin(dx+c)}{\cos(dx+c)+1}+\frac{8855\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{29491\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{55583\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{31007\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{142541\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-\frac{8855\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}}-\frac{1155\sin(dx+c)^{15}}{(\cos(dx+c)+1)^{15}}}{a^2+\frac{8a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{28a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{56a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{70a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{56a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{28a^2\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}+\frac{8a^2\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}+\frac{a^2\sin(dx+c)^{16}}{(\cos(dx+c)+1)^{16}}}-\frac{1155\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{6720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/6720*((1155*sin(d*x + c)/(cos(d*x + c) + 1) + 8855*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 + 29491*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 55583*sin(d*x
+ c)^7/(cos(d*x + c) + 1)^7 - 31007*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 1
42541*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - 8855*sin(d*x + c)^13/(cos(d*x
 + c) + 1)^13 - 1155*sin(d*x + c)^15/(cos(d*x + c) + 1)^15)/(a^2 + 8*a^2*si
n(d*x + c)^2/(cos(d*x + c) + 1)^2 + 28*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1
)^4 + 56*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 70*a^2*sin(d*x + c)^8/(c
os(d*x + c) + 1)^8 + 56*a^2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 28*a^2*
sin(d*x + c)^12/(cos(d*x + c) + 1)^12 + 8*a^2*sin(d*x + c)^14/(cos(d*x + c)
 + 1)^14 + a^2*sin(d*x + c)^16/(cos(d*x + c) + 1)^16) - 1155*arctan(sin(d*x
 + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A]
time = 3.55, size = 90, normalized size = 0.54

$$\frac{1155\,dx+\left(1680\,\cos\left(dx+c\right)^7-3840\,\cos\left(dx+c\right)^6-280\,\cos\left(dx+c\right)^5+6144\,\cos\left(dx+c\right)^4-3710\,\cos\left(dx+c\right)^3-768\,\cos\left(dx+c\right)^2+1155\,\cos\left(dx+c\right)-1536\right)\sin\left(dx+c\right)}{13440\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/13440*(1155*d*x + (1680*cos(d*x + c)^7 - 3840*cos(d*x + c)^6 - 280*cos(d*
x + c)^5 + 6144*cos(d*x + c)^4 - 3710*cos(d*x + c)^3 - 768*cos(d*x + c)^2 +
 1155*cos(d*x + c) - 1536)*sin(d*x + c))/(a^2*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**8/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.49, size = 139, normalized size = 0.83

$$\frac{\frac{1155\,(dx+c)}{a^2}+\frac{2\left(1155\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}+8855\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}-142541\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+31007\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-55583\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-29491\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-8855\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-1155\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{8}a^{2}}}{13440\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/13440*(1155*(d*x + c)/a^2 + 2*(1155*tan(1/2*d*x + 1/2*c)^15 + 8855*tan(1/
2*d*x + 1/2*c)^13 - 142541*tan(1/2*d*x + 1/2*c)^11 + 31007*tan(1/2*d*x + 1/
2*c)^9 - 55583*tan(1/2*d*x + 1/2*c)^7 - 29491*tan(1/2*d*x + 1/2*c)^5 - 8855
*tan(1/2*d*x + 1/2*c)^3 - 1155*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)
^2 + 1)^8*a^2))/d
```

Mupad [**B**]
time = 3.93, size = 133, normalized size = 0.80

$$\frac{11\,x}{128\,a^2}-\frac{-\frac{11\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{64}-\frac{253\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{192}+\frac{20363\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{960}-\frac{31007\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{6720}+\frac{55583\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{6720}+\frac{4213\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{960}+\frac{253\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{192}+\frac{11\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64}}{a^2\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^8/(a + a/cos(c + d*x))^2,x)
```

```
[Out] (11*x)/(128*a^2) - ((11*tan(c/2 + (d*x)/2))/64 + (253*tan(c/2 + (d*x)/2)^3)
/192 + (4213*tan(c/2 + (d*x)/2)^5)/960 + (55583*tan(c/2 + (d*x)/2)^7)/6720
- (31007*tan(c/2 + (d*x)/2)^9)/6720 + (20363*tan(c/2 + (d*x)/2)^11)/960 - (
253*tan(c/2 + (d*x)/2)^13)/192 - (11*tan(c/2 + (d*x)/2)^15)/64)/(a^2*d*(tan
(c/2 + (d*x)/2)^2 + 1)^8)
```

3.84 $\int \frac{\sin^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=104

$$\frac{3x}{16a^2}-\frac{3\cos(c+dx)\sin(c+dx)}{16a^2d}-\frac{\cos(c+dx)\sin^3(c+dx)}{8a^2d}-\frac{(a-a\cos(c+dx))^3\sin^3(c+dx)}{6a^5d}-\frac{\sin^5(c+dx)}{10a^2d}$$

```
[Out] 3/16*x/a^2-3/16*cos(d*x+c)*sin(d*x+c)/a^2/d-1/8*cos(d*x+c)*sin(d*x+c)^3/a^2
/d-1/6*(a-a*cos(d*x+c))^3*sin(d*x+c)^3/a^5/d-1/10*sin(d*x+c)^5/a^2/d
```

Rubi [A]
time = 0.22, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3957, 2954, 2949, 2748, 2715, 8}

$$-\frac{\sin^3(c+dx)(a-a\cos(c+dx))^3}{6a^5d}-\frac{\sin^5(c+dx)}{10a^2d}-\frac{\sin^3(c+dx)\cos(c+dx)}{8a^2d}-\frac{3\sin(c+dx)\cos(c+dx)}{16a^2d}+\frac{3x}{16a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^6/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (3*x)/(16*a^2) - (3*Cos[c + d*x]*Sin[c + d*x])/(16*a^2*d) - (Cos[c + d*x]*S
in[c + d*x]^3)/(8*a^2*d) - ((a - a*Cos[c + d*x])^3*Sin[c + d*x]^3)/(6*a^5*d
) - Sin[c + d*x]^5/(10*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2949

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(
```

```
p + 1))*((a + b*Sin[e + f*x])^(m + 1)/(2*b*f*g*(m + 1))), x] + Dist[a/(2*g^
2), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; Fr
eeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[m - p, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= \int \frac{\cos^2(c+dx)\sin^6(c+dx)}{(-a-a\cos(c+dx))^2}\,dx \\
&= \frac{\int \cos^2(c+dx)(-a+a\cos(c+dx))^2\sin^2(c+dx)\,dx}{a^4} \\
&= -\frac{(a-a\cos(c+dx))^3\sin^3(c+dx)}{6a^5d} - \frac{\int (-a+a\cos(c+dx))\sin^4(c+dx)\,dx}{2a^3} \\
&= -\frac{(a-a\cos(c+dx))^3\sin^3(c+dx)}{6a^5d} - \frac{\sin^5(c+dx)}{10a^2d} + \frac{\int \sin^4(c+dx)\,dx}{2a^2} \\
&= -\frac{\cos(c+dx)\sin^3(c+dx)}{8a^2d} - \frac{(a-a\cos(c+dx))^3\sin^3(c+dx)}{6a^5d} - \frac{\sin^5(c+dx)}{10a^2d} \\
&= -\frac{3\cos(c+dx)\sin(c+dx)}{16a^2d} - \frac{\cos(c+dx)\sin^3(c+dx)}{8a^2d} \\
&= \frac{3x}{16a^2} - \frac{3\cos(c+dx)\sin(c+dx)}{16a^2d} - \frac{\cos(c+dx)\sin^3(c+dx)}{8a^2d}
\end{aligned}
$$

Mathematica [A]
time = 0.59, size = 111, normalized size = 1.07

$$\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(360dx-480\sin(c+dx)+30\sin(2(c+dx))+80\sin(3(c+dx))-90\sin(4(c+dx))+48\sin(5(c+dx))-10\sin(6(c+dx))+25\tan\left(\frac{c}{2}\right)\right)}{480a^2d(1+\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^6/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]^4*Sec[c + d*x]^2*(360*d*x - 480*Sin[c + d*x] + 30*Sin[2*(
c + d*x)] + 80*Sin[3*(c + d*x)] - 90*Sin[4*(c + d*x)] + 48*Sin[5*(c + d*x)]
 - 10*Sin[6*(c + d*x)] + 25*Tan[c/2]))/(480*a^2*d*(1 + Sec[c + d*x])^2)
```

Maple [A]
time = 0.14, size = 115, normalized size = 1.11

method	result
risch	$\frac{3x}{16a^2}-\frac{\sin(dx+c)}{4a^2d}-\frac{\sin(6dx+6c)}{192a^2d}+\frac{\sin(5dx+5c)}{40a^2d}-\frac{3\sin(4dx+4c)}{64a^2d}+\frac{\sin(3dx+3c)}{24a^2d}+\frac{\sin(2dx+2c)}{64a^2d}$
derivativedivides	$\frac{\frac{32\left(\frac{3\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{256}-\frac{205\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{768}-\frac{29\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{640}-\frac{99\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{640}-\frac{17\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{256}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{256}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}+}{a^2d}$
default	$\frac{\frac{32\left(\frac{3\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{256}-\frac{205\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{768}-\frac{29\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{640}-\frac{99\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{640}-\frac{17\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{256}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{256}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}+}{a^2d}$
norman	$\frac{\frac{3x}{16a}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{17\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad}-\frac{99\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20ad}-\frac{29\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20ad}-\frac{205\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24ad}+\frac{3\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad}+}{\left(1+\tan^2\left(\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^6/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 32/d/a^2*((3/256*tan(1/2*d*x+1/2*c)^11-205/768*tan(1/2*d*x+1/2*c)^9-29/640*
tan(1/2*d*x+1/2*c)^7-99/640*tan(1/2*d*x+1/2*c)^5-17/256*tan(1/2*d*x+1/2*c)^
3-3/256*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^6+3/256*arctan(tan(1/2
*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(95) = 190$.
time = 0.49, size = 292, normalized size = 2.81

$$-\frac{\frac{\frac{45\sin(dx+c)}{\cos(dx+c)+1}+\frac{255\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{594\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{174\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{1025\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{45\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}}{a^2+\frac{6a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{20a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{15a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{6a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{a^2\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}}-\frac{45\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/120*((45*sin(d*x + c)/(cos(d*x + c) + 1) + 255*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 594*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 174*sin(d*x + c)^7/(
cos(d*x + c) + 1)^7 + 1025*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 45*sin(d*x
 + c)^11/(cos(d*x + c) + 1)^11)/(a^2 + 6*a^2*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 15*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 20*a^2*sin(d*x + c)^6/
(cos(d*x + c) + 1)^6 + 15*a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 6*a^2*s
```

```
in(d*x + c)^10/(cos(d*x + c) + 1)^10 + a^2*sin(d*x + c)^12/(cos(d*x + c) +
1)^12) - 45*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A]
time = 3.75, size = 71, normalized size = 0.68

$$\frac{45\,dx-\left(40\,\cos\left(dx+c\right)^{5}-96\,\cos\left(dx+c\right)^{4}+50\,\cos\left(dx+c\right)^{3}+32\,\cos\left(dx+c\right)^{2}-45\,\cos\left(dx+c\right)+64\right)\sin\left(dx+c\right)}{240\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/240*(45*d*x - (40*cos(d*x + c)^5 - 96*cos(d*x + c)^4 + 50*cos(d*x + c)^3
+ 32*cos(d*x + c)^2 - 45*cos(d*x + c) + 64)*sin(d*x + c))/(a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sin^{6}\left(c+dx\right)}{\sec^{2}\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**6/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(sin(c + d*x)**6/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [A]
time = 0.51, size = 113, normalized size = 1.09

$$\frac{\frac{45\,(dx+c)}{a^{2}}+\frac{2\left(45\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-1025\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-174\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-594\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-255\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-45\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{6}a^{2}}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/240*(45*(d*x + c)/a^2 + 2*(45*tan(1/2*d*x + 1/2*c)^11 - 1025*tan(1/2*d*x
+ 1/2*c)^9 - 174*tan(1/2*d*x + 1/2*c)^7 - 594*tan(1/2*d*x + 1/2*c)^5 - 255*
tan(1/2*d*x + 1/2*c)^3 - 45*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2
+ 1)^6*a^2))/d
```

Mupad [B]
time = 3.75, size = 107, normalized size = 1.03

$$\frac{3\,x}{16\,a^{2}}-\frac{-\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}+\frac{205\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}+\frac{29\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{20}+\frac{99\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{20}+\frac{17\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{8}+\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}}{a^{2}\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6/(a + a/cos(c + d*x))^2,x)
```

```
[Out] (3*x)/(16*a^2) - ((3*tan(c/2 + (d*x)/2))/8 + (17*tan(c/2 + (d*x)/2)^3)/8 +
(99*tan(c/2 + (d*x)/2)^5)/20 + (29*tan(c/2 + (d*x)/2)^7)/20 + (205*tan(c/2
+ (d*x)/2)^9)/24 - (3*tan(c/2 + (d*x)/2)^11)/8)/(a^2*d*(tan(c/2 + (d*x)/2)^
2 + 1)^6)
```

3.85 $\int \frac{\sin^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=87

$$\frac{7x}{8a^2}-\frac{2\sin(c+dx)}{a^2d}+\frac{7\cos(c+dx)\sin(c+dx)}{8a^2d}+\frac{\cos^3(c+dx)\sin(c+dx)}{4a^2d}+\frac{2\sin^3(c+dx)}{3a^2d}$$

```
[Out] 7/8*x/a^2-2*sin(d*x+c)/a^2/d+7/8*cos(d*x+c)*sin(d*x+c)/a^2/d+1/4*cos(d*x+c)
^3*sin(d*x+c)/a^2/d+2/3*sin(d*x+c)^3/a^2/d
```

Rubi [A]
time = 0.17, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3957, 2948, 2836, 2715, 8, 2713}

$$\frac{2\sin^3(c+dx)}{3a^2d}-\frac{2\sin(c+dx)}{a^2d}+\frac{\sin(c+dx)\cos^3(c+dx)}{4a^2d}+\frac{7\sin(c+dx)\cos(c+dx)}{8a^2d}+\frac{7x}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (7*x)/(8*a^2) - (2*Sin[c + d*x])/(a^2*d) + (7*Cos[c + d*x]*Sin[c + d*x])/(8
*a^2*d) + (Cos[c + d*x]^3*Sin[c + d*x])/(4*a^2*d) + (2*Sin[c + d*x]^3)/(3*a
^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2836

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
```

```
Q[m, 0] && RationalQ[n]
```

Rule 2948

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= \int \frac{\cos^2(c+dx)\sin^4(c+dx)}{(-a-a\cos(c+dx))^2}\,dx \\
&= \frac{\int \cos^2(c+dx)(-a+a\cos(c+dx))^2\,dx}{a^4} \\
&= \frac{\int \left(a^2\cos^2(c+dx)-2a^2\cos^3(c+dx)+a^2\cos^4(c+dx)\right)\,dx}{a^4} \\
&= \frac{\int \cos^2(c+dx)\,dx}{a^2}+\frac{\int \cos^4(c+dx)\,dx}{a^2}-\frac{2\int \cos^3(c+dx)\,dx}{a^2} \\
&= \frac{\cos(c+dx)\sin(c+dx)}{2a^2d}+\frac{\cos^3(c+dx)\sin(c+dx)}{4a^2d}+\frac{\int 1\,dx}{2a^2}+\frac{3\int \cos^2(c+dx)}{4a^2} \\
&= \frac{x}{2a^2}-\frac{2\sin(c+dx)}{a^2d}+\frac{7\cos(c+dx)\sin(c+dx)}{8a^2d}+\frac{\cos^3(c+dx)\sin(c+dx)}{4a^2d}+ \\
&= \frac{7x}{8a^2}-\frac{2\sin(c+dx)}{a^2d}+\frac{7\cos(c+dx)\sin(c+dx)}{8a^2d}+\frac{\cos^3(c+dx)\sin(c+dx)}{4a^2d}+
\end{aligned}$$

Mathematica [A]
time = 0.37, size = 91, normalized size = 1.05

$$\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(84dx-144\sin(c+dx)+48\sin(2(c+dx))-16\sin(3(c+dx))+3\sin(4(c+dx))+2\tan\left(\frac{c}{2}\right)\right)}{24a^2d(1+\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]^4*Sec[c + d*x]^2*(84*d*x - 144*Sin[c + d*x] + 48*Sin[2*(c
 + d*x)] - 16*Sin[3*(c + d*x)] + 3*Sin[4*(c + d*x)] + 2*Tan[c/2]))/(24*a^2*
d*(1 + Sec[c + d*x])^2)
```

Maple [A]
time = 0.12, size = 89, normalized size = 1.02

method	result
risch	$\frac{7x}{8a^2}-\frac{3\sin(dx+c)}{2a^2d}+\frac{\sin(4dx+4c)}{32a^2d}-\frac{\sin(3dx+3c)}{6a^2d}+\frac{\sin(2dx+2c)}{2a^2d}$
derivativedivides	$\frac{\frac{8\left(-\frac{25\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32}-\frac{83\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96}-\frac{77\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96}-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{32}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{7\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}}{a^2d}$
default	$\frac{\frac{8\left(-\frac{25\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32}-\frac{83\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96}-\frac{77\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96}-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{32}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{7\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}}{a^2d}$
norman	$\frac{\frac{7x}{8a}-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}-\frac{77\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12ad}-\frac{83\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12ad}-\frac{25\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ad}+\frac{7x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}+\frac{21x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a}+\frac{7x(}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 8/d/a^2*((-25/32*tan(1/2*d*x+1/2*c)^7-83/96*tan(1/2*d*x+1/2*c)^5-77/96*tan(
1/2*d*x+1/2*c)^3-7/32*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^4+7/32*a
rctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(79) = 158$.
time = 0.48, size = 206, normalized size = 2.37

$$-\frac{\frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}+\frac{77\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{83\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{75\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^2+\frac{4a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}-\frac{21\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/12*((21*sin(d*x + c)/(cos(d*x + c) + 1) + 77*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 + 83*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 75*sin(d*x + c)^7/(cos(
d*x + c) + 1)^7)/(a^2 + 4*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a^2*s
in(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1
)^6 + a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) - 21*arctan(sin(d*x + c)/(co
s(d*x + c) + 1))/a^2)/d
```

Fricas [A]
time = 4.13, size = 50, normalized size = 0.57

$$\frac{21\,dx+\left(6\,\cos\left(dx+c\right)^3-16\,\cos\left(dx+c\right)^2+21\,\cos\left(dx+c\right)-32\right)\sin\left(dx+c\right)}{24\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(21*d*x + (6*cos(d*x + c)^3 - 16*cos(d*x + c)^2 + 21*cos(d*x + c) - 32
)*sin(d*x + c))/(a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sin^4(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(sin(c + d*x)**4/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [A]
time = 0.44, size = 87, normalized size = 1.00

$$\frac{\frac{21\,(dx+c)}{a^2}-\frac{2\left(75\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+83\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+77\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+21\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4a^2}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(21*(d*x + c)/a^2 - 2*(75*tan(1/2*d*x + 1/2*c)^7 + 83*tan(1/2*d*x + 1/
2*c)^5 + 77*tan(1/2*d*x + 1/2*c)^3 + 21*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x
 + 1/2*c)^2 + 1)^4*a^2))/d
```

Mupad [B]
time = 4.51, size = 81, normalized size = 0.93

$$\frac{7\,x}{8\,a^2}-\frac{\frac{25\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\frac{83\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{12}+\frac{77\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12}+\frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{a^2\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(a + a/cos(c + d*x))^2,x)
```

```
[Out] (7*x)/(8*a^2) - ((7*tan(c/2 + (d*x)/2))/4 + (77*tan(c/2 + (d*x)/2)^3)/12 +
(83*tan(c/2 + (d*x)/2)^5)/12 + (25*tan(c/2 + (d*x)/2)^7)/4)/(a^2*d*(tan(c/2
 + (d*x)/2)^2 + 1)^4)
```

3.86 $\int \frac{\sin^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=69

$$-\frac{5x}{2a^2}+\frac{2\sin(c+dx)}{a^2d}-\frac{\cos(c+dx)\sin(c+dx)}{2a^2d}+\frac{2\sin(c+dx)}{a^2d(1+\cos(c+dx))}$$

```
[Out] -5/2*x/a^2+2*sin(d*x+c)/a^2/d-1/2*cos(d*x+c)*sin(d*x+c)/a^2/d+2*sin(d*x+c)/
a^2/d/(1+cos(d*x+c))
```

Rubi [A]
time = 0.23, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3957, 2953, 3029, 2788, 2717, 2715, 8, 2727}

$$\frac{2\sin(c+dx)}{a^2d}-\frac{\sin(c+dx)\cos(c+dx)}{2a^2d}+\frac{2\sin(c+dx)}{a^2d(\cos(c+dx)+1)}-\frac{5x}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-5*x)/(2*a^2) + (2*Sin[c + d*x])/(a^2*d) - (Cos[c + d*x]*Sin[c + d*x])/(2*
a^2*d) + (2*Sin[c + d*x])/(a^2*d*(1 + Cos[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
+ f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 2953

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 3029

```
Int[sin[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^n*c^n,
 Int[Tan[e + f*x]^p*(a + b*Sin[e + f*x])^(m - n), x], x] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[p + 2*n,
0] && IntegerQ[n]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= \int \frac{\cos^2(c+dx)\sin^2(c+dx)}{(-a-a\cos(c+dx))^2}\,dx \\
&= \frac{\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))}{-a-a\cos(c+dx)}\,dx}{a^2} \\
&= \frac{\int (-a+a\cos(c+dx))^2\cot^2(c+dx)\,dx}{a^4} \\
&= \frac{\int \left(-2+2\cos(c+dx)-\cos^2(c+dx)+\frac{2}{1+\cos(c+dx)}\right)\,dx}{a^2} \\
&= -\frac{2x}{a^2}-\frac{\int \cos^2(c+dx)\,dx}{a^2}+\frac{2\int \cos(c+dx)\,dx}{a^2}+\frac{2\int \frac{1}{1+\cos(c+dx)}\,dx}{a^2} \\
&= -\frac{2x}{a^2}+\frac{2\sin(c+dx)}{a^2d}-\frac{\cos(c+dx)\sin(c+dx)}{2a^2d}+\frac{2\sin(c+dx)}{a^2d(1+\cos(c+dx))}-\frac{\int 1\,a}{2a^2} \\
&= -\frac{5x}{2a^2}+\frac{2\sin(c+dx)}{a^2d}-\frac{\cos(c+dx)\sin(c+dx)}{2a^2d}+\frac{2\sin(c+dx)}{a^2d(1+\cos(c+dx))}
\end{aligned}
$$

Mathematica [A]
time = 0.22, size = 121, normalized size = 1.75

$$
-\frac{\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(60dx\cos\left(\frac{dx}{2}\right)+60dx\cos\left(c+\frac{dx}{2}\right)-119\sin\left(\frac{dx}{2}\right)-25\sin\left(c+\frac{dx}{2}\right)-21\sin\left(c+\frac{3dx}{2}\right)-21\sin\left(2c+\frac{3dx}{2}\right)+3\sin\left(2c+\frac{5dx}{2}\right)+3\sin\left(3c+\frac{5dx}{2}\right)\right)}{48a^2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/48*(Sec[c/2]*Sec[(c + d*x)/2]*(60*d*x*Cos[(d*x)/2] + 60*d*x*Cos[c + (d*x
)/2] - 119*Sin[(d*x)/2] - 25*Sin[c + (d*x)/2] - 21*Sin[c + (3*d*x)/2] - 21*
Sin[2*c + (3*d*x)/2] + 3*Sin[2*c + (5*d*x)/2] + 3*Sin[3*c + (5*d*x)/2]))/(a
^2*d)
```

Maple [A]
time = 0.12, size = 73, normalized size = 1.06

method	result	size
derivativedivides	$\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{2\left(-\frac{5\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-5\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2d}$	73
default	$\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{2\left(-\frac{5\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-5\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2d}$	73

risch	$-\frac{5x}{2a^2}-\frac{i\mathrm{e}^{i(dx+c)}}{a^2d}+\frac{i\mathrm{e}^{-i(dx+c)}}{a^2d}+\frac{4i}{a^2d(\mathrm{e}^{i(dx+c)}+1)}-\frac{\sin(2dx+2c)}{4a^2d}$	83
norman	$\frac{-\frac{5x}{2a}+\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{9\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{5x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{5x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2a}$	116

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^2*(tan(1/2*d*x+1/2*c)-(-5/2*tan(1/2*d*x+1/2*c)^3-3/2*tan(1/2*d*x+1/2*
c))/(1+tan(1/2*d*x+1/2*c)^2)^2-5/2*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(65) = 130$.
time = 0.47, size = 140, normalized size = 2.03

$$\frac{\frac{\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2+\frac{2a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{5\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{2\sin(dx+c)}{a^2(\cos(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] ((3*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x + c)^3/(cos(d*x + c) + 1)^3
)/(a^2 + 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4) - 5*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 2*sin(
d*x + c)/(a^2*(cos(d*x + c) + 1)))/d
```

Fricas [A]
time = 2.56, size = 61, normalized size = 0.88

$$-\frac{5\,dx\cos\left(dx+c\right)+5\,dx+\left(\cos\left(dx+c\right)^2-3\,\cos\left(dx+c\right)-8\right)\sin\left(dx+c\right)}{2\left(a^2d\cos\left(dx+c\right)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(5*d*x*cos(d*x + c) + 5*d*x + (cos(d*x + c)^2 - 3*cos(d*x + c) - 8)*si
n(d*x + c))/(a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sin^2(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(sin(c + d*x)**2/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [A]
time = 0.46, size = 75, normalized size = 1.09

$$-\frac{\frac{5\,(dx+c)}{a^2}-\frac{4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2}-\frac{2\left(5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(5*(d*x + c)/a^2 - 4*tan(1/2*d*x + 1/2*c)/a^2 - 2*(5*tan(1/2*d*x + 1/2
*c)^3 + 3*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2))/d
```

Mupad [B]
time = 1.03, size = 91, normalized size = 1.32

$$\frac{4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-5\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(c+d\,x\right)+10\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-4\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a^2\,d\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(a + a/cos(c + d*x))^2,x)
```

```
[Out] (4*sin(c/2 + (d*x)/2) - 5*cos(c/2 + (d*x)/2)*(c + d*x) + 10*cos(c/2 + (d*x)
/2)^2*sin(c/2 + (d*x)/2) - 4*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2))/(2*a^
2*d*cos(c/2 + (d*x)/2))
```

3.87 $\int \frac{\csc^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=73

$$-\frac{\cot^3(c+dx)}{3a^2d}-\frac{2\cot^5(c+dx)}{5a^2d}-\frac{2\csc^3(c+dx)}{3a^2d}+\frac{2\csc^5(c+dx)}{5a^2d}$$

```
[Out] -1/3*cot(d*x+c)^3/a^2/d-2/5*cot(d*x+c)^5/a^2/d-2/3*csc(d*x+c)^3/a^2/d+2/5*c
sc(d*x+c)^5/a^2/d
```

Rubi [A]
time = 0.15, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3957, 2790, 2687, 30, 2686, 14}

$$-\frac{2\cot^5(c+dx)}{5a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{2\csc^5(c+dx)}{5a^2d}-\frac{2\csc^3(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/3*Cot[c + d*x]^3/(a^2*d) - (2*Cot[c + d*x]^5)/(5*a^2*d) - (2*Csc[c + d*x
]^3)/(3*a^2*d) + (2*Csc[c + d*x]^5)/(5*a^2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rule 2790

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((g_.)*tan[(e_.) + (f_.)*(x
_)])^(p_.), x_Symbol] :> Dist[a^(2*m), Int[ExpandIntegrand[(g*Tan[e + f*x])
^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; Fr
eeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cot^2(c+dx)}{(-a-a\cos(c+dx))^2}\,dx$$
$$= \frac{\int \left(a^2\cot^4(c+dx)\csc^2(c+dx) - 2a^2\cot^3(c+dx)\csc^3(c+dx) + a^2\cot^2(c+dx\right.}{a^4}$$
$$= \frac{\int \cot^4(c+dx)\csc^2(c+dx)\,dx}{a^2} + \frac{\int \cot^2(c+dx)\csc^4(c+dx)\,dx}{a^2} - \frac{2\int \cot^3(c+}{}$$
$$= \frac{\text{Subst}\left(\int x^4\,dx, x, -\cot(c+dx)\right)}{a^2d} + \frac{\text{Subst}\left(\int x^2(1+x^2)\,dx, x, -\cot(c+dx)\right)}{a^2d} +$$
$$= -\frac{\cot^5(c+dx)}{5a^2d} + \frac{\text{Subst}\left(\int (x^2+x^4)\,dx, x, -\cot(c+dx)\right)}{a^2d} + \frac{2\text{Subst}\left(\int (-x^2+\right.}{}$$
$$= -\frac{\cot^3(c+dx)}{3a^2d} - \frac{2\cot^5(c+dx)}{5a^2d} - \frac{2\csc^3(c+dx)}{3a^2d} + \frac{2\csc^5(c+dx)}{5a^2d}$$

Mathematica [A]
time = 0.30, size = 105, normalized size = 1.44

$$\frac{\csc(c)\csc(c+dx)\sec^2(c+dx)(-80\sin(c)+80\sin(dx)+55\sin(c+dx)+44\sin(2(c+dx))+11\sin(3(c+dx))-60\sin(2c+dx)+16\sin(c+2dx)+4\sin(2c+3dx))}{240a^2d(1+\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Csc[c]*Csc[c + d*x]*Sec[c + d*x]^2*(-80*Sin[c] + 80*Sin[d*x] + 55*Sin[c +
d*x] + 44*Sin[2*(c + d*x)] + 11*Sin[3*(c + d*x)] - 60*Sin[2*c + d*x] + 16*S
in[c + 2*d*x] + 4*Sin[2*c + 3*d*x]))/(240*a^2*d*(1 + Sec[c + d*x])^2)
```

Maple [A]
time = 0.08, size = 60, normalized size = 0.82

method	result	size
derivativedivides	$\frac{\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{8d\,a^2}$	60
default	$\frac{\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{8d\,a^2}$	60
norman	$\frac{-\frac{1}{8ad}-\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)}{24ad}+\frac{\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)}{40ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a}$	82
risch	$-\frac{2i\left(15\,e^{4i(dx+c)}+20\,e^{3i(dx+c)}+20\,e^{2i(dx+c)}+4\,e^{i(dx+c)}+1\right)}{15a^2d\left(e^{i(dx+c)}+1\right)^5\left(e^{i(dx+c)}-1\right)}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d/a^2*(1/5*tan(1/2*d*x+1/2*c)^5-1/3*tan(1/2*d*x+1/2*c)^3-tan(1/2*d*x+1/
2*c)-1/tan(1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.27, size = 90, normalized size = 1.23

$$-\frac{\frac{\frac{15\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^2}+\frac{15\,(\cos(dx+c)+1)}{a^2\sin(dx+c)}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/120*((15*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 - 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^2 + 15*(cos(d*x + c) +
1)/(a^2*sin(d*x + c)))/d
```

Fricas [A]
time = 3.06, size = 71, normalized size = 0.97

$$-\frac{\cos\left(dx+c\right)^3+2\,\cos\left(dx+c\right)^2+8\,\cos\left(dx+c\right)+4}{15\,\left(a^2d\cos\left(dx+c\right)^2+2\,a^2d\cos\left(dx+c\right)+a^2d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/15*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + 8*cos(d*x + c) + 4)/((a^2*d*cos(
d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^2{(c+dx)}}{\sec^2{(c+dx)}+2\sec{(c+dx)}+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(csc(c + d*x)**2/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [A]
time = 0.48, size = 74, normalized size = 1.01

$$-\frac{\frac{15}{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{3\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-5\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-15\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{10}}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/120*(15/(a^2*tan(1/2*d*x + 1/2*c)) - (3*a^8*tan(1/2*d*x + 1/2*c)^5 - 5*a
^8*tan(1/2*d*x + 1/2*c)^3 - 15*a^8*tan(1/2*d*x + 1/2*c))/a^10)/d
```

Mupad [B]
time = 0.98, size = 71, normalized size = 0.97

$$-\frac{8\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-4\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+14\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-3}{120\,a^2\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^2*(a + a/cos(c + d*x))^2),x)
```

```
[Out] -(14*cos(c/2 + (d*x)/2)^2 - 4*cos(c/2 + (d*x)/2)^4 + 8*cos(c/2 + (d*x)/2)^6
 - 3)/(120*a^2*d*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2))
```

3.88 $\int \frac{\csc^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=91

$$-\frac{\cot^3(c+dx)}{3a^2d}-\frac{3\cot^5(c+dx)}{5a^2d}-\frac{2\cot^7(c+dx)}{7a^2d}-\frac{2\csc^5(c+dx)}{5a^2d}+\frac{2\csc^7(c+dx)}{7a^2d}$$

```
[Out] -1/3*cot(d*x+c)^3/a^2/d-3/5*cot(d*x+c)^5/a^2/d-2/7*cot(d*x+c)^7/a^2/d-2/5*c
sc(d*x+c)^5/a^2/d+2/7*csc(d*x+c)^7/a^2/d
```

Rubi [A]
time = 0.24, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3957, 2954, 2952, 2687, 14, 2686, 276}

$$-\frac{2\cot^7(c+dx)}{7a^2d}-\frac{3\cot^5(c+dx)}{5a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{2\csc^7(c+dx)}{7a^2d}-\frac{2\csc^5(c+dx)}{5a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/3*Cot[c + d*x]^3/(a^2*d) - (3*Cot[c + d*x]^5)/(5*a^2*d) - (2*Cot[c + d*x
]^7)/(7*a^2*d) - (2*Csc[c + d*x]^5)/(5*a^2*d) + (2*Csc[c + d*x]^7)/(7*a^2*d
)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= \int \frac{\cot^2(c+dx)\csc^2(c+dx)}{(-a-a\cos(c+dx))^2}\,dx \\
&= \frac{\int (-a+a\cos(c+dx))^2\cot^2(c+dx)\csc^6(c+dx)\,dx}{a^4} \\
&= \frac{\int \left(a^2\cot^4(c+dx)\csc^4(c+dx)-2a^2\cot^3(c+dx)\csc^5(c+dx)+a^2\cot^2(c+dx\right.}{a^4} \\
&= \frac{\int \cot^4(c+dx)\csc^4(c+dx)\,dx}{a^2}+\frac{\int \cot^2(c+dx)\csc^6(c+dx)\,dx}{a^2}-\frac{2\int \cot^3(c+}{} \\
&= \frac{\text{Subst}\left(\int x^4(1+x^2)\,dx,x,-\cot(c+dx)\right)}{a^2d}+\frac{\text{Subst}\left(\int x^2(1+x^2)^2\,dx,x,-\cot(c\right.}{a^2d} \\
&= \frac{\text{Subst}\left(\int (x^4+x^6)\,dx,x,-\cot(c+dx)\right)}{a^2d}+\frac{\text{Subst}\left(\int (x^2+2x^4+x^6)\,dx,x,-\text{co}\right.}{a^2d} \\
&= -\frac{\cot^3(c+dx)}{3a^2d}-\frac{3\cot^5(c+dx)}{5a^2d}-\frac{2\cot^7(c+dx)}{7a^2d}-\frac{2\csc^5(c+dx)}{5a^2d}+\frac{2\csc^7(c+}{7a^2d}
\end{aligned}$$

Mathematica [A]
time = 0.45, size = 149, normalized size = 1.64

$$-\frac{\csc(c)\csc^3(c+dx)\sec^2(c+dx)(1344\sin(c)-1456\sin(dx)-714\sin(c+dx)-408\sin(2(c+dx))+153\sin(3(c+dx))+204\sin(4(c+dx))+51\sin(5(c+dx))+1680\sin(2c+dx)+128\sin(c+2dx)-48\sin(2c+3dx)-64\sin(3c+4dx)-16\sin(4c+5dx))}{13440a^2d(1+\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/13440*(Csc[c]*Csc[c + d*x]^3*Sec[c + d*x]^2*(1344*Sin[c] - 1456*Sin[d*x]
 - 714*Sin[c + d*x] - 408*Sin[2*(c + d*x)] + 153*Sin[3*(c + d*x)] + 204*Sin
[4*(c + d*x)] + 51*Sin[5*(c + d*x)] + 1680*Sin[2*c + d*x] + 128*Sin[c + 2*d
*x] - 48*Sin[2*c + 3*d*x] - 64*Sin[3*c + 4*d*x] - 16*Sin[4*c + 5*d*x]))/(a^
2*d*(1 + Sec[c + d*x])^2)
```

Maple [A]
time = 0.10, size = 86, normalized size = 0.95

method	result	size
derivativedivides	$\frac{\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}}{32d\,a^2}$	86
default	$\frac{\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}}{32d\,a^2}$	86
risch	$\frac{4i\left(105\,e^{6i(dx+c)}+84\,e^{5i(dx+c)}+91\,e^{4i(dx+c)}-8\,e^{3i(dx+c)}+3\,e^{2i(dx+c)}+4\,e^{i(dx+c)}+1\right)}{105a^2d\left(e^{i(dx+c)}+1\right)^7\left(e^{i(dx+c)}-1\right)^3}$	104
norman	$\frac{-\frac{1}{96ad}-\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{32ad}-\frac{\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)}{16ad}-\frac{\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)}{48ad}+\frac{\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)}{160ad}+\frac{\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)}{224ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3a}$	120

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/32/d/a^2*(1/7*tan(1/2*d*x+1/2*c)^7+1/5*tan(1/2*d*x+1/2*c)^5-2/3*tan(1/2*d
*x+1/2*c)^3-2*tan(1/2*d*x+1/2*c)-1/tan(1/2*d*x+1/2*c)-1/3/tan(1/2*d*x+1/2*c
)^3)
```

Maxima [A]
time = 0.27, size = 134, normalized size = 1.47

$$-\frac{\frac{\frac{210\sin(dx+c)}{\cos(dx+c)+1}+\frac{70\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^2}+\frac{35\left(\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)(\cos(dx+c)+1)^3}{a^2\sin(dx+c)^3}}{3360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3360*((210*sin(d*x + c)/(cos(d*x + c) + 1) + 70*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/(c
os(d*x + c) + 1)^7)/a^2 + 35*(3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)*(c
os(d*x + c) + 1)^3/(a^2*sin(d*x + c)^3))/d
```

Fricas [A]
time = 3.11, size = 108, normalized size = 1.19

$$\frac{2\cos(dx+c)^5+4\cos(dx+c)^4-\cos(dx+c)^3-6\cos(dx+c)^2+24\cos(dx+c)+12}{105\left(a^2d\cos(dx+c)^4+2a^2d\cos(dx+c)^3-2a^2d\cos(dx+c)-a^2d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/105*(2*cos(d*x + c)^5 + 4*cos(d*x + c)^4 - cos(d*x + c)^3 - 6*cos(d*x + c
)^2 + 24*cos(d*x + c) + 12)/((a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3
 - 2*a^2*d*cos(d*x + c) - a^2*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\csc^4(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(csc(c + d*x)**4/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [A]
time = 0.50, size = 105, normalized size = 1.15

$$-\frac{\frac{35\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)}{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3}-\frac{15a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+21a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-70a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-210a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{14}}}{3360d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3360*(35*(3*tan(1/2*d*x + 1/2*c)^2 + 1)/(a^2*tan(1/2*d*x + 1/2*c)^3) - (
15*a^12*tan(1/2*d*x + 1/2*c)^7 + 21*a^12*tan(1/2*d*x + 1/2*c)^5 - 70*a^12*t
an(1/2*d*x + 1/2*c)^3 - 210*a^12*tan(1/2*d*x + 1/2*c))/a^14)/d
```

Mupad [B]
time = 1.07, size = 121, normalized size = 1.33

$$-\frac{64\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-96\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8+24\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6+4\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4+54\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2-15}{3360a^2d\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^7\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^9\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^4*(a + a/cos(c + d*x))^2),x)
```

```
[Out] -(54*cos(c/2 + (d*x)/2)^2 + 4*cos(c/2 + (d*x)/2)^4 + 24*cos(c/2 + (d*x)/2)^
6 - 96*cos(c/2 + (d*x)/2)^8 + 64*cos(c/2 + (d*x)/2)^10 - 15)/(3360*a^2*d*(c
os(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2) - cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*
x)/2)))
```

3.89 $\int \frac{\csc^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=109

$$-\frac{\cot^3(c+dx)}{3a^2d}-\frac{4\cot^5(c+dx)}{5a^2d}-\frac{5\cot^7(c+dx)}{7a^2d}-\frac{2\cot^9(c+dx)}{9a^2d}-\frac{2\csc^7(c+dx)}{7a^2d}+\frac{2\csc^9(c+dx)}{9a^2d}$$

```
[Out] -1/3*cot(d*x+c)^3/a^2/d-4/5*cot(d*x+c)^5/a^2/d-5/7*cot(d*x+c)^7/a^2/d-2/9*c
ot(d*x+c)^9/a^2/d-2/7*csc(d*x+c)^7/a^2/d+2/9*csc(d*x+c)^9/a^2/d
```

Rubi [A]
time = 0.26, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3957, 2954, 2952, 2687, 276, 2686, 14}

$$-\frac{2\cot^9(c+dx)}{9a^2d}-\frac{5\cot^7(c+dx)}{7a^2d}-\frac{4\cot^5(c+dx)}{5a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{2\csc^9(c+dx)}{9a^2d}-\frac{2\csc^7(c+dx)}{7a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/3*Cot[c + d*x]^3/(a^2*d) - (4*Cot[c + d*x]^5)/(5*a^2*d) - (5*Cot[c + d*x
]^7)/(7*a^2*d) - (2*Cot[c + d*x]^9)/(9*a^2*d) - (2*Csc[c + d*x]^7)/(7*a^2*d
) + (2*Csc[c + d*x]^9)/(9*a^2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= \int \frac{\cot^2(c+dx)\csc^4(c+dx)}{(-a-a\cos(c+dx))^2}\,dx \\
&= \frac{\int (-a+a\cos(c+dx))^2\cot^2(c+dx)\csc^8(c+dx)\,dx}{a^4} \\
&= \frac{\int \left(a^2\cot^4(c+dx)\csc^6(c+dx)-2a^2\cot^3(c+dx)\csc^7(c+dx)+a^2\cot^2(c+a\right.}{a^4} \\
&= \frac{\int \cot^4(c+dx)\csc^6(c+dx)\,dx}{a^2}+\frac{\int \cot^2(c+dx)\csc^8(c+dx)\,dx}{a^2}-\frac{2\int \cot^3(c-}{} \\
&= \frac{\text{Subst}\left(\int x^4(1+x^2)^2\,dx,x,-\cot(c+dx)\right)}{a^2d}+\frac{\text{Subst}\left(\int x^2(1+x^2)^3\,dx,x,-\text{co}\right.}{a^2d} \\
&= \frac{\text{Subst}\left(\int (x^4+2x^6+x^8)\,dx,x,-\cot(c+dx)\right)}{a^2d}+\frac{\text{Subst}\left(\int (x^2+3x^4+3x^6+\right.}{a^2d} \\
&= -\frac{\cot^3(c+dx)}{3a^2d}-\frac{4\cot^5(c+dx)}{5a^2d}-\frac{5\cot^7(c+dx)}{7a^2d}-\frac{2\cot^9(c+dx)}{9a^2d}-\frac{2\csc^7(c}{7a^2}
\end{aligned}$$

Mathematica [A]

time = 0.65, size = 191, normalized size = 1.75

$$\frac{\csc(c)\csc^5(c+dx)\sec^2(c+dx)(-61440\sin(c)+84480\sin(dx)+25875\sin(c+dx)+11500\sin(2(c+dx))-10925\sin(3(c+dx))-9200\sin(4(c+dx))+575\sin(5(c+dx))+2300\sin(6(c+dx))+575\sin(7(c+dx))-107520\sin(2c+dx)-10240\sin(c+2dx)+9728\sin(2c+3dx)+8192\sin(3c+4dx)-512\sin(4c+5dx)-2048\sin(5c+6dx)-512\sin(6c+7dx))}{1290240a^2d(1+\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Csc[c]*Csc[c + d*x]^5*Sec[c + d*x]^2*(-61440*Sin[c] + 84480*Sin[d*x] + 258
75*Sin[c + d*x] + 11500*Sin[2*(c + d*x)] - 10925*Sin[3*(c + d*x)] - 9200*Si
n[4*(c + d*x)] + 575*Sin[5*(c + d*x)] + 2300*Sin[6*(c + d*x)] + 575*Sin[7*(
c + d*x)] - 107520*Sin[2*c + d*x] - 10240*Sin[c + 2*d*x] + 9728*Sin[2*c + 3
*d*x] + 8192*Sin[3*c + 4*d*x] - 512*Sin[4*c + 5*d*x] - 2048*Sin[5*c + 6*d*x
] - 512*Sin[6*c + 7*d*x]))/(1290240*a^2*d*(1 + Sec[c + d*x])^2)
```

Maple [A]
time = 0.12, size = 112, normalized size = 1.03

method	result
derivativedivides	$\frac{\frac{\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}+\frac{3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{5\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{}{\tan}}{128d\,a^2}$
default	$\frac{\frac{\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}+\frac{3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{5\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{}{\tan}}{128d\,a^2}$
risch	$-\frac{16i\left(210\,e^{8i(dx+c)}+120\,e^{7i(dx+c)}+165\,e^{6i(dx+c)}-20\,e^{5i(dx+c)}+19\,e^{4i(dx+c)}+16\,e^{3i(dx+c)}-e^{2i(dx+c)}-4\,e^{i(dx+c)}-1\right)}{315a^2d\left(e^{i(dx+c)}+1\right)^9\left(e^{i(dx+c)}-1\right)^5}$
norman	$\frac{-\frac{1}{640ad}-\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{128ad}-\frac{\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)}{128ad}-\frac{5\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128ad}-\frac{5\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{384ad}+\frac{\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)}{640ad}+\frac{3\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{896ad}+\frac{\tan^{14}\left(\frac{dx}{2}\right.}{1152a}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/128/d/a^2*(1/9*tan(1/2*d*x+1/2*c)^9+3/7*tan(1/2*d*x+1/2*c)^7+1/5*tan(1/2*
d*x+1/2*c)^5-5/3*tan(1/2*d*x+1/2*c)^3-5*tan(1/2*d*x+1/2*c)-1/5/tan(1/2*d*x+
1/2*c)^5-1/tan(1/2*d*x+1/2*c)-1/tan(1/2*d*x+1/2*c)^3)
```

Maxima [A]
time = 0.28, size = 174, normalized size = 1.60

$$-\frac{\frac{\frac{1575\sin(dx+c)}{\cos(dx+c)+1}+\frac{525\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{63\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{135\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{35\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^2}+\frac{63\left(\frac{5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{5\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+1\right)(\cos(dx+c)+1)^5}{a^2\sin(dx+c)^5}}{40320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/40320*((1575*sin(d*x + c)/(cos(d*x + c) + 1) + 525*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 - 63*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 135*sin(d*x + c)^
7/(cos(d*x + c) + 1)^7 - 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/a^2 + 63*(
5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 5*sin(d*x + c)^4/(cos(d*x + c) + 1)
^4 + 1)*(cos(d*x + c) + 1)^5/(a^2*sin(d*x + c)^5))/d
```

Fricas [A]
time = 2.82, size = 169, normalized size = 1.55

$$\frac{8\,\cos\left(dx+c\right)^{7}+16\,\cos\left(dx+c\right)^{6}-12\,\cos\left(dx+c\right)^{5}-40\,\cos\left(dx+c\right)^{4}-5\,\cos\left(dx+c\right)^{3}+30\,\cos\left(dx+c\right)^{2}-40\,\cos\left(dx+c\right)-20}{315\,\left(a^{2}d\cos\left(dx+c\right)^{6}+2\,a^{2}d\cos\left(dx+c\right)^{5}-a^{2}d\cos\left(dx+c\right)^{4}-4\,a^{2}d\cos\left(dx+c\right)^{3}-a^{2}d\cos\left(dx+c\right)^{2}+2\,a^{2}d\cos\left(dx+c\right)+a^{2}d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/315*(8*cos(d*x + c)^7 + 16*cos(d*x + c)^6 - 12*cos(d*x + c)^5 - 40*cos(d*
x + c)^4 - 5*cos(d*x + c)^3 + 30*cos(d*x + c)^2 - 40*cos(d*x + c) - 20)/((a
^2*d*cos(d*x + c)^6 + 2*a^2*d*cos(d*x + c)^5 - a^2*d*cos(d*x + c)^4 - 4*a^2
*d*cos(d*x + c)^3 - a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)*si
n(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^{6}\left(c+dx\right)}{\sec^{2}\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(csc(c + d*x)**6/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [A]
time = 0.54, size = 134, normalized size = 1.23

$$-\frac{\frac{63\left(5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)}{a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}}-\frac{35\,a^{16}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+135\,a^{16}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+63\,a^{16}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-525\,a^{16}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-1575\,a^{16}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{18}}}{40320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/40320*(63*(5*tan(1/2*d*x + 1/2*c)^4 + 5*tan(1/2*d*x + 1/2*c)^2 + 1)/(a^2
*tan(1/2*d*x + 1/2*c)^5) - (35*a^16*tan(1/2*d*x + 1/2*c)^9 + 135*a^16*tan(1
/2*d*x + 1/2*c)^7 + 63*a^16*tan(1/2*d*x + 1/2*c)^5 - 525*a^16*tan(1/2*d*x +
 1/2*c)^3 - 1575*a^16*tan(1/2*d*x + 1/2*c))/a^18)/d
```

Mupad [B]
time = 2.18, size = 106, normalized size = 0.97

$$-\frac{\frac{375\cos(c+d\,x)}{8}-\frac{5\cos(2\,c+2\,d\,x)}{2}+\frac{19\cos(3\,c+3\,d\,x)}{8}+2\cos\left(4\,c+4\,d\,x\right)-\frac{\cos(5\,c+5\,d\,x)}{8}-\frac{\cos(6\,c+6\,d\,x)}{2}-\frac{\cos(7\,c+7\,d\,x)}{8}+15}{40320\,a^2\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^6*(a + a/cos(c + d*x))^2),x)
```

```
[Out] -((375*cos(c + d*x))/8 - (5*cos(2*c + 2*d*x))/2 + (19*cos(3*c + 3*d*x))/8 +
 2*cos(4*c + 4*d*x) - cos(5*c + 5*d*x)/8 - cos(6*c + 6*d*x)/2 - cos(7*c + 7
*d*x)/8 + 15)/(40320*a^2*d*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^5)
```

3.90 $\int \frac{\csc^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=125

$$-\frac{\cot^3(c+dx)}{3a^2d}-\frac{\cot^5(c+dx)}{a^2d}-\frac{9\cot^7(c+dx)}{7a^2d}-\frac{7\cot^9(c+dx)}{9a^2d}-\frac{2\cot^{11}(c+dx)}{11a^2d}-\frac{2\csc^9(c+dx)}{9a^2d}+\frac{2\csc^{11}(c+dx)}{11a^2d}$$

```
[Out] -1/3*cot(d*x+c)^3/a^2/d-cot(d*x+c)^5/a^2/d-9/7*cot(d*x+c)^7/a^2/d-7/9*cot(d
*x+c)^9/a^2/d-2/11*cot(d*x+c)^11/a^2/d-2/9*csc(d*x+c)^9/a^2/d+2/11*csc(d*x+
c)^11/a^2/d
```

Rubi [A]

time = 0.27, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3957, 2954, 2952, 2687, 276, 2686, 14}

$$-\frac{2\cot^{11}(c+dx)}{11a^2d}-\frac{7\cot^9(c+dx)}{9a^2d}-\frac{9\cot^7(c+dx)}{7a^2d}-\frac{\cot^5(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{2\csc^{11}(c+dx)}{11a^2d}-\frac{2\csc^9(c+dx)}{9a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^8/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/3*Cot[c + d*x]^3/(a^2*d) - Cot[c + d*x]^5/(a^2*d) - (9*Cot[c + d*x]^7)/(
7*a^2*d) - (7*Cot[c + d*x]^9)/(9*a^2*d) - (2*Cot[c + d*x]^11)/(11*a^2*d) -
(2*Csc[c + d*x]^9)/(9*a^2*d) + (2*Csc[c + d*x]^11)/(11*a^2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= \int \frac{\cot^2(c+dx)\csc^6(c+dx)}{(-a-a\cos(c+dx))^2}\,dx\\
&= \frac{\int (-a+a\cos(c+dx))^2\cot^2(c+dx)\csc^{10}(c+dx)\,dx}{a^4}\\
&= \frac{\int \left(a^2\cot^4(c+dx)\csc^8(c+dx)-2a^2\cot^3(c+dx)\csc^9(c+dx)+a^2\cot^2(c+dx\right.}{a^4}\\
&= \frac{\int \cot^4(c+dx)\csc^8(c+dx)\,dx}{a^2}+\frac{\int \cot^2(c+dx)\csc^{10}(c+dx)\,dx}{a^2}-\frac{2\int \cot^3(c+}{}\\
&= \frac{\operatorname{Subst}\left(\int x^4\left(1+x^2\right)^3\,dx,x,-\cot(c+dx)\right)}{a^2d}+\frac{\operatorname{Subst}\left(\int x^2\left(1+x^2\right)^4\,dx,x,-\cot\right.}{a^2d}\\
&= \frac{\operatorname{Subst}\left(\int \left(x^4+3x^6+3x^8+x^{10}\right)dx,x,-\cot(c+dx)\right)}{a^2d}+\frac{\operatorname{Subst}\left(\int \left(x^2+4x^4+6\right.\right.}{}\\
&= -\frac{\cot^3(c+dx)}{3a^2d}-\frac{\cot^5(c+dx)}{a^2d}-\frac{9\cot^7(c+dx)}{7a^2d}-\frac{7\cot^9(c+dx)}{9a^2d}-\frac{2\cot^{11}(c+}{11a^2d}
\end{aligned}
$$

Mathematica [A]
time = 0.92, size = 233, normalized size = 1.86

$$-\frac{\csc(c)\csc^7(c+dx)\sec^2(c+dx)(630784\sin(c)-1103872\sin(dx)-218834\sin(c+dx)-79576\sin(2(c+dx))+119364\sin(3(c+dx))+79576\sin(4(c+dx))-28420\sin(5(c+dx))-34104\sin(6(c+dx))-1421\sin(7(c+dx))+5684\sin(8(c+dx))+1421\sin(9(c+dx))+1419264\sin(2c+dx)+114688\sin(c+2dx)-172032\sin(2c+3dx)-114688\sin(3c+4dx)+40960\sin(4c+5dx)+49152\sin(5c+6dx)+2048\sin(6c+7dx)-8192\sin(7c+8dx)-2048\sin(8c+9dx))}{22708224a^2d(1+\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^8/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/22708224*(Csc[c]*Csc[c + d*x]^7*Sec[c + d*x]^2*(630784*Sin[c] - 1103872*
Sin[d*x] - 218834*Sin[c + d*x] - 79576*Sin[2*(c + d*x)] + 119364*Sin[3*(c +
 d*x)] + 79576*Sin[4*(c + d*x)] - 28420*Sin[5*(c + d*x)] - 34104*Sin[6*(c +
 d*x)] - 1421*Sin[7*(c + d*x)] + 5684*Sin[8*(c + d*x)] + 1421*Sin[9*(c + d*
x)] + 1419264*Sin[2*c + d*x] + 114688*Sin[c + 2*d*x] - 172032*Sin[2*c + 3*d
*x] - 114688*Sin[3*c + 4*d*x] + 40960*Sin[4*c + 5*d*x] + 49152*Sin[5*c + 6*
d*x] + 2048*Sin[6*c + 7*d*x] - 8192*Sin[7*c + 8*d*x] - 2048*Sin[8*c + 9*d*x
]))/(a^2*d*(1 + Sec[c + d*x])^2)
```

Maple [A]
time = 0.12, size = 112, normalized size = 0.90

method	result
derivativedivides	$\frac{\frac{\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{11}+\frac{5\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}+\frac{8\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{14\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-14\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{8}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right.}}{512d\,a^{2}}$
default	$\frac{\frac{\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{11}+\frac{5\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}+\frac{8\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{14\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-14\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{8}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right.}}{512d\,a^{2}}$
risch	$\frac{32i\left(693\,\mathrm{e}^{10i(dx+c)}+308\,\mathrm{e}^{9i(dx+c)}+539\,\mathrm{e}^{8i(dx+c)}-56\,\mathrm{e}^{7i(dx+c)}+84\,\mathrm{e}^{6i(dx+c)}+56\,\mathrm{e}^{5i(dx+c)}-20\,\mathrm{e}^{4i(dx+c)}-24\,\mathrm{e}^{3i(dx+c)}-\right.}{693a^{2}d\left(\mathrm{e}^{i(dx+c)}+1\right)^{11}\left(\mathrm{e}^{i(dx+c)}-1\right)^{7}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^8/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/512/d/a^2*(1/11*tan(1/2*d*x+1/2*c)^11+5/9*tan(1/2*d*x+1/2*c)^9+8/7*tan(1/
2*d*x+1/2*c)^7-14/3*tan(1/2*d*x+1/2*c)^3-14*tan(1/2*d*x+1/2*c)-8/3/tan(1/2*
d*x+1/2*c)^3-1/tan(1/2*d*x+1/2*c)^5-1/7/tan(1/2*d*x+1/2*c)^7)
```

Maxima [A]
time = 0.27, size = 174, normalized size = 1.39

$$-\frac{\frac{\frac{9702\sin(dx+c)}{\cos(dx+c)+1}+\frac{3234\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{792\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{385\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{63\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}}{a^2}+\frac{33\left(\frac{21\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{56\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+3\right)(\cos(dx+c)+1)^7}{a^2\sin(dx+c)^7}}{354816\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/354816*((9702*sin(d*x + c)/(cos(d*x + c) + 1) + 3234*sin(d*x + c)^3/(cos
(d*x + c) + 1)^3 - 792*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 385*sin(d*x +
c)^9/(cos(d*x + c) + 1)^9 - 63*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)/a^2 +
 33*(21*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 56*sin(d*x + c)^4/(cos(d*x +
c) + 1)^4 + 3)*(cos(d*x + c) + 1)^7/(a^2*sin(d*x + c)^7))/d
```

Fricas [A]
time = 4.60, size = 204, normalized size = 1.63

$$\frac{16\cos(dx+c)^9+32\cos(dx+c)^8-40\cos(dx+c)^7-112\cos(dx+c)^6+14\cos(dx+c)^5+140\cos(dx+c)^4+35\cos(dx+c)^3-70\cos(dx+c)^2+56\cos(dx+c)+28}{693\left(a^2d\cos(dx+c)^8+2a^2d\cos(dx+c)^7-2a^2d\cos(dx+c)^6-6a^2d\cos(dx+c)^5+6a^2d\cos(dx+c)^3+2a^2d\cos(dx+c)^2-2a^2d\cos(dx+c)-a^2d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/693*(16*cos(d*x + c)^9 + 32*cos(d*x + c)^8 - 40*cos(d*x + c)^7 - 112*cos(
d*x + c)^6 + 14*cos(d*x + c)^5 + 140*cos(d*x + c)^4 + 35*cos(d*x + c)^3 - 7
0*cos(d*x + c)^2 + 56*cos(d*x + c) + 28)/((a^2*d*cos(d*x + c)^8 + 2*a^2*d*c
os(d*x + c)^7 - 2*a^2*d*cos(d*x + c)^6 - 6*a^2*d*cos(d*x + c)^5 + 6*a^2*d*c
os(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 - 2*a^2*d*cos(d*x + c) - a^2*d)*sin(
d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**8/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.49, size = 134, normalized size = 1.07

$$-\frac{\frac{33\left(56\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+21\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+3\right)}{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7}-\frac{63a^{20}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}+385a^{20}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9+792a^{20}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-3234a^{20}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-9702a^{20}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{22}}}{354816\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/354816*(33*(56*tan(1/2*d*x + 1/2*c)^4 + 21*tan(1/2*d*x + 1/2*c)^2 + 3)/(
a^2*tan(1/2*d*x + 1/2*c)^7) - (63*a^20*tan(1/2*d*x + 1/2*c)^11 + 385*a^20*t
an(1/2*d*x + 1/2*c)^9 + 792*a^20*tan(1/2*d*x + 1/2*c)^7 - 3234*a^20*tan(1/2
*d*x + 1/2*c)^3 - 9702*a^20*tan(1/2*d*x + 1/2*c))/a^22)/d
```

Mupad [B]
time = 1.69, size = 201, normalized size = 1.61

$$-\frac{99\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{18}+693\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1848\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^4+9702\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^8+3234\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-792\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}-385\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}-63\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{18}}{354816\,a^2\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^8*(a + a/cos(c + d*x))^2),x)
```

```
[Out] -(99*cos(c/2 + (d*x)/2)^18 - 63*sin(c/2 + (d*x)/2)^18 - 385*cos(c/2 + (d*x)
/2)^2*sin(c/2 + (d*x)/2)^16 - 792*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^1
4 + 3234*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^10 + 9702*cos(c/2 + (d*x)/
2)^10*sin(c/2 + (d*x)/2)^8 + 1848*cos(c/2 + (d*x)/2)^14*sin(c/2 + (d*x)/2)^
4 + 693*cos(c/2 + (d*x)/2)^16*sin(c/2 + (d*x)/2)^2)/(354816*a^2*d*cos(c/2 +
 (d*x)/2)^11*sin(c/2 + (d*x)/2)^7)
```

3.91 $\int \frac{\sin^{11}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=139

$$\frac{2(a-a\cos(c+dx))^6}{3a^9d}-\frac{16(a-a\cos(c+dx))^7}{7a^{10}d}+\frac{25(a-a\cos(c+dx))^8}{8a^{11}d}-\frac{19(a-a\cos(c+dx))^9}{9a^{12}d}+\frac{7(a-a\cos}{10a}$$

```
[Out] 2/3*(a-a*cos(d*x+c))^6/a^9/d-16/7*(a-a*cos(d*x+c))^7/a^10/d+25/8*(a-a*cos(d
*x+c))^8/a^11/d-19/9*(a-a*cos(d*x+c))^9/a^12/d+7/10*(a-a*cos(d*x+c))^10/a^1
3/d-1/11*(a-a*cos(d*x+c))^11/a^14/d
```

Rubi [A]

time = 0.14, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3957, 2915, 12, 90}

$$-\frac{(a-a\cos(c+dx))^{11}}{11a^{14}d}+\frac{7(a-a\cos(c+dx))^{10}}{10a^{13}d}-\frac{19(a-a\cos(c+dx))^9}{9a^{12}d}+\frac{25(a-a\cos(c+dx))^8}{8a^{11}d}-\frac{16(a-a\cos(c+dx))^7}{7a^{10}d}+\frac{2(a-a\cos(c+dx))^6}{3a^9d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^11/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (2*(a - a*Cos[c + d*x])^6)/(3*a^9*d) - (16*(a - a*Cos[c + d*x])^7)/(7*a^10*
d) + (25*(a - a*Cos[c + d*x])^8)/(8*a^11*d) - (19*(a - a*Cos[c + d*x])^9)/(
9*a^12*d) + (7*(a - a*Cos[c + d*x])^10)/(10*a^13*d) - (a - a*Cos[c + d*x])^
11/(11*a^14*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^{11}(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= -\int \frac{\cos^3(c+dx)\sin^{11}(c+dx)}{(-a-a\cos(c+dx))^3}\,dx\\ &= \frac{\operatorname{Subst}\left(\int \frac{(-a-x)^5x^3(-a+x)^2}{a^3}\,dx, x, -a\cos(c+dx)\right)}{a^{11}d}\\ &= \frac{\operatorname{Subst}\left(\int (-a-x)^5x^3(-a+x)^2\,dx, x, -a\cos(c+dx)\right)}{a^{14}d}\\ &= \frac{\operatorname{Subst}\left(\int \left(-4a^5(-a-x)^5-16a^4(-a-x)^6-25a^3(-a-x)^7-19a^2(-a-x)^8\right.\right.}{a^{14}d}\\ &= \frac{2(a-a\cos(c+dx))^6}{3a^9d}-\frac{16(a-a\cos(c+dx))^7}{7a^{10}d}+\frac{25(a-a\cos(c+dx))^8}{8a^{11}d}-\frac{19}{}\end{aligned}$$

Mathematica [A]
time = 2.69, size = 120, normalized size = 0.86

$$\frac{-1615571+2273040\cos(c+dx)-1496880\cos(2(c+dx))+535920\cos(3(c+dx))+110880\cos(4(c+dx))-293832\cos(5(c+dx))+212520\cos(6(c+dx))-67320\cos(7(c+dx))-27720\cos(8(c+dx))+40040\cos(9(c+dx))-16632\cos(10(c+dx))+2520\cos(11(c+dx))}{28385280a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^11/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-1615571 + 2273040*Cos[c + d*x] - 1496880*Cos[2*(c + d*x)] + 535920*Cos[3*
(c + d*x)] + 110880*Cos[4*(c + d*x)] - 293832*Cos[5*(c + d*x)] + 212520*Cos
[6*(c + d*x)] - 67320*Cos[7*(c + d*x)] - 27720*Cos[8*(c + d*x)] + 40040*Cos
[9*(c + d*x)] - 16632*Cos[10*(c + d*x)] + 2520*Cos[11*(c + d*x)])/(28385280
*a^3*d)
```

Maple [A]
time = 0.15, size = 90, normalized size = 0.65

method	result
derivativedivides	$-\frac{-\frac{5}{8\sec(dx+c)^8}+\frac{3}{10\sec(dx+c)^{10}}+\frac{5}{7\sec(dx+c)^7}-\frac{1}{11\sec(dx+c)^{11}}+\frac{1}{4\sec(dx+c)^4}-\frac{3}{5\sec(dx+c)^5}+\frac{1}{6\sec(dx+c)^6}-\frac{1}{9\sec(dx+c)^9}}{d\,a^3}$
default	$-\frac{-\frac{5}{8\sec(dx+c)^8}+\frac{3}{10\sec(dx+c)^{10}}+\frac{5}{7\sec(dx+c)^7}-\frac{1}{11\sec(dx+c)^{11}}+\frac{1}{4\sec(dx+c)^4}-\frac{3}{5\sec(dx+c)^5}+\frac{1}{6\sec(dx+c)^6}-\frac{1}{9\sec(dx+c)^9}}{d\,a^3}$

risch	$\frac{41\cos(dx+c)}{512a^3d}+\frac{\cos(11dx+11c)}{11264d\,a^3}-\frac{3\cos(10dx+10c)}{5120d\,a^3}+\frac{13\cos(9dx+9c)}{9216d\,a^3}-\frac{\cos(8dx+8c)}{1024d\,a^3}-\frac{17\cos(7dx+7c)}{7168d\,a^3}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^11/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d/a^3*(-5/8/sec(d*x+c)^8+3/10/sec(d*x+c)^10+5/7/sec(d*x+c)^7-1/11/sec(d*
x+c)^11+1/4/sec(d*x+c)^4-3/5/sec(d*x+c)^5+1/6/sec(d*x+c)^6-1/9/sec(d*x+c)^9
)
```

Maxima [A]
time = 0.27, size = 89, normalized size = 0.64

$$\frac{2520\cos(dx+c)^{11}-8316\cos(dx+c)^{10}+3080\cos(dx+c)^9+17325\cos(dx+c)^8-19800\cos(dx+c)^7-4620\cos(dx+c)^6+16632\cos(dx+c)^5-6930\cos(dx+c)^4}{27720\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^11/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/27720*(2520*cos(d*x + c)^11 - 8316*cos(d*x + c)^10 + 3080*cos(d*x + c)^9
+ 17325*cos(d*x + c)^8 - 19800*cos(d*x + c)^7 - 4620*cos(d*x + c)^6 + 16632
*cos(d*x + c)^5 - 6930*cos(d*x + c)^4)/(a^3*d)
```

Fricas [A]
time = 4.21, size = 89, normalized size = 0.64

$$\frac{2520\cos(dx+c)^{11}-8316\cos(dx+c)^{10}+3080\cos(dx+c)^9+17325\cos(dx+c)^8-19800\cos(dx+c)^7-4620\cos(dx+c)^6+16632\cos(dx+c)^5-6930\cos(dx+c)^4}{27720\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^11/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/27720*(2520*cos(d*x + c)^11 - 8316*cos(d*x + c)^10 + 3080*cos(d*x + c)^9
+ 17325*cos(d*x + c)^8 - 19800*cos(d*x + c)^7 - 4620*cos(d*x + c)^6 + 16632
*cos(d*x + c)^5 - 6930*cos(d*x + c)^4)/(a^3*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**11/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6191 deep
```

Giac [A]
time = 0.55, size = 207, normalized size = 1.49

$$\frac{32\left(\frac{209(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{1045(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{3135(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}-\frac{6270(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}+\frac{8778(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5}-\frac{13398(\cos(dx+c)-1)^6}{(\cos(dx+c)+1)^6}-\frac{2310(\cos(dx+c)-1)^7}{(\cos(dx+c)+1)^7}-\frac{9240(\cos(dx+c)-1)^8}{(\cos(dx+c)+1)^8}-19\right)}{3465\,a^3d\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^11/(a+a*sec(d*x+c))^3,x, algorithm="giac")

[Out] 32/3465*(209*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1045*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 3135*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 6270*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 8778*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 13398*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 2310*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 - 9240*(cos(d*x + c) - 1)^8/(cos(d*x + c) + 1)^8 - 19)/(a^3*d*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^11)

Mupad [B]
time = 0.09, size = 110, normalized size = 0.79

$$-\frac{\frac{\cos(c+d\,x)^4}{4\,a^3}-\frac{3\,\cos(c+d\,x)^5}{5\,a^3}+\frac{\cos(c+d\,x)^6}{6\,a^3}+\frac{5\,\cos(c+d\,x)^7}{7\,a^3}-\frac{5\,\cos(c+d\,x)^8}{8\,a^3}-\frac{\cos(c+d\,x)^9}{9\,a^3}+\frac{3\,\cos(c+d\,x)^{10}}{10\,a^3}-\frac{\cos(c+d\,x)^{11}}{11\,a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(c + d*x)^11/(a + a/cos(c + d*x))^3,x)

[Out] -(cos(c + d*x)^4/(4*a^3) - (3*cos(c + d*x)^5)/(5*a^3) + cos(c + d*x)^6/(6*a^3) + (5*cos(c + d*x)^7)/(7*a^3) - (5*cos(c + d*x)^8)/(8*a^3) - cos(c + d*x)^9/(9*a^3) + (3*cos(c + d*x)^10)/(10*a^3) - cos(c + d*x)^11/(11*a^3))/d

3.92 $\int \frac{\sin^9(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=109

$$-\frac{\cos^4(c+dx)}{4a^3d}+\frac{3\cos^5(c+dx)}{5a^3d}-\frac{\cos^6(c+dx)}{3a^3d}-\frac{2\cos^7(c+dx)}{7a^3d}+\frac{3\cos^8(c+dx)}{8a^3d}-\frac{\cos^9(c+dx)}{9a^3d}$$

```
[Out] -1/4*cos(d*x+c)^4/a^3/d+3/5*cos(d*x+c)^5/a^3/d-1/3*cos(d*x+c)^6/a^3/d-2/7*c
os(d*x+c)^7/a^3/d+3/8*cos(d*x+c)^8/a^3/d-1/9*cos(d*x+c)^9/a^3/d
```

Rubi [A]
time = 0.13, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3957, 2915, 12, 76}

$$-\frac{\cos^9(c+dx)}{9a^3d}+\frac{3\cos^8(c+dx)}{8a^3d}-\frac{2\cos^7(c+dx)}{7a^3d}-\frac{\cos^6(c+dx)}{3a^3d}+\frac{3\cos^5(c+dx)}{5a^3d}-\frac{\cos^4(c+dx)}{4a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^9/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/4*Cos[c + d*x]^4/(a^3*d) + (3*Cos[c + d*x]^5)/(5*a^3*d) - Cos[c + d*x]^6
/(3*a^3*d) - (2*Cos[c + d*x]^7)/(7*a^3*d) + (3*Cos[c + d*x]^8)/(8*a^3*d) -
Cos[c + d*x]^9/(9*a^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^9(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= -\int \frac{\cos^3(c+dx)\sin^9(c+dx)}{(-a-a\cos(c+dx))^3}\,dx \\
&= \frac{\mathrm{Subst}\left(\int \frac{(-a-x)^4x^3(-a+x)}{a^3}\,dx, x, -a\cos(c+dx)\right)}{a^9d} \\
&= \frac{\mathrm{Subst}\left(\int (-a-x)^4x^3(-a+x)\,dx, x, -a\cos(c+dx)\right)}{a^{12}d} \\
&= \frac{\mathrm{Subst}\left(\int \left(-a^5x^3-3a^4x^4-2a^3x^5+2a^2x^6+3ax^7+x^8\right)\,dx, x, -a\cos(c+dx)\ldots\right.}{a^{12}d} \\
&= -\frac{\cos^4(c+dx)}{4a^3d}+\frac{3\cos^5(c+dx)}{5a^3d}-\frac{\cos^6(c+dx)}{3a^3d}-\frac{2\cos^7(c+dx)}{7a^3d}+\frac{3\cos^8(c+\ldots}{8a^3d\ldots}
\end{aligned}$$

Mathematica [A]
time = 1.85, size = 100, normalized size = 0.92

$$-\frac{34771-52920\cos(c+dx)+37800\cos(2(c+dx))-18480\cos(3(c+dx))+3780\cos(4(c+dx))+3024\cos(5(c+dx))-4200\cos(6(c+dx))+2700\cos(7(c+dx))-945\cos(8(c+dx))+140\cos(9(c+dx))}{322560a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^9/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/322560*(34771 - 52920*Cos[c + d*x] + 37800*Cos[2*(c + d*x)] - 18480*Cos[
3*(c + d*x)] + 3780*Cos[4*(c + d*x)] + 3024*Cos[5*(c + d*x)] - 4200*Cos[6*(
c + d*x)] + 2700*Cos[7*(c + d*x)] - 945*Cos[8*(c + d*x)] + 140*Cos[9*(c + d
*x)])/(a^3*d)
```

Maple [A]
time = 0.19, size = 69, normalized size = 0.63

method	result
derivativedivides	$\frac{-\frac{1}{9\sec(dx+c)^9}-\frac{1}{3\sec(dx+c)^6}+\frac{3}{8\sec(dx+c)^8}-\frac{1}{4\sec(dx+c)^4}+\frac{3}{5\sec(dx+c)^5}-\frac{2}{7\sec(dx+c)^7}}{d\,a^3}$
default	$\frac{-\frac{1}{9\sec(dx+c)^9}-\frac{1}{3\sec(dx+c)^6}+\frac{3}{8\sec(dx+c)^8}-\frac{1}{4\sec(dx+c)^4}+\frac{3}{5\sec(dx+c)^5}-\frac{2}{7\sec(dx+c)^7}}{d\,a^3}$
risch	$\frac{21\cos(dx+c)}{128a^3d}-\frac{\cos(9dx+9c)}{2304d\,a^3}+\frac{3\cos(8dx+8c)}{1024d\,a^3}-\frac{15\cos(7dx+7c)}{1792d\,a^3}+\frac{5\cos(6dx+6c)}{384d\,a^3}-\frac{3\cos(5dx+5c)}{320d\,a^3}-\frac{3\cos(4\ldots}{256\ldots}$

norman	$\frac{\frac{128}{315ad}+\frac{32\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{32\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{64\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{128\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{35ad}+\frac{512\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{35ad}+\frac{512\left(\tan^{6}\left(\frac{d}{}\right.\right.}{15ad}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^9/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(-1/9/sec(d*x+c)^9-1/3/sec(d*x+c)^6+3/8/sec(d*x+c)^8-1/4/sec(d*x+c)
^4+3/5/sec(d*x+c)^5-2/7/sec(d*x+c)^7)
```

Maxima [A]
time = 0.27, size = 69, normalized size = 0.63

$$-\frac{280\cos(dx+c)^9-945\cos(dx+c)^8+720\cos(dx+c)^7+840\cos(dx+c)^6-1512\cos(dx+c)^5+630\cos(dx+c)^4}{2520a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^9/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2520*(280*cos(d*x + c)^9 - 945*cos(d*x + c)^8 + 720*cos(d*x + c)^7 + 840
*cos(d*x + c)^6 - 1512*cos(d*x + c)^5 + 630*cos(d*x + c)^4)/(a^3*d)
```

Fricas [A]
time = 2.75, size = 69, normalized size = 0.63

$$-\frac{280\cos(dx+c)^9-945\cos(dx+c)^8+720\cos(dx+c)^7+840\cos(dx+c)^6-1512\cos(dx+c)^5+630\cos(dx+c)^4}{2520a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^9/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2520*(280*cos(d*x + c)^9 - 945*cos(d*x + c)^8 + 720*cos(d*x + c)^7 + 840
*cos(d*x + c)^6 - 1512*cos(d*x + c)^5 + 630*cos(d*x + c)^4)/(a^3*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**9/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [A]
time = 0.58, size = 185, normalized size = 1.70

$$\frac{32\left(\frac{36(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{144(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{336(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}-\frac{504(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}+\frac{630(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5}-\frac{105(\cos(dx+c)-1)^6}{(\cos(dx+c)+1)^6}+\frac{315(\cos(dx+c)-1)^7}{(\cos(dx+c)+1)^7}-4\right)}{315a^3d\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^9/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 32/315*(36*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 144*(cos(d*x + c) - 1)^2
/(cos(d*x + c) + 1)^2 + 336*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 504
*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 630*(cos(d*x + c) - 1)^5/(cos(
d*x + c) + 1)^5 - 105*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 315*(cos(
d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 - 4)/(a^3*d*((cos(d*x + c) - 1)/(cos(d
*x + c) + 1) - 1)^9)
```

Mupad [B]
time = 0.92, size = 84, normalized size = 0.77

$$-\frac{\frac{\cos(c+d\,x)^4}{4\,a^3}-\frac{3\cos(c+d\,x)^5}{5\,a^3}+\frac{\cos(c+d\,x)^6}{3\,a^3}+\frac{2\cos(c+d\,x)^7}{7\,a^3}-\frac{3\cos(c+d\,x)^8}{8\,a^3}+\frac{\cos(c+d\,x)^9}{9\,a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^9/(a + a/cos(c + d*x))^3,x)
```

```
[Out] -(cos(c + d*x)^4/(4*a^3) - (3*cos(c + d*x)^5)/(5*a^3) + cos(c + d*x)^6/(3*a
^3) + (2*cos(c + d*x)^7)/(7*a^3) - (3*cos(c + d*x)^8)/(8*a^3) + cos(c + d*x
)^9/(9*a^3))/d
```

3.93 $\int \frac{\sin^7(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=73

$$-\frac{\cos^4(c+dx)}{4a^3d}+\frac{3\cos^5(c+dx)}{5a^3d}-\frac{\cos^6(c+dx)}{2a^3d}+\frac{\cos^7(c+dx)}{7a^3d}$$

```
[Out] -1/4*cos(d*x+c)^4/a^3/d+3/5*cos(d*x+c)^5/a^3/d-1/2*cos(d*x+c)^6/a^3/d+1/7*c
os(d*x+c)^7/a^3/d
```

Rubi [A]

time = 0.12, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3957, 2915, 12, 45}

$$\frac{\cos^7(c+dx)}{7a^3d}-\frac{\cos^6(c+dx)}{2a^3d}+\frac{3\cos^5(c+dx)}{5a^3d}-\frac{\cos^4(c+dx)}{4a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^7/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/4*Cos[c + d*x]^4/(a^3*d) + (3*Cos[c + d*x]^5)/(5*a^3*d) - Cos[c + d*x]^6
/(2*a^3*d) + Cos[c + d*x]^7/(7*a^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^7(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= -\int \frac{\cos^3(c+dx)\sin^7(c+dx)}{(-a-a\cos(c+dx))^3}\,dx \\
&= \frac{\text{Subst}\left(\int \frac{(-a-x)^3x^3}{a^3}\,dx, x, -a\cos(c+dx)\right)}{a^7d} \\
&= \frac{\text{Subst}\left(\int (-a-x)^3x^3\,dx, x, -a\cos(c+dx)\right)}{a^{10}d} \\
&= \frac{\text{Subst}\left(\int \left(-a^3x^3-3a^2x^4-3ax^5-x^6\right)\,dx, x, -a\cos(c+dx)\right)}{a^{10}d} \\
&= -\frac{\cos^4(c+dx)}{4a^3d}+\frac{3\cos^5(c+dx)}{5a^3d}-\frac{\cos^6(c+dx)}{2a^3d}+\frac{\cos^7(c+dx)}{7a^3d}
\end{aligned}$$

Mathematica [A]
time = 1.10, size = 80, normalized size = 1.10

$$\frac{-2421+4060\cos(c+dx)-3220\cos(2(c+dx))+2100\cos(3(c+dx))-1120\cos(4(c+dx))+476\cos(5(c+dx))-140\cos(6(c+dx))+20\cos(7(c+dx))}{8960a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^7/(a + a*Sec[c + d*x])^3,x]
[Out] (-2421 + 4060*Cos[c + d*x] - 3220*Cos[2*(c + d*x)] + 2100*Cos[3*(c + d*x)]
- 1120*Cos[4*(c + d*x)] + 476*Cos[5*(c + d*x)] - 140*Cos[6*(c + d*x)] + 20*
Cos[7*(c + d*x)])/(8960*a^3*d)
```

Maple [A]
time = 0.14, size = 50, normalized size = 0.68

method	result
derivativedivides	$-\frac{-\frac{1}{7\sec(dx+c)^7}-\frac{3}{5\sec(dx+c)^5}+\frac{1}{4\sec(dx+c)^4}+\frac{1}{2\sec(dx+c)^6}}{d\,a^3}$
default	$-\frac{-\frac{1}{7\sec(dx+c)^7}-\frac{3}{5\sec(dx+c)^5}+\frac{1}{4\sec(dx+c)^4}+\frac{1}{2\sec(dx+c)^6}}{d\,a^3}$
risch	$\frac{29\cos(dx+c)}{64a^3d}+\frac{\cos(7dx+7c)}{448d\,a^3}-\frac{\cos(6dx+6c)}{64d\,a^3}+\frac{17\cos(5dx+5c)}{320d\,a^3}-\frac{\cos(4dx+4c)}{8d\,a^3}+\frac{15\cos(3dx+3c)}{64d\,a^3}-\frac{23\cos(2c}{64d}$
norman	$\frac{\frac{16\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{52\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{52}{35ad}+\frac{56\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{24\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{52\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5ad}+\frac{156\left(\tan^4\left(\frac{dx}{2}-\right.\right.}{5ad}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^7/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d/a^3*(-1/7/sec(d*x+c)^7-3/5/sec(d*x+c)^5+1/4/sec(d*x+c)^4+1/2/sec(d*x+c
)^6)
```

Maxima [A]
time = 0.28, size = 49, normalized size = 0.67

$$\frac{20\,\cos\left(dx+c\right)^{7}-70\,\cos\left(dx+c\right)^{6}+84\,\cos\left(dx+c\right)^{5}-35\,\cos\left(dx+c\right)^{4}}{140\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/140*(20*cos(d*x + c)^7 - 70*cos(d*x + c)^6 + 84*cos(d*x + c)^5 - 35*cos(d
*x + c)^4)/(a^3*d)
```

Fricas [A]
time = 2.51, size = 49, normalized size = 0.67

$$\frac{20\,\cos\left(dx+c\right)^{7}-70\,\cos\left(dx+c\right)^{6}+84\,\cos\left(dx+c\right)^{5}-35\,\cos\left(dx+c\right)^{4}}{140\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/140*(20*cos(d*x + c)^7 - 70*cos(d*x + c)^6 + 84*cos(d*x + c)^5 - 35*cos(d
*x + c)^4)/(a^3*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**7/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(65) = 130.
time = 0.55, size = 163, normalized size = 2.23

$$\frac{4\left(\frac{91\,(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{273\,(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{455\,(\cos(dx+c)-1)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{490\,(\cos(dx+c)-1)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{210\,(\cos(dx+c)-1)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{140\,(\cos(dx+c)-1)^{6}}{(\cos(dx+c)+1)^{6}}-13\right)}{35\,a^{3}d\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 4/35*(91*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 273*(cos(d*x + c) - 1)^2/(
cos(d*x + c) + 1)^2 + 455*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 490*(
cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 210*(cos(d*x + c) - 1)^5/(cos(d*
x + c) + 1)^5 - 140*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 13)/(a^3*d*
((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^7)
```

Mupad [B]
time = 0.07, size = 58, normalized size = 0.79

$$-\frac{\frac{\cos(c+d\,x)^4}{4\,a^3}-\frac{3\cos(c+d\,x)^5}{5\,a^3}+\frac{\cos(c+d\,x)^6}{2\,a^3}-\frac{\cos(c+d\,x)^7}{7\,a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^7/(a + a/cos(c + d*x))^3,x)
```

```
[Out] -(cos(c + d*x)^4/(4*a^3) - (3*cos(c + d*x)^5)/(5*a^3) + cos(c + d*x)^6/(2*a
^3) - cos(c + d*x)^7/(7*a^3))/d
```

3.94 $\int \frac{\sin^5(c+dx)}{(a+a\sec(c+dx))^3} \, dx$

Optimal. Leaf size=102

$$-\frac{4\cos(c+dx)}{a^3d}+\frac{2\cos^2(c+dx)}{a^3d}-\frac{4\cos^3(c+dx)}{3a^3d}+\frac{3\cos^4(c+dx)}{4a^3d}-\frac{\cos^5(c+dx)}{5a^3d}+\frac{4\log(1+\cos(c+dx))}{a^3d}$$

```
[Out] -4*cos(d*x+c)/a^3/d+2*cos(d*x+c)^2/a^3/d-4/3*cos(d*x+c)^3/a^3/d+3/4*cos(d*x
+c)^4/a^3/d-1/5*cos(d*x+c)^5/a^3/d+4*ln(1+cos(d*x+c))/a^3/d
```

Rubi [A]
time = 0.13, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3957, 2915, 12, 90}

$$-\frac{\cos^5(c+dx)}{5a^3d}+\frac{3\cos^4(c+dx)}{4a^3d}-\frac{4\cos^3(c+dx)}{3a^3d}+\frac{2\cos^2(c+dx)}{a^3d}-\frac{4\cos(c+dx)}{a^3d}+\frac{4\log(\cos(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^5/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-4*Cos[c + d*x])/(a^3*d) + (2*Cos[c + d*x]^2)/(a^3*d) - (4*Cos[c + d*x]^3)
/(3*a^3*d) + (3*Cos[c + d*x]^4)/(4*a^3*d) - Cos[c + d*x]^5/(5*a^3*d) + (4*L
og[1 + Cos[c + d*x]])/(a^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= -\int \frac{\cos^3(c+dx)\sin^5(c+dx)}{(-a-a\cos(c+dx))^3}\,dx\\ &= \frac{\text{Subst}\left(\int \frac{(-a-x)^2x^3}{a^3(-a+x)}\,dx, x, -a\cos(c+dx)\right)}{a^5d}\\ &= \frac{\text{Subst}\left(\int \frac{(-a-x)^2x^3}{-a+x}\,dx, x, -a\cos(c+dx)\right)}{a^8d}\\ &= \frac{\text{Subst}\left(\int \left(4a^4-\frac{4a^5}{a-x}+4a^3x+4a^2x^2+3ax^3+x^4\right)\,dx, x, -a\cos(c+dx)\right)}{a^8d}\\ &= -\frac{4\cos(c+dx)}{a^3d}+\frac{2\cos^2(c+dx)}{a^3d}-\frac{4\cos^3(c+dx)}{3a^3d}+\frac{3\cos^4(c+dx)}{4a^3d}-\frac{\cos^5(c\cdots}{5a^3\cdots}\end{aligned}$$

Mathematica [A]
time = 0.54, size = 73, normalized size = 0.72

$$\frac{3857-4920\cos(c+dx)+1320\cos(2(c+dx))-380\cos(3(c+dx))+90\cos(4(c+dx))-12\cos(5(c+dx))+7680\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{960a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^5/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (3857 - 4920*Cos[c + d*x] + 1320*Cos[2*(c + d*x)] - 380*Cos[3*(c + d*x)] +
90*Cos[4*(c + d*x)] - 12*Cos[5*(c + d*x)] + 7680*Log[Cos[(c + d*x)/2]])/(96
0*a^3*d)
```

Maple [A]
time = 0.15, size = 79, normalized size = 0.77

method	result
derivativedivides	$\frac{-\frac{1}{5\sec(dx+c)^5}+\frac{3}{4\sec(dx+c)^4}-\frac{4}{3\sec(dx+c)^3}+\frac{2}{\sec(dx+c)^2}-\frac{4}{\sec(dx+c)}-4\ln(\sec(dx+c))+4\ln(1+\sec(dx+c))}{d\,a^3}$
default	$\frac{-\frac{1}{5\sec(dx+c)^5}+\frac{3}{4\sec(dx+c)^4}-\frac{4}{3\sec(dx+c)^3}+\frac{2}{\sec(dx+c)^2}-\frac{4}{\sec(dx+c)}-4\ln(\sec(dx+c))+4\ln(1+\sec(dx+c))}{d\,a^3}$
norman	$\frac{-\frac{32\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{166}{15ad}-\frac{78\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{154\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{278\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5a^2}-\frac{4\ln\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3d}$

risch	$-\frac{4ix}{a^3}-\frac{41\,\mathrm{e}^{i(dx+c)}}{16a^3d}-\frac{41\,\mathrm{e}^{-i(dx+c)}}{16a^3d}-\frac{8ic}{a^3d}+\frac{8\ln\left(\mathrm{e}^{i(dx+c)}+1\right)}{a^3d}-\frac{\cos(5dx+5c)}{80d\,a^3}+\frac{3\cos(4dx+4c)}{32d\,a^3}-\frac{19\cos(3dx\text{-}}{48d\,a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^5/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(-1/5/sec(d*x+c)^5+3/4/sec(d*x+c)^4-4/3/sec(d*x+c)^3+2/sec(d*x+c)^2
-4/sec(d*x+c)-4*ln(sec(d*x+c))+4*ln(1+sec(d*x+c)))
```

Maxima [A]
time = 0.27, size = 73, normalized size = 0.72

$$-\frac{\frac{12\,\cos(dx+c)^5-45\,\cos(dx+c)^4+80\,\cos(dx+c)^3-120\,\cos(dx+c)^2+240\,\cos(dx+c)}{a^3}-\frac{240\,\log(\cos(dx+c)+1)}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/60*((12*cos(d*x + c)^5 - 45*cos(d*x + c)^4 + 80*cos(d*x + c)^3 - 120*cos
(d*x + c)^2 + 240*cos(d*x + c))/a^3 - 240*log(cos(d*x + c) + 1)/a^3)/d
```

Fricas [A]
time = 2.76, size = 70, normalized size = 0.69

$$-\frac{12\,\cos\left(dx+c\right)^5-45\,\cos\left(dx+c\right)^4+80\,\cos\left(dx+c\right)^3-120\,\cos\left(dx+c\right)^2+240\,\cos\left(dx+c\right)-240\,\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{60\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/60*(12*cos(d*x + c)^5 - 45*cos(d*x + c)^4 + 80*cos(d*x + c)^3 - 120*cos(
d*x + c)^2 + 240*cos(d*x + c) - 240*log(1/2*cos(d*x + c) + 1/2))/(a^3*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**5/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.54, size = 172, normalized size = 1.69

$$-\frac{\frac{60\,\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^3}+\frac{\frac{85\,(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{20\,(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{200\,(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{205\,(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}-\frac{137\,(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5}-29}{a^3\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/15*(60*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^3 + (85*(c
os(d*x + c) - 1)/(cos(d*x + c) + 1) - 20*(cos(d*x + c) - 1)^2/(cos(d*x + c)
 + 1)^2 - 200*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 205*(cos(d*x + c)
 - 1)^4/(cos(d*x + c) + 1)^4 - 137*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^
5 - 29)/(a^3*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^5))/d
```

Mupad [B]
time = 0.90, size = 82, normalized size = 0.80

$$\frac{\frac{4\,\ln(\cos(c+d\,x)+1)}{a^3}-\frac{4\,\cos(c+d\,x)}{a^3}+\frac{2\cos(c+d\,x)^2}{a^3}-\frac{4\cos(c+d\,x)^3}{3\,a^3}+\frac{3\cos(c+d\,x)^4}{4\,a^3}-\frac{\cos(c+d\,x)^5}{5\,a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5/(a + a/cos(c + d*x))^3,x)
```

```
[Out] ((4*log(cos(c + d*x) + 1))/a^3 - (4*cos(c + d*x))/a^3 + (2*cos(c + d*x)^2)/
a^3 - (4*cos(c + d*x)^3)/(3*a^3) + (3*cos(c + d*x)^4)/(4*a^3) - cos(c + d*x
)^5/(5*a^3))/d
```

3.95 $\int \frac{\sin^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=89

$$\frac{5\cos(c+dx)}{a^3d}-\frac{3\cos^2(c+dx)}{2a^3d}+\frac{\cos^3(c+dx)}{3a^3d}-\frac{2}{d\,(a^3+a^3\cos(c+dx))}-\frac{7\log(1+\cos(c+dx))}{a^3d}$$

```
[Out] 5*cos(d*x+c)/a^3/d-3/2*cos(d*x+c)^2/a^3/d+1/3*cos(d*x+c)^3/a^3/d-2/d/(a^3+a
^3*cos(d*x+c))-7*ln(1+cos(d*x+c))/a^3/d
```

Rubi [A]
time = 0.13, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3957, 2915, 12, 78}

$$\frac{\cos^3(c+dx)}{3a^3d}-\frac{3\cos^2(c+dx)}{2a^3d}+\frac{5\cos(c+dx)}{a^3d}-\frac{2}{d\,(a^3\cos(c+dx)+a^3)}-\frac{7\log(\cos(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (5*Cos[c + d*x])/(a^3*d) - (3*Cos[c + d*x]^2)/(2*a^3*d) + Cos[c + d*x]^3/(3
*a^3*d) - 2/(d*(a^3 + a^3*Cos[c + d*x])) - (7*Log[1 + Cos[c + d*x]])/(a^3*d
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= -\int \frac{\cos^3(c+dx)\sin^3(c+dx)}{(-a-a\cos(c+dx))^3}\,dx\\ &= \frac{\text{Subst}\left(\int \frac{(-a-x)x^3}{a^3(-a+x)^2}\,dx, x, -a\cos(c+dx)\right)}{a^3d}\\ &= \frac{\text{Subst}\left(\int \frac{(-a-x)x^3}{(-a+x)^2}\,dx, x, -a\cos(c+dx)\right)}{a^6d}\\ &= \frac{\text{Subst}\left(\int \left(-5a^2-\frac{2a^4}{(a-x)^2}+\frac{7a^3}{a-x}-3ax-x^2\right)dx, x, -a\cos(c+dx)\right)}{a^6d}\\ &= \frac{5\cos(c+dx)}{a^3d}-\frac{3\cos^2(c+dx)}{2a^3d}+\frac{\cos^3(c+dx)}{3a^3d}-\frac{2}{d\left(a^3+a^3\cos(c+dx)\right)}-\frac{7\,\mathrm{l}}{}\end{aligned}$$

Mathematica [A]
time = 0.27, size = 99, normalized size = 1.11

$$-\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\left(389-184\cos(2(c+dx))+28\cos(3(c+dx))-4\cos(4(c+dx))+1344\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\cos(c+dx)\left(-19+1344\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)\right)}{24a^3d(1+\cos(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/(a + a*Sec[c + d*x])^3,x]
[Out] -1/24*(Cos[(c + d*x)/2]^4*(389 - 184*Cos[2*(c + d*x)] + 28*Cos[3*(c + d*x)]
 - 4*Cos[4*(c + d*x)] + 1344*Log[Cos[(c + d*x)/2]] + Cos[c + d*x]*(-19 + 13
44*Log[Cos[(c + d*x)/2]])))/(a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A]
time = 0.13, size = 72, normalized size = 0.81

method	result
derivativedivides	$-\frac{-\frac{1}{3\sec(dx+c)^3}+\frac{3}{2\sec(dx+c)^2}-\frac{5}{\sec(dx+c)}-7\ln(\sec(dx+c))-\frac{2}{1+\sec(dx+c)}+7\ln(1+\sec(dx+c))}{d\,a^3}$
default	$-\frac{-\frac{1}{3\sec(dx+c)^3}+\frac{3}{2\sec(dx+c)^2}-\frac{5}{\sec(dx+c)}-7\ln(\sec(dx+c))-\frac{2}{1+\sec(dx+c)}+7\ln(1+\sec(dx+c))}{d\,a^3}$

norman	$\frac{\frac{34\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{41}{3ad}+\frac{24\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 a^2}+\frac{7\ln\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3 d}$
risch	$\frac{7ix}{a^3}+\frac{21\,\mathrm{e}^{i(dx+c)}}{8a^3d}+\frac{21\,\mathrm{e}^{-i(dx+c)}}{8a^3d}+\frac{14ic}{a^3d}-\frac{4\,\mathrm{e}^{i(dx+c)}}{a^3d\left(\mathrm{e}^{i(dx+c)}+1\right)^2}-\frac{14\ln\left(\mathrm{e}^{i(dx+c)}+1\right)}{a^3d}+\frac{\cos(3dx+3c)}{12d\,a^3}-\frac{3\cos(2d}{4d\,c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d/a^3*(-1/3/sec(d*x+c)^3+3/2/sec(d*x+c)^2-5/sec(d*x+c)-7*ln(sec(d*x+c))-
2/(1+sec(d*x+c))+7*ln(1+sec(d*x+c)))
```

Maxima [A]
time = 0.27, size = 72, normalized size = 0.81

$$-\frac{\frac{12}{a^3\cos(dx+c)+a^3}-\frac{2\cos(dx+c)^3-9\cos(dx+c)^2+30\cos(dx+c)}{a^3}+\frac{42\log(\cos(dx+c)+1)}{a^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/6*(12/(a^3*cos(d*x + c) + a^3) - (2*cos(d*x + c)^3 - 9*cos(d*x + c)^2 +
30*cos(d*x + c))/a^3 + 42*log(cos(d*x + c) + 1)/a^3)/d
```

Fricas [A]
time = 2.62, size = 82, normalized size = 0.92

$$\frac{4\cos(dx+c)^4-14\cos(dx+c)^3+42\cos(dx+c)^2-84(\cos(dx+c)+1)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+69\cos(dx+c)-15}{12\left(a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(4*cos(d*x + c)^4 - 14*cos(d*x + c)^3 + 42*cos(d*x + c)^2 - 84*(cos(d*
x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) + 69*cos(d*x + c) - 15)/(a^3*d*cos(
d*x + c) + a^3*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.51, size = 94, normalized size = 1.06

$$-\frac{7 \log\left(|-\cos\left(dx+c\right)-1|\right)}{a^3 d}-\frac{2}{a^3 d(\cos\left(dx+c\right)+1)}+\frac{2\,a^6 d^5 \cos\left(dx+c\right)^3-9\,a^6 d^5 \cos\left(dx+c\right)^2+30\,a^6 d^5 \cos\left(dx+c\right)}{6\,a^9 d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -7*log(abs(-cos(d*x + c) - 1))/(a^3*d) - 2/(a^3*d*(cos(d*x + c) + 1)) + 1/6
*(2*a^6*d^5*cos(d*x + c)^3 - 9*a^6*d^5*cos(d*x + c)^2 + 30*a^6*d^5*cos(d*x
+ c))/(a^9*d^6)
```

Mupad [B]
time = 0.89, size = 75, normalized size = 0.84

$$-\frac{\frac{2}{a^3\,\cos(c+d\,x)+a^3}+\frac{7\,\ln(\cos(c+d\,x)+1)}{a^3}-\frac{5\,\cos(c+d\,x)}{a^3}+\frac{3\cos(c+d\,x)^2}{2\,a^3}-\frac{\cos(c+d\,x)^3}{3\,a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(a + a/cos(c + d*x))^3,x)
```

```
[Out] -(2/(a^3*cos(c + d*x) + a^3) + (7*log(cos(c + d*x) + 1))/a^3 - (5*cos(c + d
*x))/a^3 + (3*cos(c + d*x)^2)/(2*a^3) - cos(c + d*x)^3/(3*a^3))/d
```

3.96 $\int \frac{\sin(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=75

$$-\frac{\cos(c+dx)}{a^3d}-\frac{1}{2ad(a+a\cos(c+dx))^2}+\frac{3}{d\left(a^3+a^3\cos(c+dx)\right)}+\frac{3\log(1+\cos(c+dx))}{a^3d}$$

```
[Out] -cos(d*x+c)/a^3/d-1/2/a/d/(a+a*cos(d*x+c))^2+3/d/(a^3+a^3*cos(d*x+c))+3*ln(
1+cos(d*x+c))/a^3/d
```

Rubi [A]
time = 0.09, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3957, 2912, 12, 45}

$$-\frac{\cos(c+dx)}{a^3d}+\frac{3}{d\left(a^3\cos(c+dx)+a^3\right)}+\frac{3\log(\cos(c+dx)+1)}{a^3d}-\frac{1}{2ad(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(Cos[c + d*x]/(a^3*d)) - 1/(2*a*d*(a + a*Cos[c + d*x])^2) + 3/(d*(a^3 + a^
3*Cos[c + d*x])) + (3*Log[1 + Cos[c + d*x]])/(a^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
```

```
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= -\int \frac{\cos^3(c+dx)\sin(c+dx)}{(-a-a\cos(c+dx))^3}\,dx\\ &= \frac{\text{Subst}\left(\int \frac{x^3}{a^3(-a+x)^3}\,dx, x, -a\cos(c+dx)\right)}{ad}\\ &= \frac{\text{Subst}\left(\int \frac{x^3}{(-a+x)^3}\,dx, x, -a\cos(c+dx)\right)}{a^4d}\\ &= \frac{\text{Subst}\left(\int \left(1-\frac{a^3}{(a-x)^3}+\frac{3a^2}{(a-x)^2}-\frac{3a}{a-x}\right)\,dx, x, -a\cos(c+dx)\right)}{a^4d}\\ &= -\frac{\cos(c+dx)}{a^3d}-\frac{1}{2ad(a+a\cos(c+dx))^2}+\frac{3}{d\left(a^3+a^3\cos(c+dx)\right)}+\frac{3\log(1-}{}\end{aligned}$$

Mathematica [A]
time = 0.21, size = 103, normalized size = 1.37

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(21-2\cos(3(c+dx))+72\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\cos(2(c+dx))\left(-5+24\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)+\cos(c+dx)\left(22+96\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)\right)}{4a^3d(1+\cos(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + a*Sec[c + d*x])^3,x]
[Out] (Cos[(c + d*x)/2]^2*(21 - 2*Cos[3*(c + d*x)] + 72*Log[Cos[(c + d*x)/2]] + C
os[2*(c + d*x)]*(-5 + 24*Log[Cos[(c + d*x)/2]]) + Cos[c + d*x]*(22 + 96*Log
[Cos[(c + d*x)/2]])))/(4*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A]
time = 0.06, size = 63, normalized size = 0.84

method	result	size
derivativedivides	$\frac{-\frac{1}{\sec(dx+c)}-3\ln(\sec(dx+c))-\frac{1}{2(1+\sec(dx+c))^2}-\frac{2}{1+\sec(dx+c)}+3\ln(1+\sec(dx+c))}{a^3d}$	63
default	$\frac{-\frac{1}{\sec(dx+c)}-3\ln(\sec(dx+c))-\frac{1}{2(1+\sec(dx+c))^2}-\frac{2}{1+\sec(dx+c)}+3\ln(1+\sec(dx+c))}{a^3d}$	63
norman	$\frac{\frac{9\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad}-\frac{\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{13}{4ad}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2}-\frac{3\ln\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3d}$	90
risch	$-\frac{3ix}{a^3}-\frac{e^{i(dx+c)}}{2a^3d}-\frac{e^{-i(dx+c)}}{2a^3d}-\frac{6ic}{a^3d}+\frac{6\,e^{3i(dx+c)}+10\,e^{2i(dx+c)}+6\,e^{i(dx+c)}}{a^3d\left(e^{i(dx+c)}+1\right)^4}+\frac{6\ln\left(e^{i(dx+c)}+1\right)}{a^3d}$	128

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3/d*(-1/sec(d*x+c)-3*ln(sec(d*x+c))-1/2/(1+sec(d*x+c))^2-2/(1+sec(d*x+c
))+3*ln(1+sec(d*x+c)))
```

Maxima [A]
time = 0.26, size = 71, normalized size = 0.95

$$\frac{\frac{6\cos(dx+c)+5}{a^3\cos(dx+c)^2+2\,a^3\cos(dx+c)+a^3}-\frac{2\cos(dx+c)}{a^3}+\frac{6\log(\cos(dx+c)+1)}{a^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((6*cos(d*x + c) + 5)/(a^3*cos(d*x + c)^2 + 2*a^3*cos(d*x + c) + a^3) -
 2*cos(d*x + c)/a^3 + 6*log(cos(d*x + c) + 1)/a^3)/d
```

Fricas [A]
time = 3.05, size = 96, normalized size = 1.28

$$-\frac{2\cos(dx+c)^3+4\cos(dx+c)^2-6\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-4\cos(dx+c)-5}{2\left(a^3d\cos(dx+c)^2+2\,a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*cos(d*x + c)^3 + 4*cos(d*x + c)^2 - 6*(cos(d*x + c)^2 + 2*cos(d*x +
 c) + 1)*log(1/2*cos(d*x + c) + 1/2) - 4*cos(d*x + c) - 5)/(a^3*d*cos(d*x +
 c)^2 + 2*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sin(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(sin(c + d*x)/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x)
 + 1), x)/a**3
```

Giac [A]
time = 0.49, size = 63, normalized size = 0.84

$$-\frac{\cos(dx+c)}{a^3d}+\frac{3\,\log\left(|-\cos(dx+c)-1|\right)}{a^3d}+\frac{6\,\cos(dx+c)+5}{2\,a^3d(\cos(dx+c)+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -cos(d*x + c)/(a^3*d) + 3*log(abs(-cos(d*x + c) - 1))/(a^3*d) + 1/2*(6*cos(
d*x + c) + 5)/(a^3*d*(cos(d*x + c) + 1)^2)
```

Mupad [B]
time = 0.08, size = 59, normalized size = 0.79

$$\frac{3\,\ln\left(\cos\left(c+d\,x\right)+1\right)}{a^3\,d}-\frac{\cos\left(c+d\,x\right)}{a^3\,d}+\frac{3\,\cos\left(c+d\,x\right)+\frac{5}{2}}{a^3\,d\,{\left(\cos\left(c+d\,x\right)+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(a + a/cos(c + d*x))^3,x)
```

```
[Out] (3*log(cos(c + d*x) + 1))/(a^3*d) - cos(c + d*x)/(a^3*d) + (3*cos(c + d*x)
+ 5/2)/(a^3*d*(cos(c + d*x) + 1)^2)
```

3.97 $\int \frac{\csc(c+dx)}{(a+a\sec(c+dx))^3} dx$

Optimal. Leaf size=82

$$-\frac{\tanh^{-1}(\cos(c+dx))}{8a^3d}-\frac{1}{6d(a+a\cos(c+dx))^3}+\frac{5}{8ad(a+a\cos(c+dx))^2}-\frac{7}{8d\,(a^3+a^3\cos(c+dx))}$$

```
[Out] -1/8*arctanh(cos(d*x+c))/a^3/d-1/6/d/(a+a*cos(d*x+c))^3+5/8/a/d/(a+a*cos(d*
x+c))^2-7/8/d/(a^3+a^3*cos(d*x+c))
```

Rubi [A]
time = 0.11, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3957, 2915, 12, 90, 212}

$$-\frac{7}{8d\,(a^3\cos(c+dx)+a^3)}-\frac{\tanh^{-1}(\cos(c+dx))}{8a^3d}+\frac{5}{8ad(a\cos(c+dx)+a)^2}-\frac{1}{6d(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/8*ArcTanh[Cos[c + d*x]]/(a^3*d) - 1/(6*d*(a + a*Cos[c + d*x])^3) + 5/(8*
a*d*(a + a*Cos[c + d*x])^2) - 7/(8*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
```

```
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= -\int \frac{\cos^2(c+dx)\cot(c+dx)}{(-a-a\cos(c+dx))^3}\,dx\\ &= \frac{a\text{Subst}\left(\int \frac{x^3}{a^3(-a-x)(-a+x)^4}\,dx, x, -a\cos(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \frac{x^3}{(-a-x)(-a+x)^4}\,dx, x, -a\cos(c+dx)\right)}{a^2d}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{a^2}{2(a-x)^4}+\frac{5a}{4(a-x)^3}-\frac{7}{8(a-x)^2}+\frac{1}{8(a^2-x^2)}\right)\,dx, x, -a\cos(c+dx)\right)}{a^2d}\\ &= -\frac{1}{6d(a+a\cos(c+dx))^3}+\frac{5}{8ad(a+a\cos(c+dx))^2}-\frac{7}{8d\,(a^3+a^3\cos(c+dx)}\\ &= -\frac{\tanh^{-1}(\cos(c+dx))}{8a^3d}-\frac{1}{6d(a+a\cos(c+dx))^3}+\frac{5}{8ad(a+a\cos(c+dx))^2}-\end{aligned}$$

Mathematica [A]
time = 0.21, size = 97, normalized size = 1.18

$$-\frac{\left(2-15\cos^2\left(\frac{1}{2}(c+dx)\right)+42\cos^4\left(\frac{1}{2}(c+dx)\right)+12\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\right)\sec^3(c+dx)}{12a^3d(1+\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/12*((2 - 15*Cos[(c + d*x)/2]^2 + 42*Cos[(c + d*x)/2]^4 + 12*Cos[(c + d*x
)/2]^6*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]]))*Sec[c + d*x]^3)/(a^
3*d*(1 + Sec[c + d*x])^3)
```

Maple [A]
time = 0.12, size = 67, normalized size = 0.82

method	result	size
derivativedivides	$\frac{\frac{\ln(-1+\cos(dx+c))}{16}-\frac{1}{6(1+\cos(dx+c))^3}+\frac{5}{8(1+\cos(dx+c))^2}-\frac{7}{8(1+\cos(dx+c))}-\frac{\ln(1+\cos(dx+c))}{16}}{d\,a^3}$	67
default	$\frac{\frac{\ln(-1+\cos(dx+c))}{16}-\frac{1}{6(1+\cos(dx+c))^3}+\frac{5}{8(1+\cos(dx+c))^2}-\frac{7}{8(1+\cos(dx+c))}-\frac{\ln(1+\cos(dx+c))}{16}}{d\,a^3}$	67
norman	$\frac{-\frac{3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16ad}+\frac{3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32ad}-\frac{\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)}{48ad}}{a^2}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^3d}$	82
risch	$-\frac{21\,e^{5i(dx+c)}+54\,e^{4i(dx+c)}+82\,e^{3i(dx+c)}+54\,e^{2i(dx+c)}+21\,e^{i(dx+c)}}{12a^3d\left(e^{i(dx+c)}+1\right)^6}-\frac{\ln\left(e^{i(dx+c)}+1\right)}{8a^3d}+\frac{\ln\left(e^{i(dx+c)}-1\right)}{8a^3d}$	119

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(1/16*ln(-1+cos(d*x+c))-1/6/(1+cos(d*x+c))^3+5/8/(1+cos(d*x+c))^2-7
/8/(1+cos(d*x+c))-1/16*ln(1+cos(d*x+c)))
```

Maxima [A]
time = 0.28, size = 98, normalized size = 1.20

$$-\frac{\frac{2\left(21\cos(dx+c)^2+27\cos(dx+c)+10\right)}{a^3\cos(dx+c)^3+3\,a^3\cos(dx+c)^2+3\,a^3\cos(dx+c)+a^3}+\frac{3\,\log(\cos(dx+c)+1)}{a^3}-\frac{3\,\log(\cos(dx+c)-1)}{a^3}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/48*(2*(21*cos(d*x + c)^2 + 27*cos(d*x + c) + 10)/(a^3*cos(d*x + c)^3 + 3
*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) + a^3) + 3*log(cos(d*x + c) + 1)/a
^3 - 3*log(cos(d*x + c) - 1)/a^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(74) = 148$.
time = 2.77, size = 151, normalized size = 1.84

$$-\frac{42\,\cos(dx+c)^2+3\left(\cos(dx+c)^3+3\,\cos(dx+c)^2+3\,\cos(dx+c)+1\right)\log\left(\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)-3\left(\cos(dx+c)^3+3\,\cos(dx+c)^2+3\,\cos(dx+c)+1\right)\log\left(-\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)+54\,\cos(dx+c)+20}{48\left(a^3d\cos(dx+c)^3+3\,a^3d\cos(dx+c)^2+3\,a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/48*(42*cos(d*x + c)^2 + 3*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x
 + c) + 1)*log(1/2*cos(d*x + c) + 1/2) - 3*(cos(d*x + c)^3 + 3*cos(d*x + c)
^2 + 3*cos(d*x + c) + 1)*log(-1/2*cos(d*x + c) + 1/2) + 54*cos(d*x + c) + 2
0)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) +
a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(csc(c + d*x)/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x)
 + 1), x)/a**3
```

Giac [A]
time = 0.55, size = 113, normalized size = 1.38

$$\frac{\frac{6\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^3}+\frac{\frac{18a^6(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{9a^6(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{2a^6(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}}{a^9}}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/96*(6*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a^3 + (18*a^6*(co
s(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*a^6*(cos(d*x + c) - 1)^2/(cos(d*x +
c) + 1)^2 + 2*a^6*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)/a^9)/d
```

Mupad [B]
time = 0.11, size = 83, normalized size = 1.01

$$-\frac{\frac{7\cos(c+d\,x)^2}{8}+\frac{9\cos(c+d\,x)}{8}+\frac{5}{12}}{d\,\left(a^3\cos\left(c+d\,x\right)^3+3\,a^3\cos\left(c+d\,x\right)^2+3\,a^3\,\cos\left(c+d\,x\right)+a^3\right)}-\frac{\mathrm{atanh}(\cos\left(c+d\,x\right))}{8\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)*(a + a/cos(c + d*x))^3),x)
```

```
[Out] - ((9*cos(c + d*x))/8 + (7*cos(c + d*x)^2)/8 + 5/12)/(d*(3*a^3*cos(c + d*x)
 + a^3 + 3*a^3*cos(c + d*x)^2 + a^3*cos(c + d*x)^3)) - atanh(cos(c + d*x))/
(8*a^3*d)
```

3.98 $\int \frac{\csc^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=126

$$\frac{\tanh^{-1}(\cos(c+dx))}{32a^3d}-\frac{a}{16d(a+a\cos(c+dx))^4}+\frac{1}{6d(a+a\cos(c+dx))^3}-\frac{3}{32ad(a+a\cos(c+dx))^2}-\frac{}{32d\,(a^3}$$

```
[Out] 1/32*arctanh(cos(d*x+c))/a^3/d-1/16*a/d/(a+a*cos(d*x+c))^4+1/6/d/(a+a*cos(d
*x+c))^3-3/32/a/d/(a+a*cos(d*x+c))^2-1/32/d/(a^3-a^3*cos(d*x+c))-1/16/d/(a^
3+a^3*cos(d*x+c))
```

Rubi [A]
time = 0.10, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3957, 2786, 90, 212}

$$-\frac{1}{32d\,(a^3-a^3\cos(c+dx))}-\frac{1}{16d\,(a^3\cos(c+dx)+a^3)}+\frac{\tanh^{-1}(\cos(c+dx))}{32a^3d}-\frac{a}{16d(a\cos(c+dx)+a)^4}+\frac{1}{6d(a\cos(c+dx)+a)^3}-\frac{3}{32ad(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] ArcTanh[Cos[c + d*x]]/(32*a^3*d) - a/(16*d*(a + a*Cos[c + d*x])^4) + 1/(6*d
*(a + a*Cos[c + d*x])^3) - 3/(32*a*d*(a + a*Cos[c + d*x])^2) - 1/(32*d*(a^3
 - a^3*Cos[c + d*x])) - 1/(16*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= -\int \frac{\cot^3(c+dx)}{(-a-a\cos(c+dx))^3}\,dx \\
&= \frac{\text{Subst}\left(\int \frac{x^3}{(-a-x)^2(-a+x)^5}\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{a}{4(a-x)^5}+\frac{1}{2(a-x)^4}-\frac{3}{16a(a-x)^3}-\frac{1}{16a^2(a-x)^2}+\frac{1}{32a^2(a+x)^2}-\frac{1}{32a^2(a^2-x^2)}\right.\right.}{d} \\
&= -\frac{a}{16d(a+a\cos(c+dx))^4}+\frac{1}{6d(a+a\cos(c+dx))^3}-\frac{3}{32ad(a+a\cos(c+dx)} \\
&= \frac{\tanh^{-1}(\cos(c+dx))}{32a^3d}-\frac{a}{16d(a+a\cos(c+dx))^4}+\frac{1}{6d(a+a\cos(c+dx))^3}-\frac{}{3}
\end{aligned}
$$

Mathematica [A]
time = 0.39, size = 138, normalized size = 1.10

$$
-\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\left(12\csc^2\left(\frac{1}{2}(c+dx)\right)+24\left(-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+24\sec^2\left(\frac{1}{2}(c+dx)\right)+18\sec^4\left(\frac{1}{2}(c+dx)\right)-16\sec^6\left(\frac{1}{2}(c+dx)\right)+3\sec^8\left(\frac{1}{2}(c+dx)\right)\right)\sec^3(c+dx)}{96a^3d(1+\sec(c+dx))^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/96*(Cos[(c + d*x)/2]^6*(12*Csc[(c + d*x)/2]^2 + 24*(-Log[Cos[(c + d*x)/2
]] + Log[Sin[(c + d*x)/2]]) + 24*Sec[(c + d*x)/2]^2 + 18*Sec[(c + d*x)/2]^4
 - 16*Sec[(c + d*x)/2]^6 + 3*Sec[(c + d*x)/2]^8)*Sec[c + d*x]^3)/(a^3*d*(1
+ Sec[c + d*x])^3)
```

Maple [A]
time = 0.14, size = 91, normalized size = 0.72

method	result
derivativedivides	$\frac{\frac{1}{-32+32\cos(dx+c)}-\frac{\ln(-1+\cos(dx+c))}{64}-\frac{1}{16(1+\cos(dx+c))^4}+\frac{1}{6(1+\cos(dx+c))^3}-\frac{3}{32(1+\cos(dx+c))^2}-\frac{1}{16(1+\cos(dx+c))}+\frac{\ln(1+\text{co}}{6}}{d\,a^3}$
default	$\frac{\frac{1}{-32+32\cos(dx+c)}-\frac{\ln(-1+\cos(dx+c))}{64}-\frac{1}{16(1+\cos(dx+c))^4}+\frac{1}{6(1+\cos(dx+c))^3}-\frac{3}{32(1+\cos(dx+c))^2}-\frac{1}{16(1+\cos(dx+c))}+\frac{\ln(1+\text{co}}{6}}{d\,a^3}$

norman	$\frac{-\frac{1}{64ad}-\frac{\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)}{32ad}+\frac{\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)}{64ad}+\frac{\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)}{192ad}-\frac{\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)}{256ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a^2}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32a^3d}$
risch	$-\frac{3\,e^{9i(dx+c)}+18\,e^{8i(dx+c)}-88\,e^{7i(dx+c)}-162\,e^{6i(dx+c)}-310\,e^{5i(dx+c)}-162\,e^{4i(dx+c)}-88\,e^{3i(dx+c)}+18\,e^{2i(dx+c)}+3\,e^{i(d}}{48a^3d\left(e^{i(dx+c)}+1\right)^8\left(e^{i(dx+c)}-1\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(1/32/(-1+cos(d*x+c))-1/64*ln(-1+cos(d*x+c))-1/16/(1+cos(d*x+c))^4+
1/6/(1+cos(d*x+c))^3-3/32/(1+cos(d*x+c))^2-1/16/(1+cos(d*x+c))+1/64*ln(1+co
s(d*x+c)))
```

Maxima [A]
time = 0.27, size = 146, normalized size = 1.16

$$-\frac{\frac{2\left(3\cos(dx+c)^4+9\cos(dx+c)^3-25\cos(dx+c)^2-27\cos(dx+c)-8\right)}{a^3\cos(dx+c)^5+3a^3\cos(dx+c)^4+2a^3\cos(dx+c)^3-2a^3\cos(dx+c)^2-3a^3\cos(dx+c)-a^3}-\frac{3\log(\cos(dx+c)+1)}{a^3}+\frac{3\log(\cos(dx+c)-1)}{a^3}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/192*(2*(3*cos(d*x + c)^4 + 9*cos(d*x + c)^3 - 25*cos(d*x + c)^2 - 27*cos
(d*x + c) - 8)/(a^3*cos(d*x + c)^5 + 3*a^3*cos(d*x + c)^4 + 2*a^3*cos(d*x +
 c)^3 - 2*a^3*cos(d*x + c)^2 - 3*a^3*cos(d*x + c) - a^3) - 3*log(cos(d*x +
c) + 1)/a^3 + 3*log(cos(d*x + c) - 1)/a^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(115) = 230$.
time = 2.69, size = 240, normalized size = 1.90

$$-\frac{6\cos(dx+c)^4+18\cos(dx+c)^3-50\cos(dx+c)^2-3\left(\cos(dx+c)^5+3\cos(dx+c)^4+2\cos(dx+c)^3-2\cos(dx+c)^2-3\cos(dx+c)-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+3\left(\cos(dx+c)^5+3\cos(dx+c)^4+2\cos(dx+c)^3-2\cos(dx+c)^2-3\cos(dx+c)-1\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-54\cos(dx+c)-16}{192\left(a^3d\cos(dx+c)^5+3a^3d\cos(dx+c)^4+2a^3d\cos(dx+c)^3-2a^3d\cos(dx+c)^2-3a^3d\cos(dx+c)-a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/192*(6*cos(d*x + c)^4 + 18*cos(d*x + c)^3 - 50*cos(d*x + c)^2 - 3*(cos(d
*x + c)^5 + 3*cos(d*x + c)^4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 - 3*cos(
d*x + c) - 1)*log(1/2*cos(d*x + c) + 1/2) + 3*(cos(d*x + c)^5 + 3*cos(d*x +
 c)^4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 - 3*cos(d*x + c) - 1)*log(-1/2*
cos(d*x + c) + 1/2) - 54*cos(d*x + c) - 16)/(a^3*d*cos(d*x + c)^5 + 3*a^3*d
*cos(d*x + c)^4 + 2*a^3*d*cos(d*x + c)^3 - 2*a^3*d*cos(d*x + c)^2 - 3*a^3*d
*cos(d*x + c) - a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^3(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(csc(c + d*x)**3/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Giac [A]
time = 0.56, size = 182, normalized size = 1.44

$$\frac{\frac{12\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)(\cos(dx+c)+1)}{a^3(\cos(dx+c)-1)}-\frac{12\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^3}+\frac{\frac{24a^9(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{12a^9(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{4a^9(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}-\frac{3a^9(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}}{a^{12}}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/768*(12*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)*(cos(d*x + c) + 1)/(a
^3*(cos(d*x + c) - 1)) - 12*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1
))/a^3 + (24*a^9*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 12*a^9*(cos(d*x +
c) - 1)^2/(cos(d*x + c) + 1)^2 - 4*a^9*(cos(d*x + c) - 1)^3/(cos(d*x + c) +
 1)^3 - 3*a^9*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4)/a^12)/d
```

Mupad [B]
time = 0.17, size = 130, normalized size = 1.03

$$\frac{\mathrm{atanh}(\cos(c+d\,x))}{32\,a^3\,d}-\frac{-\frac{\cos(c+d\,x)^4}{32}-\frac{3\cos(c+d\,x)^3}{32}+\frac{25\cos(c+d\,x)^2}{96}+\frac{9\cos(c+d\,x)}{32}+\frac{1}{12}}{d\,\left(-a^3\cos(c+d\,x)^5-3\,a^3\cos(c+d\,x)^4-2\,a^3\cos(c+d\,x)^3+2\,a^3\cos(c+d\,x)^2+3\,a^3\,\cos(c+d\,x)+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^3*(a + a/cos(c + d*x))^3),x)
```

```
[Out] atanh(cos(c + d*x))/(32*a^3*d) - ((9*cos(c + d*x))/32 + (25*cos(c + d*x)^2)
/96 - (3*cos(c + d*x)^3)/32 - cos(c + d*x)^4/32 + 1/12)/(d*(3*a^3*cos(c + d
*x) + a^3 + 2*a^3*cos(c + d*x)^2 - 2*a^3*cos(c + d*x)^3 - 3*a^3*cos(c + d*x
)^4 - a^3*cos(c + d*x)^5))
```

3.99 $\int \frac{\csc^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=128

$$\frac{3\tanh^{-1}(\cos(c+dx))}{128a^3d} - \frac{1}{128ad(a-a\cos(c+dx))^2} - \frac{a^2}{40d(a+a\cos(c+dx))^5} + \frac{3a}{64d(a+a\cos(c+dx))^4} - \frac{}{64a}$$

```
[Out] 3/128*arctanh(cos(d*x+c))/a^3/d-1/128/a/d/(a-a*cos(d*x+c))^2-1/40*a^2/d/(a+
a*cos(d*x+c))^5+3/64*a/d/(a+a*cos(d*x+c))^4-1/64/a/d/(a+a*cos(d*x+c))^2-3/1
28/d/(a^3+a^3*cos(d*x+c))
```

Rubi [A]

time = 0.15, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3957, 2915, 12, 90, 212}

$$-\frac{3}{128d(a^3\cos(c+dx)+a^3)} + \frac{3\tanh^{-1}(\cos(c+dx))}{128a^3d} - \frac{a^2}{40d(a\cos(c+dx)+a)^5} + \frac{3a}{64d(a\cos(c+dx)+a)^4} - \frac{1}{128ad(a-a\cos(c+dx))^2} - \frac{1}{64ad(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (3*ArcTanh[Cos[c + d*x]])/(128*a^3*d) - 1/(128*a*d*(a - a*Cos[c + d*x])^2)
- a^2/(40*d*(a + a*Cos[c + d*x])^5) + (3*a)/(64*d*(a + a*Cos[c + d*x])^4) -
 1/(64*a*d*(a + a*Cos[c + d*x])^2) - 3/(128*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
```

```
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= -\int \frac{\cot^3(c+dx)\csc^2(c+dx)}{(-a-a\cos(c+dx))^3}\,dx \\
&= \frac{a^5\text{Subst}\left(\int \frac{x^3}{a^3(-a-x)^3(-a+x)^6}\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= \frac{a^2\text{Subst}\left(\int \frac{x^3}{(-a-x)^3(-a+x)^6}\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= \frac{a^2\text{Subst}\left(\int \left(-\frac{1}{8(a-x)^6}+\frac{3}{16a(a-x)^5}-\frac{1}{32a^3(a-x)^3}-\frac{3}{128a^4(a-x)^2}+\frac{1}{64a^3(a+x)^3}-\frac{}{128a^4}\right.\right.}{d} \\
&= -\frac{1}{128ad(a-a\cos(c+dx))^2}-\frac{a^2}{40d(a+a\cos(c+dx))^5}+\frac{3a}{64d(a+a\cos(c+d}\\
&= \frac{3\tanh^{-1}(\cos(c+dx))}{128a^3d}-\frac{1}{128ad(a-a\cos(c+dx))^2}-\frac{a^2}{40d(a+a\cos(c+dx))^{}}
\end{aligned}$$

Mathematica [A]
time = 3.00, size = 137, normalized size = 1.07

$$-\frac{\left(4-15\cos^2\left(\frac{1}{2}(c+dx)\right)+60\cos^8\left(\frac{1}{2}(c+dx)\right)+10\cos^6\left(\frac{1}{2}(c+dx)\right)\left(2+\cot^4\left(\frac{1}{2}(c+dx)\right)\right)-120\cos^{10}\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\right)\sec^4\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)}{640a^3d(1+\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^5/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/640*((4 - 15*Cos[(c + d*x)/2]^2 + 60*Cos[(c + d*x)/2]^8 + 10*Cos[(c + d*
x)/2]^6*(2 + Cot[(c + d*x)/2]^4) - 120*Cos[(c + d*x)/2]^10*(Log[Cos[(c + d*
x)/2]] - Log[Sin[(c + d*x)/2]]))*Sec[(c + d*x)/2]^4*Sec[c + d*x]^3)/(a^3*d*
(1 + Sec[c + d*x])^3)
```

Maple [A]
time = 0.15, size = 91, normalized size = 0.71

method	result
derivativedivides	$\frac{-\frac{1}{128(-1+\cos(dx+c))^2}-\frac{3\ln(-1+\cos(dx+c))}{256}-\frac{1}{40(1+\cos(dx+c))^5}+\frac{3}{64(1+\cos(dx+c))^4}-\frac{1}{64(1+\cos(dx+c))^2}-\frac{3}{128(1+\cos(dx+c))}+\frac{3}{\ldots}}{d\,a^3}$
default	$\frac{-\frac{1}{128(-1+\cos(dx+c))^2}-\frac{3\ln(-1+\cos(dx+c))}{256}-\frac{1}{40(1+\cos(dx+c))^5}+\frac{3}{64(1+\cos(dx+c))^4}-\frac{1}{64(1+\cos(dx+c))^2}-\frac{3}{128(1+\cos(dx+c))}+\frac{3}{\ldots}}{d\,a^3}$
norman	$\frac{-\frac{1}{512ad}-\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{256ad}-\frac{3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{256ad}+\frac{3\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{512ad}+\frac{\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)}{256ad}-\frac{\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)}{1024ad}-\frac{\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)}{1280ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a^2}-\frac{3\ln\left(\tan \ldots\right.}{128\ldots}$
risch	$-\frac{15\,e^{13i(dx+c)}+90\,e^{12i(dx+c)}+170\,e^{11i(dx+c)}-30\,e^{10i(dx+c)}+1521\,e^{9i(dx+c)}+1476\,e^{8i(dx+c)}+3756\,e^{7i(dx+c)}+1476\,e^{6i(\ldots}}{320a^3d\left(e^{i(dx+c)}+1\right)^{10}\left(e^{i(dx+c)}-1\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(-1/128/(-1+cos(d*x+c))^2-3/256*ln(-1+cos(d*x+c))-1/40/(1+cos(d*x+c
))^5+3/64/(1+cos(d*x+c))^4-1/64/(1+cos(d*x+c))^2-3/128/(1+cos(d*x+c))+3/256
*ln(1+cos(d*x+c)))
```

Maxima [A]
time = 0.28, size = 188, normalized size = 1.47

$$-\frac{\frac{2\left(15\cos(dx+c)^6+45\cos(dx+c)^5+20\cos(dx+c)^4-60\cos(dx+c)^3+61\cos(dx+c)^2+63\cos(dx+c)+16\right)}{a^3\cos(dx+c)^7+3a^3\cos(dx+c)^6+a^3\cos(dx+c)^5-5a^3\cos(dx+c)^4-5a^3\cos(dx+c)^3+a^3\cos(dx+c)^2+3a^3\cos(dx+c)+a^3}-\frac{15\log(\cos(dx+c)+1)}{a^3}+\frac{15\log(\cos(dx+c)-1)}{a^3}}{1280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/1280*(2*(15*cos(d*x + c)^6 + 45*cos(d*x + c)^5 + 20*cos(d*x + c)^4 - 60*
cos(d*x + c)^3 + 61*cos(d*x + c)^2 + 63*cos(d*x + c) + 16)/(a^3*cos(d*x + c
)^7 + 3*a^3*cos(d*x + c)^6 + a^3*cos(d*x + c)^5 - 5*a^3*cos(d*x + c)^4 - 5*
a^3*cos(d*x + c)^3 + a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) + a^3) - 15*lo
g(cos(d*x + c) + 1)/a^3 + 15*log(cos(d*x + c) - 1)/a^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 317 vs. 2(117) = 234.
time = 2.73, size = 317, normalized size = 2.48

$$-\frac{30\cos(dx+c)^6+90\cos(dx+c)^5+40\cos(dx+c)^4-120\cos(dx+c)^3+122\cos(dx+c)^2-15\left(\cos(dx+c)^7+3\cos(dx+c)^6+\cos(dx+c)^5-5\cos(dx+c)^4-5\cos(dx+c)^3+\cos(dx+c)^2+3\cos(dx+c)+1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+15\left(\cos(dx+c)^7+3\cos(dx+c)^6+\cos(dx+c)^5-5\cos(dx+c)^4-5\cos(dx+c)^3+\cos(dx+c)^2+3\cos(dx+c)+1\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+126\cos(dx+c)+32}{1280\left(a^3d\cos(dx+c)^7+3a^3d\cos(dx+c)^6+a^3d\cos(dx+c)^5-5a^3d\cos(dx+c)^4-5a^3d\cos(dx+c)^3+a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/1280*(30*cos(d*x + c)^6 + 90*cos(d*x + c)^5 + 40*cos(d*x + c)^4 - 120*co
s(d*x + c)^3 + 122*cos(d*x + c)^2 - 15*(cos(d*x + c)^7 + 3*cos(d*x + c)^6 +
 cos(d*x + c)^5 - 5*cos(d*x + c)^4 - 5*cos(d*x + c)^3 + cos(d*x + c)^2 + 3*
```

```
cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) + 15*(cos(d*x + c)^7 + 3*cos(
d*x + c)^6 + cos(d*x + c)^5 - 5*cos(d*x + c)^4 - 5*cos(d*x + c)^3 + cos(d*x
 + c)^2 + 3*cos(d*x + c) + 1)*log(-1/2*cos(d*x + c) + 1/2) + 126*cos(d*x +
c) + 32)/(a^3*d*cos(d*x + c)^7 + 3*a^3*d*cos(d*x + c)^6 + a^3*d*cos(d*x + c
)^5 - 5*a^3*d*cos(d*x + c)^4 - 5*a^3*d*cos(d*x + c)^3 + a^3*d*cos(d*x + c)^
2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^5(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(csc(c + d*x)**5/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Giac [A]
time = 0.61, size = 232, normalized size = 1.81

$$\frac{\frac{10\left(\frac{2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{9(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-1\right)(\cos(dx+c)+1)^2}{a^3(\cos(dx+c)-1)^2}-\frac{60\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^3}+\frac{\frac{60a^{12}(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{30a^{12}(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{20a^{12}(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}-\frac{5a^{12}(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}+\frac{4a^{12}(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5}}{a^{15}}}{5120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/5120*(10*(2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*(cos(d*x + c) - 1)^
2/(cos(d*x + c) + 1)^2 - 1)*(cos(d*x + c) + 1)^2/(a^3*(cos(d*x + c) - 1)^2)
 - 60*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a^3 + (60*a^12*(cos
(d*x + c) - 1)/(cos(d*x + c) + 1) + 30*a^12*(cos(d*x + c) - 1)^2/(cos(d*x +
 c) + 1)^2 - 20*a^12*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 5*a^12*(co
s(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 4*a^12*(cos(d*x + c) - 1)^5/(cos(d
*x + c) + 1)^5)/a^15)/d
```

Mupad [B]
time = 1.09, size = 173, normalized size = 1.35

$$\frac{3\,\mathrm{atanh}(\cos(c+d\,x))}{128\,a^3\,d}-\frac{\frac{3\cos(c+d\,x)^6}{128}+\frac{9\cos(c+d\,x)^5}{128}+\frac{\cos(c+d\,x)^4}{32}-\frac{3\cos(c+d\,x)^3}{32}+\frac{61\cos(c+d\,x)^2}{640}+\frac{63\,\cos(c+d\,x)}{640}+\frac{1}{40}}{d\,\left(a^3\cos(c+d\,x)^7+3\,a^3\cos(c+d\,x)^6+a^3\cos(c+d\,x)^5-5\,a^3\cos(c+d\,x)^4-5\,a^3\cos(c+d\,x)^3+a^3\cos(c+d\,x)^2+3\,a^3\,\cos(c+d\,x)+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^5*(a + a/cos(c + d*x))^3),x)
```

```
[Out] (3*atanh(cos(c + d*x)))/(128*a^3*d) - ((63*cos(c + d*x))/640 + (61*cos(c +
d*x)^2)/640 - (3*cos(c + d*x)^3)/32 + cos(c + d*x)^4/32 + (9*cos(c + d*x)^5
)/128 + (3*cos(c + d*x)^6)/128 + 1/40)/(d*(3*a^3*cos(c + d*x) + a^3 + a^3*c
os(c + d*x)^2 - 5*a^3*cos(c + d*x)^3 - 5*a^3*cos(c + d*x)^4 + a^3*cos(c + d
*x)^5 + 3*a^3*cos(c + d*x)^6 + a^3*cos(c + d*x)^7))
```

3.100 $\int \frac{\sin^8(c+dx)}{(a+a\sec(c+dx))^3} dx$

Optimal. Leaf size=157

$$-\frac{29x}{128a^3}-\frac{29\cos(c+dx)\sin(c+dx)}{128a^3d}-\frac{29\cos^3(c+dx)\sin(c+dx)}{192a^3d}+\frac{23\cos^5(c+dx)\sin(c+dx)}{48a^3d}+\frac{\cos^7(c+dx)}{8a}$$

```
[Out] -29/128*x/a^3-29/128*cos(d*x+c)*sin(d*x+c)/a^3/d-29/192*cos(d*x+c)^3*sin(d*
x+c)/a^3/d+23/48*cos(d*x+c)^5*sin(d*x+c)/a^3/d+1/8*cos(d*x+c)^7*sin(d*x+c)/
a^3/d+4/3*sin(d*x+c)^3/a^3/d-7/5*sin(d*x+c)^5/a^3/d+3/7*sin(d*x+c)^7/a^3/d
```

Rubi [A]
time = 0.33, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3957, 2954, 2952, 2644, 14, 2648, 2715, 8, 276}

$$\frac{3\sin^7(c+dx)}{7a^3d}-\frac{7\sin^5(c+dx)}{5a^3d}+\frac{4\sin^3(c+dx)}{3a^3d}+\frac{\sin(c+dx)\cos^7(c+dx)}{8a^3d}+\frac{23\sin(c+dx)\cos^5(c+dx)}{48a^3d}-\frac{29\sin(c+dx)\cos^3(c+dx)}{192a^3d}-\frac{29\sin(c+dx)\cos(c+dx)}{128a^3d}-\frac{29x}{128a^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^8/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-29*x)/(128*a^3) - (29*Cos[c + d*x]*Sin[c + d*x])/(128*a^3*d) - (29*Cos[c
+ d*x]^3*Sin[c + d*x])/(192*a^3*d) + (23*Cos[c + d*x]^5*Sin[c + d*x])/(48*a
^3*d) + (Cos[c + d*x]^7*Sin[c + d*x])/(8*a^3*d) + (4*Sin[c + d*x]^3)/(3*a^3
*d) - (7*Sin[c + d*x]^5)/(5*a^3*d) + (3*Sin[c + d*x]^7)/(7*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
```

```
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= -\int \frac{\cos^3(c+dx)\sin^8(c+dx)}{(-a-a\cos(c+dx))^3}\,dx \\
&= -\frac{\int \cos^3(c+dx)(-a+a\cos(c+dx))^3\sin^2(c+dx)\,dx}{a^6} \\
&= -\frac{\int \left(-a^3\cos^3(c+dx)\sin^2(c+dx)+3a^3\cos^4(c+dx)\sin^2(c+dx)-3a^3\cos^5(c\right.}{a^6} \\
&= \frac{\int \cos^3(c+dx)\sin^2(c+dx)\,dx}{a^3} - \frac{\int \cos^6(c+dx)\sin^2(c+dx)\,dx}{a^3} - \frac{3\int \cos^4(c+}{} \\
&= \frac{\cos^5(c+dx)\sin(c+dx)}{2a^3d} + \frac{\cos^7(c+dx)\sin(c+dx)}{8a^3d} - \frac{\int \cos^6(c+dx)\,dx}{8a^3} - \frac{\int \mathrm{co}}{} \\
&= -\frac{\cos^3(c+dx)\sin(c+dx)}{8a^3d} + \frac{23\cos^5(c+dx)\sin(c+dx)}{48a^3d} + \frac{\cos^7(c+dx)\sin(c+}{8a^3d} \\
&= -\frac{3\cos(c+dx)\sin(c+dx)}{16a^3d} - \frac{29\cos^3(c+dx)\sin(c+dx)}{192a^3d} + \frac{23\cos^5(c+dx)\sin(}{48a^3d} \\
&= -\frac{3x}{16a^3} - \frac{29\cos(c+dx)\sin(c+dx)}{128a^3d} - \frac{29\cos^3(c+dx)\sin(c+dx)}{192a^3d} + \frac{23\cos^5(c-}{} \\
&= -\frac{29x}{128a^3} - \frac{29\cos(c+dx)\sin(c+dx)}{128a^3d} - \frac{29\cos^3(c+dx)\sin(c+dx)}{192a^3d} + \frac{23\cos^5(c}{}
\end{aligned}
$$

Mathematica [A]

time = 3.07, size = 131, normalized size = 0.83

$$
\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(-24360dx+38640\sin(c+dx)-6720\sin(2(c+dx))-3920\sin(3(c+dx))+5880\sin(4(c+dx))-4368\sin(5(c+dx))+2240\sin(6(c+dx))-720\sin(7(c+dx))+105\sin(8(c+dx))+294\tan\left(\frac{c}{2}\right)\right)}{13440a^3d(1+\sec(c+dx))^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^8/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]^6*Sec[c + d*x]^3*(-24360*d*x + 38640*Sin[c + d*x] - 6720*
Sin[2*(c + d*x)] - 3920*Sin[3*(c + d*x)] + 5880*Sin[4*(c + d*x)] - 4368*Sin
[5*(c + d*x)] + 2240*Sin[6*(c + d*x)] - 720*Sin[7*(c + d*x)] + 105*Sin[8*(c
 + d*x)] + 294*Tan[c/2]))/(13440*a^3*d*(1 + Sec[c + d*x])^3)
```

Maple [A]

time = 0.16, size = 142, normalized size = 0.90

method	result
risch	$-\frac{29x}{128a^3}+\frac{23\sin(dx+c)}{64a^3d}+\frac{\sin(8dx+8c)}{1024a^3d}-\frac{3\sin(7dx+7c)}{448a^3d}+\frac{\sin(6dx+6c)}{48a^3d}-\frac{13\sin(5dx+5c)}{320a^3d}+\frac{7\sin(4dx+4c)}{128a^3d}-$
derivativedivides	$\frac{-\frac{64\left(-\frac{29\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4096}-\frac{667\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12288}-\frac{11107\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{61440}-\frac{146537\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{430080}-\frac{72669\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{143360}-\frac{1759\left(\tan^{1}\right.}{20}\right.}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}}{a^3d}$

default	$-\frac{64\left(-\frac{29\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4096}-\frac{667\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12288}-\frac{11107\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{61440}-\frac{146537\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{430080}-\frac{72669\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{143360}-\frac{1759\left(\tan\right.}{}\cdots}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}}{a^3 d}$
norman	$-\frac{29x}{128a}+\frac{29\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64ad}+\frac{667\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192ad}+\frac{11107\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{960ad}+\frac{146537\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6720ad}+\frac{72669\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2240ad}+\frac{1759\left(\right.}{}\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^8/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 64/d/a^3*(-(-29/4096*tan(1/2*d*x+1/2*c)-667/12288*tan(1/2*d*x+1/2*c)^3-1110
7/61440*tan(1/2*d*x+1/2*c)^5-146537/430080*tan(1/2*d*x+1/2*c)^7-72669/14336
0*tan(1/2*d*x+1/2*c)^9-1759/20480*tan(1/2*d*x+1/2*c)^11-1143/4096*tan(1/2*d
*x+1/2*c)^13+29/4096*tan(1/2*d*x+1/2*c)^15)/(1+tan(1/2*d*x+1/2*c)^2)^8-29/4
096*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 378 vs. $2(141) = 282$.
time = 0.49, size = 378, normalized size = 2.41

$$\frac{\frac{\frac{3045\sin(dx+c)}{\cos(dx+c)+1}+\frac{23345\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{77749\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{146537\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{218007\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{36939\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}+\frac{120015\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}}-\frac{3045\sin(dx+c)^{15}}{(\cos(dx+c)+1)^{15}}}{a^3+\frac{8a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{28a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{56a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{70a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{56a^3\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{28a^3\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}+\frac{8a^3\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}+\frac{a^3\sin(dx+c)^{16}}{(\cos(dx+c)+1)^{16}}}-\frac{3045\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{6720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/6720*((3045*sin(d*x + c)/(cos(d*x + c) + 1) + 23345*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 + 77749*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 146537*sin(d*x
 + c)^7/(cos(d*x + c) + 1)^7 + 218007*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 +
 36939*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 120015*sin(d*x + c)^13/(cos(
d*x + c) + 1)^13 - 3045*sin(d*x + c)^15/(cos(d*x + c) + 1)^15)/(a^3 + 8*a^3
*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 28*a^3*sin(d*x + c)^4/(cos(d*x + c)
+ 1)^4 + 56*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 70*a^3*sin(d*x + c)^8
/(cos(d*x + c) + 1)^8 + 56*a^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 28*a
^3*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 + 8*a^3*sin(d*x + c)^14/(cos(d*x +
 c) + 1)^14 + a^3*sin(d*x + c)^16/(cos(d*x + c) + 1)^16) - 3045*arctan(sin(
d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

Fricas [A]
time = 2.95, size = 91, normalized size = 0.58

$$-\frac{3045\,dx-\left(1680\,\cos(dx+c)^7-5760\,\cos(dx+c)^6+6440\,\cos(dx+c)^5-1536\,\cos(dx+c)^4-2030\,\cos(dx+c)^3+2432\,\cos(dx+c)^2-3045\,\cos(dx+c)+4864\right)\sin(dx+c)}{13440\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/13440*(3045*d*x - (1680*cos(d*x + c)^7 - 5760*cos(d*x + c)^6 + 6440*cos(
d*x + c)^5 - 1536*cos(d*x + c)^4 - 2030*cos(d*x + c)^3 + 2432*cos(d*x + c)^
2 - 3045*cos(d*x + c) + 4864)*sin(d*x + c))/(a^3*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**8/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.54, size = 139, normalized size = 0.89

$$-\frac{\frac{3045\,(dx+c)}{a^3}+\frac{2\left(3045\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}-120015\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}-36939\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-218007\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-146537\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-77749\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-23345\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-3045\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{8}a^3}}{13440\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/13440*(3045*(d*x + c)/a^3 + 2*(3045*tan(1/2*d*x + 1/2*c)^15 - 120015*tan
(1/2*d*x + 1/2*c)^13 - 36939*tan(1/2*d*x + 1/2*c)^11 - 218007*tan(1/2*d*x +
 1/2*c)^9 - 146537*tan(1/2*d*x + 1/2*c)^7 - 77749*tan(1/2*d*x + 1/2*c)^5 -
23345*tan(1/2*d*x + 1/2*c)^3 - 3045*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1
/2*c)^2 + 1)^8*a^3))/d
```

Mupad [B]
time = 3.87, size = 132, normalized size = 0.84

$$\frac{-\frac{29\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{64}+\frac{1143\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{64}+\frac{1759\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{320}+\frac{72669\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{2240}+\frac{146537\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{6720}+\frac{11107\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{960}+\frac{667\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{192}+\frac{29\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64}}{a^3\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^8}-\frac{29\,x}{128\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^8/(a + a/cos(c + d*x))^3,x)
```

```
[Out] ((29*tan(c/2 + (d*x)/2))/64 + (667*tan(c/2 + (d*x)/2)^3)/192 + (11107*tan(c
/2 + (d*x)/2)^5)/960 + (146537*tan(c/2 + (d*x)/2)^7)/6720 + (72669*tan(c/2
+ (d*x)/2)^9)/2240 + (1759*tan(c/2 + (d*x)/2)^11)/320 + (1143*tan(c/2 + (d*
x)/2)^13)/64 - (29*tan(c/2 + (d*x)/2)^15)/64)/(a^3*d*(tan(c/2 + (d*x)/2)^2
+ 1)^8) - (29*x)/(128*a^3)
```

3.101 $\int \frac{\sin^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=129

$$-\frac{23x}{16a^3}+\frac{4\sin(c+dx)}{a^3d}-\frac{23\cos(c+dx)\sin(c+dx)}{16a^3d}-\frac{23\cos^3(c+dx)\sin(c+dx)}{24a^3d}-\frac{\cos^5(c+dx)\sin(c+dx)}{6a^3d}-$$

```
[Out] -23/16*x/a^3+4*sin(d*x+c)/a^3/d-23/16*cos(d*x+c)*sin(d*x+c)/a^3/d-23/24*cos
(d*x+c)^3*sin(d*x+c)/a^3/d-1/6*cos(d*x+c)^5*sin(d*x+c)/a^3/d-7/3*sin(d*x+c)
^3/a^3/d+3/5*sin(d*x+c)^5/a^3/d
```

Rubi [A]
time = 0.22, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3957, 2948, 2836, 2713, 2715, 8}

$$\frac{3\sin^5(c+dx)}{5a^3d}-\frac{7\sin^3(c+dx)}{3a^3d}+\frac{4\sin(c+dx)}{a^3d}-\frac{\sin(c+dx)\cos^5(c+dx)}{6a^3d}-\frac{23\sin(c+dx)\cos^3(c+dx)}{24a^3d}-\frac{23\sin(c+dx)\cos(c+dx)}{16a^3d}-\frac{23x}{16a^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^6/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-23*x)/(16*a^3) + (4*Sin[c + d*x])/(a^3*d) - (23*Cos[c + d*x]*Sin[c + d*x]
)/(16*a^3*d) - (23*Cos[c + d*x]^3*Sin[c + d*x])/(24*a^3*d) - (Cos[c + d*x]^
5*Sin[c + d*x])/(6*a^3*d) - (7*Sin[c + d*x]^3)/(3*a^3*d) + (3*Sin[c + d*x]^
5)/(5*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2836

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
```

```
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2948

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= -\int \frac{\cos^3(c+dx)\sin^6(c+dx)}{(-a-a\cos(c+dx))^3}\,dx \\
&= -\frac{\int \cos^3(c+dx)(-a+a\cos(c+dx))^3\,dx}{a^6} \\
&= -\frac{\int (-a^3\cos^3(c+dx)+3a^3\cos^4(c+dx)-3a^3\cos^5(c+dx)+a^3\cos^6(c+dx))}{a^6} \\
&= \frac{\int \cos^3(c+dx)\,dx}{a^3} - \frac{\int \cos^6(c+dx)\,dx}{a^3} - \frac{3\int \cos^4(c+dx)\,dx}{a^3} + \frac{3\int \cos^5(c+dx}{a^3} \\
&= -\frac{3\cos^3(c+dx)\sin(c+dx)}{4a^3d} - \frac{\cos^5(c+dx)\sin(c+dx)}{6a^3d} - \frac{5\int \cos^4(c+dx)\,dx}{6a^3} - \\
&= \frac{4\sin(c+dx)}{a^3d} - \frac{9\cos(c+dx)\sin(c+dx)}{8a^3d} - \frac{23\cos^3(c+dx)\sin(c+dx)}{24a^3d} - \cos^5(\\
&= -\frac{9x}{8a^3} + \frac{4\sin(c+dx)}{a^3d} - \frac{23\cos(c+dx)\sin(c+dx)}{16a^3d} - \frac{23\cos^3(c+dx)\sin(c+dx}{24a^3d} \\
&= -\frac{23x}{16a^3} + \frac{4\sin(c+dx)}{a^3d} - \frac{23\cos(c+dx)\sin(c+dx)}{16a^3d} - \frac{23\cos^3(c+dx)\sin(c+}{24a^3d}
\end{aligned}$$

Mathematica [A]
time = 1.15, size = 111, normalized size = 0.86

$$\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(-2760dx+5040\sin(c+dx)-1890\sin(2(c+dx))+760\sin(3(c+dx))-270\sin(4(c+dx))+72\sin(5(c+dx))-10\sin(6(c+dx))+9\tan\left(\frac{c}{2}\right)\right)}{240a^3d(1+\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^6/(a + a*Sec[c + d*x])^3,x]
```

[Out] (Cos[(c + d*x)/2]^6*Sec[c + d*x]^3*(-2760*d*x + 5040*Sin[c + d*x] - 1890*Sin[2*(c + d*x)] + 760*Sin[3*(c + d*x)] - 270*Sin[4*(c + d*x)] + 72*Sin[5*(c + d*x)] - 10*Sin[6*(c + d*x)] + 9*Tan[c/2]))/(240*a^3*d*(1 + Sec[c + d*x])^3)

Maple [A]
time = 0.15, size = 116, normalized size = 0.90

method	result
risch	$-\frac{23x}{16a^3}+\frac{21\sin(dx+c)}{8a^3d}-\frac{\sin(6dx+6c)}{192a^3d}+\frac{3\sin(5dx+5c)}{80a^3d}-\frac{9\sin(4dx+4c)}{64a^3d}+\frac{19\sin(3dx+3c)}{48a^3d}-\frac{63\sin(2dx+2c)}{64a^3d}$
derivativedivides	$-\frac{16\left(-\frac{105\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128}-\frac{211\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128}-\frac{969\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{320}-\frac{759\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{320}-\frac{391\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{384}-\frac{23\tan\left(\frac{dx}{2}\right.}{128}\right.}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}}{a^3d}$
default	$-\frac{16\left(-\frac{105\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128}-\frac{211\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128}-\frac{969\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{320}-\frac{759\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{320}-\frac{391\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{384}-\frac{23\tan\left(\frac{dx}{2}\right.}{128}\right.}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}}{a^3d}$
norman	$\frac{-\frac{23x}{16a}+\frac{23\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{391\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24ad}+\frac{759\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20ad}+\frac{969\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20ad}+\frac{211\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad}+\frac{105\left(\tan^{11}\left(\frac{dx}{2}\right.\right.}{8ad}}{\left(1+\right.}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(d*x+c)^6/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 16/d/a^3*(-(-105/128*tan(1/2*d*x+1/2*c)^11-211/128*tan(1/2*d*x+1/2*c)^9-969/320*tan(1/2*d*x+1/2*c)^7-759/320*tan(1/2*d*x+1/2*c)^5-391/384*tan(1/2*d*x+1/2*c)^3-23/128*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^6-23/128*arctan(tan(1/2*d*x+1/2*c)))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(117) = 234$.
time = 0.50, size = 292, normalized size = 2.26

$$\frac{\frac{\frac{345\sin(dx+c)}{\cos(dx+c)+1}+\frac{1955\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{4554\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5814\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{3165\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{1575\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}}{a^3+\frac{6a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{20a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{15a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{6a^3\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{a^3\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}}-\frac{345\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="maxima")

[Out] 1/120*((345*sin(d*x + c)/(cos(d*x + c) + 1) + 1955*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 4554*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5814*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 3165*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 1575*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)/(a^3 + 6*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 15*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 20*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 15*a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 6*

```
a^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + a^3*sin(d*x + c)^12/(cos(d*x +
c) + 1)^12) - 345*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

Fricas [A]
time = 3.00, size = 70, normalized size = 0.54

$$-\frac{345\,dx+\left(40\,\cos\left(dx+c\right)^{5}-144\,\cos\left(dx+c\right)^{4}+230\,\cos\left(dx+c\right)^{3}-272\,\cos\left(dx+c\right)^{2}+345\,\cos\left(dx+c\right)-544\right)\sin\left(dx+c\right)}{240\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/240*(345*d*x + (40*cos(d*x + c)^5 - 144*cos(d*x + c)^4 + 230*cos(d*x + c
)^3 - 272*cos(d*x + c)^2 + 345*cos(d*x + c) - 544)*sin(d*x + c))/(a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sin^{6}\left(c+dx\right)}{\sec^{3}\left(c+dx\right)+3\sec^{2}\left(c+dx\right)+3\sec\left(c+dx\right)+1}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**6/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(sin(c + d*x)**6/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Giac [A]
time = 0.51, size = 113, normalized size = 0.88

$$-\frac{\frac{345\,(dx+c)}{a^{3}}-\frac{2\left(1575\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+3165\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+5814\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+4554\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+1955\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+345\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{6}a^{3}}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/240*(345*(d*x + c)/a^3 - 2*(1575*tan(1/2*d*x + 1/2*c)^11 + 3165*tan(1/2*
d*x + 1/2*c)^9 + 5814*tan(1/2*d*x + 1/2*c)^7 + 4554*tan(1/2*d*x + 1/2*c)^5
+ 1955*tan(1/2*d*x + 1/2*c)^3 + 345*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1
/2*c)^2 + 1)^6*a^3))/d
```

Mupad [B]
time = 3.67, size = 106, normalized size = 0.82

$$\frac{\frac{105\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}+\frac{211\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{8}+\frac{969\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{20}+\frac{759\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{20}+\frac{391\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{24}+\frac{23\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}}{a^{3}\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)^{6}}-\frac{23\,x}{16\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6/(a + a/cos(c + d*x))^3,x)
```

```
[Out] ((23*tan(c/2 + (d*x)/2))/8 + (391*tan(c/2 + (d*x)/2)^3)/24 + (759*tan(c/2 +
 (d*x)/2)^5)/20 + (969*tan(c/2 + (d*x)/2)^7)/20 + (211*tan(c/2 + (d*x)/2)^9
)/8 + (105*tan(c/2 + (d*x)/2)^11)/8)/(a^3*d*(tan(c/2 + (d*x)/2)^2 + 1)^6) -
 (23*x)/(16*a^3)
```

3.102 $\int \frac{\sin^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=108

$$\frac{51x}{8a^3}-\frac{7\sin(c+dx)}{a^3d}+\frac{19\cos(c+dx)\sin(c+dx)}{8a^3d}+\frac{\cos^3(c+dx)\sin(c+dx)}{4a^3d}-\frac{4\sin(c+dx)}{a^3d(1+\cos(c+dx))}+\frac{\sin^3(c+d}{a^3d}$$

```
[Out] 51/8*x/a^3-7*sin(d*x+c)/a^3/d+19/8*cos(d*x+c)*sin(d*x+c)/a^3/d+1/4*cos(d*x+
c)^3*sin(d*x+c)/a^3/d-4*sin(d*x+c)/a^3/d/(1+cos(d*x+c))+sin(d*x+c)^3/a^3/d
```

Rubi [A]
time = 0.23, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3957, 2954, 2951, 2727, 2717, 2715, 8, 2713}

$$\frac{\sin^3(c+dx)}{a^3d}-\frac{7\sin(c+dx)}{a^3d}+\frac{\sin(c+dx)\cos^3(c+dx)}{4a^3d}+\frac{19\sin(c+dx)\cos(c+dx)}{8a^3d}-\frac{4\sin(c+dx)}{a^3d(\cos(c+dx)+1)}+\frac{51x}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (51*x)/(8*a^3) - (7*Sin[c + d*x])/(a^3*d) + (19*Cos[c + d*x]*Sin[c + d*x])/
(8*a^3*d) + (Cos[c + d*x]^3*Sin[c + d*x])/(4*a^3*d) - (4*Sin[c + d*x])/(a^3
*d*(1 + Cos[c + d*x])) + Sin[c + d*x]^3/(a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= -\int \frac{\cos^3(c+dx)\sin^4(c+dx)}{(-a-a\cos(c+dx))^3}\,dx \\
&= -\frac{\int \cos(c+dx)(-a+a\cos(c+dx))^3\cot^2(c+dx)\,dx}{a^6} \\
&= \frac{\int \left(4a+\frac{4a}{-1-\cos(c+dx)}-4a\cos(c+dx)+4a\cos^2(c+dx)-3a\cos^3(c+dx)+a\cos\right.}{a^4} \\
&= \frac{4x}{a^3}+\frac{\int \cos^4(c+dx)\,dx}{a^3}-\frac{3\int \cos^3(c+dx)\,dx}{a^3}+\frac{4\int \frac{1}{-1-\cos(c+dx)}\,dx}{a^3}-\frac{4\int \cos(c}{a} \\
&= \frac{4x}{a^3}-\frac{4\sin(c+dx)}{a^3d}+\frac{2\cos(c+dx)\sin(c+dx)}{a^3d}+\frac{\cos^3(c+dx)\sin(c+dx)}{4a^3d}-\frac{}{a^3} \\
&= \frac{6x}{a^3}-\frac{7\sin(c+dx)}{a^3d}+\frac{19\cos(c+dx)\sin(c+dx)}{8a^3d}+\frac{\cos^3(c+dx)\sin(c+dx)}{4a^3d}- \\
&= \frac{51x}{8a^3}-\frac{7\sin(c+dx)}{a^3d}+\frac{19\cos(c+dx)\sin(c+dx)}{8a^3d}+\frac{\cos^3(c+dx)\sin(c+dx)}{4a^3d}-
\end{aligned}
$$

Mathematica [A]

time = 0.48, size = 173, normalized size = 1.60

$$
\frac{\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(2040dx\cos\left(\frac{dx}{2}\right)+2040dx\cos\left(c+\frac{dx}{2}\right)-3563\sin\left(\frac{dx}{2}\right)-997\sin\left(c+\frac{dx}{2}\right)-800\sin\left(c+\frac{3dx}{2}\right)-800\sin\left(2c+\frac{3dx}{2}\right)+160\sin\left(2c+\frac{5dx}{2}\right)+160\sin\left(3c+\frac{5dx}{2}\right)-35\sin\left(3c+\frac{7dx}{2}\right)-35\sin\left(4c+\frac{7dx}{2}\right)+5\sin\left(4c+\frac{9dx}{2}\right)+5\sin\left(5c+\frac{9dx}{2}\right)\right)}{640a^3d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a + a*Sec[c + d*x])^3,x]

[Out] (Sec[c/2]*Sec[(c + d*x)/2]*(2040*d*x*Cos[(d*x)/2] + 2040*d*x*Cos[c + (d*x)/
2] - 3563*Sin[(d*x)/2] - 997*Sin[c + (d*x)/2] - 800*Sin[c + (3*d*x)/2] - 80
0*Sin[2*c + (3*d*x)/2] + 160*Sin[2*c + (5*d*x)/2] + 160*Sin[3*c + (5*d*x)/2
] - 35*Sin[3*c + (7*d*x)/2] - 35*Sin[4*c + (7*d*x)/2] + 5*Sin[4*c + (9*d*x)
/2] + 5*Sin[5*c + (9*d*x)/2]))/(640*a^3*d)
```

Maple [A]

time = 0.14, size = 100, normalized size = 0.93

method	result
derivativedivides	$\frac{-4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{4\left(-\frac{77\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}-\frac{149\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}-\frac{123\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}-\frac{35\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{51\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}}{a^3d}$
default	$\frac{-4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{4\left(-\frac{77\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}-\frac{149\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}-\frac{123\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}-\frac{35\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{51\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}}{a^3d}$

risch	$\frac{51x}{8a^3}+\frac{25i\mathrm{e}^{i(dx+c)}}{8a^3d}-\frac{25i\mathrm{e}^{-i(dx+c)}}{8a^3d}-\frac{8i}{a^3d\left(\mathrm{e}^{i(dx+c)}+1\right)}+\frac{\sin(4dx+4c)}{32a^3d}-\frac{\sin(3dx+3c)}{4a^3d}+\frac{5\sin(2dx+2c)}{4a^3d}$
norman	$\frac{\frac{51x}{8a}-\frac{51\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}-\frac{187\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ad}-\frac{245\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ad}-\frac{141\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ad}-\frac{4\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{51x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{2a}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 4/d/a^3*(-tan(1/2*d*x+1/2*c)+(-77/16*tan(1/2*d*x+1/2*c)^7-149/16*tan(1/2*d*
x+1/2*c)^5-123/16*tan(1/2*d*x+1/2*c)^3-35/16*tan(1/2*d*x+1/2*c))/(1+tan(1/2
*d*x+1/2*c)^2)^4+51/16*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(102) = 204$.
time = 0.49, size = 227, normalized size = 2.10

$$-\frac{\frac{\frac{35\sin(dx+c)}{\cos(dx+c)+1}+\frac{123\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{149\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{77\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^3+\frac{4a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}-\frac{51\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}+\frac{16\sin(dx+c)}{a^3(\cos(dx+c)+1)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*((35*sin(d*x + c)/(cos(d*x + c) + 1) + 123*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 + 149*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 77*sin(d*x + c)^7/(cos
(d*x + c) + 1)^7)/(a^3 + 4*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a^3*
sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*a^3*sin(d*x + c)^6/(cos(d*x + c) +
1)^6 + a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) - 51*arctan(sin(d*x + c)/(c
os(d*x + c) + 1))/a^3 + 16*sin(d*x + c)/(a^3*(cos(d*x + c) + 1)))/d
```

Fricas [A]
time = 3.09, size = 83, normalized size = 0.77

$$\frac{51\,dx\cos(dx+c)+51\,dx+\left(2\cos(dx+c)^4-6\cos(dx+c)^3+11\cos(dx+c)^2-29\cos(dx+c)-80\right)\sin(dx+c)}{8\left(a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/8*(51*d*x*cos(d*x + c) + 51*d*x + (2*cos(d*x + c)^4 - 6*cos(d*x + c)^3 +
11*cos(d*x + c)^2 - 29*cos(d*x + c) - 80)*sin(d*x + c))/(a^3*d*cos(d*x + c)
 + a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sin^4(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(sin(c + d*x)**4/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Giac [A]
time = 0.52, size = 101, normalized size = 0.94

$$\frac{\frac{51\,(dx+c)}{a^3}-\frac{32\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^3}-\frac{2\left(77\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+149\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+123\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+35\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4a^3}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(51*(d*x + c)/a^3 - 32*tan(1/2*d*x + 1/2*c)/a^3 - 2*(77*tan(1/2*d*x + 1
/2*c)^7 + 149*tan(1/2*d*x + 1/2*c)^5 + 123*tan(1/2*d*x + 1/2*c)^3 + 35*tan(
1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*a^3))/d
```

Mupad [B]
time = 2.66, size = 98, normalized size = 0.91

$$\frac{51\,x}{8\,a^3}-\frac{4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^3\,d}-\frac{\frac{77\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\frac{149\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}+\frac{123\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}+\frac{35\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{a^3\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(a + a/cos(c + d*x))^3,x)
```

```
[Out] (51*x)/(8*a^3) - (4*tan(c/2 + (d*x)/2))/(a^3*d) - ((35*tan(c/2 + (d*x)/2))/
4 + (123*tan(c/2 + (d*x)/2)^3)/4 + (149*tan(c/2 + (d*x)/2)^5)/4 + (77*tan(c
/2 + (d*x)/2)^7)/4)/(a^3*d*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

3.103 $\int \frac{\sin^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=97

$$-\frac{11x}{2a^3}+\frac{3\sin(c+dx)}{a^3d}-\frac{\cos(c+dx)\sin(c+dx)}{2a^3d}-\frac{2\sin(c+dx)}{3a^3d(1+\cos(c+dx))^2}+\frac{19\sin(c+dx)}{3a^3d(1+\cos(c+dx))}$$

```
[Out] -11/2*x/a^3+3*sin(d*x+c)/a^3/d-1/2*cos(d*x+c)*sin(d*x+c)/a^3/d-2/3*sin(d*x+
c)/a^3/d/(1+cos(d*x+c))^2+19/3*sin(d*x+c)/a^3/d/(1+cos(d*x+c))
```

Rubi [A]
time = 0.22, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3957, 2953, 3045, 2717, 2715, 8, 2729, 2727}

$$\frac{3\sin(c+dx)}{a^3d}-\frac{\sin(c+dx)\cos(c+dx)}{2a^3d}+\frac{19\sin(c+dx)}{3a^3d(\cos(c+dx)+1)}-\frac{2\sin(c+dx)}{3a^3d(\cos(c+dx)+1)^2}-\frac{11x}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-11*x)/(2*a^3) + (3*Sin[c + d*x])/(a^3*d) - (Cos[c + d*x]*Sin[c + d*x])/(2
*a^3*d) - (2*Sin[c + d*x])/(3*a^3*d*(1 + Cos[c + d*x])^2) + (19*Sin[c + d*x
])/(3*a^3*d*(1 + Cos[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2953

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 3045

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= -\int \frac{\cos^3(c+dx)\sin^2(c+dx)}{(-a-a\cos(c+dx))^3}\,dx \\
&= -\frac{\int \frac{\cos^3(c+dx)(-a+a\cos(c+dx))}{(-a-a\cos(c+dx))^2}\,dx}{a^2} \\
&= -\frac{\int \left(\frac{5}{a}-\frac{3\cos(c+dx)}{a}+\frac{\cos^2(c+dx)}{a}+\frac{2}{a(1+\cos(c+dx))^2}-\frac{7}{a(1+\cos(c+dx))}\right)dx}{a^2} \\
&= -\frac{5x}{a^3}-\frac{\int \cos^2(c+dx)\,dx}{a^3}-\frac{2\int \frac{1}{(1+\cos(c+dx))^2}\,dx}{a^3}+\frac{3\int \cos(c+dx)\,dx}{a^3}+\frac{7\int \frac{1}{1+c}}{} \\
&= -\frac{5x}{a^3}+\frac{3\sin(c+dx)}{a^3d}-\frac{\cos(c+dx)\sin(c+dx)}{2a^3d}-\frac{2\sin(c+dx)}{3a^3d(1+\cos(c+dx))^2}+\frac{}{a^3} \\
&= -\frac{11x}{2a^3}+\frac{3\sin(c+dx)}{a^3d}-\frac{\cos(c+dx)\sin(c+dx)}{2a^3d}-\frac{2\sin(c+dx)}{3a^3d(1+\cos(c+dx))^2}+\frac{}{3}
\end{aligned}
$$

Mathematica [A]
time = 0.29, size = 177, normalized size = 1.82

$$-\frac{\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(1980dx\cos\left(\frac{dx}{2}\right)+1980dx\cos\left(c+\frac{dx}{2}\right)+660dx\cos\left(c+\frac{3dx}{2}\right)+660dx\cos\left(2c+\frac{3dx}{2}\right)-3216\sin\left(\frac{dx}{2}\right)+1326\sin\left(c+\frac{dx}{2}\right)-2012\sin\left(c+\frac{3dx}{2}\right)-498\sin\left(2c+\frac{3dx}{2}\right)-135\sin\left(2c+\frac{5dx}{2}\right)-135\sin\left(3c+\frac{5dx}{2}\right)+15\sin\left(3c+\frac{7dx}{2}\right)+15\sin\left(4c+\frac{7dx}{2}\right)\right)}{960a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/960*(Sec[c/2]*Sec[(c + d*x)/2]^3*(1980*d*x*Cos[(d*x)/2] + 1980*d*x*Cos[c
 + (d*x)/2] + 660*d*x*Cos[c + (3*d*x)/2] + 660*d*x*Cos[2*c + (3*d*x)/2] - 3
216*Sin[(d*x)/2] + 1326*Sin[c + (d*x)/2] - 2012*Sin[c + (3*d*x)/2] - 498*Si
n[2*c + (3*d*x)/2] - 135*Sin[2*c + (5*d*x)/2] - 135*Sin[3*c + (5*d*x)/2] +
15*Sin[3*c + (7*d*x)/2] + 15*Sin[4*c + (7*d*x)/2]))/(a^3*d)
```

Maple [A]
time = 0.12, size = 87, normalized size = 0.90

method	result
derivativedivides	$\frac{-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{2\left(-\frac{7\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-11\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^3}$
default	$\frac{-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{2\left(-\frac{7\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-11\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^3}$
risch	$-\frac{11x}{2a^3}+\frac{i\mathrm{e}^{2i(dx+c)}}{8a^3d}-\frac{3i\mathrm{e}^{i(dx+c)}}{2a^3d}+\frac{3i\mathrm{e}^{-i(dx+c)}}{2a^3d}-\frac{i\mathrm{e}^{-2i(dx+c)}}{8a^3d}+\frac{2i\left(21\,\mathrm{e}^{2i(dx+c)}+36\,\mathrm{e}^{i(dx+c)}+19\right)}{3a^3d\left(\mathrm{e}^{i(dx+c)}+1\right)^3}$
norman	$\frac{-\frac{11x}{2a}+\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{56\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{16\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{3ad}-\frac{11x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{11x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(-1/3*tan(1/2*d*x+1/2*c)^3+6*tan(1/2*d*x+1/2*c)-2*(-7/2*tan(1/2*d*x
+1/2*c)^3-5/2*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^2-11*arctan(tan(
1/2*d*x+1/2*c)))
```

Maxima [A]
time = 0.48, size = 164, normalized size = 1.69

$$\frac{\frac{3\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}+\frac{7\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^3+\frac{2a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{18\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^3}-\frac{33\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/3*(3*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 7*sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3)/(a^3 + 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*x + c)
^4/(cos(d*x + c) + 1)^4) + (18*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x +
c)^3/(cos(d*x + c) + 1)^3)/a^3 - 33*arctan(sin(d*x + c)/(cos(d*x + c) + 1))
/a^3)/d
```

Fricas [A]
time = 2.78, size = 99, normalized size = 1.02

$$-\frac{33\,dx\cos\left(dx+c\right)^{2}+66\,dx\cos\left(dx+c\right)+33\,dx+\left(3\,\cos\left(dx+c\right)^{3}-12\,\cos\left(dx+c\right)^{2}-71\,\cos\left(dx+c\right)-52\right)\sin\left(dx+c\right)}{6\,\left(a^{3}d\cos\left(dx+c\right)^{2}+2\,a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/6*(33*d*x*cos(d*x + c)^2 + 66*d*x*cos(d*x + c) + 33*d*x + (3*cos(d*x + c
)^3 - 12*cos(d*x + c)^2 - 71*cos(d*x + c) - 52)*sin(d*x + c))/(a^3*d*cos(d*
x + c)^2 + 2*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sin^{2}\left(c+dx\right)}{\sec^{3}\left(c+dx\right)+3\sec^{2}\left(c+dx\right)+3\sec\left(c+dx\right)+1}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(sin(c + d*x)**2/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Giac [A]
time = 0.50, size = 96, normalized size = 0.99

$$-\frac{\frac{33\,\left(dx+c\right)}{a^{3}}-\frac{6\,\left(7\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{2}a^{3}}+\frac{2\,\left(a^{6}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-18\,a^{6}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^{9}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/6*(33*(d*x + c)/a^3 - 6*(7*tan(1/2*d*x + 1/2*c)^3 + 5*tan(1/2*d*x + 1/2*
c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^3) + 2*(a^6*tan(1/2*d*x + 1/2*c)^3 -
18*a^6*tan(1/2*d*x + 1/2*c))/a^9)/d
```

Mupad [B]
time = 1.06, size = 115, normalized size = 1.19

$$-\frac{2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-38\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-42\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+12\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+33\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(c+d\,x)}{6\,a^3\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(a + a/cos(c + d*x))^3,x)
```

```
[Out] -(2*sin(c/2 + (d*x)/2) - 38*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) - 42*co
s(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) + 12*cos(c/2 + (d*x)/2)^6*sin(c/2 + (
d*x)/2) + 33*cos(c/2 + (d*x)/2)^3*(c + d*x))/(6*a^3*d*cos(c/2 + (d*x)/2)^3)
```

3.104 $\int \frac{\csc^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=89

$$\frac{3\cot^5(c+dx)}{5a^3d}+\frac{4\cot^7(c+dx)}{7a^3d}-\frac{\csc^3(c+dx)}{a^3d}+\frac{7\csc^5(c+dx)}{5a^3d}-\frac{4\csc^7(c+dx)}{7a^3d}$$

```
[Out] 3/5*cot(d*x+c)^5/a^3/d+4/7*cot(d*x+c)^7/a^3/d-csc(d*x+c)^3/a^3/d+7/5*csc(d*
x+c)^5/a^3/d-4/7*csc(d*x+c)^7/a^3/d
```

Rubi [A]
time = 0.26, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3957, 2954, 2952, 2687, 30, 2686, 276, 14}

$$\frac{4\cot^7(c+dx)}{7a^3d}+\frac{3\cot^5(c+dx)}{5a^3d}-\frac{4\csc^7(c+dx)}{7a^3d}+\frac{7\csc^5(c+dx)}{5a^3d}-\frac{\csc^3(c+dx)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (3*Cot[c + d*x]^5)/(5*a^3*d) + (4*Cot[c + d*x]^7)/(7*a^3*d) - Csc[c + d*x]^
3/(a^3*d) + (7*Csc[c + d*x]^5)/(5*a^3*d) - (4*Csc[c + d*x]^7)/(7*a^3*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
&& !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= -\int \frac{\cos(c+dx)\cot^2(c+dx)}{(-a-a\cos(c+dx))^3}\,dx \\
&= -\frac{\int (-a+a\cos(c+dx))^3\cot^3(c+dx)\csc^5(c+dx)\,dx}{a^6} \\
&= \frac{\int \left(-a^3\cot^6(c+dx)\csc^2(c+dx)+3a^3\cot^5(c+dx)\csc^3(c+dx)-3a^3\cot^4(c+\right.}{a^6} \\
&= -\frac{\int \cot^6(c+dx)\csc^2(c+dx)\,dx}{a^3}+\frac{\int \cot^3(c+dx)\csc^5(c+dx)\,dx}{a^3}+\frac{3\int \cot^5(c}{} \\
&= -\frac{\operatorname{Subst}\left(\int x^6\,dx,x,-\cot(c+dx)\right)}{a^3d}-\frac{\operatorname{Subst}\left(\int x^4(-1+x^2)\,dx,x,\csc(c+dx)\right)}{a^3d} \\
&= \frac{\cot^7(c+dx)}{7a^3d}-\frac{\operatorname{Subst}\left(\int (-x^4+x^6)\,dx,x,\csc(c+dx)\right)}{a^3d}-\frac{3\operatorname{Subst}\left(\int (x^2-2x^4\right.}{} \\
&= \frac{3\cot^5(c+dx)}{5a^3d}+\frac{4\cot^7(c+dx)}{7a^3d}-\frac{\csc^3(c+dx)}{a^3d}+\frac{7\csc^5(c+dx)}{5a^3d}-\frac{4\csc^7(c+d}{7a^3d}
\end{aligned}
$$

Mathematica [A]

time = 0.40, size = 137, normalized size = 1.54

$$
\frac{\csc(c)\csc(c+dx)\sec^3(c+dx)(-840\sin(c)+448\sin(dx)+602\sin(c+dx)+602\sin(2(c+dx))+258\sin(3(c+dx))+43\sin(4(c+dx))-560\sin(2c+dx)+168\sin(c+2dx)-280\sin(3c+2dx)-48\sin(2c+3dx)-8\sin(3c+4dx))}{2240a^3d(1+\sec(c+dx))^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Csc[c]*Csc[c + d*x]*Sec[c + d*x]^3*(-840*Sin[c] + 448*Sin[d*x] + 602*Sin[c
 + d*x] + 602*Sin[2*(c + d*x)] + 258*Sin[3*(c + d*x)] + 43*Sin[4*(c + d*x)]
 - 560*Sin[2*c + d*x] + 168*Sin[c + 2*d*x] - 280*Sin[3*c + 2*d*x] - 48*Sin[
2*c + 3*d*x] - 8*Sin[3*c + 4*d*x]))/(2240*a^3*d*(1 + Sec[c + d*x])^3)
```

Maple [A]

time = 0.10, size = 60, normalized size = 0.67

method	result	size
derivativedivides	$\frac{-\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{16d\,a^3}$	60
default	$\frac{-\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{16d\,a^3}$	60
norman	$\frac{-\frac{1}{16ad}-\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)}{40ad}-\frac{\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)}{112ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a^2}$	82

risch	$-\frac{2i\left(35\,\mathrm{e}^{6i(dx+c)}+70\,\mathrm{e}^{5i(dx+c)}+105\,\mathrm{e}^{4i(dx+c)}+56\,\mathrm{e}^{3i(dx+c)}+21\,\mathrm{e}^{2i(dx+c)}-6\,\mathrm{e}^{i(dx+c)}-1\right)}{35a^3d\left(\mathrm{e}^{i(dx+c)}+1\right)^7\left(\mathrm{e}^{i(dx+c)}-1\right)}$	104

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/d/a^3*(-1/7*tan(1/2*d*x+1/2*c)^7+2/5*tan(1/2*d*x+1/2*c)^5-2*tan(1/2*d*
x+1/2*c)-1/tan(1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.28, size = 90, normalized size = 1.01

$$-\frac{\frac{\frac{70\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{14\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^3}+\frac{35\,(\cos(dx+c)+1)}{a^3\sin(dx+c)}}{560\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/560*((70*sin(d*x + c)/(cos(d*x + c) + 1) - 14*sin(d*x + c)^5/(cos(d*x +
c) + 1)^5 + 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^3 + 35*(cos(d*x + c) +
 1)/(a^3*sin(d*x + c)))/d
```

Fricas [A]
time = 3.69, size = 95, normalized size = 1.07

$$\frac{\cos\left(dx+c\right)^4+3\,\cos\left(dx+c\right)^3-15\,\cos\left(dx+c\right)^2-18\,\cos\left(dx+c\right)-6}{35\,\left(a^3d\cos\left(dx+c\right)^3+3\,a^3d\cos\left(dx+c\right)^2+3\,a^3d\cos\left(dx+c\right)+a^3d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/35*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 - 15*cos(d*x + c)^2 - 18*cos(d*x +
c) - 6)/((a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x +
 c) + a^3*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\csc^2\left(c+dx\right)}{\sec^3\left(c+dx\right)+3\sec^2\left(c+dx\right)+3\sec\left(c+dx\right)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(csc(c + d*x)**2/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Giac [A]
time = 0.53, size = 73, normalized size = 0.82

$$-\frac{\frac{35}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\frac{5\,a^{18}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-14\,a^{18}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+70\,a^{18}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{21}}}{560\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/560*(35/(a^3*tan(1/2*d*x + 1/2*c)) + (5*a^18*tan(1/2*d*x + 1/2*c)^7 - 14
*a^18*tan(1/2*d*x + 1/2*c)^5 + 70*a^18*tan(1/2*d*x + 1/2*c))/a^21)/d
```

Mupad [B]
time = 1.01, size = 84, normalized size = 0.94

$$-\frac{-16\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+8\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+72\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-34\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+5}{560\,a^3\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^2*(a + a/cos(c + d*x))^3),x)
```

```
[Out] -(72*cos(c/2 + (d*x)/2)^4 - 34*cos(c/2 + (d*x)/2)^2 + 8*cos(c/2 + (d*x)/2)^
6 - 16*cos(c/2 + (d*x)/2)^8 + 5)/(560*a^3*d*cos(c/2 + (d*x)/2)^7*sin(c/2 +
(d*x)/2))
```

3.105 $\int \frac{\csc^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=103

$$\frac{3\cot^5(c+dx)}{5a^3d}+\frac{\cot^7(c+dx)}{a^3d}+\frac{4\cot^9(c+dx)}{9a^3d}-\frac{3\csc^5(c+dx)}{5a^3d}+\frac{\csc^7(c+dx)}{a^3d}-\frac{4\csc^9(c+dx)}{9a^3d}$$

```
[Out] 3/5*cot(d*x+c)^5/a^3/d+cot(d*x+c)^7/a^3/d+4/9*cot(d*x+c)^9/a^3/d-3/5*csc(d*
x+c)^5/a^3/d+csc(d*x+c)^7/a^3/d-4/9*csc(d*x+c)^9/a^3/d
```

Rubi [A]
time = 0.26, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3957, 2954, 2952, 2687, 14, 2686, 276}

$$\frac{4\cot^9(c+dx)}{9a^3d}+\frac{\cot^7(c+dx)}{a^3d}+\frac{3\cot^5(c+dx)}{5a^3d}-\frac{4\csc^9(c+dx)}{9a^3d}+\frac{\csc^7(c+dx)}{a^3d}-\frac{3\csc^5(c+dx)}{5a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (3*Cot[c + d*x]^5)/(5*a^3*d) + Cot[c + d*x]^7/(a^3*d) + (4*Cot[c + d*x]^9)/
(9*a^3*d) - (3*Csc[c + d*x]^5)/(5*a^3*d) + Csc[c + d*x]^7/(a^3*d) - (4*Csc[
c + d*x]^9)/(9*a^3*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= -\int \frac{\cot^3(c+dx)\csc(c+dx)}{(-a-a\cos(c+dx))^3}\,dx \\
&= -\frac{\int (-a+a\cos(c+dx))^3\cot^3(c+dx)\csc^7(c+dx)\,dx}{a^6} \\
&= \frac{\int \left(-a^3\cot^6(c+dx)\csc^4(c+dx)+3a^3\cot^5(c+dx)\csc^5(c+dx)-3a^3\cot^4(c+\right.}{a^6} \\
&= -\frac{\int \cot^6(c+dx)\csc^4(c+dx)\,dx}{a^3}+\frac{\int \cot^3(c+dx)\csc^7(c+dx)\,dx}{a^3}+\frac{3\int \cot^5(c\cdot}{} \\
&= -\frac{\mathrm{Subst}\left(\int x^6(-1+x^2)\,dx,x,\csc(c+dx)\right)}{a^3d}-\frac{\mathrm{Subst}\left(\int x^6(1+x^2)\,dx,x,-\cot(c\right.}{a^3d} \\
&= -\frac{\mathrm{Subst}\left(\int (-x^6+x^8)\,dx,x,\csc(c+dx)\right)}{a^3d}-\frac{\mathrm{Subst}\left(\int (x^6+x^8)\,dx,x,-\cot(c+\right.}{a^3d} \\
&= \frac{3\cot^5(c+dx)}{5a^3d}+\frac{\cot^7(c+dx)}{a^3d}+\frac{4\cot^9(c+dx)}{9a^3d}-\frac{3\csc^5(c+dx)}{5a^3d}+\frac{\csc^7(c+dx}{a^3d}
\end{aligned}$$

Mathematica [A]
time = 0.49, size = 175, normalized size = 1.70

$$-\frac{\csc(c)\csc^3(2(c+dx))(5376\sin(c)-1152\sin(dx)-1764\sin(c+dx)-1323\sin(2(c+dx))+98\sin(3(c+dx))+588\sin(4(c+dx))+294\sin(5(c+dx))+49\sin(6(c+dx))+3456\sin(2c+dx)-1152\sin(c+2dx)+2880\sin(3c+2dx)-128\sin(2c+3dx)-768\sin(3c+4dx)-384\sin(4c+5dx)-64\sin(5c+6dx))}{5760a^3d(1+\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/5760*(Csc[c]*Csc[2*(c + d*x)]^3*(5376*Sin[c] - 1152*Sin[d*x] - 1764*Sin[
c + d*x] - 1323*Sin[2*(c + d*x)] + 98*Sin[3*(c + d*x)] + 588*Sin[4*(c + d*x
)] + 294*Sin[5*(c + d*x)] + 49*Sin[6*(c + d*x)] + 3456*Sin[2*c + d*x] - 115
2*Sin[c + 2*d*x] + 2880*Sin[3*c + 2*d*x] - 128*Sin[2*c + 3*d*x] - 768*Sin[3
*c + 4*d*x] - 384*Sin[4*c + 5*d*x] - 64*Sin[5*c + 6*d*x]))/(a^3*d*(1 + Sec[
c + d*x])^3)
```

Maple [A]
time = 0.12, size = 60, normalized size = 0.58

method	result	si
derivativedivides	$\frac{-\frac{\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}+\frac{3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}}{64d\,a^3}$	60
default	$\frac{-\frac{\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}+\frac{3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}}{64d\,a^3}$	60
norman	$\frac{-\frac{1}{192ad}-\frac{3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64ad}+\frac{3\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{320ad}-\frac{\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)}{576ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3a^2}$	82
risch	$\frac{4i\left(45\,e^{8i(dx+c)}+54\,e^{7i(dx+c)}+84\,e^{6i(dx+c)}+18\,e^{5i(dx+c)}+18\,e^{4i(dx+c)}+2\,e^{3i(dx+c)}+12\,e^{2i(dx+c)}+6\,e^{i(dx+c)}+1\right)}{45a^3d\left(e^{i(dx+c)}+1\right)^9\left(e^{i(dx+c)}-1\right)^3}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/64/d/a^3*(-1/9*tan(1/2*d*x+1/2*c)^9+3/5*tan(1/2*d*x+1/2*c)^5-3*tan(1/2*d*
x+1/2*c)-1/3/tan(1/2*d*x+1/2*c)^3)
```

Maxima [A]
time = 0.28, size = 92, normalized size = 0.89

$$-\frac{\frac{\frac{135\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{27\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5\,\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^3}+\frac{15\,(\cos(dx+c)+1)^3}{a^3\sin(dx+c)^3}}{2880\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2880*((135*sin(d*x + c)/(cos(d*x + c) + 1) - 27*sin(d*x + c)^5/(cos(d*x
+ c) + 1)^5 + 5*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/a^3 + 15*(cos(d*x + c)
 + 1)^3/(a^3*sin(d*x + c)^3))/d
```

Fricas [A]
time = 3.14, size = 146, normalized size = 1.42

$$\frac{2\cos(dx+c)^6+6\cos(dx+c)^5+3\cos(dx+c)^4-7\cos(dx+c)^3+3\cos(dx+c)^2+6\cos(dx+c)+2}{45\left(a^3d\cos(dx+c)^5+3a^3d\cos(dx+c)^4+2a^3d\cos(dx+c)^3-2a^3d\cos(dx+c)^2-3a^3d\cos(dx+c)-a^3d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/45*(2*cos(d*x + c)^6 + 6*cos(d*x + c)^5 + 3*cos(d*x + c)^4 - 7*cos(d*x +
c)^3 + 3*cos(d*x + c)^2 + 6*cos(d*x + c) + 2)/((a^3*d*cos(d*x + c)^5 + 3*a^
3*d*cos(d*x + c)^4 + 2*a^3*d*cos(d*x + c)^3 - 2*a^3*d*cos(d*x + c)^2 - 3*a^
3*d*cos(d*x + c) - a^3*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^4(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(csc(c + d*x)**4/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Giac [A]
time = 0.53, size = 73, normalized size = 0.71

$$-\frac{\frac{15}{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3}+\frac{5a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9-27a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+135a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{27}}}{2880\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2880*(15/(a^3*tan(1/2*d*x + 1/2*c)^3) + (5*a^24*tan(1/2*d*x + 1/2*c)^9 -
 27*a^24*tan(1/2*d*x + 1/2*c)^5 + 135*a^24*tan(1/2*d*x + 1/2*c))/a^27)/d
```

Mupad [B]
time = 1.11, size = 105, normalized size = 1.02

$$-\frac{15\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+135\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^4-27\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^8+5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}}{2880\,a^3\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^9\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^4*(a + a/cos(c + d*x))^3),x)
```

```
[Out] -(15*cos(c/2 + (d*x)/2)^12 + 5*sin(c/2 + (d*x)/2)^12 - 27*cos(c/2 + (d*x)/2
)^4*sin(c/2 + (d*x)/2)^8 + 135*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^4)/(
2880*a^3*d*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^3)
```

3.106 $\int \frac{\csc^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=127

$$\frac{3\cot^5(c+dx)}{5a^3d}+\frac{10\cot^7(c+dx)}{7a^3d}+\frac{11\cot^9(c+dx)}{9a^3d}+\frac{4\cot^{11}(c+dx)}{11a^3d}-\frac{3\csc^7(c+dx)}{7a^3d}+\frac{7\csc^9(c+dx)}{9a^3d}-\frac{4\csc^{11}(c+dx)}{11a^3d}$$

```
[Out] 3/5*cot(d*x+c)^5/a^3/d+10/7*cot(d*x+c)^7/a^3/d+11/9*cot(d*x+c)^9/a^3/d+4/11
*cot(d*x+c)^11/a^3/d-3/7*csc(d*x+c)^7/a^3/d+7/9*csc(d*x+c)^9/a^3/d-4/11*csc
(d*x+c)^11/a^3/d
```

Rubi [A]
time = 0.29, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3957, 2954, 2952, 2687, 276, 2686, 14}

$$\frac{4\cot^{11}(c+dx)}{11a^3d}+\frac{11\cot^9(c+dx)}{9a^3d}+\frac{10\cot^7(c+dx)}{7a^3d}+\frac{3\cot^5(c+dx)}{5a^3d}-\frac{4\csc^{11}(c+dx)}{11a^3d}+\frac{7\csc^9(c+dx)}{9a^3d}-\frac{3\csc^7(c+dx)}{7a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (3*Cot[c + d*x]^5)/(5*a^3*d) + (10*Cot[c + d*x]^7)/(7*a^3*d) + (11*Cot[c +
d*x]^9)/(9*a^3*d) + (4*Cot[c + d*x]^11)/(11*a^3*d) - (3*Csc[c + d*x]^7)/(7*
a^3*d) + (7*Csc[c + d*x]^9)/(9*a^3*d) - (4*Csc[c + d*x]^11)/(11*a^3*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= -\int \frac{\cot^3(c+dx)\csc^3(c+dx)}{(-a-a\cos(c+dx))^3}\,dx \\
&= -\frac{\int (-a+a\cos(c+dx))^3\cot^3(c+dx)\csc^9(c+dx)\,dx}{a^6} \\
&= \frac{\int \left(-a^3\cot^6(c+dx)\csc^6(c+dx)+3a^3\cot^5(c+dx)\csc^7(c+dx)-3a^3\cot^4(c\right.}{a^6} \\
&= -\frac{\int \cot^6(c+dx)\csc^6(c+dx)\,dx}{a^3}+\frac{\int \cot^3(c+dx)\csc^9(c+dx)\,dx}{a^3}+\frac{3\int \cot^5(}{} \\
&= -\frac{\text{Subst}\left(\int x^8(-1+x^2)\,dx,x,\csc(c+dx)\right)}{a^3d}-\frac{\text{Subst}\left(\int x^6(1+x^2)^2\,dx,x,-\text{co}\right.}{a^3d} \\
&= -\frac{\text{Subst}\left(\int (-x^8+x^{10})\,dx,x,\csc(c+dx)\right)}{a^3d}-\frac{\text{Subst}\left(\int (x^6+2x^8+x^{10})\,dx,x,\right.}{a^3d} \\
&= \frac{3\cot^5(c+dx)}{5a^3d}+\frac{10\cot^7(c+dx)}{7a^3d}+\frac{11\cot^9(c+dx)}{9a^3d}+\frac{4\cot^{11}(c+dx)}{11a^3d}-\frac{3\csc^7}{}
\end{aligned}$$

Mathematica [A]
time = 0.81, size = 223, normalized size = 1.76

$$\frac{\csc(c)\csc^5(c+dx)\sec^3(c+dx)(-3886080\sin(c)+563200\sin(dx)+524150\sin(c+dx)+314490\sin(2(c+dx))-162010\sin(3(c+dx))-238250\sin(4(c+dx))-47650\sin(5(c+dx))+47650\sin(6(c+dx))+28590\sin(7(c+dx))+4765\sin(8(c+dx))-2027520\sin(2c+dx)+1486848\sin(c+2dx)-2365440\sin(3c+2dx)+452608\sin(2c+3dx)+665600\sin(3c+4dx)+133120\sin(4c+5dx)-133120\sin(5c+6dx)-79872\sin(6c+7dx)-13312\sin(7c+8dx))}{56770560a^3d(1+\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Csc[c]*Csc[c + d*x]^5*Sec[c + d*x]^3*(-3886080*Sin[c] + 563200*Sin[d*x] +
524150*Sin[c + d*x] + 314490*Sin[2*(c + d*x)] - 162010*Sin[3*(c + d*x)] - 2
38250*Sin[4*(c + d*x)] - 47650*Sin[5*(c + d*x)] + 47650*Sin[6*(c + d*x)] +
28590*Sin[7*(c + d*x)] + 4765*Sin[8*(c + d*x)] - 2027520*Sin[2*c + d*x] + 1
486848*Sin[c + 2*d*x] - 2365440*Sin[3*c + 2*d*x] + 452608*Sin[2*c + 3*d*x]
+ 665600*Sin[3*c + 4*d*x] + 133120*Sin[4*c + 5*d*x] - 133120*Sin[5*c + 6*d*
x] - 79872*Sin[6*c + 7*d*x] - 13312*Sin[7*c + 8*d*x]))/(56770560*a^3*d*(1 +
 Sec[c + d*x])^3)
```

Maple [A]
time = 0.13, size = 112, normalized size = 0.88

method	result
derivativedivides	$\frac{-\frac{\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{11}-\frac{2\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}+\frac{2\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{6\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}-}{256d\,a^{3}}$
default	$\frac{-\frac{\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{11}-\frac{2\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}+\frac{2\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{6\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}-}{256d\,a^{3}}$
risch	$-\frac{16i\left(2310\,e^{10i(dx+c)}+1980\,e^{9i(dx+c)}+3795\,e^{8i(dx+c)}+550\,e^{7i(dx+c)}+1452\,e^{6i(dx+c)}+442\,e^{5i(dx+c)}+650\,e^{4i(dx+c)}+130\right.}{3465a^{3}d\left(e^{i(dx+c)}+1\right)^{11}\left(e^{i(dx+c)}-1\right)^{5}}$
norman	$\frac{-\frac{1}{1280ad}-\frac{\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}{384ad}+\frac{\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)}{128ad}-\frac{3\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128ad}+\frac{3\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{640ad}+\frac{\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)}{896ad}-\frac{\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)}{1152ad}-\frac{\tan^{16}\left(\frac{dx}{2}+\right.}{2816ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}a^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/256/d/a^3*(-1/11*tan(1/2*d*x+1/2*c)^11-2/9*tan(1/2*d*x+1/2*c)^9+2/7*tan(1
/2*d*x+1/2*c)^7+6/5*tan(1/2*d*x+1/2*c)^5-6*tan(1/2*d*x+1/2*c)+2/tan(1/2*d*x
+1/2*c)-2/3/tan(1/2*d*x+1/2*c)^3-1/5/tan(1/2*d*x+1/2*c)^5)
```

Maxima [A]
time = 0.27, size = 174, normalized size = 1.37

$$-\frac{\frac{\frac{20790\sin(dx+c)}{\cos(dx+c)+1}-\frac{4158\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{990\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{770\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{315\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}}{a^3}+\frac{231\left(\frac{10\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{30\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+3\right)(\cos(dx+c)+1)^5}{a^3\sin(dx+c)^5}}{887040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/887040*((20790*sin(d*x + c)/(cos(d*x + c) + 1) - 4158*sin(d*x + c)^5/(co
s(d*x + c) + 1)^5 - 990*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 770*sin(d*x +
 c)^9/(cos(d*x + c) + 1)^9 + 315*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)/a^3
 + 231*(10*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 30*sin(d*x + c)^4/(cos(d*x
 + c) + 1)^4 + 3)*(cos(d*x + c) + 1)^5/(a^3*sin(d*x + c)^5))/d
```

Fricas [**A**]
time = 3.45, size = 191, normalized size = 1.50

$$\frac{104 \cos(dx+c)^8 + 312 \cos(dx+c)^7 + 52 \cos(dx+c)^6 - 676 \cos(dx+c)^5 - 585 \cos(dx+c)^4 + 325 \cos(dx+c)^3 - 25 \cos(dx+c)^2 - 150 \cos(dx+c) - 50}{3465 \left(a^3 d \cos(dx+c)^7 + 3 a^3 d \cos(dx+c)^6 + a^3 d \cos(dx+c)^5 - 5 a^3 d \cos(dx+c)^4 - 5 a^3 d \cos(dx+c)^3 + a^3 d \cos(dx+c)^2 + 3 a^3 d \cos(dx+c) + a^3 d\right) \sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/3465*(104*cos(d*x + c)^8 + 312*cos(d*x + c)^7 + 52*cos(d*x + c)^6 - 676*c
os(d*x + c)^5 - 585*cos(d*x + c)^4 + 325*cos(d*x + c)^3 - 25*cos(d*x + c)^2
 - 150*cos(d*x + c) - 50)/((a^3*d*cos(d*x + c)^7 + 3*a^3*d*cos(d*x + c)^6 +
 a^3*d*cos(d*x + c)^5 - 5*a^3*d*cos(d*x + c)^4 - 5*a^3*d*cos(d*x + c)^3 + a
^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)*sin(d*x + c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^6(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\, dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(csc(c + d*x)**6/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Giac [**A**]
time = 0.58, size = 134, normalized size = 1.06

$$\frac{\frac{231 \left(30 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4 - 10 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 3\right)}{a^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5} - \frac{315 a^{30} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^{11} + 770 a^{30} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^9 - 990 a^{30} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^7 - 4158 a^{30} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 + 20790 a^{30} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{a^{33}}}{887040\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/887040*(231*(30*tan(1/2*d*x + 1/2*c)^4 - 10*tan(1/2*d*x + 1/2*c)^2 - 3)/(
a^3*tan(1/2*d*x + 1/2*c)^5) - (315*a^30*tan(1/2*d*x + 1/2*c)^11 + 770*a^30*
tan(1/2*d*x + 1/2*c)^9 - 990*a^30*tan(1/2*d*x + 1/2*c)^7 - 4158*a^30*tan(1/
2*d*x + 1/2*c)^5 + 20790*a^30*tan(1/2*d*x + 1/2*c))/a^33)/d
```

Mupad [B]
time = 1.43, size = 201, normalized size = 1.58

$$-\frac{693\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}+2310\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}-6930\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}+20790\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}-4158\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-990\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+770\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+315\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}}{887040\,a^{3}\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^6*(a + a/cos(c + d*x))^3),x)
```

```
[Out] -(693*cos(c/2 + (d*x)/2)^16 + 315*sin(c/2 + (d*x)/2)^16 + 770*cos(c/2 + (d*
x)/2)^2*sin(c/2 + (d*x)/2)^14 - 990*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)
^12 - 4158*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^10 + 20790*cos(c/2 + (d*
x)/2)^10*sin(c/2 + (d*x)/2)^6 - 6930*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/
2)^4 + 2310*cos(c/2 + (d*x)/2)^14*sin(c/2 + (d*x)/2)^2)/(887040*a^3*d*cos(c
/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2)^5)
```

3.107 $\int \frac{\csc^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=145

$$\frac{3\cot^5(c+dx)}{5a^3d}+\frac{13\cot^7(c+dx)}{7a^3d}+\frac{7\cot^9(c+dx)}{3a^3d}+\frac{15\cot^{11}(c+dx)}{11a^3d}+\frac{4\cot^{13}(c+dx)}{13a^3d}-\frac{\csc^9(c+dx)}{3a^3d}+\frac{7\csc^{11}(}{11}$$

```
[Out] 3/5*cot(d*x+c)^5/a^3/d+13/7*cot(d*x+c)^7/a^3/d+7/3*cot(d*x+c)^9/a^3/d+15/11
*cot(d*x+c)^11/a^3/d+4/13*cot(d*x+c)^13/a^3/d-1/3*csc(d*x+c)^9/a^3/d+7/11*c
sc(d*x+c)^11/a^3/d-4/13*csc(d*x+c)^13/a^3/d
```

Rubi [A]
time = 0.30, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3957, 2954, 2952, 2687, 276, 2686, 14}

$$\frac{4\cot^{13}(c+dx)}{13a^3d}+\frac{15\cot^{11}(c+dx)}{11a^3d}+\frac{7\cot^9(c+dx)}{3a^3d}+\frac{13\cot^7(c+dx)}{7a^3d}+\frac{3\cot^5(c+dx)}{5a^3d}-\frac{4\csc^{13}(c+dx)}{13a^3d}+\frac{7\csc^{11}(c+dx)}{11a^3d}-\frac{\csc^9(c+dx)}{3a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^8/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (3*Cot[c + d*x]^5)/(5*a^3*d) + (13*Cot[c + d*x]^7)/(7*a^3*d) + (7*Cot[c + d
*x]^9)/(3*a^3*d) + (15*Cot[c + d*x]^11)/(11*a^3*d) + (4*Cot[c + d*x]^13)/(1
3*a^3*d) - Csc[c + d*x]^9/(3*a^3*d) + (7*Csc[c + d*x]^11)/(11*a^3*d) - (4*C
sc[c + d*x]^13)/(13*a^3*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= -\int \frac{\cot^3(c+dx)\csc^5(c+dx)}{(-a-a\cos(c+dx))^3}\,dx \\
&= -\frac{\int (-a+a\cos(c+dx))^3\cot^3(c+dx)\csc^{11}(c+dx)\,dx}{a^6} \\
&= \frac{\int \left(-a^3\cot^6(c+dx)\csc^8(c+dx)+3a^3\cot^5(c+dx)\csc^9(c+dx)-3a^3\cot^4(c+\right.}{a^6} \\
&= -\frac{\int \cot^6(c+dx)\csc^8(c+dx)\,dx}{a^3}+\frac{\int \cot^3(c+dx)\csc^{11}(c+dx)\,dx}{a^3}+\frac{3\int \cot^5(c}{} \\
&= -\frac{\mathrm{Subst}\left(\int x^{10}\left(-1+x^2\right)\,dx,x,\csc(c+dx)\right)}{a^3d}-\frac{\mathrm{Subst}\left(\int x^6\left(1+x^2\right)^3\,dx,x,-\cot\right.}{a^3d} \\
&= -\frac{\mathrm{Subst}\left(\int \left(-x^{10}+x^{12}\right)\,dx,x,\csc(c+dx)\right)}{a^3d}-\frac{\mathrm{Subst}\left(\int \left(x^6+3x^8+3x^{10}+x^{12}\right)\right.}{a^3d} \\
&= \frac{3\cot^5(c+dx)}{5a^3d}+\frac{13\cot^7(c+dx)}{7a^3d}+\frac{7\cot^9(c+dx)}{3a^3d}+\frac{15\cot^{11}(c+dx)}{11a^3d}+\frac{4\cot^{13}}{13}
\end{aligned}$$

Mathematica [A]
time = 1.15, size = 265, normalized size = 1.83

$$-\frac{\csc(c)\csc^7(c+dx)\sec^3(c+dx)(49201152\sin(c)-6336512\sin(dx)-2764580\sin(c+dx)-1382290\sin(2(c+dx))+1275960\sin(3(c+dx))+1336720\sin(4(c+dx))-60760\sin(5(c+dx))-524055\sin(6(c+dx))-167090\sin(7(c+dx))+60760\sin(8(c+dx))+45570\sin(9(c+dx))+7595\sin(10(c+dx))+20500480\sin(2c+dx)-23668736\sin(c+2dx)+30750720\sin(3c+2dx)-6537216\sin(2c+3dx)-6848512\sin(3c+4dx)+311296\sin(4c+5dx)+2684928\sin(5c+6dx)+856064\sin(6c+7dx)-311296\sin(7c+8dx)-233472\sin(8c+9dx)-38912\sin(9c+10dx))}{984023040a^3d(1+\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^8/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/984023040*(Csc[c]*Csc[c + d*x]^7*Sec[c + d*x]^3*(49201152*Sin[c] - 63365
12*Sin[d*x] - 2764580*Sin[c + d*x] - 1382290*Sin[2*(c + d*x)] + 1275960*Sin
[3*(c + d*x)] + 1336720*Sin[4*(c + d*x)] - 60760*Sin[5*(c + d*x)] - 524055*
Sin[6*(c + d*x)] - 167090*Sin[7*(c + d*x)] + 60760*Sin[8*(c + d*x)] + 45570
*Sin[9*(c + d*x)] + 7595*Sin[10*(c + d*x)] + 20500480*Sin[2*c + d*x] - 2366
8736*Sin[c + 2*d*x] + 30750720*Sin[3*c + 2*d*x] - 6537216*Sin[2*c + 3*d*x]
- 6848512*Sin[3*c + 4*d*x] + 311296*Sin[4*c + 5*d*x] + 2684928*Sin[5*c + 6*
d*x] + 856064*Sin[6*c + 7*d*x] - 311296*Sin[7*c + 8*d*x] - 233472*Sin[8*c +
 9*d*x] - 38912*Sin[9*c + 10*d*x]))/(a^3*d*(1 + Sec[c + d*x])^3)
```

Maple [A]
time = 0.14, size = 138, normalized size = 0.95

method	result
derivativedivides	$\frac{-\frac{\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{13}-\frac{4\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{11}-\frac{\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{8\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{14\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-14\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{7\tan\left(\frac{d}{2}\right.}}{1024d\,a^3}$
default	$\frac{-\frac{\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{13}-\frac{4\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{11}-\frac{\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{8\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{14\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-14\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{7\tan\left(\frac{d}{2}\right.}}{1024d\,a^3}$
risch	$\frac{32i\left(15015\,\mathrm{e}^{12i(dx+c)}+10010\,\mathrm{e}^{11i(dx+c)}+24024\,\mathrm{e}^{10i(dx+c)}+3094\,\mathrm{e}^{9i(dx+c)}+11557\,\mathrm{e}^{8i(dx+c)}+3192\,\mathrm{e}^{7i(dx+c)}+3344\,\mathrm{e}^{6i(d}\right.}{15015a^3d\left(\mathrm{e}^{i(dx+c)}+1\right)^{13}\left(\mathrm{e}^{i(dx+c)}-1\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^8/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/1024/d/a^3*(-1/13*tan(1/2*d*x+1/2*c)^13-4/11*tan(1/2*d*x+1/2*c)^11-1/3*ta
n(1/2*d*x+1/2*c)^9+8/7*tan(1/2*d*x+1/2*c)^7+14/5*tan(1/2*d*x+1/2*c)^5-14*ta
n(1/2*d*x+1/2*c)-1/7/tan(1/2*d*x+1/2*c)^7-4/5/tan(1/2*d*x+1/2*c)^5-1/tan(1/
2*d*x+1/2*c)^3+8/tan(1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.28, size = 214, normalized size = 1.48

$$-\frac{\frac{\frac{210210\sin(dx+c)}{\cos(dx+c)+1}-\frac{42042\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{17160\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{5005\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{5460\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}+\frac{1155\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}}}{a^3}+\frac{429\left(\frac{28\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{35\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{280\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+5\right)(\cos(dx+c)+1)^7}{a^3\sin(dx+c)^7}}{15375360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/15375360*((210210*sin(d*x + c)/(cos(d*x + c) + 1) - 42042*sin(d*x + c)^5
/(cos(d*x + c) + 1)^5 - 17160*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 5005*si
n(d*x + c)^9/(cos(d*x + c) + 1)^9 + 5460*sin(d*x + c)^11/(cos(d*x + c) + 1)
^11 + 1155*sin(d*x + c)^13/(cos(d*x + c) + 1)^13)/a^3 + 429*(28*sin(d*x + c
)^2/(cos(d*x + c) + 1)^2 + 35*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 280*sin
(d*x + c)^6/(cos(d*x + c) + 1)^6 + 5)*(cos(d*x + c) + 1)^7/(a^3*sin(d*x + c
)^7))/d
```

Fricas [**A**]
time = 3.88, size = 214, normalized size = 1.48

$$\frac{304 \cos\left(dx+c\right)^{10}+912 \cos\left(dx+c\right)^{9}-152 \cos\left(dx+c\right)^{8}-2888 \cos\left(dx+c\right)^{7}-1862 \cos\left(dx+c\right)^{6}+2926 \cos\left(dx+c\right)^{5}+3325 \cos\left(dx+c\right)^{4}-665 \cos\left(dx+c\right)^{3}-35 \cos\left(dx+c\right)^{2}+210 \cos\left(dx+c\right)+70}{15015 \left(a^{3}d\cos\left(dx+c\right)^{9}+3\,a^{3}d\cos\left(dx+c\right)^{8}-8\,a^{3}d\cos\left(dx+c\right)^{6}-6\,a^{3}d\cos\left(dx+c\right)^{5}+6\,a^{3}d\cos\left(dx+c\right)^{4}+8\,a^{3}d\cos\left(dx+c\right)^{3}-3\,a^{3}d\cos\left(dx+c\right)-a^{3}d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/15015*(304*cos(d*x + c)^10 + 912*cos(d*x + c)^9 - 152*cos(d*x + c)^8 - 28
88*cos(d*x + c)^7 - 1862*cos(d*x + c)^6 + 2926*cos(d*x + c)^5 + 3325*cos(d*
x + c)^4 - 665*cos(d*x + c)^3 - 35*cos(d*x + c)^2 + 210*cos(d*x + c) + 70)/
((a^3*d*cos(d*x + c)^9 + 3*a^3*d*cos(d*x + c)^8 - 8*a^3*d*cos(d*x + c)^6 -
6*a^3*d*cos(d*x + c)^5 + 6*a^3*d*cos(d*x + c)^4 + 8*a^3*d*cos(d*x + c)^3 -
3*a^3*d*cos(d*x + c) - a^3*d)*sin(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**8/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.53, size = 163, normalized size = 1.12

$$\frac{\frac{429 \left(280 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-35 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-28 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-5\right)}{a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}}-\frac{1155\,a^{36}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+5460\,a^{36}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+5005\,a^{36}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-17160\,a^{36}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-42042\,a^{36}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+210210\,a^{36}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{39}}}{15375360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/15375360*(429*(280*tan(1/2*d*x + 1/2*c)^6 - 35*tan(1/2*d*x + 1/2*c)^4 - 2
8*tan(1/2*d*x + 1/2*c)^2 - 5)/(a^3*tan(1/2*d*x + 1/2*c)^7) - (1155*a^36*tan
(1/2*d*x + 1/2*c)^13 + 5460*a^36*tan(1/2*d*x + 1/2*c)^11 + 5005*a^36*tan(1/
```

```
2*d*x + 1/2*c)^9 - 17160*a^36*tan(1/2*d*x + 1/2*c)^7 - 42042*a^36*tan(1/2*d
*x + 1/2*c)^5 + 210210*a^36*tan(1/2*d*x + 1/2*c))/a^39)/d
```

Mupad [B]
time = 2.13, size = 249, normalized size = 1.72

$$-\frac{2145\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{20}+12012\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{18}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+15015\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}-120120\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+210210\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}-42042\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-17160\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+5005\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}+5460\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{18}+1155\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{20}}{15375360\,a^3\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^8*(a + a/cos(c + d*x))^3),x)
```

```
[Out] -(2145*cos(c/2 + (d*x)/2)^20 + 1155*sin(c/2 + (d*x)/2)^20 + 5460*cos(c/2 +
(d*x)/2)^2*sin(c/2 + (d*x)/2)^18 + 5005*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x
)/2)^16 - 17160*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^14 - 42042*cos(c/2
+ (d*x)/2)^8*sin(c/2 + (d*x)/2)^12 + 210210*cos(c/2 + (d*x)/2)^12*sin(c/2 +
 (d*x)/2)^8 - 120120*cos(c/2 + (d*x)/2)^14*sin(c/2 + (d*x)/2)^6 + 15015*cos
(c/2 + (d*x)/2)^16*sin(c/2 + (d*x)/2)^4 + 12012*cos(c/2 + (d*x)/2)^18*sin(c
/2 + (d*x)/2)^2)/(15375360*a^3*d*cos(c/2 + (d*x)/2)^13*sin(c/2 + (d*x)/2)^7
)
```

3.108 $\int (a+a\sec(c+dx))(e\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=157

$$-\frac{ae^{5/2}\text{ArcTan}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{ae^{5/2}\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{6ae^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+}}{5d\sqrt{\sin(c+dx)}}$$

```
[Out] -a*e^(5/2)*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))/d+a*e^(5/2)*arctanh((e*sin(
d*x+c))^(1/2)/e^(1/2))/d-2/3*a*e*(e*sin(d*x+c))^(3/2)/d-2/5*a*e*cos(d*x+c)*
(e*sin(d*x+c))^(3/2)/d-6/5*a*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/
2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x
+c))^(1/2)/d/sin(d*x+c)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3957, 2917, 2644, 327, 335, 304, 209, 212, 2715, 2721, 2719}

$$-\frac{ae^{5/2}\text{ArcTan}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{ae^{5/2}\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{6ae^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}}-\frac{2ae(e\sin(c+dx))^{3/2}}{3d}-\frac{2ae\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] -((a*e^(5/2)*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d) + (a*e^(5/2)*ArcTanh[
Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d + (6*a*e^2*EllipticE[(c - Pi/2 + d*x)/2, 2
]*Sqrt[e*Sin[c + d*x]])/(5*d*Sqrt[Sin[c + d*x]]) - (2*a*e*(e*Sin[c + d*x])^
(3/2))/(3*d) - (2*a*e*Cos[c + d*x]*(e*Sin[c + d*x])^(3/2))/(5*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
```

```
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
```

```
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+a\sec(c+dx))(e\sin(c+dx))^{5/2}\,dx = -\int (-a-a\cos(c+dx))\sec(c+dx)(e\sin(c+dx))^{5/2}\,dx$$
$$= a\int (e\sin(c+dx))^{5/2}\,dx + a\int \sec(c+dx)(e\sin(c+dx))^{5/2}\,dx$$
$$= -\frac{2ae\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d} + \frac{a\text{Subst}\left(\int \frac{x^{5/2}}{1-\frac{x^2}{e^2}}\,dx, x, e\sin\right.}{de}$$
$$= -\frac{2ae(e\sin(c+dx))^{3/2}}{3d} - \frac{2ae\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d} + \frac{(a}{}$$
$$= \frac{6ae^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}} - \frac{2ae(e\sin(c+dx))}{3d}$$
$$= \frac{6ae^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}} - \frac{2ae(e\sin(c+dx))}{3d}$$
$$= -\frac{ae^{5/2}\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{ae^{5/2}\tanh^{-1}\left(\frac{\sqrt{e\sin(c+}}{\sqrt{e}}\right.}{d}$$

Mathematica [A]
time = 0.21, size = 106, normalized size = 0.68

$$-\frac{a(e\sin(c+dx))^{5/2}\left(15\text{ArcTan}\left(\sqrt{\sin(c+dx)}\right)-15\tanh^{-1}\left(\sqrt{\sin(c+dx)}\right)+18E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)+10\sin^{\frac{3}{2}}(c+dx)+3\sqrt{\sin(c+dx)}\sin(2(c+dx))\right)}{15d\sin^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] -1/15*(a*(e*Sin[c + d*x])^(5/2)*(15*ArcTan[Sqrt[Sin[c + d*x]]] - 15*ArcTanh
[Sqrt[Sin[c + d*x]]] + 18*EllipticE[(-2*c + Pi - 2*d*x)/4, 2] + 10*Sin[c +
d*x]^(3/2) + 3*Sqrt[Sin[c + d*x]]*Sin[2*(c + d*x)]))/(d*Sin[c + d*x]^(5/2))
```

Maple [A]
time = 0.22, size = 210, normalized size = 1.34

method	result
default	$\frac{-\frac{2ae(e\sin(dx+c))^{\frac{3}{2}}}{3}+a\,e^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)-a\,e^{\frac{5}{2}}\arctan\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)-\frac{a\,e^{3}\left(6\sqrt{-\sin(dx+c}\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-2/3*a*e*(e*sin(d*x+c))^(3/2)+a*e^(5/2)*arctanh((e*sin(d*x+c))^(1/2)/e^(1/
2))-a*e^(5/2)*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))-1/5*a*e^3*(6*(-sin(d*x+c
)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1
)^(1/2),1/2*2^(1/2))-3*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x
+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-2*sin(d*x+c)^4+2*sin
(d*x+c)^2)/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] e^(5/2)*integrate((a*sec(d*x + c) + a)*sin(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.43, size = 228, normalized size = 1.45

$$\frac{36i\sqrt{2}\sqrt{-i}\,ae^{\frac{5}{2}}\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c)))-36i\sqrt{2}\sqrt{i}\,ae^{\frac{5}{2}}\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)-i\sin(dx+c)))-30\,a\arctan\left(\frac{2\left(76\cos(dx+c)^{2}+425(\sin(dx+c)-1)\sqrt{\sin(dx+c)}-152\sin(dx+c)-152\right)}{361\cos(dx+c)^{2}+978\sin(dx+c)-722}\right)e^{\frac{5}{2}}+15\,ae^{\frac{5}{2}}\log\left(\frac{\cos(dx+c)^{2}-4(\sin(dx+c)+1)\sqrt{\sin(dx+c)}-6\sin(dx+c)-2}{\cos(dx+c)^{2}+2\sin(dx+c)-2}\right)-8\left(3a\cos(dx+c)e^{\frac{5}{2}}+5ae^{\frac{5}{2}}\right)\sin(dx+c)^{\frac{3}{2}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/60*(36*I*sqrt(2)*sqrt(-I)*a*e^(5/2)*weierstrassZeta(4, 0, weierstrassPInv
erse(4, 0, cos(d*x + c) + I*sin(d*x + c))) - 36*I*sqrt(2)*sqrt(I)*a*e^(5/2)
*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x +
 c))) - 30*a*arctan(2*(76*cos(d*x + c)^2 + 425*(sin(d*x + c) - 1)*sqrt(sin(
d*x + c)) - 152*sin(d*x + c) - 152)/(361*cos(d*x + c)^2 + 978*sin(d*x + c)
```

```
- 722))*e^(5/2) + 15*a*e^(5/2)*log((cos(d*x + c)^2 - 4*(sin(d*x + c) + 1)*s
qrt(sin(d*x + c)) - 6*sin(d*x + c) - 2)/(cos(d*x + c)^2 + 2*sin(d*x + c) -
2)) - 8*(3*a*cos(d*x + c)*e^(5/2) + 5*a*e^(5/2))*sin(d*x + c)^(3/2))/d
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))**(5/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)*(e*sin(d*x + c))^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(e\,\sin\left(c+d\,x\right)\right)}^{5/2}\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(5/2)*(a + a/cos(c + d*x)),x)
[Out] int((e*sin(c + d*x))^(5/2)*(a + a/cos(c + d*x)), x)
```

3.109 $\int (a+a\sec(c+dx))(e\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=154

$$\frac{ae^{3/2}\text{ArcTan}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{ae^{3/2}\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{2ae^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+\cdots}}{3d\sqrt{e\sin(c+dx)}}$$

```
[Out] a*e^(3/2)*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))/d+a*e^(3/2)*arctanh((e*sin(d
*x+c))^(1/2)/e^(1/2))/d-2/3*a*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1
/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c
)^(1/2)/d/(e*sin(d*x+c))^(1/2)-2*a*e*(e*sin(d*x+c))^(1/2)/d-2/3*a*e*cos(d*x
+c)*(e*sin(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.14, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.478$, Rules used = {3957, 2917, 2644, 327, 335, 218, 212, 209, 2715, 2721, 2720}

$$\frac{ae^{3/2}\text{ArcTan}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{ae^{3/2}\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{2ae^2\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{3d\sqrt{e\sin(c+dx)}}-\frac{2ae\sqrt{e\sin(c+dx)}}{d}-\frac{2ae\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*(e*Sin[c + d*x])^(3/2),x]
```

```
[Out] (a*e^(3/2)*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d + (a*e^(3/2)*ArcTanh[Sqr
t[e*Sin[c + d*x]]/Sqrt[e]])/d + (2*a*e^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*S
qrt[Sin[c + d*x]])/(3*d*Sqrt[e*Sin[c + d*x]]) - (2*a*e*Sqrt[e*Sin[c + d*x]]
)/d - (2*a*e*Cos[c + d*x]*Sqrt[e*Sin[c + d*x]])/(3*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
```

```
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
```

```
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))(e\sin(c+dx))^{3/2}\,dx &= -\int (-a-a\cos(c+dx))\sec(c+dx)(e\sin(c+dx))^{3/2}\,dx \\ &= a\int (e\sin(c+dx))^{3/2}\,dx + a\int \sec(c+dx)(e\sin(c+dx))^{3/2}\,dx \\ &= -\frac{2ae\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d} + \frac{a\text{Subst}\left(\int \frac{x^{3/2}}{1-\frac{x^2}{e^2}}\,dx, x, e\sin\right.}{de} \\ &= -\frac{2ae\sqrt{e\sin(c+dx)}}{d} - \frac{2ae\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d} + \frac{(ae}{} \\ &= \frac{2ae^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{3d\sqrt{e\sin(c+dx)}} - \frac{2ae\sqrt{e\sin(c+dx)}}{d} \\ &= \frac{2ae^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{3d\sqrt{e\sin(c+dx)}} - \frac{2ae\sqrt{e\sin(c+dx)}}{d} \\ &= \frac{ae^{3/2}\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{ae^{3/2}\tanh^{-1}\left(\frac{\sqrt{e\sin(c+d}}{\sqrt{e}}\right.}{d}\end{aligned}$$

Mathematica [A]
time = 0.38, size = 170, normalized size = 1.10

$$\frac{a(e\sin(c+dx))^{3/2}\left(12\text{ArcTan}\left(\sqrt{\sin(c+dx)}\right)+6\tanh^{-1}\left(\sqrt{\sin(c+dx)}\right)-8F\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)-3\log\left(1-\sqrt{\sin(c+dx)}\right)+3\log\left(1+\sqrt{\sin(c+dx)}\right)-24\sqrt{\sin(c+dx)}-8\cos(c+dx)\sec(2(c+dx))\sqrt{\sin(c+dx)}+16\cos(c+dx)\sec(2(c+dx))\sin^{\frac{5}{2}}(c+dx)\right)}{12d\sin^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*(e*Sin[c + d*x])^(3/2),x]
```

```
[Out] (a*(e*Sin[c + d*x])^(3/2)*(12*ArcTan[Sqrt[Sin[c + d*x]]] + 6*ArcTanh[Sqrt[S
in[c + d*x]]] - 8*EllipticF[(-2*c + Pi - 2*d*x)/4, 2] - 3*Log[1 - Sqrt[Sin[
c + d*x]]] + 3*Log[1 + Sqrt[Sin[c + d*x]]] - 24*Sqrt[Sin[c + d*x]] - 8*Cos[
c + d*x]*Sec[2*(c + d*x)]*Sqrt[Sin[c + d*x]] + 16*Cos[c + d*x]*Sec[2*(c + d
*x)]*Sin[c + d*x]^(5/2)))/(12*d*Sin[c + d*x]^(3/2))
```

Maple [A]
time = 0.20, size = 154, normalized size = 1.00

method	result
default	$\frac{a\,e^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{e\sin\left(dx+c\right)}}{\sqrt{e}}\right)+a\,e^{\frac{3}{2}}\arctan\left(\frac{\sqrt{e\sin\left(dx+c\right)}}{\sqrt{e}}\right)-2ae\sqrt{e\sin\left(dx+c\right)}-\frac{a\,e^{2}\left(\sqrt{-\sin\left(dx\right.}\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (a*e^(3/2)*arctanh((e*sin(d*x+c))^(1/2)/e^(1/2))+a*e^(3/2)*arctan((e*sin(d*
x+c))^(1/2)/e^(1/2))-2*a*e*(e*sin(d*x+c))^(1/2)-1/3*a*e^2*((-sin(d*x+c)+1)^
(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/
2),1/2*2^(1/2))-2*sin(d*x+c)^3+2*sin(d*x+c))/cos(d*x+c)/(e*sin(d*x+c))^(1/2
))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] e^(3/2)*integrate((a*sec(d*x + c) + a)*sin(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 10.86, size = 221, normalized size = 1.44

$$\frac{4\sqrt{2}\sqrt{-i}\,ae^{\frac{3}{2}}\text{weierstrassPInverse}(4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right))+4\sqrt{2}\sqrt{i}\,ae^{\frac{3}{2}}\text{weierstrassPInverse}(4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right))+6\,a\arctan\left(-\frac{2\left(76\cos(dx+c)^{2}-425\,(\sin(dx+c)-1)\sqrt{\sin\left(dx+c\right)}-152\sin(dx+c)-152\right)}{361\cos(dx+c)^{2}+978\sin(dx+c)-722}\right)e^{\frac{3}{2}}+3\,ae^{\frac{3}{2}}\log\left(\frac{\cos(dx+c)^{2}-4\,(\sin(dx+c)+1)\sqrt{\sin\left(dx+c\right)}-6\sin(dx+c)-2}{\cos(dx+c)^{2}+2\sin(dx+c)-2}\right)-8\left(a\cos\left(dx+c\right)e^{\frac{3}{2}}+3\,ae^{\frac{3}{2}}\right)\sqrt{\sin\left(dx+c\right)}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/12*(4*sqrt(2)*sqrt(-I)*a*e^(3/2)*weierstrassPInverse(4, 0, cos(d*x + c) +
 I*sin(d*x + c)) + 4*sqrt(2)*sqrt(I)*a*e^(3/2)*weierstrassPInverse(4, 0, co
s(d*x + c) - I*sin(d*x + c)) + 6*a*arctan(-2*(76*cos(d*x + c)^2 - 425*(sin(
d*x + c) - 1)*sqrt(sin(d*x + c)) - 152*sin(d*x + c) - 152)/(361*cos(d*x + c
```

```
)^2 + 978*sin(d*x + c) - 722))*e^(3/2) + 3*a*e^(3/2)*log((cos(d*x + c)^2 -
4*(sin(d*x + c) + 1)*sqrt(sin(d*x + c)) - 6*sin(d*x + c) - 2)/(cos(d*x + c)
^2 + 2*sin(d*x + c) - 2)) - 8*(a*cos(d*x + c)*e^(3/2) + 3*a*e^(3/2))*sqrt(s
in(d*x + c)))/d
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)*(e*sin(d*x + c))^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\sin\left(c+d\,x\right))^{3/2}\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(3/2)*(a + a/cos(c + d*x)),x)
```

```
[Out] int((e*sin(c + d*x))^(3/2)*(a + a/cos(c + d*x)), x)
```

3.110 $\int (a+a\sec(c+dx))\sqrt{e\sin(c+dx)}\,dx$

Optimal. Leaf size=104

$$-\frac{a\sqrt{e}\,\text{ArcTan}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{a\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}$$

```
[Out] -a*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/d+a*arctanh((e*sin(d*x+c))^
(1/2)/e^(1/2))*e^(1/2)/d-2*a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+
1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))
^(1/2)/d/sin(d*x+c)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3957, 2917, 2644, 335, 304, 209, 212, 2721, 2719}

$$-\frac{a\sqrt{e}\,\text{ArcTan}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{a\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Sqrt[e*Sin[c + d*x]],x]
```

```
[Out] -((a*Sqrt[e]*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d) + (a*Sqrt[e]*ArcTanh[
Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d + (2*a*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sq
rt[e*Sin[c + d*x]])/(d*Sqrt[Sin[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
```

```
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+a\sec(c+dx))\sqrt{e\sin(c+dx)}\,dx = -\int (-a-a\cos(c+dx))\sec(c+dx)\sqrt{e\sin(c+dx)}\,dx$$
$$= a\int \sqrt{e\sin(c+dx)}\,dx + a\int \sec(c+dx)\sqrt{e\sin(c+dx)}\,dx$$
$$= \frac{a\text{Subst}\left(\int \frac{\sqrt{x}}{1-\frac{x^2}{e^2}}\,dx, x, e\sin(c+dx)\right)}{de} + \frac{\left(a\sqrt{e\sin(c+dx)}\right)\int \sqrt{}}{\sqrt{\sin(c+dx}}$$
$$= \frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}} + \frac{(2a)\text{Subst}\left(\int \frac{x^2}{1-\frac{x^4}{e^2}}\,dx\right.}{d}$$
$$= \frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}} + \frac{(ae)\text{Subst}\left(\int \frac{1}{e-x^2}\,dx,\right.}{d}$$
$$= -\frac{a\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{a\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e\sin(c+}}{\sqrt{e}}\right.}{d}$$

Mathematica [A]
time = 0.08, size = 69, normalized size = 0.66

$$\frac{a\left(-\text{ArcTan}\left(\sqrt{\sin(c+dx)}\right)+\tanh^{-1}\left(\sqrt{\sin(c+dx)}\right)-2E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Sqrt[e*Sin[c + d*x]],x]
[Out] (a*(-ArcTan[Sqrt[Sin[c + d*x]]] + ArcTanh[Sqrt[Sin[c + d*x]]] - 2*EllipticE
[(-2*c + Pi - 2*d*x)/4, 2])*Sqrt[e*Sin[c + d*x]])/(d*Sqrt[Sin[c + d*x]])
```

Maple [A]
time = 0.18, size = 142, normalized size = 1.37

method	result
default	$\frac{a\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)-a\sqrt{e}\,\arctan\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)-\frac{ae\sqrt{-\sin(dx+c)+1}\sqrt{2\sin(dx}}{}}{c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (a*e^(1/2)*arctanh((e*sin(d*x+c))^(1/2)/e^(1/2))-a*e^(1/2)*arctan((e*sin(d*
x+c))^(1/2)/e^(1/2))-a*e*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d
*x+c)^(1/2)*(2*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-EllipticF((-sin
(d*x+c)+1)^(1/2),1/2*2^(1/2)))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(1/2)*integrate((a*sec(d*x + c) + a)*sqrt(sin(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.93, size = 200, normalized size = 1.92

$$\frac{4i\sqrt{2}\sqrt{-i}\,ae^{\frac{1}{2}}\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c)))-4i\sqrt{2}\sqrt{i}\,ae^{\frac{1}{2}}\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)-i\sin(dx+c)))-2a\arctan\left(\frac{2\left(76\cos(dx+c)^2+425(\sin(dx+c)-1)\sqrt{\sin(dx+c)}-152\sin(dx+c)-152\right)}{361\cos(dx+c)^2+978\sin(dx+c)-722}\right)e^{\frac{1}{2}}+ae^{\frac{1}{2}}\log\left(\frac{\cos(dx+c)^2-4(\sin(dx+c)+1)\sqrt{\sin(dx+c)}-6\sin(dx+c)-2}{\cos(dx+c)^2+2\sin(dx+c)-2}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(4*I*sqrt(2)*sqrt(-I)*a*e^(1/2)*weierstrassZeta(4, 0, weierstrassPInver
se(4, 0, cos(d*x + c) + I*sin(d*x + c))) - 4*I*sqrt(2)*sqrt(I)*a*e^(1/2)*we
ierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)
)) - 2*a*arctan(2*(76*cos(d*x + c)^2 + 425*(sin(d*x + c) - 1)*sqrt(sin(d*x
+ c)) - 152*sin(d*x + c) - 152)/(361*cos(d*x + c)^2 + 978*sin(d*x + c) - 72
2))*e^(1/2) + a*e^(1/2)*log((cos(d*x + c)^2 - 4*(sin(d*x + c) + 1)*sqrt(sin
(d*x + c)) - 6*sin(d*x + c) - 2)/(cos(d*x + c)^2 + 2*sin(d*x + c) - 2)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sqrt{e\sin(c+dx)}\,dx+\int\sqrt{e\sin(c+dx)}\sec(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))**(1/2),x)
```

```
[Out] a*(Integral(sqrt(e*sin(c + d*x)), x) + Integral(sqrt(e*sin(c + d*x))*sec(c
+ d*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)*sqrt(e*sin(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{e\,\sin\left(c+d\,x\right)}\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(1/2)*(a + a/cos(c + d*x)),x)
[Out] int((e*sin(c + d*x))^(1/2)*(a + a/cos(c + d*x)), x)
```

3.111 $\int \frac{a+a\sec(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx$

Optimal. Leaf size=103

$$\frac{a\text{ArcTan}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}}+\frac{a\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}}+\frac{2aF\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{d\sqrt{e\sin(c+dx)}}$$

```
[Out] a*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))/d/e^(1/2)+a*arctanh((e*sin(d*x+c))^(
1/2)/e^(1/2))/d/e^(1/2)-2*a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1
/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2
)/d/(e*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3957, 2917, 2644, 335, 218, 212, 209, 2721, 2720}

$$\frac{a\text{ArcTan}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}}+\frac{a\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}}+\frac{2a\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{d\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/Sqrt[e*Sin[c + d*x]],x]
```

```
[Out] (a*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/(d*Sqrt[e]) + (a*ArcTanh[Sqrt[e*Si
n[c + d*x]]/Sqrt[e]])/(d*Sqrt[e]) + (2*a*EllipticF[(c - Pi/2 + d*x)/2, 2]*S
qrt[Sin[c + d*x]])/(d*Sqrt[e*Sin[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
```

```
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+a\sec(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx &= -\int \frac{(-a-a\cos(c+dx))\sec(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx \\
&= a\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx + a\int \frac{\sec(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx \\
&= \frac{a\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{e^2}\right)}\,dx, x, e\sin(c+dx)\right)}{de} + \frac{\left(a\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx}}}{\sqrt{e\sin(c+dx)}} \\
&= \frac{2aF\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{d\sqrt{e\sin(c+dx)}} + \frac{(2a)\text{Subst}\left(\int \frac{1}{1-\frac{x^4}{e^2}}\,dx, x, \sqrt{e\sin(c+dx}\right.}{de} \\
&= \frac{2aF\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{d\sqrt{e\sin(c+dx)}} + \frac{a\text{Subst}\left(\int \frac{1}{e-x^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{d} \\
&= \frac{a\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}} + \frac{a\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}} + \frac{2aF\left(\frac{1}{2}\left(c-\frac{\pi}{2}+\right.\right.}{d\sqrt{e\,\text{s}}}
\end{aligned}
$$

Mathematica [A]
time = 1.97, size = 193, normalized size = 1.87

$$
\frac{4a\cos\left(\frac{1}{2}(c+dx)\right)\left(4F\left(\text{ArcSin}\left(\frac{1}{\sqrt{\tan\left(\frac{1}{4}(c+dx)\right)}}\right)\middle|-1\right)+\sqrt{2}\left(\Pi\left(-1-\sqrt{2};\text{ArcSin}\left(\frac{1}{\sqrt{\tan\left(\frac{1}{4}(c+dx)\right)}}\right)\middle|-1\right)-\Pi\left(1-\sqrt{2};\text{ArcSin}\left(\frac{1}{\sqrt{\tan\left(\frac{1}{4}(c+dx)\right)}}\right)\middle|-1\right)-\Pi\left(-1+\sqrt{2};\text{ArcSin}\left(\frac{1}{\sqrt{\tan\left(\frac{1}{4}(c+dx)\right)}}\right)\middle|-1\right)+\Pi\left(1+\sqrt{2};\text{ArcSin}\left(\frac{1}{\sqrt{\tan\left(\frac{1}{4}(c+dx)\right)}}\right)\middle|-1\right)\right)\right)}{d\sqrt{1-\cot^2\left(\frac{1}{4}(c+dx)\right)}\sqrt{e\sin(c+dx)}\sqrt{\tan\left(\frac{1}{4}(c+dx)\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])/Sqrt[e*Sin[c + d*x]],x]
```

```
[Out] (4*a*Cos[(c + d*x)/2]*(4*EllipticF[ArcSin[1/Sqrt[Tan[(c + d*x)/4]]], -1] +
Sqrt[2]*(EllipticPi[-1 - Sqrt[2], ArcSin[1/Sqrt[Tan[(c + d*x)/4]]], -1] - E
llipticPi[1 - Sqrt[2], ArcSin[1/Sqrt[Tan[(c + d*x)/4]]], -1] - EllipticPi[-
1 + Sqrt[2], ArcSin[1/Sqrt[Tan[(c + d*x)/4]]], -1] + EllipticPi[1 + Sqrt[2]
, ArcSin[1/Sqrt[Tan[(c + d*x)/4]]], -1])))/(d*Sqrt[1 - Cot[(c + d*x)/4]^2]*
Sqrt[e*Sin[c + d*x]]*Sqrt[Tan[(c + d*x)/4]])
```

Maple [A]
time = 0.17, size = 117, normalized size = 1.14

method	result

default	$\frac{\frac{a \,\mathrm{arctanh}\left(\frac{\sqrt{e \sin \left(d x +c \right)}}{\sqrt{e}}\right)}{\sqrt{e}}+\frac{a \arctan \left(\frac{\sqrt{e \sin \left(d x +c \right)}}{\sqrt{e}}\right)}{\sqrt{e}}-\frac{a \sqrt{-\sin \left(d x +c \right)+1}\, \sqrt{2 \sin \left(d x +c \right)+2}}{\cos \left(d x +c \right) \sqrt{e}}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (a/e^(1/2)*arctanh((e*sin(d*x+c))^(1/2)/e^(1/2))+a/e^(1/2)*arctan((e*sin(d*
x+c))^(1/2)/e^(1/2))-a*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x
+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))/cos(d*x+c)/(e*sin(d*
x+c))^(1/2))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-1/2)*integrate((a*sec(d*x + c) + a)/sqrt(sin(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 4.06, size = 144, normalized size = 1.40

$$\frac{\left(4 \sqrt{2} \sqrt{-i}\, a \mathrm{weierstrassPInverse}\left(4, 0, \cos\left(dx+c\right)+i \sin\left(dx+c\right)\right)+4 \sqrt{2} \sqrt{i}\, a \mathrm{weierstrassPInverse}\left(4, 0, \cos\left(dx+c\right)-i \sin\left(dx+c\right)\right)+2 a \arctan\left(\frac{\sin\left(dx+c\right)-1}{2 \sqrt{\sin\left(dx+c\right)}}\right)+a \log\left(\frac{\cos\left(dx+c\right)^{2}-4 \left(\sin\left(dx+c\right)+1\right) \sqrt{\sin\left(dx+c\right)}-6 \sin\left(dx+c\right)-2}{\cos\left(dx+c\right)^{2}+2 \sin\left(dx+c\right)-2}\right)\right) e^{\left(-\frac{1}{2}\right)}}{4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(4*sqrt(2)*sqrt(-I)*a*weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*
x + c)) + 4*sqrt(2)*sqrt(I)*a*weierstrassPInverse(4, 0, cos(d*x + c) - I*si
n(d*x + c)) + 2*a*arctan(1/2*(sin(d*x + c) - 1)/sqrt(sin(d*x + c))) + a*log
((cos(d*x + c)^2 - 4*(sin(d*x + c) + 1)*sqrt(sin(d*x + c)) - 6*sin(d*x + c)
 - 2)/(cos(d*x + c)^2 + 2*sin(d*x + c) - 2)))*e^(-1/2)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a \left(\int \frac{1}{\sqrt{e \sin{\left(c + d x \right)}}}\, dx + \int \frac{\sec{\left(c + d x \right)}}{\sqrt{e \sin{\left(c + d x \right)}}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))**(1/2),x)
```

```
[Out] a*(Integral(1/sqrt(e*sin(c + d*x)), x) + Integral(sec(c + d*x)/sqrt(e*sin(c
 + d*x)), x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)/sqrt(e*sin(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+\frac{a}{\cos(c+d\,x)}}{\sqrt{e\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/(e*sin(c + d*x))^(1/2),x)
```

```
[Out] int((a + a/cos(c + d*x))/(e*sin(c + d*x))^(1/2), x)
```

3.112 $\int \frac{a+a\sec(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=155

$$-\frac{a\text{ArcTan}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}}+\frac{a\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}}-\frac{2a}{de\sqrt{e\sin(c+dx)}}-\frac{2a\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}-\frac{2a}{}$$

```
[Out] -a*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))/d/e^(3/2)+a*arctanh((e*sin(d*x+c))^
(1/2)/e^(1/2))/d/e^(3/2)-2*a/d/e/(e*sin(d*x+c))^(1/2)-2*a*cos(d*x+c)/d/e/(e
*sin(d*x+c))^(1/2)+2*a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi
+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)
/d/e^2/sin(d*x+c)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3957, 2917, 2644, 331, 335, 304, 209, 212, 2716, 2721, 2719}

$$-\frac{a\text{ArcTan}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}}+\frac{a\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}}-\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{de^2\sqrt{\sin(c+dx)}}-\frac{2a}{de\sqrt{e\sin(c+dx)}}-\frac{2a\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/(e*Sin[c + d*x])^(3/2),x]
```

```
[Out] -((a*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/(d*e^(3/2))) + (a*ArcTanh[Sqrt[e
*Sin[c + d*x]]/Sqrt[e]])/(d*e^(3/2)) - (2*a)/(d*e*Sqrt[e*Sin[c + d*x]]) - (
2*a*Cos[c + d*x])/(d*e*Sqrt[e*Sin[c + d*x]]) - (2*a*EllipticE[(c - Pi/2 + d
*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(d*e^2*Sqrt[Sin[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
```

```
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
```

```
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+a\sec(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx &= -\int \frac{(-a-a\cos(c+dx))\sec(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx \\
&= a\int \frac{1}{(e\sin(c+dx))^{3/2}}\,dx + a\int \frac{\sec(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx \\
&= -\frac{2a\cos(c+dx)}{de\sqrt{e\sin(c+dx)}} - \frac{a\int \sqrt{e\sin(c+dx)}\,dx}{e^2} + \frac{a\mathrm{Subst}\left(\int \frac{1}{x^{3/2}\left(1-\frac{x^2}{e^2}\right)}\,dx, x, e\sin
\end{aligned}$$

$$\begin{aligned}
&= -\frac{2a}{de\sqrt{e\sin(c+dx)}} - \frac{2a\cos(c+dx)}{de\sqrt{e\sin(c+dx)}} + \frac{a\mathrm{Subst}\left(\int \frac{\sqrt{x}}{1-\frac{x^2}{e^2}}\,dx, x, e\sin(c+dx)\right)}{de^3} \\
&= -\frac{2a}{de\sqrt{e\sin(c+dx)}} - \frac{2a\cos(c+dx)}{de\sqrt{e\sin(c+dx)}} - \frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+}}{de^2\sqrt{\sin(c+dx)}} \\
&= -\frac{2a}{de\sqrt{e\sin(c+dx)}} - \frac{2a\cos(c+dx)}{de\sqrt{e\sin(c+dx)}} - \frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+}}{de^2\sqrt{\sin(c+dx)}} \\
&= -\frac{a\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}} + \frac{a\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}} - \frac{2a}{de\sqrt{e\sin(c+dx}}
\end{aligned}$$

Mathematica [A]
time = 0.26, size = 143, normalized size = 0.92

$$-\frac{a(1+\cos(c+dx))\sec\left(\frac{1}{2}(c+dx)\right)\left(\mathrm{ArcTan}\left(\sqrt{\sin(c+dx)}\right)\sec\left(\frac{1}{2}(c+dx)\right)-\tanh^{-1}\left(\sqrt{\sin(c+dx)}\right)\sec\left(\frac{1}{2}(c+dx)\right)-2E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)\sec\left(\frac{1}{2}(c+dx)\right)+2\csc\left(\frac{1}{2}(c+dx)\right)\sqrt{\sin(c+dx)}\right)\sin^{\frac{3}{2}}(c+dx)}{2d(e\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])/(e*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/2*(a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]*(ArcTan[Sqrt[Sin[c + d*x]]]*Sec
[(c + d*x)/2] - ArcTanh[Sqrt[Sin[c + d*x]]]*Sec[(c + d*x)/2] - 2*EllipticE[
(-2*c + Pi - 2*d*x)/4, 2]*Sec[(c + d*x)/2] + 2*Csc[(c + d*x)/2]*Sqrt[Sin[c
+ d*x]])*Sin[c + d*x]^(3/2))/(d*(e*Sin[c + d*x])^(3/2))
```

Maple [A]
time = 0.18, size = 201, normalized size = 1.30

method	result
default	$\frac{-\frac{2a}{e\sqrt{e\sin(dx+c)}}+\frac{a\,\mathrm{arctanh}\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)}{e^{\frac{3}{2}}}-\frac{a\arctan\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)}{e^{\frac{3}{2}}}+\frac{a\left(2\sqrt{-\sin(dx+c)+\cdots\right.}{\cdots}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-2*a/e/(e*sin(d*x+c))^(1/2)+a/e^(3/2)*arctanh((e*sin(d*x+c))^(1/2)/e^(1/2)
)-a/e^(3/2)*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))+a/e*(2*(-sin(d*x+c)+1)^(1/
2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),
1/2*2^(1/2))-(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*
EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-2*cos(d*x+c)^2)/cos(d*x+c)/(e*
sin(d*x+c))^(1/2))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] e^(-3/2)*integrate((a*sec(d*x + c) + a)/sin(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.41, size = 202, normalized size = 1.30

$$\frac{\left(-4i\sqrt{2}\sqrt{-i}\,a\sin(dx+c)\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c)))+4i\sqrt{2}\sqrt{i}\,a\sin(dx+c)\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)-i\sin(dx+c)))-2a\arctan\left(\frac{\sin(dx+c)-1}{2\sqrt{\sin(dx+c)}}\right)\sin(dx+c)+a\log\left(\frac{\cos(dx+c)^2-4(\sin(dx+c)+1)\sqrt{\sin(dx+c)}-6\sin(dx+c)-2}{\cos(dx+c)^2+2\sin(dx+c)-2}\right)\sin(dx+c)-8(a\cos(dx+c)+a)\sqrt{\sin(dx+c)}\right)e^{\left(-\frac{3}{2}\right)}}{4d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(-4*I*sqrt(2)*sqrt(-I)*a*sin(d*x + c)*weierstrassZeta(4, 0, weierstrass
PInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) + 4*I*sqrt(2)*sqrt(I)*a*sin(
d*x + c)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*s
in(d*x + c))) - 2*a*arctan(1/2*(sin(d*x + c) - 1)/sqrt(sin(d*x + c)))*sin(d
```

```
*x + c) + a*log((cos(d*x + c)^2 - 4*(sin(d*x + c) + 1)*sqrt(sin(d*x + c)) -
 6*sin(d*x + c) - 2)/(cos(d*x + c)^2 + 2*sin(d*x + c) - 2))*sin(d*x + c) -
8*(a*cos(d*x + c) + a)*sqrt(sin(d*x + c)))*e^(-3/2)/(d*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{1}{(e\sin{(c+dx)})^{\frac{3}{2}}}\,dx+\int \frac{\sec{(c+dx)}}{(e\sin{(c+dx)})^{\frac{3}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))**(3/2),x)
[Out] a*(Integral((e*sin(c + d*x))**(-3/2), x) + Integral(sec(c + d*x)/(e*sin(c +
 d*x))**(3/2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)/(e*sin(d*x + c))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+\frac{a}{\cos(c+d\,x)}}{(e\,\sin(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/(e*sin(c + d*x))^(3/2),x)
[Out] int((a + a/cos(c + d*x))/(e*sin(c + d*x))^(3/2), x)
```

3.113 $\int \frac{a+a\sec(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=160

$$\frac{a\text{ArcTan}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}}+\frac{a\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}}-\frac{2a}{3de(e\sin(c+dx))^{3/2}}-\frac{2a\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}}$$

```
[Out] a*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))/d/e^(5/2)+a*arctanh((e*sin(d*x+c))^(
1/2)/e^(1/2))/d/e^(5/2)-2/3*a/d/e/(e*sin(d*x+c))^(3/2)-2/3*a*cos(d*x+c)/d/e
/(e*sin(d*x+c))^(3/2)-2/3*a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1
/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2
)/d/e^2/(e*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3957, 2917, 2644, 331, 335, 218, 212, 209, 2716, 2721, 2720}

$$\frac{a\text{ArcTan}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}}+\frac{a\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}}+\frac{2a\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{3de^2\sqrt{e\sin(c+dx)}}-\frac{2a}{3de(e\sin(c+dx))^{3/2}}-\frac{2a\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] (a*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/(d*e^(5/2)) + (a*ArcTanh[Sqrt[e*Si
n[c + d*x]]/Sqrt[e]])/(d*e^(5/2)) - (2*a)/(3*d*e*(e*Sin[c + d*x])^(3/2)) -
(2*a*Cos[c + d*x])/(3*d*e*(e*Sin[c + d*x])^(3/2)) + (2*a*EllipticF[(c - Pi/
2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*d*e^2*Sqrt[e*Sin[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
```

```
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
```

```
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+a\sec(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx &= -\int \frac{(-a-a\cos(c+dx))\sec(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx \\
&= a\int \frac{1}{(e\sin(c+dx))^{5/2}}\,dx + a\int \frac{\sec(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx \\
&= -\frac{2a\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}} + \frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{3e^2} + \frac{a\mathrm{Subst}\left(\int \frac{1}{x^{5/2}\left(1-\frac{x^2}{e^2}\right)}\,dx, x, e\right.}{de} \\
&= -\frac{2a}{3de(e\sin(c+dx))^{3/2}} - \frac{2a\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}} + \frac{a\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{e^2}\right)}\,dx, x, e\,\mathrm{s}\right.}{de^3} \\
&= -\frac{2a}{3de(e\sin(c+dx))^{3/2}} - \frac{2a\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}} + \frac{2aF\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(}}{3de^2\sqrt{e\sin(c+dx)}} \\
&= -\frac{2a}{3de(e\sin(c+dx))^{3/2}} - \frac{2a\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}} + \frac{2aF\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(}}{3de^2\sqrt{e\sin(c+dx)}} \\
&= \frac{a\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}} + \frac{a\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}} - \frac{2a}{3de(e\sin(c+dx)}
\end{aligned}$$

Mathematica [A]
time = 0.24, size = 120, normalized size = 0.75

$$-\frac{a(1+\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\left(-3\mathrm{ArcTan}\left(\sqrt{\sin(c+dx)}\right)-3\tanh^{-1}\left(\sqrt{\sin(c+dx)}\right)+2F\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)+\csc^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\sin(c+dx)}\right)\sqrt{\sin(c+dx)}}{6de^2\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])/(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] -1/6*(a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*(-3*ArcTan[Sqrt[Sin[c + d*x]]
] - 3*ArcTanh[Sqrt[Sin[c + d*x]]] + 2*EllipticF[(-2*c + Pi - 2*d*x)/4, 2] +
 Csc[(c + d*x)/2]^2*Sqrt[Sin[c + d*x]])*Sqrt[Sin[c + d*x]])/(d*e^2*Sqrt[e*S
in[c + d*x]])
```

Maple [A]
time = 0.20, size = 164, normalized size = 1.02

method	result
default	$\frac{-\frac{2a}{3e(e\sin(dx+c))^{\frac{3}{2}}}+\frac{a\,\mathrm{arctanh}\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)}{e^{\frac{5}{2}}}+\frac{a\arctan\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)}{e^{\frac{5}{2}}}-\frac{a\left(\sqrt{-\sin(dx+c)+1}\,\sqrt{2\mathrm{si}}\right.}{}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-2/3*a/e/(e*sin(d*x+c))^(3/2)+a/e^(5/2)*arctanh((e*sin(d*x+c))^(1/2)/e^(1/
2))+a/e^(5/2)*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))-1/3*a/e^2*((-sin(d*x+c)+
1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(5/2)*EllipticF((-sin(d*x+c)+1)^
(1/2),1/2*2^(1/2))-2*sin(d*x+c)^3+2*sin(d*x+c))/sin(d*x+c)^2/cos(d*x+c)/(e*
sin(d*x+c))^(1/2))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] e^(-5/2)*integrate((a*sec(d*x + c) + a)/sin(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 4.33, size = 219, normalized size = 1.37

$$\frac{4\sqrt{-i}\left(\sqrt{2}\,a\cos(dx+c)-\sqrt{2}\,a\right)\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c))+4\sqrt{i}\left(\sqrt{2}\,a\cos(dx+c)-\sqrt{2}\,a\right)\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)-i\sin(dx+c))+6\,(a\cos(dx+c)-a)\arctan\left(\frac{\sin(dx+c)-1}{2\sqrt{\sin(dx+c)}}\right)+3\,(a\cos(dx+c)-a)\log\left(\frac{\cos(dx+c)^2-4\,(\sin(dx+c)+1)\sqrt{\sin(dx+c)}-6\sin(dx+c)-2}{\cos(dx+c)^2+2\sin(dx+c)-2}\right)+8\,a\sqrt{\sin(dx+c)}}{12\left(d\cos(dx+c)\,e^{\frac{5}{2}}-de^{\frac{5}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12*(4*sqrt(-I)*(sqrt(2)*a*cos(d*x + c) - sqrt(2)*a)*weierstrassPInverse(4
, 0, cos(d*x + c) + I*sin(d*x + c)) + 4*sqrt(I)*(sqrt(2)*a*cos(d*x + c) - s
qrt(2)*a)*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) + 6*(a*c
os(d*x + c) - a)*arctan(1/2*(sin(d*x + c) - 1)/sqrt(sin(d*x + c))) + 3*(a*c
os(d*x + c) - a)*log((cos(d*x + c)^2 - 4*(sin(d*x + c) + 1)*sqrt(sin(d*x +
```

```
c)) - 6*sin(d*x + c) - 2)/(cos(d*x + c)^2 + 2*sin(d*x + c) - 2)) + 8*a*sqrt
(sin(d*x + c)))/(d*cos(d*x + c)*e^(5/2) - d*e^(5/2))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)/(e*sin(d*x + c))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+\frac{a}{\cos(c+d\,x)}}{{\left(e\,\sin\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/(e*sin(c + d*x))^(5/2),x)
[Out] int((a + a/cos(c + d*x))/(e*sin(c + d*x))^(5/2), x)
```

3.114 $\int (a+a\sec(c+dx))^2(e\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=194

$$-\frac{2a^2e^{5/2}\text{ArcTan}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{2a^2e^{5/2}\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}-\frac{9a^2e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin}}{5d\sqrt{\sin(c+dx)}}$$

```
[Out] -2*a^2*e^(5/2)*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))/d+2*a^2*e^(5/2)*arctanh
((e*sin(d*x+c))^(1/2)/e^(1/2))/d-4/3*a^2*e*(e*sin(d*x+c))^(3/2)/d-2/5*a^2*e
*cos(d*x+c)*(e*sin(d*x+c))^(3/2)/d+a^2*e*sec(d*x+c)*(e*sin(d*x+c))^(3/2)/d+
9/5*a^2*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*E
llipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/d/sin(d*x+
c)^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3957, 2952, 2715, 2721, 2719, 2644, 327, 335, 304, 209, 212, 2646}

$$-\frac{2a^2e^{5/2}\text{ArcTan}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{2a^2e^{5/2}\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}-\frac{9a^2e^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}}-\frac{4a^2e(e\sin(c+dx))^{3/2}}{3d}-\frac{2a^2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d}+\frac{a^2e\sec(c+dx)(e\sin(c+dx))^{3/2}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*a^2*e^(5/2)*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d + (2*a^2*e^(5/2)*Ar
cTanh[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d - (9*a^2*e^2*EllipticE[(c - Pi/2 + d
*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(5*d*Sqrt[Sin[c + d*x]]) - (4*a^2*e*(e*Sin[
c + d*x])^(3/2))/(3*d) - (2*a^2*e*Cos[c + d*x]*(e*Sin[c + d*x])^(3/2))/(5*d
) + (a^2*e*Sec[c + d*x]*(e*Sin[c + d*x])^(3/2))/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
```

] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 327

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 335

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 2644

Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[(m - 1)/2] && LtQ[0, m, n])

Rule 2646

Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n + 1)/(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Sin[e + f*x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2719

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+a\sec(c+dx))^2(e\sin(c+dx))^{5/2}\,dx = \int (-a-a\cos(c+dx))^2\sec^2(c+dx)(e\sin(c+dx))^{5/2}\,dx$$

$$= \int \left(a^2(e\sin(c+dx))^{5/2}+2a^2\sec(c+dx)(e\sin(c+dx))^{5/2}+a^2\right.$$

$$= a^2\int (e\sin(c+dx))^{5/2}\,dx + a^2\int \sec^2(c+dx)(e\sin(c+dx))^{5/2}$$

$$= -\frac{2a^2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d}+\frac{a^2e\sec(c+dx)(e\sin(c+}{d}$$

$$= -\frac{4a^2e(e\sin(c+dx))^{3/2}}{3d}-\frac{2a^2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d}+$$

$$= -\frac{9a^2e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}}-\frac{4a^2e(e\sin(c+}{3d}$$

$$= -\frac{9a^2e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}}-\frac{4a^2e(e\sin(c+}{3d}$$

$$= -\frac{2a^2e^{5/2}\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{2a^2e^{5/2}\tanh^{-1}\left(\frac{\sqrt{e\sin}}{\sqrt{}}\right.}{d}$$

Mathematica [A]
time = 21.94, size = 248, normalized size = 1.28

$$\frac{16a^2\cos^4\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)(e\sin(c+dx))^{5/2}\left(-30\text{ArcTan}\left(\sqrt{\sin(c+dx)}\right)\sqrt{\cos^2(c+dx)}-27\sqrt{\cos^2(c+dx)}\,E\left(\text{ArcSin}\left(\sqrt{\sin(c+dx)}\right)\middle|-1\right)+27\sqrt{\cos^2(c+dx)}\,F\left(\text{ArcSin}\left(\sqrt{\sin(c+dx)}\right)\middle|-1\right)-15\sqrt{\cos^2(c+dx)}\log\left(1-\sqrt{\sin(c+dx)}\right)+15\sqrt{\cos^2(c+dx)}\log\left(1+\sqrt{\sin(c+dx)}\right)+9\sin^{\frac{3}{2}}(c+dx)-20\sqrt{\cos^2(c+dx)}\sin^{\frac{3}{2}}(c+dx)+6\sin^{\frac{7}{2}}(c+dx)\right)\sin^4\left(\frac{1}{2}\text{ArcSin}(\sin(c+dx))\right)}{15d\sin^{\frac{13}{2}}(c+dx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] (16*a^2*Cos[(c + d*x)/2]^4*Sec[c + d*x]*(e*Sin[c + d*x])^(5/2)*(-30*ArcTan[
Sqrt[Sin[c + d*x]]]*Sqrt[Cos[c + d*x]^2] - 27*Sqrt[Cos[c + d*x]^2]*Elliptic
E[ArcSin[Sqrt[Sin[c + d*x]]], -1] + 27*Sqrt[Cos[c + d*x]^2]*EllipticF[ArcSi
n[Sqrt[Sin[c + d*x]]], -1] - 15*Sqrt[Cos[c + d*x]^2]*Log[1 - Sqrt[Sin[c + d
*x]]] + 15*Sqrt[Cos[c + d*x]^2]*Log[1 + Sqrt[Sin[c + d*x]]] + 9*Sin[c + d*x
]^(3/2) - 20*Sqrt[Cos[c + d*x]^2]*Sin[c + d*x]^(3/2) + 6*Sin[c + d*x]^(7/2)
)*Sin[ArcSin[Sin[c + d*x]]/2]^4)/(15*d*Sin[c + d*x]^(13/2))
```

Maple [A]
time = 0.30, size = 265, normalized size = 1.37

method	result
default	$a^2\left(60\sqrt{e\sin(dx+c)}\,e^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)\cos(dx+c)-60\sqrt{e\sin(dx+c)}\,e^{\frac{5}{2}}\arctan\left(\frac{\sqrt{e\sin(d}}{\sqrt{e}}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/30/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*a^2*(60*(e*sin(d*x+c))^(1/2)*e^(5/2)*a
rctanh((e*sin(d*x+c))^(1/2)/e^(1/2))*cos(d*x+c)-60*(e*sin(d*x+c))^(1/2)*e^(
5/2)*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))*cos(d*x+c)+54*(-sin(d*x+c)+1)^(1/
2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),
1/2*2^(1/2))*e^3-27*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)
^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*e^3+12*cos(d*x+c)^4*e^3
+40*cos(d*x+c)^3*e^3-42*e^3*cos(d*x+c)^2-40*cos(d*x+c)*e^3+30*e^3)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] e^(5/2)*integrate((a*sec(d*x + c) + a)^2*sin(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.93, size = 287, normalized size = 1.48

$$\frac{-27i\sqrt{2}\sqrt{-i}\,a^2\cos(dx+c)\,e^{\frac{5}{2}}\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c)))+27i\sqrt{2}\sqrt{i}\,a^2\cos(dx+c)\,e^{\frac{5}{2}}\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)-i\sin(dx+c)))-30\,a^2\arctan\left(\frac{2\left(76\cos(dx+c)^2+425(\sin(dx+c)-1)\sqrt{\sin(dx+c)}-152\sin(dx+c)-152\right)}{361\cos(dx+c)^2+978\sin(dx+c)-722}\right)\cos(dx+c)\,e^{\frac{5}{2}}+15\,a^2\cos(dx+c)\,e^{\frac{5}{2}}\log\left(\frac{\cos(dx+c)^2-4(\sin(dx+c)+1)\sqrt{\sin(dx+c)}-6\sin(dx+c)-2}{\cos(dx+c)^2+2\sin(dx+c)-2}\right)-2\left(6\,a^2\cos(dx+c)^2e^{\frac{5}{2}}+20\,a^2\cos(dx+c)\,e^{\frac{5}{2}}-15\,a^2e^{\frac{5}{2}}\right)\sin(dx+c)^{\frac{3}{2}}}{30\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/30*(-27*I*sqrt(2)*sqrt(-I)*a^2*cos(d*x + c)*e^(5/2)*weierstrassZeta(4, 0,
 weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) + 27*I*sqrt(2)*s
qrt(I)*a^2*cos(d*x + c)*e^(5/2)*weierstrassZeta(4, 0, weierstrassPInverse(4
, 0, cos(d*x + c) - I*sin(d*x + c))) - 30*a^2*arctan(2*(76*cos(d*x + c)^2 +
 425*(sin(d*x + c) - 1)*sqrt(sin(d*x + c)) - 152*sin(d*x + c) - 152)/(361*c
os(d*x + c)^2 + 978*sin(d*x + c) - 722))*cos(d*x + c)*e^(5/2) + 15*a^2*cos(
d*x + c)*e^(5/2)*log((cos(d*x + c)^2 - 4*(sin(d*x + c) + 1)*sqrt(sin(d*x +
c)) - 6*sin(d*x + c) - 2)/(cos(d*x + c)^2 + 2*sin(d*x + c) - 2)) - 2*(6*a^2
*cos(d*x + c)^2*e^(5/2) + 20*a^2*cos(d*x + c)*e^(5/2) - 15*a^2*e^(5/2))*sin
(d*x + c)^(3/2))/(d*cos(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(e*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*(e*sin(d*x + c))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\sin(c+d\,x))^{5/2}\left(a+\frac{a}{\cos(c+d\,x)}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(5/2)*(a + a/cos(c + d*x))^2,x)
```

```
[Out] int((e*sin(c + d*x))^(5/2)*(a + a/cos(c + d*x))^2, x)
```

3.115 $\int (a+a\sec(c+dx))^2(e\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=192

$$\frac{2a^2e^{3/2}\text{ArcTan}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{2a^2e^{3/2}\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}-\frac{a^2e^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(\ldots}}{3d\sqrt{e\sin(c+dx)}}$$

```
[Out] 2*a^2*e^(3/2)*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))/d+2*a^2*e^(3/2)*arctanh(
(e*sin(d*x+c))^(1/2)/e^(1/2))/d+1/3*a^2*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(
1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))
*sin(d*x+c)^(1/2)/d/(e*sin(d*x+c))^(1/2)-4*a^2*e*(e*sin(d*x+c))^(1/2)/d-2/3
*a^2*e*cos(d*x+c)*(e*sin(d*x+c))^(1/2)/d+a^2*e*sec(d*x+c)*(e*sin(d*x+c))^(1
/2)/d
```

Rubi [A]
time = 0.26, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3957, 2952, 2715, 2721, 2720, 2644, 327, 335, 218, 212, 209, 2646}

$$\frac{2a^2e^{3/2}\text{ArcTan}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{2a^2e^{3/2}\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}-\frac{a^2e^2\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{3d\sqrt{e\sin(c+dx)}}-\frac{4a^2e\sqrt{e\sin(c+dx)}}{d}-\frac{2a^2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}+\frac{a^2e\sec(c+dx)\sqrt{e\sin(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*(e*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*a^2*e^(3/2)*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d + (2*a^2*e^(3/2)*Arc
Tanh[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d - (a^2*e^2*EllipticF[(c - Pi/2 + d*x)
/2, 2]*Sqrt[Sin[c + d*x]])/(3*d*Sqrt[e*Sin[c + d*x]]) - (4*a^2*e*Sqrt[e*Sin
[c + d*x]])/d - (2*a^2*e*Cos[c + d*x]*Sqrt[e*Sin[c + d*x]])/(3*d) + (a^2*e*
Sec[c + d*x]*Sqrt[e*Sin[c + d*x]])/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
```

```
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2646

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Sin[e + f*
x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+a\sec(c+dx))^2(e\sin(c+dx))^{3/2}\,dx = \int (-a-a\cos(c+dx))^2\sec^2(c+dx)(e\sin(c+dx))^{3/2}\,dx$$

$$= \int \left(a^2(e\sin(c+dx))^{3/2}+2a^2\sec(c+dx)(e\sin(c+dx))^{3/2}+\right.$$

$$= a^2\int (e\sin(c+dx))^{3/2}\,dx + a^2\int \sec^2(c+dx)(e\sin(c+dx))^{3/}$$

$$= -\frac{2a^2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}+\frac{a^2e\sec(c+dx)\sqrt{e\sin(c}}{d}$$

$$= -\frac{4a^2e\sqrt{e\sin(c+dx)}}{d}-\frac{2a^2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d}+$$

$$= -\frac{a^2e^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{3d\sqrt{e\sin(c+dx)}}-\frac{4a^2e\sqrt{e\sin(c+}}{d}$$

$$= -\frac{a^2e^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{3d\sqrt{e\sin(c+dx)}}-\frac{4a^2e\sqrt{e\sin(c+}}{d}$$

$$= \frac{2a^2e^{3/2}\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{2a^2e^{3/2}\tanh^{-1}\left(\frac{\sqrt{e\sin(}}{\sqrt{}}\right.}{d}$$

Mathematica [A]
time = 30.40, size = 220, normalized size = 1.15

$$\frac{16a^2e\cos^4\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt{e\sin(c+dx)}\left(6\text{ArcTan}\left(\sqrt{\sin(c+dx)}\right)\sqrt{\cos^2(c+dx)}-\sqrt{\cos^2(c+dx)}F\left(\text{ArcSin}\left(\sqrt{\sin(c+dx)}\right)\middle|-1\right)-3\sqrt{\cos^2(c+dx)}\log\left(1-\sqrt{\sin(c+dx)}\right)+3\sqrt{\cos^2(c+dx)}\log\left(1+\sqrt{\sin(c+dx)}\right)+\sqrt{\sin(c+dx)}-12\sqrt{\cos^2(c+dx)}\sqrt{\sin(c+dx)}+2\sin^{\frac{5}{2}}(c+dx)\right)\sin^4\left(\frac{1}{2}\text{ArcSin}(\sin(c+dx))\right)}{3d\sin^{\frac{9}{2}}(c+dx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*(e*Sin[c + d*x])^(3/2),x]
```

```
[Out] (16*a^2*e*Cos[(c + d*x)/2]^4*Sec[c + d*x]*Sqrt[e*Sin[c + d*x]]*(6*ArcTan[Sq
rt[Sin[c + d*x]]]*Sqrt[Cos[c + d*x]^2] - Sqrt[Cos[c + d*x]^2]*EllipticF[Arc
Sin[Sqrt[Sin[c + d*x]]], -1] - 3*Sqrt[Cos[c + d*x]^2]*Log[1 - Sqrt[Sin[c +
d*x]]] + 3*Sqrt[Cos[c + d*x]^2]*Log[1 + Sqrt[Sin[c + d*x]]] + Sqrt[Sin[c +
d*x]] - 12*Sqrt[Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]] + 2*Sin[c + d*x]^(5/2))*
Sin[ArcSin[Sin[c + d*x]]/2]^4)/(3*d*Sin[c + d*x]^(9/2))
```

Maple [A]
time = 0.22, size = 201, normalized size = 1.05

method	result
default	$a^2\left(12\cos(dx+c)e^{\frac{3}{2}}\sqrt{e\sin(dx+c)}\ \text{arctanh}\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)+12\cos(dx+c)e^{\frac{3}{2}}\sqrt{e\sin(dx+c)}\ \text{arctan}\left(\frac{\sqrt{e}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*a^2*(12*cos(d*x+c)*e^(3/2)*(e*sin(d*x+c
))^(1/2)*arctanh((e*sin(d*x+c))^(1/2)/e^(1/2))+12*cos(d*x+c)*e^(3/2)*(e*sin
(d*x+c))^(1/2)*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))+sin(d*x+c)^(1/2)*Ellipt
icF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+
2)^(1/2)*e^2-4*cos(d*x+c)^2*e^2*sin(d*x+c)-24*e^2*sin(d*x+c)*cos(d*x+c)+6*e
^2*sin(d*x+c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] e^(3/2)*integrate((a*sec(d*x + c) + a)^2*sin(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 9.16, size = 279, normalized size = 1.45

$$-\frac{\sqrt{2}\sqrt{-i}\,a^2\cos(dx+c)\,e^{\frac{3}{2}}\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\,\sin(dx+c))+\sqrt{2}\sqrt{i}\,a^2\cos(dx+c)\,e^{\frac{3}{2}}\text{weierstrassPInverse}(4,0,\cos(dx+c)-i\,\sin(dx+c))-6\,a^2\arctan\left(-\frac{2\left(76\,\cos(dx+c)^2-425\,(\sin(dx+c)-1)\sqrt{\sin(dx+c)}-152\,\sin(dx+c)-152\right)}{361\,\cos(dx+c)^2+978\,\sin(dx+c)-722}\right)\cos(dx+c)\,e^{\frac{3}{2}}-3\,a^2\cos(dx+c)\,e^{\frac{3}{2}}\log\left(\frac{\cos(dx+c)^2-4\,(\sin(dx+c)+1)\sqrt{\sin(dx+c)}-6\,\sin(dx+c)-2}{\cos(dx+c)^2+2\,\sin(dx+c)-2}\right)+2\left(2\,a^2\cos(dx+c)^2\,e^{\frac{3}{2}}+12\,a^2\cos(dx+c)\,e^{\frac{3}{2}}-3\,a^2e^{\frac{3}{2}}\right)\sqrt{\sin(dx+c)}}{6\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(3/2),x, algorithm="fricas")

[Out] -1/6*(sqrt(2)*sqrt(-I)*a^2*cos(d*x + c)*e^(3/2)*weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*sqrt(I)*a^2*cos(d*x + c)*e^(3/2)*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) - 6*a^2*arctan(-2*(76*cos(d*x + c)^2 - 425*(sin(d*x + c) - 1)*sqrt(sin(d*x + c)) - 152*sin(d*x + c) - 152)/(361*cos(d*x + c)^2 + 978*sin(d*x + c) - 722))*cos(d*x + c)*e^(3/2) - 3*a^2*cos(d*x + c)*e^(3/2)*log((cos(d*x + c)^2 - 4*(sin(d*x + c) + 1)*sqrt(sin(d*x + c)) - 6*sin(d*x + c) - 2)/(cos(d*x + c)^2 + 2*sin(d*x + c) - 2)) + 2*(2*a^2*cos(d*x + c)^2*e^(3/2) + 12*a^2*cos(d*x + c)*e^(3/2) - 3*a^2*e^(3/2))*sqrt(sin(d*x + c)))/(d*cos(d*x + c))

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))**2*(e*sin(d*x+c))**(3/2),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 5008 deep

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((a*sec(d*x + c) + a)^2*(e*sin(d*x + c))^(3/2), x)

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\sin(c+d\,x))^{3/2}\left(a+\frac{a}{\cos(c+d\,x)}\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*sin(c + d*x))^(3/2)*(a + a/cos(c + d*x))^2,x)

[Out] int((e*sin(c + d*x))^(3/2)*(a + a/cos(c + d*x))^2, x)

3.116 $\int (a+a\sec(c+dx))^2\sqrt{e\sin(c+dx)}\,dx$

Optimal. Leaf size=138

$$-\frac{2a^2\sqrt{e}\,\text{ArcTan}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{2a^2\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{a^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(}}{d\sqrt{\sin(c+dx)}}$$

```
[Out] a^2*sec(d*x+c)*(e*sin(d*x+c))^(3/2)/d/e-2*a^2*arctan((e*sin(d*x+c))^(1/2)/e
^(1/2))*e^(1/2)/d+2*a^2*arctanh((e*sin(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/d-a^2
*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(co
s(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/d/sin(d*x+c)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3957, 2952, 2721, 2719, 2644, 335, 304, 209, 212, 2651}

$$-\frac{2a^2\sqrt{e}\,\text{ArcTan}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{2a^2\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{a^2\sec(c+dx)(e\sin(c+dx))^{3/2}}{de}+\frac{a^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Sqrt[e*Sin[c + d*x]],x]
```

```
[Out] (-2*a^2*Sqrt[e]*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d + (2*a^2*Sqrt[e]*Ar
cTanh[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d + (a^2*EllipticE[(c - Pi/2 + d*x)/2,
 2]*Sqrt[e*Sin[c + d*x]])/(d*Sqrt[Sin[c + d*x]]) + (a^2*Sec[c + d*x]*(e*Sin
[c + d*x])^(3/2))/(d*e)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
```

```
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2651

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(b*Sin[e + f*x])^(n + 1))*((a*Cos[e + f*x])^(m + 1)
/(a*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x]
)^n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
```

```
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+a\sec(c+dx))^2\sqrt{e\sin(c+dx)}\,dx=\int (-a-a\cos(c+dx))^2\sec^2(c+dx)\sqrt{e\sin(c+dx)}\,dx$$

$$=\int \left(a^2\sqrt{e\sin(c+dx)}+2a^2\sec(c+dx)\sqrt{e\sin(c+dx)}+a^2\text{se}\right.$$

$$=a^2\int \sqrt{e\sin(c+dx)}\,dx+a^2\int \sec^2(c+dx)\sqrt{e\sin(c+dx)}\,dx$$

$$=\frac{a^2\sec(c+dx)(e\sin(c+dx))^{3/2}}{de}-\frac{1}{2}a^2\int \sqrt{e\sin(c+dx)}\,dx+$$

$$=\frac{2a^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}+\frac{a^2\sec(c+dx)(e\sin}{de}$$

$$=\frac{a^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}+\frac{a^2\sec(c+dx)(e\sin(}{de}$$

$$=-\frac{2a^2\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{2a^2\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e\sin(}}{\sqrt{}}\right.}{d}$$

Mathematica [A]
time = 11.60, size = 206, normalized size = 1.49

$$\frac{16a^2\cos^4\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt{e\sin(c+dx)}\left(-2\text{ArcTan}\left(\sqrt{\sin(c+dx)}\right)\sqrt{\cos^2(c+dx)}+\sqrt{\cos^2(c+dx)}\,E\left(\text{ArcSin}\left(\sqrt{\sin(c+dx)}\right)\middle|-1\right)-\sqrt{\cos^2(c+dx)}\,F\left(\text{ArcSin}\left(\sqrt{\sin(c+dx)}\right)\middle|-1\right)-\sqrt{\cos^2(c+dx)}\log\left(1-\sqrt{\sin(c+dx)}\right)+\sqrt{\cos^2(c+dx)}\log\left(1+\sqrt{\sin(c+dx)}\right)+\sin^{\frac{3}{2}}(c+dx)\right)\sin^4\left(\frac{1}{2}\text{ArcSin}(\sin(c+dx))\right)}{d\sin^{\frac{9}{2}}(c+dx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Sqrt[e*Sin[c + d*x]],x]
```

```
[Out] (16*a^2*Cos[(c + d*x)/2]^4*Sec[c + d*x]*Sqrt[e*Sin[c + d*x]]*(-2*ArcTan[Sqr
t[Sin[c + d*x]]]*Sqrt[Cos[c + d*x]^2] + Sqrt[Cos[c + d*x]^2]*EllipticE[ArcS
in[Sqrt[Sin[c + d*x]]], -1] - Sqrt[Cos[c + d*x]^2]*EllipticF[ArcSin[Sqrt[Si
n[c + d*x]]], -1] - Sqrt[Cos[c + d*x]^2]*Log[1 - Sqrt[Sin[c + d*x]]] + Sqrt
[Cos[c + d*x]^2]*Log[1 + Sqrt[Sin[c + d*x]]] + Sin[c + d*x]^(3/2))*Sin[ArcS
in[Sin[c + d*x]]/2]^4)/(d*Sin[c + d*x]^(9/2))
```

Maple [A]
time = 0.24, size = 219, normalized size = 1.59

method	result
default	$\frac{a^2\left(-2\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}\,(dx+c)\right)\operatorname{EllipticE}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*a^2*(-2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*
x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))
*e+(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF(
(-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*e+4*cos(d*x+c)*e^(1/2)*(e*sin(d*x+c))^(1
/2)*arctanh((e*sin(d*x+c))^(1/2)/e^(1/2))-4*cos(d*x+c)*e^(1/2)*(e*sin(d*x+c
))^(1/2)*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))-2*cos(d*x+c)^2*e+2*e)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(1/2)*integrate((a*sec(d*x + c) + a)^2*sqrt(sin(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.48, size = 255, normalized size = 1.85

$$\frac{i\sqrt{2}\,\sqrt{-i}\,a^2\cos(dx+c)\,e^{\frac{1}{2}}\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c)))-i\sqrt{2}\,\sqrt{i}\,a^2\cos(dx+c)\,e^{\frac{1}{2}}\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(dx+c)-i\sin(dx+c)))-2\,a^2\arctan\left(\frac{2\left(76\cos(dx+c)^2+425(\sin(dx+c)-1)\sqrt{\sin(dx+c)}-152\sin(dx+c)-152\right)}{361\cos(dx+c)^2+978\sin(dx+c)-722}\right)\cos(dx+c)\,e^{\frac{1}{2}}+a^2\cos(dx+c)\,e^{\frac{1}{2}}\log\left(\frac{\cos(dx+c)^2-4(\sin(dx+c)+1)\sqrt{\sin(dx+c)}-6\sin(dx+c)-2}{\cos(dx+c)^2+2\sin(dx+c)-2}\right)+2\,a^2e^{\frac{1}{2}}\sin(dx+c)^{\frac{3}{2}}}{2\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(I*sqrt(2)*sqrt(-I)*a^2*cos(d*x + c)*e^(1/2)*weierstrassZeta(4, 0, weie
rstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) - I*sqrt(2)*sqrt(I)*a
^2*cos(d*x + c)*e^(1/2)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos
(d*x + c) - I*sin(d*x + c))) - 2*a^2*arctan(2*(76*cos(d*x + c)^2 + 425*(sin
(d*x + c) - 1)*sqrt(sin(d*x + c)) - 152*sin(d*x + c) - 152)/(361*cos(d*x +
c)^2 + 978*sin(d*x + c) - 722))*cos(d*x + c)*e^(1/2) + a^2*cos(d*x + c)*e^(
1/2)*log((cos(d*x + c)^2 - 4*(sin(d*x + c) + 1)*sqrt(sin(d*x + c)) - 6*sin(
d*x + c) - 2)/(cos(d*x + c)^2 + 2*sin(d*x + c) - 2)) + 2*a^2*e^(1/2)*sin(d*
x + c)^(3/2))/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int \sqrt{e\sin(c+dx)}\,dx+\int 2\sqrt{e\sin(c+dx)}\sec(c+dx)\,dx+\int \sqrt{e\sin(c+dx)}\sec^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(e*sin(d*x+c))**(1/2),x)
```

```
[Out] a**2*(Integral(sqrt(e*sin(c + d*x)), x) + Integral(2*sqrt(e*sin(c + d*x))*s
ec(c + d*x), x) + Integral(sqrt(e*sin(c + d*x))*sec(c + d*x)**2, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*sqrt(e*sin(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{e\,\sin\left(c+d\,x\right)}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(1/2)*(a + a/cos(c + d*x))^2,x)
```

```
[Out] int((e*sin(c + d*x))^(1/2)*(a + a/cos(c + d*x))^2, x)
```

3.117 $\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx$

Optimal. Leaf size=139

$$\frac{2a^2\text{ArcTan}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}}+\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}}+\frac{3a^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{d\sqrt{e\sin(c+dx)}}$$

```
[Out] 2*a^2*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))/d/e^(1/2)+2*a^2*arctanh((e*sin(d
*x+c))^(1/2)/e^(1/2))/d/e^(1/2)-3*a^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/s
in(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d
*x+c)^(1/2)/d/(e*sin(d*x+c))^(1/2)+a^2*sec(d*x+c)*(e*sin(d*x+c))^(1/2)/d/e
```

Rubi [A]

time = 0.22, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3957, 2952, 2721, 2720, 2644, 335, 218, 212, 209, 2651}

$$\frac{2a^2\text{ArcTan}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}}+\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}}+\frac{a^2\sec(c+dx)\sqrt{e\sin(c+dx)}}{de}+\frac{3a^2\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{d\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/Sqrt[e*Sin[c + d*x]],x]
```

```
[Out] (2*a^2*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/(d*Sqrt[e]) + (2*a^2*ArcTanh[S
qrt[e*Sin[c + d*x]]/Sqrt[e]])/(d*Sqrt[e]) + (3*a^2*EllipticF[(c - Pi/2 + d*
x)/2, 2]*Sqrt[Sin[c + d*x]])/(d*Sqrt[e*Sin[c + d*x]]) + (a^2*Sec[c + d*x]*S
qrt[e*Sin[c + d*x]])/(d*e)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
```

```
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2651

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(b*Sin[e + f*x])^(n + 1))*((a*Cos[e + f*x])^(m + 1)
/(a*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x]
)^n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
```

```
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx &= \int \frac{(-a-a\cos(c+dx))^2\sec^2(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx \\
&= \int \left(\frac{a^2}{\sqrt{e\sin(c+dx)}}+\frac{2a^2\sec(c+dx)}{\sqrt{e\sin(c+dx)}}+\frac{a^2\sec^2(c+dx)}{\sqrt{e\sin(c+dx)}}\right)dx \\
&= a^2\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx + a^2\int \frac{\sec^2(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx + \left(2a^2\right)\int \frac{\sec(c+dx}{\sqrt{e\sin(c+d}} \\
&= \frac{a^2\sec(c+dx)\sqrt{e\sin(c+dx)}}{de} + \frac{1}{2}a^2\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx + \frac{\left(2a^2\right)\operatorname{Subst}\Big(\int}{} \\
&= \frac{2a^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{d\sqrt{e\sin(c+dx)}} + \frac{a^2\sec(c+dx)\sqrt{e\sin(c+dx)}}{de} + \frac{(4}{} \\
&= \frac{3a^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{d\sqrt{e\sin(c+dx)}} + \frac{a^2\sec(c+dx)\sqrt{e\sin(c+dx)}}{de} + \frac{(2}{} \\
&= \frac{2a^2\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}} + \frac{2a^2\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}} + \frac{3a^2F\left(\frac{1}{2}(c\right.}{}
\end{aligned}$$

Mathematica [A]
time = 72.92, size = 179, normalized size = 1.29

$$\frac{a^2\cos^4\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sec^4\left(\frac{1}{2}\operatorname{ArcSin}(\sin(c+dx))\right)\left(2\operatorname{ArcTan}\left(\sqrt{\sin(c+dx)}\right)\sqrt{\cos^2(c+dx)}+3\sqrt{\cos^2(c+dx)}\,F\left(\operatorname{ArcSin}\left(\sqrt{\sin(c+dx)}\right)\middle|-1\right)-\sqrt{\cos^2(c+dx)}\log\left(1-\sqrt{\sin(c+dx)}\right)+\sqrt{\cos^2(c+dx)}\log\left(1+\sqrt{\sin(c+dx)}\right)+\sqrt{\sin(c+dx)}\right)\sqrt{\sin(c+dx)}}{d\sqrt{e\sin(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/Sqrt[e*Sin[c + d*x]],x]
```

```
[Out] (a^2*Cos[(c + d*x)/2]^4*Sec[c + d*x]*Sec[ArcSin[Sin[c + d*x]]/2]^4*(2*ArcTa
n[Sqrt[Sin[c + d*x]]]*Sqrt[Cos[c + d*x]^2] + 3*Sqrt[Cos[c + d*x]^2]*Ellipti
cF[ArcSin[Sqrt[Sin[c + d*x]]], -1] - Sqrt[Cos[c + d*x]^2]*Log[1 - Sqrt[Sin[
c + d*x]]] + Sqrt[Cos[c + d*x]^2]*Log[1 + Sqrt[Sin[c + d*x]]] + Sqrt[Sin[c
+ d*x]])*Sqrt[Sin[c + d*x]])/(d*Sqrt[e*Sin[c + d*x]])
```

Maple [A]
time = 0.20, size = 163, normalized size = 1.17

method	result
default	$-\frac{a^2\left(3\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}\,(dx+c)\right)\text{EllipticF}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right.\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/e^(1/2)/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*a^2*(3*(-sin(d*x+c)+1)^(1/2)*(
2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*
2^(1/2))*e^(1/2)-4*cos(d*x+c)*(e*sin(d*x+c))^(1/2)*arctanh((e*sin(d*x+c))^(
1/2)/e^(1/2))-4*cos(d*x+c)*(e*sin(d*x+c))^(1/2)*arctan((e*sin(d*x+c))^(1/2)
/e^(1/2))-2*e^(1/2)*sin(d*x+c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-1/2)*integrate((a*sec(d*x + c) + a)^2/sqrt(sin(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.94, size = 197, normalized size = 1.42

$$\frac{\left(3\sqrt{2}\sqrt{-i}\,a^2\cos(dx+c)\,\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c))+3\sqrt{2}\sqrt{i}\,a^2\cos(dx+c)\,\text{weierstrassPInverse}(4,0,\cos(dx+c)-i\sin(dx+c))+2a^2\arctan\left(\frac{\sin(dx+c)-1}{2\sqrt{\sin(dx+c)}}\right)\cos(dx+c)+a^2\cos(dx+c)\log\left(\frac{\cos(dx+c)^2-4(\sin(dx+c)+1)\sqrt{\sin(dx+c)}-6\sin(dx+c)-2}{\cos(dx+c)^2+2\sin(dx+c)-2}\right)+2a^2\sqrt{\sin(dx+c)}\right)e^{\left(-\frac{1}{2}\right)}}{2d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(3*sqrt(2)*sqrt(-I)*a^2*cos(d*x + c)*weierstrassPInverse(4, 0, cos(d*x
+ c) + I*sin(d*x + c)) + 3*sqrt(2)*sqrt(I)*a^2*cos(d*x + c)*weierstrassPInv
erse(4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*a^2*arctan(1/2*(sin(d*x + c)
- 1)/sqrt(sin(d*x + c)))*cos(d*x + c) + a^2*cos(d*x + c)*log((cos(d*x + c)^
2 - 4*(sin(d*x + c) + 1)*sqrt(sin(d*x + c)) - 6*sin(d*x + c) - 2)/(cos(d*x
+ c)^2 + 2*sin(d*x + c) - 2)) + 2*a^2*sqrt(sin(d*x + c)))*e^(-1/2)/(d*cos(d
*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int\frac{1}{\sqrt{e\sin(c+dx)}}\,dx+\int\frac{2\sec(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx+\int\frac{\sec^2(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2/(e*sin(d*x+c))**(1/2),x)
```

```
[Out] a**2*(Integral(1/sqrt(e*sin(c + d*x)), x) + Integral(2*sec(c + d*x)/sqrt(e*
sin(c + d*x)), x) + Integral(sec(c + d*x)**2/sqrt(e*sin(c + d*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/sqrt(e*sin(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^2}{\sqrt{e\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/(e*sin(c + d*x))^(1/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^2/(e*sin(c + d*x))^(1/2), x)
```

3.118 $\int \frac{(a+a\sec(c+dx))^2}{(e\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=224

$$-\frac{2a^2\text{ArcTan}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}}+\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}}-\frac{4a^2}{de\sqrt{e\sin(c+dx)}}-\frac{2a^2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}$$

```
[Out] -2*a^2*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))/d/e^(3/2)+2*a^2*arctanh((e*sin(
d*x+c))^(1/2)/e^(1/2))/d/e^(3/2)+3*a^2*sec(d*x+c)*(e*sin(d*x+c))^(3/2)/d/e^
3-4*a^2/d/e/(e*sin(d*x+c))^(1/2)-2*a^2*cos(d*x+c)/d/e/(e*sin(d*x+c))^(1/2)-
2*a^2*sec(d*x+c)/d/e/(e*sin(d*x+c))^(1/2)+5*a^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^
2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1
/2))*(e*sin(d*x+c))^(1/2)/d/e^2/sin(d*x+c)^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3957, 2952, 2716, 2721, 2719, 2644, 331, 335, 304, 209, 212, 2650, 2651}

$$-\frac{2a^2\text{ArcTan}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}}+\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}}+\frac{3a^2\sec(c+dx)(e\sin(c+dx))^{3/2}}{de^3}-\frac{5a^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{de^2\sqrt{\sin(c+dx)}}-\frac{4a^2}{de\sqrt{e\sin(c+dx)}}-\frac{2a^2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}-\frac{2a^2\sec(c+dx)}{de\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/(e*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*a^2*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/(d*e^(3/2)) + (2*a^2*ArcTanh[
Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/(d*e^(3/2)) - (4*a^2)/(d*e*Sqrt[e*Sin[c + d*
x]]) - (2*a^2*Cos[c + d*x])/(d*e*Sqrt[e*Sin[c + d*x]]) - (2*a^2*Sec[c + d*x
])/(d*e*Sqrt[e*Sin[c + d*x]]) - (5*a^2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqr
t[e*Sin[c + d*x]])/(d*e^2*Sqrt[Sin[c + d*x]]) + (3*a^2*Sec[c + d*x]*(e*Sin[
c + d*x])^(3/2))/(d*e^3)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a
/b, 0]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2650

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2651

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[-(b*Sin[e + f*x])^(n + 1))*((a*Cos[e + f*x])^(m + 1)
/(a*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x]
)^n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
```

```
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\sec(c+dx))^2}{(e\sin(c+dx))^{3/2}}\,dx &= \int \frac{(-a-a\cos(c+dx))^2\sec^2(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx \\
&= \int \left(\frac{a^2}{(e\sin(c+dx))^{3/2}}+\frac{2a^2\sec(c+dx)}{(e\sin(c+dx))^{3/2}}+\frac{a^2\sec^2(c+dx)}{(e\sin(c+dx))^{3/2}}\right)dx \\
&= a^2\int \frac{1}{(e\sin(c+dx))^{3/2}}\,dx + a^2\int \frac{\sec^2(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx + \left(2a^2\right)\int \frac{\sec(c+}{(e\sin(c+d} \\
&= -\frac{2a^2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}} - \frac{2a^2\sec(c+dx)}{de\sqrt{e\sin(c+dx)}} - \frac{a^2\int\sqrt{e\sin(c+dx)}\,dx}{e^2} + \frac{(3a^2)}{} \\
&= -\frac{4a^2}{de\sqrt{e\sin(c+dx)}} - \frac{2a^2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}} - \frac{2a^2\sec(c+dx)}{de\sqrt{e\sin(c+dx)}} + \frac{3a^2\sec(c+}{} \\
&= -\frac{4a^2}{de\sqrt{e\sin(c+dx)}} - \frac{2a^2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}} - \frac{2a^2\sec(c+dx)}{de\sqrt{e\sin(c+dx)}} - \frac{2a^2E\left(\frac{1}{2}(c\right.}{} \\
&= -\frac{4a^2}{de\sqrt{e\sin(c+dx)}} - \frac{2a^2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}} - \frac{2a^2\sec(c+dx)}{de\sqrt{e\sin(c+dx)}} - \frac{5a^2E\left(\frac{1}{2}(c\right.}{} \\
&= -\frac{2a^2\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}} + \frac{2a^2\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}} - \frac{4}{de\sqrt{e\sin}}
\end{aligned}
$$

Mathematica [A]
time = 17.68, size = 260, normalized size = 1.16

$$
-\frac{\left(1+\cos\left(2\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\right)^2\cos(c+dx)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2\sin^{\frac{3}{2}}(c+dx)\left(-1+\sin^2(c+dx)\right)\left(-2\text{ArcTan}\left(\sqrt{\sin(c+dx)}\right)-5E\left(\text{ArcSin}\left(\sqrt{\sin(c+dx)}\right)\middle|-1\right)+5F\left(\text{ArcSin}\left(\sqrt{\sin(c+dx)}\right)\middle|-1\right)-\log\left(1-\sqrt{\sin(c+dx)}\right)+\log\left(1+\sqrt{\sin(c+dx)}\right)-\frac{4}{\sqrt{\sin(c+dx)}}+\frac{-4+5\sin^2(c+dx)}{\sqrt{\sin(c+dx)}\sqrt{1-\sin^2(c+dx)}}\right)}{4d\left(1+\cos\left(2\left(\frac{c}{2}+\frac{1}{2}(-c+\text{ArcSin}(\sin(c+dx)))\right)\right)\right)^2(e\sin(c+dx))^{3/2}\sqrt{1-\sin^2(c+dx)}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/(e*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/4*((1 + Cos[2*(c/2 + (d*x)/2)])^2*Cos[c + d*x]*Sec[c/2 + (d*x)/2]^4*(a +
 a*Sec[c + d*x])^2*Sin[c + d*x]^(3/2)*(-1 + Sin[c + d*x]^2)*(-2*ArcTan[Sqrt
[Sin[c + d*x]]] - 5*EllipticE[ArcSin[Sqrt[Sin[c + d*x]]], -1] + 5*EllipticF
[ArcSin[Sqrt[Sin[c + d*x]]], -1] - Log[1 - Sqrt[Sin[c + d*x]]] + Log[1 + Sq
rt[Sin[c + d*x]]] - 4/Sqrt[Sin[c + d*x]] + (-4 + 5*Sin[c + d*x]^2)/(Sqrt[Si
n[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])))/(d*(1 + Cos[2*(c/2 + (-c + ArcSin[S
in[c + d*x]])/2)])^2*(e*Sin[c + d*x])^(3/2)*Sqrt[1 - Sin[c + d*x]^2])
```

Maple [A]
time = 0.24, size = 238, normalized size = 1.06

method	result
default	$\frac{a^2\left(10\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}\,(dx+c)\right)\text{EllipticE}\left(\sqrt{-\sin(dx+c)+1}\,,\frac{\sqrt{2}}{2}\right)\sqrt{-\sin(dx+c)+1}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/e^(5/2)/(e*sin(d*x+c))^(1/2)/cos(d*x+c)*a^2*(10*(2*sin(d*x+c)+2)^(1/2)*
sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*(-sin(d*x+c)+
1)^(1/2)*e^(3/2)-5*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(
d*x+c)+1)^(1/2),1/2*2^(1/2))*(-sin(d*x+c)+1)^(1/2)*e^(3/2)-10*e^(3/2)*cos(d
*x+c)^2-8*e^(3/2)*cos(d*x+c)+4*arctanh((e*sin(d*x+c))^(1/2)/e^(1/2))*(e*sin
(d*x+c))^(1/2)*cos(d*x+c)*e-4*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))*(e*sin(d
*x+c))^(1/2)*cos(d*x+c)*e+2*e^(3/2))/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.37, size = 262, normalized size = 1.17

$$\frac{\left(-5i\sqrt{2}\sqrt{-i}\,a^2\cos(dx+c)\sin(dx+c)\,\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c)))+5i\sqrt{2}\sqrt{i}\,a^2\cos(dx+c)\sin(dx+c)\,\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(dx+c)-i\sin(dx+c)))-2a^2\arctan\left(\frac{\sin(dx+c)-1}{2\sqrt{\sin(dx+c)}}\right)\cos(dx+c)\sin(dx+c)+a^2\cos(dx+c)\log\left(\frac{\cos(dx+c)^2-4(\sin(dx+c)+1)\sqrt{\sin(dx+c)}-6\sin(dx+c)-2}{\cos(dx+c)^2+2\sin(dx+c)-2}\right)\sin(dx+c)-2\left(5a^2\cos(dx+c)^2+4a^2\cos(dx+c)-a^2\right)\sqrt{\sin(dx+c)}\right)e^{\left(-\frac{3}{2}\right)}}{2d\cos(dx+c)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(-5*I*sqrt(2)*sqrt(-I)*a^2*cos(d*x + c)*sin(d*x + c)*weierstrassZeta(4,
 0, weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) + 5*I*sqrt(2)
*sqrt(I)*a^2*cos(d*x + c)*sin(d*x + c)*weierstrassZeta(4, 0, weierstrassPIn
verse(4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*a^2*arctan(1/2*(sin(d*x + c
) - 1)/sqrt(sin(d*x + c)))*cos(d*x + c)*sin(d*x + c) + a^2*cos(d*x + c)*log
((cos(d*x + c)^2 - 4*(sin(d*x + c) + 1)*sqrt(sin(d*x + c)) - 6*sin(d*x + c)
 - 2)/(cos(d*x + c)^2 + 2*sin(d*x + c) - 2))*sin(d*x + c) - 2*(5*a^2*cos(d*
x + c)^2 + 4*a^2*cos(d*x + c) - a^2)*sqrt(sin(d*x + c)))*e^(-3/2)/(d*cos(d*
x + c)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int\frac{1}{(e\sin(c+dx))^{\frac{3}{2}}}\,dx+\int\frac{2\sec(c+dx)}{(e\sin(c+dx))^{\frac{3}{2}}}\,dx+\int\frac{\sec^2(c+dx)}{(e\sin(c+dx))^{\frac{3}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2/(e*sin(d*x+c))**(3/2),x)
```

```
[Out] a**2*(Integral((e*sin(c + d*x))**(-3/2), x) + Integral(2*sec(c + d*x)/(e*si
n(c + d*x))**(3/2), x) + Integral(sec(c + d*x)**2/(e*sin(c + d*x))**(3/2),
x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/(e*sin(d*x + c))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}{(e\,\sin(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/(e*sin(c + d*x))^(3/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^2/(e*sin(c + d*x))^(3/2), x)
```

3.119 $\int \frac{(a+a\sec(c+dx))^2}{(e\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=234

$$\frac{2a^2\text{ArcTan}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}}+\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}}-\frac{4a^2}{3de(e\sin(c+dx))^{3/2}}-\frac{2a^2\cos(c+dx)}{3de(e\sin(c+dx))^{3}}$$

```
[Out] 2*a^2*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))/d/e^(5/2)+2*a^2*arctanh((e*sin(d
*x+c))^(1/2)/e^(1/2))/d/e^(5/2)-4/3*a^2/d/e/(e*sin(d*x+c))^(3/2)-2/3*a^2*co
s(d*x+c)/d/e/(e*sin(d*x+c))^(3/2)-2/3*a^2*sec(d*x+c)/d/e/(e*sin(d*x+c))^(3/
2)-7/3*a^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*El
lipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/d/e^2/(e*sin(d*
x+c))^(1/2)+5/3*a^2*sec(d*x+c)*(e*sin(d*x+c))^(1/2)/d/e^3
```

Rubi [A]

time = 0.29, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3957, 2952, 2716, 2721, 2720, 2644, 331, 335, 218, 212, 209, 2650, 2651}

$$\frac{2a^2\text{ArcTan}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}}+\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}}+\frac{5a^2\sec(c+dx)\sqrt{e\sin(c+dx)}}{3de^3}+\frac{7a^2\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{3de^2\sqrt{e\sin(c+dx)}}-\frac{4a^2}{3de(e\sin(c+dx))^{3/2}}-\frac{2a^2\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}}-\frac{2a^2\sec(c+dx)}{3de(e\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*a^2*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/(d*e^(5/2)) + (2*a^2*ArcTanh[S
qrt[e*Sin[c + d*x]]/Sqrt[e]])/(d*e^(5/2)) - (4*a^2)/(3*d*e*(e*Sin[c + d*x])
^(3/2)) - (2*a^2*Cos[c + d*x])/(3*d*e*(e*Sin[c + d*x])^(3/2)) - (2*a^2*Sec[
c + d*x])/(3*d*e*(e*Sin[c + d*x])^(3/2)) + (7*a^2*EllipticF[(c - Pi/2 + d*x
)/2, 2]*Sqrt[Sin[c + d*x]])/(3*d*e^2*Sqrt[e*Sin[c + d*x]]) + (5*a^2*Sec[c +
 d*x]*Sqrt[e*Sin[c + d*x]])/(3*d*e^3)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2650

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2651

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(b*Sin[e + f*x])^(n + 1))*((a*Cos[e + f*x])^(m + 1)
/(a*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x]
)^n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
```

```
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\sec(c+dx))^2}{(e\sin(c+dx))^{5/2}}\,dx &= \int \frac{(-a-a\cos(c+dx))^2\sec^2(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx \\
&= \int \left(\frac{a^2}{(e\sin(c+dx))^{5/2}}+\frac{2a^2\sec(c+dx)}{(e\sin(c+dx))^{5/2}}+\frac{a^2\sec^2(c+dx)}{(e\sin(c+dx))^{5/2}}\right)dx \\
&= a^2\int \frac{1}{(e\sin(c+dx))^{5/2}}\,dx+a^2\int \frac{\sec^2(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx+\left(2a^2\right)\int \frac{\sec(c+
}{(e\sin(c+
} \\
&= -\frac{2a^2\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}}-\frac{2a^2\sec(c+dx)}{3de(e\sin(c+dx))^{3/2}}+\frac{a^2\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{3e^2}+ \\
&= -\frac{4a^2}{3de(e\sin(c+dx))^{3/2}}-\frac{2a^2\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}}-\frac{2a^2\sec(c+dx)}{3de(e\sin(c+dx))^{3/2}}+\frac{5a^2}{} \\
&= -\frac{4a^2}{3de(e\sin(c+dx))^{3/2}}-\frac{2a^2\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}}-\frac{2a^2\sec(c+dx)}{3de(e\sin(c+dx))^{3/2}}+\frac{2a^2}{} \\
&= -\frac{4a^2}{3de(e\sin(c+dx))^{3/2}}-\frac{2a^2\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}}-\frac{2a^2\sec(c+dx)}{3de(e\sin(c+dx))^{3/2}}+\frac{7a^2}{} \\
&= \frac{2a^2\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}}+\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}}-\frac{4}{3de(e\sin(}
\end{aligned}
$$

Mathematica [A]
time = 40.43, size = 241, normalized size = 1.03

$$
\frac{a^2\cos^2\left(\frac{1}{2}(c+dx)\right)\cot\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sec^4\left(\frac{1}{2}\text{ArcSin}(\sin(c+dx))\right)\left(-1-8\sqrt{\cos^2(c+dx)}-7\cos(2(c+dx))+12\text{ArcTan}\left(\sqrt{\sin(c+dx)}\right)\sqrt{\cos^2(c+dx)}\sin^{\frac{3}{2}}(c+dx)+14\sqrt{\cos^2(c+dx)}F\left(\text{ArcSin}\left(\sqrt{\sin(c+dx)}\right)\middle|-1\right)\sin^{\frac{3}{2}}(c+dx)-6\sqrt{\cos^2(c+dx)}\log\left(1-\sqrt{\sin(c+dx)}\right)\sin^{\frac{3}{2}}(c+dx)+6\sqrt{\cos^2(c+dx)}\log\left(1+\sqrt{\sin(c+dx)}\right)\sin^{\frac{3}{2}}(c+dx)\right)}{12de^2\sqrt{e\sin(c+dx)}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] (a^2*Cos[(c + d*x)/2]^2*Cot[(c + d*x)/2]*Sec[c + d*x]*Sec[ArcSin[Sin[c + d*
x]]/2]^4*(-1 - 8*Sqrt[Cos[c + d*x]^2] - 7*Cos[2*(c + d*x)] + 12*ArcTan[Sqrt
[Sin[c + d*x]]]*Sqrt[Cos[c + d*x]^2]*Sin[c + d*x]^(3/2) + 14*Sqrt[Cos[c + d
*x]^2]*EllipticF[ArcSin[Sqrt[Sin[c + d*x]]], -1]*Sin[c + d*x]^(3/2) - 6*Sqr
t[Cos[c + d*x]^2]*Log[1 - Sqrt[Sin[c + d*x]]]*Sin[c + d*x]^(3/2) + 6*Sqrt[C
os[c + d*x]^2]*Log[1 + Sqrt[Sin[c + d*x]]]*Sin[c + d*x]^(3/2)))/(12*d*e^2*S
qrt[e*Sin[c + d*x]])
```

Maple [A]
time = 0.28, size = 301, normalized size = 1.29

method	result
default	$\frac{a^{2}\left(7\sqrt{-\sin\left(dx+c\right)+1}\,\sqrt{2\sin\left(dx+c\right)+2}\,\left(\sin^{\frac{7}{2}}\left(dx+c\right)\right)\operatorname{EllipticF}\left(\sqrt{-\sin\left(dx+c\right)+1},\frac{\sqrt{2}}{2}\right)e^{\frac{7}{2}}-1}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/e^(9/2)/(e*sin(d*x+c))^(3/2)/cos(d*x+c)/(cos(d*x+c)^2-1)*a^2*(7*(-sin(d
*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticF((-sin(d*x+
c)+1)^(1/2),1/2*2^(1/2))*e^(7/2)-14*e^(7/2)*cos(d*x+c)^4-8*e^(7/2)*cos(d*x+
c)^3+12*(e*sin(d*x+c))^(3/2)*arctanh((e*sin(d*x+c))^(1/2)/e^(1/2))*cos(d*x+
c)^3*e^2+12*(e*sin(d*x+c))^(3/2)*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))*cos(d
*x+c)^3*e^2+20*e^(7/2)*cos(d*x+c)^2+8*e^(7/2)*cos(d*x+c)-12*(e*sin(d*x+c))^
(3/2)*arctanh((e*sin(d*x+c))^(1/2)/e^(1/2))*cos(d*x+c)*e^2-12*(e*sin(d*x+c)
)^(3/2)*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))*cos(d*x+c)*e^2-6*e^(7/2))/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.65, size = 291, normalized size = 1.24

$$\frac{7\sqrt{-i}\left(\sqrt{2}\,a^{2}\cos\left(dx+c\right)^{2}-\sqrt{2}\,a^{2}\cos\left(dx+c\right)\right)\mathrm{weierstrassPInverse}(4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+7\sqrt{i}\left(\sqrt{2}\,a^{2}\cos\left(dx+c\right)^{2}-\sqrt{2}\,a^{2}\cos\left(dx+c\right)\right)\mathrm{weierstrassPInverse}(4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))+6\left(a^{2}\cos\left(dx+c\right)^{2}-a^{2}\cos\left(dx+c\right)\right)\arctan\left(\frac{\sin(dx+c)-1}{2\sqrt{\sin\left(dx+c\right)}}\right)+3\left(a^{2}\cos\left(dx+c\right)^{2}-a^{2}\cos\left(dx+c\right)\right)\log\left(\frac{\cos(dx+c)^{2}-4\left(\sin(dx+c)+1\right)\sqrt{\sin\left(dx+c\right)}-6\sin(dx+c)-2}{\cos(dx+c)^{2}+2\sin(dx+c)-2}\right)+2\left(7\,a^{2}\cos\left(dx+c\right)-3\,a^{2}\right)\sqrt{\sin\left(dx+c\right)}}{6\left(d\cos\left(dx+c\right)^{2}e^{\frac{5}{2}}-d\cos\left(dx+c\right)e^{\frac{5}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*(7*sqrt(-I)*(sqrt(2)*a^2*cos(d*x + c)^2 - sqrt(2)*a^2*cos(d*x + c))*wei
erstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c)) + 7*sqrt(I)*(sqrt(2)*
a^2*cos(d*x + c)^2 - sqrt(2)*a^2*cos(d*x + c))*weierstrassPInverse(4, 0, co
s(d*x + c) - I*sin(d*x + c)) + 6*(a^2*cos(d*x + c)^2 - a^2*cos(d*x + c))*ar
ctan(1/2*(sin(d*x + c) - 1)/sqrt(sin(d*x + c))) + 3*(a^2*cos(d*x + c)^2 - a
^2*cos(d*x + c))*log((cos(d*x + c)^2 - 4*(sin(d*x + c) + 1)*sqrt(sin(d*x +
c)) - 6*sin(d*x + c) - 2)/(cos(d*x + c)^2 + 2*sin(d*x + c) - 2)) + 2*(7*a^2
*cos(d*x + c) - 3*a^2)*sqrt(sin(d*x + c)))/(d*cos(d*x + c)^2*e^(5/2) - d*co
s(d*x + c)*e^(5/2))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2/(e*sin(d*x+c))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^2/(e*sin(d*x + c))^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}{\left(e\,\sin\left(c+d\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/(e*sin(c + d*x))^(5/2),x)
[Out] int((a + a/cos(c + d*x))^2/(e*sin(c + d*x))^(5/2), x)
```

3.120 $\int \frac{(e\sin(c+dx))^{7/2}}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=139

$$-\frac{4e^4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{21ad\sqrt{e\sin(c+dx)}}-\frac{2e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21ad}+\frac{2e^3\cos^3(c+dx)\sqrt{e\sin(c+dx)}}{7ad}$$

```
[Out] 2/5*e*(e*sin(d*x+c))^(5/2)/a/d+4/21*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)
/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin
(d*x+c)^(1/2)/a/d/(e*sin(d*x+c))^(1/2)-2/21*e^3*cos(d*x+c)*(e*sin(d*x+c))^(
1/2)/a/d+2/7*e^3*cos(d*x+c)^3*(e*sin(d*x+c))^(1/2)/a/d
```

Rubi [A]
time = 0.19, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3957, 2918, 2644, 30, 2648, 2649, 2721, 2720}

$$-\frac{4e^4\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{21ad\sqrt{e\sin(c+dx)}}+\frac{2e^3\cos^3(c+dx)\sqrt{e\sin(c+dx)}}{7ad}-\frac{2e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21ad}+\frac{2e(e\sin(c+dx))^{5/2}}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(7/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-4*e^4*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(21*a*d*Sqrt[e
*Sin[c + d*x]]) - (2*e^3*Cos[c + d*x]*Sqrt[e*Sin[c + d*x]])/(21*a*d) + (2*e
^3*Cos[c + d*x]^3*Sqrt[e*Sin[c + d*x]])/(7*a*d) + (2*e*(e*Sin[c + d*x])^(5/
2))/(5*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
```

```
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2649

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(b*Sin[e + f*x])^(n + 1)*((a*Cos[e + f*x])^(m - 1)/
(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^{7/2}}{a+a\sec(c+dx)}\,dx = -\int \frac{\cos(c+dx)(e\sin(c+dx))^{7/2}}{-a-a\cos(c+dx)}\,dx$$

$$= \frac{e^2\int \cos(c+dx)(e\sin(c+dx))^{3/2}\,dx}{a} - \frac{e^2\int \cos^2(c+dx)(e\sin(c+dx))^{3/2}\,dx}{a}$$

$$= \frac{2e^3\cos^3(c+dx)\sqrt{e\sin(c+dx)}}{7ad} + \frac{e\text{Subst}\left(\int x^{3/2}\,dx, x, e\sin(c+dx)\right)}{ad} - \frac{e^4\int \cdots}{\sqrt{e \cdots}}$$

$$= -\frac{2e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21ad} + \frac{2e^3\cos^3(c+dx)\sqrt{e\sin(c+dx)}}{7ad} + \frac{2e(e\sin(c \cdots}{5a \cdots}$$

$$= -\frac{2e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21ad} + \frac{2e^3\cos^3(c+dx)\sqrt{e\sin(c+dx)}}{7ad} + \frac{2e(e\sin(c \cdots}{5a \cdots}$$

$$= -\frac{4e^4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{21ad\sqrt{e\sin(c+dx)}} - \frac{2e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21ad} + \frac{2e^{\cdots}}{\cdots}$$

Mathematica [A]
time = 0.47, size = 122, normalized size = 0.88

$$\frac{e^3\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(40F\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)+(42+25\cos(c+dx)-42\cos(2(c+dx))+15\cos(3(c+dx)))\sqrt{\sin(c+dx)}\right)\sqrt{e\sin(c+dx)}}{105ad(1+\sec(c+dx))\sqrt{\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sin[c + d*x])^(7/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (e^3*Cos[(c + d*x)/2]^2*Sec[c + d*x]*(40*EllipticF[(-2*c + Pi - 2*d*x)/4, 2
] + (42 + 25*Cos[c + d*x] - 42*Cos[2*(c + d*x)] + 15*Cos[3*(c + d*x)])*Sqrt
[Sin[c + d*x]])*Sqrt[e*Sin[c + d*x]])/(105*a*d*(1 + Sec[c + d*x])*Sqrt[Sin[
c + d*x]])
```

Maple [A]
time = 0.19, size = 128, normalized size = 0.92

method	result
default	$\frac{\frac{2e(e\sin(dx+c))^{\frac{5}{2}}}{5a}+\frac{2e^4\left(3\left(\sin^5(dx+c)\right)+\sqrt{-\sin(dx+c)+1}\sqrt{2\sin(dx+c)+2}\left(\sqrt{\sin}(dx+c)\right)\text{EllipticF}\left(\sqrt{-\sin \cdots}\right.\right.}{21a\cos(dx+c)\sqrt{e\sin(dx+c)}}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] (2/5/a*e*(e*sin(d*x+c))^(5/2)+2/21*e^4*(3*sin(d*x+c)^5+(-sin(d*x+c)+1)^(1/2
)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1
/2*2^(1/2))-5*sin(d*x+c)^3+2*sin(d*x+c))/a/cos(d*x+c)/(e*sin(d*x+c))^(1/2))
/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] e^(7/2)*integrate(sin(d*x + c)^(7/2)/(a*sec(d*x + c) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.95, size = 114, normalized size = 0.82

$$-\frac{2\left(5\sqrt{2}\sqrt{-i}\,e^{\frac{7}{2}}\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c))+5\sqrt{2}\sqrt{i}\,e^{\frac{7}{2}}\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)-i\sin(dx+c))-\left(15\cos(dx+c)^{3}e^{\frac{7}{2}}-21\cos(dx+c)^{2}e^{\frac{7}{2}}-5\cos(dx+c)e^{\frac{7}{2}}+21e^{\frac{7}{2}}\right)\sqrt{\sin(dx+c)}\right)}{105\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -2/105*(5*sqrt(2)*sqrt(-I)*e^(7/2)*weierstrassPInverse(4, 0, cos(d*x + c) +
 I*sin(d*x + c)) + 5*sqrt(2)*sqrt(I)*e^(7/2)*weierstrassPInverse(4, 0, cos(
d*x + c) - I*sin(d*x + c)) - (15*cos(d*x + c)^3*e^(7/2) - 21*cos(d*x + c)^2
*e^(7/2) - 5*cos(d*x + c)*e^(7/2) + 21*e^(7/2))*sqrt(sin(d*x + c)))/(a*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(7/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*sin(d*x + c))^(7/2)/(a*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)\,{\left(e\,\sin\left(c+d\,x\right)\right)}^{7/2}}{a\,\left(\cos\left(c+d\,x\right)+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(7/2)/(a + a/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e*sin(c + d*x))^(7/2))/(a*(cos(c + d*x) + 1)), x)
```

3.121 $\int \frac{(e\sin(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=104

$$-\frac{4e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5ad\sqrt{\sin(c+dx)}}+\frac{2e(e\sin(c+dx))^{3/2}}{3ad}-\frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5ad}$$

```
[Out] 2/3*e*(e*sin(d*x+c))^(3/2)/a/d-2/5*e*cos(d*x+c)*(e*sin(d*x+c))^(3/2)/a/d+4/
5*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ellipti
cE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/a/d/sin(d*x+c)^(
1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3957, 2918, 2644, 30, 2649, 2721, 2719}

$$-\frac{4e^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5ad\sqrt{\sin(c+dx)}}+\frac{2e(e\sin(c+dx))^{3/2}}{3ad}-\frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(5/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-4*e^2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(5*a*d*Sqrt[
Sin[c + d*x]]) + (2*e*(e*Sin[c + d*x])^(3/2))/(3*a*d) - (2*e*Cos[c + d*x]*(
e*Sin[c + d*x])^(3/2))/(5*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2649

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(b*Sin[e + f*x])^(n + 1)*((a*Cos[e + f*x])^(m - 1)/
(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
```

```
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e\sin(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx &= -\int \frac{\cos(c+dx)(e\sin(c+dx))^{5/2}}{-a-a\cos(c+dx)}\,dx \\
&= \frac{e^2\int \cos(c+dx)\sqrt{e\sin(c+dx)}\,dx}{a} - \frac{e^2\int \cos^2(c+dx)\sqrt{e\sin(c+dx)}\,dx}{a} \\
&= -\frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5ad} + \frac{e\text{Subst}\left(\int \sqrt{x}\,dx, x, e\sin(c+dx)\right)}{ad} - \frac{(2e^2)\int
\end{aligned}$$

$$\begin{aligned}
&= \frac{2e(e\sin(c+dx))^{3/2}}{3ad} - \frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5ad} - \frac{\left(2e^2\sqrt{e\sin(c+dx)}\right)\int}{5a\sqrt{\sin(c+}} \\
&= -\frac{4e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5ad\sqrt{\sin(c+dx)}} + \frac{2e(e\sin(c+dx))^{3/2}}{3ad} - \frac{2e\cos(c+da}{}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 3.14, size = 232, normalized size = 2.23

$$\frac{2\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)(e\sin(c+dx))^{5/2}\left(\frac{2e^{-idx}\sqrt{2-2e^{2i(c+dx)}}\left(3\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};e^{2i(c+dx)}\right)+e^{2idx}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};e^{2i(c+dx)}\right)\right)\sec(c)}{\sqrt{-ie^{-i(c+dx)}\left(-1+e^{2i(c+dx)}\right)}}+\sqrt{\sin(c+dx)}\,(10\cos(dx)\sin(c)-3\cos(2dx)\sin(2c)+10\cos(c)\sin(dx)-3\cos(2c)\sin(2dx)-12\tan(c))\right)}{15ad(1+\sec(c+dx))\sin^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sin[c + d*x])^(5/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (2*Cos[(c + d*x)/2]^2*Sec[c + d*x]*(e*Sin[c + d*x])^(5/2)*((2*Sqrt[2 - 2*E^
((2*I)*(c + d*x))]*(3*Hypergeometric2F1[-1/4, 1/2, 3/4, E^((2*I)*(c + d*x))
] + E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, E^((2*I)*(c + d*x))])*Se
c[c])/(E^(I*d*x)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/E^(I*(c + d*x))]) +
 Sqrt[Sin[c + d*x]]*(10*Cos[d*x]*Sin[c] - 3*Cos[2*d*x]*Sin[2*c] + 10*Cos[c]
*Sin[d*x] - 3*Cos[2*c]*Sin[2*d*x] - 12*Tan[c])))/(15*a*d*(1 + Sec[c + d*x])
*Sin[c + d*x]^(5/2))
```

Maple [A]
time = 0.31, size = 173, normalized size = 1.66

method	result
default	$\frac{2e^3\left(6\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}\,(dx+c)\right)\text{EllipticE}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15/a/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*e^3*(6*(-sin(d*x+c)+1)^(1/2)*(2*sin(
d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2
))-3*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*Elliptic
F((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+3*cos(d*x+c)^4-5*cos(d*x+c)^3-3*cos(d*
x+c)^2+5*cos(d*x+c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] e^(5/2)*integrate(sin(d*x + c)^(5/2)/(a*sec(d*x + c) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.96, size = 95, normalized size = 0.91

$$-\frac{2\left(3i\sqrt{2}\sqrt{-i}\,e^{\frac{5}{2}}\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c)))-3i\sqrt{2}\sqrt{i}\,e^{\frac{5}{2}}\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(dx+c)-i\sin(dx+c)))+\left(3\cos(dx+c)e^{\frac{5}{2}}-5e^{\frac{5}{2}}\right)\sin(dx+c)^{\frac{3}{2}}\right)}{15\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -2/15*(3*I*sqrt(2)*sqrt(-I)*e^(5/2)*weierstrassZeta(4, 0, weierstrassPInver
se(4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*sqrt(I)*e^(5/2)*weie
rstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)))
 + (3*cos(d*x + c)*e^(5/2) - 5*e^(5/2))*sin(d*x + c)^(3/2))/(a*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(5/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*sin(d*x + c))^(5/2)/(a*sec(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)\,{\left(e\,\sin\left(c+d\,x\right)\right)}^{5/2}}{a\,\left(\cos\left(c+d\,x\right)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(5/2)/(a + a/cos(c + d*x)),x)
```

```
[Out] int((cos(c + d*x)*(e*sin(c + d*x))^(5/2))/(a*(cos(c + d*x) + 1)), x)
```

3.122 $\int \frac{(e\sin(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=102

$$-\frac{4e^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{3ad\sqrt{e\sin(c+dx)}}+\frac{2e\sqrt{e\sin(c+dx)}}{ad}-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3ad}$$

```
[Out] 4/3*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ellip
ticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/a/d/(e*sin(d*x+c))
^(1/2)+2*e*(e*sin(d*x+c))^(1/2)/a/d-2/3*e*cos(d*x+c)*(e*sin(d*x+c))^(1/2)/a
/d
```

Rubi [A]
time = 0.15, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3957, 2918, 2644, 30, 2649, 2721, 2720}

$$-\frac{4e^2\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{3ad\sqrt{e\sin(c+dx)}}+\frac{2e\sqrt{e\sin(c+dx)}}{ad}-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(3/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-4*e^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*a*d*Sqrt[e*
Sin[c + d*x]]) + (2*e*Sqrt[e*Sin[c + d*x]])/(a*d) - (2*e*Cos[c + d*x]*Sqrt[
e*Sin[c + d*x]])/(3*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2649

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(b*Sin[e + f*x])^(n + 1)*((a*Cos[e + f*x])^(m - 1)/
(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
```

```
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e\sin(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx &= -\int \frac{\cos(c+dx)(e\sin(c+dx))^{3/2}}{-a-a\cos(c+dx)}\,dx \\
&= \frac{e^2\int \frac{\cos(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx}{a} - \frac{e^2\int \frac{\cos^2(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx}{a} \\
&= -\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3ad} + \frac{e\text{Subst}\left(\int \frac{1}{\sqrt{x}}\,dx, x, e\sin(c+dx)\right)}{ad} - \frac{(2e^2)\int}{} \\
&= \frac{2e\sqrt{e\sin(c+dx)}}{ad} - \frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3ad} - \frac{\left(2e^2\sqrt{\sin(c+dx)}\right)\int \frac{}{\sqrt{s}}}{3a\sqrt{e\sin(c+d}} \\
&= -\frac{4e^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{3ad\sqrt{e\sin(c+dx)}} + \frac{2e\sqrt{e\sin(c+dx)}}{ad} - \frac{2e\cos(c+dx)\sqrt{}}{3a}
\end{aligned}$$

Mathematica [**A**]
time = 13.52, size = 69, normalized size = 0.68

$$-\frac{2\left(-2F\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)+(-3+\cos(c+dx))\sqrt{\sin(c+dx)}\right)(e\sin(c+dx))^{3/2}}{3ad\sin^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sin[c + d*x])^(3/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-2*(-2*EllipticF[(-2*c + Pi - 2*d*x)/4, 2] + (-3 + Cos[c + d*x])*Sqrt[Sin[
c + d*x]])*(e*Sin[c + d*x])^(3/2))/(3*a*d*Sin[c + d*x]^(3/2))
```

Maple [**A**]
time = 0.18, size = 112, normalized size = 1.10

method	result
default	$\frac{2e^{2}\left(\sqrt{-\sin\left(dx+c\right)+1}\,\sqrt{2\sin\left(dx+c\right)+2}\,\left(\sqrt{\sin}\left(dx+c\right)\right)\mathrm{EllipticF}\left(\sqrt{-\sin\left(dx+c\right)+1},\frac{\sqrt{2}}{2}\right)\right.}{3a\cos\left(dx+c\right)\sqrt{e\sin\left(dx+c\right)}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/a/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*e^2*((-sin(d*x+c)+1)^(1/2)*(2*sin(d*x
+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-
cos(d*x+c)^2*sin(d*x+c)+3*cos(d*x+c)*sin(d*x+c))/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] e^(3/2)*integrate(sin(d*x + c)^(3/2)/(a*sec(d*x + c) + a), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.82, size = 86, normalized size = 0.84

$$-\frac{2\left(\sqrt{2}\sqrt{-i}\,e^{\frac{3}{2}}\mathrm{weierstrassPInverse}(4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right))+\sqrt{2}\sqrt{i}\,e^{\frac{3}{2}}\mathrm{weierstrassPInverse}(4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right))+\left(\cos\left(dx+c\right)e^{\frac{3}{2}}-3\,e^{\frac{3}{2}}\right)\sqrt{\sin\left(dx+c\right)}\right)}{3\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -2/3*(sqrt(2)*sqrt(-I)*e^(3/2)*weierstrassPInverse(4, 0, cos(d*x + c) + I*s
in(d*x + c)) + sqrt(2)*sqrt(I)*e^(3/2)*weierstrassPInverse(4, 0, cos(d*x +
c) - I*sin(d*x + c)) + (cos(d*x + c)*e^(3/2) - 3*e^(3/2))*sqrt(sin(d*x + c)
))/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(e\sin(c+dx))^{\frac{3}{2}}}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(3/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral((e*sin(c + d*x))**(3/2)/(sec(c + d*x) + 1), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*sin(d*x + c))^(3/2)/(a*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)\,\left(e\,\sin(c+d\,x)\right)^{3/2}}{a\,\left(\cos(c+d\,x)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(3/2)/(a + a/cos(c + d*x)),x)
```

```
[Out] int((cos(c + d*x)*(e*sin(c + d*x))^(3/2))/(a*(cos(c + d*x) + 1)), x)
```

3.123 $\int \frac{\sqrt{e\sin(c+dx)}}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=95

$$-\frac{2e}{ad\sqrt{e\sin(c+dx)}}+\frac{2e\cos(c+dx)}{ad\sqrt{e\sin(c+dx)}}+\frac{4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{ad\sqrt{\sin(c+dx)}}$$

```
[Out] -2*e/a/d/(e*sin(d*x+c))^(1/2)+2*e*cos(d*x+c)/a/d/(e*sin(d*x+c))^(1/2)-4*(si
n(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/
2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/a/d/sin(d*x+c)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3957, 2918, 2644, 30, 2647, 2721, 2719}

$$-\frac{2e}{ad\sqrt{e\sin(c+dx)}}+\frac{2e\cos(c+dx)}{ad\sqrt{e\sin(c+dx)}}+\frac{4E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{ad\sqrt{\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Sin[c + d*x]]/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-2*e)/(a*d*Sqrt[e*Sin[c + d*x]]) + (2*e*Cos[c + d*x])/(a*d*Sqrt[e*Sin[c +
d*x]]) + (4*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(a*d*Sqr
t[Sin[c + d*x]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
```

```
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{e\sin(c+dx)}}{a+a\sec(c+dx)}\,dx &= -\int \frac{\cos(c+dx)\sqrt{e\sin(c+dx)}}{-a-a\cos(c+dx)}\,dx \\
&= \frac{e^2\int \frac{\cos(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx}{a} - \frac{e^2\int \frac{\cos^2(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx}{a} \\
&= \frac{2e\cos(c+dx)}{ad\sqrt{e\sin(c+dx)}} + \frac{2\int \sqrt{e\sin(c+dx)}\,dx}{a} + \frac{e\mathrm{Subst}\left(\int \frac{1}{x^{3/2}}\,dx, x, e\sin(c+dx)\right)}{ad} \\
&= -\frac{2e}{ad\sqrt{e\sin(c+dx)}} + \frac{2e\cos(c+dx)}{ad\sqrt{e\sin(c+dx)}} + \frac{\left(2\sqrt{e\sin(c+dx)}\right)\int \sqrt{\sin(c+dx)}}{a\sqrt{\sin(c+dx)}} \\
&= -\frac{2e}{ad\sqrt{e\sin(c+dx)}} + \frac{2e\cos(c+dx)}{ad\sqrt{e\sin(c+dx)}} + \frac{4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+d}}{ad\sqrt{\sin(c+dx)}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.42, size = 249, normalized size = 2.62

$$\frac{2\left(3-9e^{2ic}+6e^{i(c+dx)}-9e^{2i(c+dx)}+3e^{2i(2c+dx)}+6e^{i(3c+dx)}+12e^{2ic}\sqrt{1-e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};e^{2i(c+dx)}\right)+4e^{2i(c+dx)}\sqrt{1-e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};e^{2i(c+dx)}\right)\right)\sqrt{e\sin(c+dx)}}{3ad\left(1+ie^{ic}\right)\left(i+e^{ic}\right)\left(-1+e^{i(c+dx)}\right)\left(1+e^{i(c+dx)}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Sin[c + d*x]]/(a + a*Sec[c + d*x]),x]
```

```
[Out] (2*(3 - 9*E^((2*I)*c) + 6*E^(I*(c + d*x)) - 9*E^((2*I)*(c + d*x)) + 3*E^((2
*I)*(2*c + d*x)) + 6*E^(I*(3*c + d*x)) + 12*E^((2*I)*c)*Sqrt[1 - E^((2*I)*(
c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, E^((2*I)*(c + d*x))] + 4*E^((2
*I)*(c + d*x))*Sqrt[1 - E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/
4, E^((2*I)*(c + d*x))])*Sqrt[e*Sin[c + d*x]])/(3*a*d*(1 + I*E^(I*c))*(I +
E^(I*c))*(-1 + E^(I*(c + d*x)))*(1 + E^(I*(c + d*x))))
```

Maple [A]
time = 0.17, size = 148, normalized size = 1.56

method	result
default	$\frac{2e\left(\sqrt{-\sin\left(dx+c\right)+1}\,\sqrt{2\sin\left(dx+c\right)+2}\,\left(\sqrt{\sin}\left(dx+c\right)\right)\mathrm{EllipticF}\left(\sqrt{-\sin\left(dx+c\right)+1},\frac{\sqrt{2}}{2}\right)-\right.}{a\cos(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/a/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*e*((-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+
2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-2*(-
sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin
(d*x+c)+1)^(1/2),1/2*2^(1/2))+cos(d*x+c)^2-cos(d*x+c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] e^(1/2)*integrate(sqrt(sin(d*x + c))/(a*sec(d*x + c) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.28, size = 121, normalized size = 1.27

$$-\frac{2\left(e^{\frac{1}{2}}\sin\left(dx+c\right)^{\frac{3}{2}}+\sqrt{-i}\left(-i\sqrt{2}\cos\left(dx+c\right)e^{\frac{1}{2}}-i\sqrt{2}\,e^{\frac{1}{2}}\right)\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)))+\sqrt{i}\left(i\sqrt{2}\cos\left(dx+c\right)e^{\frac{1}{2}}+i\sqrt{2}\,e^{\frac{1}{2}}\right)\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)))\right)}{ad\cos\left(dx+c\right)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -2*(e^(1/2)*sin(d*x + c)^(3/2) + sqrt(-I)*(-I*sqrt(2)*cos(d*x + c)*e^(1/2)
- I*sqrt(2)*e^(1/2))*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*
x + c) + I*sin(d*x + c))) + sqrt(I)*(I*sqrt(2)*cos(d*x + c)*e^(1/2) + I*sqr
t(2)*e^(1/2))*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c)
- I*sin(d*x + c))))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{e\sin(c+dx)}}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(1/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(sqrt(e*sin(c + d*x))/(sec(c + d*x) + 1), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*sin(d*x + c))/(a*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)\,\sqrt{e\,\sin(c+d\,x)}}{a\,(\cos(c+d\,x)+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(1/2)/(a + a/cos(c + d*x)),x)
```

```
[Out] int((cos(c + d*x)*(e*sin(c + d*x))^(1/2))/(a*(cos(c + d*x) + 1)), x)
```

3.124 $\int \frac{1}{(a+a\sec(c+dx))\sqrt{e\sin(c+dx)}}\,dx$

Optimal. Leaf size=101

$$-\frac{2e}{3ad(e\sin(c+dx))^{3/2}}+\frac{2e\cos(c+dx)}{3ad(e\sin(c+dx))^{3/2}}+\frac{4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{3ad\sqrt{e\sin(c+dx)}}$$

```
[Out] -2/3*e/a/d/(e*sin(d*x+c))^(3/2)+2/3*e*cos(d*x+c)/a/d/(e*sin(d*x+c))^(3/2)-4
/3*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(
cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/a/d/(e*sin(d*x+c))^(1/2
)
```

Rubi [A]
time = 0.16, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3957, 2918, 2644, 30, 2647, 2721, 2720}

$$-\frac{2e}{3ad(e\sin(c+dx))^{3/2}}+\frac{2e\cos(c+dx)}{3ad(e\sin(c+dx))^{3/2}}+\frac{4\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{3ad\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[c + d*x])*Sqrt[e*Sin[c + d*x]]),x]
```

```
[Out] (-2*e)/(3*a*d*(e*Sin[c + d*x])^(3/2)) + (2*e*Cos[c + d*x])/(3*a*d*(e*Sin[c
+ d*x])^(3/2)) + (4*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3
*a*d*Sqrt[e*Sin[c + d*x]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
```

```
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(c+dx))\sqrt{e\sin(c+dx)}}\,dx = -\int \frac{\cos(c+dx)}{(-a-a\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx$$

$$= \frac{e^2\int \frac{\cos(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx}{a} - \frac{e^2\int \frac{\cos^2(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx}{a}$$

$$= \frac{2e\cos(c+dx)}{3ad(e\sin(c+dx))^{3/2}} + \frac{2\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{3a} + \frac{e\text{Subst}\left(\int \frac{1}{x^{5/2}} \cdots\right.}{\cdots}$$

$$= -\frac{2e}{3ad(e\sin(c+dx))^{3/2}} + \frac{2e\cos(c+dx)}{3ad(e\sin(c+dx))^{3/2}} + \frac{\left(2\sqrt{\sin(c+\cdots}\right.}{3a\cdots}$$

$$= -\frac{2e}{3ad(e\sin(c+dx))^{3/2}} + \frac{2e\cos(c+dx)}{3ad(e\sin(c+dx))^{3/2}} + \frac{4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}\cdots\right.\right.}{3ad\sqrt{\cdots}}$$

Mathematica [A]
time = 0.36, size = 77, normalized size = 0.76

$$\frac{2\cot\left(\frac{1}{2}(c+dx)\right)\left(-1+\cos(c+dx)-2F\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)\sin^{\frac{3}{2}}(c+dx)\right)}{3ad(1+\cos(c+dx))\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[c + d*x])*Sqrt[e*Sin[c + d*x]]),x]
```

```
[Out] (2*Cot[(c + d*x)/2]*(-1 + Cos[c + d*x] - 2*EllipticF[(-2*c + Pi - 2*d*x)/4,
 2]*Sin[c + d*x]^(3/2)))/(3*a*d*(1 + Cos[c + d*x])*Sqrt[e*Sin[c + d*x]])
```

Maple [A]
time = 0.18, size = 121, normalized size = 1.20

method	result
default	$\frac{-\frac{2e}{3a(e\sin(dx+c))^{\frac{3}{2}}}-\frac{2\left(\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\left(\sin^{\frac{5}{2}}(dx+c)\right)\mathrm{EllipticF}\left(\sqrt{-\sin(dx+c)+\right.}{3a\sin(dx+c)^{2}\cos(dx+c)\sqrt{e\sin(dx+c)}}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-2/3/a*e/(e*sin(d*x+c))^(3/2)-2/3*((-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^
(1/2)*sin(d*x+c)^(5/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+sin(d*x
+c)^3-sin(d*x+c))/a/sin(d*x+c)^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-1/2)*integrate(1/((a*sec(d*x + c) + a)*sqrt(sin(d*x + c))), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.69, size = 104, normalized size = 1.03

$$\frac{2\left(\sqrt{-i}\left(\sqrt{2}\cos(dx+c)+\sqrt{2}\right)\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c))+\sqrt{i}\left(\sqrt{2}\cos(dx+c)+\sqrt{2}\right)\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)-i\sin(dx+c))-\sqrt{\sin(dx+c)}\right)}{3\left(ad\cos(dx+c)e^{\frac{1}{2}}+ade^{\frac{1}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(sqrt(-I)*(sqrt(2)*cos(d*x + c) + sqrt(2))*weierstrassPInverse(4, 0, co
s(d*x + c) + I*sin(d*x + c)) + sqrt(I)*(sqrt(2)*cos(d*x + c) + sqrt(2))*wei
erstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) - sqrt(sin(d*x + c)))
/(a*d*cos(d*x + c)*e^(1/2) + a*d*e^(1/2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sqrt{e\sin{(c+dx)}}\sec{(c+dx)}+\sqrt{e\sin{(c+dx)}}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(e*sin(c + d*x))*sec(c + d*x) + sqrt(e*sin(c + d*x))), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)*sqrt(e*sin(d*x + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)}{a\,\sqrt{e\,\sin\left(c+d\,x\right)}\,\left(\cos\left(c+d\,x\right)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*sin(c + d*x))^(1/2)*(a + a/cos(c + d*x))),x)
```

```
[Out] int(cos(c + d*x)/(a*(e*sin(c + d*x))^(1/2)*(cos(c + d*x) + 1)), x)
```

3.125 $\int \frac{1}{(a+a\sec(c+dx))(e\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=135

$$-\frac{2e}{5ad(e\sin(c+dx))^{5/2}}+\frac{2e\cos(c+dx)}{5ad(e\sin(c+dx))^{5/2}}-\frac{4\cos(c+dx)}{5ade\sqrt{e\sin(c+dx)}}-\frac{4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+d}}{5ade^2\sqrt{\sin(c+dx)}}$$

```
[Out] -2/5*e/a/d/(e*sin(d*x+c))^(5/2)+2/5*e*cos(d*x+c)/a/d/(e*sin(d*x+c))^(5/2)-4
/5*cos(d*x+c)/a/d/e/(e*sin(d*x+c))^(1/2)+4/5*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^
(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)
)*(e*sin(d*x+c))^(1/2)/a/d/e^2/sin(d*x+c)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3957, 2918, 2644, 30, 2647, 2716, 2721, 2719}

$$-\frac{4E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5ade^2\sqrt{\sin(c+dx)}}-\frac{2e}{5ad(e\sin(c+dx))^{5/2}}+\frac{2e\cos(c+dx)}{5ad(e\sin(c+dx))^{5/2}}-\frac{4\cos(c+dx)}{5ade\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[c + d*x])*(e*Sin[c + d*x])^(3/2)),x]
```

```
[Out] (-2*e)/(5*a*d*(e*Sin[c + d*x])^(5/2)) + (2*e*Cos[c + d*x])/(5*a*d*(e*Sin[c
+ d*x])^(5/2)) - (4*Cos[c + d*x])/(5*a*d*e*Sqrt[e*Sin[c + d*x]]) - (4*Ellip
ticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(5*a*d*e^2*Sqrt[Sin[c + d
*x]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
```

```
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(c+dx))(e\sin(c+dx))^{3/2}}\,dx = -\int \frac{\cos(c+dx)}{(-a-a\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx$$

$$= \frac{e^2\int \frac{\cos(c+dx)}{(e\sin(c+dx))^{7/2}}\,dx}{a} - \frac{e^2\int \frac{\cos^2(c+dx)}{(e\sin(c+dx))^{7/2}}\,dx}{a}$$

$$= \frac{2e\cos(c+dx)}{5ad(e\sin(c+dx))^{5/2}} + \frac{2\int \frac{1}{(e\sin(c+dx))^{3/2}}\,dx}{5a} + \frac{\text{eSubst}\left(\int \frac{1}{x^{7/2}} \right.}{}$$

$$= -\frac{2e}{5ad(e\sin(c+dx))^{5/2}} + \frac{2e\cos(c+dx)}{5ad(e\sin(c+dx))^{5/2}} - \frac{4\cos(c}{5ade\sqrt{e\sin}}$$

$$= -\frac{2e}{5ad(e\sin(c+dx))^{5/2}} + \frac{2e\cos(c+dx)}{5ad(e\sin(c+dx))^{5/2}} - \frac{4\cos(c}{5ade\sqrt{e\sin}}$$

$$= -\frac{2e}{5ad(e\sin(c+dx))^{5/2}} + \frac{2e\cos(c+dx)}{5ad(e\sin(c+dx))^{5/2}} - \frac{4\cos(c}{5ade\sqrt{e\sin}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.75, size = 124, normalized size = 0.92

$$\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)(\cos(c+dx)+i\sin(c+dx))\left(-6-9\cos(c+dx)+2\sqrt{1-e^{2i(c+dx)}}(1+\cos(c+dx))\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};e^{2i(c+dx)}\right)+3i\sin(c+dx)\right)}{15ade\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[c + d*x])*(e*Sin[c + d*x])^(3/2)),x]
```

```
[Out] (Sec[(c + d*x)/2]^2*(Cos[c + d*x] + I*Sin[c + d*x])*(-6 - 9*Cos[c + d*x] +
2*Sqrt[1 - E^((2*I)*(c + d*x))]*(1 + Cos[c + d*x])*Hypergeometric2F1[1/2, 3
/4, 7/4, E^((2*I)*(c + d*x))] + (3*I)*Sin[c + d*x]))/(15*a*d*e*Sqrt[e*Sin[c
 + d*x]])
```

Maple [A]
time = 0.20, size = 187, normalized size = 1.39

method	result
default	$-\frac{2e}{5a(e\sin(dx+c))^{\frac{5}{2}}}+\frac{\frac{4\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\left(\sin^{\frac{7}{2}}(dx+c)\right)\text{EllipticE}\left(\sqrt{-\sin(dx+c)+1}\right.}{5}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-2/5/a*e/(e*sin(d*x+c))^(5/2)+2/5/e*(2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)
+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-(-s
in(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticF((-sin(
d*x+c)+1)^(1/2),1/2*2^(1/2))+2*sin(d*x+c)^5-3*sin(d*x+c)^3+sin(d*x+c))/a/si
n(d*x+c)^3/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.62, size = 155, normalized size = 1.15

$$-\frac{2\left(\sqrt{-i}\left(i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\sin\left(dx+c\right)\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)))+\sqrt{i}\left(-i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right)\sin\left(dx+c\right)\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)))+\left(2\cos\left(dx+c\right)^{2}+2\cos\left(dx+c\right)+1\right)\sqrt{\sin\left(dx+c\right)}\right)}{5\left(ad\cos\left(dx+c\right)e^{\frac{3}{2}}+ade^{\frac{3}{2}}\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/5*(sqrt(-I)*(I*sqrt(2)*cos(d*x + c) + I*sqrt(2))*sin(d*x + c)*weierstras
sZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) + sqr
t(I)*(-I*sqrt(2)*cos(d*x + c) - I*sqrt(2))*sin(d*x + c)*weierstrassZeta(4,
0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c))) + (2*cos(d*x +
 c)^2 + 2*cos(d*x + c) + 1)*sqrt(sin(d*x + c)))/((a*d*cos(d*x + c)*e^(3/2)
+ a*d*e^(3/2))*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\left(e\sin\left(c+dx\right)\right)^{\frac{3}{2}}\sec\left(c+dx\right)+\left(e\sin\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(1/((e*sin(c + d*x))**(3/2)*sec(c + d*x) + (e*sin(c + d*x))**(3/2))
, x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)*(e*sin(d*x + c))^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)}{a\,{\left(e\,\sin\left(c+d\,x\right)\right)}^{3/2}\,\left(\cos\left(c+d\,x\right)+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*sin(c + d*x))^(3/2)*(a + a/cos(c + d*x))),x)
```

```
[Out] int(cos(c + d*x)/(a*(e*sin(c + d*x))^(3/2)*(cos(c + d*x) + 1)), x)
```

3.126 $\int \frac{1}{(a+a\sec(c+dx))(e\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=135

$$-\frac{2e}{7ad(e\sin(c+dx))^{7/2}}+\frac{2e\cos(c+dx)}{7ad(e\sin(c+dx))^{7/2}}-\frac{4\cos(c+dx)}{21ade(e\sin(c+dx))^{3/2}}+\frac{4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{21ade^2\sqrt{e\sin(c+dx)}}$$

```
[Out] -2/7*e/a/d/(e*sin(d*x+c))^(7/2)+2/7*e*cos(d*x+c)/a/d/(e*sin(d*x+c))^(7/2)-4
/21*cos(d*x+c)/a/d/e/(e*sin(d*x+c))^(3/2)-4/21*(sin(1/2*c+1/4*Pi+1/2*d*x)^2
)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/
2))*sin(d*x+c)^(1/2)/a/d/e^2/(e*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3957, 2918, 2644, 30, 2647, 2716, 2721, 2720}

$$\frac{4\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{21ade^2\sqrt{e\sin(c+dx)}}-\frac{2e}{7ad(e\sin(c+dx))^{7/2}}+\frac{2e\cos(c+dx)}{7ad(e\sin(c+dx))^{7/2}}-\frac{4\cos(c+dx)}{21ade(e\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[c + d*x])*(e*Sin[c + d*x])^(5/2)),x]
```

```
[Out] (-2*e)/(7*a*d*(e*Sin[c + d*x])^(7/2)) + (2*e*Cos[c + d*x])/(7*a*d*(e*Sin[c
+ d*x])^(7/2)) - (4*Cos[c + d*x])/(21*a*d*e*(e*Sin[c + d*x])^(3/2)) + (4*El
lipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(21*a*d*e^2*Sqrt[e*Sin[c
 + d*x]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
```

```
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(c+dx))(e\sin(c+dx))^{5/2}}\,dx = -\int \frac{\cos(c+dx)}{(-a-a\cos(c+dx))(e\sin(c+dx))^{5/2}}\,dx$$
$$= \frac{e^2\int \frac{\cos(c+dx)}{(e\sin(c+dx))^{9/2}}\,dx}{a} - \frac{e^2\int \frac{\cos^2(c+dx)}{(e\sin(c+dx))^{9/2}}\,dx}{a}$$
$$= \frac{2e\cos(c+dx)}{7ad(e\sin(c+dx))^{7/2}} + \frac{2\int \frac{1}{(e\sin(c+dx))^{5/2}}\,dx}{7a} + \frac{e\text{Subst}\left(\int \frac{1}{x^{9/2}}\,dx\right.}{}$$
$$= -\frac{2e}{7ad(e\sin(c+dx))^{7/2}} + \frac{2e\cos(c+dx)}{7ad(e\sin(c+dx))^{7/2}} - \frac{4\cos(c}{21ade(e\sin(}$$
$$= -\frac{2e}{7ad(e\sin(c+dx))^{7/2}} + \frac{2e\cos(c+dx)}{7ad(e\sin(c+dx))^{7/2}} - \frac{4\cos(c}{21ade(e\sin(}$$
$$= -\frac{2e}{7ad(e\sin(c+dx))^{7/2}} + \frac{2e\cos(c+dx)}{7ad(e\sin(c+dx))^{7/2}} - \frac{4\cos(c}{21ade(e\sin(}$$

Mathematica [A]
time = 0.85, size = 91, normalized size = 0.67

$$-\frac{2\left(4+2\cos(c+dx)+\cos(2(c+dx))+\csc^2\left(\frac{1}{2}(c+dx)\right)F\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)\sin^{\frac{7}{2}}(c+dx)\right)}{21ade(1+\cos(c+dx))(e\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[c + d*x])*(e*Sin[c + d*x])^(5/2)),x]
```

```
[Out] (-2*(4 + 2*Cos[c + d*x] + Cos[2*(c + d*x)] + Csc[(c + d*x)/2]^2*EllipticF[(
-2*c + Pi - 2*d*x)/4, 2]*Sin[c + d*x]^(7/2)))/(21*a*d*e*(1 + Cos[c + d*x])*
(e*Sin[c + d*x])^(3/2))
```

Maple [A]
time = 0.19, size = 136, normalized size = 1.01

method	result
default	$\frac{-\frac{2e}{7a(e\sin(dx+c))^{\frac{7}{2}}}-\frac{2\left(\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\left(\sin^{\frac{9}{2}}(dx+c)\right)\text{EllipticF}\left(\sqrt{-\sin(dx+c)+1}\right.\right.}{21e^2a\sin(dx+c)^4\cos(dx+c)\sqrt{e\sin(dx+c)}}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-2/7/a*e/(e*sin(d*x+c))^(7/2)-2/21/e^2*((-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c
)+2)^(1/2)*sin(d*x+c)^(9/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-2*
sin(d*x+c)^5+5*sin(d*x+c)^3-3*sin(d*x+c))/a/sin(d*x+c)^4/cos(d*x+c)/(e*sin(
d*x+c))^(1/2))/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.79, size = 205, normalized size = 1.52

$$\frac{2\left(\sqrt{-i}\left(\sqrt{2}\cos\left(dx+c\right)^{3}+\sqrt{2}\cos\left(dx+c\right)^{2}-\sqrt{2}\cos\left(dx+c\right)-\sqrt{2}\right)\mathrm{weierstrassPInverse}(4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right))+\sqrt{i}\left(\sqrt{2}\cos\left(dx+c\right)^{3}+\sqrt{2}\cos\left(dx+c\right)^{2}-\sqrt{2}\cos\left(dx+c\right)-\sqrt{2}\right)\mathrm{weierstrassPInverse}(4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right))+\left(2\cos\left(dx+c\right)^{2}+2\cos\left(dx+c\right)+3\right)\sqrt{\sin\left(dx+c\right)}\right)}{21\left(ad\cos\left(dx+c\right)^{3}e^{\frac{5}{2}}+ad\cos\left(dx+c\right)^{2}e^{\frac{5}{2}}-ad\cos\left(dx+c\right)e^{\frac{5}{2}}-ade^{\frac{5}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/21*(sqrt(-I)*(sqrt(2)*cos(d*x + c)^3 + sqrt(2)*cos(d*x + c)^2 - sqrt(2)*c
os(d*x + c) - sqrt(2))*weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x +
 c)) + sqrt(I)*(sqrt(2)*cos(d*x + c)^3 + sqrt(2)*cos(d*x + c)^2 - sqrt(2)*c
os(d*x + c) - sqrt(2))*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x +
 c)) + (2*cos(d*x + c)^2 + 2*cos(d*x + c) + 3)*sqrt(sin(d*x + c)))/(a*d*cos
(d*x + c)^3*e^(5/2) + a*d*cos(d*x + c)^2*e^(5/2) - a*d*cos(d*x + c)*e^(5/2)
 - a*d*e^(5/2))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5010 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)*(e*sin(d*x + c))^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)}{a\,{\left(e\,\sin\left(c+d\,x\right)\right)}^{5/2}\,\left(\cos\left(c+d\,x\right)+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*sin(c + d*x))^(5/2)*(a + a/cos(c + d*x))),x)
```

```
[Out] int(cos(c + d*x)/(a*(e*sin(c + d*x))^(5/2)*(cos(c + d*x) + 1)), x)
```

3.127 $\int \frac{(e\sin(c+dx))^{7/2}}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=162

$$\frac{52e^4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{21a^2d\sqrt{e\sin(c+dx)}}-\frac{4e^3\sqrt{e\sin(c+dx)}}{a^2d}+\frac{26e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21a^2d}+\frac{2e^3\cos^3}{}$$

```
[Out] 4/5*e*(e*sin(d*x+c))^(5/2)/a^2/d-52/21*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1
/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*
sin(d*x+c)^(1/2)/a^2/d/(e*sin(d*x+c))^(1/2)-4*e^3*(e*sin(d*x+c))^(1/2)/a^2/
d+26/21*e^3*cos(d*x+c)*(e*sin(d*x+c))^(1/2)/a^2/d+2/7*e^3*cos(d*x+c)^3*(e*s
in(d*x+c))^(1/2)/a^2/d
```

Rubi [A]
time = 0.38, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3957, 2954, 2952, 2649, 2721, 2720, 2644, 14}

$$\frac{52e^4\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{21a^2d\sqrt{e\sin(c+dx)}}-\frac{4e^3\sqrt{e\sin(c+dx)}}{a^2d}+\frac{2e^3\cos^3(c+dx)\sqrt{e\sin(c+dx)}}{7a^2d}+\frac{26e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21a^2d}+\frac{4e(e\sin(c+dx))^{5/2}}{5a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(7/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (52*e^4*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(21*a^2*d*Sqrt
[e*Sin[c + d*x]]) - (4*e^3*Sqrt[e*Sin[c + d*x]])/(a^2*d) + (26*e^3*Cos[c +
d*x]*Sqrt[e*Sin[c + d*x]])/(21*a^2*d) + (2*e^3*Cos[c + d*x]^3*Sqrt[e*Sin[c
+ d*x]])/(7*a^2*d) + (4*e*(e*Sin[c + d*x])^(5/2))/(5*a^2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2649

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(b*Sin[e + f*x])^(n + 1)*((a*Cos[e + f*x])^(m - 1)/
```

```
(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)(e\sin(c+dx))^{7/2}}{(-a-a\cos(c+dx))^2}\,dx$$

$$= \frac{e^4\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx}{a^4}$$

$$= \frac{e^4\int \left(\frac{a^2\cos^2(c+dx)}{\sqrt{e\sin(c+dx)}} - \frac{2a^2\cos^3(c+dx)}{\sqrt{e\sin(c+dx)}} + \frac{a^2\cos^4(c+dx)}{\sqrt{e\sin(c+dx)}}\right)dx}{a^4}$$

$$= \frac{e^4\int \frac{\cos^2(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx}{a^2} + \frac{e^4\int \frac{\cos^4(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx}{a^2} - \frac{(2e^4)\int \frac{\cos^3(c+dx)}{\sqrt{e\sin(c+dx)}}}{a^2}$$

$$= \frac{2e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{3a^2d} + \frac{2e^3\cos^3(c+dx)\sqrt{e\sin(c+dx)}}{7a^2d} - \frac{(2e^3)\,\mathrm{Su}}{}$$

$$= \frac{26e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21a^2d} + \frac{2e^3\cos^3(c+dx)\sqrt{e\sin(c+dx)}}{7a^2d} - \frac{(2e^3)\,\mathrm{S}}{}$$

$$= \frac{4e^4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{3a^2d\sqrt{e\sin(c+dx)}} - \frac{4e^3\sqrt{e\sin(c+dx)}}{a^2d} + \frac{26e^3\cos(c+}{}$$

$$= \frac{52e^4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{21a^2d\sqrt{e\sin(c+dx)}} - \frac{4e^3\sqrt{e\sin(c+dx)}}{a^2d} + \frac{26e^3\cos(c}{}$$

<u>Mathematica</u> [A]
time = 1.03, size = 94, normalized size = 0.58

$$-\frac{e^3\left(520F\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)+(756-305\cos(c+dx)+84\cos(2(c+dx))-15\cos(3(c+dx)))\sqrt{\sin(c+dx)}\right)\sqrt{e\sin(c+dx)}}{210a^2d\sqrt{\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sin[c + d*x])^(7/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/210*(e^3*(520*EllipticF[(-2*c + Pi - 2*d*x)/4, 2] + (756 - 305*Cos[c + d
*x] + 84*Cos[2*(c + d*x)] - 15*Cos[3*(c + d*x)])*Sqrt[Sin[c + d*x]])*Sqrt[e
*Sin[c + d*x]])/(a^2*d*Sqrt[Sin[c + d*x]])
```

<u>Maple</u> [A]
time = 0.23, size = 145, normalized size = 0.90

method	result

default	$-\frac{2e^4\left(-15\left(\cos^4(dx+c)\right)\sin(dx+c)+65\sqrt{-\sin\left(dx+c\right)+1}\,\sqrt{2\sin\left(dx+c\right)+2}\,\left(\sqrt{\sin}\,(dx+c)\right)\text{EllipticF}\left(\sqrt{-\right.\right.}{105a^2\cos(dx+c)\sqrt{e\sin}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/105/a^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*e^4*(-15*cos(d*x+c)^4*sin(d*x+c)
+65*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF
((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+42*cos(d*x+c)^3*sin(d*x+c)-65*cos(d*x+c
)^2*sin(d*x+c)+168*cos(d*x+c)*sin(d*x+c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] e^(7/2)*integrate(sin(d*x + c)^(7/2)/(a*sec(d*x + c) + a)^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.92, size = 113, normalized size = 0.70

$$\frac{2\left(65\sqrt{2}\sqrt{-i}\,e^{\frac{7}{2}}\text{weierstrassPInverse}(4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+65\sqrt{2}\sqrt{i}\,e^{\frac{7}{2}}\text{weierstrassPInverse}(4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))+\left(15\,\cos\left(dx+c\right)^3e^{\frac{7}{2}}-42\,\cos\left(dx+c\right)^2e^{\frac{7}{2}}+65\,\cos\left(dx+c\right)e^{\frac{7}{2}}-168\,e^{\frac{7}{2}}\right)\sqrt{\sin\left(dx+c\right)}\right)}{105\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 2/105*(65*sqrt(2)*sqrt(-I)*e^(7/2)*weierstrassPInverse(4, 0, cos(d*x + c) +
 I*sin(d*x + c)) + 65*sqrt(2)*sqrt(I)*e^(7/2)*weierstrassPInverse(4, 0, cos
(d*x + c) - I*sin(d*x + c)) + (15*cos(d*x + c)^3*e^(7/2) - 42*cos(d*x + c)^
2*e^(7/2) + 65*cos(d*x + c)*e^(7/2) - 168*e^(7/2))*sqrt(sin(d*x + c)))/(a^2
*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(7/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate((e*sin(d*x + c))^(7/2)/(a*sec(d*x + c) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^2\,{\left(e\,\sin\left(c+d\,x\right)\right)}^{7/2}}{a^2\,{\left(\cos\left(c+d\,x\right)+1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(7/2)/(a + a/cos(c + d*x))^2,x)
[Out] int((cos(c + d*x)^2*(e*sin(c + d*x))^(7/2))/(a^2*(cos(c + d*x) + 1)^2), x)
```

3.128 $\int \frac{(e\sin(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=187

$$\frac{4e^3}{a^2d\sqrt{e\sin(c+dx)}}-\frac{2e^3\cos(c+dx)}{a^2d\sqrt{e\sin(c+dx)}}-\frac{2e^3\cos^3(c+dx)}{a^2d\sqrt{e\sin(c+dx)}}-\frac{44e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5a^2d\sqrt{\sin(c+dx)}}$$

```
[Out] 4/3*e*(e*sin(d*x+c))^(3/2)/a^2/d-12/5*e*cos(d*x+c)*(e*sin(d*x+c))^(3/2)/a^2
/d+4*e^3/a^2/d/(e*sin(d*x+c))^(1/2)-2*e^3*cos(d*x+c)/a^2/d/(e*sin(d*x+c))^(
1/2)-2*e^3*cos(d*x+c)^3/a^2/d/(e*sin(d*x+c))^(1/2)+44/5*e^2*(sin(1/2*c+1/4*
Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1
/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/a^2/d/sin(d*x+c)^(1/2)
```

Rubi [A]
time = 0.42, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3957, 2954, 2952, 2647, 2721, 2719, 2644, 14, 2649}

$$\frac{4e^3}{a^2d\sqrt{e\sin(c+dx)}}-\frac{2e^3\cos^3(c+dx)}{a^2d\sqrt{e\sin(c+dx)}}-\frac{2e^3\cos(c+dx)}{a^2d\sqrt{e\sin(c+dx)}}-\frac{44e^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5a^2d\sqrt{\sin(c+dx)}}+\frac{4e(e\sin(c+dx))^{3/2}}{3a^2d}-\frac{12e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(5/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (4*e^3)/(a^2*d*Sqrt[e*Sin[c + d*x]]) - (2*e^3*Cos[c + d*x])/(a^2*d*Sqrt[e*S
in[c + d*x]]) - (2*e^3*Cos[c + d*x]^3)/(a^2*d*Sqrt[e*Sin[c + d*x]]) - (44*e
^2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(5*a^2*d*Sqrt[Sin
[c + d*x]]) + (4*e*(e*Sin[c + d*x])^(3/2))/(3*a^2*d) - (12*e*Cos[c + d*x]*(
e*Sin[c + d*x])^(3/2))/(5*a^2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
```

```
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2649

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(b*Sin[e + f*x])^(n + 1)*((a*Cos[e + f*x])^(m - 1)/
(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)(e\sin(c+dx))^{5/2}}{(-a-a\cos(c+dx))^2}\,dx$$

$$= \frac{e^4\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{(e\sin(c+dx))^{3/2}}\,dx}{a^4}$$

$$= \frac{e^4\int \left(\frac{a^2\cos^2(c+dx)}{(e\sin(c+dx))^{3/2}} - \frac{2a^2\cos^3(c+dx)}{(e\sin(c+dx))^{3/2}} + \frac{a^2\cos^4(c+dx)}{(e\sin(c+dx))^{3/2}}\right)dx}{a^4}$$

$$= \frac{e^4\int \frac{\cos^2(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx}{a^2} + \frac{e^4\int \frac{\cos^4(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx}{a^2} - \frac{(2e^4)\int \frac{\cos^3(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx}{a^2}$$

$$= -\frac{2e^3\cos(c+dx)}{a^2d\sqrt{e\sin(c+dx)}} - \frac{2e^3\cos^3(c+dx)}{a^2d\sqrt{e\sin(c+dx)}} - \frac{(2e^2)\int \sqrt{e\sin(c+dx)}\,dx}{a^2} - \frac{(6$$

$$= -\frac{2e^3\cos(c+dx)}{a^2d\sqrt{e\sin(c+dx)}} - \frac{2e^3\cos^3(c+dx)}{a^2d\sqrt{e\sin(c+dx)}} - \frac{12e\cos(c+dx)(e\sin(c+dx))^{3/}}{5a^2d}$$

$$= \frac{4e^3}{a^2d\sqrt{e\sin(c+dx)}} - \frac{2e^3\cos(c+dx)}{a^2d\sqrt{e\sin(c+dx)}} - \frac{2e^3\cos^3(c+dx)}{a^2d\sqrt{e\sin(c+dx)}} - \frac{4e^2E\left(\frac{1}{2}(c\right.}{}$$

$$= \frac{4e^3}{a^2d\sqrt{e\sin(c+dx)}} - \frac{2e^3\cos(c+dx)}{a^2d\sqrt{e\sin(c+dx)}} - \frac{2e^3\cos^3(c+dx)}{a^2d\sqrt{e\sin(c+dx)}} - \frac{44e^2E\left(\frac{1}{2}(\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.96, size = 249, normalized size = 1.33

$$\frac{4\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)(e\sin(c+dx))^{5/2}\left(\frac{352ie^{2i(2c+dx)}\left(3\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};e^{2i(c+dx)}\right)+e^{2idx}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};e^{2i(c+dx)}\right)\right)}{(1+e^{2ic})\left(1-e^{2i(c+dx)}\right)^{5/2}}+\csc^2(c+dx)\left(20\cos(dx)\sin(c)-3\cos(2dx)\sin(2c)+\sec\left(\frac{c}{2}\right)\left(-36\sec(c)\sin\left(\frac{3c}{2}\right)+60\sec\left(\frac{1}{2}(c+dx)\right)\sin\left(\frac{dx}{2}\right)\right)+20\cos(c)\sin(dx)-3\cos(2c)\sin(2dx)-96\sec(c)\tan\left(\frac{c}{2}\right)\right)\right)}{15a^2d(1+\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sin[c + d*x])^(5/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (4*Cos[(c + d*x)/2]^4*Sec[c + d*x]^2*(e*Sin[c + d*x])^(5/2)*(((352*I)*E^((2
*I)*(2*c + d*x))*(3*Hypergeometric2F1[-1/4, 1/2, 3/4, E^((2*I)*(c + d*x))]
+ E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, E^((2*I)*(c + d*x))]))/((1
 + E^((2*I)*c))*(1 - E^((2*I)*(c + d*x)))^(5/2)) + Csc[c + d*x]^2*(20*Cos[d
*x]*Sin[c] - 3*Cos[2*d*x]*Sin[2*c] + Sec[c/2]*(-36*Sec[c]*Sin[(3*c)/2] + 60
*Sec[(c + d*x)/2]*Sin[(d*x)/2]) + 20*Cos[c]*Sin[d*x] - 3*Cos[2*c]*Sin[2*d*x
] - 96*Sec[c]*Tan[c/2])))/(15*a^2*d*(1 + Sec[c + d*x])^2)
```

Maple [A]
time = 0.22, size = 173, normalized size = 0.93

method	result
default	$\frac{2e^3\left(66\sqrt{-\sin\left(dx+c\right)+1}\,\sqrt{2\sin\left(dx+c\right)+2}\,\left(\sqrt{\sin}\,(dx+c)\right)\operatorname{EllipticE}\left(\sqrt{-\sin\left(dx+c\right)+1},\frac{\sqrt{2}}{2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/15/a^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*e^3*(66*(-sin(d*x+c)+1)^(1/2)*(2*s
in(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(
1/2))-33*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*Elli
pticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+3*cos(d*x+c)^4-10*cos(d*x+c)^3-33*
cos(d*x+c)^2+40*cos(d*x+c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] e^(5/2)*integrate(sin(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.12, size = 152, normalized size = 0.81

$$-\frac{2\left(\left(3\cos\left(dx+c\right)^2e^{\frac{5}{2}}-7\cos\left(dx+c\right)e^{\frac{5}{2}}-40e^{\frac{5}{2}}\right)\sin\left(dx+c\right)^{\frac{3}{2}}+33\sqrt{-i}\left(i\sqrt{2}\cos\left(dx+c\right)e^{\frac{5}{2}}+i\sqrt{2}e^{\frac{5}{2}}\right)\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)))+33\sqrt{i}\left(-i\sqrt{2}\cos\left(dx+c\right)e^{\frac{5}{2}}-i\sqrt{2}e^{\frac{5}{2}}\right)\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)))\right)}{15\left(a^2d\cos\left(dx+c\right)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -2/15*((3*cos(d*x + c)^2*e^(5/2) - 7*cos(d*x + c)*e^(5/2) - 40*e^(5/2))*sin
(d*x + c)^(3/2) + 33*sqrt(-I)*(I*sqrt(2)*cos(d*x + c)*e^(5/2) + I*sqrt(2)*e
^(5/2))*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) + I*si
n(d*x + c))) + 33*sqrt(I)*(-I*sqrt(2)*cos(d*x + c)*e^(5/2) - I*sqrt(2)*e^(5
/2))*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d
*x + c))))/(a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(5/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*sin(d*x + c))^(5/2)/(a*sec(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^2\,{\left(e\,\sin\left(c+d\,x\right)\right)}^{5/2}}{a^2\,{\left(\cos\left(c+d\,x\right)+1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(5/2)/(a + a/cos(c + d*x))^2,x)
```

```
[Out] int((cos(c + d*x)^2*(e*sin(c + d*x))^(5/2))/(a^2*(cos(c + d*x) + 1)^2), x)
```

3.129 $\int \frac{(e\sin(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=189

$$\frac{4e^3}{3a^2d(e\sin(c+dx))^{3/2}}-\frac{2e^3\cos(c+dx)}{3a^2d(e\sin(c+dx))^{3/2}}-\frac{2e^3\cos^3(c+dx)}{3a^2d(e\sin(c+dx))^{3/2}}-\frac{4e^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+}}{a^2d\sqrt{e\sin(c+dx)}}$$

```
[Out] 4/3*e^3/a^2/d/(e*sin(d*x+c))^(3/2)-2/3*e^3*cos(d*x+c)/a^2/d/(e*sin(d*x+c))^
(3/2)-2/3*e^3*cos(d*x+c)^3/a^2/d/(e*sin(d*x+c))^(3/2)+4*e^2*(sin(1/2*c+1/4*
Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1
/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/a^2/d/(e*sin(d*x+c))^(1/2)+4*e*(e*sin(d*x
+c))^(1/2)/a^2/d-4/3*e*cos(d*x+c)*(e*sin(d*x+c))^(1/2)/a^2/d
```

Rubi [A]

time = 0.42, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3957, 2954, 2952, 2647, 2721, 2720, 2644, 14, 2649}

$$\frac{4e^3}{3a^2d(e\sin(c+dx))^{3/2}}-\frac{2e^3\cos^3(c+dx)}{3a^2d(e\sin(c+dx))^{3/2}}-\frac{2e^3\cos(c+dx)}{3a^2d(e\sin(c+dx))^{3/2}}-\frac{4e^2\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{a^2d\sqrt{e\sin(c+dx)}}+\frac{4e\sqrt{e\sin(c+dx)}}{a^2d}-\frac{4e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(3/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (4*e^3)/(3*a^2*d*(e*Sin[c + d*x])^(3/2)) - (2*e^3*Cos[c + d*x])/(3*a^2*d*(e
*Sin[c + d*x])^(3/2)) - (2*e^3*Cos[c + d*x]^3)/(3*a^2*d*(e*Sin[c + d*x])^(3
/2)) - (4*e^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^2*d*S
qrt[e*Sin[c + d*x]]) + (4*e*Sqrt[e*Sin[c + d*x]])/(a^2*d) - (4*e*Cos[c + d*
x]*Sqrt[e*Sin[c + d*x]])/(3*a^2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
```

```
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2649

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(b*Sin[e + f*x])^(n + 1)*((a*Cos[e + f*x])^(m - 1)/
(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)(e\sin(c+dx))^{3/2}}{(-a-a\cos(c+dx))^2}\,dx$$
$$= \frac{e^4\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{(e\sin(c+dx))^{5/2}}\,dx}{a^4}$$
$$= \frac{e^4\int \left(\frac{a^2\cos^2(c+dx)}{(e\sin(c+dx))^{5/2}} - \frac{2a^2\cos^3(c+dx)}{(e\sin(c+dx))^{5/2}} + \frac{a^2\cos^4(c+dx)}{(e\sin(c+dx))^{5/2}}\right)dx}{a^4}$$
$$= \frac{e^4\int \frac{\cos^2(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx}{a^2} + \frac{e^4\int \frac{\cos^4(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx}{a^2} - \frac{(2e^4)\int \frac{\cos^3(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx}{a^2}$$
$$= -\frac{2e^3\cos(c+dx)}{3a^2d(e\sin(c+dx))^{3/2}} - \frac{2e^3\cos^3(c+dx)}{3a^2d(e\sin(c+dx))^{3/2}} - \frac{(2e^2)\int \frac{1}{\sqrt{e\sin(c+dx)}}\,d}{3a^2}$$
$$= -\frac{2e^3\cos(c+dx)}{3a^2d(e\sin(c+dx))^{3/2}} - \frac{2e^3\cos^3(c+dx)}{3a^2d(e\sin(c+dx))^{3/2}} - \frac{4e\cos(c+dx)\sqrt{e\sin(c+}}{3a^2d}$$
$$= \frac{4e^3}{3a^2d(e\sin(c+dx))^{3/2}} - \frac{2e^3\cos(c+dx)}{3a^2d(e\sin(c+dx))^{3/2}} - \frac{2e^3\cos^3(c+dx)}{3a^2d(e\sin(c+dx))^{3/2}} - \frac{4e}{}$$
$$= \frac{4e^3}{3a^2d(e\sin(c+dx))^{3/2}} - \frac{2e^3\cos(c+dx)}{3a^2d(e\sin(c+dx))^{3/2}} - \frac{2e^3\cos^3(c+dx)}{3a^2d(e\sin(c+dx))^{3/2}} - \frac{4e}{}$$

Mathematica [A]
time = 1.24, size = 119, normalized size = 0.63

$$\frac{2\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left((15+10\cos(c+dx)-\cos(2(c+dx)))\csc(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)+\frac{24F\left(\frac{1}{4}(-2c+\pi-2dx)|2\right)}{\sin^{\frac{3}{2}}(c+dx)}\right)(e\sin(c+dx))^{3/2}}{3a^2d(1+\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sin[c + d*x])^(3/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*Cos[(c + d*x)/2]^4*Sec[c + d*x]^2*((15 + 10*Cos[c + d*x] - Cos[2*(c + d*
x)])*Csc[c + d*x]*Sec[(c + d*x)/2]^2 + (24*EllipticF[(-2*c + Pi - 2*d*x)/4,
 2])/Sin[c + d*x]^(3/2))*(e*Sin[c + d*x])^(3/2))/(3*a^2*d*(1 + Sec[c + d*x]
)^2)
```

Maple [A]
time = 0.22, size = 153, normalized size = 0.81

method	result

default	$-\frac{2e^{3}\left(3\sqrt{-\sin\left(dx+c\right)+1}\,\sqrt{2\sin\left(dx+c\right)+2}\,\left(\sin^{\frac{7}{2}}\left(dx+c\right)\right)\mathrm{EllipticF}\left(\sqrt{-\sin\left(dx+c\right)+1},\frac{\sqrt{2}}{2}\right)-}{3a^{2}\left(e\sin(dx+c)\right)^{\frac{3}{2}}\cos(dx+c)(\cos^{2}(dx+c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/a^2/(e*sin(d*x+c))^(3/2)/cos(d*x+c)/(cos(d*x+c)^2-1)*e^3*(3*(-sin(d*x+
c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticF((-sin(d*x+c)+
1)^(1/2),1/2*2^(1/2))-cos(d*x+c)^6+6*cos(d*x+c)^5+4*cos(d*x+c)^4-14*cos(d*x
+c)^3-3*cos(d*x+c)^2+8*cos(d*x+c))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] e^(3/2)*integrate(sin(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.79, size = 141, normalized size = 0.75

$$-\frac{2\left(3\sqrt{-i}\left(\sqrt{2}\cos\left(dx+c\right)e^{\frac{3}{2}}+\sqrt{2}\,e^{\frac{3}{2}}\right)\mathrm{weierstrassPInverse}(4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right))+3\sqrt{i}\left(\sqrt{2}\cos\left(dx+c\right)e^{\frac{3}{2}}+\sqrt{2}\,e^{\frac{3}{2}}\right)\mathrm{weierstrassPInverse}(4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right))+\left(\cos\left(dx+c\right)^{2}e^{\frac{3}{2}}-5\cos\left(dx+c\right)e^{\frac{3}{2}}-8\,e^{\frac{3}{2}}\right)\sqrt{\sin\left(dx+c\right)}\right)}{3\left(a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -2/3*(3*sqrt(-I)*(sqrt(2)*cos(d*x + c)*e^(3/2) + sqrt(2)*e^(3/2))*weierstra
ssPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c)) + 3*sqrt(I)*(sqrt(2)*cos(d*
x + c)*e^(3/2) + sqrt(2)*e^(3/2))*weierstrassPInverse(4, 0, cos(d*x + c) -
I*sin(d*x + c)) + (cos(d*x + c)^2*e^(3/2) - 5*cos(d*x + c)*e^(3/2) - 8*e^(3
/2))*sqrt(sin(d*x + c)))/(a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(3/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate((e*sin(d*x + c))^(3/2)/(a*sec(d*x + c) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^2\,{\left(e\,\sin\left(c+d\,x\right)\right)}^{3/2}}{a^2\,{\left(\cos\left(c+d\,x\right)+1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(3/2)/(a + a/cos(c + d*x))^2,x)
[Out] int((cos(c + d*x)^2*(e*sin(c + d*x))^(3/2))/(a^2*(cos(c + d*x) + 1)^2), x)
```

3.130 $\int \frac{\sqrt{e\sin(c+dx)}}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=188

$$\frac{4e^3}{5a^2d(e\sin(c+dx))^{5/2}}-\frac{2e^3\cos(c+dx)}{5a^2d(e\sin(c+dx))^{5/2}}-\frac{2e^3\cos^3(c+dx)}{5a^2d(e\sin(c+dx))^{5/2}}-\frac{4e}{a^2d\sqrt{e\sin(c+dx)}}+\frac{16e\cos(c+}{5a^2d\sqrt{e\sin(c}$$

```
[Out] 4/5*e^3/a^2/d/(e*sin(d*x+c))^(5/2)-2/5*e^3*cos(d*x+c)/a^2/d/(e*sin(d*x+c))^
(5/2)-2/5*e^3*cos(d*x+c)^3/a^2/d/(e*sin(d*x+c))^(5/2)-4*e/a^2/d/(e*sin(d*x+
c))^(1/2)+16/5*e*cos(d*x+c)/a^2/d/(e*sin(d*x+c))^(1/2)-28/5*(sin(1/2*c+1/4*
Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1
/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/a^2/d/sin(d*x+c)^(1/2)
```

Rubi [A]

time = 0.40, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3957, 2954, 2952, 2647, 2716, 2721, 2719, 2644, 14}

$$\frac{4e^3}{5a^2d(e\sin(c+dx))^{5/2}}-\frac{2e^3\cos^3(c+dx)}{5a^2d(e\sin(c+dx))^{5/2}}-\frac{2e^3\cos(c+dx)}{5a^2d(e\sin(c+dx))^{5/2}}-\frac{4e}{a^2d\sqrt{e\sin(c+dx)}}+\frac{16e\cos(c+dx)}{5a^2d\sqrt{e\sin(c+dx)}}+\frac{28E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5a^2d\sqrt{\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Sin[c + d*x]]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (4*e^3)/(5*a^2*d*(e*Sin[c + d*x])^(5/2)) - (2*e^3*Cos[c + d*x])/(5*a^2*d*(e
*Sin[c + d*x])^(5/2)) - (2*e^3*Cos[c + d*x]^3)/(5*a^2*d*(e*Sin[c + d*x])^(5
/2)) - (4*e)/(a^2*d*Sqrt[e*Sin[c + d*x]]) + (16*e*Cos[c + d*x])/(5*a^2*d*Sq
rt[e*Sin[c + d*x]]) + (28*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d
*x]])/(5*a^2*d*Sqrt[Sin[c + d*x]])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{e\sin(c+dx)}}{(a+a\sec(c+dx))^2}\,dx &= \int \frac{\cos^2(c+dx)\sqrt{e\sin(c+dx)}}{(-a-a\cos(c+dx))^2}\,dx \\
&= \frac{e^4\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{(e\sin(c+dx))^{7/2}}\,dx}{a^4} \\
&= \frac{e^4\int \left(\frac{a^2\cos^2(c+dx)}{(e\sin(c+dx))^{7/2}} - \frac{2a^2\cos^3(c+dx)}{(e\sin(c+dx))^{7/2}} + \frac{a^2\cos^4(c+dx)}{(e\sin(c+dx))^{7/2}}\right)dx}{a^4} \\
&= \frac{e^4\int \frac{\cos^2(c+dx)}{(e\sin(c+dx))^{7/2}}\,dx}{a^2} + \frac{e^4\int \frac{\cos^4(c+dx)}{(e\sin(c+dx))^{7/2}}\,dx}{a^2} - \frac{(2e^4)\int \frac{\cos^3(c+dx)}{(e\sin(c+dx))^{7/2}}\,dx}{a^2} \\
&= -\frac{2e^3\cos(c+dx)}{5a^2d(e\sin(c+dx))^{5/2}} - \frac{2e^3\cos^3(c+dx)}{5a^2d(e\sin(c+dx))^{5/2}} - \frac{(2e^2)\int \frac{1}{(e\sin(c+dx))^{3/2}}\,dx}{5a^2} - \\
&= -\frac{2e^3\cos(c+dx)}{5a^2d(e\sin(c+dx))^{5/2}} - \frac{2e^3\cos^3(c+dx)}{5a^2d(e\sin(c+dx))^{5/2}} + \frac{16e\cos(c+dx)}{5a^2d\sqrt{e\sin(c+dx)}} + \\
&= \frac{4e^3}{5a^2d(e\sin(c+dx))^{5/2}} - \frac{2e^3\cos(c+dx)}{5a^2d(e\sin(c+dx))^{5/2}} - \frac{2e^3\cos^3(c+dx)}{5a^2d(e\sin(c+dx))^{5/2}} - \\
&= \frac{4e^3}{5a^2d(e\sin(c+dx))^{5/2}} - \frac{2e^3\cos(c+dx)}{5a^2d(e\sin(c+dx))^{5/2}} - \frac{2e^3\cos^3(c+dx)}{5a^2d(e\sin(c+dx))^{5/2}} -
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.90, size = 222, normalized size = 1.18

$$
\frac{4\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\sqrt{e\sin(c+dx)}\left(\frac{56ie^{2ic}\left(3\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};e^{2i(c+dx)}\right)+e^{2idx}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};e^{2i(c+dx)}\right)\right)}{(1+e^{2ic})\sqrt{1-e^{2i(c+dx)}}} + \frac{3}{4}\sec(c)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(49\sin\left(\frac{1}{2}(c-dx)\right)+35\sin\left(\frac{1}{2}(3c+dx)\right)-23\sin\left(\frac{1}{2}(c+3dx)\right)+5\sin\left(\frac{1}{2}(5c+3dx)\right)\right)\right)}{15a^2d(1+\sec(c+dx))^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Sin[c + d*x]]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (4*Cos[(c + d*x)/2]^4*Sec[c + d*x]^2*Sqrt[e*Sin[c + d*x]]*(((56*I)*E^((2*I)
*c)*(3*Hypergeometric2F1[-1/4, 1/2, 3/4, E^((2*I)*(c + d*x))] + E^((2*I)*d*
x)*Hypergeometric2F1[1/2, 3/4, 7/4, E^((2*I)*(c + d*x))]))/((1 + E^((2*I)*c
))*Sqrt[1 - E^((2*I)*(c + d*x))]) + (3*Sec[c]*Sec[(c + d*x)/2]^3*(49*Sin[(c
 - d*x)/2] + 35*Sin[(3*c + d*x)/2] - 23*Sin[(c + 3*d*x)/2] + 5*Sin[(5*c + 3
*d*x)/2]))/4))/(15*a^2*d*(1 + Sec[c + d*x])^2)
```

Maple [A]
time = 0.24, size = 205, normalized size = 1.09

method	result
default	$-\frac{2e\left(-\frac{2e^2}{5(e\sin(dx+c))^{\frac{5}{2}}}+\frac{2}{\sqrt{e\sin(dx+c)}}\right)}{a^2}-\frac{2e\left(14\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\left(\sin^{\frac{7}{2}}(dx+c)\right)\mathrm{El}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-2*e/a^2*(-2/5*e^2/(e*sin(d*x+c))^(5/2)+2/(e*sin(d*x+c))^(1/2))-2/5*e*(14*
(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticE((-s
in(d*x+c)+1)^(1/2),1/2*2^(1/2))-7*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1
/2)*sin(d*x+c)^(7/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+9*sin(d*x
+c)^5-11*sin(d*x+c)^3+2*sin(d*x+c))/a^2/sin(d*x+c)^3/cos(d*x+c)/(e*sin(d*x+
c))^(1/2))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] e^(1/2)*integrate(sqrt(sin(d*x + c))/(a*sec(d*x + c) + a)^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.72, size = 184, normalized size = 0.98

$$-\frac{2\left(\left(9\cos(dx+c)e^{\frac{1}{2}}+8e^{\frac{1}{2}}\right)\sin(dx+c)^{\frac{3}{2}}+7\sqrt{-i}\left(-i\sqrt{2}\cos(dx+c)^2e^{\frac{1}{2}}-2i\sqrt{2}\cos(dx+c)e^{\frac{1}{2}}-i\sqrt{2}e^{\frac{1}{2}}\right)\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c)))+7\sqrt{i}\left(i\sqrt{2}\cos(dx+c)^2e^{\frac{1}{2}}+2i\sqrt{2}\cos(dx+c)e^{\frac{1}{2}}+i\sqrt{2}e^{\frac{1}{2}}\right)\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)-i\sin(dx+c)))\right)}{5\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -2/5*((9*cos(d*x + c)*e^(1/2) + 8*e^(1/2))*sin(d*x + c)^(3/2) + 7*sqrt(-I)*
(-I*sqrt(2)*cos(d*x + c)^2*e^(1/2) - 2*I*sqrt(2)*cos(d*x + c)*e^(1/2) - I*s
qrt(2)*e^(1/2))*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c
) + I*sin(d*x + c))) + 7*sqrt(I)*(I*sqrt(2)*cos(d*x + c)^2*e^(1/2) + 2*I*sq
rt(2)*cos(d*x + c)*e^(1/2) + I*sqrt(2)*e^(1/2))*weierstrassZeta(4, 0, weier
strassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^2*d*cos(d*x + c)^2
 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{e\sin(c+dx)}}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(1/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(sqrt(e*sin(c + d*x))/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a*
*2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*sin(d*x + c))/(a*sec(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^2\,\sqrt{e\,\sin\left(c+d\,x\right)}}{a^2\,{\left(\cos\left(c+d\,x\right)+1\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(1/2)/(a + a/cos(c + d*x))^2,x)
```

```
[Out] int((cos(c + d*x)^2*(e*sin(c + d*x))^(1/2))/(a^2*(cos(c + d*x) + 1)^2), x)
```

3.131 $\int \frac{1}{(a+a\sec(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx$

Optimal. Leaf size=190

$$\frac{4e^3}{7a^2d(e\sin(c+dx))^{7/2}}-\frac{2e^3\cos(c+dx)}{7a^2d(e\sin(c+dx))^{7/2}}-\frac{2e^3\cos^3(c+dx)}{7a^2d(e\sin(c+dx))^{7/2}}-\frac{4e}{3a^2d(e\sin(c+dx))^{3/2}}+\frac{16e\cos}{21a^2d(e\sin}$$

```
[Out] 4/7*e^3/a^2/d/(e*sin(d*x+c))^(7/2)-2/7*e^3*cos(d*x+c)/a^2/d/(e*sin(d*x+c))^
(7/2)-2/7*e^3*cos(d*x+c)^3/a^2/d/(e*sin(d*x+c))^(7/2)-4/3*e/a^2/d/(e*sin(d*
x+c))^(3/2)+16/21*e*cos(d*x+c)/a^2/d/(e*sin(d*x+c))^(3/2)-20/21*(sin(1/2*c+
1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*
Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/a^2/d/(e*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.43, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3957, 2954, 2952, 2647, 2716, 2721, 2720, 2644, 14}

$$\frac{4e^3}{7a^2d(e\sin(c+dx))^{7/2}}-\frac{2e^3\cos^3(c+dx)}{7a^2d(e\sin(c+dx))^{7/2}}-\frac{2e^3\cos(c+dx)}{7a^2d(e\sin(c+dx))^{7/2}}-\frac{4e}{3a^2d(e\sin(c+dx))^{3/2}}+\frac{16e\cos(c+dx)}{21a^2d(e\sin(c+dx))^{3/2}}+\frac{20\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{21a^2d\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[c + d*x])^2*Sqrt[e*Sin[c + d*x]]),x]
```

```
[Out] (4*e^3)/(7*a^2*d*(e*Sin[c + d*x])^(7/2)) - (2*e^3*Cos[c + d*x])/(7*a^2*d*(e
*Sin[c + d*x])^(7/2)) - (2*e^3*Cos[c + d*x]^3)/(7*a^2*d*(e*Sin[c + d*x])^(7
/2)) - (4*e)/(3*a^2*d*(e*Sin[c + d*x])^(3/2)) + (16*e*Cos[c + d*x])/(21*a^2
*d*(e*Sin[c + d*x])^(3/2)) + (20*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[
c + d*x]])/(21*a^2*d*Sqrt[e*Sin[c + d*x]])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
```

```
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx = \int \frac{\cos^2(c+dx)}{(-a-a\cos(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx$$

$$= \frac{e^4\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{(e\sin(c+dx))^{9/2}}\,dx}{a^4}$$

$$= \frac{e^4\int \left(\frac{a^2\cos^2(c+dx)}{(e\sin(c+dx))^{9/2}} - \frac{2a^2\cos^3(c+dx)}{(e\sin(c+dx))^{9/2}} + \frac{a^2\cos^4(c+dx)}{(e\sin(c+dx))^{9/2}}\right)dx}{a^4}$$

$$= \frac{e^4\int \frac{\cos^2(c+dx)}{(e\sin(c+dx))^{9/2}}\,dx}{a^2} + \frac{e^4\int \frac{\cos^4(c+dx)}{(e\sin(c+dx))^{9/2}}\,dx}{a^2} - \frac{(2e^4)\int \frac{\cos^3(c+}{(e\sin(c+d}}{a^2}$$

$$= -\frac{2e^3\cos(c+dx)}{7a^2d(e\sin(c+dx))^{7/2}} - \frac{2e^3\cos^3(c+dx)}{7a^2d(e\sin(c+dx))^{7/2}} - \frac{(2e^2)\int \frac{}{(e}}{}$$

$$= -\frac{2e^3\cos(c+dx)}{7a^2d(e\sin(c+dx))^{7/2}} - \frac{2e^3\cos^3(c+dx)}{7a^2d(e\sin(c+dx))^{7/2}} + \frac{16e\,\mathrm{co}}{21a^2d(e\,\mathrm{si}}$$

$$= \frac{4e^3}{7a^2d(e\sin(c+dx))^{7/2}} - \frac{2e^3\cos(c+dx)}{7a^2d(e\sin(c+dx))^{7/2}} - \frac{2e^3\cos^3(}{7a^2d(e\sin(}$$

$$= \frac{4e^3}{7a^2d(e\sin(c+dx))^{7/2}} - \frac{2e^3\cos(c+dx)}{7a^2d(e\sin(c+dx))^{7/2}} - \frac{2e^3\cos^3(}{7a^2d(e\sin(}$$

Mathematica [A]
time = 0.94, size = 82, normalized size = 0.43

$$-\frac{\csc^3(c+dx)\left(16(8+11\cos(c+dx))\sin^4\left(\frac{1}{2}(c+dx)\right)+40F\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)\sin^{\frac{7}{2}}(c+dx)\right)}{42a^2d\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[c + d*x])^2*Sqrt[e*Sin[c + d*x]]),x]
```

```
[Out] -1/42*(Csc[c + d*x]^3*(16*(8 + 11*Cos[c + d*x])*Sin[(c + d*x)/2]^4 + 40*Ell
ipticF[(-2*c + Pi - 2*d*x)/4, 2]*Sin[c + d*x]^(7/2)))/(a^2*d*Sqrt[e*Sin[c +
 d*x]])
```

Maple [A]
time = 0.22, size = 148, normalized size = 0.78

method	result

default	$\frac{\frac{4e^{3}\left(7\left(\cos^{2}(dx+c)\right)-4\right)}{21a^{2}(e\sin(dx+c))^{\frac{7}{2}}}-\frac{2\left(5\sqrt{-\sin\left(dx+c\right)+1}\,\sqrt{2\sin\left(dx+c\right)+2}\,\left(\sin^{\frac{9}{2}}(dx+c)\right)\text{EllipticF}\left(\sqrt{-\sin\left(dx+c\right)}\cdots\right.\right.}{21a^{2}\sin(dx+c)^{4}\cos(dx+c)\sqrt{e\sin\left(dx+c\right)}\cdots}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (4/21/a^2*e^3/(e*sin(d*x+c))^(7/2)*(7*cos(d*x+c)^2-4)-2/21*(5*(-sin(d*x+c)+
1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(9/2)*EllipticF((-sin(d*x+c)+1)^
(1/2),1/2*2^(1/2))+11*sin(d*x+c)^5-17*sin(d*x+c)^3+6*sin(d*x+c))/a^2/sin(d*
x+c)^4/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.97, size = 162, normalized size = 0.85

$$\frac{2\left(5\sqrt{-i}\left(\sqrt{2}\cos\left(dx+c\right)^{2}+2\sqrt{2}\cos\left(dx+c\right)+\sqrt{2}\right)\text{weierstrassPInverse}(4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right))+5\sqrt{i}\left(\sqrt{2}\cos\left(dx+c\right)^{2}+2\sqrt{2}\cos\left(dx+c\right)+\sqrt{2}\right)\text{weierstrassPInverse}(4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right))-(11\cos\left(dx+c\right)+8)\sqrt{\sin\left(dx+c\right)}\right)}{21\left(a^{2}d\cos\left(dx+c\right)^{2}e^{\frac{1}{2}}+2a^{2}d\cos\left(dx+c\right)e^{\frac{1}{2}}+a^{2}de^{\frac{1}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/21*(5*sqrt(-I)*(sqrt(2)*cos(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c) + sqrt(2)
)*weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(I)*(sqr
t(2)*cos(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c) + sqrt(2))*weierstrassPInverse
(4, 0, cos(d*x + c) - I*sin(d*x + c)) - (11*cos(d*x + c) + 8)*sqrt(sin(d*x
+ c)))/(a^2*d*cos(d*x + c)^2*e^(1/2) + 2*a^2*d*cos(d*x + c)*e^(1/2) + a^2*d
*e^(1/2))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sqrt{e\sin\left(c+dx\right)}\sec^{2}(c+dx)+2\sqrt{e\sin\left(c+dx\right)}\sec(c+dx)+\sqrt{e\sin\left(c+dx\right)}}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**2/(e*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(e*sin(c + d*x))*sec(c + d*x)**2 + 2*sqrt(e*sin(c + d*x))*s
ec(c + d*x) + sqrt(e*sin(c + d*x))), x)/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^2*sqrt(e*sin(d*x + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^2}{a^2\,\sqrt{e\,\sin\left(c+d\,x\right)}\,{\left(\cos\left(c+d\,x\right)+1\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*sin(c + d*x))^(1/2)*(a + a/cos(c + d*x))^2),x)
```

```
[Out] int(cos(c + d*x)^2/(a^2*(e*sin(c + d*x))^(1/2)*(cos(c + d*x) + 1)^2), x)
```

3.132 $\int \frac{1}{(a+a\sec(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=224

$$\frac{4e^3}{9a^2d(e\sin(c+dx))^{9/2}}-\frac{2e^3\cos(c+dx)}{9a^2d(e\sin(c+dx))^{9/2}}-\frac{2e^3\cos^3(c+dx)}{9a^2d(e\sin(c+dx))^{9/2}}-\frac{4e}{5a^2d(e\sin(c+dx))^{5/2}}+\frac{16e\cos(}{45a^2d(e\sin(}$$

```
[Out] 4/9*e^3/a^2/d/(e*sin(d*x+c))^(9/2)-2/9*e^3*cos(d*x+c)/a^2/d/(e*sin(d*x+c))^
(9/2)-2/9*e^3*cos(d*x+c)^3/a^2/d/(e*sin(d*x+c))^(9/2)-4/5*e/a^2/d/(e*sin(d*
x+c))^(5/2)+16/45*e*cos(d*x+c)/a^2/d/(e*sin(d*x+c))^(5/2)-4/15*cos(d*x+c)/a
^2/d/e/(e*sin(d*x+c))^(1/2)+4/15*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/
2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x
+c))^(1/2)/a^2/d/e^2/sin(d*x+c)^(1/2)
```

Rubi [A]
time = 0.46, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3957, 2954, 2952, 2647, 2716, 2721, 2719, 2644, 14}

$$\frac{4e^3}{9a^2d(e\sin(c+dx))^{9/2}}-\frac{2e^3\cos^3(c+dx)}{9a^2d(e\sin(c+dx))^{9/2}}-\frac{2e^3\cos(c+dx)}{9a^2d(e\sin(c+dx))^{9/2}}-\frac{4E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{15a^2de^2\sqrt{\sin(c+dx)}}-\frac{4e}{5a^2d(e\sin(c+dx))^{5/2}}+\frac{16e\cos(c+dx)}{45a^2d(e\sin(c+dx))^{5/2}}-\frac{4\cos(c+dx)}{15a^2de\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[c + d*x])^2*(e*Sin[c + d*x])^(3/2)),x]
```

```
[Out] (4*e^3)/(9*a^2*d*(e*Sin[c + d*x])^(9/2)) - (2*e^3*Cos[c + d*x])/(9*a^2*d*(e
*Sin[c + d*x])^(9/2)) - (2*e^3*Cos[c + d*x]^3)/(9*a^2*d*(e*Sin[c + d*x])^(9
/2)) - (4*e)/(5*a^2*d*(e*Sin[c + d*x])^(5/2)) + (16*e*Cos[c + d*x])/(45*a^2
*d*(e*Sin[c + d*x])^(5/2)) - (4*Cos[c + d*x])/(15*a^2*d*e*Sqrt[e*Sin[c + d*
x]]) - (4*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(15*a^2*d*
e^2*Sqrt[Sin[c + d*x]])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(a+a\sec(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx &= \int \frac{\cos^2(c+dx)}{(-a-a\cos(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx \\
&= \frac{e^4\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{(e\sin(c+dx))^{11/2}}\,dx}{a^4} \\
&= \frac{e^4\int \left(\frac{a^2\cos^2(c+dx)}{(e\sin(c+dx))^{11/2}}-\frac{2a^2\cos^3(c+dx)}{(e\sin(c+dx))^{11/2}}+\frac{a^2\cos^4(c+dx)}{(e\sin(c+dx))^{11/2}}\right)dx}{a^4} \\
&= \frac{e^4\int \frac{\cos^2(c+dx)}{(e\sin(c+dx))^{11/2}}\,dx}{a^2}+\frac{e^4\int \frac{\cos^4(c+dx)}{(e\sin(c+dx))^{11/2}}\,dx}{a^2}-\frac{(2e^4)\int \frac{\cos^3(}{(e\sin(c+}}{a^2} \\
&= -\frac{2e^3\cos(c+dx)}{9a^2d(e\sin(c+dx))^{9/2}}-\frac{2e^3\cos^3(c+dx)}{9a^2d(e\sin(c+dx))^{9/2}}-\frac{(2e^2)\int \frac{}{(e}}{} \\
&= -\frac{2e^3\cos(c+dx)}{9a^2d(e\sin(c+dx))^{9/2}}-\frac{2e^3\cos^3(c+dx)}{9a^2d(e\sin(c+dx))^{9/2}}+\frac{16e\,co}{45a^2d(e\,si} \\
&= \frac{4e^3}{9a^2d(e\sin(c+dx))^{9/2}}-\frac{2e^3\cos(c+dx)}{9a^2d(e\sin(c+dx))^{9/2}}-\frac{2e^3\cos^3(}{9a^2d(e\sin(c} \\
&= \frac{4e^3}{9a^2d(e\sin(c+dx))^{9/2}}-\frac{2e^3\cos(c+dx)}{9a^2d(e\sin(c+dx))^{9/2}}-\frac{2e^3\cos^3(}{9a^2d(e\sin(c} \\
&= \frac{4e^3}{9a^2d(e\sin(c+dx))^{9/2}}-\frac{2e^3\cos(c+dx)}{9a^2d(e\sin(c+dx))^{9/2}}-\frac{2e^3\cos^3(}{9a^2d(e\sin(c}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.96, size = 163, normalized size = 0.73

$$
\frac{\sec^4\left(\frac{1}{2}(c+dx)\right)(\cos(c+dx)+i\sin(c+dx))\left(-31-40\cos(c+dx)-19\cos(2(c+dx))+e^{-2i(c+dx)}\left(1+e^{i(c+dx)}\right)^4\sqrt{1-e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};e^{2i(c+dx)}\right)+16i\sin(c+dx)+13i\sin(2(c+dx))\right)}{180a^2de\sqrt{e\sin(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[c + d*x])^2*(e*Sin[c + d*x])^(3/2)),x]
```

```
[Out] (Sec[(c + d*x)/2]^4*(Cos[c + d*x] + I*Sin[c + d*x])*(-31 - 40*Cos[c + d*x]
- 19*Cos[2*(c + d*x)] + ((1 + E^(I*(c + d*x)))^4*Sqrt[1 - E^((2*I)*(c + d*x
))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((2*I)*(c + d*x))])/E^((2*I)*(c + d*
x)) + (16*I)*Sin[c + d*x] + (13*I)*Sin[2*(c + d*x)]))/(180*a^2*d*e*Sqrt[e*S
in[c + d*x]])
```

Maple [A]
time = 0.24, size = 213, normalized size = 0.95

method	result
default	$\frac{\frac{4e^{3}\left(9\left(\cos^{2}(dx+c)\right)-4\right)}{45a^{2}(e\sin(dx+c))^{\frac{9}{2}}}+\frac{\frac{4\sqrt{-\sin\left(dx+c\right)+1}\,\sqrt{2\sin\left(dx+c\right)+2}\,\left(\sin^{\frac{11}{2}}(dx+c)\right)\mathrm{EllipticE}\left(\sqrt{-\sin\left(dx+c\right)}\right.}{15}}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (4/45*e^3/a^2/(e*sin(d*x+c))^(9/2)*(9*cos(d*x+c)^2-4)+2/45/e*(6*(-sin(d*x+c
)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(11/2)*EllipticE((-sin(d*x+c)+
1)^(1/2),1/2*2^(1/2))-3*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*
x+c)^(11/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+6*sin(d*x+c)^7-19*
sin(d*x+c)^5+23*sin(d*x+c)^3-10*sin(d*x+c))/a^2/sin(d*x+c)^5/cos(d*x+c)/(e*
sin(d*x+c))^(1/2))/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.08, size = 213, normalized size = 0.95

$$-\frac{2\left(3\sqrt{-i}\left(i\sqrt{2}\cos\left(dx+c\right)^{2}+2i\sqrt{2}\cos\left(dx+c\right)+i\sqrt{2}\right)\sin\left(dx+c\right)\mathrm{weierstrassZeta}\left(4,0,\mathrm{weierstrassPInverse}\left(4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)+3\sqrt{i}\left(-i\sqrt{2}\cos\left(dx+c\right)^{2}-2i\sqrt{2}\cos\left(dx+c\right)-i\sqrt{2}\right)\sin\left(dx+c\right)\mathrm{weierstrassZeta}\left(4,0,\mathrm{weierstrassPInverse}\left(4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)+\left(6\cos\left(dx+c\right)^{3}+12\cos\left(dx+c\right)^{2}+19\cos\left(dx+c\right)+8\right)\sqrt{\sin\left(dx+c\right)}\right)}{45\left(a^{2}d\cos\left(dx+c\right)^{2}e^{\frac{3}{2}}+2a^{2}d\cos\left(dx+c\right)e^{\frac{3}{2}}+a^{2}de^{\frac{3}{2}}\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/45*(3*sqrt(-I)*(I*sqrt(2)*cos(d*x + c)^2 + 2*I*sqrt(2)*cos(d*x + c) + I*
sqrt(2))*sin(d*x + c)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d
*x + c) + I*sin(d*x + c))) + 3*sqrt(I)*(-I*sqrt(2)*cos(d*x + c)^2 - 2*I*sqr
t(2)*cos(d*x + c) - I*sqrt(2))*sin(d*x + c)*weierstrassZeta(4, 0, weierstra
ssPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c))) + (6*cos(d*x + c)^3 + 12*c
os(d*x + c)^2 + 19*cos(d*x + c) + 8)*sqrt(sin(d*x + c)))/((a^2*d*cos(d*x +
c)^2*e^(3/2) + 2*a^2*d*cos(d*x + c)*e^(3/2) + a^2*d*e^(3/2))*sin(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**2/(e*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^2*(e*sin(d*x + c))^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^2}{a^2\,{\left(e\,\sin\left(c+d\,x\right)\right)}^{3/2}\,{\left(\cos\left(c+d\,x\right)+1\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*sin(c + d*x))^(3/2)*(a + a/cos(c + d*x))^2),x)
```

```
[Out] int(cos(c + d*x)^2/(a^2*(e*sin(c + d*x))^(3/2)*(cos(c + d*x) + 1)^2), x)
```

3.133 $\int \frac{1}{(a+a\sec(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=224

$$\frac{4e^3}{11a^2d(e\sin(c+dx))^{11/2}}-\frac{2e^3\cos(c+dx)}{11a^2d(e\sin(c+dx))^{11/2}}-\frac{2e^3\cos^3(c+dx)}{11a^2d(e\sin(c+dx))^{11/2}}-\frac{4e}{7a^2d(e\sin(c+dx))^{7/2}}+\frac{1}{77a^2}$$

```
[Out] 4/11*e^3/a^2/d/(e*sin(d*x+c))^(11/2)-2/11*e^3*cos(d*x+c)/a^2/d/(e*sin(d*x+c
))^(11/2)-2/11*e^3*cos(d*x+c)^3/a^2/d/(e*sin(d*x+c))^(11/2)-4/7*e/a^2/d/(e*
sin(d*x+c))^(7/2)+16/77*e*cos(d*x+c)/a^2/d/(e*sin(d*x+c))^(7/2)-4/231*cos(d
*x+c)/a^2/d/e/(e*sin(d*x+c))^(3/2)-4/231*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2
)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*si
n(d*x+c)^(1/2)/a^2/d/e^2/(e*sin(d*x+c))^(1/2)
```

Rubi [A]

time = 0.46, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3957, 2954, 2952, 2647, 2716, 2721, 2720, 2644, 14}

$$\frac{4e^3}{11a^2d(e\sin(c+dx))^{11/2}}-\frac{2e^3\cos^3(c+dx)}{11a^2d(e\sin(c+dx))^{11/2}}-\frac{2e^3\cos(c+dx)}{11a^2d(e\sin(c+dx))^{11/2}}+\frac{4\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{231a^2de^2\sqrt{e\sin(c+dx)}}-\frac{4e}{7a^2d(e\sin(c+dx))^{7/2}}+\frac{16e\cos(c+dx)}{77a^2d(e\sin(c+dx))^{7/2}}-\frac{4\cos(c+dx)}{231a^2de(e\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[c + d*x])^2*(e*Sin[c + d*x])^(5/2)),x]
```

```
[Out] (4*e^3)/(11*a^2*d*(e*Sin[c + d*x])^(11/2)) - (2*e^3*Cos[c + d*x])/(11*a^2*d
*(e*Sin[c + d*x])^(11/2)) - (2*e^3*Cos[c + d*x]^3)/(11*a^2*d*(e*Sin[c + d*x
])^(11/2)) - (4*e)/(7*a^2*d*(e*Sin[c + d*x])^(7/2)) + (16*e*Cos[c + d*x])/(
77*a^2*d*(e*Sin[c + d*x])^(7/2)) - (4*Cos[c + d*x])/(231*a^2*d*e*(e*Sin[c +
 d*x])^(3/2)) + (4*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(23
1*a^2*d*e^2*Sqrt[e*Sin[c + d*x]])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx = \int \frac{\cos^2(c+dx)}{(-a-a\cos(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx$$

$$= \frac{e^4\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{(e\sin(c+dx))^{13/2}}\,dx}{a^4}$$

$$= \frac{e^4\int \left(\frac{a^2\cos^2(c+dx)}{(e\sin(c+dx))^{13/2}} - \frac{2a^2\cos^3(c+dx)}{(e\sin(c+dx))^{13/2}} + \frac{a^2\cos^4(c+dx)}{(e\sin(c+dx))^{13/2}}\right)dx}{a^4}$$

$$= \frac{e^4\int \frac{\cos^2(c+dx)}{(e\sin(c+dx))^{13/2}}\,dx}{a^2} + \frac{e^4\int \frac{\cos^4(c+dx)}{(e\sin(c+dx))^{13/2}}\,dx}{a^2} - \frac{(2e^4)\int \frac{\cos}{(e\sin(}}{a^2}$$

$$= -\frac{2e^3\cos(c+dx)}{11a^2d(e\sin(c+dx))^{11/2}} - \frac{2e^3\cos^3(c+dx)}{11a^2d(e\sin(c+dx))^{11/2}} - \frac{(2e^2}{}$$

$$= -\frac{2e^3\cos(c+dx)}{11a^2d(e\sin(c+dx))^{11/2}} - \frac{2e^3\cos^3(c+dx)}{11a^2d(e\sin(c+dx))^{11/2}} + \frac{}{77a^2}$$

$$= \frac{4e^3}{11a^2d(e\sin(c+dx))^{11/2}} - \frac{2e^3\cos(c+dx)}{11a^2d(e\sin(c+dx))^{11/2}} - \frac{2e}{11a^2d}$$

$$= \frac{4e^3}{11a^2d(e\sin(c+dx))^{11/2}} - \frac{2e^3\cos(c+dx)}{11a^2d(e\sin(c+dx))^{11/2}} - \frac{2e}{11a^2d}$$

$$= \frac{4e^3}{11a^2d(e\sin(c+dx))^{11/2}} - \frac{2e^3\cos(c+dx)}{11a^2d(e\sin(c+dx))^{11/2}} - \frac{2e}{11a^2d}$$

Mathematica [A]
time = 0.65, size = 113, normalized size = 0.50

$$-\frac{\csc\left(\frac{1}{2}(c+dx)\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\left(52+97\cos(c+dx)+4\cos(2(c+dx))+\cos(3(c+dx))+\csc^4\left(\frac{1}{2}(c+dx)\right)F\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)\sin^{\frac{11}{2}}(c+dx)\right)}{1848a^2de^2\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[c + d*x])^2*(e*Sin[c + d*x])^(5/2)),x]
```

```
[Out] -1/1848*(Csc[(c + d*x)/2]*Sec[(c + d*x)/2]^5*(52 + 97*Cos[c + d*x] + 4*Cos[
2*(c + d*x)] + Cos[3*(c + d*x)] + Csc[(c + d*x)/2]^4*EllipticF[(-2*c + Pi -
 2*d*x)/4, 2]*Sin[c + d*x]^(11/2)))/(a^2*d*e^2*Sqrt[e*Sin[c + d*x]])
```

Maple [A]
time = 0.26, size = 160, normalized size = 0.71

method	result

default	$\frac{\frac{4e^3\left(11\left(\cos^2(dx+c)\right)-4\right)}{77a^2(e\sin(dx+c))^{\frac{11}{2}}}-\frac{2\left(\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\left(\sin^{\frac{13}{2}}(dx+c)\right)\mathrm{EllipticF}\left(\sqrt{-\sin(dx+c)}\right.\right.}{231e^2a^2\sin(dx+c)^6\cos(dx+c)\sqrt{e\sin(dx}}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (4/77*e^3/a^2/(e*sin(d*x+c))^(11/2)*(11*cos(d*x+c)^2-4)-2/231/e^2*((-sin(d*
x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(13/2)*EllipticF((-sin(d*x+
c)+1)^(1/2),1/2*2^(1/2))-2*sin(d*x+c)^7+47*sin(d*x+c)^5-87*sin(d*x+c)^3+42*
sin(d*x+c))/a^2/sin(d*x+c)^6/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.74, size = 226, normalized size = 1.01

$$\frac{2\left(\sqrt{-i}\left(\sqrt{2}\cos(dx+c)^4+2\sqrt{2}\cos(dx+c)^3-2\sqrt{2}\cos(dx+c)-\sqrt{2}\right)\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c))+\sqrt{i}\left(\sqrt{2}\cos(dx+c)^4+2\sqrt{2}\cos(dx+c)^3-2\sqrt{2}\cos(dx+c)-\sqrt{2}\right)\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)-i\sin(dx+c))+\left(2\cos(dx+c)^3+4\cos(dx+c)^2+47\cos(dx+c)+24\right)\sqrt{\sin(dx+c)}\right)}{231\left(a^2d\cos(dx+c)^4e^{\frac{5}{2}}+2a^2d\cos(dx+c)^3e^{\frac{5}{2}}-2a^2d\cos(dx+c)e^{\frac{5}{2}}-a^2de^{\frac{5}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/231*(sqrt(-I)*(sqrt(2)*cos(d*x + c)^4 + 2*sqrt(2)*cos(d*x + c)^3 - 2*sqrt
(2)*cos(d*x + c) - sqrt(2))*weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(
d*x + c)) + sqrt(I)*(sqrt(2)*cos(d*x + c)^4 + 2*sqrt(2)*cos(d*x + c)^3 - 2*
sqrt(2)*cos(d*x + c) - sqrt(2))*weierstrassPInverse(4, 0, cos(d*x + c) - I*
sin(d*x + c)) + (2*cos(d*x + c)^3 + 4*cos(d*x + c)^2 + 47*cos(d*x + c) + 24
)*sqrt(sin(d*x + c)))/(a^2*d*cos(d*x + c)^4*e^(5/2) + 2*a^2*d*cos(d*x + c)^
3*e^(5/2) - 2*a^2*d*cos(d*x + c)*e^(5/2) - a^2*d*e^(5/2))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**2/(e*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^2*(e*sin(d*x + c))^(5/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^2}{a^2\,{\left(e\,\sin\left(c+d\,x\right)\right)}^{5/2}\,{\left(\cos\left(c+d\,x\right)+1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*sin(c + d*x))^(5/2)*(a + a/cos(c + d*x))^2),x)
```

```
[Out] int(cos(c + d*x)^2/(a^2*(e*sin(c + d*x))^(5/2)*(cos(c + d*x) + 1)^2), x)
```

3.134 $\int (a+a\sec(c+dx))^3(e\sin(c+dx))^m\,dx$

Optimal. Leaf size=247

$$\frac{a^3\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)\sqrt{\cos^2(c+dx)}}+\frac{3a^3\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)(e\sin(c}{de(1+m)}$$

```
[Out] 3*a^3*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*(e*sin(d*x+c))^(1+
m)/d/e/(1+m)+a^3*hypergeom([2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*(e*sin(
d*x+c))^(1+m)/d/e/(1+m)+a^3*cos(d*x+c)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*
m],sin(d*x+c)^2)*(e*sin(d*x+c))^(1+m)/d/e/(1+m)/(cos(d*x+c)^2)^(1/2)+3*a^3*
hypergeom([3/2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*sec(d*x+c)*(e*sin(d*x+
c))^(1+m)*(cos(d*x+c)^2)^(1/2)/d/e/(1+m)
```

Rubi [A]

time = 0.25, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3957, 2952, 2722, 2644, 371, 2657}

$$\frac{3a^3(e\sin(c+dx))^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{de(m+1)}+\frac{a^3(e\sin(c+dx))^{m+1}\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{de(m+1)}+\frac{a^3\cos(c+dx)(e\sin(c+dx))^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{de(m+1)\sqrt{\cos^2(c+dx)}}+\frac{3a^3\sqrt{\cos^2(c+dx)}\sec(c+dx)(e\sin(c+dx))^{m+1}\,{}_2F_1\left(\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{de(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*(e*Sin[c + d*x])^m,x]
```

```
[Out] (a^3*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]
^2]*(e*Sin[c + d*x])^(1 + m))/(d*e*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (3*a^3*H
ypergeometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])^
(1 + m))/(d*e*(1 + m)) + (a^3*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, Si
n[c + d*x]^2]*(e*Sin[c + d*x])^(1 + m))/(d*e*(1 + m)) + (3*a^3*Sqrt[Cos[c +
 d*x]^2]*Hypergeometric2F1[3/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*Sec[c
 + d*x]*(e*Sin[c + d*x])^(1 + m))/(d*e*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+a\sec(c+dx))^3(e\sin(c+dx))^m\,dx = -\int (-a-a\cos(c+dx))^3\sec^3(c+dx)(e\sin(c+dx))^m\,dx$$

$$= -\int \left(-a^3(e\sin(c+dx))^m - 3a^3\sec(c+dx)(e\sin(c+dx))^m -\right.$$

$$= a^3\int (e\sin(c+dx))^m\,dx + a^3\int \sec^3(c+dx)(e\sin(c+dx))^m\,d$$

$$= \frac{a^3\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)(e\sin(c+dx))^{1+}}{de(1+m)\sqrt{\cos^2(c+dx)}}$$

$$= \frac{a^3\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)(e\sin(c+dx))^{1+}}{de(1+m)\sqrt{\cos^2(c+dx)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.45, size = 287, normalized size = 1.16

$$\frac{2^{-3-m}a^3e^{i(c+dx)}\left(-ie^{-i(c+dx)}\left(-1+e^{2i(c+dx)}\right)\right)^{1+m}(1+\cos(c+dx))^3\,{}_2F_1\left(1,\frac{2+m}{2};1-\frac{m}{2};e^{2i(c+dx)}\right)\sec^6\left(\frac{1}{2}(c+dx)\right)\sin^{-m}(c+dx)(e\sin(c+dx))^m}{dm}+\frac{a^3(1+\cos(c+dx))^3\left(3\cos(c+dx)\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)+3\sqrt{\cos^2(c+dx)}\,{}_2F_1\left(\frac{3}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)+\cos(c+dx)\,{}_2F_1\left(2,\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)\right)\sec^6\left(\frac{1}{2}(c+dx)\right)(e\sin(c+dx))^m\tan(c+dx)}{8d(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*(e*Sin[c + d*x])^m,x]
```

```
[Out] (2^(-3 - m)*a^3*E^(I*(c + d*x))*(((-I)*(-1 + E^((2*I)*(c + d*x))))/E^(I*(c
+ d*x)))^(1 + m)*(1 + Cos[c + d*x])^3*Hypergeometric2F1[1, (2 + m)/2, 1 - m
/2, E^((2*I)*(c + d*x))]*Sec[(c + d*x)/2]^6*(e*Sin[c + d*x])^m)/(d*m*Sin[c
+ d*x]^m) + (a^3*(1 + Cos[c + d*x])^3*(3*Cos[c + d*x]*Hypergeometric2F1[1,
(1 + m)/2, (3 + m)/2, Sin[c + d*x]^2] + 3*Sqrt[Cos[c + d*x]^2]*Hypergeometr
ic2F1[3/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2] + Cos[c + d*x]*Hypergeomet
ric2F1[2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2])*Sec[(c + d*x)/2]^6*(e*Sin[
c + d*x])^m*Tan[c + d*x])/(8*d*(1 + m))
```

Maple [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int \left(a+a\sec\left(dx+c\right)\right)^{3}\left(e\sin\left(dx+c\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*(e*sin(d*x+c))^m,x)
```

```
[Out] int((a+a*sec(d*x+c))^3*(e*sin(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(e*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3*(e*sin(d*x + c))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(e*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) +
a^3)*(e*sin(d*x + c))^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int(e\sin(c+dx))^m\,dx+\int 3(e\sin(c+dx))^m\sec(c+dx)\,dx+\int 3(e\sin(c+dx))^m\sec^2(c+dx)\,dx+\int(e\sin(c+dx))^m\sec^3(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*(e*sin(d*x+c))**m,x)
```

```
[Out] a**3*(Integral((e*sin(c + d*x))**m, x) + Integral(3*(e*sin(c + d*x))**m*sec
(c + d*x), x) + Integral(3*(e*sin(c + d*x))**m*sec(c + d*x)**2, x) + Integr
al((e*sin(c + d*x))**m*sec(c + d*x)**3, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(e*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3*(e*sin(d*x + c))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int(e\,\sin(c+d\,x))^m\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^m*(a + a/cos(c + d*x))^3,x)
```

```
[Out] int((e*sin(c + d*x))^m*(a + a/cos(c + d*x))^3, x)
```

3.135 $\int (a+a\sec(c+dx))^2(e\sin(c+dx))^m\,dx$

Optimal. Leaf size=195

$$\frac{a^2\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)\sqrt{\cos^2(c+dx)}}+\frac{2a^2\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)(e\sin(c}{de(1+m)}$$

```
[Out] 2*a^2*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*(e*sin(d*x+c))^(1+
m)/d/e/(1+m)+a^2*cos(d*x+c)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+
c)^2)*(e*sin(d*x+c))^(1+m)/d/e/(1+m)/(cos(d*x+c)^2)^(1/2)+a^2*hypergeom([3/
2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*sec(d*x+c)*(e*sin(d*x+c))^(1+m)*(co
s(d*x+c)^2)^(1/2)/d/e/(1+m)
```

Rubi [A]
time = 0.20, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3957, 2952, 2722, 2644, 371, 2657}

$$\frac{2a^2(e\sin(c+dx))^{m+1}{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{de(m+1)}+\frac{a^2\cos(c+dx)(e\sin(c+dx))^{m+1}{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{de(m+1)\sqrt{\cos^2(c+dx)}}+\frac{a^2\sqrt{\cos^2(c+dx)}\sec(c+dx)(e\sin(c+dx))^{m+1}{}_2F_1\left(\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{de(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*(e*Sin[c + d*x])^m,x]
```

```
[Out] (a^2*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]
^2]*(e*Sin[c + d*x])^(1 + m))/(d*e*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (2*a^2*H
ypergeometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])^
(1 + m))/(d*e*(1 + m)) + (a^2*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[3/2, (
1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*(e*Sin[c + d*x])^(1 + m))
/(d*e*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+a\sec(c+dx))^2(e\sin(c+dx))^m\,dx = \int (-a-a\cos(c+dx))^2\sec^2(c+dx)(e\sin(c+dx))^m\,dx$$

$$= \int \left(a^2(e\sin(c+dx))^m + 2a^2\sec(c+dx)(e\sin(c+dx))^m + a^2\,\mathrm{s}\right.$$

$$= a^2\int (e\sin(c+dx))^m\,dx + a^2\int \sec^2(c+dx)(e\sin(c+dx))^m\,d$$

$$= \frac{a^2\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)(e\sin(c+dx))^{1+}}{de(1+m)\sqrt{\cos^2(c+dx)}}$$

$$= \frac{a^2\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)(e\sin(c+dx))^{1+}}{de(1+m)\sqrt{\cos^2(c+dx)}}$$

Mathematica [C] Result contains complex when optimal does not.

time = 2.68, size = 230, normalized size = 1.18

$$\frac{2^{-2-m}a^2e^{i(c+dx)}\left(-ie^{-i(c+dx)}\left(-1+e^{2i(c+dx)}\right)\right)^{1+m}(1+\cos(c+dx))^2\,{}_2F_1\left(1,\frac{2+m}{2};1-\frac{m}{2};e^{2i(c+dx)}\right)\sec^4\left(\frac{1}{2}(c+dx)\right)\sin^{-m}(c+dx)(e\sin(c+dx))^m}{dm}+\frac{a^2\left(2\cos(c+dx)\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)+\sqrt{\cos^2(c+dx)}\,{}_2F_1\left(\frac{3}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)\right)(e\sin(c+dx))^m\tan(c+dx)}{d(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*(e*Sin[c + d*x])^m,x]
```

```
[Out] (2^(-2 - m)*a^2*E^(I*(c + d*x))*(((-I)*(-1 + E^((2*I)*(c + d*x))))/E^(I*(c
+ d*x)))^(1 + m)*(1 + Cos[c + d*x])^2*Hypergeometric2F1[1, (2 + m)/2, 1 - m
/2, E^((2*I)*(c + d*x))]*Sec[(c + d*x)/2]^4*(e*Sin[c + d*x])^m)/(d*m*Sin[c
+ d*x]^m) + (a^2*(2*Cos[c + d*x]*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2,
 Sin[c + d*x]^2] + Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[3/2, (1 + m)/2, (
3 + m)/2, Sin[c + d*x]^2])*(e*Sin[c + d*x])^m*Tan[c + d*x])/(d*(1 + m))
```

Maple [F]
time = 0.24, size = 0, normalized size = 0.00

$$\int (a+a\sec(dx+c))^2(e\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^m,x)
```

```
[Out] int((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*(e*sin(d*x + c))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)*(e*sin(d*x + c))^m
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int (e\sin(c+dx))^m\, dx + \int 2(e\sin(c+dx))^m \sec(c+dx)\, dx + \int (e\sin(c+dx))^m \sec^2(c+dx)\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(e*sin(d*x+c))**m,x)
```

```
[Out] a**2*(Integral((e*sin(c + d*x))**m, x) + Integral(2*(e*sin(c + d*x))**m*sec
(c + d*x), x) + Integral((e*sin(c + d*x))**m*sec(c + d*x)**2, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*(e*sin(d*x + c))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\sin(c+d\,x))^m\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^m*(a + a/cos(c + d*x))^2,x)
```

```
[Out] int((e*sin(c + d*x))^m*(a + a/cos(c + d*x))^2, x)
```

3.136 $\int (a + a\sec(c + dx))(e\sin(c + dx))^m \, dx$

Optimal. Leaf size=119

$$\frac{a\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)\sqrt{\cos^2(c+dx)}}+\frac{a\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)(e\sin(c+}{de(1+m)}$$

```
[Out] a*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*(e*sin(d*x+c))^(1+m)/d
/e/(1+m)+a*cos(d*x+c)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*
(e*sin(d*x+c))^(1+m)/d/e/(1+m)/(cos(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3957, 2917, 2644, 371, 2722}

$$\frac{a(e\sin(c+dx))^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{de(m+1)}+\frac{a\cos(c+dx)(e\sin(c+dx))^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{de(m+1)\sqrt{\cos^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*(e*Sin[c + d*x])^m,x]
```

```
[Out] (a*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2
]*(e*Sin[c + d*x])^(1 + m))/(d*e*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (a*Hyperge
ometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])^(1 + m
))/(d*e*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))(e\sin(c+dx))^m\,dx &= -\int (-a-a\cos(c+dx))\sec(c+dx)(e\sin(c+dx))^m\,dx\\ &= a\int (e\sin(c+dx))^m\,dx + a\int \sec(c+dx)(e\sin(c+dx))^m\,dx\\ &= \frac{a\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)\sqrt{\cos^2(c+dx)}}\\ &= \frac{a\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)\sqrt{\cos^2(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 97, normalized size = 0.82

$$\frac{a(e\sin(c+dx))^m\left({}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)\sin(c+dx)+\sqrt{\cos^2(c+dx)}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)\tan(c+dx)\right)}{d(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*(e*Sin[c + d*x])^m,x]
[Out] (a*(e*Sin[c + d*x])^m*(Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[c + d
*x]^2]*Sin[c + d*x] + Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[1/2, (1 + m)/2
, (3 + m)/2, Sin[c + d*x]^2]*Tan[c + d*x]))/(d*(1 + m))
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \left(a+a\sec\left(dx+c\right)\right)\left(e\sin\left(dx+c\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(e*sin(d*x+c))^m,x)
```

```
[Out] int((a+a*sec(d*x+c))*(e*sin(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \left(e\sin\left(c+dx\right)\right)^{m}\,dx+\int \left(e\sin\left(c+dx\right)\right)^{m}\sec\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))**m,x)
```

```
[Out] a*(Integral((e*sin(c + d*x))**m, x) + Integral((e*sin(c + d*x))**m*sec(c +
d*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

<u>Mupad</u> [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\sin\left(c+d\,x\right))^m\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^m*(a + a/cos(c + d*x)),x)
[Out] int((e*sin(c + d*x))^m*(a + a/cos(c + d*x)), x)
```

3.137 $\int \frac{(e \sin(c+dx))^m}{a+a \sec(c+dx)} \, dx$

Optimal. Leaf size=100

$$-\frac{e(e\sin(c+dx))^{-1+m}}{ad(1-m)}+\frac{e\cos(c+dx)\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}(-1+m);\frac{1+m}{2};\sin^2(c+dx)\right)(e\sin(c+dx))^{-1+m}}{ad(1-m)\sqrt{\cos^2(c+dx)}}$$

```
[Out] -e*(e*sin(d*x+c))^(-1+m)/a/d/(1-m)+e*cos(d*x+c)*hypergeom([-1/2, -1/2+1/2*m
],[1/2+1/2*m],sin(d*x+c)^2)*(e*sin(d*x+c))^(-1+m)/a/d/(1-m)/(cos(d*x+c)^2)^
(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3957, 2918, 2644, 30, 2657}

$$\frac{e\cos(c+dx)(e\sin(c+dx))^{m-1}\,{}_2F_1\left(-\frac{1}{2},\frac{m-1}{2};\frac{m+1}{2};\sin^2(c+dx)\right)}{ad(1-m)\sqrt{\cos^2(c+dx)}}-\frac{e(e\sin(c+dx))^{m-1}}{ad(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^m/(a + a*Sec[c + d*x]),x]
```

```
[Out] -((e*(e*Sin[c + d*x])^(-1 + m))/(a*d*(1 - m))) + (e*Cos[c + d*x]*Hypergeome
tric2F1[-1/2, (-1 + m)/2, (1 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])^(-1 +
 m))/(a*d*(1 - m)*Sqrt[Cos[c + d*x]^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{(e\sin(c+dx))^m}{a+a\sec(c+dx)}\,dx &= -\int \frac{\cos(c+dx)(e\sin(c+dx))^m}{-a-a\cos(c+dx)}\,dx\\ &= \frac{e^2\int \cos(c+dx)(e\sin(c+dx))^{-2+m}\,dx}{a} - \frac{e^2\int \cos^2(c+dx)(e\sin(c+dx))^{-2+m}\,d}{a}\\ &= \frac{e\cos(c+dx)\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}(-1+m);\frac{1+m}{2};\sin^2(c+dx)\right)(e\sin(c+dx))^{-1+m}}{ad(1-m)\sqrt{\cos^2(c+dx)}} + \frac{e\mathrm{Sul}}{}\\ &= -\frac{e(e\sin(c+dx))^{-1+m}}{ad(1-m)} + \frac{e\cos(c+dx)\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}(-1+m);\frac{1+m}{2};\sin^2(c+dx)\right)(}{ad(1-m)\sqrt{\cos^2(c+dx)}}\end{aligned}$$

Mathematica [F]
time = 19.79, size = 0, normalized size = 0.00

$$\int \frac{(e\sin(c+dx))^m}{a+a\sec(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e*Sin[c + d*x])^m/(a + a*Sec[c + d*x]),x]
```

```
[Out] Integrate[(e*Sin[c + d*x])^m/(a + a*Sec[c + d*x]), x]
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(e\sin(dx+c))^m}{a+a\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^m/(a+a*sec(d*x+c)),x)
[Out] int((e*sin(d*x+c))^m/(a+a*sec(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate((e*sin(d*x + c))^m/(a*sec(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] integral((e*sin(d*x + c))^m/(a*sec(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(e \sin (c+dx))^m}{\sec (c+dx)+1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**m/(a+a*sec(d*x+c)),x)
[Out] Integral((e*sin(c + d*x))**m/(sec(c + d*x) + 1), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((e*sin(d*x + c))^m/(a*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos (c+d\,x)\,{\left(e\,\sin (c+d\,x)\right)}^m}{a\,\left(\cos (c+d\,x)+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^m/(a + a/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e*sin(c + d*x))^m)/(a*(cos(c + d*x) + 1)), x)
```

3.138 $\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=207

$$\frac{2e^3(e\sin(c+dx))^{-3+m}}{a^2d(3-m)}-\frac{e^3\cos(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2}(-3+m);\frac{1}{2}(-1+m);\sin^2(c+dx)\right)(e\sin(c+dx))^{-3+m}}{a^2d(3-m)\sqrt{\cos^2(c+dx)}}-$$

```
[Out] 2*e^3*(e*sin(d*x+c))^(-3+m)/a^2/d/(3-m)-2*e*(e*sin(d*x+c))^(-1+m)/a^2/d/(1-
m)-e^3*cos(d*x+c)*hypergeom([-3/2, -3/2+1/2*m],[-1/2+1/2*m],sin(d*x+c)^2)*(
e*sin(d*x+c))^(-3+m)/a^2/d/(3-m)/(cos(d*x+c)^2)^(1/2)-e^3*cos(d*x+c)*hyperg
eom([-1/2, -3/2+1/2*m],[-1/2+1/2*m],sin(d*x+c)^2)*(e*sin(d*x+c))^(-3+m)/a^2
/d/(3-m)/(cos(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.38, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3957, 2954, 2952, 2657, 2644, 14}

$$-\frac{e^3\cos(c+dx)(e\sin(c+dx))^{m-3}\,{}_2F_1\left(-\frac{3}{2},\frac{m-3}{2};\frac{m-1}{2};\sin^2(c+dx)\right)}{a^2d(3-m)\sqrt{\cos^2(c+dx)}}-\frac{e^3\cos(c+dx)(e\sin(c+dx))^{m-3}\,{}_2F_1\left(-\frac{1}{2},\frac{m-3}{2};\frac{m-1}{2};\sin^2(c+dx)\right)}{a^2d(3-m)\sqrt{\cos^2(c+dx)}}+\frac{2e^3(e\sin(c+dx))^{m-3}}{a^2d(3-m)}-\frac{2e(e\sin(c+dx))^{m-1}}{a^2d(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^m/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*e^3*(e*Sin[c + d*x])^(-3 + m))/(a^2*d*(3 - m)) - (e^3*Cos[c + d*x]*Hyper
geometric2F1[-3/2, (-3 + m)/2, (-1 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])
^(-3 + m))/(a^2*d*(3 - m)*Sqrt[Cos[c + d*x]^2]) - (e^3*Cos[c + d*x]*Hyperge
ometric2F1[-1/2, (-3 + m)/2, (-1 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])^(
-3 + m))/(a^2*d*(3 - m)*Sqrt[Cos[c + d*x]^2]) - (2*e*(e*Sin[c + d*x])^(-1 +
 m))/(a^2*d*(1 - m))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)(e\sin(c+dx))^m}{(-a-a\cos(c+dx))^2}\,dx$$
$$= \frac{e^4\int \cos^2(c+dx)(-a+a\cos(c+dx))^2(e\sin(c+dx))^{-4+m}\,dx}{a^4}$$
$$= \frac{e^4\int \left(a^2\cos^2(c+dx)(e\sin(c+dx))^{-4+m} - 2a^2\cos^3(c+dx)(e\sin(c+dx))^{-4+m}\right.}{a^4}$$
$$= \frac{e^4\int \cos^2(c+dx)(e\sin(c+dx))^{-4+m}\,dx}{a^2} + \frac{e^4\int \cos^4(c+dx)(e\sin(c+dx))^{-4+m}}{a^2}$$
$$= -\frac{e^3\cos(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2}(-3+m);\frac{1}{2}(-1+m);\sin^2(c+dx)\right)(e\sin(c+dx))^{-}}{a^2d(3-m)\sqrt{\cos^2(c+dx)}}$$
$$= -\frac{e^3\cos(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2}(-3+m);\frac{1}{2}(-1+m);\sin^2(c+dx)\right)(e\sin(c+dx))^{-}}{a^2d(3-m)\sqrt{\cos^2(c+dx)}}$$
$$= \frac{2e^3(e\sin(c+dx))^{-3+m}}{a^2d(3-m)} - \frac{e^3\cos(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2}(-3+m);\frac{1}{2}(-1+m);\sin^2(c\right.}{a^2d(3-m)\sqrt{\cos^2(c+dx)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 17.28, size = 2272, normalized size = 10.98

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sin[c + d*x])^m/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2^(2 - m)*E^(I*(c + d*x))*(((-I)*(-1 + E^((2*I)*(c + d*x))))/E^(I*(c + d*x
)))^(1 + m)*Cos[c/2 + (d*x)/2]^4*Hypergeometric2F1[1, (2 + m)/2, 1 - m/2, E
^((2*I)*(c + d*x))]*Sec[c + d*x]^2*(e*Sin[c + d*x])^m)/(d*m*(a + a*Sec[c +
d*x])^2*Sin[c + d*x]^m) + ((AppellF1[(1 + m)/2, 1 - m, 2*m, (3 + m)/2, Tan[
(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2] - 6*AppellF1[(1 + m)/2, 2 - m, 2*m, (3
 + m)/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2] + 12*AppellF1[(1 + m)/2,
3 - m, 2*m, (3 + m)/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2] - 8*AppellF
1[(1 + m)/2, 4 - m, 2*m, (3 + m)/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2
])*Sec[(c + d*x)/2]^4*(e*Sin[c + d*x])^m*Tan[(c + d*x)/4])/(a^2*d*((AppellF
1[(1 + m)/2, 1 - m, 2*m, (3 + m)/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2
] - 6*AppellF1[(1 + m)/2, 2 - m, 2*m, (3 + m)/2, Tan[(c + d*x)/4]^2, -Tan[(
c + d*x)/4]^2] + 12*AppellF1[(1 + m)/2, 3 - m, 2*m, (3 + m)/2, Tan[(c + d*x
)/4]^2, -Tan[(c + d*x)/4]^2] - 8*AppellF1[(1 + m)/2, 4 - m, 2*m, (3 + m)/2,
 Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2])*Sec[(c + d*x)/4]^2 + 4*m*(Appell
F1[(1 + m)/2, 1 - m, 2*m, (3 + m)/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^
```

```
2] - 6*AppellF1[(1 + m)/2, 2 - m, 2*m, (3 + m)/2, Tan[(c + d*x)/4]^2, -Tan[
(c + d*x)/4]^2] + 12*AppellF1[(1 + m)/2, 3 - m, 2*m, (3 + m)/2, Tan[(c + d*
x)/4]^2, -Tan[(c + d*x)/4]^2] - 8*AppellF1[(1 + m)/2, 4 - m, 2*m, (3 + m)/2
, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2])*Cot[c + d*x]*Tan[(c + d*x)/4] +
 4*m*(AppellF1[(1 + m)/2, 1 - m, 2*m, (3 + m)/2, Tan[(c + d*x)/4]^2, -Tan[(
c + d*x)/4]^2] - 6*AppellF1[(1 + m)/2, 2 - m, 2*m, (3 + m)/2, Tan[(c + d*x)
/4]^2, -Tan[(c + d*x)/4]^2] + 12*AppellF1[(1 + m)/2, 3 - m, 2*m, (3 + m)/2,
 Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2] - 8*AppellF1[(1 + m)/2, 4 - m, 2*
m, (3 + m)/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2])*Tan[(c + d*x)/4]^2
- (2*(1 + m)*(2*m*AppellF1[(3 + m)/2, 1 - m, 1 + 2*m, (5 + m)/2, Tan[(c + d
*x)/4]^2, -Tan[(c + d*x)/4]^2] + (-1 + m)*AppellF1[(3 + m)/2, 2 - m, 2*m, (
5 + m)/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2] - 12*m*AppellF1[(3 + m)/
2, 2 - m, 1 + 2*m, (5 + m)/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2] - 6*
(-2 + m)*AppellF1[(3 + m)/2, 3 - m, 2*m, (5 + m)/2, Tan[(c + d*x)/4]^2, -Ta
n[(c + d*x)/4]^2] + 24*m*AppellF1[(3 + m)/2, 3 - m, 1 + 2*m, (5 + m)/2, Tan
[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2] + 12*(-3 + m)*AppellF1[(3 + m)/2, 4 -
 m, 2*m, (5 + m)/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2] - 16*m*AppellF
1[(3 + m)/2, 4 - m, 1 + 2*m, (5 + m)/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/
4]^2] - 8*(-4 + m)*AppellF1[(3 + m)/2, 5 - m, 2*m, (5 + m)/2, Tan[(c + d*x)
/4]^2, -Tan[(c + d*x)/4]^2])*Sec[(c + d*x)/4]^2*Tan[(c + d*x)/4]^2)/(3 + m)
 + 2*m*(AppellF1[(1 + m)/2, 1 - m, 2*m, (3 + m)/2, Tan[(c + d*x)/4]^2, -Tan
[(c + d*x)/4]^2] - 6*AppellF1[(1 + m)/2, 2 - m, 2*m, (3 + m)/2, Tan[(c + d*
x)/4]^2, -Tan[(c + d*x)/4]^2] + 12*AppellF1[(1 + m)/2, 3 - m, 2*m, (3 + m)/
2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2] - 8*AppellF1[(1 + m)/2, 4 - m,
2*m, (3 + m)/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2])*Sec[(c + d*x)/2]*
Tan[(c + d*x)/4]^2)) - (2*(3 + m)*(AppellF1[(1 + m)/2, 1 - m, 2*m, (3 + m)/
2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2] - 2*AppellF1[(1 + m)/2, 2 - m,
2*m, (3 + m)/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2])*Sec[(c + d*x)/2]*
(e*Sin[c + d*x])^m*Tan[(c + d*x)/2])/(a^2*d*(1 + m)*((3 + m)*AppellF1[(1 +
m)/2, 1 - m, 2*m, (3 + m)/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2] - 2*(
(3 + m)*AppellF1[(1 + m)/2, 2 - m, 2*m, (3 + m)/2, Tan[(c + d*x)/4]^2, -Tan
[(c + d*x)/4]^2] + (2*m*AppellF1[(3 + m)/2, 1 - m, 1 + 2*m, (5 + m)/2, Tan[
(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2] + (-1 + m)*AppellF1[(3 + m)/2, 2 - m,
2*m, (5 + m)/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2] - 4*m*AppellF1[(3
+ m)/2, 2 - m, 1 + 2*m, (5 + m)/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2]
 + 4*AppellF1[(3 + m)/2, 3 - m, 2*m, (5 + m)/2, Tan[(c + d*x)/4]^2, -Tan[(c
 + d*x)/4]^2] - 2*m*AppellF1[(3 + m)/2, 3 - m, 2*m, (5 + m)/2, Tan[(c + d*x
)/4]^2, -Tan[(c + d*x)/4]^2])*Tan[(c + d*x)/4]^2)))
```

Maple [F]

time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{(e \sin (dx+c))^m}{(a+a \sec (dx+c))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^2,x)
```

```
[Out] int((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((e*sin(d*x + c))^m/(a*sec(d*x + c) + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((e*sin(d*x + c))^m/(a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(e\sin{(c+dx)})^m}{\sec^2{(c+dx)}+2\sec{(c+dx)}+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**m/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral((e*sin(c + d*x))**m/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**
2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*sin(d*x + c))^m/(a*sec(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^2\,{\left(e\,\sin\left(c+d\,x\right)\right)}^m}{a^2\,{\left(\cos\left(c+d\,x\right)+1\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^m/(a + a/cos(c + d*x))^2,x)
[Out] int((cos(c + d*x)^2*(e*sin(c + d*x))^m)/(a^2*(cos(c + d*x) + 1)^2), x)
```

3.139 $\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=236

$$-\frac{4e^5(e\sin(c+dx))^{-5+m}}{a^3d(5-m)}+\frac{e^5\cos(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{1}{2}(-5+m);\frac{1}{2}(-3+m);\sin^2(c+dx)\right)(e\sin(c+dx))^{-5+m}}{a^3d(5-m)\sqrt{\cos^2(c+dx)}}$$

```
[Out] -4*e^5*(e*sin(d*x+c))^(-5+m)/a^3/d/(5-m)+7*e^3*(e*sin(d*x+c))^(-3+m)/a^3/d/
(3-m)-3*e*(e*sin(d*x+c))^(-1+m)/a^3/d/(1-m)+e^5*cos(d*x+c)*hypergeom([-5/2,
 -5/2+1/2*m],[-3/2+1/2*m],sin(d*x+c)^2)*(e*sin(d*x+c))^(-5+m)/a^3/d/(5-m)/(
cos(d*x+c)^2)^(1/2)+3*e^5*cos(d*x+c)*hypergeom([-3/2, -5/2+1/2*m],[-3/2+1/2
*m],sin(d*x+c)^2)*(e*sin(d*x+c))^(-5+m)/a^3/d/(5-m)/(cos(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.45, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3957, 2954, 2952, 2644, 14, 2657, 276}

$$\frac{e^5\cos(c+dx)(e\sin(c+dx))^{m-5}\,{}_2F_1\left(-\frac{5}{2},\frac{m-5}{2};\frac{m-3}{2};\sin^2(c+dx)\right)}{a^3d(5-m)\sqrt{\cos^2(c+dx)}}+\frac{3e^5\cos(c+dx)(e\sin(c+dx))^{m-5}\,{}_2F_1\left(-\frac{3}{2},\frac{m-5}{2};\frac{m-3}{2};\sin^2(c+dx)\right)}{a^3d(5-m)\sqrt{\cos^2(c+dx)}}-\frac{4e^5(e\sin(c+dx))^{m-5}}{a^3d(5-m)}+\frac{7e^3(e\sin(c+dx))^{m-3}}{a^3d(3-m)}-\frac{3e(e\sin(c+dx))^{m-1}}{a^3d(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^m/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-4*e^5*(e*Sin[c + d*x])^(-5 + m))/(a^3*d*(5 - m)) + (e^5*Cos[c + d*x]*Hype
rgeometric2F1[-5/2, (-5 + m)/2, (-3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x]
)^(-5 + m))/(a^3*d*(5 - m)*Sqrt[Cos[c + d*x]^2]) + (3*e^5*Cos[c + d*x]*Hype
rgeometric2F1[-3/2, (-5 + m)/2, (-3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x]
)^(-5 + m))/(a^3*d*(5 - m)*Sqrt[Cos[c + d*x]^2]) + (7*e^3*(e*Sin[c + d*x])^
(-3 + m))/(a^3*d*(3 - m)) - (3*e*(e*Sin[c + d*x])^(-1 + m))/(a^3*d*(1 - m))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
```

```
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^3}\,dx = -\int \frac{\cos^3(c+dx)(e\sin(c+dx))^m}{(-a-a\cos(c+dx))^3}\,dx$$

$$= -\frac{e^6\int \cos^3(c+dx)(-a+a\cos(c+dx))^3(e\sin(c+dx))^{-6+m}\,dx}{a^6}$$

$$= -\frac{e^6\int \left(-a^3\cos^3(c+dx)(e\sin(c+dx))^{-6+m} + 3a^3\cos^4(c+dx)(e\sin(c+dx))^{-6}\right.}{}$$

$$= \frac{e^6\int \cos^3(c+dx)(e\sin(c+dx))^{-6+m}\,dx}{a^3} - \frac{e^6\int \cos^6(c+dx)(e\sin(c+dx))^{-6+m}}{a^3}$$

$$= \frac{e^5\cos(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{1}{2}(-5+m);\frac{1}{2}(-3+m);\sin^2(c+dx)\right)(e\sin(c+dx))^{-5+}}{a^3d(5-m)\sqrt{\cos^2(c+dx)}}$$

$$= \frac{e^5\cos(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{1}{2}(-5+m);\frac{1}{2}(-3+m);\sin^2(c+dx)\right)(e\sin(c+dx))^{-5+}}{a^3d(5-m)\sqrt{\cos^2(c+dx)}}$$

$$= -\frac{4e^5(e\sin(c+dx))^{-5+m}}{a^3d(5-m)} + \frac{e^5\cos(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{1}{2}(-5+m);\frac{1}{2}(-3+m);\sin^2\right.}{a^3d(5-m)\sqrt{\cos^2(c+d}}$$

Mathematica [F]
time = 6.41, size = 0, normalized size = 0.00

$$\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e*Sin[c + d*x])^m/(a + a*Sec[c + d*x])^3,x]
[Out] Integrate[(e*Sin[c + d*x])^m/(a + a*Sec[c + d*x])^3, x]
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(e\sin(dx+c))^m}{(a+a\sec(dx+c))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^3,x)
[Out] int((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((e*sin(d*x + c))^m/(a*sec(d*x + c) + a)^3, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((e*sin(d*x + c))^m/(a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*
a^3*sec(d*x + c) + a^3), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(e\sin{(c+dx)})^m}{\sec^3{(c+dx)}+3\sec^2{(c+dx)}+3\sec{(c+dx)}+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**m/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral((e*sin(c + d*x))**m/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c
 + d*x) + 1), x)/a**3
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((e*sin(d*x + c))^m/(a*sec(d*x + c) + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos{(c+d\,x)}^3\,{(e\,\sin{(c+d\,x)})}^m}{a^3\,{(\cos{(c+d\,x)}+1)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^m/(a + a/cos(c + d*x))^3,x)
```

```
[Out] int((cos(c + d*x)^3*(e*sin(c + d*x))^m)/(a^3*(cos(c + d*x) + 1)^3), x)
```

3.140 $\int (a+a\sec(c+dx))^{3/2}(e\sin(c+dx))^m\,dx$

Optimal. Leaf size=106

$$\frac{2aeF_1\left(-\frac{1}{2};\frac{1-m}{2},\frac{1}{2}(-2-m);\frac{1}{2};\cos(c+dx),-\cos(c+dx)\right)(1-\cos(c+dx))^{\frac{1-m}{2}}(1+\cos(c+dx))^{-m/2}\sqrt{a+}}{d}$$

```
[Out] 2*a*e*AppellF1(-1/2,-1-1/2*m,1/2-1/2*m,1/2,-cos(d*x+c),cos(d*x+c))*(1-cos(d
*x+c))^(1/2-1/2*m)*(e*sin(d*x+c))^(-1+m)*(a+a*sec(d*x+c))^(1/2)/d/((1+cos(d
*x+c))^(1/2*m))
```

Rubi [A]
time = 0.24, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3961, 2965, 140, 138}

$$\frac{2ae\sqrt{a\sec(c+dx)+a}\,(1-\cos(c+dx))^{\frac{1-m}{2}}(\cos(c+dx)+1)^{-m/2}F_1\left(-\frac{1}{2};\frac{1-m}{2},\frac{1}{2}(-m-2);\frac{1}{2};\cos(c+dx),-\cos(c+dx)\right)(e\sin(c+dx))^{m-1}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)*(e*Sin[c + d*x])^m,x]
```

```
[Out] (2*a*e*AppellF1[-1/2, (1 - m)/2, (-2 - m)/2, 1/2, Cos[c + d*x], -Cos[c + d*
x]]*(1 - Cos[c + d*x])^((1 - m)/2)*Sqrt[a + a*Sec[c + d*x]]*(e*Sin[c + d*x]
)^(-1 + m))/(d*(1 + Cos[c + d*x])^(m/2))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 2965

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[g*((g*Cos
[e + f*x])^(p - 1)/(f*(a + b*Sin[e + f*x])^((p - 1)/2)*(a - b*Sin[e + f*x])
^((p - 1)/2))), Subst[Int[(d*x)^n*(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p -
 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && E
```

```
qQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 3961

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[Sin[e + f*x]^FracPart[m]*((a + b*Csc[e + f*x]
)^FracPart[m]/(b + a*Sin[e + f*x])^FracPart[m]), Int[(g*Cos[e + f*x])^p*((b
 + a*Sin[e + f*x])^m/Sin[e + f*x]^m), x], x] /; FreeQ[{a, b, e, f, g, m, p}
, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

Rubi steps

$$\int (a+a\sec(c+dx))^{3/2}(e\sin(c+dx))^m\,dx = \frac{\left(\sqrt{-\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}\right)\int \frac{(-a-a\cos(c+dx))^{3/2}(e}{(-\cos(c+dx))}}{\sqrt{-a-a\cos(c+dx)}}$$

$$= -\frac{\left(e\sqrt{-\cos(c+dx)}\,(-a-a\cos(c+dx))^{-\frac{1}{2}+\frac{1-m}{2}}(-a+a\cos\right.}{}$$

$$= \frac{\left(ae\sqrt{-\cos(c+dx)}\,(1+\cos(c+dx))^{-m/2}(-a-a\cos(c+d\right.}{}$$

$$= \frac{\left(ae(1-\cos(c+dx))^{\frac{1}{2}-\frac{m}{2}}\sqrt{-\cos(c+dx)}\,(1+\cos(c+dx))\right.}{}$$

$$= \frac{2aeF_1\left(-\frac{1}{2};\frac{1-m}{2},\frac{1}{2}(-2-m);\frac{1}{2};\cos(c+dx),-\cos(c+dx)\right)(1}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1243 vs. 2(106) = 212.
time = 8.43, size = 1243, normalized size = 11.73

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)*(e*Sin[c + d*x])^m,x]
```

```
[Out] (4*(3 + m)*(AppellF1[(1 + m)/2, -1/2, 1 + m, (3 + m)/2, Tan[(c + d*x)/2]^2,
 -Tan[(c + d*x)/2]^2] + AppellF1[(1 + m)/2, 1/2, m, (3 + m)/2, Tan[(c + d*x
)/2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[(1 + m)/2, 3/2, m, (3 + m)/2, Tan
[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[(c + d*x)/2]^3*(a*(1 + Sec[c + d
*x]))^(3/2)*Sin[(c + d*x)/2]*(e*Sin[c + d*x])^m)/(d*(1 + m)*(6*AppellF1[(1
+ m)/2, 3/2, m, (3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*m*A
```

```
ppellF1[(1 + m)/2, 3/2, m, (3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]
^2] - 2*AppellF1[(3 + m)/2, -1/2, 2 + m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2] - 2*m*AppellF1[(3 + m)/2, -1/2, 2 + m, (5 + m)/2, Tan[(c
+ d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[(3 + m)/2, 1/2, 1 + m, (5 + m)
/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*m*AppellF1[(3 + m)/2, 1/2,
 1 + m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[(3 +
 m)/2, 3/2, m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 4*m*Ap
pellF1[(3 + m)/2, 3/2, 1 + m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)
/2]^2] + 6*AppellF1[(3 + m)/2, 5/2, m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[
(c + d*x)/2]^2] + 6*AppellF1[(1 + m)/2, 3/2, m, (3 + m)/2, Tan[(c + d*x)/2]
^2, -Tan[(c + d*x)/2]^2]*Cos[c + d*x] + 2*m*AppellF1[(1 + m)/2, 3/2, m, (3
+ m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[c + d*x] + 2*AppellF1[
(3 + m)/2, -1/2, 2 + m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
*Cos[c + d*x] + 2*m*AppellF1[(3 + m)/2, -1/2, 2 + m, (5 + m)/2, Tan[(c + d*
x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[c + d*x] + AppellF1[(3 + m)/2, 1/2, 1 + m
, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[c + d*x] + 2*m*Ap
pellF1[(3 + m)/2, 1/2, 1 + m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)
/2]^2]*Cos[c + d*x] - AppellF1[(3 + m)/2, 3/2, m, (5 + m)/2, Tan[(c + d*x)/
2]^2, -Tan[(c + d*x)/2]^2]*Cos[c + d*x] + 4*m*AppellF1[(3 + m)/2, 3/2, 1 +
m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[c + d*x] - 6*App
ellF1[(3 + m)/2, 5/2, m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2
]*Cos[c + d*x] + (3 + m)*AppellF1[(1 + m)/2, -1/2, 1 + m, (3 + m)/2, Tan[(c
 + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 + Cos[c + d*x]) + (3 + m)*AppellF1[(1
 + m)/2, 1/2, m, (3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 + C
os[c + d*x])))
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \left(a+a\sec\left(dx+c\right)\right)^{\frac{3}{2}}\left(e\sin\left(dx+c\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*(e*sin(d*x+c))^m,x)
[Out] int((a+a*sec(d*x+c))^(3/2)*(e*sin(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(e*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^(3/2)*(e*sin(d*x + c))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(e*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((a*sec(d*x + c) + a)^(3/2)*(e*sin(d*x + c))^m, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(e*sin(d*x+c))**m,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6437 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(e*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^(3/2)*(e*sin(d*x + c))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\sin\left(c+d\,x\right))^m\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^m*(a + a/cos(c + d*x))^(3/2),x)
[Out] int((e*sin(c + d*x))^m*(a + a/cos(c + d*x))^(3/2), x)
```

3.141 $\int \sqrt{a+a\sec(c+dx)}\,(e\sin(c+dx))^m\,dx$

Optimal. Leaf size=107

$$-\frac{2eF_1\left(\frac{1}{2};\frac{1-m}{2},-\frac{m}{2};\frac{3}{2};\cos(c+dx),-\cos(c+dx)\right)(1-\cos(c+dx))^{\frac{1-m}{2}}\cos(c+dx)(1+\cos(c+dx))^{-m/2}\sqrt{a}}{d}$$

```
[Out] -2*e*AppellF1(1/2,-1/2*m,1/2-1/2*m,3/2,-cos(d*x+c),cos(d*x+c))*(1-cos(d*x+c
))^(1/2-1/2*m)*cos(d*x+c)*(e*sin(d*x+c))^(-1+m)*(a+a*sec(d*x+c))^(1/2)/d/((
1+cos(d*x+c))^(1/2*m))
```

Rubi [A]
time = 0.21, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3961, 2965, 140, 138}

$$-\frac{2e\cos(c+dx)\sqrt{a\sec(c+dx)+a}\,(1-\cos(c+dx))^{\frac{1-m}{2}}(\cos(c+dx)+1)^{-m/2}F_1\left(\frac{1}{2};\frac{1-m}{2},-\frac{m}{2};\frac{3}{2};\cos(c+dx),-\cos(c+dx)\right)(e\sin(c+dx))^{m-1}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]*(e*Sin[c + d*x])^m,x]
```

```
[Out] (-2*e*AppellF1[1/2, (1 - m)/2, -1/2*m, 3/2, Cos[c + d*x], -Cos[c + d*x]]*(1
 - Cos[c + d*x])^((1 - m)/2)*Cos[c + d*x]*Sqrt[a + a*Sec[c + d*x]]*(e*Sin[c
 + d*x])^(-1 + m))/(d*(1 + Cos[c + d*x])^(m/2))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 2965

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[g*((g*Cos
[e + f*x])^(p - 1)/(f*(a + b*Sin[e + f*x])^((p - 1)/2)*(a - b*Sin[e + f*x])
^((p - 1)/2))), Subst[Int[(d*x)^n*(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p -
 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && E
```

```
qQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 3961

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[Sin[e + f*x]^FracPart[m]*((a + b*Csc[e + f*x]
)^FracPart[m]/(b + a*Sin[e + f*x])^FracPart[m]), Int[(g*Cos[e + f*x])^p*((b
 + a*Sin[e + f*x])^m/Sin[e + f*x]^m), x], x] /; FreeQ[{a, b, e, f, g, m, p}
, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

Rubi steps

$$\int \sqrt{a+a\sec(c+dx)}\,(e\sin(c+dx))^m\,dx = \frac{\left(\sqrt{-\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}\right)\int \frac{\sqrt{-a-a\cos(c+}}{\sqrt{-\cos(c}}}{\sqrt{-a-a\cos(c+dx)}}$$

$$= -\frac{\left(e\sqrt{-\cos(c+dx)}\,(-a-a\cos(c+dx))^{-\frac{1}{2}+\frac{1-m}{2}}(-a+a\cos\right.}{}$$

$$= -\frac{\left(e\sqrt{-\cos(c+dx)}\,(1+\cos(c+dx))^{-m/2}(-a-a\cos(c+d\right.}{}$$

$$= -\frac{\left(e(1-\cos(c+dx))^{\frac{1}{2}-\frac{m}{2}}\sqrt{-\cos(c+dx)}\,(1+\cos(c+dx))^{\right.}}{}$$

$$= -\frac{2eF_1\left(\frac{1}{2};\frac{1-m}{2},-\frac{m}{2};\frac{3}{2};\cos(c+dx),-\cos(c+dx)\right)(1-\cos(c}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 433 vs. 2(107) = 214.
time = 1.91, size = 433, normalized size = 4.05

$$\frac{4(3+m)\left(F_1\left(\frac{1+m}{2};-\frac{1}{2},1+m;\frac{3+m}{2};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)+F_1\left(\frac{1+m}{2};\frac{1}{2},m;\frac{3+m}{2};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\right)\cos^3\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sec(c+dx))}\,\sin\left(\frac{1}{2}(c+dx)\right)(e\sin(c+dx))^m}{d(1+m)\left(\left(2(1+m)F_1\left(\frac{3+m}{2};-\frac{1}{2},2+m;\frac{5+m}{2};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)+(1+2m)F_1\left(\frac{3+m}{2};\frac{1}{2},1+m;\frac{5+m}{2};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)-F_1\left(\frac{3+m}{2};\frac{3}{2},m;\frac{5+m}{2};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\right)(-1+\cos(c+dx))+(3+m)F_1\left(\frac{1+m}{2};-\frac{1}{2},1+m;\frac{3+m}{2};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)(1+\cos(c+dx))+(3+m)F_1\left(\frac{1+m}{2};\frac{1}{2},m;\frac{3+m}{2};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)(1+\cos(c+dx))\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]*(e*Sin[c + d*x])^m,x]
```

```
[Out] (4*(3 + m)*(AppellF1[(1 + m)/2, -1/2, 1 + m, (3 + m)/2, Tan[(c + d*x)/2]^2,
 -Tan[(c + d*x)/2]^2] + AppellF1[(1 + m)/2, 1/2, m, (3 + m)/2, Tan[(c + d*x
)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[(c + d*x)/2]^3*Sqrt[a*(1 + Sec[c + d*x])]
*Sin[(c + d*x)/2]*(e*Sin[c + d*x])^m)/(d*(1 + m)*((2*(1 + m)*AppellF1[(3 +
m)/2, -1/2, 2 + m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (1
```

```
 + 2*m)*AppellF1[(3 + m)/2, 1/2, 1 + m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan
[(c + d*x)/2]^2] - AppellF1[(3 + m)/2, 3/2, m, (5 + m)/2, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2])*(-1 + Cos[c + d*x]) + (3 + m)*AppellF1[(1 + m)/2,
-1/2, 1 + m, (3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 + Cos[c
 + d*x]) + (3 + m)*AppellF1[(1 + m)/2, 1/2, m, (3 + m)/2, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2]*(1 + Cos[c + d*x]))
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \sqrt{a+a\sec\left(dx+c\right)}\,\left(e\sin\left(dx+c\right)\right)^{m}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)*(e*sin(d*x+c))^m,x)
[Out] int((a+a*sec(d*x+c))^(1/2)*(e*sin(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*(e*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate(sqrt(a*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*(e*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral(sqrt(a*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\,\left(e\sin\left(c+dx\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2)*(e*sin(d*x+c))**m,x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(e*sin(c + d*x))**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*(e*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate(sqrt(a*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\sin\left(c+d\,x\right))^m\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^m*(a + a/cos(c + d*x))^(1/2),x)
[Out] int((e*sin(c + d*x))^m*(a + a/cos(c + d*x))^(1/2), x)
```

3.142 $\int \frac{(e\sin(c+dx))^m}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=115

$$-\frac{2eF_1\left(\frac{3}{2};\frac{1-m}{2},\frac{2-m}{2};\frac{5}{2};\cos(c+dx),-\cos(c+dx)\right)(1-\cos(c+dx))^{\frac{1-m}{2}}\cos(c+dx)(1+\cos(c+dx))^{1-\frac{m}{2}}(e\sin(c+dx))^{m-1}}{3d\sqrt{a+a\sec(c+dx)}}$$

```
[Out] -2/3*e*AppellF1(3/2,1-1/2*m,1/2-1/2*m,5/2,-cos(d*x+c),cos(d*x+c))*(1-cos(d*
x+c))^(1/2-1/2*m)*cos(d*x+c)*(1+cos(d*x+c))^(1-1/2*m)*(e*sin(d*x+c))^(-1+m)
/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3961, 2965, 140, 138}

$$-\frac{2e\cos(c+dx)(1-\cos(c+dx))^{\frac{1-m}{2}}(\cos(c+dx)+1)^{1-\frac{m}{2}}F_1\left(\frac{3}{2};\frac{1-m}{2},\frac{2-m}{2};\frac{5}{2};\cos(c+dx),-\cos(c+dx)\right)(e\sin(c+dx))^{m-1}}{3d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^m/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-2*e*AppellF1[3/2, (1 - m)/2, (2 - m)/2, 5/2, Cos[c + d*x], -Cos[c + d*x]]
*(1 - Cos[c + d*x])^((1 - m)/2)*Cos[c + d*x]*(1 + Cos[c + d*x])^(1 - m/2)*(
e*Sin[c + d*x])^(-1 + m))/(3*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 2965

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[g*((g*Cos
[e + f*x])^(p - 1)/(f*(a + b*Sin[e + f*x])^((p - 1)/2)*(a - b*Sin[e + f*x])
```

```
^((p - 1)/2))), Subst[Int[(d*x)^n*(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p -
 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 3961

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[Sin[e + f*x]^FracPart[m]*((a + b*Csc[e + f*x]
)^FracPart[m]/(b + a*Sin[e + f*x])^FracPart[m]), Int[(g*Cos[e + f*x])^p*((b
 + a*Sin[e + f*x])^m/Sin[e + f*x]^m), x], x] /; FreeQ[{a, b, e, f, g, m, p}
, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^m}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{-a-a\cos(c+dx)}\int \frac{\sqrt{-\cos(c+dx)}\,(e\sin(c+dx))^m}{\sqrt{-a-a\cos(c+dx)}}\,dx}{\sqrt{-\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{\left(e(-a-a\cos(c+dx))^{\frac{1}{2}+\frac{1-m}{2}}(-a+a\cos(c+dx))^{\frac{1-m}{2}}(e\sin(c+dx))^{-1+m}\right)}{d\sqrt{-\cos(c+dx)}\ \cdots}$$

$$= -\frac{\left(e(1+\cos(c+dx))^{1-\frac{m}{2}}(-a-a\cos(c+dx))^{-\frac{1}{2}+\frac{1-m}{2}+\frac{m}{2}}(-a+a\cos(c+dx\cdots\right.}{d\sqrt{-\cos\cdots}}$$

$$= -\frac{\left(e(1-\cos(c+dx))^{\frac{1}{2}-\frac{m}{2}}(1+\cos(c+dx))^{1-\frac{m}{2}}(-a-a\cos(c+dx))^{-\frac{1}{2}+\frac{1-m}{2}+\cdots}\right.}{\cdots}$$

$$= -\frac{2eF_1\left(\frac{3}{2};\frac{1-m}{2},\frac{2-m}{2};\frac{5}{2};\cos(c+dx),-\cos(c+dx)\right)(1-\cos(c+dx))^{\frac{1-m}{2}}\cos(\cdots}{3d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 277 vs. 2(115) = 230.
time = 1.37, size = 277, normalized size = 2.41

$$\frac{4(3+m)F_1\left(\frac{1+m}{2};-\frac{1}{2},1+m;\frac{3+m}{2};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\cos^3\left(\frac{1}{2}(c+dx)\right)\sin\left(\frac{1}{2}(c+dx)\right)(e\sin(c+dx))^m}{d(1+m)\left(\left(2(1+m)F_1\left(\frac{3+m}{2};-\frac{1}{2},2+m;\frac{5+m}{2};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)+F_1\left(\frac{3+m}{2};\frac{1}{2},1+m;\frac{5+m}{2};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\right)(-1+\cos(c+dx))+(3+m)F_1\left(\frac{1+m}{2};-\frac{1}{2},1+m;\frac{3+m}{2};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)(1+\cos(c+dx))\right)\sqrt{a(1+\sec(c+dx))}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sin[c + d*x])^m/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (4*(3 + m)*AppellF1[(1 + m)/2, -1/2, 1 + m, (3 + m)/2, Tan[(c + d*x)/2]^2,
-Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^3*Sin[(c + d*x)/2]*(e*Sin[c + d*x])^m
)/(d*(1 + m)*((2*(1 + m)*AppellF1[(3 + m)/2, -1/2, 2 + m, (5 + m)/2, Tan[(c
```

```
 + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[(3 + m)/2, 1/2, 1 + m, (5 + m
)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(-1 + Cos[c + d*x]) + (3 + m
)*AppellF1[(1 + m)/2, -1/2, 1 + m, (3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c +
 d*x)/2]^2]*(1 + Cos[c + d*x]))*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(e \sin \left(dx+c\right)\right)^{m}}{\sqrt{a+a \sec \left(dx+c\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^(1/2),x)
[Out] int((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate((e*sin(d*x + c))^m/sqrt(a*sec(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral((e*sin(d*x + c))^m/sqrt(a*sec(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(e \sin \left(c+dx\right)\right)^{m}}{\sqrt{a \left(\sec \left(c+dx\right)+1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**m/(a+a*sec(d*x+c))**(1/2),x)
[Out] Integral((e*sin(c + d*x))**m/sqrt(a*(sec(c + d*x) + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*sin(d*x + c))^m/sqrt(a*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(e\,\sin\left(c+d\,x\right)\right)^m}{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^m/(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int((e*sin(c + d*x))^m/(a + a/cos(c + d*x))^(1/2), x)
```

3.143 $\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=120

$$-\frac{2eF_1\left(\frac{5}{2};\frac{1-m}{2},\frac{4-m}{2};\frac{7}{2};\cos(c+dx),-\cos(c+dx)\right)(1-\cos(c+dx))^{\frac{1-m}{2}}\cos^2(c+dx)(1+\cos(c+dx))^{1-\frac{m}{2}}(e$$
$$5ad\sqrt{a+a\sec(c+dx)}$$

```
[Out] -2/5*e*AppellF1(5/2,2-1/2*m,1/2-1/2*m,7/2,-cos(d*x+c),cos(d*x+c))*(1-cos(d*
x+c))^(1/2-1/2*m)*cos(d*x+c)^2*(1+cos(d*x+c))^(1-1/2*m)*(e*sin(d*x+c))^(-1+
m)/a/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3961, 2965, 140, 138}

$$-\frac{2e\cos^2(c+dx)(1-\cos(c+dx))^{\frac{1-m}{2}}(\cos(c+dx)+1)^{1-\frac{m}{2}}F_1\left(\frac{5}{2};\frac{1-m}{2},\frac{4-m}{2};\frac{7}{2};\cos(c+dx),-\cos(c+dx)\right)(e\sin(c+dx))^{m-1}}{5ad\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^m/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*e*AppellF1[5/2, (1 - m)/2, (4 - m)/2, 7/2, Cos[c + d*x], -Cos[c + d*x]]
*(1 - Cos[c + d*x])^((1 - m)/2)*Cos[c + d*x]^2*(1 + Cos[c + d*x])^(1 - m/2)
*(e*Sin[c + d*x])^(-1 + m))/(5*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 2965

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[g*((g*Cos
[e + f*x])^(p - 1)/(f*(a + b*Sin[e + f*x])^((p - 1)/2)*(a - b*Sin[e + f*x])
^((p - 1)/2))), Subst[Int[(d*x)^n*(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p -
```

```
 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 3961

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[Sin[e + f*x]^FracPart[m]*((a + b*Csc[e + f*x]
)^FracPart[m]/(b + a*Sin[e + f*x])^FracPart[m]), Int[(g*Cos[e + f*x])^p*((b
 + a*Sin[e + f*x])^m/Sin[e + f*x]^m), x], x] /; FreeQ[{a, b, e, f, g, m, p}
, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

Rubi steps

$$\begin{aligned}
\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^{3/2}}\,dx &= \frac{\sqrt{-a-a\cos(c+dx)}\int \frac{(-\cos(c+dx))^{3/2}(e\sin(c+dx))^m}{(-a-a\cos(c+dx))^{3/2}}\,dx}{\sqrt{-\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} \\
&= -\frac{\left(e(-a-a\cos(c+dx))^{\frac{1}{2}+\frac{1-m}{2}}(-a+a\cos(c+dx))^{\frac{1-m}{2}}(e\sin(c+dx))^{-1+m}\right.}{d\sqrt{-\cos(c+dx)}} \\
&= \frac{\left(e(1+\cos(c+dx))^{1-\frac{m}{2}}(-a-a\cos(c+dx))^{-\frac{1}{2}+\frac{1-m}{2}+\frac{m}{2}}(-a+a\cos(c+dx)\right.}{ad\sqrt{-\cos}} \\
&= \frac{\left(e(1-\cos(c+dx))^{\frac{1}{2}-\frac{m}{2}}(1+\cos(c+dx))^{1-\frac{m}{2}}(-a-a\cos(c+dx))^{-\frac{1}{2}+\frac{1-m}{2}+}\right.}{} \\
&= -\frac{2eF_1\left(\frac{5}{2};\frac{1-m}{2},\frac{4-m}{2};\frac{7}{2};\cos(c+dx),-\cos(c+dx)\right)(1-\cos(c+dx))^{\frac{1-m}{2}}\cos^2}{5ad\sqrt{a+a\sec(c+dx)}}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 484 vs. 2(120) = 240.

time = 1.95, size = 484, normalized size = 4.03

$$\frac{4(3+m)\left(F_1\left(\frac{1+m}{2};-\frac{1}{2},m;\frac{3+m}{2};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)-2F_1\left(\frac{1+m}{2};-\frac{1}{2},1+m;\frac{3+m}{2};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\right)\cos^3\left(\frac{1}{2}(c+dx)\right)\sin\left(\frac{1}{2}(c+dx)\right)(e\sin(c+dx))^m}{d(1+m)\left(-4(3+m)F_1\left(\frac{1+m}{2};-\frac{1}{2},1+m;\frac{3+m}{2};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\cos^2\left(\frac{1}{2}(c+dx)\right)+\left(2mF_1\left(\frac{3+m}{2};-\frac{1}{2},1+m;\frac{5+m}{2};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)-4(1+m)F_1\left(\frac{3+m}{2};-\frac{1}{2},2+m;\frac{5+m}{2};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)+F_1\left(\frac{3+m}{2};\frac{1}{2},m;\frac{5+m}{2};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)-2F_1\left(\frac{3+m}{2};\frac{1}{2},1+m;\frac{5+m}{2};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\right)(-1+\cos(c+dx))+(3+m)F_1\left(\frac{1+m}{2};-\frac{1}{2},m;\frac{3+m}{2};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)(1+\cos(c+dx))\right)(a(1+\sec(c+dx)))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sin[c + d*x])^m/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (4*(3 + m)*(AppellF1[(1 + m)/2, -1/2, m, (3 + m)/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2] - 2*AppellF1[(1 + m)/2, -1/2, 1 + m, (3 + m)/2, Tan[(c +
d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[(c + d*x)/2]^3*Sin[(c + d*x)/2]*(e*Sin
[c + d*x])^m)/(d*(1 + m)*(-4*(3 + m)*AppellF1[(1 + m)/2, -1/2, 1 + m, (3 +
m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2 + (2*m*Ap
```

```
pellF1[(3 + m)/2, -1/2, 1 + m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x
)/2]^2] - 4*(1 + m)*AppellF1[(3 + m)/2, -1/2, 2 + m, (5 + m)/2, Tan[(c + d*
x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[(3 + m)/2, 1/2, m, (5 + m)/2, Tan[
(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*AppellF1[(3 + m)/2, 1/2, 1 + m, (5
 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(-1 + Cos[c + d*x]) + (3
 + m)*AppellF1[(1 + m)/2, -1/2, m, (3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c +
 d*x)/2]^2]*(1 + Cos[c + d*x]))*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(e \sin \left(dx+c\right)\right)^{m}}{\left(a+a \sec \left(dx+c\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^(3/2),x)
[Out] int((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((e*sin(d*x + c))^m/(a*sec(d*x + c) + a)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(a*sec(d*x + c) + a)*(e*sin(d*x + c))^m/(a^2*sec(d*x + c)^2 +
2*a^2*sec(d*x + c) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(e \sin \left(c+dx\right)\right)^{m}}{\left(a \left(\sec \left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**m/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((e*sin(c + d*x))**m/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(tan(sageVARc/2)^
3*t_noste
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,\sin\left(c+d\,x\right))^m}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^m/(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int((e*sin(c + d*x))^m/(a + a/cos(c + d*x))^(3/2), x)
```

3.144 $\int (a+a\sec(c+dx))^n(e\sin(c+dx))^m\,dx$

Optimal. Leaf size=130

$$-\frac{eF_1\left(1-n;\frac{1-m}{2},\frac{1}{2}(1-m-2n);2-n;\cos(c+dx),-\cos(c+dx)\right)(1-\cos(c+dx))^{\frac{1-m}{2}}\cos(c+dx)(1+\text{co}}{d(1-n)}$$

```
[Out] -e*AppellF1(1-n,1/2-1/2*m-n,1/2-1/2*m,2-n,-cos(d*x+c),cos(d*x+c))*(1-cos(d*
x+c))^(1/2-1/2*m)*cos(d*x+c)*(1+cos(d*x+c))^(1/2-1/2*m-n)*(a+a*sec(d*x+c))^
n*(e*sin(d*x+c))^(-1+m)/d/(1-n)
```

Rubi [A]
time = 0.20, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3961, 2965, 140, 138}

$$-\frac{e\cos(c+dx)(1-\cos(c+dx))^{\frac{1-m}{2}}(a\sec(c+dx)+a)^n(e\sin(c+dx))^{m-1}(\cos(c+dx)+1)^{\frac{1}{2}(-m-2n+1)}F_1\left(1-n;\frac{1-m}{2},\frac{1}{2}(-m-2n+1);2-n;\cos(c+dx),-\cos(c+dx)\right)}{d(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n*(e*Sin[c + d*x])^m,x]
```

```
[Out] -((e*AppellF1[1 - n, (1 - m)/2, (1 - m - 2*n)/2, 2 - n, Cos[c + d*x], -Cos[
c + d*x]]*(1 - Cos[c + d*x])^((1 - m)/2)*Cos[c + d*x]*(1 + Cos[c + d*x])^((
1 - m - 2*n)/2)*(a + a*Sec[c + d*x])^n*(e*Sin[c + d*x])^(-1 + m))/(d*(1 - n
)))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 2965

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[g*((g*Cos
[e + f*x])^(p - 1)/(f*(a + b*Sin[e + f*x])^((p - 1)/2)*(a - b*Sin[e + f*x])
^((p - 1)/2))), Subst[Int[(d*x)^n*(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p -
```

```
 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 3961

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[Sin[e + f*x]^FracPart[m]*((a + b*Csc[e + f*x]
)^FracPart[m]/(b + a*Sin[e + f*x])^FracPart[m]), Int[(g*Cos[e + f*x])^p*((b
 + a*Sin[e + f*x])^m/Sin[e + f*x]^m), x], x] /; FreeQ[{a, b, e, f, g, m, p}
, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

Rubi steps

$$\int (a+a\sec(c+dx))^n(e\sin(c+dx))^m\,dx = ((-\cos(c+dx))^n(-a-a\cos(c+dx))^{-n}(a+a\sec(c+dx))^n)$$

$$= -\frac{\left(e(-\cos(c+dx))^n(-a-a\cos(c+dx))^{\frac{1-m}{2}-n}(-a+a\cos(c\right.}{}$$

$$= -\frac{\left(e(-\cos(c+dx))^n(1+\cos(c+dx))^{\frac{1}{2}-\frac{m}{2}-n}(-a-a\cos(c+\right.}{}$$

$$= -\frac{\left(e(1-\cos(c+dx))^{\frac{1}{2}-\frac{m}{2}}(-\cos(c+dx))^n(1+\cos(c+dx))^{\frac{1}{2}-}\right.}{}$$

$$= -\frac{eF_1\left(1-n;\frac{1-m}{2},\frac{1}{2}(1-m-2n);2-n;\cos(c+dx),-\cos(c+\right.}{}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 276 vs. 2(130) = 260.
time = 1.32, size = 276, normalized size = 2.12

$$\frac{4(3+m)F_1\left(\frac{1+m}{2};n,1+m;\frac{3+m}{2};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\cos^3\left(\frac{1}{2}(c+dx)\right)(a(1+\sec(c+dx)))^n\sin\left(\frac{1}{2}(c+dx)\right)(e\sin(c+dx))^m}{d(1+m)\left((3+m)F_1\left(\frac{1+m}{2};n,1+m;\frac{3+m}{2};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)(1+\cos(c+dx))-4\left((1+m)F_1\left(\frac{3+m}{2};n,2+m;\frac{5+m}{2};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)-nF_1\left(\frac{3+m}{2};1+n,1+m;\frac{5+m}{2};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\right)\sin^2\left(\frac{1}{2}(c+dx)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^n*(e*Sin[c + d*x])^m,x]
```

```
[Out] (4*(3 + m)*AppellF1[(1 + m)/2, n, 1 + m, (3 + m)/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^3*(a*(1 + Sec[c + d*x]))^n*Sin[(c + d*x)
/2]*(e*Sin[c + d*x])^m)/(d*(1 + m)*((3 + m)*AppellF1[(1 + m)/2, n, 1 + m, (
3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 + Cos[c + d*x]) - 4*(
(1 + m)*AppellF1[(3 + m)/2, n, 2 + m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(
c + d*x)/2]^2] - n*AppellF1[(3 + m)/2, 1 + n, 1 + m, (5 + m)/2, Tan[(c + d*
x)/2]^2, -Tan[(c + d*x)/2]^2])*Sin[(c + d*x)/2]^2))
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \left(a+a\sec \left(dx+c\right)\right)^{n} \left(e\sin \left(dx+c\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*(e*sin(d*x+c))^m,x)
[Out] int((a+a*sec(d*x+c))^n*(e*sin(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*(e*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n*(e*sin(d*x + c))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*(e*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((a*sec(d*x + c) + a)^n*(e*sin(d*x + c))^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sec \left(c+dx\right)+1\right)\right)^{n} \left(e\sin \left(c+dx\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*(e*sin(d*x+c))**m,x)
[Out] Integral((a*(sec(c + d*x) + 1))**n*(e*sin(c + d*x))**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*(e*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*(e*sin(d*x + c))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\sin\left(c+d\,x\right))^m\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^m*(a + a/cos(c + d*x))^n,x)
```

```
[Out] int((e*sin(c + d*x))^m*(a + a/cos(c + d*x))^n, x)
```

3.145 $\int (a + a\sec(c + dx))^n \sin^7(c + dx)\, dx$

Optimal. Leaf size=180

$$-\frac{(3-n)(8-n)(16-n)\,{}_2F_1(6,4+n;5+n;1+\sec(c+dx))(a+a\sec(c+dx))^{4+n}}{42a^4d(1-n)(4+n)}-\frac{\cos^7(c+dx)(1-\sec(c}{a}$$

```
[Out] -1/42*(3-n)*(8-n)*(16-n)*hypergeom([6, 4+n],[5+n],1+sec(d*x+c))*(a+a*sec(d*
x+c))^(4+n)/a^4/d/(-n^2-3*n+4)-cos(d*x+c)^7*(1-sec(d*x+c))^2*(a+a*sec(d*x+c
))^(4+n)/a^4/d/(1-n)+1/42*cos(d*x+c)^7*(a+a*sec(d*x+c))^(4+n)*(48-6*n-(n^2-
25*n+108)*sec(d*x+c))/a^4/d/(1-n)
```

Rubi [A]
time = 0.12, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3958, 102, 150, 67}

$$-\frac{(3-n)(8-n)(16-n)(a\sec(c+dx)+a)^{n+4}\,{}_2F_1(6,n+4;n+5;\sec(c+dx)+1)}{42a^4d(1-n)(n+4)}+\frac{\cos^7(c+dx)\left(6(8-n)-\left(n^2-25n+108\right)\sec(c+dx)\right)(a\sec(c+dx)+a)^{n+4}}{42a^4d(1-n)}-\frac{\cos^7(c+dx)(1-\sec(c+dx))^2(a\sec(c+dx)+a)^{n+4}}{a^4d(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^7,x]
```

```
[Out] -1/42*((3 - n)*(8 - n)*(16 - n)*Hypergeometric2F1[6, 4 + n, 5 + n, 1 + Sec[
c + d*x]]*(a + a*Sec[c + d*x])^(4 + n))/(a^4*d*(1 - n)*(4 + n)) - (Cos[c +
d*x]^7*(1 - Sec[c + d*x])^2*(a + a*Sec[c + d*x])^(4 + n))/(a^4*d*(1 - n)) +
 (Cos[c + d*x]^7*(a + a*Sec[c + d*x])^(4 + n)*(6*(8 - n) - (108 - 25*n + n^
2)*Sec[c + d*x]))/(42*a^4*d*(1 - n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(
n + 2) - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h)
 + d*e*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(
f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2))))*x)/(
b^2*(b*c - a*d)^2*(m + 1)*(m + 2)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x]
 + Dist[f*(h/b^2) - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1
) - d*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)
)))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), Int[(a + b*x)^(m + 2)*(c + d*x)^n,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[
m + n + 3, 0] &&  !LtQ[n, -2]))
```

Rule 3958

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> Dist[-(f*b^(p - 1))^(-1), Subst[Int[(-a + b*x)^((p - 1)/2)
*((a + b*x)^(m + (p - 1)/2)/x^(p + 1)), x], x, Csc[e + f*x]], x] /; FreeQ[{
a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+a\sec(c+dx))^n \sin^7(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{(-a-ax)^3(a-ax)^{3+n}}{x^8}\,dx, x, -\sec(c+dx)\right)}{a^6 d}$$

$$= -\frac{\cos^7(c+dx)(1-\sec(c+dx))^2(a+a\sec(c+dx))^{4+n}}{a^4 d(1-n)} - \frac{\text{Subst}\left(\right.}{}$$

$$= -\frac{\cos^7(c+dx)(1-\sec(c+dx))^2(a+a\sec(c+dx))^{4+n}}{a^4 d(1-n)} + \frac{\cos^7(c+}{}$$

$$= -\frac{(3-n)(8-n)(16-n)\,{}_2F_1(6,4+n;5+n;1+\sec(c+dx))(a+}{42a^4 d(1-n)(4+n)}$$

Mathematica [A]
time = 0.99, size = 113, normalized size = 0.63

$$\frac{\left((4+n)\cos^5(c+dx)\left(42+\left(24-25n+n^2\right)\cos(c+dx)+6(-1+n)\cos^2(c+dx)\right)-\left(-384+200n-27n^2+n^3\right)\,{}_2F_1(6,4+n;5+n;1+\sec(c+dx))\right)(1+\sec(c+dx))^4(a(1+\sec(c+dx)))^n}{42d(-1+n)(4+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^7,x]
```

```
[Out] (((4 + n)*Cos[c + d*x]^5*(42 + (24 - 25*n + n^2)*Cos[c + d*x] + 6*(-1 + n)*
Cos[c + d*x]^2) - (-384 + 200*n - 27*n^2 + n^3)*Hypergeometric2F1[6, 4 + n,
```

```
 5 + n, 1 + Sec[c + d*x]])*(1 + Sec[c + d*x])^4*(a*(1 + Sec[c + d*x]))^n)/(
42*d*(-1 + n)*(4 + n))
```

Maple [F]
time = 0.32, size = 0, normalized size = 0.00

$$\int (a + a\sec(dx+c))^n \left(\sin^7(dx+c)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*sin(d*x+c)^7,x)
[Out] int((a+a*sec(d*x+c))^n*sin(d*x+c)^7,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^7,x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^7, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^7,x, algorithm="fricas")
[Out] integral(-(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*(a*sec
(d*x + c) + a)^n*sin(d*x + c), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*sin(d*x+c)**7,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 8010 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^7,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^7, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(c+d\,x\right)^7\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^n \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^7*(a + a/cos(c + d*x))^n,x)
```

```
[Out] int(sin(c + d*x)^7*(a + a/cos(c + d*x))^n, x)
```

3.146 $\int (a + a\sec(c + dx))^n \sin^5(c + dx)\, dx$

Optimal. Leaf size=123

$$\frac{(12-n)\cos^4(c+dx)(a+a\sec(c+dx))^{3+n}}{20a^3d} - \frac{\cos^5(c+dx)(a+a\sec(c+dx))^{3+n}}{5a^3d} + \frac{(32-13n+n^2)\ {}_2F_1(4,3\cdots}{}$$

```
[Out] 1/20*(12-n)*cos(d*x+c)^4*(a+a*sec(d*x+c))^(3+n)/a^3/d-1/5*cos(d*x+c)^5*(a+a
*sec(d*x+c))^(3+n)/a^3/d+1/20*(n^2-13*n+32)*hypergeom([4, 3+n],[4+n],1+sec(
d*x+c))*(a+a*sec(d*x+c))^(3+n)/a^3/d/(3+n)
```

Rubi [A]

time = 0.08, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3958, 91, 79, 67}

$$\frac{(n^2-13n+32)(a\sec(c+dx)+a)^{n+3}\,{}_2F_1(4,n+3;n+4;\sec(c+dx)+1)}{20a^3d(n+3)} - \frac{\cos^5(c+dx)(a\sec(c+dx)+a)^{n+3}}{5a^3d} + \frac{(12-n)\cos^4(c+dx)(a\sec(c+dx)+a)^{n+3}}{20a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^5,x]
```

```
[Out] ((12 - n)*Cos[c + d*x]^4*(a + a*Sec[c + d*x])^(3 + n))/(20*a^3*d) - (Cos[c
+ d*x]^5*(a + a*Sec[c + d*x])^(3 + n))/(5*a^3*d) + ((32 - 13*n + n^2)*Hyper
geometric2F1[4, 3 + n, 4 + n, 1 + Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3 + n
))/(20*a^3*d*(3 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
```

```
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 3958

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> Dist[-(f*b^(p - 1))^(-1), Subst[Int[(-a + b*x)^((p - 1)/2)
*((a + b*x)^(m + (p - 1)/2)/x^(p + 1)), x], x, Csc[e + f*x]], x] /; FreeQ[{
a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+a\sec(c+dx))^n \sin^5(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{(-a-ax)^2(a-ax)^{2+n}}{x^6}\,dx, x, -\sec(c+dx)\right)}{a^4 d}$$

$$= -\frac{\cos^5(c+dx)(a+a\sec(c+dx))^{3+n}}{5a^3 d} - \frac{\text{Subst}\left(\int \frac{(a-ax)^{2+n}(a^3(12-n)\cdots}{x^5}\cdots\right.}{5a\cdots}$$

$$= \frac{(12-n)\cos^4(c+dx)(a+a\sec(c+dx))^{3+n}}{20a^3 d} - \frac{\cos^5(c+dx)(a+a\cdots}{5a^3\cdots}$$

$$= \frac{(12-n)\cos^4(c+dx)(a+a\sec(c+dx))^{3+n}}{20a^3 d} - \frac{\cos^5(c+dx)(a+a\cdots}{5a^3\cdots}$$

Mathematica [A]
time = 0.35, size = 84, normalized size = 0.68

$$-\frac{((3+n)\cos^4(c+dx)(-12+n+4\cos(c+dx)) - (32-13n+n^2)\,{}_2F_1(4,3+n;4+n;1+\sec(c+dx)))\,(1+\sec(c+dx))^3(a(1+\sec(c+dx)))^n}{20d(3+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^5,x]
```

```
[Out] -1/20*(((3 + n)*Cos[c + d*x]^4*(-12 + n + 4*Cos[c + d*x]) - (32 - 13*n + n^
2)*Hypergeometric2F1[4, 3 + n, 4 + n, 1 + Sec[c + d*x]])*(1 + Sec[c + d*x])
^3*(a*(1 + Sec[c + d*x]))^n)/(d*(3 + n))
```

Maple [F]
time = 0.29, size = 0, normalized size = 0.00

$$\int (a+a\sec(dx+c))^n \left(\sin^5(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*sin(d*x+c)^5,x)
```

```
[Out] int((a+a*sec(d*x+c))^n*sin(d*x+c)^5,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^5, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] integral((cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*(a*sec(d*x + c) + a)^n*sin
(d*x + c), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*sin(d*x+c)**5,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^5,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin (c+d\,x)^5\,\left(a+\frac{a}{\cos (c+d\,x)}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5*(a + a/cos(c + d*x))^n,x)
[Out] int(sin(c + d*x)^5*(a + a/cos(c + d*x))^n, x)
```

3.147 $\int (a+a\sec(c+dx))^n \sin^3(c+dx)\,dx$

Optimal. Leaf size=83

$$\frac{\cos^3(c+dx)(a+a\sec(c+dx))^{2+n}}{3a^2d}-\frac{(4-n)\,{}_2F_1(3,2+n;3+n;1+\sec(c+dx))(a+a\sec(c+dx))^{2+n}}{3a^2d(2+n)}$$

```
[Out] 1/3*cos(d*x+c)^3*(a+a*sec(d*x+c))^(2+n)/a^2/d-1/3*(4-n)*hypergeom([3, 2+n],
[3+n],1+sec(d*x+c))*(a+a*sec(d*x+c))^(2+n)/a^2/d/(2+n)
```

Rubi [A]
time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3958, 79, 67}

$$\frac{\cos^3(c+dx)(a\sec(c+dx)+a)^{n+2}}{3a^2d}-\frac{(4-n)(a\sec(c+dx)+a)^{n+2}\,{}_2F_1(3,n+2;n+3;\sec(c+dx)+1)}{3a^2d(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^3,x]
```

```
[Out] (Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(2 + n))/(3*a^2*d) - ((4 - n)*Hypergeo
metric2F1[3, 2 + n, 3 + n, 1 + Sec[c + d*x]]*(a + a*Sec[c + d*x])^(2 + n))/
(3*a^2*d*(2 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 3958

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> Dist[-(f*b^(p - 1))^(-1), Subst[Int[(-a + b*x)^((p - 1)/2)
*((a + b*x)^(m + (p - 1)/2)/x^(p + 1)), x], x, Csc[e + f*x]], x] /; FreeQ[{
```

```
a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+a\sec(c+dx))^n\sin^3(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{(-a-ax)(a-ax)^{1+n}}{x^4}\,dx, x, -\sec(c+dx)\right)}{a^2d}$$

$$= \frac{\cos^3(c+dx)(a+a\sec(c+dx))^{2+n}}{3a^2d} + \frac{(4-n)\text{Subst}\left(\int \frac{(a-ax)^{1+n}}{x^3}\,dx\right.}{3ad}$$

$$= \frac{\cos^3(c+dx)(a+a\sec(c+dx))^{2+n}}{3a^2d} - \frac{(4-n)\,{}_2F_1(3,2+n;3+n;}{3}$$

Mathematica [A]
time = 0.10, size = 67, normalized size = 0.81

$$\frac{((2+n)\cos^3(c+dx)+(-4+n)\,{}_2F_1(3,2+n;3+n;1+\sec(c+dx)))\,(1+\sec(c+dx))^2(a(1+\sec(c+dx)))^n}{3d(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^3,x]
[Out] (((2 + n)*Cos[c + d*x]^3 + (-4 + n)*Hypergeometric2F1[3, 2 + n, 3 + n, 1 + Sec[c + d*x]])*(1 + Sec[c + d*x])^2*(a*(1 + Sec[c + d*x]))^n)/(3*d*(2 + n))
```

Maple [F]
time = 0.28, size = 0, normalized size = 0.00

$$\int (a+a\sec(dx+c))^n\left(\sin^3(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*sin(d*x+c)^3,x)
[Out] int((a+a*sec(d*x+c))^n*sin(d*x+c)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral(-(cos(d*x + c)^2 - 1)*(a*sec(d*x + c) + a)^n*sin(d*x + c), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*sin(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Evaluation time: 0.47sym2poly/r2sym(const gen & e,const i
ndex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(c+d\,x\right)^3\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3*(a + a/cos(c + d*x))^n,x)
```

```
[Out] int(sin(c + d*x)^3*(a + a/cos(c + d*x))^n, x)
```

3.148 $\int (a+a\sec(c+dx))^n \sin(c+dx)\,dx$

Optimal. Leaf size=42

$$\frac{{}_2F_1(2,1+n;2+n;1+\sec(c+dx))(a+a\sec(c+dx))^{1+n}}{ad(1+n)}$$

```
[Out] hypergeom([2, 1+n],[2+n],1+sec(d*x+c))*(a+a*sec(d*x+c))^(1+n)/a/d/(1+n)
```

Rubi [A]
time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3958, 67}

$$\frac{(a\sec(c+dx)+a)^{n+1}\,{}_2F_1(2,n+1;n+2;\sec(c+dx)+1)}{ad(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n*Sin[c + d*x],x]
```

```
[Out] (Hypergeometric2F1[2, 1 + n, 2 + n, 1 + Sec[c + d*x]]*(a + a*Sec[c + d*x])^
(1 + n))/(a*d*(1 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 3958

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> Dist[-(f*b^(p - 1))^(-1), Subst[Int[(-a + b*x)^((p - 1)/2)
*((a + b*x)^(m + (p - 1)/2)/x^(p + 1)), x], x, Csc[e + f*x]], x] /; FreeQ[{
a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))^n \sin(c+dx)\,dx &= -\frac{\text{Subst}\left(\int \frac{(a-ax)^n}{x^2}\,dx, x, -\sec(c+dx)\right)}{d}\\ &= \frac{{}_2F_1(2,1+n;2+n;1+\sec(c+dx))(a+a\sec(c+dx))^{1+n}}{ad(1+n)}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 42, normalized size = 1.00

$$\frac{{}_2F_1(2,1+n;2+n;1+\sec(c+dx))(a(1+\sec(c+dx)))^{1+n}}{ad(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Sin[c + d*x],x]
```

```
[Out] (Hypergeometric2F1[2, 1 + n, 2 + n, 1 + Sec[c + d*x]]*(a*(1 + Sec[c + d*x])
)^(1 + n))/(a*d*(1 + n))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \left(a+a\sec\left(dx+c\right)\right)^{n}\sin\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*sin(d*x+c),x)
```

```
[Out] int((a+a*sec(d*x+c))^n*sin(d*x+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*sin(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n*sin(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sec\left(c+dx\right)+1\right)\right)^{n}\sin\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*sin(d*x+c),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**n*sin(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*sin(d*x + c), x)
```

Mupad [B]
time = 1.18, size = 64, normalized size = 1.52

$$\frac{\cos\left(c+d\,x\right)\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^n\,{}_2\mathrm{F}_1(1-n,-n;\ 2-n;\ -\cos\left(c+d\,x\right))}{d\left(\cos\left(c+d\,x\right)+1\right)^n\,\left(n-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a + a/cos(c + d*x))^n,x)
```

```
[Out] (cos(c + d*x)*(a + a/cos(c + d*x))^n*hypergeom([1 - n, -n], 2 - n, -cos(c +
 d*x)))/(d*(cos(c + d*x) + 1)^n*(n - 1))
```

3.149 $\int \csc(c+dx)(a+a\sec(c+dx))^n\,dx$

Optimal. Leaf size=40

$$-\frac{{}_2F_1\left(1,n;1+n;\frac{1}{2}(1+\sec(c+dx))\right)(a+a\sec(c+dx))^n}{2dn}$$

```
[Out] -1/2*hypergeom([1, n],[1+n],1/2+1/2*sec(d*x+c))*(a+a*sec(d*x+c))^n/d/n
```

Rubi [A]
time = 0.04, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3958, 70}

$$-\frac{(a\sec(c+dx)+a)^n\,{}_2F_1\left(1,n;n+1;\frac{1}{2}(\sec(c+dx)+1)\right)}{2dn}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] -1/2*(Hypergeometric2F1[1, n, 1 + n, (1 + Sec[c + d*x])/2]*(a + a*Sec[c + d
*x])^n)/(d*n)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3958

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> Dist[-(f*b^(p - 1))^(-1), Subst[Int[(-a + b*x)^((p - 1)/2)
*((a + b*x)^(m + (p - 1)/2)/x^(p + 1)), x], x, Csc[e + f*x]], x] /; FreeQ[{
a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \csc(c+dx)(a+a\sec(c+dx))^n\,dx &= -\frac{a^2\text{Subst}\left(\int \frac{(a-ax)^{-1+n}}{-a-ax}\,dx, x, -\sec(c+dx)\right)}{d}\\ &= -\frac{{}_2F_1\left(1,n;1+n;\frac{1}{2}(1+\sec(c+dx))\right)(a+a\sec(c+dx))^n}{2dn}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 92 vs. 2(40) = 80.
time = 0.42, size = 92, normalized size = 2.30

$$\frac{2^{-1+n}\,{}_2F_1\left(1,1-n;2-n;\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{-1+n}(1+\sec(c+dx))^{-n}(a(1+\sec(c+dx)))^n}{d(-1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] (2^(-1 + n)*Hypergeometric2F1[1, 1 - n, 2 - n, Cos[c + d*x]*Sec[(c + d*x)/2
]^2]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(-1 + n)*(a*(1 + Sec[c + d*x]))^n)/(
d*(-1 + n)*(1 + Sec[c + d*x])^n)
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \csc(dx+c)\left(a+a\sec(dx+c)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+a*sec(d*x+c))^n,x)
```

```
[Out] int(csc(d*x+c)*(a+a*sec(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*csc(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n*csc(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a(\sec(c+dx)+1)\right)^n\csc(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c))**n,x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**n*csc(c + d*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*csc(d*x + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^n}{\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^n/sin(c + d*x),x)
```

```
[Out] int((a + a/cos(c + d*x))^n/sin(c + d*x), x)
```

3.150 $\int \csc^3(c+dx)(a+a\sec(c+dx))^n\,dx$

Optimal. Leaf size=112

$$-\frac{a(2-n)(a+a\sec(c+dx))^{-1+n}}{4d(1-n)}+\frac{a(a+a\sec(c+dx))^{-1+n}}{2d(1-\sec(c+dx))}-\frac{(2+n)\,{}_2F_1\left(1,n;1+n;\frac{1}{2}(1+\sec(c+dx))\right)}{8dn}$$

```
[Out] -1/4*a*(2-n)*(a+a*sec(d*x+c))^(-1+n)/d/(1-n)+1/2*a*(a+a*sec(d*x+c))^(-1+n)/
d/(1-sec(d*x+c))-1/8*(2+n)*hypergeom([1, n],[1+n],1/2+1/2*sec(d*x+c))*(a+a*
sec(d*x+c))^n/d/n
```

Rubi [A]
time = 0.07, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3958, 91, 80, 70}

$$-\frac{(n+2)(a\sec(c+dx)+a)^n\,{}_2F_1\left(1,n;n+1;\frac{1}{2}(\sec(c+dx)+1)\right)}{8dn}-\frac{a(2-n)(a\sec(c+dx)+a)^{n-1}}{4d(1-n)}+\frac{a(a\sec(c+dx)+a)^{n-1}}{2d(1-\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] -1/4*(a*(2 - n)*(a + a*Sec[c + d*x])^(-1 + n))/(d*(1 - n)) + (a*(a + a*Sec[
c + d*x])^(-1 + n))/(2*d*(1 - Sec[c + d*x])) - ((2 + n)*Hypergeometric2F1[1
, n, 1 + n, (1 + Sec[c + d*x])/2]*(a + a*Sec[c + d*x])^n)/(8*d*n)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p
+ 1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && Sum
SimplerQ[p, 1]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
```

```
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 3958

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> Dist[-(f*b^(p - 1))^(-1), Subst[Int[(-a + b*x)^((p - 1)/2)
*((a + b*x)^(m + (p - 1)/2)/x^(p + 1)), x], x, Csc[e + f*x]], x] /; FreeQ[{
a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \csc^3(c+dx)(a+a\sec(c+dx))^n\,dx = -\frac{a^4\text{Subst}\left(\int \frac{x^2(a-ax)^{-2+n}}{(-a-ax)^2}\,dx, x, -\sec(c+dx)\right)}{d}$$

$$= \frac{a(a+a\sec(c+dx))^{-1+n}}{2d(1-\sec(c+dx))} + \frac{\text{Subst}\left(\int \frac{(a-ax)^{-2+n}(-a^3n+2a^3x)}{-a-ax}\,dx, x, -\text{s}\right.}{2d}$$

$$= -\frac{a(2-n)(a+a\sec(c+dx))^{-1+n}}{4d(1-n)} + \frac{a(a+a\sec(c+dx))^{-1+n}}{2d(1-\sec(c+dx))} - \frac{(a}{}$$

$$= -\frac{a(2-n)(a+a\sec(c+dx))^{-1+n}}{4d(1-n)} + \frac{a(a+a\sec(c+dx))^{-1+n}}{2d(1-\sec(c+dx))} - \frac{(2}{}$$

Mathematica [A]
time = 1.43, size = 179, normalized size = 1.60

$$\frac{\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))^{-n}(a(1+\sec(c+dx)))^n\left(2^{1+n}\,{}_2F_1\left(1,1-n;2-n;\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^n+2^n\,{}_2F_1\left(2,1-n;2-n;\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^n+(1+\sec(c+dx))^n\right)}{8d(-1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] (Cos[c + d*x]*Sec[(c + d*x)/2]^2*(a*(1 + Sec[c + d*x]))^n*(2^(1 + n)*Hyperg
eometric2F1[1, 1 - n, 2 - n, Cos[c + d*x]*Sec[(c + d*x)/2]^2]*(Cos[(c + d*x
)/2]^2*Sec[c + d*x])^n + 2^n*Hypergeometric2F1[2, 1 - n, 2 - n, Cos[c + d*x
]*Sec[(c + d*x)/2]^2]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n + (1 + Sec[c + d*
x])^n))/(8*d*(-1 + n)*(1 + Sec[c + d*x])^n)
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \left(\csc^3(dx+c)\right)\left(a+a\sec(dx+c)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(a+a*sec(d*x+c))^n,x)
```

```
[Out] int(csc(d*x+c)^3*(a+a*sec(d*x+c))^n,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sec(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*csc(d*x + c)^3, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sec(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n*csc(d*x + c)^3, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec (c+dx)+1))^n \csc^3 (c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(a+a*sec(d*x+c))**n,x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**n*csc(c + d*x)**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sec(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*csc(d*x + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^n}{\sin\left(c+d\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^n/sin(c + d*x)^3,x)
[Out] int((a + a/cos(c + d*x))^n/sin(c + d*x)^3, x)
```

3.151 $\int \csc^5(c+dx)(a+a\sec(c+dx))^n\,dx$

Optimal. Leaf size=240

$$\frac{a^2(12+9n+n^2)\ {}_2F_1\left(1,-2+n;-1+n;\frac{1}{2}(1+\sec(c+dx))\right)(a+a\sec(c+dx))^{-2+n}}{16d(2-n)}+\frac{a^2(3+n)\sec^2(c+a}{4d(1-n)(}$$

```
[Out] 1/16*a^2*(n^2+9*n+12)*hypergeom([1, -2+n],[-1+n],1/2+1/2*sec(d*x+c))*(a+a*s
ec(d*x+c))^(-2+n)/d/(2-n)+1/4*a^2*(3+n)*sec(d*x+c)^2*(a+a*sec(d*x+c))^(-2+n
)/d/(1-n)/(1-sec(d*x+c))^2-a^2*sec(d*x+c)^3*(a+a*sec(d*x+c))^(-2+n)/d/(1-n)
/(1-sec(d*x+c))^2-1/8*a^2*(a+a*sec(d*x+c))^(-2+n)*(12+4*n-7*n^2-n^3-2*(1-n)
*(6+n)*sec(d*x+c))/d/(n^2-3*n+2)/(1-sec(d*x+c))
```

Rubi [A]
time = 0.17, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3958, 102, 154, 151, 70}

$$\frac{a^2(n^2+9n+12)\,(a\sec(c+dx)+a)^{n-2}\,{}_2F_1\left(1,n-2;n-1;\frac{1}{2}(\sec(c+dx)+1)\right)}{16d(2-n)}-\frac{a^2(-2(1-n)(n+6)\sec(c+dx)-n^3-7n^2+4n+12)\,(a\sec(c+dx)+a)^{n-2}}{8d\,(n^2-3n+2)\,(1-\sec(c+dx))}-\frac{a^2\sec^3(c+dx)(a\sec(c+dx)+a)^{n-2}}{d(1-n)(1-\sec(c+dx))^2}+\frac{a^2(n+3)\sec^2(c+dx)(a\sec(c+dx)+a)^{n-2}}{4d(1-n)(1-\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] (a^2*(12 + 9*n + n^2)*Hypergeometric2F1[1, -2 + n, -1 + n, (1 + Sec[c + d*x
])/2]*(a + a*Sec[c + d*x])^(-2 + n))/(16*d*(2 - n)) + (a^2*(3 + n)*Sec[c +
d*x]^2*(a + a*Sec[c + d*x])^(-2 + n))/(4*d*(1 - n)*(1 - Sec[c + d*x])^2) -
(a^2*Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(-2 + n))/(d*(1 - n)*(1 - Sec[c +
d*x])^2) - (a^2*(a + a*Sec[c + d*x])^(-2 + n)*(12 + 4*n - 7*n^2 - n^3 - 2*(
1 - n)*(6 + n)*Sec[c + d*x]))/(8*d*(2 - 3*n + n^2)*(1 - Sec[c + d*x]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)
*(c + d*x)^(n + 1), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

Rule 3958

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> Dist[-(f*b^(p - 1))^(-1), Subst[Int[(-a + b*x)^((p - 1)/2)
*((a + b*x)^(m + (p - 1)/2)/x^(p + 1)), x], x, Csc[e + f*x]], x] /; FreeQ[{
a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \csc^5(c+dx)(a+a\sec(c+dx))^n\,dx = -\frac{a^6\text{Subst}\left(\int \frac{x^4(a-ax)^{-3+n}}{(-a-ax)^3}\,dx, x, -\sec(c+dx)\right)}{d}$$

$$= -\frac{a^2\sec^3(c+dx)(a+a\sec(c+dx))^{-2+n}}{d(1-n)(1-\sec(c+dx))^2} + \frac{a^4\text{Subst}\left(\int \frac{x^2(a-ax)^{-3+}}{(-a-}\right.}{}$$

$$= \frac{a^2(3+n)\sec^2(c+dx)(a+a\sec(c+dx))^{-2+n}}{4d(1-n)(1-\sec(c+dx))^2} - \frac{a^2\sec^3(c+dx)(a}{d(1-n)(1}$$

$$= \frac{a^2(3+n)\sec^2(c+dx)(a+a\sec(c+dx))^{-2+n}}{4d(1-n)(1-\sec(c+dx))^2} - \frac{a^2\sec^3(c+dx)(a}{d(1-n)(1}$$

$$= \frac{a^2(12+9n+n^2)\ {}_2F_1\left(1,-2+n;-1+n;\frac{1}{2}(1+\sec(c+dx))\right)(a+}{16d(2-n)}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 492 vs. $2(240) = 480$.
time = 4.17, size = 492, normalized size = 2.05

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^5*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] -1/64*(Cos[c + d*x]*(a*(1 + Sec[c + d*x]))^n*(2^(1 + n)*Cot[(c + d*x)/2]^4*
Sec[c + d*x]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(-1 + n) - 3*2^n*n*Cot[(c +
d*x)/2]^4*Sec[c + d*x]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(-1 + n) + 2^n*n^2
*Cot[(c + d*x)/2]^4*Sec[c + d*x]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(-1 + n)
 - 3*2^(2 + n)*(-2 + n)*Hypergeometric2F1[1, 1 - n, 2 - n, Cos[c + d*x]*Sec
[(c + d*x)/2]^2]*Sec[c + d*x]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(-1 + n) -
2^n*(-18 + 7*n + n^2)*Hypergeometric2F1[2, 1 - n, 2 - n, Cos[c + d*x]*Sec[(
c + d*x)/2]^2]*Sec[c + d*x]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(-1 + n) + 32
*Sec[c + d*x]*(1 + Sec[c + d*x])^n - 12*n*Sec[c + d*x]*(1 + Sec[c + d*x])^n
 - 12*Sec[(c + d*x)/2]^2*Sec[c + d*x]*(1 + Sec[c + d*x])^n + 2*n*Sec[(c + d
*x)/2]^2*Sec[c + d*x]*(1 + Sec[c + d*x])^n - 2*Sec[(c + d*x)/2]^4*Sec[c + d
*x]*(1 + Sec[c + d*x])^n + 2*n*Sec[(c + d*x)/2]^4*Sec[c + d*x]*(1 + Sec[c +
 d*x])^n))/(d*(-2 + n)*(-1 + n)*(1 + Sec[c + d*x])^n)
```

Maple [**F**]
time = 0.10, size = 0, normalized size = 0.00

$$\int \left(\csc^5(dx+c)\right)(a+a\sec(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5*(a+a*sec(d*x+c))^n,x)
```

```
[Out] int(csc(d*x+c)^5*(a+a*sec(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+a*sec(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*csc(d*x + c)^5, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+a*sec(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n*csc(d*x + c)^5, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5*(a+a*sec(d*x+c))**n,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+a*sec(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*csc(d*x + c)^5, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^n}{\sin\left(c+d\,x\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^n/sin(c + d*x)^5,x)
[Out] int((a + a/cos(c + d*x))^n/sin(c + d*x)^5, x)
```

3.152 $\int (a + a\sec(c + dx))^n \sin^4(c + dx)\,dx$

Optimal. Leaf size=230

$$-\frac{F_1\left(1-n;-\frac{1}{2},\frac{1}{2}-n;2-n;\cos(c+dx),-\cos(c+dx)\right)(1+\cos(c+dx))^{\frac{1}{2}-n}(n-n\cos(c+dx))\cot(c+dx}{d(1-n)\sqrt{1-\cos(c+dx)}}$$

```
[Out] -cos(d*x+c)*(a+a*sec(d*x+c))^n*sin(d*x+c)/d+2^(1/2+n)*AppellF1(1/2,-4+n,1/2
-n,3/2,1-cos(d*x+c),1/2-1/2*cos(d*x+c))*cos(d*x+c)^n*(1+cos(d*x+c))^(-1/2-n
)*(a+a*sec(d*x+c))^n*sin(d*x+c)/d-AppellF1(1-n,1/2-n,-1/2,2-n,-cos(d*x+c),c
os(d*x+c))*(1+cos(d*x+c))^(1/2-n)*(n-n*cos(d*x+c))*cot(d*x+c)*(a+a*sec(d*x+
c))^n/d/(1-n)/(1-cos(d*x+c))^(1/2)
```

Rubi [A]

time = 0.47, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3961, 2960, 2866, 2865, 2864, 138, 3125, 3087, 140}

$$\frac{2^{n+\frac{1}{2}}\sin(c+dx)\cos^n(c+dx)(\cos(c+dx)+1)^{-n-\frac{1}{2}}(a\sec(c+dx)+a)^nF_1\left(\frac{1}{2};n-4,\frac{1}{2}-n;\frac{3}{2};1-\cos(c+dx),\frac{1}{2}(1-\cos(c+dx))\right)}{d}-\frac{\cot(c+dx)(n-n\cos(c+dx))(\cos(c+dx)+1)^{\frac{1}{2}-n}(a\sec(c+dx)+a)^nF_1\left(1-n;-\frac{1}{2},\frac{1}{2}-n;2-n;\cos(c+dx),-\cos(c+dx)\right)}{d(1-n)\sqrt{1-\cos(c+dx)}}-\frac{\sin(c+dx)\cos(c+dx)(a\sec(c+dx)+a)^n}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^4,x]
```

```
[Out] -((AppellF1[1 - n, -1/2, 1/2 - n, 2 - n, Cos[c + d*x], -Cos[c + d*x]]*(1 +
Cos[c + d*x])^(1/2 - n)*(n - n*Cos[c + d*x])*Cot[c + d*x]*(a + a*Sec[c + d*
x])^n)/(d*(1 - n)*Sqrt[1 - Cos[c + d*x]])) - (Cos[c + d*x]*(a + a*Sec[c + d
*x])^n*Sin[c + d*x])/d + (2^(1/2 + n)*AppellF1[1/2, -4 + n, 1/2 - n, 3/2, 1
 - Cos[c + d*x], (1 - Cos[c + d*x])/2]*Cos[c + d*x]^n*(1 + Cos[c + d*x])^(-
1/2 - n)*(a + a*Sec[c + d*x])^n*Sin[c + d*x])/d
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 2864

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(-b)*(d/b)^n*(Cos[e + f*x]/(f*Sqrt[a + b*Sin[e
 + f*x]]*Sqrt[a - b*Sin[e + f*x]])), Subst[Int[(a - x)^n*((2*a - x)^(m - 1/
2)/Sqrt[x]), x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n},
 x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2865

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(d/b)^IntPart[n]*((d*Sin[e + f*x])^FracPart[n
]/(b*Sin[e + f*x])^FracPart[n]), Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x]
)^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !In
tegerQ[m] && GtQ[a, 0] &&  !GtQ[d/b, 0]
```

Rule 2866

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Sin[e + f*x])^FracPart[m
]/(1 + (b/a)*Sin[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Sin[e + f*x])^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 2960

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 3087

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])^(p_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol]
:> Dist[Sqrt[a + b*Sin[e + f*x]]*(Sqrt[c + d*Sin[e + f*x]]/(f*Cos[e + f*x])
), Subst[Int[(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2)*(A + B*x)^p, x], x, Si
n[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n, p}, x] && EqQ[b*c +
 a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3125

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1
)/(d*f*(m + n + 2))), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x
])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1)
```

```
) + C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 3961

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[Sin[e + f*x]^FracPart[m]*((a + b*Csc[e + f*x]
)^FracPart[m]/(b + a*Sin[e + f*x])^FracPart[m]), Int[(g*Cos[e + f*x])^p*((b
 + a*Sin[e + f*x])^m/Sin[e + f*x]^m), x], x] /; FreeQ[{a, b, e, f, g, m, p}
, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

Rubi steps

$$\int (a + a\sec(c+dx))^n \sin^4(c+dx)\,dx = \left((-\cos(c+dx))^n(-a - a\cos(c+dx))^{-n}(a + a\sec(c+dx))^n\right)\int (-$$

$$= \left((-\cos(c+dx))^n(-a - a\cos(c+dx))^{-n}(a + a\sec(c+dx))^n\right)\int (-$$

$$= -\frac{\cos(c+dx)(a + a\sec(c+dx))^n\sin(c+dx)}{d} + \left((-\cos(c+dx))^n(\right.$$

$$= -\frac{\cos(c+dx)(a + a\sec(c+dx))^n\sin(c+dx)}{d} + \left(\cos^n(c+dx)(1+\right.$$

$$= -\frac{\cos(c+dx)(a + a\sec(c+dx))^n\sin(c+dx)}{d} - \frac{\left((-\cos(c+dx))^n(\right.}{}$$

$$= -\frac{\cos(c+dx)(a + a\sec(c+dx))^n\sin(c+dx)}{d} + \frac{2^{\frac{1}{2}+n}F_1\left(\frac{1}{2}; -4+n,\right.}{}$$

$$= -\frac{F_1\left(1-n; -\frac{1}{2}, \frac{1}{2}-n; 2-n; \cos(c+dx), -\cos(c+dx)\right)(1+\cos(c}{d(1-n)\sqrt{1-}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.64, size = 7069, normalized size = 30.73

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^4,x]
[Out] Result too large to show
```

Maple [F]
time = 0.32, size = 0, normalized size = 0.00

$$\int (a + a\sec(dx + c))^n (\sin^4(dx + c))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*sin(d*x+c)^4,x)
[Out] int((a+a*sec(d*x+c))^n*sin(d*x+c)^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^4,x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^4,x, algorithm="fricas")
[Out] integral((cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*(a*sec(d*x + c) + a)^n, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*sin(d*x+c)**4,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^4,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(c+d\,x\right)^4\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4*(a + a/cos(c + d*x))^n,x)
```

```
[Out] int(sin(c + d*x)^4*(a + a/cos(c + d*x))^n, x)
```

3.153 $\int (a + a\sec(c + dx))^n \sin^2(c + dx)\, dx$

Optimal. Leaf size=95

$$-\frac{F_1\left(1-n;-\frac{1}{2},-\frac{1}{2}-n;2-n;\cos(c+dx),-\cos(c+dx)\right)\sqrt{1-\cos(c+dx)}\,(1+\cos(c+dx))^{\frac{1}{2}-n}\cot(c+$$
$$d(1-n)$$

```
[Out] -AppellF1(1-n,-1/2-n,-1/2,2-n,-cos(d*x+c),cos(d*x+c))*(1+cos(d*x+c))^(1/2-n
)*cot(d*x+c)*(a+a*sec(d*x+c))^n*(1-cos(d*x+c))^(1/2)/d/(1-n)
```

Rubi [A]
time = 0.24, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3961, 2953, 3087, 140, 138}

$$-\frac{\sqrt{1-\cos(c+dx)}\cot(c+dx)(\cos(c+dx)+1)^{\frac{1}{2}-n}(a\sec(c+dx)+a)^n F_1\left(1-n;-\frac{1}{2},-n-\frac{1}{2};2-n;\cos(c+dx),-\cos(c+dx)\right)}{d(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^2,x]
```

```
[Out] -((AppellF1[1 - n, -1/2, -1/2 - n, 2 - n, Cos[c + d*x], -Cos[c + d*x]]*Sqrt
[1 - Cos[c + d*x]]*(1 + Cos[c + d*x])^(1/2 - n)*Cot[c + d*x]*(a + a*Sec[c +
 d*x])^n)/(d*(1 - n)))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 2953

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 3087

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])^(p_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol]
:> Dist[Sqrt[a + b*Sin[e + f*x]]*(Sqrt[c + d*Sin[e + f*x]]/(f*Cos[e + f*x])
), Subst[Int[(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2)*(A + B*x)^p, x], x, Si
n[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n, p}, x] && EqQ[b*c +
 a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3961

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[Sin[e + f*x]^FracPart[m]*((a + b*Csc[e + f*x]
)^FracPart[m]/(b + a*Sin[e + f*x])^FracPart[m]), Int[(g*Cos[e + f*x])^p*((b
 + a*Sin[e + f*x])^m/Sin[e + f*x]^m), x], x] /; FreeQ[{a, b, e, f, g, m, p}
, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

Rubi steps

$$\int (a+a\sec(c+dx))^n \sin^2(c+dx)\,dx = \left((-\cos(c+dx))^n(-a-a\cos(c+dx))^{-n}(a+a\sec(c+dx))^n\right)\int (-$$

$$= \frac{\left((-\cos(c+dx))^n(-a-a\cos(c+dx))^{-n}(a+a\sec(c+dx))^n\right)\int (-}{a^2}$$

$$= -\frac{\left((-\cos(c+dx))^n(-a-a\cos(c+dx))^{\frac{1}{2}-n}\sqrt{-a+a\cos(c+dx)}\right.}{}$$

$$= -\frac{\left((-\cos(c+dx))^n(1+\cos(c+dx))^{-\frac{1}{2}-n}(-a-a\cos(c+dx))\sqrt{-\right.}{}$$

$$= -\frac{\left((-\cos(c+dx))^n(1+\cos(c+dx))^{-\frac{1}{2}-n}(-a-a\cos(c+dx))(-a\right.}{}$$

$$= -\frac{F_1\left(1-n;-\frac{1}{2},-\frac{1}{2}-n;2-n;\cos(c+dx),-\cos(c+dx)\right)\sqrt{1-c}}{d(1-}$$

Mathematica [C] Result contains complex when optimal does not.
time = 15.30, size = 4297, normalized size = 45.23

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^2,x]
```

```
[Out] (2^(3 + n)*Cos[(c + d*x)/2]^5*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n*(a*(1 + S
ec[c + d*x]))^n*Sin[(c + d*x)/2]*(Cos[2*(c + d*x)]*(-1/4*(1 + Sec[c + d*x])
^n - ((1 + Sec[c + d*x])^n*Sin[c + d*x]^2)/2 - ((1 + Sec[c + d*x])^n*Sin[c
+ d*x]^4)/4) + (I/4)*(1 + Sec[c + d*x])^n*Sin[2*(c + d*x)] + (I/2)*(1 + Sec
[c + d*x])^n*Sin[c + d*x]^2*Sin[2*(c + d*x)] + (I/4)*(1 + Sec[c + d*x])^n*S
in[c + d*x]^4*Sin[2*(c + d*x)] + Cos[c + d*x]^4*(-1/4*(Cos[2*(c + d*x)]*(1
+ Sec[c + d*x])^n) + (I/4)*(1 + Sec[c + d*x])^n*Sin[2*(c + d*x)]) + Cos[c +
 d*x]^3*((-I)*Cos[2*(c + d*x)]*(1 + Sec[c + d*x])^n*Sin[c + d*x] - (1 + Sec
[c + d*x])^n*Sin[c + d*x]*Sin[2*(c + d*x)]) + Cos[c + d*x]^2*(Cos[2*(c + d*
x)]*((1 + Sec[c + d*x])^n/2 + (3*(1 + Sec[c + d*x])^n*Sin[c + d*x]^2)/2) -
(I/2)*(1 + Sec[c + d*x])^n*Sin[2*(c + d*x)] - ((3*I)/2)*(1 + Sec[c + d*x])^
n*Sin[c + d*x]^2*Sin[2*(c + d*x)]) + Cos[c + d*x]*(Cos[2*(c + d*x)]*(I*(1 +
 Sec[c + d*x])^n*Sin[c + d*x] + I*(1 + Sec[c + d*x])^n*Sin[c + d*x]^3) + (1
 + Sec[c + d*x])^n*Sin[c + d*x]*Sin[2*(c + d*x)] + (1 + Sec[c + d*x])^n*Sin
[c + d*x]^3*Sin[2*(c + d*x)]))*((3*AppellF1[1/2, n, 2, 3/2, Tan[(c + d*x)/2
]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2)/(3*AppellF1[1/2, n, 2, 3/2, T
an[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(-2*AppellF1[3/2, n, 3, 5/2, Ta
n[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + n*AppellF1[3/2, 1 + n, 2, 5/2, Tan
[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2) - AppellF1[1/2,
n, 3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]/(AppellF1[1/2, n, 3, 3/
2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (2*(-3*AppellF1[3/2, n, 4, 5/
2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + n*AppellF1[3/2, 1 + n, 3, 5/2
, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)/3)))/(d*(1
+ Sec[c + d*x])^n*(2^(2 + n)*Cos[(c + d*x)/2]^6*(Cos[(c + d*x)/2]^2*Sec[c +
 d*x])^n*((3*AppellF1[1/2, n, 2, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]
^2]*Sec[(c + d*x)/2]^2)/(3*AppellF1[1/2, n, 2, 3/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2] + 2*(-2*AppellF1[3/2, n, 3, 5/2, Tan[(c + d*x)/2]^2, -Tan
[(c + d*x)/2]^2] + n*AppellF1[3/2, 1 + n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[
(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2) - AppellF1[1/2, n, 3, 3/2, Tan[(c + d*
x)/2]^2, -Tan[(c + d*x)/2]^2]/(AppellF1[1/2, n, 3, 3/2, Tan[(c + d*x)/2]^2,
 -Tan[(c + d*x)/2]^2] + (2*(-3*AppellF1[3/2, n, 4, 5/2, Tan[(c + d*x)/2]^2,
 -Tan[(c + d*x)/2]^2] + n*AppellF1[3/2, 1 + n, 3, 5/2, Tan[(c + d*x)/2]^2,
-Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)/3)) - 5*2^(2 + n)*Cos[(c + d*x)/2
]^4*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n*Sin[(c + d*x)/2]^2*((3*AppellF1[1/2
, n, 2, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2)/(
3*AppellF1[1/2, n, 2, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(-2
*AppellF1[3/2, n, 3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + n*Appe
llF1[3/2, 1 + n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c +
 d*x)/2]^2) - AppellF1[1/2, n, 3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2]/(AppellF1[1/2, n, 3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (
2*(-3*AppellF1[3/2, n, 4, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + n
*AppellF1[3/2, 1 + n, 3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan
[(c + d*x)/2]^2)/3)) + 2^(3 + n)*Cos[(c + d*x)/2]^5*(Cos[(c + d*x)/2]^2*Sec
[c + d*x])^n*Sin[(c + d*x)/2]*((3*AppellF1[1/2, n, 2, 3/2, Tan[(c + d*x)/2]
^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(3*AppellF1[1
```

```
/2, n, 2, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(-2*AppellF1[3/
2, n, 3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + n*AppellF1[3/2, 1
+ n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)
+ (3*Sec[(c + d*x)/2]^2*((-2*AppellF1[3/2, n, 3, 5/2, Tan[(c + d*x)/2]^2, -
Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/3 + (n*AppellF1[3/
2, 1 + n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]
^2*Tan[(c + d*x)/2])/3))/(3*AppellF1[1/2, n, 2, 3/2, Tan[(c + d*x)/2]^2, -T
an[(c + d*x)/2]^2] + 2*(-2*AppellF1[3/2, n, 3, 5/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2] + n*AppellF1[3/2, 1 + n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan
[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2) - (-(AppellF1[3/2, n, 4, 5/2, Tan[(c
+ d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]) + (n
*AppellF1[3/2, 1 + n, 3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[
(c + d*x)/2]^2*Tan[(c + d*x)/2])/3)/(AppellF1[1/2, n, 3, 3/2, Tan[(c + d*x)
/2]^2, -Tan[(c + d*x)/2]^2] + (2*(-3*AppellF1[3/2, n, 4, 5/2, Tan[(c + d*x)
/2]^2, -Tan[(c + d*x)/2]^2] + n*AppellF1[3/2, 1 + n, 3, 5/2, Tan[(c + d*x)/
2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)/3) - (3*AppellF1[1/2, n, 2,
 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*(2*(-2*Ap
pellF1[3/2, n, 3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + n*AppellF
1[3/2, 1 + n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Sec[(c + d*
x)/2]^2*Tan[(c + d*x)/2] + 3*((-2*AppellF1[3/2, n, 3, 5/2, Tan[(c + d*x)/2]
^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/3 + (n*Appell
F1[3/2, 1 + n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan...
```

Maple [F]
time = 0.25, size = 0, normalized size = 0.00

$$\int (a + a\sec(dx + c))^n \left(\sin^2(dx + c)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*sin(d*x+c)^2,x)
[Out] int((a+a*sec(d*x+c))^n*sin(d*x+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^2,x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^2,x, algorithm="fricas")
[Out] integral(-(cos(d*x + c)^2 - 1)*(a*sec(d*x + c) + a)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(c+dx)+1))^n \sin^2(c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*sin(d*x+c)**2,x)
[Out] Integral((a*(sec(c + d*x) + 1))**n*sin(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^2,x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(c+d\,x)^2\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2*(a + a/cos(c + d*x))^n,x)
[Out] int(sin(c + d*x)^2*(a + a/cos(c + d*x))^n, x)
```

3.154 $\int \csc^2(c+dx)(a+a\sec(c+dx))^n\,dx$

Optimal. Leaf size=98

$$-\frac{\cot(c+dx)(a+a\sec(c+dx))^n}{d}+\frac{2^{-\frac{1}{2}+n}n\,{}_2F_1\left(\frac{1}{2},\frac{3}{2}-n;\frac{3}{2};\frac{1}{2}(1-\sec(c+dx))\right)(1+\sec(c+dx))^{-\frac{1}{2}-n}(a+a}{d}$$

```
[Out] -cot(d*x+c)*(a+a*sec(d*x+c))^n/d+2^(-1/2+n)*n*hypergeom([1/2, 3/2-n],[3/2],
1/2-1/2*sec(d*x+c))*(1+sec(d*x+c))^(-1/2-n)*(a+a*sec(d*x+c))^n*tan(d*x+c)/d
```

Rubi [A]
time = 0.10, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3960, 3913, 3912, 71}

$$\frac{2^{n-\frac{1}{2}}n\tan(c+dx)(\sec(c+dx)+1)^{-n-\frac{1}{2}}(a\sec(c+dx)+a)^n\,{}_2F_1\left(\frac{1}{2},\frac{3}{2}-n;\frac{3}{2};\frac{1}{2}(1-\sec(c+dx))\right)}{d}-\frac{\cot(c+dx)(a\sec(c+dx)+a)^n}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] -((Cot[c + d*x]*(a + a*Sec[c + d*x])^n)/d) + (2^(-1/2 + n)*n*Hypergeometric
2F1[1/2, 3/2 - n, 3/2, (1 - Sec[c + d*x])/2]*(1 + Sec[c + d*x])^(-1/2 - n)*
(a + a*Sec[c + d*x])^n*Tan[c + d*x])/d
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
```

```
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)/cos[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[Tan[e + f*x]*((a + b*Csc[e + f*x])^m/f), x] + Dist[b*m, I
nt[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
m}, x]
```

Rubi steps

$$\int \csc^2(c+dx)(a+a\sec(c+dx))^n\,dx = -\frac{\cot(c+dx)(a+a\sec(c+dx))^n}{d} + (an)\int \sec(c+dx)(a+a\sec($$
$$= -\frac{\cot(c+dx)(a+a\sec(c+dx))^n}{d} + \left(n(1+\sec(c+dx))^{-n}(a+a\right.$$
$$= -\frac{\cot(c+dx)(a+a\sec(c+dx))^n}{d} - \frac{\left(n(1+\sec(c+dx))^{-\frac{1}{2}-n}(a+\right.}{}$$
$$= -\frac{\cot(c+dx)(a+a\sec(c+dx))^n}{d} + \frac{2^{-\frac{1}{2}+n}n\,{}_2F_1\left(\frac{1}{2},\frac{3}{2}-n;\frac{3}{2};\frac{1}{2}(1-\right.}{}$$

Mathematica [A]
time = 0.80, size = 142, normalized size = 1.45

$$-\frac{2^{-1+n}\left(\cot^2\left(\frac{1}{2}(c+dx)\right)\,{}_2F_1\left(-\frac{1}{2},n;\frac{1}{2};\tan^2\left(\frac{1}{2}(c+dx)\right)\right)-{}_2F_1\left(\frac{1}{2},n;\frac{3}{2};\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\right)\left(\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\right)^n\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^n(1+\sec(c+dx))^{-n}(a(1+\sec(c+dx)))^n\tan\left(\frac{1}{2}(c+dx)\right)}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]^2*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] -((2^(-1 + n)*(Cot[(c + d*x)/2]^2*Hypergeometric2F1[-1/2, n, 1/2, Tan[(c +
d*x)/2]^2] - Hypergeometric2F1[1/2, n, 3/2, Tan[(c + d*x)/2]^2])*(Cos[c + d
*x]*Sec[(c + d*x)/2]^2)^n*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n*(a*(1 + Sec[c
 + d*x]))^n*Tan[(c + d*x)/2])/(d*(1 + Sec[c + d*x])^n))
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \left(\csc^2(dx+c)\right)(a+a\sec(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+a*sec(d*x+c))^n,x)
```

```
[Out] int(csc(d*x+c)^2*(a+a*sec(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sec(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sec(d*x+c))^n,x, algorithm="fricas")
[Out] integral((a*sec(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(c + dx) + 1))^n \csc^2(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+a*sec(d*x+c))**n,x)
[Out] Integral((a*(sec(c + d*x) + 1))**n*csc(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sec(d*x+c))^n,x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + \frac{a}{\cos(c + d\,x)}\right)^n}{\sin(c + d\,x)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^n/sin(c + d*x)^2,x)
[Out] int((a + a/cos(c + d*x))^n/sin(c + d*x)^2, x)
```

3.155 $\int \csc^4(c+dx)(a+a\sec(c+dx))^n\,dx$

Optimal. Leaf size=349

$$\frac{(2-n+n^2)\cos(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d(3-2n)\left(1-4n^2\right)(1-\cos(c+dx))^2}-\frac{a^4\cos(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d(3-2n)(a-a\cos(c+dx))^2(a+a\cos(c+dx)}$$

```
[Out] (n^2-n+2)*cos(d*x+c)*(a+a*sec(d*x+c))^n*sin(d*x+c)/d/(3-2*n)/(-4*n^2+1)/(1-
cos(d*x+c))^2-a^4*cos(d*x+c)*(a+a*sec(d*x+c))^n*sin(d*x+c)/d/(3-2*n)/(a-a*c
os(d*x+c))^2/(a+a*cos(d*x+c))^2-a^3*(4-n)*cos(d*x+c)*(a+a*sec(d*x+c))^n*sin
(d*x+c)/d/(4*n^2-8*n+3)/(a-a*cos(d*x+c))^2/(a+a*cos(d*x+c))+n*(-n^2-3*n+7)*
cos(d*x+c)*((1+cos(d*x+c))/(1-cos(d*x+c)))^(-1/2-n)*hypergeom([1-n, -1/2-n]
,[2-n],-2*cos(d*x+c)/(1-cos(d*x+c)))*(a+a*sec(d*x+c))^n*sin(d*x+c)/d/(-8*n^
4+20*n^3-10*n^2-5*n+3)/(1-cos(d*x+c))^2
```

Rubi [A]
time = 0.39, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3961, 2962, 136, 160, 12, 134}

$$-\frac{a^4\sin(c+dx)\cos(c+dx)(a\sec(c+dx)+a)^n}{d(3-2n)(a-a\cos(c+dx))^2(a\cos(c+dx)+a)^2}-\frac{a^3(4-n)\sin(c+dx)\cos(c+dx)(a\sec(c+dx)+a)^n}{d\left(4n^2-8n+3\right)(a-a\cos(c+dx))^2(a\cos(c+dx)+a)}+\frac{n(-n^2-3n+7)\sin(c+dx)\cos(c+dx)\left(\frac{\cos(c+dx)+1}{1-\cos(c+dx)}\right)^{-n-\frac{1}{2}}(a\sec(c+dx)+a)^n\,{}_2F_1\left(-n-\frac{1}{2},1-n;2-n;-\frac{2\cos(c+dx)}{1-\cos(c+dx)}\right)}{d(1-2n)(3-2n)(1-n)(2n+1)(1-\cos(c+dx))^2}+\frac{(n^2-n+2)\sin(c+dx)\cos(c+dx)(a\sec(c+dx)+a)^n}{d(3-2n)\left(1-4n^2\right)(1-\cos(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] ((2 - n + n^2)*Cos[c + d*x]*(a + a*Sec[c + d*x])^n*Sin[c + d*x])/(d*(3 - 2*
n)*(1 - 4*n^2)*(1 - Cos[c + d*x])^2) - (a^4*Cos[c + d*x]*(a + a*Sec[c + d*x
])^n*Sin[c + d*x])/(d*(3 - 2*n)*(a - a*Cos[c + d*x])^2*(a + a*Cos[c + d*x])
^2) - (a^3*(4 - n)*Cos[c + d*x]*(a + a*Sec[c + d*x])^n*Sin[c + d*x])/(d*(3
- 8*n + 4*n^2)*(a - a*Cos[c + d*x])^2*(a + a*Cos[c + d*x])) + (n*(7 - 3*n -
 n^2)*Cos[c + d*x]*((1 + Cos[c + d*x])/(1 - Cos[c + d*x]))^(-1/2 - n)*Hyper
geometric2F1[-1/2 - n, 1 - n, 2 - n, (-2*Cos[c + d*x])/(1 - Cos[c + d*x])]*
(a + a*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 - 2*n)*(3 - 2*n)*(1 - n)*(1 + 2*
n)*(1 - Cos[c + d*x])^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
```

```
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] && !IntegerQ[n]
```

Rule 136

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && ILtQ[m + n + p + 2,
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] && !SumSimp
lerQ[p, 1]))
```

Rule 160

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) && !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 2962

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[Cos[e + f*x]/(a^(
p - 2)*f*Sqrt[a + b*Sin[e + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[(d*x
)^n*(a + b*x)^(m + p/2 - 1/2)*(a - b*x)^(p/2 - 1/2), x], x, Sin[e + f*x]],
x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[p/2]
 && !IntegerQ[m]
```

Rule 3961

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[Sin[e + f*x]^FracPart[m]*((a + b*Csc[e + f*x]
)^FracPart[m]/(b + a*Sin[e + f*x])^FracPart[m]), Int[(g*Cos[e + f*x])^p*((b
 + a*Sin[e + f*x])^m/Sin[e + f*x]^m), x], x] /; FreeQ[{a, b, e, f, g, m, p}
, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

Rubi steps

$$\int \csc^4(c+dx)(a+a\sec(c+dx))^n\,dx = ((-\cos(c+dx))^n(-a-a\cos(c+dx))^{-n}(a+a\sec(c+dx))^n)\int($$

$$= -\frac{\left(a^6(-\cos(c+dx))^n(-a-a\cos(c+dx))^{-\frac{1}{2}-n}(a+a\sec(c+dx)\right.}{d\sqrt{-a+a\,\mathrm{c}}}$$

$$= -\frac{a^4\cos(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d(3-2n)(a-a\cos(c+dx))^2(a+a\cos(c+dx))^2} - \frac{\left(a^3(-\cos(\right.}{}$$

$$= -\frac{a^4\cos(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d(3-2n)(a-a\cos(c+dx))^2(a+a\cos(c+dx))^2} - \frac{a^3(4-n}{d(1-2n)(}$$

$$= \frac{(2-n+n^2)\cos(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d(1-2n)(3-2n)(1+2n)(1-\cos(c+dx))^2} - \frac{a^4\,\mathrm{co}}{d(3-}$$

$$= \frac{(2-n+n^2)\cos(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d(1-2n)(3-2n)(1+2n)(1-\cos(c+dx))^2} - \frac{a^4\,\mathrm{co}}{d(3-}$$

$$= \frac{(2-n+n^2)\cos(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d(1-2n)(3-2n)(1+2n)(1-\cos(c+dx))^2} - \frac{a^4\,\mathrm{co}}{d(3-}$$

Mathematica [**A**]
time = 4.26, size = 350, normalized size = 1.00

$$\frac{(a(1+\sec(c+dx)))^n\left(-2\cot^2\left(\frac{1}{2}(c+dx)\right)\,{}_2F_1\left(-\frac{1}{2},n;\frac{1}{2};\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\right)^n(1+\sec(c+dx))^{-n}\left(3\,2^n\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^n+2(1+\sec(c+dx))^n+n(1+\sec(c+dx))^n\right)+\frac{-\cos(c+dx)(4n\cos(c+dx)+(-3+n)(3+\cos(2(c+dx))))\csc^4\left(\frac{1}{2}(c+dx)\right)\sec^2\left(\frac{1}{2}(c+dx)\right)+24\,{}_2F_1\left(\frac{1}{2},n;\frac{3}{2};\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\right)^n(1+\sec(c+dx))^{-n}\left(-3\,2^n\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^n-2(1+\sec(c+dx))^n+n\left(2^{1+n}\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^n+(1+\sec(c+dx))^n\right)\right)}{4(-3+2n)}\right)\tan\left(\frac{1}{2}(c+dx)\right)}{24d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]^4*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] ((a*(1 + Sec[c + d*x]))^n*((-2*Cot[(c + d*x)/2]^2*Hypergeometric2F1[-1/2, n
, 1/2, Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n*(3*2^n*(Cos[
(c + d*x)/2]^2*Sec[c + d*x])^n + 2*(1 + Sec[c + d*x])^n + n*(1 + Sec[c + d*
x])^n))/(1 + Sec[c + d*x])^n + (-(Cos[c + d*x]*(4*n*Cos[c + d*x] + (-3 + n)
*(3 + Cos[2*(c + d*x)]))*Csc[(c + d*x)/2]^4*Sec[(c + d*x)/2]^2) + (24*Hyper
geometric2F1[1/2, n, 3/2, Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2
]^2)^n*(-3*2^n*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n - 2*(1 + Sec[c + d*x])^n
 + n*(2^(1 + n)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n + (1 + Sec[c + d*x])^n)
))/(1 + Sec[c + d*x])^n)/(4*(-3 + 2*n)))*Tan[(c + d*x)/2])/(24*d)
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \left(\csc^{4}\left(dx+c\right)\right)\left(a+a\sec\left(dx+c\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(a+a*sec(d*x+c))^n,x)
```

```
[Out] int(csc(d*x+c)^4*(a+a*sec(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sec(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*csc(d*x + c)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sec(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n*csc(d*x + c)^4, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(a+a*sec(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sec(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*csc(d*x + c)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^n}{\sin\left(c+d\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^n/sin(c + d*x)^4,x)
```

```
[Out] int((a + a/cos(c + d*x))^n/sin(c + d*x)^4, x)
```

3.156 $\int (a+a\sec(c+dx))^n \sin^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=105

$$-\frac{F_1\left(1-n;-\frac{1}{4},-\frac{1}{4}-n;2-n;\cos(c+dx),-\cos(c+dx)\right)\cos(c+dx)(1+\cos(c+dx))^{-\frac{1}{4}-n}(a+a\sec(c+dx))^n\sqrt{\sin(c+dx)}}{d(1-n)\sqrt[4]{1-\cos(c+dx)}}$$

```
[Out] -AppellF1(1-n,-1/4-n,-1/4,2-n,-cos(d*x+c),cos(d*x+c))*cos(d*x+c)*(1+cos(d*x
+c))^(-1/4-n)*(a+a*sec(d*x+c))^n*sin(d*x+c)^(1/2)/d/(1-n)/(1-cos(d*x+c))^(1
/4)
```

Rubi [A]
time = 0.17, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3961, 2965, 140, 138}

$$-\frac{\sqrt{\sin(c+dx)}\cos(c+dx)(\cos(c+dx)+1)^{-n-\frac{1}{4}}(a\sec(c+dx)+a)^n F_1\left(1-n;-\frac{1}{4},-n-\frac{1}{4};2-n;\cos(c+dx),-\cos(c+dx)\right)}{d(1-n)\sqrt[4]{1-\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^(3/2),x]
```

```
[Out] -((AppellF1[1 - n, -1/4, -1/4 - n, 2 - n, Cos[c + d*x], -Cos[c + d*x]]*Cos[
c + d*x]*(1 + Cos[c + d*x])^(-1/4 - n)*(a + a*Sec[c + d*x])^n*Sqrt[Sin[c +
d*x]])/(d*(1 - n)*(1 - Cos[c + d*x])^(1/4)))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 2965

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[g*((g*Cos
[e + f*x])^(p - 1)/(f*(a + b*Sin[e + f*x])^((p - 1)/2)*(a - b*Sin[e + f*x])
^((p - 1)/2))), Subst[Int[(d*x)^n*(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p -
```

```
 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 3961

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[Sin[e + f*x]^FracPart[m]*((a + b*Csc[e + f*x]
)^FracPart[m]/(b + a*Sin[e + f*x])^FracPart[m]), Int[(g*Cos[e + f*x])^p*((b
 + a*Sin[e + f*x])^m/Sin[e + f*x]^m), x], x] /; FreeQ[{a, b, e, f, g, m, p}
, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

Rubi steps

$$\int (a+a\sec(c+dx))^n \sin^{\frac{3}{2}}(c+dx)\,dx = \left((-\cos(c+dx))^n(-a-a\cos(c+dx))^{-n}(a+a\sec(c+dx))^n\right)\int($$

$$= -\frac{\left((-\cos(c+dx))^n(-a-a\cos(c+dx))^{-\frac{1}{4}-n}(a+a\sec(c+dx))^{n}\right.}{d\sqrt[4]{-}}$$

$$= -\frac{\left((-\cos(c+dx))^n(1+\cos(c+dx))^{-\frac{1}{4}-n}(a+a\sec(c+dx))^n\sqrt{s}\right.}{d\sqrt[4]{-a-}}$$

$$= -\frac{\left((-\cos(c+dx))^n(1+\cos(c+dx))^{-\frac{1}{4}-n}(a+a\sec(c+dx))^n\sqrt{s}\right.}{d\sqrt[4]{1-}}$$

$$= -\frac{F_1\left(1-n;-\frac{1}{4},-\frac{1}{4}-n;2-n;\cos(c+dx),-\cos(c+dx)\right)\cos(c-}{d(1-n)\sqrt[4]{1-}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 382 vs. 2(105) = 210.
time = 11.20, size = 382, normalized size = 3.64

$$\frac{10\left(F_1\left(\frac{1}{4};n,\frac{3}{2};\frac{5}{4};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)-F_1\left(\frac{1}{4};n,\frac{5}{2};\frac{5}{4};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\right)(1+\cos(c+dx))(a(1+\sec(c+dx)))^n\sin^{\frac{5}{2}}(c+dx)}{d\left(2\left(3F_1\left(\frac{5}{4};n,\frac{5}{2};\frac{9}{4};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)-5F_1\left(\frac{5}{4};n,\frac{7}{2};\frac{9}{4};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)-2nF_1\left(\frac{5}{4};1+n,\frac{3}{2};\frac{9}{4};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)+2nF_1\left(\frac{5}{4};1+n,\frac{5}{2};\frac{9}{4};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\right)(-1+\cos(c+dx))+5F_1\left(\frac{1}{4};n,\frac{3}{2};\frac{5}{4};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)(1+\cos(c+dx))-5F_1\left(\frac{1}{4};n,\frac{5}{2};\frac{5}{4};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)(1+\cos(c+dx))\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^(3/2),x]
```

```
[Out] (10*(AppellF1[1/4, n, 3/2, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] -
AppellF1[1/4, n, 5/2, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(1 + C
os[c + d*x])*(a*(1 + Sec[c + d*x]))^n*Sin[c + d*x]^(5/2))/(d*(2*(3*AppellF1
[5/4, n, 5/2, 9/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 5*AppellF1[5/
4, n, 7/2, 9/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*n*AppellF1[5/4
, 1 + n, 3/2, 9/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*n*AppellF1[
```

```
5/4, 1 + n, 5/2, 9/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(-1 + Cos[c
 + d*x]) + 5*AppellF1[1/4, n, 3/2, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/
2]^2]*(1 + Cos[c + d*x]) - 5*AppellF1[1/4, n, 5/2, 5/4, Tan[(c + d*x)/2]^2,
 -Tan[(c + d*x)/2]^2]*(1 + Cos[c + d*x])))
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \left(a+a\sec \left(dx+c\right)\right)^{n} \left(\sin^{\frac{3}{2}}\left(dx+c\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*sin(d*x+c)^(3/2),x)
[Out] int((a+a*sec(d*x+c))^n*sin(d*x+c)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^(3/2),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^(3/2),x, algorithm="fricas")
[Out] integral((a*sec(d*x + c) + a)^n*sin(d*x + c)^(3/2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*sin(d*x+c)**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6437 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^(3/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(c+d\,x\right)^{3/2}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^(3/2)*(a + a/cos(c + d*x))^n,x)
[Out] int(sin(c + d*x)^(3/2)*(a + a/cos(c + d*x))^n, x)
```

3.157 $\int (a + a\sec(c + dx))^n \sqrt{\sin(c + dx)}\, dx$

Optimal. Leaf size=105

$$-\frac{F_1\left(1-n;\frac{1}{4},\frac{1}{4}-n;2-n;\cos(c+dx),-\cos(c+dx)\right)\sqrt[4]{1-\cos(c+dx)}\cos(c+dx)(1+\cos(c+dx))^{\frac{1}{4}-n}(}{d(1-n)\sqrt{\sin(c+dx)}}$$

```
[Out] -AppellF1(1-n,1/4-n,1/4,2-n,-cos(d*x+c),cos(d*x+c))*(1-cos(d*x+c))^(1/4)*co
s(d*x+c)*(1+cos(d*x+c))^(1/4-n)*(a+a*sec(d*x+c))^n/d/(1-n)/sin(d*x+c)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3961, 2965, 140, 138}

$$-\frac{\sqrt[4]{1-\cos(c+dx)}\cos(c+dx)(\cos(c+dx)+1)^{\frac{1}{4}-n}(a\sec(c+dx)+a)^n F_1\left(1-n;\frac{1}{4},\frac{1}{4}-n;2-n;\cos(c+dx),-\cos(c+dx)\right)}{d(1-n)\sqrt{\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n*Sqrt[Sin[c + d*x]],x]
```

```
[Out] -((AppellF1[1 - n, 1/4, 1/4 - n, 2 - n, Cos[c + d*x], -Cos[c + d*x]]*(1 - C
os[c + d*x])^(1/4)*Cos[c + d*x]*(1 + Cos[c + d*x])^(1/4 - n)*(a + a*Sec[c +
 d*x])^n)/(d*(1 - n)*Sqrt[Sin[c + d*x]]))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 2965

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[g*((g*Cos
[e + f*x])^(p - 1)/(f*(a + b*Sin[e + f*x])^((p - 1)/2)*(a - b*Sin[e + f*x])
^((p - 1)/2))), Subst[Int[(d*x)^n*(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p -
 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && E
```

```
qQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 3961

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[Sin[e + f*x]^FracPart[m]*((a + b*Csc[e + f*x]
)^FracPart[m]/(b + a*Sin[e + f*x])^FracPart[m]), Int[(g*Cos[e + f*x])^p*((b
 + a*Sin[e + f*x])^m/Sin[e + f*x]^m), x], x] /; FreeQ[{a, b, e, f, g, m, p}
, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

Rubi steps

$$\int (a+a\sec(c+dx))^n\sqrt{\sin(c+dx)}\,dx = \left((-\cos(c+dx))^n(-a-a\cos(c+dx))^{-n}(a+a\sec(c+dx))^n\right)\int$$

$$= -\frac{\left((-\cos(c+dx))^n(-a-a\cos(c+dx))^{\frac{1}{4}-n}\sqrt[4]{-a+a\cos(c+d}\right.}{d\sqrt{}}$$

$$= -\frac{\left((-\cos(c+dx))^n(1+\cos(c+dx))^{\frac{1}{4}-n}\sqrt[4]{-a+a\cos(c+dx)}\right.}{d\sqrt{\sin}}$$

$$= -\frac{\left(\sqrt[4]{1-\cos(c+dx)}\,(-\cos(c+dx))^n(1+\cos(c+dx))^{\frac{1}{4}-n}(a+\right.}{d\sqrt{\sin(}}$$

$$= -\frac{F_1\left(1-n;\frac{1}{4},\frac{1}{4}-n;2-n;\cos(c+dx),-\cos(c+dx)\right)\sqrt[4]{1-\mathrm{co}}}{d(1-n)\sqrt{}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 214 vs. 2(105) = 210.
time = 1.98, size = 214, normalized size = 2.04

$$\frac{14F_1\left(\frac{3}{4};n,\frac{3}{2};\frac{7}{4};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)(1+\cos(c+dx))(a(1+\sec(c+dx)))^n\sin^{\frac{3}{2}}(c+dx)}{d\left(6\left(3F_1\left(\frac{7}{4};n,\frac{5}{2};\frac{11}{4};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)-2nF_1\left(\frac{7}{4};1+n,\frac{3}{2};\frac{11}{4};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\right)(-1+\cos(c+dx))+21F_1\left(\frac{3}{4};n,\frac{3}{2};\frac{7}{4};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)(1+\cos(c+dx))\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Sqrt[Sin[c + d*x]],x]
```

```
[Out] (14*AppellF1[3/4, n, 3/2, 7/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1
+ Cos[c + d*x])*(a*(1 + Sec[c + d*x]))^n*Sin[c + d*x]^(3/2))/(d*(6*(3*Appel
lF1[7/4, n, 5/2, 11/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*n*Appel
lF1[7/4, 1 + n, 3/2, 11/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(-1 +
Cos[c + d*x]) + 21*AppellF1[3/4, n, 3/2, 7/4, Tan[(c + d*x)/2]^2, -Tan[(c +
 d*x)/2]^2]*(1 + Cos[c + d*x])))
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \left(a+a\sec\left(dx+c\right)\right)^{n}\left(\sqrt{\sin}\left(dx+c\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*sin(d*x+c)^(1/2),x)
```

```
[Out] int((a+a*sec(d*x+c))^n*sin(d*x+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*sqrt(sin(d*x + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n*sqrt(sin(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sec\left(c+dx\right)+1\right)\right)^{n}\sqrt{\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*sin(d*x+c)**(1/2),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**n*sqrt(sin(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*sqrt(sin(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\sin\left(c+d\,x\right)}\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^(1/2)*(a + a/cos(c + d*x))^n,x)
```

```
[Out] int(sin(c + d*x)^(1/2)*(a + a/cos(c + d*x))^n, x)
```

3.158 $\int \frac{(a+a\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}\,dx$

Optimal. Leaf size=105

$$-\frac{F_1\left(1-n;\frac{3}{4},\frac{3}{4}-n;2-n;\cos(c+dx),-\cos(c+dx)\right)(1-\cos(c+dx))^{3/4}\cos(c+dx)(1+\cos(c+dx))^{\frac{3}{4}-n}}{d(1-n)\sin^{\frac{3}{2}}(c+dx)}$$

```
[Out] -AppellF1(1-n,3/4-n,3/4,2-n,-cos(d*x+c),cos(d*x+c))*(1-cos(d*x+c))^(3/4)*co
s(d*x+c)*(1+cos(d*x+c))^(3/4-n)*(a+a*sec(d*x+c))^n/d/(1-n)/sin(d*x+c)^(3/2)
```

Rubi [A]

time = 0.16, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3961, 2965, 140, 138}

$$-\frac{(1-\cos(c+dx))^{3/4}\cos(c+dx)(\cos(c+dx)+1)^{\frac{3}{4}-n}(a\sec(c+dx)+a)^n F_1\left(1-n;\frac{3}{4},\frac{3}{4}-n;2-n;\cos(c+dx),-\cos(c+dx)\right)}{d(1-n)\sin^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n/Sqrt[Sin[c + d*x]],x]
```

```
[Out] -((AppellF1[1 - n, 3/4, 3/4 - n, 2 - n, Cos[c + d*x], -Cos[c + d*x]]*(1 - C
os[c + d*x])^(3/4)*Cos[c + d*x]*(1 + Cos[c + d*x])^(3/4 - n)*(a + a*Sec[c +
 d*x])^n)/(d*(1 - n)*Sin[c + d*x]^(3/2)))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 2965

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[g*((g*Cos
[e + f*x])^(p - 1)/(f*(a + b*Sin[e + f*x])^((p - 1)/2)*(a - b*Sin[e + f*x])
^((p - 1)/2))), Subst[Int[(d*x)^n*(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p -
```

```
 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 3961

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[Sin[e + f*x]^FracPart[m]*((a + b*Csc[e + f*x]
)^FracPart[m]/(b + a*Sin[e + f*x])^FracPart[m]), Int[(g*Cos[e + f*x])^p*((b
 + a*Sin[e + f*x])^m/Sin[e + f*x]^m), x], x] /; FreeQ[{a, b, e, f, g, m, p}
, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}\,dx &= \left((-\cos(c+dx))^n(-a-a\cos(c+dx))^{-n}(a+a\sec(c+dx))^n\right)\int \frac{(-\cos(c+
\\
&= -\frac{\left((-\cos(c+dx))^n(-a-a\cos(c+dx))^{\frac{3}{4}-n}(-a+a\cos(c+dx))^{3/4}(a+a\sec
}{d\sin^{\frac{3}{2}}(c+dx)}
\\
&= -\frac{\left((-\cos(c+dx))^n(1+\cos(c+dx))^{\frac{3}{4}-n}(-a+a\cos(c+dx))^{3/4}(a+a\sec(c
}{d\sin^{\frac{3}{2}}(c+dx)}
\\
&= -\frac{\left((1-\cos(c+dx))^{3/4}(-\cos(c+dx))^n(1+\cos(c+dx))^{\frac{3}{4}-n}(a+a\sec(c+dx
}{d\sin^{\frac{3}{2}}(c+dx)}
\\
&= -\frac{F_1\left(1-n;\frac{3}{4},\frac{3}{4}-n;2-n;\cos(c+dx),-\cos(c+dx)\right)(1-\cos(c+dx))^{3/4}\cos
}{d(1-n)\sin^{\frac{3}{2}}(c+dx)}
\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 212 vs. 2(105) = 210.
time = 2.12, size = 212, normalized size = 2.02

$$\frac{10F_1\left(\frac{1}{4};n,\frac{1}{2};\frac{5}{4};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)(1+\cos(c+dx))(a(1+\sec(c+dx)))^n\sqrt{\sin(c+dx)}}{d\left(2\left(F_1\left(\frac{5}{4};n,\frac{3}{2};\frac{9}{4};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)-2nF_1\left(\frac{5}{4};1+n,\frac{1}{2};\frac{9}{4};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\right)(-1+\cos(c+dx))+5F_1\left(\frac{1}{4};n,\frac{1}{2};\frac{5}{4};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)(1+\cos(c+dx))\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^n/Sqrt[Sin[c + d*x]],x]
```

```
[Out] (10*AppellF1[1/4, n, 1/2, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1
+ Cos[c + d*x])*(a*(1 + Sec[c + d*x]))^n*Sqrt[Sin[c + d*x]])/(d*(2*(AppellF
1[5/4, n, 3/2, 9/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*n*AppellF1
[5/4, 1 + n, 1/2, 9/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(-1 + Cos[
```

```
c + d*x]) + 5*AppellF1[1/4, n, 1/2, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)
/2]^2]*(1 + Cos[c + d*x])))
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(a+a\sec\left(dx+c\right)\right)^{n}}{\sqrt{\sin\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n/sin(d*x+c)^(1/2),x)
[Out] int((a+a*sec(d*x+c))^n/sin(d*x+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n/sin(d*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n/sqrt(sin(d*x + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n/sin(d*x+c)^(1/2),x, algorithm="fricas")
[Out] integral((a*sec(d*x + c) + a)^n/sqrt(sin(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{n}}{\sqrt{\sin\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n/sin(d*x+c)**(1/2),x)
[Out] Integral((a*(sec(c + d*x) + 1))**n/sqrt(sin(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n/sin(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n/sqrt(sin(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^n}{\sqrt{\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^n/sin(c + d*x)^(1/2),x)
[Out] int((a + a/cos(c + d*x))^n/sin(c + d*x)^(1/2), x)
```

3.159 $\int \frac{(a+a\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=105

$$-\frac{F_1\left(1-n;\frac{5}{4},\frac{5}{4}-n;2-n;\cos(c+dx),-\cos(c+dx)\right)(1-\cos(c+dx))^{5/4}\cos(c+dx)(1+\cos(c+dx))^{\frac{5}{4}-n}}{d(1-n)\sin^{\frac{5}{2}}(c+dx)}$$

```
[Out] -AppellF1(1-n,5/4-n,5/4,2-n,-cos(d*x+c),cos(d*x+c))*(1-cos(d*x+c))^(5/4)*co
s(d*x+c)*(1+cos(d*x+c))^(5/4-n)*(a+a*sec(d*x+c))^n/d/(1-n)/sin(d*x+c)^(5/2)
```

Rubi [A]
time = 0.18, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3961, 2965, 140, 138}

$$-\frac{(1-\cos(c+dx))^{5/4}\cos(c+dx)(\cos(c+dx)+1)^{\frac{5}{4}-n}(a\sec(c+dx)+a)^n F_1\left(1-n;\frac{5}{4},\frac{5}{4}-n;2-n;\cos(c+dx),-\cos(c+dx)\right)}{d(1-n)\sin^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n/Sin[c + d*x]^(3/2),x]
```

```
[Out] -((AppellF1[1 - n, 5/4, 5/4 - n, 2 - n, Cos[c + d*x], -Cos[c + d*x]]*(1 - C
os[c + d*x])^(5/4)*Cos[c + d*x]*(1 + Cos[c + d*x])^(5/4 - n)*(a + a*Sec[c +
 d*x])^n)/(d*(1 - n)*Sin[c + d*x]^(5/2)))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 2965

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[g*((g*Cos
[e + f*x])^(p - 1)/(f*(a + b*Sin[e + f*x])^((p - 1)/2)*(a - b*Sin[e + f*x])
^((p - 1)/2))), Subst[Int[(d*x)^n*(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p -
 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && E
```

```
qQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 3961

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[Sin[e + f*x]^FracPart[m]*((a + b*Csc[e + f*x]
)^FracPart[m]/(b + a*Sin[e + f*x])^FracPart[m]), Int[(g*Cos[e + f*x])^p*((b
 + a*Sin[e + f*x])^m/Sin[e + f*x]^m), x], x] /; FreeQ[{a, b, e, f, g, m, p}
, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx = \left((-\cos(c+dx))^n(-a-a\cos(c+dx))^{-n}(a+a\sec(c+dx))^n\right)\int \frac{(-\cos(c+ \ldots}{\ldots}$$

$$= -\frac{\left((-\cos(c+dx))^n(-a-a\cos(c+dx))^{\frac{5}{4}-n}(-a+a\cos(c+dx))^{5/4}(a+a\sec \ldots\right.}{d\sin^{\frac{5}{2}}(c+dx) \ldots}$$

$$= -\left(-\frac{\left((-\cos(c+dx))^n(1+\cos(c+dx))^{\frac{1}{4}-n}(-a-a\cos(c+dx))(-a+a\cos \ldots\right.}{ad\,\mathrm{si}\ldots}\right.$$

$$= -\frac{\left(\sqrt[4]{1-\cos(c+dx)}\,(-\cos(c+dx))^n(1+\cos(c+dx))^{\frac{1}{4}-n}(-a-a\cos(c+ \ldots\right.}{\ldots}$$

$$= -\frac{F_1\left(1-n;\frac{5}{4},\frac{5}{4}-n;2-n;\cos(c+dx),-\cos(c+dx)\right)(1-\cos(c+dx))^{5/4}\,\mathrm{co}\ldots}{d(1-n)\sin^{\frac{5}{2}}(c+dx)}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 212 vs. 2(105) = 210.
time = 2.57, size = 212, normalized size = 2.02

$$-\frac{6F_1\left(-\frac{1}{4};n,-\frac{1}{2};\frac{3}{4};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)(1+\cos(c+dx))(a(1+\sec(c+dx)))^n}{d\left(-2\left(F_1\left(\frac{3}{4};n,\frac{1}{2};\frac{7}{4};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)+2nF_1\left(\frac{3}{4};1+n,-\frac{1}{2};\frac{7}{4};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\right)(-1+\cos(c+dx))+3F_1\left(-\frac{1}{4};n,-\frac{1}{2};\frac{3}{4};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)(1+\cos(c+dx))\right)\sqrt{\sin(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^n/Sin[c + d*x]^(3/2),x]
```

```
[Out] (-6*AppellF1[-1/4, n, -1/2, 3/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(
1 + Cos[c + d*x])*(a*(1 + Sec[c + d*x]))^n)/(d*(-2*(AppellF1[3/4, n, 1/2, 7
/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*n*AppellF1[3/4, 1 + n, -1/
2, 7/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(-1 + Cos[c + d*x]) + 3*A
```

```
ppellF1[-1/4, n, -1/2, 3/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 + C
os[c + d*x]))*Sqrt[Sin[c + d*x]])
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(a + a \sec (dx + c))^{n}}{\sin (dx + c)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n/sin(d*x+c)^(3/2),x)
[Out] int((a+a*sec(d*x+c))^n/sin(d*x+c)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n/sin(d*x+c)^(3/2),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n/sin(d*x + c)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n/sin(d*x+c)^(3/2),x, algorithm="fricas")
[Out] integral(-(a*sec(d*x + c) + a)^n*sqrt(sin(d*x + c))/(cos(d*x + c)^2 - 1), x
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sec (c + dx) + 1))^{n}}{\sin^{\frac{3}{2}} (c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n/sin(d*x+c)**(3/2),x)
[Out] Integral((a*(sec(c + d*x) + 1))**n/sin(c + d*x)**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n/sin(d*x+c)^(3/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n/sin(d*x + c)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^n}{\sin\left(c+d\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^n/sin(c + d*x)^(3/2),x)
[Out] int((a + a/cos(c + d*x))^n/sin(c + d*x)^(3/2), x)
```

3.160 $\int (a+b\sec(c+dx))\sin^7(c+dx)\,dx$

Optimal. Leaf size=119

$$-\frac{a\cos(c+dx)}{d}+\frac{3b\cos^2(c+dx)}{2d}+\frac{a\cos^3(c+dx)}{d}-\frac{3b\cos^4(c+dx)}{4d}-\frac{3a\cos^5(c+dx)}{5d}+\frac{b\cos^6(c+dx)}{6d}+\frac{a\cos^7(}{7}$$

```
[Out] -a*cos(d*x+c)/d+3/2*b*cos(d*x+c)^2/d+a*cos(d*x+c)^3/d-3/4*b*cos(d*x+c)^4/d-
3/5*a*cos(d*x+c)^5/d+1/6*b*cos(d*x+c)^6/d+1/7*a*cos(d*x+c)^7/d-b*ln(cos(d*x
+c))/d
```

Rubi [A]
time = 0.08, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3957, 2916, 12, 780}

$$\frac{a\cos^7(c+dx)}{7d}-\frac{3a\cos^5(c+dx)}{5d}+\frac{a\cos^3(c+dx)}{d}-\frac{a\cos(c+dx)}{d}+\frac{b\cos^6(c+dx)}{6d}-\frac{3b\cos^4(c+dx)}{4d}+\frac{3b\cos^2(c+dx)}{2d}-\frac{b\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])*Sin[c + d*x]^7,x]
```

```
[Out] -((a*Cos[c + d*x])/d) + (3*b*Cos[c + d*x]^2)/(2*d) + (a*Cos[c + d*x]^3)/d -
 (3*b*Cos[c + d*x]^4)/(4*d) - (3*a*Cos[c + d*x]^5)/(5*d) + (b*Cos[c + d*x]^
6)/(6*d) + (a*Cos[c + d*x]^7)/(7*d) - (b*Log[Cos[c + d*x]])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 780

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
```

```
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+b\sec(c+dx))\sin^7(c+dx)\,dx = -\int (-b-a\cos(c+dx))\sin^6(c+dx)\tan(c+dx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{a(-b+x)(a^2-x^2)^3}{x}\,dx, x, -a\cos(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(-b+x)(a^2-x^2)^3}{x}\,dx, x, -a\cos(c+dx)\right)}{a^6 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^6 - \frac{a^6 b}{x} + 3a^4bx - 3a^4x^2 - 3a^2bx^3 + 3a^2x^4 + bx^5 - x^6\right)\right.}{a^6 d}$$
$$= -\frac{a\cos(c+dx)}{d} + \frac{3b\cos^2(c+dx)}{2d} + \frac{a\cos^3(c+dx)}{d} - \frac{3b\cos^4(c+d}{4d}$$

Mathematica [A]
time = 0.10, size = 115, normalized size = 0.97

$$-\frac{35a\cos(c+dx)}{64d}+\frac{7a\cos(3(c+dx))}{64d}-\frac{7a\cos(5(c+dx))}{320d}+\frac{a\cos(7(c+dx))}{448d}-\frac{b\left(-3\cos^2(c+dx)+\frac{3}{2}\cos^4(c+dx)-\frac{1}{3}\cos^6(c+dx)+2\log(\cos(c+dx))\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])*Sin[c + d*x]^7,x]
[Out] (-35*a*Cos[c + d*x])/(64*d) + (7*a*Cos[3*(c + d*x)])/(64*d) - (7*a*Cos[5*(c
 + d*x)])/(320*d) + (a*Cos[7*(c + d*x)])/(448*d) - (b*(-3*Cos[c + d*x]^2 +
(3*Cos[c + d*x]^4)/2 - Cos[c + d*x]^6/3 + 2*Log[Cos[c + d*x]]))/(2*d)
```

Maple [A]
time = 0.12, size = 87, normalized size = 0.73

method	result
derivativedivides	$\frac{b\left(-\frac{\left(\sin^6(dx+c)\right)}{6}-\frac{\left(\sin^4(dx+c)\right)}{4}-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)-\frac{a\left(\frac{16}{5}+\sin^6(dx+c)+\frac{6\left(\sin^4(dx+c)\right)}{5}+\frac{8\left(\sin^2(dx+c)\right)}{5}\right)\text{co}}{7}}{d}$
default	$\frac{b\left(-\frac{\left(\sin^6(dx+c)\right)}{6}-\frac{\left(\sin^4(dx+c)\right)}{4}-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)-\frac{a\left(\frac{16}{5}+\sin^6(dx+c)+\frac{6\left(\sin^4(dx+c)\right)}{5}+\frac{8\left(\sin^2(dx+c)\right)}{5}\right)\text{co}}{7}}{d}$
risch	$ibx+\frac{2ibc}{d}+\frac{29b\,\mathrm{e}^{2i(dx+c)}}{128d}+\frac{29b\,\mathrm{e}^{-2i(dx+c)}}{128d}-\frac{b\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{d}-\frac{35a\cos(dx+c)}{64d}+\frac{a\cos(7dx+7c)}{448d}+\frac{b\cos}{}$

norman	$\frac{-\frac{32a}{35d}-\frac{128b\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{2b\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{14b\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{(96a+70b)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}-\frac{(96a+128b)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*sin(d*x+c)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b*(-1/6*sin(d*x+c)^6-1/4*sin(d*x+c)^4-1/2*sin(d*x+c)^2-ln(cos(d*x+c)))
-1/7*a*(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c))
```

Maxima [A]
time = 0.27, size = 91, normalized size = 0.76

$$\frac{60\,a\cos\left(dx+c\right)^7+70\,b\cos\left(dx+c\right)^6-252\,a\cos\left(dx+c\right)^5-315\,b\cos\left(dx+c\right)^4+420\,a\cos\left(dx+c\right)^3+630\,b\cos\left(dx+c\right)^2-420\,a\cos\left(dx+c\right)-420\,b\log\left(\cos\left(dx+c\right)\right)}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/420*(60*a*cos(d*x + c)^7 + 70*b*cos(d*x + c)^6 - 252*a*cos(d*x + c)^5 - 3
15*b*cos(d*x + c)^4 + 420*a*cos(d*x + c)^3 + 630*b*cos(d*x + c)^2 - 420*a*c
os(d*x + c) - 420*b*log(cos(d*x + c)))/d
```

Fricas [A]
time = 3.23, size = 93, normalized size = 0.78

$$\frac{60\,a\cos\left(dx+c\right)^7+70\,b\cos\left(dx+c\right)^6-252\,a\cos\left(dx+c\right)^5-315\,b\cos\left(dx+c\right)^4+420\,a\cos\left(dx+c\right)^3+630\,b\cos\left(dx+c\right)^2-420\,a\cos\left(dx+c\right)-420\,b\log\left(-\cos\left(dx+c\right)\right)}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/420*(60*a*cos(d*x + c)^7 + 70*b*cos(d*x + c)^6 - 252*a*cos(d*x + c)^5 - 3
15*b*cos(d*x + c)^4 + 420*a*cos(d*x + c)^3 + 630*b*cos(d*x + c)^2 - 420*a*c
os(d*x + c) - 420*b*log(-cos(d*x + c)))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)**7,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 317 vs. 2(109) = 218.
time = 0.46, size = 317, normalized size = 2.66

$$\frac{420\,b\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-420\,b\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{384\,a+1089\,b-\frac{2688\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{8463\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{8064\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{28749\,b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{13440\,a(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}-\frac{56035\,b(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{56035\,b(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}-\frac{28749\,b(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5}+\frac{8463\,b(\cos(dx+c)-1)^6}{(\cos(dx+c)+1)^6}-\frac{1089\,b(\cos(dx+c)-1)^7}{(\cos(dx+c)+1)^7}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^7}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/420*(420*b*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 420*b*l
og(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (384*a + 1089*b - 268
8*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 8463*b*(cos(d*x + c) - 1)/(cos(
d*x + c) + 1) + 8064*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 28749*b*
(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 13440*a*(cos(d*x + c) - 1)^3/(c
os(d*x + c) + 1)^3 - 56035*b*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 56
035*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 28749*b*(cos(d*x + c) - 1
)^5/(cos(d*x + c) + 1)^5 + 8463*b*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6
 - 1089*b*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7)/((cos(d*x + c) - 1)/(c
os(d*x + c) + 1) - 1)^7)/d
```

Mupad [B]
time = 0.93, size = 89, normalized size = 0.75

$$-\frac{a\,\cos\left(c+d\,x\right)-a\,{\cos\left(c+d\,x\right)}^3+\frac{3\,a\,{\cos\left(c+d\,x\right)}^5}{5}-\frac{a\,{\cos\left(c+d\,x\right)}^7}{7}-\frac{3\,b\,{\cos\left(c+d\,x\right)}^2}{2}+\frac{3\,b\,{\cos\left(c+d\,x\right)}^4}{4}-\frac{b\,{\cos\left(c+d\,x\right)}^6}{6}+b\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^7*(a + b/cos(c + d*x)),x)
```

```
[Out] -(a*cos(c + d*x) - a*cos(c + d*x)^3 + (3*a*cos(c + d*x)^5)/5 - (a*cos(c + d
*x)^7)/7 - (3*b*cos(c + d*x)^2)/2 + (3*b*cos(c + d*x)^4)/4 - (b*cos(c + d*x
)^6)/6 + b*log(cos(c + d*x)))/d
```

3.161 $\int (a+b\sec(c+dx))\sin^5(c+dx)\,dx$

Optimal. Leaf size=87

$$-\frac{a\cos(c+dx)}{d}+\frac{b\cos^2(c+dx)}{d}+\frac{2a\cos^3(c+dx)}{3d}-\frac{b\cos^4(c+dx)}{4d}-\frac{a\cos^5(c+dx)}{5d}-\frac{b\log(\cos(c+dx))}{d}$$

```
[Out] -a*cos(d*x+c)/d+b*cos(d*x+c)^2/d+2/3*a*cos(d*x+c)^3/d-1/4*b*cos(d*x+c)^4/d-
1/5*a*cos(d*x+c)^5/d-b*ln(cos(d*x+c))/d
```

Rubi [A]
time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3957, 2916, 12, 780}

$$-\frac{a\cos^5(c+dx)}{5d}+\frac{2a\cos^3(c+dx)}{3d}-\frac{a\cos(c+dx)}{d}-\frac{b\cos^4(c+dx)}{4d}+\frac{b\cos^2(c+dx)}{d}-\frac{b\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])*Sin[c + d*x]^5,x]
```

```
[Out] -((a*Cos[c + d*x])/d) + (b*Cos[c + d*x]^2)/d + (2*a*Cos[c + d*x]^3)/(3*d) -
 (b*Cos[c + d*x]^4)/(4*d) - (a*Cos[c + d*x]^5)/(5*d) - (b*Log[Cos[c + d*x]]
)/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 780

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
```

```
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+b\sec(c+dx))\sin^5(c+dx)\,dx = -\int (-b-a\cos(c+dx))\sin^4(c+dx)\tan(c+dx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{a(-b+x)(a^2-x^2)^2}{x}\,dx, x, -a\cos(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int \frac{(-b+x)(a^2-x^2)^2}{x}\,dx, x, -a\cos(c+dx)\right)}{a^4d}$$
$$= \frac{\text{Subst}\left(\int \left(a^4-\frac{a^4b}{x}+2a^2bx-2a^2x^2-bx^3+x^4\right)\,dx, x, -a\cos(c+\right.}{a^4d}$$
$$= -\frac{a\cos(c+dx)}{d}+\frac{b\cos^2(c+dx)}{d}+\frac{2a\cos^3(c+dx)}{3d}-\frac{b\cos^4(c+dx}{4d}$$

Mathematica [A]
time = 0.06, size = 83, normalized size = 0.95

$$-\frac{5a\cos(c+dx)}{8d}+\frac{5a\cos(3(c+dx))}{48d}-\frac{a\cos(5(c+dx))}{80d}-\frac{b\left(-\cos^2(c+dx)+\frac{1}{4}\cos^4(c+dx)+\log(\cos(c+dx))\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])*Sin[c + d*x]^5,x]
```

```
[Out] (-5*a*Cos[c + d*x])/(8*d) + (5*a*Cos[3*(c + d*x)])/(48*d) - (a*Cos[5*(c + d
*x)])/(80*d) - (b*(-Cos[c + d*x]^2 + Cos[c + d*x]^4/4 + Log[Cos[c + d*x]]))
/d
```

Maple [A]
time = 0.09, size = 67, normalized size = 0.77

method	result
derivativedivides	$\frac{b\left(-\frac{\left(\sin^4(dx+c)\right)}{4}-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)-\frac{a\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)}{5}}{d}$
default	$\frac{b\left(-\frac{\left(\sin^4(dx+c)\right)}{4}-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)-\frac{a\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)}{5}}{d}$
risch	$ibx+\frac{3b\,e^{2i(dx+c)}}{16d}+\frac{3b\,e^{-2i(dx+c)}}{16d}+\frac{2ibc}{d}-\frac{b\ln\left(e^{2i(dx+c)}+1\right)}{d}-\frac{5a\cos(dx+c)}{8d}-\frac{a\cos(5dx+5c)}{80d}-\frac{b\cos(4d}{32}$

norman	$\frac{-\frac{16a}{15d}-\frac{2b\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{10b\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{(16a+6b)\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{2(16a+15b)\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}}+\frac{b\ln\left(1+\tan^{2}\left(\frac{dx}{2}+\right.\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*sin(d*x+c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b*(-1/4*sin(d*x+c)^4-1/2*sin(d*x+c)^2-ln(cos(d*x+c)))-1/5*a*(8/3+sin(d
*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))
```

Maxima [A]
time = 0.27, size = 69, normalized size = 0.79

$$-\frac{12\,a\cos\left(dx+c\right)^{5}+15\,b\cos\left(dx+c\right)^{4}-40\,a\cos\left(dx+c\right)^{3}-60\,b\cos\left(dx+c\right)^{2}+60\,a\cos\left(dx+c\right)+60\,b\log\left(\cos\left(dx+c\right)\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/60*(12*a*cos(d*x + c)^5 + 15*b*cos(d*x + c)^4 - 40*a*cos(d*x + c)^3 - 60
*b*cos(d*x + c)^2 + 60*a*cos(d*x + c) + 60*b*log(cos(d*x + c)))/d
```

Fricas [A]
time = 3.55, size = 71, normalized size = 0.82

$$-\frac{12\,a\cos\left(dx+c\right)^{5}+15\,b\cos\left(dx+c\right)^{4}-40\,a\cos\left(dx+c\right)^{3}-60\,b\cos\left(dx+c\right)^{2}+60\,a\cos\left(dx+c\right)+60\,b\log\left(-\cos\left(dx+c\right)\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/60*(12*a*cos(d*x + c)^5 + 15*b*cos(d*x + c)^4 - 40*a*cos(d*x + c)^3 - 60
*b*cos(d*x + c)^2 + 60*a*cos(d*x + c) + 60*b*log(-cos(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec\left(c+dx\right)\right)\sin^{5}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)**5,x)
```

```
[Out] Integral((a + b*sec(c + d*x))*sin(c + d*x)**5, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 248 vs. 2(81) = 162.
time = 0.49, size = 248, normalized size = 2.85

$$\frac{60\,b\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-60\,b\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{64\,a+137\,b-\frac{320\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{805\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{640\,a(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{1970\,b(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{1970\,b(\cos(dx+c)-1)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{805\,b(\cos(dx+c)-1)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{137\,b(\cos(dx+c)-1)^{5}}{(\cos(dx+c)+1)^{5}}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^{5}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/60*(60*b*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 60*b*log(
abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (64*a + 137*b - 320*a*(c
os(d*x + c) - 1)/(cos(d*x + c) + 1) - 805*b*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1) + 640*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 1970*b*(cos(d*x
+ c) - 1)^2/(cos(d*x + c) + 1)^2 - 1970*b*(cos(d*x + c) - 1)^3/(cos(d*x + c
) + 1)^3 + 805*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 137*b*(cos(d*x
 + c) - 1)^5/(cos(d*x + c) + 1)^5)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) -
 1)^5)/d
```

Mupad [B]
time = 0.91, size = 67, normalized size = 0.77

$$-\frac{a\,\cos\left(c+d\,x\right)-\frac{2\,a\,{\cos\left(c+d\,x\right)}^3}{3}+\frac{a\,{\cos\left(c+d\,x\right)}^5}{5}-b\,{\cos\left(c+d\,x\right)}^2+\frac{b\,{\cos\left(c+d\,x\right)}^4}{4}+b\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5*(a + b/cos(c + d*x)),x)
```

```
[Out] -(a*cos(c + d*x) - (2*a*cos(c + d*x)^3)/3 + (a*cos(c + d*x)^5)/5 - b*cos(c
+ d*x)^2 + (b*cos(c + d*x)^4)/4 + b*log(cos(c + d*x)))/d
```

3.162 $\int (a+b\sec(c+dx))\sin^3(c+dx)\,dx$

Optimal. Leaf size=58

$$-\frac{a\cos(c+dx)}{d}+\frac{b\cos^2(c+dx)}{2d}+\frac{a\cos^3(c+dx)}{3d}-\frac{b\log(\cos(c+dx))}{d}$$

```
[Out] -a*cos(d*x+c)/d+1/2*b*cos(d*x+c)^2/d+1/3*a*cos(d*x+c)^3/d-b*ln(cos(d*x+c))/
d
```

Rubi [A]
time = 0.06, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3957, 2916, 12, 780}

$$\frac{a\cos^3(c+dx)}{3d}-\frac{a\cos(c+dx)}{d}+\frac{b\cos^2(c+dx)}{2d}-\frac{b\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])*Sin[c + d*x]^3,x]
```

```
[Out] -((a*Cos[c + d*x])/d) + (b*Cos[c + d*x]^2)/(2*d) + (a*Cos[c + d*x]^3)/(3*d)
 - (b*Log[Cos[c + d*x]])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 780

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
```

```
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+b\sec(c+dx))\sin^3(c+dx)\,dx = -\int (-b-a\cos(c+dx))\sin^2(c+dx)\tan(c+dx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{a(-b+x)(a^2-x^2)}{x}\,dx, x, -a\cos(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \frac{(-b+x)(a^2-x^2)}{x}\,dx, x, -a\cos(c+dx)\right)}{a^2d}$$
$$= \frac{\text{Subst}\left(\int \left(a^2-\frac{a^2b}{x}+bx-x^2\right)\,dx, x, -a\cos(c+dx)\right)}{a^2d}$$
$$= -\frac{a\cos(c+dx)}{d}+\frac{b\cos^2(c+dx)}{2d}+\frac{a\cos^3(c+dx)}{3d}-\frac{b\log(\cos(c+d}{d}$$

Mathematica [A]
time = 0.04, size = 57, normalized size = 0.98

$$-\frac{3a\cos(c+dx)}{4d}+\frac{a\cos(3(c+dx))}{12d}-\frac{b\left(-\frac{1}{2}\cos^2(c+dx)+\log(\cos(c+dx))\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])*Sin[c + d*x]^3,x]
[Out] (-3*a*Cos[c + d*x])/(4*d) + (a*Cos[3*(c + d*x)])/(12*d) - (b*(-1/2*Cos[c +
d*x]^2 + Log[Cos[c + d*x]]))/d
```

Maple [A]
time = 0.07, size = 47, normalized size = 0.81

method	result
derivativedivides	$\frac{b\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)-\frac{a\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}}{d}$
default	$\frac{b\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)-\frac{a\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}}{d}$
risch	$ibx+\frac{b\,e^{2i(dx+c)}}{8d}+\frac{b\,e^{-2i(dx+c)}}{8d}+\frac{2ibc}{d}-\frac{b\ln\left(e^{2i(dx+c)}+1\right)}{d}-\frac{3a\cos(dx+c)}{4d}+\frac{a\cos(3dx+3c)}{12d}$

norman	$\frac{-\frac{4a}{3d}-\frac{2b\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{(4a+2b)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{b\ln\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\right.\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*sin(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b*(-1/2*sin(d*x+c)^2-ln(cos(d*x+c)))-1/3*a*(2+sin(d*x+c)^2)*cos(d*x+c)
)
```

Maxima [A]
time = 0.27, size = 47, normalized size = 0.81

$$\frac{2\,a\cos\left(dx+c\right)^3+3\,b\cos\left(dx+c\right)^2-6\,a\cos\left(dx+c\right)-6\,b\log\left(\cos\left(dx+c\right)\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(2*a*cos(d*x + c)^3 + 3*b*cos(d*x + c)^2 - 6*a*cos(d*x + c) - 6*b*log(c
os(d*x + c)))/d
```

Fricas [A]
time = 3.66, size = 49, normalized size = 0.84

$$\frac{2\,a\cos\left(dx+c\right)^3+3\,b\cos\left(dx+c\right)^2-6\,a\cos\left(dx+c\right)-6\,b\log\left(-\cos\left(dx+c\right)\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*cos(d*x + c)^3 + 3*b*cos(d*x + c)^2 - 6*a*cos(d*x + c) - 6*b*log(-
cos(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec\left(c+dx\right)\right)\sin^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)**3,x)
```

```
[Out] Integral((a + b*sec(c + d*x))*sin(c + d*x)**3, x)
```

Giac [A]
time = 0.46, size = 66, normalized size = 1.14

$$-\frac{b\log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d}+\frac{2\,ad^2\cos\left(dx+c\right)^3+3\,bd^2\cos\left(dx+c\right)^2-6\,ad^2\cos\left(dx+c\right)}{6\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -b*log(abs(cos(d*x + c))/abs(d))/d + 1/6*(2*a*d^2*cos(d*x + c)^3 + 3*b*d^2*
cos(d*x + c)^2 - 6*a*d^2*cos(d*x + c))/d^3
```

Mupad [B]
time = 0.06, size = 45, normalized size = 0.78

$$-\frac{a\,\cos\left(c+d\,x\right)-\frac{a\cos(c+d\,x)^3}{3}-\frac{b\cos(c+d\,x)^2}{2}+b\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3*(a + b/cos(c + d*x)),x)
```

```
[Out] -(a*cos(c + d*x) - (a*cos(c + d*x)^3)/3 - (b*cos(c + d*x)^2)/2 + b*log(cos(
c + d*x)))/d
```

3.163 $\int (a+b\sec(c+dx))\sin(c+dx)\,dx$

Optimal. Leaf size=26

$$-\frac{a\cos(c+dx)}{d}-\frac{b\log(\cos(c+dx))}{d}$$

```
[Out] -a*cos(d*x+c)/d-b*ln(cos(d*x+c))/d
```

Rubi [A]
time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.176$, Rules used = {3957, 2800, 45}

$$-\frac{a\cos(c+dx)}{d}-\frac{b\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])*Sin[c + d*x],x]
```

```
[Out] -((a*Cos[c + d*x])/d) - (b*Log[Cos[c + d*x]])/d
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+b\sec(c+dx))\sin(c+dx)\,dx = -\int (-b-a\cos(c+dx))\tan(c+dx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{-b+x}{x}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(1-\frac{b}{x}\right)\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= -\frac{a\cos(c+dx)}{d} - \frac{b\log(\cos(c+dx))}{d}$$

Mathematica [A]
time = 0.02, size = 37, normalized size = 1.42

$$-\frac{a\cos(c)\cos(dx)}{d} - \frac{b\log(\cos(c+dx))}{d} + \frac{a\sin(c)\sin(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])*Sin[c + d*x],x]
[Out] -((a*Cos[c]*Cos[d*x])/d) - (b*Log[Cos[c + d*x]])/d + (a*Sin[c]*Sin[d*x])/d
```

Maple [A]
time = 0.05, size = 26, normalized size = 1.00

method	result	size
derivativedivides	$\frac{-\frac{a}{\sec(dx+c)}+b\ln(\sec(dx+c))}{d}$	26
default	$\frac{-\frac{a}{\sec(dx+c)}+b\ln(\sec(dx+c))}{d}$	26
risch	$ibx+\frac{2ibc}{d}-\frac{b\ln(e^{2i(dx+c)}+1)}{d}-\frac{a\cos(dx+c)}{d}$	45
norman	$\frac{2a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{b\ln\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*sin(d*x+c),x,method=_RETURNVERBOSE)
[Out] 1/d*(-a/sec(d*x+c)+b*ln(sec(d*x+c)))
```

Maxima [A]
time = 0.27, size = 23, normalized size = 0.88

$$-\frac{a\cos\left(dx+c\right)+b\log\left(\cos\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -(a*cos(d*x + c) + b*log(cos(d*x + c)))/d
```

Fricas [A]
time = 4.82, size = 25, normalized size = 0.96

$$-\frac{a\cos(dx+c)+b\log(-\cos(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -(a*cos(d*x + c) + b*log(-cos(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int(a+b\sec(c+dx))\sin(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*sin(c + d*x), x)
```

Giac [A]
time = 0.44, size = 32, normalized size = 1.23

$$-\frac{a\cos(dx+c)}{d}-\frac{b\log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -a*cos(d*x + c)/d - b*log(abs(cos(d*x + c))/abs(d))/d
```

Mupad [B]
time = 0.04, size = 23, normalized size = 0.88

$$-\frac{a\,\cos(c+d\,x)+b\,\ln(\cos(c+d\,x))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a + b/cos(c + d*x)),x)
```

```
[Out] -(a*cos(c + d*x) + b*log(cos(c + d*x)))/d
```

3.164 $\int \csc(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=26

$$-\frac{a\tanh^{-1}(\cos(c+dx))}{d}+\frac{b\log(\tan(c+dx))}{d}$$

```
[Out] -a*arctanh(cos(d*x+c))/d+b*ln(tan(d*x+c))/d
```

Rubi [A]
time = 0.05, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3957, 2913, 2700, 29, 3855}

$$\frac{b\log(\tan(c+dx))}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + b*Sec[c + d*x]),x]
```

```
[Out] -((a*ArcTanh[Cos[c + d*x]])/d) + (b*Log[Tan[c + d*x]])/d
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2913

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \csc(c+dx)(a+b\sec(c+dx))\,dx &= -\int (-b-a\cos(c+dx))\csc(c+dx)\sec(c+dx)\,dx \\
&= a\int \csc(c+dx)\,dx + b\int \csc(c+dx)\sec(c+dx)\,dx \\
&= -\frac{a\tanh^{-1}(\cos(c+dx))}{d} + \frac{b\text{Subst}\left(\int \frac{1}{x}\,dx, x, \tan(c+dx)\right)}{d} \\
&= -\frac{a\tanh^{-1}(\cos(c+dx))}{d} + \frac{b\log(\tan(c+dx))}{d}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 63 vs. 2(26) = 52.
time = 0.02, size = 63, normalized size = 2.42

$$-\frac{a\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{a\log\left(\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{b(\log(\cos(c+dx))-\log(\sin(c+dx)))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*(a + b*Sec[c + d*x]),x]
[Out] -((a*Log[Cos[c/2 + (d*x)/2]])/d) + (a*Log[Sin[c/2 + (d*x)/2]])/d - (b*(Log[
Cos[c + d*x]] - Log[Sin[c + d*x]]))/d
```

Maple [A]
time = 0.07, size = 33, normalized size = 1.27

method	result	size
derivativedivides	$\frac{b\ln(\tan(dx+c))+a\ln(\csc(dx+c)-\cot(dx+c))}{d}$	33
default	$\frac{b\ln(\tan(dx+c))+a\ln(\csc(dx+c)-\cot(dx+c))}{d}$	33
norman	$\frac{(a+b)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}$	55
risch	$\frac{a\ln(e^{i(dx+c)}-1)}{d}+\frac{\ln(e^{i(dx+c)}-1)b}{d}-\frac{a\ln(e^{i(dx+c)}+1)}{d}+\frac{\ln(e^{i(dx+c)}+1)b}{d}-\frac{b\ln(e^{2i(dx+c)}+1)}{d}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b*ln(tan(d*x+c))+a*ln(csc(d*x+c)-cot(d*x+c)))
```

Maxima [A]
time = 0.28, size = 45, normalized size = 1.73

$$-\frac{(a-b)\log\left(\cos\left(dx+c\right)+1\right)-(a+b)\log\left(\cos\left(dx+c\right)-1\right)+2\,b\log\left(\cos\left(dx+c\right)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*((a - b)*log(cos(d*x + c) + 1) - (a + b)*log(cos(d*x + c) - 1) + 2*b*l
og(cos(d*x + c)))/d
```

Fricas [A]
time = 3.33, size = 51, normalized size = 1.96

$$-\frac{2\,b\log\left(-\cos\left(dx+c\right)\right)+(a-b)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-(a+b)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*log(-cos(d*x + c)) + (a - b)*log(1/2*cos(d*x + c) + 1/2) - (a + b
)*log(-1/2*cos(d*x + c) + 1/2))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec\left(c+dx\right)\right)\csc\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*csc(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(26) = 52.
time = 0.45, size = 61, normalized size = 2.35

$$\frac{(a+b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-2\,b\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*((a + b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 2*b*log(ab
s(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)))/d
```

Mupad [B]
time = 0.11, size = 63, normalized size = 2.42

$$\frac{\frac{a\ln(\cos(c+dx)-1)}{2}-b\ln\left(\cos\left(c+dx\right)\right)-\frac{a\ln(\cos(c+dx)+1)}{2}+\frac{b\ln(\cos(c+dx)-1)}{2}+\frac{b\ln(\cos(c+dx)+1)}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))/sin(c + d*x),x)
```

```
[Out] ((a*log(cos(c + d*x) - 1))/2 - b*log(cos(c + d*x)) - (a*log(cos(c + d*x) +
1))/2 + (b*log(cos(c + d*x) - 1))/2 + (b*log(cos(c + d*x) + 1))/2)/d
```

3.165 $\int \csc^3(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=64

$$-\frac{a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{b\cot^2(c+dx)}{2d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d}+\frac{b\log(\tan(c+dx))}{d}$$

```
[Out] -1/2*a*arctanh(cos(d*x+c))/d-1/2*b*cot(d*x+c)^2/d-1/2*a*cot(d*x+c)*csc(d*x+
c)/d+b*ln(tan(d*x+c))/d
```

Rubi [A]
time = 0.08, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3957, 2913, 2700, 14, 3853, 3855}

$$-\frac{a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d}-\frac{b\cot^2(c+dx)}{2d}+\frac{b\log(\tan(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*(a + b*Sec[c + d*x]),x]
```

```
[Out] -1/2*(a*ArcTanh[Cos[c + d*x]])/d - (b*Cot[c + d*x]^2)/(2*d) - (a*Cot[c + d*
x]*Csc[c + d*x])/(2*d) + (b*Log[Tan[c + d*x]])/d
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2913

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc^3(c+dx)(a+b\sec(c+dx))\,dx = -\int (-b-a\cos(c+dx))\csc^3(c+dx)\sec(c+dx)\,dx$$
$$= a\int \csc^3(c+dx)\,dx + b\int \csc^3(c+dx)\sec(c+dx)\,dx$$
$$= -\frac{a\cot(c+dx)\csc(c+dx)}{2d} + \frac{1}{2}a\int \csc(c+dx)\,dx + \frac{b\text{Subst}\left(\int \frac{1+x^2}{x^3}\right.}{}$$
$$= -\frac{a\tanh^{-1}(\cos(c+dx))}{2d} - \frac{a\cot(c+dx)\csc(c+dx)}{2d} + \frac{b\text{Subst}\left(\int \left(\frac{1}{x^3}\right.\right.}{}$$
$$= -\frac{a\tanh^{-1}(\cos(c+dx))}{2d} - \frac{b\cot^2(c+dx)}{2d} - \frac{a\cot(c+dx)\csc(c+dx}{2d}$$

Mathematica [A]
time = 0.32, size = 114, normalized size = 1.78

$$-\frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d} + \frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d} - \frac{b(\csc^2(c+dx)+2\log(\cos(c+dx))-2\log(\sin(c+dx)))}{2d} + \frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*(a + b*Sec[c + d*x]),x]
[Out] -1/8*(a*Csc[(c + d*x)/2]^2)/d - (a*Log[Cos[(c + d*x)/2]])/(2*d) + (a*Log[Si
n[(c + d*x)/2]])/(2*d) - (b*(Csc[c + d*x]^2 + 2*Log[Cos[c + d*x]] - 2*Log[S
in[c + d*x]]))/(2*d) + (a*Sec[(c + d*x)/2]^2)/(8*d)
```

Maple [A]
time = 0.14, size = 61, normalized size = 0.95

method	result
derivativedivides	$\frac{b\left(-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+a\left(-\frac{\csc(dx+c)\cot(dx+c)}{2}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)}{d}$
default	$\frac{b\left(-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+a\left(-\frac{\csc(dx+c)\cot(dx+c)}{2}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)}{d}$
norman	$\frac{-\frac{a+b}{8d}+\frac{(a-b)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}+\frac{(a+2b)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}$
risch	$\frac{a\,e^{3i(dx+c)}+2b\,e^{2i(dx+c)}+e^{i(dx+c)}a}{d\left(e^{2i(dx+c)}-1\right)^2}-\frac{a\ln\left(e^{i(dx+c)}+1\right)}{2d}+\frac{\ln\left(e^{i(dx+c)}+1\right)b}{d}+\frac{a\ln\left(e^{i(dx+c)}-1\right)}{2d}+\frac{\ln\left(e^{i(dx+c)}-1\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b*(-1/2/sin(d*x+c)^2+ln(tan(d*x+c)))+a*(-1/2*csc(d*x+c)*cot(d*x+c)+1/2
*ln(csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]
time = 0.28, size = 71, normalized size = 1.11

$$-\frac{(a-2\,b)\log\left(\cos\left(dx+c\right)+1\right)-(a+2\,b)\log\left(\cos\left(dx+c\right)-1\right)+4\,b\log\left(\cos\left(dx+c\right)\right)-\frac{2\,(a\cos(dx+c)+b)}{\cos(dx+c)^2-1}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*((a - 2*b)*log(cos(d*x + c) + 1) - (a + 2*b)*log(cos(d*x + c) - 1) + 4
*b*log(cos(d*x + c)) - 2*(a*cos(d*x + c) + b)/(cos(d*x + c)^2 - 1))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 123 vs. 2(58) = 116.
time = 4.53, size = 123, normalized size = 1.92

$$\frac{2\,a\cos\left(dx+c\right)-4\,\left(b\cos\left(dx+c\right)^2-b\right)\log\left(-\cos\left(dx+c\right)\right)-\left((a-2\,b)\cos\left(dx+c\right)^2-a+2\,b\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+\left((a+2\,b)\cos\left(dx+c\right)^2-a-2\,b\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+2\,b}{4\,\left(d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*cos(d*x + c) - 4*(b*cos(d*x + c)^2 - b)*log(-cos(d*x + c)) - ((a -
 2*b)*cos(d*x + c)^2 - a + 2*b)*log(1/2*cos(d*x + c) + 1/2) + ((a + 2*b)*co
s(d*x + c)^2 - a - 2*b)*log(-1/2*cos(d*x + c) + 1/2) + 2*b)/(d*cos(d*x + c)
^2 - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))\csc^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*csc(c + d*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(58) = 116$.
time = 0.49, size = 169, normalized size = 2.64

$$\frac{2\,(a+2\,b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-8\,b\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{\left(a+b-\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{4\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{\cos(dx+c)-1}-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(2*(a + 2*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 8*b*lo
g(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (a + b - 2*a*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) - 4*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))*(
cos(d*x + c) + 1)/(cos(d*x + c) - 1) - a*(cos(d*x + c) - 1)/(cos(d*x + c) +
 1) + b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/d
```

Mupad [B]
time = 0.10, size = 76, normalized size = 1.19

$$\frac{\frac{\frac{b}{2}+\frac{a\,\cos(c+d\,x)}{2}}{\cos(c+d\,x)^2-1}+\ln\left(\cos\left(c+d\,x\right)-1\right)\,\left(\frac{a}{4}+\frac{b}{2}\right)-\ln\left(\cos\left(c+d\,x\right)+1\right)\,\left(\frac{a}{4}-\frac{b}{2}\right)-b\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))/sin(c + d*x)^3,x)
```

```
[Out] ((b/2 + (a*cos(c + d*x))/2)/(cos(c + d*x)^2 - 1) + log(cos(c + d*x) - 1)*(a
/4 + b/2) - log(cos(c + d*x) + 1)*(a/4 - b/2) - b*log(cos(c + d*x)))/d
```

3.166 $\int \csc^5(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=100

$$-\frac{3a\tanh^{-1}(\cos(c+dx))}{8d}-\frac{b\cot^2(c+dx)}{d}-\frac{b\cot^4(c+dx)}{4d}-\frac{3a\cot(c+dx)\csc(c+dx)}{8d}-\frac{a\cot(c+dx)\csc^3(c}{4d}$$

```
[Out] -3/8*a*arctanh(cos(d*x+c))/d-b*cot(d*x+c)^2/d-1/4*b*cot(d*x+c)^4/d-3/8*a*co
t(d*x+c)*csc(d*x+c)/d-1/4*a*cot(d*x+c)*csc(d*x+c)^3/d+b*ln(tan(d*x+c))/d
```

Rubi [A]
time = 0.09, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3957, 2913, 2700, 272, 45, 3853, 3855}

$$-\frac{3a\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{3a\cot(c+dx)\csc(c+dx)}{8d}-\frac{b\cot^4(c+dx)}{4d}-\frac{b\cot^2(c+dx)}{d}+\frac{b\log(\tan(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5*(a + b*Sec[c + d*x]),x]
```

```
[Out] (-3*a*ArcTanh[Cos[c + d*x]])/(8*d) - (b*Cot[c + d*x]^2)/d - (b*Cot[c + d*x]
^4)/(4*d) - (3*a*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (a*Cot[c + d*x]*Csc[c +
 d*x]^3)/(4*d) + (b*Log[Tan[c + d*x]])/d
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2913

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
```

```
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \csc^5(c+dx)(a+b\sec(c+dx))\,dx &= -\int (-b-a\cos(c+dx))\csc^5(c+dx)\sec(c+dx)\,dx \\
&= a\int \csc^5(c+dx)\,dx + b\int \csc^5(c+dx)\sec(c+dx)\,dx \\
&= -\frac{a\cot(c+dx)\csc^3(c+dx)}{4d} + \frac{1}{4}(3a)\int \csc^3(c+dx)\,dx + \frac{b\text{Subst}\Big(\int \\
&= -\frac{3a\cot(c+dx)\csc(c+dx)}{8d} - \frac{a\cot(c+dx)\csc^3(c+dx)}{4d} + \frac{1}{8}(3a)\int \\
&= -\frac{3a\tanh^{-1}(\cos(c+dx))}{8d} - \frac{3a\cot(c+dx)\csc(c+dx)}{8d} - \frac{a\cot(c+d \\
&= -\frac{3a\tanh^{-1}(\cos(c+dx))}{8d} - \frac{b\cot^2(c+dx)}{d} - \frac{b\cot^4(c+dx)}{4d} - \frac{3a\cot
\end{aligned}$$

Mathematica [A]
time = 0.38, size = 164, normalized size = 1.64

$$-\frac{3a\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d}-\frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{3a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{8d}+\frac{3a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d}-\frac{b(2\csc^2(c+dx)+\csc^4(c+dx)+4\log(\cos(c+dx))-4\log(\sin(c+dx)))}{4d}+\frac{3a\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^5*(a + b*Sec[c + d*x]),x]
```

```
[Out] (-3*a*Csc[(c + d*x)/2]^2)/(32*d) - (a*Csc[(c + d*x)/2]^4)/(64*d) - (3*a*Log
[Cos[(c + d*x)/2]])/(8*d) + (3*a*Log[Sin[(c + d*x)/2]])/(8*d) - (b*(2*Csc[c
 + d*x]^2 + Csc[c + d*x]^4 + 4*Log[Cos[c + d*x]] - 4*Log[Sin[c + d*x]]))/(4
*d) + (3*a*Sec[(c + d*x)/2]^2)/(32*d) + (a*Sec[(c + d*x)/2]^4)/(64*d)
```

Maple [A]
time = 0.16, size = 83, normalized size = 0.83

method	result
derivativedivides	$\frac{b\left(-\frac{1}{4\sin(dx+c)^4}-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+a\left(\left(-\frac{(\csc^3(dx+c))}{4}-\frac{3\csc(dx+c)}{8}\right)\cot(dx+c)+\frac{3\ln(\csc(dx+c)-\cot(dx+}{8}\right.}{d}$
default	$\frac{b\left(-\frac{1}{4\sin(dx+c)^4}-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+a\left(\left(-\frac{(\csc^3(dx+c))}{4}-\frac{3\csc(dx+c)}{8}\right)\cot(dx+c)+\frac{3\ln(\csc(dx+c)-\cot(dx+}{8}\right.}{d}$
norman	$\frac{-\frac{a+b}{64d}+\frac{(a-b)\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}+\frac{(2a-3b)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16d}-\frac{(2a+3b)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{b\ln\left(\tan\left(\right.\right.}{}$
risch	$\frac{3a\,e^{7i(dx+c)}+8b\,e^{6i(dx+c)}-11a\,e^{5i(dx+c)}-32b\,e^{4i(dx+c)}-11a\,e^{3i(dx+c)}+8b\,e^{2i(dx+c)}+3\,e^{i(dx+c)}a}{4d\left(e^{2i(dx+c)}-1\right)^4}+\frac{3a\ln\left(e^{i(dx+c)}-1\right.}{8d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b*(-1/4/sin(d*x+c)^4-1/2/sin(d*x+c)^2+ln(tan(d*x+c)))+a*((-1/4*csc(d*x
+c)^3-3/8*csc(d*x+c))*cot(d*x+c)+3/8*ln(csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]
time = 0.27, size = 110, normalized size = 1.10

$$-\frac{(3\,a-8\,b)\log\left(\cos\left(dx+c\right)+1\right)-(3\,a+8\,b)\log\left(\cos\left(dx+c\right)-1\right)+16\,b\log\left(\cos\left(dx+c\right)\right)-\frac{2\left(3\,a\cos\left(dx+c\right)^3+4\,b\cos\left(dx+c\right)^2-5\,a\cos\left(dx+c\right)-6\,b\right)}{\cos\left(dx+c\right)^4-2\,\cos\left(dx+c\right)^2+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*((3*a - 8*b)*log(cos(d*x + c) + 1) - (3*a + 8*b)*log(cos(d*x + c) - 1
) + 16*b*log(cos(d*x + c)) - 2*(3*a*cos(d*x + c)^3 + 4*b*cos(d*x + c)^2 - 5
*a*cos(d*x + c) - 6*b)/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(92) = 184$.
time = 2.74, size = 201, normalized size = 2.01

$$\frac{6\,a\cos\left(dx+c\right)^3+8\,b\cos\left(dx+c\right)^2-10\,a\cos\left(dx+c\right)-16\left(b\cos\left(dx+c\right)^4-2\,b\cos\left(dx+c\right)^2+b\right)\log\left(-\cos\left(dx+c\right)\right)-\left((3\,a-8\,b)\cos\left(dx+c\right)^4-2\,(3\,a-8\,b)\cos\left(dx+c\right)^2+3\,a-8\,b\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+\left((3\,a+8\,b)\cos\left(dx+c\right)^4-2\,(3\,a+8\,b)\cos\left(dx+c\right)^2+3\,a+8\,b\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-12\,b}{16\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(6*a*cos(d*x + c)^3 + 8*b*cos(d*x + c)^2 - 10*a*cos(d*x + c) - 16*(b*c
os(d*x + c)^4 - 2*b*cos(d*x + c)^2 + b)*log(-cos(d*x + c)) - ((3*a - 8*b)*c
os(d*x + c)^4 - 2*(3*a - 8*b)*cos(d*x + c)^2 + 3*a - 8*b)*log(1/2*cos(d*x +
 c) + 1/2) + ((3*a + 8*b)*cos(d*x + c)^4 - 2*(3*a + 8*b)*cos(d*x + c)^2 + 3
*a + 8*b)*log(-1/2*cos(d*x + c) + 1/2) - 12*b)/(d*cos(d*x + c)^4 - 2*d*cos(
d*x + c)^2 + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec(c + dx))\csc^5(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*csc(c + d*x)**5, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 266 vs. 2(92) = 184.
time = 0.48, size = 266, normalized size = 2.66

$$\frac{4\,(3\,a+8\,b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-64\,b\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)-\frac{\left(a+b-\frac{8\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{12\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{18\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{48\,b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)^2}{(\cos(dx+c)-1)^2}-\frac{8\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{12\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/64*(4*(3*a + 8*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 64*
b*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - (a + b - 8*a*(cos(
d*x + c) - 1)/(cos(d*x + c) + 1) - 12*b*(cos(d*x + c) - 1)/(cos(d*x + c) +
1) + 18*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 48*b*(cos(d*x + c) -
1)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)^2/(cos(d*x + c) - 1)^2 - 8*a*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 12*b*(cos(d*x + c) - 1)/(cos(d*x +
c) + 1) + a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - b*(cos(d*x + c) - 1
)^2/(cos(d*x + c) + 1)^2)/d
```

Mupad [B]
time = 0.98, size = 117, normalized size = 1.17

$$\frac{\ln\left(\cos\left(c+d\,x\right)-1\right)\,\left(\frac{3\,a}{16}+\frac{b}{2}\right)}{d}-\frac{-\frac{3\,a\cos\left(c+d\,x\right)^3}{8}-\frac{b\cos\left(c+d\,x\right)^2}{2}+\frac{5\,a\,\cos\left(c+d\,x\right)}{8}+\frac{3\,b}{4}}{d\,\left({\cos\left(c+d\,x\right)}^4-2\cos\left(c+d\,x\right)^2+1\right)}-\frac{\ln\left(\cos\left(c+d\,x\right)+1\right)\,\left(\frac{3\,a}{16}-\frac{b}{2}\right)}{d}-\frac{b\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))/sin(c + d*x)^5,x)
```

```
[Out] (log(cos(c + d*x) - 1)*((3*a)/16 + b/2))/d - ((3*b)/4 + (5*a*cos(c + d*x))/
8 - (3*a*cos(c + d*x)^3)/8 - (b*cos(c + d*x)^2)/2)/(d*(cos(c + d*x)^4 - 2*c
os(c + d*x)^2 + 1)) - (log(cos(c + d*x) + 1)*((3*a)/16 - b/2))/d - (b*log(c
os(c + d*x)))/d
```

3.167 $\int \csc^7(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=140

$$-\frac{5a\tanh^{-1}(\cos(c+dx))}{16d}-\frac{3b\cot^2(c+dx)}{2d}-\frac{3b\cot^4(c+dx)}{4d}-\frac{b\cot^6(c+dx)}{6d}-\frac{5a\cot(c+dx)\csc(c+dx)}{16d}-\frac{5a}{}$$

```
[Out] -5/16*a*arctanh(cos(d*x+c))/d-3/2*b*cot(d*x+c)^2/d-3/4*b*cot(d*x+c)^4/d-1/6
*b*cot(d*x+c)^6/d-5/16*a*cot(d*x+c)*csc(d*x+c)/d-5/24*a*cot(d*x+c)*csc(d*x+
c)^3/d-1/6*a*cot(d*x+c)*csc(d*x+c)^5/d+b*ln(tan(d*x+c))/d
```

Rubi [A]
time = 0.11, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3957, 2913, 2700, 272, 45, 3853, 3855}

$$-\frac{5a\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a\cot(c+dx)\csc^5(c+dx)}{6d}-\frac{5a\cot(c+dx)\csc^3(c+dx)}{24d}-\frac{5a\cot(c+dx)\csc(c+dx)}{16d}-\frac{b\cot^6(c+dx)}{6d}-\frac{3b\cot^4(c+dx)}{4d}-\frac{3b\cot^2(c+dx)}{2d}+\frac{b\log(\tan(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^7*(a + b*Sec[c + d*x]),x]
```

```
[Out] (-5*a*ArcTanh[Cos[c + d*x]])/(16*d) - (3*b*Cot[c + d*x]^2)/(2*d) - (3*b*Cot
[c + d*x]^4)/(4*d) - (b*Cot[c + d*x]^6)/(6*d) - (5*a*Cot[c + d*x]*Csc[c + d
*x])/(16*d) - (5*a*Cot[c + d*x]*Csc[c + d*x]^3)/(24*d) - (a*Cot[c + d*x]*Cs
c[c + d*x]^5)/(6*d) + (b*Log[Tan[c + d*x]])/d
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2913

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \csc^7(c+dx)(a+b\sec(c+dx))\,dx &= -\int (-b-a\cos(c+dx))\csc^7(c+dx)\sec(c+dx)\,dx \\
&= a\int \csc^7(c+dx)\,dx + b\int \csc^7(c+dx)\sec(c+dx)\,dx \\
&= -\frac{a\cot(c+dx)\csc^5(c+dx)}{6d} + \frac{1}{6}(5a)\int \csc^5(c+dx)\,dx + \frac{b\text{Subst}\Big(\\
&= -\frac{5a\cot(c+dx)\csc^3(c+dx)}{24d} - \frac{a\cot(c+dx)\csc^5(c+dx)}{6d} + \frac{1}{8}(5a) \\
&= -\frac{5a\cot(c+dx)\csc(c+dx)}{16d} - \frac{5a\cot(c+dx)\csc^3(c+dx)}{24d} - \frac{a\cot(\\
&= -\frac{5a\tanh^{-1}(\cos(c+dx))}{16d} - \frac{3b\cot^2(c+dx)}{2d} - \frac{3b\cot^4(c+dx)}{4d} - \frac{b
\end{aligned}$$

Mathematica [A]

time = 0.40, size = 216, normalized size = 1.54

$$-\frac{5a\csc^2\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{a\csc^6\left(\frac{1}{2}(c+dx)\right)}{384d}-\frac{5a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{16d}+\frac{5a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{16d}-\frac{b(6\csc^2(c+dx)+3\csc^4(c+dx)+2\csc^6(c+dx)+12\log(\cos(c+dx))-12\log(\sin(c+dx)))}{12d}+\frac{5a\sec^2\left(\frac{1}{2}(c+dx)\right)}{64d}+\frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d}+\frac{a\sec^6\left(\frac{1}{2}(c+dx)\right)}{384d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^7*(a + b*Sec[c + d*x]),x]
```

```
[Out] (-5*a*Csc[(c + d*x)/2]^2)/(64*d) - (a*Csc[(c + d*x)/2]^4)/(64*d) - (a*Csc[(
c + d*x)/2]^6)/(384*d) - (5*a*Log[Cos[(c + d*x)/2]])/(16*d) + (5*a*Log[Sin[
(c + d*x)/2]])/(16*d) - (b*(6*Csc[c + d*x]^2 + 3*Csc[c + d*x]^4 + 2*Csc[c +
 d*x]^6 + 12*Log[Cos[c + d*x]] - 12*Log[Sin[c + d*x]]))/(12*d) + (5*a*Sec[(
c + d*x)/2]^2)/(64*d) + (a*Sec[(c + d*x)/2]^4)/(64*d) + (a*Sec[(c + d*x)/2]
^6)/(384*d)
```

Maple [A]
time = 0.18, size = 103, normalized size = 0.74

method	result
derivativedivides	$\frac{b\left(-\frac{1}{6\sin(dx+c)^6}-\frac{1}{4\sin(dx+c)^4}-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+a\left(\left(-\frac{\left(\csc^5(dx+c)\right)}{6}-\frac{5\left(\csc^3(dx+c)\right)}{24}-\frac{5\csc(dx+c)}{16}\right)\cot(dx\right.}{d}$
default	$\frac{b\left(-\frac{1}{6\sin(dx+c)^6}-\frac{1}{4\sin(dx+c)^4}-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+a\left(\left(-\frac{\left(\csc^5(dx+c)\right)}{6}-\frac{5\left(\csc^3(dx+c)\right)}{24}-\frac{5\csc(dx+c)}{16}\right)\cot(dx\right.}{d}$
norman	$\frac{-\frac{a+b}{384d}+\frac{(a-b)\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{384d}+\frac{(3a-4b)\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128d}-\frac{(3a+4b)\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128d}+\frac{(15a-29b)\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128d}-\frac{(15a+29b)(\text{ta}}{12}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}$
risch	$\frac{15a\,e^{11i(dx+c)}+48b\,e^{10i(dx+c)}-85a\,e^{9i(dx+c)}-288b\,e^{8i(dx+c)}+198a\,e^{7i(dx+c)}+736b\,e^{6i(dx+c)}+198a\,e^{5i(dx+c)}-288b\,e^{4i(d}}{24d\left(e^{2i(dx+c)}-1\right)^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^7*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b*(-1/6/sin(d*x+c)^6-1/4/sin(d*x+c)^4-1/2/sin(d*x+c)^2+ln(tan(d*x+c)))
+a*((-1/6*csc(d*x+c)^5-5/24*csc(d*x+c)^3-5/16*csc(d*x+c))*cot(d*x+c)+5/16*l
n(csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]
time = 0.28, size = 143, normalized size = 1.02

$$-\frac{3(5a-16b)\log(\cos(dx+c)+1)-3(5a+16b)\log(\cos(dx+c)-1)+96b\log(\cos(dx+c))-\frac{2\left(15a\cos(dx+c)^5+24b\cos(dx+c)^4-40a\cos(dx+c)^3-60b\cos(dx+c)^2+33a\cos(dx+c)+44b\right)}{\cos(dx+c)^6-3\cos(dx+c)^4+3\cos(dx+c)^2-1}}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^7*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/96*(3*(5*a - 16*b)*log(cos(d*x + c) + 1) - 3*(5*a + 16*b)*log(cos(d*x +
c) - 1) + 96*b*log(cos(d*x + c)) - 2*(15*a*cos(d*x + c)^5 + 24*b*cos(d*x +
```

```
c)^4 - 40*a*cos(d*x + c)^3 - 60*b*cos(d*x + c)^2 + 33*a*cos(d*x + c) + 44*b
)/(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(126) = 252$.
time = 4.10, size = 284, normalized size = 2.03

$$\frac{30\,a\cos\left(dx+c\right)^{5}+48\,b\cos\left(dx+c\right)^{4}-80\,a\cos\left(dx+c\right)^{3}-120\,b\cos\left(dx+c\right)^{2}+66\,a\cos\left(dx+c\right)-96\left(b\cos\left(dx+c\right)^{6}-3\,b\cos\left(dx+c\right)^{4}+3\,b\cos\left(dx+c\right)^{2}-b\right)\log\left(-\cos\left(dx+c\right)\right)-3\left((5\,a-16\,b)\cos\left(dx+c\right)^{6}-3\,(5\,a-16\,b)\cos\left(dx+c\right)^{4}+3\,(5\,a-16\,b)\cos\left(dx+c\right)^{2}-5\,a+16\,b\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+3\left((5\,a+16\,b)\cos\left(dx+c\right)^{6}-3\,(5\,a+16\,b)\cos\left(dx+c\right)^{4}+3\,(5\,a+16\,b)\cos\left(dx+c\right)^{2}-5\,a-16\,b\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+88\,b}{96\left(d\cos\left(dx+c\right)^{6}-3\,d\cos\left(dx+c\right)^{4}+3\,d\cos\left(dx+c\right)^{2}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^7*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/96*(30*a*cos(d*x + c)^5 + 48*b*cos(d*x + c)^4 - 80*a*cos(d*x + c)^3 - 120
*b*cos(d*x + c)^2 + 66*a*cos(d*x + c) - 96*(b*cos(d*x + c)^6 - 3*b*cos(d*x
+ c)^4 + 3*b*cos(d*x + c)^2 - b)*log(-cos(d*x + c)) - 3*((5*a - 16*b)*cos(d
*x + c)^6 - 3*(5*a - 16*b)*cos(d*x + c)^4 + 3*(5*a - 16*b)*cos(d*x + c)^2 -
 5*a + 16*b)*log(1/2*cos(d*x + c) + 1/2) + 3*((5*a + 16*b)*cos(d*x + c)^6 -
 3*(5*a + 16*b)*cos(d*x + c)^4 + 3*(5*a + 16*b)*cos(d*x + c)^2 - 5*a - 16*b
)*log(-1/2*cos(d*x + c) + 1/2) + 88*b)/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)
^4 + 3*d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**7*(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(126) = 252$.
time = 0.47, size = 357, normalized size = 2.55

$$\frac{12\,(5\,a+16\,b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-384\,b\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{\left(a+b-\frac{9\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{12\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{45\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{87\,b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{110\,a(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}-\frac{352\,b(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}\right)(\cos(dx+c)+1)^3}{(\cos(dx+c)-1)^3}-\frac{45\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{87\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{9\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{12\,b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{a(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{b(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^7*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/384*(12*(5*a + 16*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) -
384*b*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (a + b - 9*a*(
cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 12*b*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1) + 45*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 87*b*(cos(d*x + c
) - 1)^2/(cos(d*x + c) + 1)^2 - 110*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) +
1)^3 - 352*b*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)*(cos(d*x + c) + 1)^
```

```
3/(cos(d*x + c) - 1)^3 - 45*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 87*b*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*a*(cos(d*x + c) - 1)^2/(cos(d*x +
 c) + 1)^2 - 12*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - a*(cos(d*x +
c) - 1)^3/(cos(d*x + c) + 1)^3 + b*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^
3)/d
```

Mupad [B]
time = 1.02, size = 148, normalized size = 1.06

$$\frac{\frac{5\,a\cos(c+d\,x)^5}{16}+\frac{b\cos(c+d\,x)^4}{2}-\frac{5\,a\cos(c+d\,x)^3}{6}-\frac{5\,b\cos(c+d\,x)^2}{4}+\frac{11\,a\,\cos(c+d\,x)}{16}+\frac{11\,b}{12}}{d\,\left(\cos\left(c+d\,x\right)^6-3\cos\left(c+d\,x\right)^4+3\cos\left(c+d\,x\right)^2-1\right)}+\frac{\ln\left(\cos\left(c+d\,x\right)-1\right)\,\left(\frac{5\,a}{32}+\frac{b}{2}\right)}{d}-\frac{\ln\left(\cos\left(c+d\,x\right)+1\right)\,\left(\frac{5\,a}{32}-\frac{b}{2}\right)}{d}-\frac{b\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))/sin(c + d*x)^7,x)
```

```
[Out] ((11*b)/12 + (11*a*cos(c + d*x))/16 - (5*a*cos(c + d*x)^3)/6 + (5*a*cos(c +
 d*x)^5)/16 - (5*b*cos(c + d*x)^2)/4 + (b*cos(c + d*x)^4)/2)/(d*(3*cos(c +
d*x)^2 - 3*cos(c + d*x)^4 + cos(c + d*x)^6 - 1)) + (log(cos(c + d*x) - 1)*(
(5*a)/32 + b/2))/d - (log(cos(c + d*x) + 1)*((5*a)/32 - b/2))/d - (b*log(co
s(c + d*x)))/d
```

3.168 $\int (a+b\sec(c+dx))\sin^6(c+dx)\,dx$

Optimal. Leaf size=127

$$\frac{5ax}{16}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}-\frac{b\sin(c+dx)}{d}-\frac{5a\cos(c+dx)\sin(c+dx)}{16d}-\frac{b\sin^3(c+dx)}{3d}-\frac{5a\cos(c+dx)\sin^3}{24d}$$

```
[Out] 5/16*a*x+b*arctanh(sin(d*x+c))/d-b*sin(d*x+c)/d-5/16*a*cos(d*x+c)*sin(d*x+c
)/d-1/3*b*sin(d*x+c)^3/d-5/24*a*cos(d*x+c)*sin(d*x+c)^3/d-1/5*b*sin(d*x+c)^
5/d-1/6*a*cos(d*x+c)*sin(d*x+c)^5/d
```

Rubi [A]

time = 0.10, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3957, 2917, 2672, 308, 212, 2715, 8}

$$-\frac{a\sin^5(c+dx)\cos(c+dx)}{6d}-\frac{5a\sin^3(c+dx)\cos(c+dx)}{24d}-\frac{5a\sin(c+dx)\cos(c+dx)}{16d}+\frac{5ax}{16}-\frac{b\sin^5(c+dx)}{5d}-\frac{b\sin^3(c+dx)}{3d}-\frac{b\sin(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])*Sin[c + d*x]^6,x]
```

```
[Out] (5*a*x)/16 + (b*ArcTanh[Sin[c + d*x]])/d - (b*Sin[c + d*x])/d - (5*a*Cos[c
+ d*x]*Sin[c + d*x])/(16*d) - (b*Sin[c + d*x]^3)/(3*d) - (5*a*Cos[c + d*x]*
Sin[c + d*x]^3)/(24*d) - (b*Sin[c + d*x]^5)/(5*d) - (a*Cos[c + d*x]*Sin[c +
 d*x]^5)/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
```

```
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int (a+b\sec(c+dx))\sin^6(c+dx)\,dx &= -\int (-b-a\cos(c+dx))\sin^5(c+dx)\tan(c+dx)\,dx \\
&= a\int \sin^6(c+dx)\,dx + b\int \sin^5(c+dx)\tan(c+dx)\,dx \\
&= -\frac{a\cos(c+dx)\sin^5(c+dx)}{6d} + \frac{1}{6}(5a)\int \sin^4(c+dx)\,dx + \frac{b\text{Subst}\left(\int \right.}{} \\
&= -\frac{5a\cos(c+dx)\sin^3(c+dx)}{24d} - \frac{a\cos(c+dx)\sin^5(c+dx)}{6d} + \frac{1}{8}(5a)\int \\
&= -\frac{b\sin(c+dx)}{d} - \frac{5a\cos(c+dx)\sin(c+dx)}{16d} - \frac{b\sin^3(c+dx)}{3d} - \frac{5a\,\mathrm{co}}{} \\
&= \frac{5ax}{16} + \frac{b\tanh^{-1}(\sin(c+dx))}{d} - \frac{b\sin(c+dx)}{d} - \frac{5a\cos(c+dx)\sin(c}{16d}
\end{aligned}$$

Mathematica [A]
time = 0.14, size = 118, normalized size = 0.93

$$\frac{5a(c+dx)}{16d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}-\frac{b\sin(c+dx)}{d}-\frac{b\sin^3(c+dx)}{3d}-\frac{b\sin^5(c+dx)}{5d}-\frac{15a\sin(2(c+dx))}{64d}+\frac{3a\sin(4(c+dx))}{64d}-\frac{a\sin(6(c+dx))}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])*Sin[c + d*x]^6,x]
```

```
[Out] (5*a*(c + d*x))/(16*d) + (b*ArcTanh[Sin[c + d*x]])/d - (b*Sin[c + d*x])/d -
 (b*Sin[c + d*x]^3)/(3*d) - (b*Sin[c + d*x]^5)/(5*d) - (15*a*Sin[2*(c + d*x
)])/(64*d) + (3*a*Sin[4*(c + d*x)])/(64*d) - (a*Sin[6*(c + d*x)])/(192*d)
```

Maple [A]
time = 0.10, size = 96, normalized size = 0.76

method	result
derivativedivides	$\frac{b\left(-\frac{\left(\sin^5(dx+c)\right)}{5}-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(-\frac{\left(\sin^5(dx+c)+\frac{5\left(\sin^3(dx+c)\right)}{4}+\frac{15\sin(dx}{8}\right.}{6}\right.}{d}$
default	$\frac{b\left(-\frac{\left(\sin^5(dx+c)\right)}{5}-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(-\frac{\left(\sin^5(dx+c)+\frac{5\left(\sin^3(dx+c)\right)}{4}+\frac{15\sin(dx}{8}\right.}{6}\right.}{d}$
risch	$\frac{5ax}{16}+\frac{11ib\,e^{i(dx+c)}}{16d}-\frac{11ib\,e^{-i(dx+c)}}{16d}-\frac{b\ln\left(e^{i(dx+c)}-i\right)}{d}+\frac{b\ln\left(e^{i(dx+c)}+i\right)}{d}-\frac{a\sin(6dx+6c)}{192d}-\frac{b\sin(5dx+5c}{80d}$
norman	$\frac{5ax}{16}+\frac{15ax\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}+\frac{75ax\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}+\frac{25ax\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}+\frac{75ax\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}+\frac{15ax\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}+\frac{5ax}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*sin(d*x+c)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b*(-1/5*sin(d*x+c)^5-1/3*sin(d*x+c)^3-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x
+c)))+a*(-1/6*(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/
16*d*x+5/16*c))
```

Maxima [A]
time = 0.27, size = 106, normalized size = 0.83

$$\frac{5\left(4\sin\left(2dx+2c\right)^{3}+60dx+60c+9\sin\left(4dx+4c\right)-48\sin\left(2dx+2c\right)\right)a-32\left(6\sin\left(dx+c\right)^{5}+10\sin\left(dx+c\right)^{3}-15\log\left(\sin\left(dx+c\right)+1\right)+15\log\left(\sin\left(dx+c\right)-1\right)+30\sin\left(dx+c\right)\right)b}{960d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/960*(5*(4*sin(2*d*x + 2*c)^3 + 60*d*x + 60*c + 9*sin(4*d*x + 4*c) - 48*si
n(2*d*x + 2*c))*a - 32*(6*sin(d*x + c)^5 + 10*sin(d*x + c)^3 - 15*log(sin(d
*x + c) + 1) + 15*log(sin(d*x + c) - 1) + 30*sin(d*x + c))*b)/d
```

Fricas [A]
time = 4.98, size = 102, normalized size = 0.80

$$\frac{75\,adx+120\,b\log\left(\sin\left(dx+c\right)+1\right)-120\,b\log\left(-\sin\left(dx+c\right)+1\right)-\left(40\,a\cos\left(dx+c\right)^{5}+48\,b\cos\left(dx+c\right)^{4}-130\,a\cos\left(dx+c\right)^{3}-176\,b\cos\left(dx+c\right)^{2}+165\,a\cos\left(dx+c\right)+368\,b\right)\sin\left(dx+c\right)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] 1/240*(75*a*d*x + 120*b*log(sin(d*x + c) + 1) - 120*b*log(-sin(d*x + c) + 1
) - (40*a*cos(d*x + c)^5 + 48*b*cos(d*x + c)^4 - 130*a*cos(d*x + c)^3 - 176
*b*cos(d*x + c)^2 + 165*a*cos(d*x + c) + 368*b)*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec(c + dx))\sin^6(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)**6,x)
```

```
[Out] Integral((a + b*sec(c + d*x))*sin(c + d*x)**6, x)
```

Giac [A]
time = 0.45, size = 228, normalized size = 1.80

$$\frac{75\,(dx+c)a + 240\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 240\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + \frac{2\left(75\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11} - 240\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11} + 425\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9} - 1520\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9} + 990\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7} - 4128\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7} - 990\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5} - 4128\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5} - 425\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3} - 1520\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3} - 75\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - 240\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{6}}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^6,x, algorithm="giac")
```

```
[Out] 1/240*(75*(d*x + c)*a + 240*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 240*b*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(75*a*tan(1/2*d*x + 1/2*c)^11 - 240*b*
tan(1/2*d*x + 1/2*c)^11 + 425*a*tan(1/2*d*x + 1/2*c)^9 - 1520*b*tan(1/2*d*x
 + 1/2*c)^9 + 990*a*tan(1/2*d*x + 1/2*c)^7 - 4128*b*tan(1/2*d*x + 1/2*c)^7
- 990*a*tan(1/2*d*x + 1/2*c)^5 - 4128*b*tan(1/2*d*x + 1/2*c)^5 - 425*a*tan(
1/2*d*x + 1/2*c)^3 - 1520*b*tan(1/2*d*x + 1/2*c)^3 - 75*a*tan(1/2*d*x + 1/2
*c) - 240*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

Mupad [B]
time = 2.17, size = 332, normalized size = 2.61

$$\frac{5\,a\,\mathrm{atan}\left(\frac{125\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64\left(\frac{125\,a^3}{64}+20\,a\,b^2\right)}+\frac{20\,a\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\frac{125\,a^3}{64}+20\,a\,b^2}\right)}{8\,d}+\frac{2\,b\,\mathrm{atanh}\left(\frac{64\,b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\frac{25\,a^2\,b}{4}+64\,b^3}+\frac{25\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\left(\frac{25\,a^2\,b}{4}+64\,b^3\right)}\right)}{d}-\frac{\left(2\,b-\frac{5\,a}{8}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+\left(\frac{38\,b}{3}-\frac{85\,a}{24}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}+\left(\frac{172\,b}{5}-\frac{33\,a}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}+\left(\frac{33\,a}{4}+\frac{172\,b}{5}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}+\left(\frac{85\,a}{24}+\frac{38\,b}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}+\left(\frac{5\,a}{8}+2\,b\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+15\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+20\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+15\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6*(a + b/cos(c + d*x)),x)
```

```
[Out] (5*a*atan((125*a^3*tan(c/2 + (d*x)/2))/(64*(20*a*b^2 + (125*a^3)/64)) + (20
*a*b^2*tan(c/2 + (d*x)/2))/(20*a*b^2 + (125*a^3)/64)))/(8*d) + (2*b*atanh((
64*b^3*tan(c/2 + (d*x)/2))/((25*a^2*b)/4 + 64*b^3) + (25*a^2*b*tan(c/2 + (d
*x)/2))/(4*((25*a^2*b)/4 + 64*b^3))))/d - (tan(c/2 + (d*x)/2)*((5*a)/8 + 2*
```

```
b) - tan(c/2 + (d*x)/2)^11*((5*a)/8 - 2*b) + tan(c/2 + (d*x)/2)^3*((85*a)/2
4 + (38*b)/3) - tan(c/2 + (d*x)/2)^9*((85*a)/24 - (38*b)/3) + tan(c/2 + (d*
x)/2)^5*((33*a)/4 + (172*b)/5) - tan(c/2 + (d*x)/2)^7*((33*a)/4 - (172*b)/5
))/(d*(6*tan(c/2 + (d*x)/2)^2 + 15*tan(c/2 + (d*x)/2)^4 + 20*tan(c/2 + (d*x
)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 + 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x
)/2)^12 + 1))
```

3.169 $\int (a+b\sec(c+dx))\sin^4(c+dx)\,dx$

Optimal. Leaf size=89

$$\frac{3ax}{8}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}-\frac{b\sin(c+dx)}{d}-\frac{3a\cos(c+dx)\sin(c+dx)}{8d}-\frac{b\sin^3(c+dx)}{3d}-\frac{a\cos(c+dx)\sin^3(c$$

```
[Out] 3/8*a*x+b*arctanh(sin(d*x+c))/d-b*sin(d*x+c)/d-3/8*a*cos(d*x+c)*sin(d*x+c)/
d-1/3*b*sin(d*x+c)^3/d-1/4*a*cos(d*x+c)*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3957, 2917, 2672, 308, 212, 2715, 8}

$$-\frac{a\sin^3(c+dx)\cos(c+dx)}{4d}-\frac{3a\sin(c+dx)\cos(c+dx)}{8d}+\frac{3ax}{8}-\frac{b\sin^3(c+dx)}{3d}-\frac{b\sin(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])*Sin[c + d*x]^4,x]
```

```
[Out] (3*a*x)/8 + (b*ArcTanh[Sin[c + d*x]])/d - (b*Sin[c + d*x])/d - (3*a*Cos[c +
 d*x]*Sin[c + d*x])/(8*d) - (b*Sin[c + d*x]^3)/(3*d) - (a*Cos[c + d*x]*Sin[
c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int (a+b\sec(c+dx))\sin^4(c+dx)\,dx &= -\int (-b-a\cos(c+dx))\sin^3(c+dx)\tan(c+dx)\,dx \\
&= a\int \sin^4(c+dx)\,dx + b\int \sin^3(c+dx)\tan(c+dx)\,dx \\
&= -\frac{a\cos(c+dx)\sin^3(c+dx)}{4d} + \frac{1}{4}(3a)\int \sin^2(c+dx)\,dx + \frac{b\text{Subst}\Big(\int \\
&= -\frac{3a\cos(c+dx)\sin(c+dx)}{8d} - \frac{a\cos(c+dx)\sin^3(c+dx)}{4d} + \frac{1}{8}(3a) \\
&= \frac{3ax}{8} - \frac{b\sin(c+dx)}{d} - \frac{3a\cos(c+dx)\sin(c+dx)}{8d} - \frac{b\sin^3(c+dx)}{3d} \\
&= \frac{3ax}{8} + \frac{b\tanh^{-1}(\sin(c+dx))}{d} - \frac{b\sin(c+dx)}{d} - \frac{3a\cos(c+dx)\sin(c
\end{aligned}$$

Mathematica [A]
time = 0.10, size = 86, normalized size = 0.97

$$\frac{3a(c+dx)}{8d} + \frac{b\tanh^{-1}(\sin(c+dx))}{d} - \frac{b\sin(c+dx)}{d} - \frac{b\sin^3(c+dx)}{3d} - \frac{a\sin(2(c+dx))}{4d} + \frac{a\sin(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sec[c + d*x])*Sin[c + d*x]^4,x]

[Out] (3*a*(c + d*x))/(8*d) + (b*ArcTanh[Sin[c + d*x]])/d - (b*Sin[c + d*x])/d - (b*Sin[c + d*x]^3)/(3*d) - (a*Sin[2*(c + d*x)])/(4*d) + (a*Sin[4*(c + d*x)])/(32*d)

Maple [A]
time = 0.08, size = 76, normalized size = 0.85

method	result
derivativedivides	$\frac{b\left(-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
default	$\frac{b\left(-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
risch	$\frac{3ax}{8}+\frac{5ib\,e^{i(dx+c)}}{8d}-\frac{5ib\,e^{-i(dx+c)}}{8d}+\frac{b\ln\left(e^{i(dx+c)}+i\right)}{d}-\frac{b\ln\left(e^{i(dx+c)}-i\right)}{d}+\frac{a\sin(4dx+4c)}{32d}+\frac{b\sin(3dx+3c)}{12d}-$
norman	$\frac{\frac{3ax}{8}+\frac{3ax\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{9ax\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}+\frac{3ax\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{3ax\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}+\frac{(3a-8b)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{(3a+8b)}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))*sin(d*x+c)^4,x,method=_RETURNVERBOSE)

[Out] 1/d*(b*(-1/3*sin(d*x+c)^3-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+a*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c))

Maxima [A]
time = 0.26, size = 81, normalized size = 0.91

$$\frac{3\,(12\,dx+12\,c+\sin(4\,dx+4\,c)-8\,\sin(2\,dx+2\,c))a-16\,\left(2\,\sin(dx+c)^3-3\,\log(\sin(dx+c)+1)+3\,\log(\sin(dx+c)-1)+6\,\sin(dx+c)\right)b}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^4,x, algorithm="maxima")

[Out] 1/96*(3*(12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*d*x + 2*c))*a - 16*(2*sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1) + 6*sin(d*x + c))*b)/d

Fricas [A]
time = 4.73, size = 79, normalized size = 0.89

$$\frac{9\,adx+12\,b\log(\sin(dx+c)+1)-12\,b\log(-\sin(dx+c)+1)+\left(6\,a\cos(dx+c)^3+8\,b\cos(dx+c)^2-15\,a\cos(dx+c)-32\,b\right)\sin(dx+c)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^4,x, algorithm="fricas")

```
[Out] 1/24*(9*a*d*x + 12*b*log(sin(d*x + c) + 1) - 12*b*log(-sin(d*x + c) + 1) +
(6*a*cos(d*x + c)^3 + 8*b*cos(d*x + c)^2 - 15*a*cos(d*x + c) - 32*b)*sin(d*
x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec(c + dx))\sin^4(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)**4,x)
```

```
[Out] Integral((a + b*sec(c + d*x))*sin(c + d*x)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(81) = 162.
time = 0.45, size = 172, normalized size = 1.93

$$\frac{9\,(dx+c)a + 24\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 24\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + \frac{2\left(9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7} - 24\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7} + 33\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5} - 104\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5} - 33\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3} - 104\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3} - 9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - 24\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{4}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/24*(9*(d*x + c)*a + 24*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 24*b*log(ab
s(tan(1/2*d*x + 1/2*c) - 1)) + 2*(9*a*tan(1/2*d*x + 1/2*c)^7 - 24*b*tan(1/2
*d*x + 1/2*c)^7 + 33*a*tan(1/2*d*x + 1/2*c)^5 - 104*b*tan(1/2*d*x + 1/2*c)^
5 - 33*a*tan(1/2*d*x + 1/2*c)^3 - 104*b*tan(1/2*d*x + 1/2*c)^3 - 9*a*tan(1/
2*d*x + 1/2*c) - 24*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)
/d
```

Mupad [B]
time = 1.88, size = 267, normalized size = 3.00

$$\frac{3\,a\,\mathrm{atan}\left(\frac{27\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\left(\frac{27\,a^3}{8}+24\,a\,b^2\right)} + \frac{24\,a\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\frac{27\,a^3}{8}+24\,a\,b^2}\right)}{4\,d} + \frac{2\,b\,\mathrm{atanh}\left(\frac{64\,b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{9\,a^2\,b+64\,b^3} + \frac{9\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{9\,a^2\,b+64\,b^3}\right)}{d} - \frac{\left(2\,b-\frac{3\,a}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7 + \left(\frac{26\,b}{3}-\frac{11\,a}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5 + \left(\frac{11\,a}{4}+\frac{26\,b}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 + \left(\frac{3\,a}{4}+2\,b\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8 + 4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 + 6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 + 4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4*(a + b/cos(c + d*x)),x)
```

```
[Out] (3*a*atan((27*a^3*tan(c/2 + (d*x)/2))/(8*(24*a*b^2 + (27*a^3)/8)) + (24*a*b
^2*tan(c/2 + (d*x)/2))/(24*a*b^2 + (27*a^3)/8)))/(4*d) + (2*b*atanh((64*b^3
*tan(c/2 + (d*x)/2))/(9*a^2*b + 64*b^3) + (9*a^2*b*tan(c/2 + (d*x)/2))/(9*a
^2*b + 64*b^3)))/d - (tan(c/2 + (d*x)/2)*((3*a)/4 + 2*b) - tan(c/2 + (d*x)/
2)^7*((3*a)/4 - 2*b) + tan(c/2 + (d*x)/2)^3*((11*a)/4 + (26*b)/3) - tan(c/2
 + (d*x)/2)^5*((11*a)/4 - (26*b)/3))/(d*(4*tan(c/2 + (d*x)/2)^2 + 6*tan(c/2
 + (d*x)/2)^4 + 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

3.170 $\int (a+b\sec(c+dx))\sin^2(c+dx)\,dx$

Optimal. Leaf size=51

$$\frac{ax}{2}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}-\frac{b\sin(c+dx)}{d}-\frac{a\cos(c+dx)\sin(c+dx)}{2d}$$

```
[Out] 1/2*a*x+b*arctanh(sin(d*x+c))/d-b*sin(d*x+c)/d-1/2*a*cos(d*x+c)*sin(d*x+c)/
d
```

Rubi [A]
time = 0.06, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3957, 2917, 2672, 327, 212, 2715, 8}

$$-\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}-\frac{b\sin(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])*Sin[c + d*x]^2,x]
```

```
[Out] (a*x)/2 + (b*ArcTanh[Sin[c + d*x]])/d - (b*Sin[c + d*x])/d - (a*Cos[c + d*x
]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
```

```
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int (a+b\sec(c+dx))\sin^2(c+dx)\,dx &= -\int (-b-a\cos(c+dx))\sin(c+dx)\tan(c+dx)\,dx \\
&= a\int \sin^2(c+dx)\,dx + b\int \sin(c+dx)\tan(c+dx)\,dx \\
&= -\frac{a\cos(c+dx)\sin(c+dx)}{2d} + \frac{1}{2}a\int 1\,dx + \frac{b\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \sin\right.}{d} \\
&= \frac{ax}{2} - \frac{b\sin(c+dx)}{d} - \frac{a\cos(c+dx)\sin(c+dx)}{2d} + \frac{b\text{Subst}\left(\int \frac{1}{1-x^2}\,dx\right.}{d} \\
&= \frac{ax}{2} + \frac{b\tanh^{-1}(\sin(c+dx))}{d} - \frac{b\sin(c+dx)}{d} - \frac{a\cos(c+dx)\sin(c+}{2d}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 54, normalized size = 1.06

$$\frac{a(c+dx)}{2d} + \frac{b\tanh^{-1}(\sin(c+dx))}{d} - \frac{b\sin(c+dx)}{d} - \frac{a\sin(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])*Sin[c + d*x]^2,x]
```

```
[Out] (a*(c + d*x))/(2*d) + (b*ArcTanh[Sin[c + d*x]])/d - (b*Sin[c + d*x])/d - (a
*Sin[2*(c + d*x)])/(4*d)
```

Maple [A]
time = 0.07, size = 55, normalized size = 1.08

method	result
derivativedivides	$\frac{b(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+a\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
default	$\frac{b(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+a\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
risch	$\frac{ax}{2}+\frac{ib\,\mathrm{e}^{i(dx+c)}}{2d}-\frac{ib\,\mathrm{e}^{-i(dx+c)}}{2d}-\frac{b\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{d}+\frac{b\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{d}-\frac{a\sin(2dx+2c)}{4d}$
norman	$\frac{ax\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{(a-2b)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{ax}{2}+\frac{ax\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{(a+2b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-$ $b\,l$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*sin(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b*(-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+a*(-1/2*cos(d*x+c)*sin(d*x+c
)+1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.27, size = 59, normalized size = 1.16

$$\frac{(2\,dx+2\,c-\sin(2\,dx+2\,c))a+2\,b(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)-2\,\sin(dx+c))}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c - sin(2*d*x + 2*c))*a + 2*b*(log(sin(d*x + c) + 1) - log(
sin(d*x + c) - 1) - 2*sin(d*x + c)))/d
```

Fricas [A]
time = 3.13, size = 55, normalized size = 1.08

$$\frac{adx+b\log(\sin(dx+c)+1)-b\log(-\sin(dx+c)+1)-(a\cos(dx+c)+2\,b)\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a*d*x + b*log(sin(d*x + c) + 1) - b*log(-sin(d*x + c) + 1) - (a*cos(d*
x + c) + 2*b)*sin(d*x + c))/d
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec(c + dx))\sin^2(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))*sin(c + d*x)**2, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(47) = 94.
time = 0.48, size = 114, normalized size = 2.24

$$\frac{(dx+c)a + 2b\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - 2b\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right) + \frac{2\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - 2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*((d*x + c)*a + 2*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*b*log(abs(tan
(1/2*d*x + 1/2*c) - 1)) + 2*(a*tan(1/2*d*x + 1/2*c)^3 - 2*b*tan(1/2*d*x + 1
/2*c)^3 - a*tan(1/2*d*x + 1/2*c) - 2*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x +
 1/2*c)^2 + 1)^2)/d
```

Mupad [**B**]
time = 1.09, size = 83, normalized size = 1.63

$$\frac{a\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{2\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} - \frac{a\,\sin\left(2\,c+2\,d\,x\right)}{4\,d} - \frac{b\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2*(a + b/cos(c + d*x)),x)
```

```
[Out] (a*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*b*atanh(sin(c/2 + (d
*x)/2)/cos(c/2 + (d*x)/2)))/d - (a*sin(2*c + 2*d*x))/(4*d) - (b*sin(c + d*x
))/d
```

3.171 $\int \csc^2(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=37

$$\frac{b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a\cot(c+dx)}{d}-\frac{b\csc(c+dx)}{d}$$

```
[Out] b*arctanh(sin(d*x+c))/d-a*cot(d*x+c)/d-b*csc(d*x+c)/d
```

Rubi [A]
time = 0.07, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.368$, Rules used = {3957, 2917, 2701, 327, 213, 3852, 8}

$$-\frac{a\cot(c+dx)}{d}-\frac{b\csc(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + b*Sec[c + d*x]),x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]])/d - (a*Cot[c + d*x])/d - (b*Csc[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \csc^2(c+dx)(a+b\sec(c+dx))\,dx &= -\int(-b-a\cos(c+dx))\csc^2(c+dx)\sec(c+dx)\,dx\\ &= a\int\csc^2(c+dx)\,dx+b\int\csc^2(c+dx)\sec(c+dx)\,dx\\ &= -\frac{a\text{Subst}\left(\int 1\,dx,x,\cot(c+dx)\right)}{d}-\frac{b\text{Subst}\left(\int\frac{x^2}{-1+x^2}\,dx,x,\csc(c+dx)\right)}{d}\\ &= -\frac{a\cot(c+dx)}{d}-\frac{b\csc(c+dx)}{d}-\frac{b\text{Subst}\left(\int\frac{1}{-1+x^2}\,dx,x,\csc(c+dx)\right)}{d}\\ &= \frac{b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a\cot(c+dx)}{d}-\frac{b\csc(c+dx)}{d}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.02, size = 41, normalized size = 1.11

$$-\frac{a\cot(c+dx)}{d}-\frac{b\csc(c+dx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\sin^2(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*(a + b*Sec[c + d*x]),x]
```

[Out] -((a*Cot[c + d*x])/d) - (b*Csc[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, Sin[c + d*x]^2])/d

Maple [A]
time = 0.06, size = 42, normalized size = 1.14

method	result	size
derivativedivides	$\frac{b\left(-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)-a\cot(dx+c)}{d}$	42
default	$\frac{b\left(-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)-a\cot(dx+c)}{d}$	42
risch	$-\frac{2i\left(b\,e^{i(dx+c)}+a\right)}{d\left(e^{2i(dx+c)}-1\right)}-\frac{b\ln\left(e^{i(dx+c)}-i\right)}{d}+\frac{b\ln\left(e^{i(dx+c)}+i\right)}{d}$	71
norman	$\frac{-\frac{a+b}{2d}+\frac{(a-b)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}$	79

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^2*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d*(b*(-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))-a*cot(d*x+c))

Maxima [A]
time = 0.28, size = 50, normalized size = 1.35

$$-\frac{b\left(\frac{2}{\sin(dx+c)}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+\frac{2\,a}{\tan(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="maxima")

[Out] -1/2*(b*(2/sin(d*x + c) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 2*a/tan(d*x + c))/d

Fricas [A]
time = 2.90, size = 63, normalized size = 1.70

$$\frac{b\log\left(\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)-b\log\left(-\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)-2\,a\cos\left(dx+c\right)-2\,b}{2\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/2*(b*log(sin(d*x + c) + 1)*sin(d*x + c) - b*log(-sin(d*x + c) + 1)*sin(d*x + c) - 2*a*cos(d*x + c) - 2*b)/(d*sin(d*x + c))

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec(c + dx))\csc^2(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*csc(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(37) = 74.
time = 0.44, size = 77, normalized size = 2.08

$$\frac{2\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 2\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - b\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - \frac{a+b}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*b*log(abs(tan(1/2*d*x + 1/2
*c) - 1)) + a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c) - (a + b)/tan(1
/2*d*x + 1/2*c))/d
```

Mupad [B]
time = 1.02, size = 60, normalized size = 1.62

$$\frac{2\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} - \frac{\frac{a}{2}+\frac{b}{2}}{d\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)} + \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{a}{2}-\frac{b}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))/sin(c + d*x)^2,x)
```

```
[Out] (2*b*atanh(tan(c/2 + (d*x)/2)))/d - (a/2 + b/2)/(d*tan(c/2 + (d*x)/2)) + (t
an(c/2 + (d*x)/2)*(a/2 - b/2))/d
```

3.172 $\int \csc^4(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=69

$$\frac{b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a\cot(c+dx)}{d}-\frac{a\cot^3(c+dx)}{3d}-\frac{b\csc(c+dx)}{d}-\frac{b\csc^3(c+dx)}{3d}$$

```
[Out] b*arctanh(sin(d*x+c))/d-a*cot(d*x+c)/d-1/3*a*cot(d*x+c)^3/d-b*csc(d*x+c)/d-
1/3*b*csc(d*x+c)^3/d
```

Rubi [A]
time = 0.08, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3957, 2917, 2701, 308, 213, 3852}

$$-\frac{a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}-\frac{b\csc^3(c+dx)}{3d}-\frac{b\csc(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*(a + b*Sec[c + d*x]),x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]])/d - (a*Cot[c + d*x])/d - (a*Cot[c + d*x]^3)/(3*d)
 - (b*Csc[c + d*x])/d - (b*Csc[c + d*x]^3)/(3*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
```

```
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc^4(c+dx)(a+b\sec(c+dx))\,dx = -\int (-b-a\cos(c+dx))\csc^4(c+dx)\sec(c+dx)\,dx$$
$$= a\int \csc^4(c+dx)\,dx + b\int \csc^4(c+dx)\sec(c+dx)\,dx$$
$$= -\frac{a\text{Subst}\left(\int (1+x^2)\,dx, x, \cot(c+dx)\right)}{d} - \frac{b\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \text{cs}\right.}{d}$$
$$= -\frac{a\cot(c+dx)}{d} - \frac{a\cot^3(c+dx)}{3d} - \frac{b\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)\,dx,\right.}{d}$$
$$= -\frac{a\cot(c+dx)}{d} - \frac{a\cot^3(c+dx)}{3d} - \frac{b\csc(c+dx)}{d} - \frac{b\csc^3(c+dx)}{3d}$$
$$= \frac{b\tanh^{-1}(\sin(c+dx))}{d} - \frac{a\cot(c+dx)}{d} - \frac{a\cot^3(c+dx)}{3d} - \frac{b\csc(c+}{d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.02, size = 69, normalized size = 1.00

$$-\frac{2a\cot(c+dx)}{3d} - \frac{a\cot(c+dx)\csc^2(c+dx)}{3d} - \frac{b\csc^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\sin^2(c+dx)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*(a + b*Sec[c + d*x]),x]
[Out] (-2*a*Cot[c + d*x])/(3*d) - (a*Cot[c + d*x]*Csc[c + d*x]^2)/(3*d) - (b*Csc[
c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, Sin[c + d*x]^2])/(3*d)
```

Maple [A]
time = 0.09, size = 63, normalized size = 0.91

method	result
derivativedivides	$\frac{b\left(-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(-\frac{2}{3}-\frac{\left(\csc^2(dx+c)\right)}{3}\right)\cot(dx+c)}{d}$
default	$\frac{b\left(-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(-\frac{2}{3}-\frac{\left(\csc^2(dx+c)\right)}{3}\right)\cot(dx+c)}{d}$
risch	$-\frac{2i\left(3b\,e^{5i(dx+c)}-10b\,e^{3i(dx+c)}-6a\,e^{2i(dx+c)}+3b\,e^{i(dx+c)}+2a\right)}{3d\left(e^{2i(dx+c)}-1\right)^3}+\frac{b\ln\left(e^{i(dx+c)}+i\right)}{d}-\frac{b\ln\left(e^{i(dx+c)}-i\right)}{d}$
norman	$\frac{-\frac{a+b}{24d}+\frac{(a-b)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}+\frac{(3a-5b)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{(3a+5b)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{b\ln\left(\tan\left(\frac{dx}{2}\right.\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(b*(-1/3/sin(d*x+c)^3-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+a*(-2/3-1
/3*csc(d*x+c)^2)*cot(d*x+c))
```

Maxima [A]
time = 0.26, size = 76, normalized size = 1.10

$$-\frac{b\left(\frac{2\left(3\sin(dx+c)^2+1\right)}{\sin(dx+c)^3}-3\log\left(\sin\left(dx+c\right)+1\right)+3\log\left(\sin\left(dx+c\right)-1\right)\right)+\frac{2\left(3\tan(dx+c)^2+1\right)a}{\tan(dx+c)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="maxima")
[Out] -1/6*(b*(2*(3*sin(d*x + c)^2 + 1)/sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1)
+ 3*log(sin(d*x + c) - 1)) + 2*(3*tan(d*x + c)^2 + 1)*a/tan(d*x + c)^3)/d
```

Fricas [A]
time = 3.79, size = 125, normalized size = 1.81

$$-\frac{4\,a\cos\left(dx+c\right)^3+6\,b\cos\left(dx+c\right)^2-3\left(b\cos\left(dx+c\right)^2-b\right)\log\left(\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)+3\left(b\cos\left(dx+c\right)^2-b\right)\log\left(-\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)-6\,a\cos\left(dx+c\right)-8\,b}{6\left(d\cos\left(dx+c\right)^2-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="fricas")
[Out] -1/6*(4*a*cos(d*x + c)^3 + 6*b*cos(d*x + c)^2 - 3*(b*cos(d*x + c)^2 - b)*lo
g(sin(d*x + c) + 1)*sin(d*x + c) + 3*(b*cos(d*x + c)^2 - b)*log(-sin(d*x +
c) + 1)*sin(d*x + c) - 6*a*cos(d*x + c) - 8*b)/((d*cos(d*x + c)^2 - d)*sin(
d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))\csc^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*csc(c + d*x)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(65) = 130.
time = 0.47, size = 133, normalized size = 1.93

$$\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+24\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-24\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-15\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+15\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a+b}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1/2*d*x + 1/2*c)^3 + 24*b*log(abs(ta
n(1/2*d*x + 1/2*c) + 1)) - 24*b*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 9*a*ta
n(1/2*d*x + 1/2*c) - 15*b*tan(1/2*d*x + 1/2*c) - (9*a*tan(1/2*d*x + 1/2*c)^
2 + 15*b*tan(1/2*d*x + 1/2*c)^2 + a + b)/tan(1/2*d*x + 1/2*c)^3)/d
```

Mupad [B]
time = 1.00, size = 101, normalized size = 1.46

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{a}{24}-\frac{b}{24}\right)}{d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\left(3\,a+5\,b\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{a}{3}+\frac{b}{3}\right)}{8\,d}+\frac{2\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{3\,a}{8}-\frac{5\,b}{8}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))/sin(c + d*x)^4,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3*(a/24 - b/24))/d - (cot(c/2 + (d*x)/2)^3*(a/3 + b/3 +
 tan(c/2 + (d*x)/2)^2*(3*a + 5*b)))/(8*d) + (2*b*atanh(tan(c/2 + (d*x)/2)))
/d + (tan(c/2 + (d*x)/2)*((3*a)/8 - (5*b)/8))/d
```

3.173 $\int \csc^6(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=101

$$\frac{b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a\cot(c+dx)}{d}-\frac{2a\cot^3(c+dx)}{3d}-\frac{a\cot^5(c+dx)}{5d}-\frac{b\csc(c+dx)}{d}-\frac{b\csc^3(c+dx)}{3d}-\frac{b\csc$$

```
[Out] b*arctanh(sin(d*x+c))/d-a*cot(d*x+c)/d-2/3*a*cot(d*x+c)^3/d-1/5*a*cot(d*x+c
)^5/d-b*csc(d*x+c)/d-1/3*b*csc(d*x+c)^3/d-1/5*b*csc(d*x+c)^5/d
```

Rubi [A]
time = 0.08, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3957, 2917, 2701, 308, 213, 3852}

$$-\frac{a\cot^5(c+dx)}{5d}-\frac{2a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}-\frac{b\csc^5(c+dx)}{5d}-\frac{b\csc^3(c+dx)}{3d}-\frac{b\csc(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6*(a + b*Sec[c + d*x]),x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]])/d - (a*Cot[c + d*x])/d - (2*a*Cot[c + d*x]^3)/(3*
d) - (a*Cot[c + d*x]^5)/(5*d) - (b*Csc[c + d*x])/d - (b*Csc[c + d*x]^3)/(3*
d) - (b*Csc[c + d*x]^5)/(5*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
```

```
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc^6(c+dx)(a+b\sec(c+dx))\,dx = -\int(-b-a\cos(c+dx))\csc^6(c+dx)\sec(c+dx)\,dx$$
$$= a\int\csc^6(c+dx)\,dx + b\int\csc^6(c+dx)\sec(c+dx)\,dx$$
$$= -\frac{a\text{Subst}\left(\int\left(1+2x^2+x^4\right)\,dx,x,\cot(c+dx)\right)}{d} - \frac{b\text{Subst}\left(\int\frac{x^6}{-1+x^2}\,d\right.}{}$$
$$= -\frac{a\cot(c+dx)}{d} - \frac{2a\cot^3(c+dx)}{3d} - \frac{a\cot^5(c+dx)}{5d} - \frac{b\text{Subst}\left(\int(1\right.}{}$$
$$= -\frac{a\cot(c+dx)}{d} - \frac{2a\cot^3(c+dx)}{3d} - \frac{a\cot^5(c+dx)}{5d} - \frac{b\csc(c+dx)}{d}$$
$$= \frac{b\tanh^{-1}(\sin(c+dx))}{d} - \frac{a\cot(c+dx)}{d} - \frac{2a\cot^3(c+dx)}{3d} - \frac{a\cot^5(}{5}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.02, size = 91, normalized size = 0.90

$$-\frac{8a\cot(c+dx)}{15d} - \frac{4a\cot(c+dx)\csc^2(c+dx)}{15d} - \frac{a\cot(c+dx)\csc^4(c+dx)}{5d} - \frac{b\csc^5(c+dx)\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};\sin^2(c+dx)\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6*(a + b*Sec[c + d*x]),x]
```

```
[Out] (-8*a*Cot[c + d*x])/(15*d) - (4*a*Cot[c + d*x]*Csc[c + d*x]^2)/(15*d) - (a*
Cot[c + d*x]*Csc[c + d*x]^4)/(5*d) - (b*Csc[c + d*x]^5*Hypergeometric2F1[-5
/2, 1, -3/2, Sin[c + d*x]^2])/(5*d)
```

Maple [A]
time = 0.11, size = 83, normalized size = 0.82

method	result
derivativedivides	$\frac{b\left(-\frac{1}{5\sin(dx+c)^5}-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(-\frac{8}{15}-\frac{\left(\csc^4(dx+c)\right)}{5}-\frac{4\left(\csc^2(dx+c)\right)}{15}\right)\cot(dx-}{d}$
default	$\frac{b\left(-\frac{1}{5\sin(dx+c)^5}-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)+a\left(-\frac{8}{15}-\frac{\left(\csc^4(dx+c)\right)}{5}-\frac{4\left(\csc^2(dx+c)\right)}{15}\right)\cot(dx-}{d}$
risch	$-\frac{2i\left(15b\,e^{9i(dx+c)}-80b\,e^{7i(dx+c)}+178b\,e^{5i(dx+c)}+80a\,e^{4i(dx+c)}-80b\,e^{3i(dx+c)}-40a\,e^{2i(dx+c)}+15b\,e^{i(dx+c)}+8a\right)}{15d\left(e^{2i(dx+c)}-1\right)^5}-\frac{b\ln}{}$
norman	$\frac{-\frac{a+b}{160d}+\frac{(a-b)\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{160d}+\frac{(5a-11b)\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16d}+\frac{(5a-7b)\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96d}-\frac{(5a+7b)\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96d}-\frac{(5a+11b)\left(\tan^{4}\right.}{16d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6*(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b*(-1/5/sin(d*x+c)^5-1/3/sin(d*x+c)^3-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d
*x+c)))+a*(-8/15-1/5*csc(d*x+c)^4-4/15*csc(d*x+c)^2)*cot(d*x+c))
```

Maxima [A]
time = 0.27, size = 96, normalized size = 0.95

$$-\frac{b\left(\frac{2\left(15\sin(dx+c)^4+5\sin(dx+c)^2+3\right)}{\sin(dx+c)^5}-15\log\left(\sin\left(dx+c\right)+1\right)+15\log\left(\sin\left(dx+c\right)-1\right)\right)+\frac{2\left(15\tan(dx+c)^4+10\tan(dx+c)^2+3\right)a}{\tan(dx+c)^5}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/30*(b*(2*(15*sin(d*x + c)^4 + 5*sin(d*x + c)^2 + 3)/sin(d*x + c)^5 - 15*
log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) + 2*(15*tan(d*x + c)^4 +
10*tan(d*x + c)^2 + 3)*a/tan(d*x + c)^5)/d
```

Fricas [A]
time = 3.43, size = 174, normalized size = 1.72

$$-\frac{16\,a\cos\left(dx+c\right)^5+30\,b\cos\left(dx+c\right)^4-40\,a\cos\left(dx+c\right)^3-70\,b\cos\left(dx+c\right)^2-15\left(b\cos\left(dx+c\right)^4-2\,b\cos\left(dx+c\right)^2+b\right)\log\left(\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)+15\left(b\cos\left(dx+c\right)^4-2\,b\cos\left(dx+c\right)^2+b\right)\log\left(-\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)+30\,a\cos\left(dx+c\right)+46\,b}{30\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^2+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/30*(16*a*cos(d*x + c)^5 + 30*b*cos(d*x + c)^4 - 40*a*cos(d*x + c)^3 - 70
*b*cos(d*x + c)^2 - 15*(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + b)*log(sin(
d*x + c) + 1)*sin(d*x + c) + 15*(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + b)
```

```
*log(-sin(d*x + c) + 1)*sin(d*x + c) + 30*a*cos(d*x + c) + 46*b)/((d*cos(d*
x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec (c+dx))\csc^6 (c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*csc(c + d*x)**6, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 194 vs. 2(93) = 186.
time = 0.46, size = 194, normalized size = 1.92

$$\frac{3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+25\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-35\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+480\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-480\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+150\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-330\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{150\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+330\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+25\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+35\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3\,a+3\,b}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/480*(3*a*tan(1/2*d*x + 1/2*c)^5 - 3*b*tan(1/2*d*x + 1/2*c)^5 + 25*a*tan(1
/2*d*x + 1/2*c)^3 - 35*b*tan(1/2*d*x + 1/2*c)^3 + 480*b*log(abs(tan(1/2*d*x
 + 1/2*c) + 1)) - 480*b*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 150*a*tan(1/2*
d*x + 1/2*c) - 330*b*tan(1/2*d*x + 1/2*c) - (150*a*tan(1/2*d*x + 1/2*c)^4 +
 330*b*tan(1/2*d*x + 1/2*c)^4 + 25*a*tan(1/2*d*x + 1/2*c)^2 + 35*b*tan(1/2*
d*x + 1/2*c)^2 + 3*a + 3*b)/tan(1/2*d*x + 1/2*c)^5)/d
```

Mupad [B]
time = 1.09, size = 142, normalized size = 1.41

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\,\left(\frac{5\,a}{96}-\frac{7\,b}{96}\right)}{d}-\frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^5\,\left(\left(10\,a+22\,b\right)\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+\left(\frac{5\,a}{3}+\frac{7\,b}{3}\right)\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\frac{a}{5}+\frac{b}{5}\right)}{32\,d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\,\left(\frac{a}{160}-\frac{b}{160}\right)}{d}+\frac{2\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(\frac{5\,a}{16}-\frac{11\,b}{16}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))/sin(c + d*x)^6,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3*((5*a)/96 - (7*b)/96))/d - (cot(c/2 + (d*x)/2)^5*(a/5
 + b/5 + tan(c/2 + (d*x)/2)^2*((5*a)/3 + (7*b)/3) + tan(c/2 + (d*x)/2)^4*(1
0*a + 22*b)))/(32*d) + (tan(c/2 + (d*x)/2)^5*(a/160 - b/160))/d + (2*b*atan
h(tan(c/2 + (d*x)/2)))/d + (tan(c/2 + (d*x)/2)*((5*a)/16 - (11*b)/16))/d
```

3.174 $\int (a+b\sec(c+dx))^2\sin^5(c+dx)\,dx$

Optimal. Leaf size=124

$$-\frac{(a^2-2b^2)\cos(c+dx)}{d}+\frac{2ab\cos^2(c+dx)}{d}+\frac{(2a^2-b^2)\cos^3(c+dx)}{3d}-\frac{ab\cos^4(c+dx)}{2d}-\frac{a^2\cos^5(c+dx)}{5d}-\frac{2ab}{}$$

```
[Out] -(a^2-2*b^2)*cos(d*x+c)/d+2*a*b*cos(d*x+c)^2/d+1/3*(2*a^2-b^2)*cos(d*x+c)^3
/d-1/2*a*b*cos(d*x+c)^4/d-1/5*a^2*cos(d*x+c)^5/d-2*a*b*ln(cos(d*x+c))/d+b^2
*sec(d*x+c)/d
```

Rubi [A]
time = 0.14, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3957, 2916, 12, 962}

$$\frac{(2a^2-b^2)\cos^3(c+dx)}{3d}-\frac{(a^2-2b^2)\cos(c+dx)}{d}-\frac{a^2\cos^5(c+dx)}{5d}-\frac{ab\cos^4(c+dx)}{2d}+\frac{2ab\cos^2(c+dx)}{d}-\frac{2ab\log(\cos(c+dx))}{d}+\frac{b^2\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2*Sin[c + d*x]^5,x]
```

```
[Out] -(((a^2 - 2*b^2)*Cos[c + d*x])/d) + (2*a*b*Cos[c + d*x]^2)/d + ((2*a^2 - b^
2)*Cos[c + d*x]^3)/(3*d) - (a*b*Cos[c + d*x]^4)/(2*d) - (a^2*Cos[c + d*x]^5
)/(5*d) - (2*a*b*Log[Cos[c + d*x]])/d + (b^2*Sec[c + d*x])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 962

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int (a+b\sec(c+dx))^2\sin^5(c+dx)\,dx &= \int (-b-a\cos(c+dx))^2\sin^3(c+dx)\tan^2(c+dx)\,dx\\ &= \frac{\text{Subst}\left(\int \frac{a^2(-b+x)^2(a^2-x^2)^2}{x^2}\,dx, x, -a\cos(c+dx)\right)}{a^5 d}\\ &= \frac{\text{Subst}\left(\int \frac{(-b+x)^2(a^2-x^2)^2}{x^2}\,dx, x, -a\cos(c+dx)\right)}{a^3 d}\\ &= \frac{\text{Subst}\left(\int \left(a^4\left(1-\frac{2b^2}{a^2}\right)+\frac{a^4b^2}{x^2}-\frac{2a^4b}{x}+4a^2bx-(2a^2-b^2)x^2-2bx\right.\right.}{a^3 d}\\ &= -\frac{(a^2-2b^2)\cos(c+dx)}{d}+\frac{2ab\cos^2(c+dx)}{d}+\frac{(2a^2-b^2)\cos^3(c+\right.}{3d}\end{aligned}$$

Mathematica [A]

time = 0.24, size = 112, normalized size = 0.90

$$-\frac{30(5a^2-14b^2)\cos(c+dx)-180ab\cos(2(c+dx))-25a^2\cos(3(c+dx))+20b^2\cos(3(c+dx))+15ab\cos(4(c+dx))+3a^2\cos(5(c+dx))+480ab\log(\cos(c+dx))-240b^2\sec(c+dx)}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Sin[c + d*x]^5,x]
```

```
[Out] -1/240*(30*(5*a^2 - 14*b^2)*Cos[c + d*x] - 180*a*b*Cos[2*(c + d*x)] - 25*a^
2*Cos[3*(c + d*x)] + 20*b^2*Cos[3*(c + d*x)] + 15*a*b*Cos[4*(c + d*x)] + 3*
a^2*Cos[5*(c + d*x)] + 480*a*b*Log[Cos[c + d*x]] - 240*b^2*Sec[c + d*x])/d
```

Maple [A]

time = 0.11, size = 120, normalized size = 0.97

method	result
derivativedivides	$\frac{b^2\left(\frac{\sin^6(dx+c)}{\cos(dx+c)}+\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)\right)+2ba\left(-\frac{\left(\sin^4(dx+c)\right)}{4}-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)-}{d}$
default	$\frac{b^2\left(\frac{\sin^6(dx+c)}{\cos(dx+c)}+\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)\right)+2ba\left(-\frac{\left(\sin^4(dx+c)\right)}{4}-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)-}{d}$

norman	$\frac{\frac{16a^2-80b^2}{15d}-\frac{32(a^2+b^2)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{4ba\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{16ba\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4(16a^2+15ba-80b^2)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}+\left(\right.}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}$
risch	$2iabx+\frac{3ba\,e^{2i(dx+c)}}{8d}-\frac{5a^2e^{i(dx+c)}}{16d}+\frac{7\,e^{i(dx+c)}b^2}{8d}-\frac{5a^2e^{-i(dx+c)}}{16d}+\frac{7\,e^{-i(dx+c)}b^2}{8d}+\frac{3ba\,e^{-2i(dx+c)}}{8d}+\frac{4ib}{c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*sin(d*x+c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^2*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d
*x+c))+2*b*a*(-1/4*sin(d*x+c)^4-1/2*sin(d*x+c)^2-ln(cos(d*x+c)))-1/5*a^2*(8
/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))
```

Maxima [A]
time = 0.26, size = 105, normalized size = 0.85

$$-\frac{6\,a^2\cos\left(dx+c\right)^5+15\,ab\cos\left(dx+c\right)^4-60\,ab\cos\left(dx+c\right)^2-10\,(2\,a^2-b^2)\cos\left(dx+c\right)^3+60\,ab\log\left(\cos\left(dx+c\right)\right)+30\,(a^2-2\,b^2)\cos\left(dx+c\right)-\frac{30\,b^2}{\cos(dx+c)}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/30*(6*a^2*cos(d*x + c)^5 + 15*a*b*cos(d*x + c)^4 - 60*a*b*cos(d*x + c)^2
 - 10*(2*a^2 - b^2)*cos(d*x + c)^3 + 60*a*b*log(cos(d*x + c)) + 30*(a^2 - 2
*b^2)*cos(d*x + c) - 30*b^2/cos(d*x + c))/d
```

Fricas [A]
time = 3.05, size = 125, normalized size = 1.01

$$-\frac{48\,a^2\cos\left(dx+c\right)^6+120\,ab\cos\left(dx+c\right)^5-480\,ab\cos\left(dx+c\right)^3-80\,(2\,a^2-b^2)\cos\left(dx+c\right)^4+480\,ab\cos\left(dx+c\right)\log\left(-\cos\left(dx+c\right)\right)+195\,ab\cos\left(dx+c\right)+240\,(a^2-2\,b^2)\cos\left(dx+c\right)^2-240\,b^2}{240\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/240*(48*a^2*cos(d*x + c)^6 + 120*a*b*cos(d*x + c)^5 - 480*a*b*cos(d*x +
c)^3 - 80*(2*a^2 - b^2)*cos(d*x + c)^4 + 480*a*b*cos(d*x + c)*log(-cos(d*x
+ c)) + 195*a*b*cos(d*x + c) + 240*(a^2 - 2*b^2)*cos(d*x + c)^2 - 240*b^2)/
(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec\left(c+dx\right)\right)^2\sin^5\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*sin(d*x+c)**5,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*sin(c + d*x)**5, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 418 vs. 2(118) = 236.
time = 0.51, size = 418, normalized size = 3.37

$$\frac{60\,ab\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-60\,ab\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{60\left(ab+b^2+\frac{ab(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1}+\frac{32\,a^2+137\,ab-100\,b^2-\frac{160\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{805\,ab(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{440\,b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{320\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{1970\,ab(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{640\,b^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{1970\,ab(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{360\,b^2(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{805\,ab(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}-\frac{60\,b^2(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}-\frac{137\,ab(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^5}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/30*(60*a*b*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 60*a*b*
log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + 60*(a*b + b^2 + a*b*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c) +
1) + 1) + (32*a^2 + 137*a*b - 100*b^2 - 160*a^2*(cos(d*x + c) - 1)/(cos(d*x
 + c) + 1) - 805*a*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 440*b^2*(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) + 320*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c
) + 1)^2 + 1970*a*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 640*b^2*(co
s(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 1970*a*b*(cos(d*x + c) - 1)^3/(cos
(d*x + c) + 1)^3 + 360*b^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 805*
a*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 60*b^2*(cos(d*x + c) - 1)^4
/(cos(d*x + c) + 1)^4 - 137*a*b*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5)/
((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^5)/d
```

Mupad [B]
time = 0.95, size = 104, normalized size = 0.84

$$-\frac{\cos\left(c+d\,x\right)\,\left(a^2-2\,b^2\right)-{\cos\left(c+d\,x\right)}^3\,\left(\frac{2\,a^2}{3}-\frac{b^2}{3}\right)+\frac{a^2\,{\cos\left(c+d\,x\right)}^5}{5}-\frac{b^2}{\cos\left(c+d\,x\right)}-2\,a\,b\,{\cos\left(c+d\,x\right)}^2+\frac{a\,b\,{\cos\left(c+d\,x\right)}^4}{2}+2\,a\,b\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5*(a + b/cos(c + d*x))^2,x)
```

```
[Out] -(cos(c + d*x)*(a^2 - 2*b^2) - cos(c + d*x)^3*((2*a^2)/3 - b^2/3) + (a^2*co
s(c + d*x)^5)/5 - b^2/cos(c + d*x) - 2*a*b*cos(c + d*x)^2 + (a*b*cos(c + d*
x)^4)/2 + 2*a*b*log(cos(c + d*x)))/d
```

3.175 $\int (a+b\sec(c+dx))^2 \sin^3(c+dx)\,dx$

Optimal. Leaf size=80

$$-\frac{(a^2-b^2)\cos(c+dx)}{d}+\frac{ab\cos^2(c+dx)}{d}+\frac{a^2\cos^3(c+dx)}{3d}-\frac{2ab\log(\cos(c+dx))}{d}+\frac{b^2\sec(c+dx)}{d}$$

```
[Out] -(a^2-b^2)*cos(d*x+c)/d+a*b*cos(d*x+c)^2/d+1/3*a^2*cos(d*x+c)^3/d-2*a*b*ln(
cos(d*x+c))/d+b^2*sec(d*x+c)/d
```

Rubi [A]
time = 0.11, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3957, 2916, 12, 908}

$$-\frac{(a^2-b^2)\cos(c+dx)}{d}+\frac{a^2\cos^3(c+dx)}{3d}+\frac{ab\cos^2(c+dx)}{d}-\frac{2ab\log(\cos(c+dx))}{d}+\frac{b^2\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2*Sin[c + d*x]^3,x]
```

```
[Out] -(((a^2 - b^2)*Cos[c + d*x])/d) + (a*b*Cos[c + d*x]^2)/d + (a^2*Cos[c + d*x
]^3)/(3*d) - (2*a*b*Log[Cos[c + d*x]])/d + (b^2*Sec[c + d*x])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int (a+b\sec(c+dx))^2\sin^3(c+dx)\,dx &= \int (-b-a\cos(c+dx))^2\sin(c+dx)\tan^2(c+dx)\,dx\\ &= \frac{\text{Subst}\left(\int \frac{a^2(-b+x)^2(a^2-x^2)}{x^2}\,dx, x, -a\cos(c+dx)\right)}{a^3d}\\ &= \frac{\text{Subst}\left(\int \frac{(-b+x)^2(a^2-x^2)}{x^2}\,dx, x, -a\cos(c+dx)\right)}{ad}\\ &= \frac{\text{Subst}\left(\int \left(a^2\left(1-\frac{b^2}{a^2}\right)+\frac{a^2b^2}{x^2}-\frac{2a^2b}{x}+2bx-x^2\right)\,dx, x, -a\cos(c+\right.}{ad}\\ &= -\frac{(a^2-b^2)\cos(c+dx)}{d}+\frac{ab\cos^2(c+dx)}{d}+\frac{a^2\cos^3(c+dx)}{3d}-\frac{2ab}{}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 72, normalized size = 0.90

$$\frac{(-9a^2+12b^2)\cos(c+dx)+6ab\cos(2(c+dx))+a^2\cos(3(c+dx))-24ab\log(\cos(c+dx))+12b^2\sec(c+dx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Sin[c + d*x]^3,x]
[Out] ((-9*a^2 + 12*b^2)*Cos[c + d*x] + 6*a*b*Cos[2*(c + d*x)] + a^2*Cos[3*(c + d
*x)] - 24*a*b*Log[Cos[c + d*x]] + 12*b^2*Sec[c + d*x])/(12*d)
```

Maple [A]
time = 0.09, size = 90, normalized size = 1.12

method	result
derivativedivides	$\frac{b^2\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)+2ba\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)-\frac{a^2\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}}{d}$
default	$\frac{b^2\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)+2ba\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)-\frac{a^2\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}}{d}$
norman	$\frac{\frac{4a^2-12b^2}{3d}-\frac{4\left(a^2+b^2\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{4ba\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2\left(4a^2+6ba-12b^2\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{2ba\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}$

risch	$2iabx+\frac{ba\,e^{2i(dx+c)}}{4d}-\frac{3a^2e^{i(dx+c)}}{8d}+\frac{e^{i(dx+c)}b^2}{2d}-\frac{3a^2e^{-i(dx+c)}}{8d}+\frac{e^{-i(dx+c)}b^2}{2d}+\frac{ba\,e^{-2i(dx+c)}}{4d}+\frac{4ibac}{d}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*sin(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^2*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+2*b*a*(-1/2*
sin(d*x+c)^2-ln(cos(d*x+c)))-1/3*a^2*(2+sin(d*x+c)^2)*cos(d*x+c))
```

Maxima [A]
time = 0.27, size = 71, normalized size = 0.89

$$\frac{a^2\cos\left(dx+c\right)^3+3\,ab\cos\left(dx+c\right)^2-6\,ab\log\left(\cos\left(dx+c\right)\right)-3\left(a^2-b^2\right)\cos\left(dx+c\right)+\frac{3\,b^2}{\cos(dx+c)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/3*(a^2*cos(d*x + c)^3 + 3*a*b*cos(d*x + c)^2 - 6*a*b*log(cos(d*x + c)) -
3*(a^2 - b^2)*cos(d*x + c) + 3*b^2/cos(d*x + c))/d
```

Fricas [A]
time = 2.82, size = 92, normalized size = 1.15

$$\frac{2\,a^2\cos\left(dx+c\right)^4+6\,ab\cos\left(dx+c\right)^3-12\,ab\cos\left(dx+c\right)\log\left(-\cos\left(dx+c\right)\right)-3\,ab\cos\left(dx+c\right)-6\left(a^2-b^2\right)\cos\left(dx+c\right)^2+6\,b^2}{6\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a^2*cos(d*x + c)^4 + 6*a*b*cos(d*x + c)^3 - 12*a*b*cos(d*x + c)*log(
-cos(d*x + c)) - 3*a*b*cos(d*x + c) - 6*(a^2 - b^2)*cos(d*x + c)^2 + 6*b^2)
/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec\left(c+dx\right)\right)^2\sin^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*sin(d*x+c)**3,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*sin(c + d*x)**3, x)
```

Giac [A]
time = 0.52, size = 100, normalized size = 1.25

$$-\frac{2\,ab\log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d}+\frac{b^2}{d\cos\left(dx+c\right)}+\frac{a^2d^5\cos\left(dx+c\right)^3+3\,abd^5\cos\left(dx+c\right)^2-3\,a^2d^5\cos\left(dx+c\right)+3\,b^2d^5\cos\left(dx+c\right)}{3\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -2*a*b*log(abs(cos(d*x + c))/abs(d))/d + b^2/(d*cos(d*x + c)) + 1/3*(a^2*d^
5*cos(d*x + c)^3 + 3*a*b*d^5*cos(d*x + c)^2 - 3*a^2*d^5*cos(d*x + c) + 3*b^
2*d^5*cos(d*x + c))/d^6
```

Mupad [B]
time = 0.92, size = 69, normalized size = 0.86

$$\frac{\frac{a^2\cos(c+d\,x)^3}{3}-\cos\left(c+d\,x\right)\,\left(a^2-b^2\right)+\frac{b^2}{\cos(c+d\,x)}+a\,b\cos\left(c+d\,x\right)^2-2\,a\,b\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3*(a + b/cos(c + d*x))^2,x)
```

```
[Out] ((a^2*cos(c + d*x)^3)/3 - cos(c + d*x)*(a^2 - b^2) + b^2/cos(c + d*x) + a*b
*cos(c + d*x)^2 - 2*a*b*log(cos(c + d*x)))/d
```

3.176 $\int (a+b\sec(c+dx))^2\sin(c+dx)\,dx$

Optimal. Leaf size=42

$$-\frac{a^2\cos(c+dx)}{d}-\frac{2ab\log(\cos(c+dx))}{d}+\frac{b^2\sec(c+dx)}{d}$$

```
[Out] -a^2*cos(d*x+c)/d-2*a*b*ln(cos(d*x+c))/d+b^2*sec(d*x+c)/d
```

Rubi [A]
time = 0.06, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3957, 2912, 12, 45}

$$-\frac{a^2\cos(c+dx)}{d}-\frac{2ab\log(\cos(c+dx))}{d}+\frac{b^2\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2*Sin[c + d*x],x]
```

```
[Out] -((a^2*Cos[c + d*x])/d) - (2*a*b*Log[Cos[c + d*x]])/d + (b^2*Sec[c + d*x])/
d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int (a+b\sec(c+dx))^2\sin(c+dx)\,dx &= \int (-b-a\cos(c+dx))^2\sec(c+dx)\tan(c+dx)\,dx\\
&= \frac{\text{Subst}\left(\int \frac{a^2(-b+x)^2}{x^2}\,dx, x, -a\cos(c+dx)\right)}{ad}\\
&= \frac{a\text{Subst}\left(\int \frac{(-b+x)^2}{x^2}\,dx, x, -a\cos(c+dx)\right)}{d}\\
&= \frac{a\text{Subst}\left(\int \left(1+\frac{b^2}{x^2}-\frac{2b}{x}\right)\,dx, x, -a\cos(c+dx)\right)}{d}\\
&= -\frac{a^2\cos(c+dx)}{d}-\frac{2ab\log(\cos(c+dx))}{d}+\frac{b^2\sec(c+dx)}{d}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 37, normalized size = 0.88

$$
\frac{-a^2\cos(c+dx)+b(-2a\log(\cos(c+dx))+b\sec(c+dx))}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Sin[c + d*x],x]
[Out] (-(a^2*Cos[c + d*x]) + b*(-2*a*Log[Cos[c + d*x]] + b*Sec[c + d*x]))/d
```

Maple [A]
time = 0.05, size = 40, normalized size = 0.95

method	result
derivativedivides	$\frac{b^2\sec(dx+c)-\frac{a^2}{\sec(dx+c)}+2ba\ln(\sec(dx+c))}{d}$
default	$\frac{b^2\sec(dx+c)-\frac{a^2}{\sec(dx+c)}+2ba\ln(\sec(dx+c))}{d}$
risch	$2iabx-\frac{a^2e^{i(dx+c)}}{2d}-\frac{a^2e^{-i(dx+c)}}{2d}+\frac{4ibac}{d}+\frac{2b^2e^{i(dx+c)}}{d(e^{2i(dx+c)}+1)}-\frac{2ba\ln(e^{2i(dx+c)}+1)}{d}$
norman	$\frac{\frac{2a^2-2b^2}{d}-\frac{2(a^2+b^2)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{2ba\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{2ba\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}+\frac{2ba\ln\left(1+\tan^2\left(\right.\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*sin(d*x+c),x,method=_RETURNVERBOSE)
[Out] 1/d*(b^2*sec(d*x+c)-a^2/sec(d*x+c)+2*b*a*ln(sec(d*x+c)))
```

Maxima [A]
time = 0.28, size = 40, normalized size = 0.95

$$-\frac{a^2\cos(dx+c)+2ab\log(\cos(dx+c))-\frac{b^2}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -(a^2*cos(d*x + c) + 2*a*b*log(cos(d*x + c)) - b^2/cos(d*x + c))/d
```

Fricas [A]
time = 2.92, size = 50, normalized size = 1.19

$$-\frac{a^2\cos(dx+c)^2+2ab\cos(dx+c)\log(-\cos(dx+c))-b^2}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c)*log(-cos(d*x + c)) - b^2)/(d*cos(
d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^2\sin(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*sin(d*x+c),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*sin(c + d*x), x)
```

Giac [A]
time = 0.47, size = 50, normalized size = 1.19

$$-\frac{a^2\cos(dx+c)}{d}-\frac{2ab\log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d}+\frac{b^2}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -a^2*cos(d*x + c)/d - 2*a*b*log(abs(cos(d*x + c))/abs(d))/d + b^2/(d*cos(d*
x + c))
```

Mupad [B]
time = 0.05, size = 40, normalized size = 0.95

$$-\frac{a^2\,\cos\left(c+d\,x\right)-\frac{b^2}{\cos\left(c+d\,x\right)}+2\,a\,b\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a + b/cos(c + d*x))^2,x)
[Out] -(a^2*cos(c + d*x) - b^2/cos(c + d*x) + 2*a*b*log(cos(c + d*x)))/d
```

3.177 $\int \csc(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=74

$$\frac{(a+b)^2\log(1-\cos(c+dx))}{2d}-\frac{2ab\log(\cos(c+dx))}{d}-\frac{(a-b)^2\log(1+\cos(c+dx))}{2d}+\frac{b^2\sec(c+dx)}{d}$$

```
[Out] 1/2*(a+b)^2*ln(1-cos(d*x+c))/d-2*a*b*ln(cos(d*x+c))/d-1/2*(a-b)^2*ln(1+cos(
d*x+c))/d+b^2*sec(d*x+c)/d
```

Rubi [A]
time = 0.13, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3957, 2916, 12, 1816}

$$-\frac{(a-b)^2\log(\cos(c+dx)+1)}{2d}+\frac{(a+b)^2\log(1-\cos(c+dx))}{2d}-\frac{2ab\log(\cos(c+dx))}{d}+\frac{b^2\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((a + b)^2*Log[1 - Cos[c + d*x]])/(2*d) - (2*a*b*Log[Cos[c + d*x]])/d - ((a
 - b)^2*Log[1 + Cos[c + d*x]])/(2*d) + (b^2*Sec[c + d*x])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
```

```
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \csc(c+dx)(a+b\sec(c+dx))^2\,dx &= \int (-b-a\cos(c+dx))^2\csc(c+dx)\sec^2(c+dx)\,dx\\ &= \frac{a\text{Subst}\left(\int \frac{a^2(-b+x)^2}{x^2(a^2-x^2)}\,dx, x, -a\cos(c+dx)\right)}{d}\\ &= \frac{a^3\text{Subst}\left(\int \frac{(-b+x)^2}{x^2(a^2-x^2)}\,dx, x, -a\cos(c+dx)\right)}{d}\\ &= \frac{a^3\text{Subst}\left(\int \left(\frac{(a-b)^2}{2a^3(a-x)}+\frac{b^2}{a^2x^2}-\frac{2b}{a^2x}+\frac{(a+b)^2}{2a^3(a+x)}\right)dx, x, -a\cos(c+dx)\right)}{d}\\ &= \frac{(a+b)^2\log(1-\cos(c+dx))}{2d}-\frac{2ab\log(\cos(c+dx))}{d}-\frac{(a-b)^2\log}{}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 91, normalized size = 1.23

$$\frac{-(a-b)^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-2ab\log(\cos(c+dx))+a^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+2ab\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+b^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+b^2\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*(a + b*Sec[c + d*x])^2,x]
[Out] (-((a - b)^2*Log[Cos[(c + d*x)/2]]) - 2*a*b*Log[Cos[c + d*x]] + a^2*Log[Sin
[(c + d*x)/2]] + 2*a*b*Log[Sin[(c + d*x)/2]] + b^2*Log[Sin[(c + d*x)/2]] +
b^2*Sec[c + d*x])/d
```

Maple [A]
time = 0.09, size = 66, normalized size = 0.89

method	result
derivativedivides	$\frac{b^2\left(\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c))\right)+2ba\ln(\tan(dx+c))+a^2\ln(\csc(dx+c)-\cot(dx+c))}{d}$
default	$\frac{b^2\left(\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c))\right)+2ba\ln(\tan(dx+c))+a^2\ln(\csc(dx+c)-\cot(dx+c))}{d}$
norman	$-\frac{2b^2}{d\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{(a^2+2ba+b^2)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2ba\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{2ba\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}$
risch	$\frac{2b^2e^{i(dx+c)}}{d(e^{2i(dx+c)}+1)}-\frac{a^2\ln(e^{i(dx+c)}+1)}{d}+\frac{2\ln(e^{i(dx+c)}+1)ba}{d}-\frac{\ln(e^{i(dx+c)}+1)b^2}{d}+\frac{a^2\ln(e^{i(dx+c)}-1)}{d}+\frac{2\ln(e^{i}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^2*(1/cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))+2*b*a*ln(tan(d*x+c))+a^2*
ln(csc(d*x+c)-cot(d*x+c)))
```

Maxima [A]
time = 0.26, size = 73, normalized size = 0.99

$$-\frac{4\,ab\log\left(\cos\left(dx+c\right)\right)+\left(a^2-2\,ab+b^2\right)\log\left(\cos\left(dx+c\right)+1\right)-\left(a^2+2\,ab+b^2\right)\log\left(\cos\left(dx+c\right)-1\right)-\frac{2\,b^2}{\cos(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(4*a*b*log(cos(d*x + c)) + (a^2 - 2*a*b + b^2)*log(cos(d*x + c) + 1) -
 (a^2 + 2*a*b + b^2)*log(cos(d*x + c) - 1) - 2*b^2/cos(d*x + c))/d
```

Fricas [A]
time = 3.25, size = 97, normalized size = 1.31

$$-\frac{4\,ab\cos\left(dx+c\right)\log\left(-\cos\left(dx+c\right)\right)+\left(a^2-2\,ab+b^2\right)\cos\left(dx+c\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-\left(a^2+2\,ab+b^2\right)\cos\left(dx+c\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-2\,b^2}{2\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(4*a*b*cos(d*x + c)*log(-cos(d*x + c)) + (a^2 - 2*a*b + b^2)*cos(d*x +
 c)*log(1/2*cos(d*x + c) + 1/2) - (a^2 + 2*a*b + b^2)*cos(d*x + c)*log(-1/2
*cos(d*x + c) + 1/2) - 2*b^2)/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec\left(c+dx\right)\right)^2\csc\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*csc(c + d*x), x)
```

Giac [A]
time = 0.49, size = 124, normalized size = 1.68

$$-\frac{4\,ab\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)-\left(a^2+2\,ab+b^2\right)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-\frac{4\left(ab+b^2+\frac{ab(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*a*b*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - (a^2 + 2
*a*b + b^2)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 4*(a*b + b^
2 + a*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x
 + c) + 1) + 1))/d
```

Mupad [B]
time = 0.97, size = 62, normalized size = 0.84

$$\frac{\frac{\ln(\cos(c+d\,x)-1)\,(a+b)^2}{2}-\frac{\ln(\cos(c+d\,x)+1)\,(a-b)^2}{2}+\frac{b^2}{\cos(c+d\,x)}-2\,a\,b\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^2/sin(c + d*x),x)
```

```
[Out] ((log(cos(c + d*x) - 1)*(a + b)^2)/2 - (log(cos(c + d*x) + 1)*(a - b)^2)/2
+ b^2/cos(c + d*x) - 2*a*b*log(cos(c + d*x)))/d
```

3.178 $\int \csc^3(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=114

$$-\frac{(2ab+(a^2+b^2)\cos(c+dx))\csc^2(c+dx)}{2d}+\frac{(a+b)(a+3b)\log(1-\cos(c+dx))}{4d}-\frac{2ab\log(\cos(c+dx))}{d}-\frac{(a$$

```
[Out] -1/2*(2*a*b+(a^2+b^2)*cos(d*x+c))*csc(d*x+c)^2/d+1/4*(a+b)*(a+3*b)*ln(1-cos
(d*x+c))/d-2*a*b*ln(cos(d*x+c))/d-1/4*(a-3*b)*(a-b)*ln(1+cos(d*x+c))/d+b^2*
sec(d*x+c)/d
```

Rubi [A]

time = 0.21, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3957, 2916, 12, 1819, 1816}

$$-\frac{\csc^2(c+dx)\left((a^2+b^2)\cos(c+dx)+2ab\right)}{2d}-\frac{2ab\log(\cos(c+dx))}{d}+\frac{(a+b)(a+3b)\log(1-\cos(c+dx))}{4d}-\frac{(a-3b)(a-b)\log(\cos(c+dx)+1)}{4d}+\frac{b^2\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -1/2*((2*a*b + (a^2 + b^2)*Cos[c + d*x])*Csc[c + d*x]^2)/d + ((a + b)*(a +
3*b)*Log[1 - Cos[c + d*x]])/(4*d) - (2*a*b*Log[Cos[c + d*x]])/d - ((a - 3*b
)*(a - b)*Log[1 + Cos[c + d*x]])/(4*d) + (b^2*Sec[c + d*x])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc^3(c+dx)(a+b\sec(c+dx))^2\,dx = \int (-b-a\cos(c+dx))^2\csc^3(c+dx)\sec^2(c+dx)\,dx$$
$$= \frac{a^3\text{Subst}\left(\int \frac{a^2(-b+x)^2}{x^2(a^2-x^2)^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^5\text{Subst}\left(\int \frac{(-b+x)^2}{x^2(a^2-x^2)^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= -\frac{a\left(2b+\frac{(a^2+b^2)\cos(c+dx)}{a}\right)\csc^2(c+dx)}{2d} - \frac{a^3\text{Subst}\left(\int \frac{-2b^2+4bx-\frac{(a^2+}{}}{x^2(a^2-x^2)}\right.}{}$$
$$= -\frac{a\left(2b+\frac{(a^2+b^2)\cos(c+dx)}{a}\right)\csc^2(c+dx)}{2d} - \frac{a^3\text{Subst}\left(\int \left(\frac{(a-3b)(-a+b)}{2a^3(a-x)}\right.\right.}{}$$
$$= -\frac{a\left(2b+\frac{(a^2+b^2)\cos(c+dx)}{a}\right)\csc^2(c+dx)}{2d} + \frac{(a+b)(a+3b)\log(1-c}{4d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 329 vs. 2(114) = 228.
time = 0.41, size = 329, normalized size = 2.89

$$-\frac{\csc^4(c+dx)\left(2a^2-2b^2+2(a^2+3b^2)\cos(2(c+dx))-a^2\cos(3(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+4ab\cos(3(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-3b^2\cos(3(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-4ab\cos(3(c+dx))\log(\cos(c+dx))+a^2\cos(3(c+dx))\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+4ab\cos(3(c+dx))\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+3b^2\cos(3(c+dx))\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\cos(c+dx)\left(8ab+(a^2-4ab+3b^2)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+4ab\log(\cos(c+dx))-a^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-4ab\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-3b^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\right)}{2d\left(\csc^2\left(\frac{1}{2}(c+dx)\right)-\sec^2\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -1/2*(Csc[c + d*x]^4*(2*a^2 - 2*b^2 + 2*(a^2 + 3*b^2)*Cos[2*(c + d*x)] - a^
2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 4*a*b*Cos[3*(c + d*x)]*Log[Cos[(
```

```
c + d*x)/2]] - 3*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2]] - 4*a*b*Cos[3*(
c + d*x)]*Log[Cos[c + d*x]] + a^2*Cos[3*(c + d*x)]*Log[Sin[(c + d*x)/2]] +
4*a*b*Cos[3*(c + d*x)]*Log[Sin[(c + d*x)/2]] + 3*b^2*Cos[3*(c + d*x)]*Log[S
in[(c + d*x)/2]] + Cos[c + d*x]*(8*a*b + (a^2 - 4*a*b + 3*b^2)*Log[Cos[(c +
 d*x)/2]] + 4*a*b*Log[Cos[c + d*x]] - a^2*Log[Sin[(c + d*x)/2]] - 4*a*b*Log
[Sin[(c + d*x)/2]] - 3*b^2*Log[Sin[(c + d*x)/2]])))/(d*(Csc[(c + d*x)/2]^2
- Sec[(c + d*x)/2]^2))
```

Maple [A]
time = 0.12, size = 116, normalized size = 1.02

method	result
derivativedivides	$\frac{b^2\left(-\frac{1}{2\sin(dx+c)^2\cos(dx+c)}+\frac{3}{2\cos(dx+c)}+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+2ba\left(-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+a^2\left(-\frac{\csc(dx+\cdots}{}\right.}{d}$
default	$\frac{b^2\left(-\frac{1}{2\sin(dx+c)^2\cos(dx+c)}+\frac{3}{2\cos(dx+c)}+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+2ba\left(-\frac{1}{2\sin(dx+c)^2}+\ln(\tan(dx+c))\right)+a^2\left(-\frac{\csc(dx+\cdots}{}\right.}{d}$
norman	$\frac{\frac{a^2+2ba+b^2}{8d}+\frac{(a^2-2ba+b^2)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{(a^2+9b^2)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{(a^2+4ba+3b^2)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{2ba\ln\left(\tan\cdots\right.}{}$
risch	$\frac{a^2e^{5i(dx+c)}+3b^2e^{5i(dx+c)}+4ba\,e^{4i(dx+c)}+2a^2e^{3i(dx+c)}-2b^2e^{3i(dx+c)}+4ba\,e^{2i(dx+c)}+a^2e^{i(dx+c)}+3b^2e^{i(dx+c)}}{d\left(e^{2i(dx+c)}-1\right)^2\left(e^{2i(dx+c)}+1\right)}-\frac{a^2\ln(\cdots}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^2*(-1/2/sin(d*x+c)^2/cos(d*x+c)+3/2/cos(d*x+c)+3/2*ln(csc(d*x+c)-cot
(d*x+c)))+2*b*a*(-1/2/sin(d*x+c)^2+ln(tan(d*x+c)))+a^2*(-1/2*csc(d*x+c)*cot
(d*x+c)+1/2*ln(csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]
time = 0.27, size = 119, normalized size = 1.04

$$-\frac{8\,ab\log\left(\cos\left(dx+c\right)\right)+\left(a^2-4\,ab+3\,b^2\right)\log\left(\cos\left(dx+c\right)+1\right)-\left(a^2+4\,ab+3\,b^2\right)\log\left(\cos\left(dx+c\right)-1\right)-\frac{2\left(2\,ab\cos(dx+c)+(a^2+3\,b^2)\cos(dx+c)^2-2\,b^2\right)}{\cos(dx+c)^3-\cos(dx+c)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/4*(8*a*b*log(cos(d*x + c)) + (a^2 - 4*a*b + 3*b^2)*log(cos(d*x + c) + 1)
 - (a^2 + 4*a*b + 3*b^2)*log(cos(d*x + c) - 1) - 2*(2*a*b*cos(d*x + c) + (a
^2 + 3*b^2)*cos(d*x + c)^2 - 2*b^2)/(cos(d*x + c)^3 - cos(d*x + c)))/d
```

Fricas [A]
time = 3.06, size = 205, normalized size = 1.80

$$\frac{4\,ab\cos\left(dx+c\right)+2\left(a^2+3\,b^2\right)\cos\left(dx+c\right)^2-4\,b^2-8\left(ab\cos\left(dx+c\right)^3-ab\cos\left(dx+c\right)\right)\log\left(-\cos\left(dx+c\right)\right)-\left(\left(a^2-4\,ab+3\,b^2\right)\cos\left(dx+c\right)^3-\left(a^2-4\,ab+3\,b^2\right)\cos\left(dx+c\right)\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+\left(\left(a^2+4\,ab+3\,b^2\right)\cos\left(dx+c\right)^3-\left(a^2+4\,ab+3\,b^2\right)\cos\left(dx+c\right)\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{4\left(d\cos\left(dx+c\right)^3-d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*a*b*cos(d*x + c) + 2*(a^2 + 3*b^2)*cos(d*x + c)^2 - 4*b^2 - 8*(a*b*c
os(d*x + c)^3 - a*b*cos(d*x + c))*log(-cos(d*x + c)) - ((a^2 - 4*a*b + 3*b^
2)*cos(d*x + c)^3 - (a^2 - 4*a*b + 3*b^2)*cos(d*x + c))*log(1/2*cos(d*x + c
) + 1/2) + ((a^2 + 4*a*b + 3*b^2)*cos(d*x + c)^3 - (a^2 + 4*a*b + 3*b^2)*co
s(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^3 - d*cos(d*x + c
))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \sec (c + dx))^2 \csc^3 (c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*csc(c + d*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 314 vs. 2(108) = 216.
time = 0.51, size = 314, normalized size = 2.75

$$-\frac{16\,ab\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{2\,ab(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-2\left(a^2+4\,ab+3\,b^2\right)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-\frac{a^2+2\,ab+b^2+\frac{6\,ab(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{14\,b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{4\,ab(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{3\,b^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+\frac{(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/8*(16*a*b*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + a^2*(co
s(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*a*b*(cos(d*x + c) - 1)/(cos(d*x + c)
 + 1) + b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*(a^2 + 4*a*b + 3*b^2)
*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - (a^2 + 2*a*b + b^2 + 6
*a*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 14*b^2*(cos(d*x + c) - 1)/(cos
(d*x + c) + 1) - a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 4*a*b*(cos
(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 3*b^2*(cos(d*x + c) - 1)^2/(cos(d*x
 + c) + 1)^2)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + (cos(d*x + c) - 1)^2
/(cos(d*x + c) + 1)^2))/d
```

Mupad [B]
time = 0.12, size = 120, normalized size = 1.05

$$\frac{\ln\left(\cos\left(c+d\,x\right)-1\right)\,\left(a+b\right)\,\left(a+3\,b\right)}{4\,d}-\frac{\ln\left(\cos\left(c+d\,x\right)+1\right)\,\left(a-b\right)\,\left(a-3\,b\right)}{4\,d}-\frac{2\,a\,b\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}-\frac{{\cos\left(c+d\,x\right)}^2\,\left(\frac{a^2}{2}+\frac{3\,b^2}{2}\right)-b^2+a\,b\,\cos\left(c+d\,x\right)}{d\,\left(\cos\left(c+d\,x\right)-{\cos\left(c+d\,x\right)}^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^2/sin(c + d*x)^3,x)
```

```
[Out] (log(cos(c + d*x) - 1)*(a + b)*(a + 3*b))/(4*d) - (log(cos(c + d*x) + 1)*(a
 - b)*(a - 3*b))/(4*d) - (2*a*b*log(cos(c + d*x)))/d - (cos(c + d*x)^2*(a^2
/2 + (3*b^2)/2) - b^2 + a*b*cos(c + d*x))/(d*(cos(c + d*x) - cos(c + d*x)^3
))
```

3.179 $\int (a+b\sec(c+dx))^2\sin^6(c+dx)\,dx$

Optimal. Leaf size=175

$$\frac{5}{16}\left(a^2-6b^2\right)x+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}-\frac{2ab\sin(c+dx)}{d}-\frac{(11a^2-18b^2)\cos(c+dx)\sin(c+dx)}{16d}+\frac{(13a^2-$$

```
[Out] 5/16*(a^2-6*b^2)*x+2*a*b*arctanh(sin(d*x+c))/d-2*a*b*sin(d*x+c)/d-1/16*(11*
a^2-18*b^2)*cos(d*x+c)*sin(d*x+c)/d+1/24*(13*a^2-6*b^2)*cos(d*x+c)^3*sin(d*
x+c)/d-1/6*a^2*cos(d*x+c)^5*sin(d*x+c)/d-2/3*a*b*sin(d*x+c)^3/d-2/5*a*b*sin
(d*x+c)^5/d+b^2*tan(d*x+c)/d
```

Rubi [A]
time = 0.34, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3957, 2990, 2672, 308, 212, 466, 1828, 1171, 396, 209}

$$\frac{(13a^2-6b^2)\sin(c+dx)\cos^3(c+dx)}{24d}-\frac{(11a^2-18b^2)\sin(c+dx)\cos(c+dx)}{16d}+\frac{5}{16}x\left(a^2-6b^2\right)-\frac{a^2\sin(c+dx)\cos^5(c+dx)}{6d}-\frac{2ab\sin^5(c+dx)}{5d}-\frac{2ab\sin^3(c+dx)}{3d}-\frac{2ab\sin(c+dx)}{d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2*Sin[c + d*x]^6,x]
```

```
[Out] (5*(a^2 - 6*b^2)*x)/16 + (2*a*b*ArcTanh[Sin[c + d*x]])/d - (2*a*b*Sin[c + d
*x])/d - ((11*a^2 - 18*b^2)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + ((13*a^2 -
6*b^2)*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) - (a^2*Cos[c + d*x]^5*Sin[c + d*
x])/(6*d) - (2*a*b*Sin[c + d*x]^3)/(3*d) - (2*a*b*Sin[c + d*x]^5)/(5*d) + (
b^2*Tan[c + d*x])/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2990

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[2*a*(b/d), I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
```

```
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+b\sec(c+dx))^2\sin^6(c+dx)\,dx=\int (-b-a\cos(c+dx))^2\sin^4(c+dx)\tan^2(c+dx)\,dx$$

$$=(2ab)\int \sin^5(c+dx)\tan(c+dx)\,dx+\int \left(b^2+a^2\cos^2(c+dx)\right)\sin$$

$$=\frac{\text{Subst}\left(\int \frac{x^6\left(a^2+b^2+b^2x^2\right)}{\left(1+x^2\right)^4}\,dx,x,\tan(c+dx)\right)}{d}+\frac{(2ab)\text{Subst}\left(\int \frac{x^6}{1-x^2}\,d\right.}{d}$$

$$=-\frac{a^2\cos^5(c+dx)\sin(c+dx)}{6d}-\frac{\text{Subst}\left(\int \frac{-a^2+6a^2x^2-6a^2x^4-6b^2x^6}{\left(1+x^2\right)^3}\,dx,\right.}{6d}$$

$$=-\frac{2ab\sin(c+dx)}{d}+\frac{\left(13a^2-6b^2\right)\cos^3(c+dx)\sin(c+dx)}{24d}-\frac{a^2\cos}{}$$

$$=\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}-\frac{2ab\sin(c+dx)}{d}-\frac{\left(11a^2-18b^2\right)\cos(c}{16d}$$

$$=\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}-\frac{2ab\sin(c+dx)}{d}-\frac{\left(11a^2-18b^2\right)\cos(c}{16d}$$

$$=\frac{5}{16}\left(a^2-6b^2\right)x+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}-\frac{2ab\sin(c+dx)}{d}-\frac{(1}{}$$

Mathematica [A]
time = 1.09, size = 193, normalized size = 1.10

$$\frac{60\left(5\left(a^2-6b^2\right)(c+dx)-32ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+32ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-2128ab\sin(c+dx)+\left(-185a^2+1410b^2-5\left(29a^2-84b^2\right)\cos(2(c+dx))+232ab\cos(3(c+dx))+35a^2\cos(4(c+dx))-30b^2\cos(4(c+dx))-24ab\cos(5(c+dx))-5a^2\cos(6(c+dx))\right)\tan(c+dx)}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Sin[c + d*x]^6,x]
```

```
[Out] (60*(5*(a^2 - 6*b^2)*(c + d*x) - 32*a*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]] + 32*a*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 2128*a*b*Sin[c +
 d*x] + (-185*a^2 + 1410*b^2 - 5*(29*a^2 - 84*b^2)*Cos[2*(c + d*x)] + 232*a
```

```
*b*Cos[3*(c + d*x)] + 35*a^2*Cos[4*(c + d*x)] - 30*b^2*Cos[4*(c + d*x)] - 2
4*a*b*Cos[5*(c + d*x)] - 5*a^2*Cos[6*(c + d*x)])*Tan[c + d*x])/(960*d)
```

Maple [A]
time = 0.12, size = 163, normalized size = 0.93

method	result
derivativedivides	$\frac{b^2\left(\frac{\sin^7(dx+c)}{\cos(dx+c)}+\left(\sin^5(dx+c)+\frac{5\left(\sin^3(dx+c)\right)}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)-\frac{15dx}{8}-\frac{15c}{8}\right)+2ba\left(-\frac{\left(\sin^5(dx+c)\right)}{5}-\frac{\left(\sin^3(dx+c)\right)}{3}\right.}{d}$
default	$\frac{b^2\left(\frac{\sin^7(dx+c)}{\cos(dx+c)}+\left(\sin^5(dx+c)+\frac{5\left(\sin^3(dx+c)\right)}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)-\frac{15dx}{8}-\frac{15c}{8}\right)+2ba\left(-\frac{\left(\sin^5(dx+c)\right)}{5}-\frac{\left(\sin^3(dx+c)\right)}{3}\right.}{d}$
risch	$\frac{5a^2x}{16}-\frac{15b^2x}{8}+\frac{ie^{-2i(dx+c)}b^2}{4d}-\frac{ie^{2i(dx+c)}b^2}{4d}+\frac{2ib^2}{d\left(e^{2i(dx+c)}+1\right)}+\frac{15ia^2e^{2i(dx+c)}}{128d}-\frac{11iba\,e^{-i(dx+c)}}{8d}+\frac{11iba}{}$
norman	$\left(-\frac{5a^2}{16}+\frac{15b^2}{8}\right)x+\left(-\frac{45a^2}{16}+\frac{135b^2}{8}\right)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{25a^2}{16}+\frac{75b^2}{8}\right)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{25a^2}{16}+\frac{75b^2}{8}\right)x\left(\tan^6\left(\frac{dx}{2}+\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*sin(d*x+c)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^2*(sin(d*x+c)^7/cos(d*x+c)+(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d
*x+c))*cos(d*x+c)-15/8*d*x-15/8*c)+2*b*a*(-1/5*sin(d*x+c)^5-1/3*sin(d*x+c)^
3-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+a^2*(-1/6*(sin(d*x+c)^5+5/4*sin(d*x
+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/16*d*x+5/16*c))
```

Maxima [A]
time = 0.50, size = 173, normalized size = 0.99

$$\frac{5\left(4\sin(2dx+2c)^3+60dx+60c+9\sin(4dx+4c)-48\sin(2dx+2c)\right)a^2-64\left(6\sin(dx+c)^5+10\sin(dx+c)^3-15\log(\sin(dx+c)+1)+15\log(\sin(dx+c)-1)+30\sin(dx+c)\right)ab-120\left(15dx+15c-\frac{9\tan(dx+c)^3+7\tan(dx+c)}{\tan(dx+c)^4+2\tan(dx+c)^2+1}-8\tan(dx+c)\right)b^2}{960d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/960*(5*(4*sin(2*d*x + 2*c)^3 + 60*d*x + 60*c + 9*sin(4*d*x + 4*c) - 48*si
n(2*d*x + 2*c))*a^2 - 64*(6*sin(d*x + c)^5 + 10*sin(d*x + c)^3 - 15*log(sin
(d*x + c) + 1) + 15*log(sin(d*x + c) - 1) + 30*sin(d*x + c))*a*b - 120*(15*
d*x + 15*c - (9*tan(d*x + c)^3 + 7*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d*
x + c)^2 + 1) - 8*tan(d*x + c))*b^2)/d
```

Fricas [A]
time = 4.02, size = 176, normalized size = 1.01

$$\frac{75\left(a^2-6b^2\right)dx\cos(dx+c)+240ab\cos(dx+c)\log(\sin(dx+c)+1)-240ab\cos(dx+c)\log(-\sin(dx+c)+1)-\left(40a^2\cos(dx+c)^6+96ab\cos(dx+c)^5-352ab\cos(dx+c)^3-10\left(13a^2-6b^2\right)\cos(dx+c)^4+736ab\cos(dx+c)+15\left(11a^2-18b^2\right)\cos(dx+c)^2-240b^2\right)\sin(dx+c)}{240d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] 1/240*(75*(a^2 - 6*b^2)*d*x*cos(d*x + c) + 240*a*b*cos(d*x + c)*log(sin(d*x
 + c) + 1) - 240*a*b*cos(d*x + c)*log(-sin(d*x + c) + 1) - (40*a^2*cos(d*x
+ c)^6 + 96*a*b*cos(d*x + c)^5 - 352*a*b*cos(d*x + c)^3 - 10*(13*a^2 - 6*b^
2)*cos(d*x + c)^4 + 736*a*b*cos(d*x + c) + 15*(11*a^2 - 18*b^2)*cos(d*x + c
)^2 - 240*b^2)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*sin(d*x+c)**6,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 379 vs. 2(163) = 326.
time = 0.50, size = 379, normalized size = 2.17

$$\frac{480\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-480\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+75\,(a^2-6\,b^2)(dx+c)-\frac{480\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}+\frac{2\left(75\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-480\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-210\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+425\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-3040\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-870\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+990\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-8256\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-660\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-990\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-8256\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+660\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-425\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-3040\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+870\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-75\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-480\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+210\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^6}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^6,x, algorithm="giac")
```

```
[Out] 1/240*(480*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 480*a*b*log(abs(tan(1/2
*d*x + 1/2*c) - 1)) + 75*(a^2 - 6*b^2)*(d*x + c) - 480*b^2*tan(1/2*d*x + 1/
2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) + 2*(75*a^2*tan(1/2*d*x + 1/2*c)^11 - 480
*a*b*tan(1/2*d*x + 1/2*c)^11 - 210*b^2*tan(1/2*d*x + 1/2*c)^11 + 425*a^2*ta
n(1/2*d*x + 1/2*c)^9 - 3040*a*b*tan(1/2*d*x + 1/2*c)^9 - 870*b^2*tan(1/2*d*
x + 1/2*c)^9 + 990*a^2*tan(1/2*d*x + 1/2*c)^7 - 8256*a*b*tan(1/2*d*x + 1/2*
c)^7 - 660*b^2*tan(1/2*d*x + 1/2*c)^7 - 990*a^2*tan(1/2*d*x + 1/2*c)^5 - 82
56*a*b*tan(1/2*d*x + 1/2*c)^5 + 660*b^2*tan(1/2*d*x + 1/2*c)^5 - 425*a^2*ta
n(1/2*d*x + 1/2*c)^3 - 3040*a*b*tan(1/2*d*x + 1/2*c)^3 + 870*b^2*tan(1/2*d*
x + 1/2*c)^3 - 75*a^2*tan(1/2*d*x + 1/2*c) - 480*a*b*tan(1/2*d*x + 1/2*c) +
 210*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

Mupad [**B**]
time = 3.10, size = 231, normalized size = 1.32

$$\frac{\frac{5\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,a^2}{8}+4\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,a\,b-\frac{15\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,b^2}{4}}{d}-\frac{\frac{15\,a^2\,\sin(c+d\,x)}{128}-\frac{5\,b^2\,\sin(c+d\,x)}{4}+\frac{3\,a^2\,\sin(3\,c+3\,d\,x)}{32}-\frac{a^2\,\sin(5\,c+5\,d\,x)}{48}+\frac{a^2\,\sin(7\,c+7\,d\,x)}{384}-\frac{15\,b^2\,\sin(3\,c+3\,d\,x)}{64}+\frac{b^2\,\sin(5\,c+5\,d\,x)}{64}+\frac{59\,a\,b\,\sin(2\,c+2\,d\,x)}{48}-\frac{2\,a\,b\,\sin(4\,c+4\,d\,x)}{15}+\frac{a\,b\,\sin(6\,c+6\,d\,x)}{80}}{d\,\cos\left(c+d\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6*(a + b/cos(c + d*x))^2,x)
```

```
[Out] ((5*a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/8 - (15*b^2*atan(sin(c
/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/4 + 4*a*b*atanh(sin(c/2 + (d*x)/2)/cos(c
/2 + (d*x)/2)))/d - ((15*a^2*sin(c + d*x))/128 - (5*b^2*sin(c + d*x))/4 + (
3*a^2*sin(3*c + 3*d*x))/32 - (a^2*sin(5*c + 5*d*x))/48 + (a^2*sin(7*c + 7*d
*x))/384 - (15*b^2*sin(3*c + 3*d*x))/64 + (b^2*sin(5*c + 5*d*x))/64 + (59*a
*b*sin(2*c + 2*d*x))/48 - (2*a*b*sin(4*c + 4*d*x))/15 + (a*b*sin(6*c + 6*d*
x))/80)/(d*cos(c + d*x))
```

3.180 $\int (a+b\sec(c+dx))^2\sin^4(c+dx)\,dx$

Optimal. Leaf size=178

$$\frac{3}{8}\left(a^2-4b^2\right)x+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}-\frac{b(28a^2+b^2)\sin(c+dx)}{6ad}-\frac{(39a^2+2b^2)\cos(c+dx)\sin(c+dx)}{24d}-($$

```
[Out] 3/8*(a^2-4*b^2)*x+2*a*b*arctanh(sin(d*x+c))/d-1/6*b*(28*a^2+b^2)*sin(d*x+c)
/a/d-1/24*(39*a^2+2*b^2)*cos(d*x+c)*sin(d*x+c)/d-1/12*(12*a^2+b^2)*(b+a*cos
(d*x+c))^2*sin(d*x+c)/a/b/d+1/4*(b+a*cos(d*x+c))^3*sin(d*x+c)/a/d+(b+a*cos(
d*x+c))^3*tan(d*x+c)/b/d
```

Rubi [A]
time = 0.39, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3957, 2973, 3128, 3112, 3102, 2814, 3855}

$$-\frac{b(28a^2+b^2)\sin(c+dx)}{6ad}-\frac{(12a^2+b^2)\sin(c+dx)(a\cos(c+dx)+b)^2}{12abd}-\frac{(39a^2+2b^2)\sin(c+dx)\cos(c+dx)}{24d}+\frac{3}{8}x(a^2-4b^2)+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{\sin(c+dx)(a\cos(c+dx)+b)^3}{4ad}+\frac{\tan(c+dx)(a\cos(c+dx)+b)^3}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2*Sin[c + d*x]^4,x]
```

```
[Out] (3*(a^2 - 4*b^2)*x)/8 + (2*a*b*ArcTanh[Sin[c + d*x]])/d - (b*(28*a^2 + b^2)
*Sin[c + d*x])/(6*a*d) - ((39*a^2 + 2*b^2)*Cos[c + d*x]*Sin[c + d*x])/(24*d
) - ((12*a^2 + b^2)*(b + a*Cos[c + d*x])^2*Sin[c + d*x])/(12*a*b*d) + ((b +
 a*Cos[c + d*x])^3*Sin[c + d*x])/(4*a*d) + ((b + a*Cos[c + d*x])^3*Tan[c +
d*x])/(b*d)
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2973

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*d*f*(n + 1))), x] + (Dis
t[1/(a*b*d*(n + 1)*(m + n + 4)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x]
)^(n + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4) + a*b*(m +
 3)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 3)*(m + n + 4))*Sin[
e + f*x]^2, x], x], x] - Simp[Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((d
*Sin[e + f*x])^(n + 2)/(b*d^2*f*(m + n + 4))), x]) /; FreeQ[{a, b, d, e, f,
 m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n]) &&  !m
< -1 && LtQ[n, -1] && NeQ[m + n + 4, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3112

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin
[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A
*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+b\sec(c+dx))^2\sin^4(c+dx)\,dx = \int (-b-a\cos(c+dx))^2\sin^2(c+dx)\tan^2(c+dx)\,dx$$
$$= \frac{(b+a\cos(c+dx))^3\sin(c+dx)}{4ad} + \frac{(b+a\cos(c+dx))^3\tan(c+dx)}{bd}$$
$$= -\frac{(12a^2+b^2)(b+a\cos(c+dx))^2\sin(c+dx)}{12abd} + \frac{(b+a\cos(c+dx)}{4ad}$$
$$= -\frac{(39a^2+2b^2)\cos(c+dx)\sin(c+dx)}{24d} - \frac{(12a^2+b^2)(b+a\cos(c+}{12abd}$$
$$= -\frac{b(28a^2+b^2)\sin(c+dx)}{6ad} - \frac{(39a^2+2b^2)\cos(c+dx)\sin(c+dx)}{24d}$$
$$= \frac{3}{8}(a^2-4b^2)x - \frac{b(28a^2+b^2)\sin(c+dx)}{6ad} - \frac{(39a^2+2b^2)\cos(c+d}{24d}$$
$$= \frac{3}{8}(a^2-4b^2)x + \frac{2ab\tanh^{-1}(\sin(c+dx))}{d} - \frac{b(28a^2+b^2)\sin(c+d}{6ad}$$

Mathematica [A]

time = 0.68, size = 157, normalized size = 0.88

$$\frac{12(3(a^2-4b^2)(c+dx)-16ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+16ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right))-208ab\sin(c+dx)+(-6(3a^2-4b^2)\cos(2(c+dx))+16ab\cos(3(c+dx))+3(-7a^2+40b^2+a^2\cos(4(c+dx))))\tan(c+dx)}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Sin[c + d*x]^4,x]
```

```
[Out] (12*(3*(a^2 - 4*b^2)*(c + d*x) - 16*a*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]] + 16*a*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 208*a*b*Sin[c +
d*x] + (-6*(3*a^2 - 4*b^2)*Cos[2*(c + d*x)] + 16*a*b*Cos[3*(c + d*x)] + 3*(
-7*a^2 + 40*b^2 + a^2*Cos[4*(c + d*x)]))*Tan[c + d*x])/(96*d)
```

Maple [A]

time = 0.11, size = 133, normalized size = 0.75

method	result
derivativedivides	$\frac{b^2\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)+2ba\left(-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan\right.}{d}$
default	$\frac{b^2\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)+2ba\left(-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan\right.}{d}$
risch	$\frac{3a^2x}{8}-\frac{3b^2x}{2}+\frac{ia^2e^{2i(dx+c)}}{8d}-\frac{ie^{2i(dx+c)}b^2}{8d}+\frac{5iba\,e^{i(dx+c)}}{4d}-\frac{5iba\,e^{-i(dx+c)}}{4d}-\frac{ia^2e^{-2i(dx+c)}}{8d}+\frac{ie^{-2i(dx+c}}{8d}$

norman	$\frac{\left(-\frac{3a^2}{8}+\frac{3b^2}{2}\right)x+\left(-\frac{9a^2}{8}+\frac{9b^2}{2}\right)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{3a^2}{4}+3b^2\right)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{3a^2}{4}-3b^2\right)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{3a^2}{8}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*sin(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^2*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-
3/2*d*x-3/2*c)+2*b*a*(-1/3*sin(d*x+c)^3-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)
))+a^2*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A]
time = 0.49, size = 125, normalized size = 0.70

$$\frac{3\,(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)-8\,\sin\left(2\,dx+2\,c\right))a^2-32\,\left(2\,\sin\left(dx+c\right)^3-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)+6\,\sin\left(dx+c\right)\right)ab-48\,\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^2+1}-2\,\tan\left(dx+c\right)\right)b^2}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/96*(3*(12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*d*x + 2*c))*a^2 - 32*(2
*sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1) + 6*sin
(d*x + c))*a*b - 48*(3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*ta
n(d*x + c))*b^2)/d
```

Fricas [A]
time = 2.88, size = 142, normalized size = 0.80

$$\frac{9\,(a^2-4\,b^2)dx\cos\left(dx+c\right)+24\,ab\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-24\,ab\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right)+\left(6\,a^2\cos\left(dx+c\right)^4+16\,ab\cos\left(dx+c\right)^3-64\,ab\cos\left(dx+c\right)-3\,(5\,a^2-4\,b^2)\cos\left(dx+c\right)^2+24\,b^2\right)\sin\left(dx+c\right)}{24\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/24*(9*(a^2 - 4*b^2)*d*x*cos(d*x + c) + 24*a*b*cos(d*x + c)*log(sin(d*x +
c) + 1) - 24*a*b*cos(d*x + c)*log(-sin(d*x + c) + 1) + (6*a^2*cos(d*x + c)^
4 + 16*a*b*cos(d*x + c)^3 - 64*a*b*cos(d*x + c) - 3*(5*a^2 - 4*b^2)*cos(d*x
 + c)^2 + 24*b^2)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec\left(c+dx\right)\right)^2\sin^4\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*sin(d*x+c)**4,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*sin(c + d*x)**4, x)
```

Giac [A]
time = 0.49, size = 285, normalized size = 1.60

$$\frac{48\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-48\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+9\,(a^2-4\,b^2)(dx+c)-\frac{48\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}+\frac{2\,\left(9\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-48\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-12\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+33\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-208\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-12\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-33\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-208\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+12\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-9\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-48\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+12\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/24*(48*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 48*a*b*log(abs(tan(1/2*d*
x + 1/2*c) - 1)) + 9*(a^2 - 4*b^2)*(d*x + c) - 48*b^2*tan(1/2*d*x + 1/2*c)/
(tan(1/2*d*x + 1/2*c)^2 - 1) + 2*(9*a^2*tan(1/2*d*x + 1/2*c)^7 - 48*a*b*tan
(1/2*d*x + 1/2*c)^7 - 12*b^2*tan(1/2*d*x + 1/2*c)^7 + 33*a^2*tan(1/2*d*x +
1/2*c)^5 - 208*a*b*tan(1/2*d*x + 1/2*c)^5 - 12*b^2*tan(1/2*d*x + 1/2*c)^5 -
 33*a^2*tan(1/2*d*x + 1/2*c)^3 - 208*a*b*tan(1/2*d*x + 1/2*c)^3 + 12*b^2*ta
n(1/2*d*x + 1/2*c)^3 - 9*a^2*tan(1/2*d*x + 1/2*c) - 48*a*b*tan(1/2*d*x + 1/
2*c) + 12*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

Mupad [B]
time = 1.23, size = 207, normalized size = 1.16

$$\frac{3\,a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{4\,d}-\frac{3\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{a^2\cos\left(c+d\,x\right)^3\,\sin\left(c+d\,x\right)}{4\,d}+\frac{b^2\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}-\frac{8\,a\,b\,\sin\left(c+d\,x\right)}{3\,d}+\frac{4\,a\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{5\,a^2\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{8\,d}+\frac{b^2\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{2\,d}+\frac{2\,a\,b\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (3*a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(4*d) - (3*b^2*atan(sin
(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (a^2*cos(c + d*x)^3*sin(c + d*x))/
(4*d) + (b^2*sin(c + d*x))/(d*cos(c + d*x)) - (8*a*b*sin(c + d*x))/(3*d) +
(4*a*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (5*a^2*cos(c + d*x
)*sin(c + d*x))/(8*d) + (b^2*cos(c + d*x)*sin(c + d*x))/(2*d) + (2*a*b*cos(
c + d*x)^2*sin(c + d*x))/(3*d)
```

3.181 $\int (a+b\sec(c+dx))^2\sin^2(c+dx)\,dx$

Optimal. Leaf size=77

$$\frac{a^2x}{2}-b^2x+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}-\frac{2ab\sin(c+dx)}{d}-\frac{a^2\cos(c+dx)\sin(c+dx)}{2d}+\frac{b^2\tan(c+dx)}{d}$$

```
[Out] 1/2*a^2*x-b^2*x+2*a*b*arctanh(sin(d*x+c))/d-2*a*b*sin(d*x+c)/d-1/2*a^2*cos(
d*x+c)*sin(d*x+c)/d+b^2*tan(d*x+c)/d
```

Rubi [A]
time = 0.09, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3957, 2801, 2715, 8, 2672, 327, 212, 3554}

$$-\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{a^2x}{2}-\frac{2ab\sin(c+dx)}{d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}-b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2*Sin[c + d*x]^2,x]
```

```
[Out] (a^2*x)/2 - b^2*x + (2*a*b*ArcTanh[Sin[c + d*x]])/d - (2*a*b*Sin[c + d*x])/
d - (a^2*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (b^2*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
```

```
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2801

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+b\sec(c+dx))^2\sin^2(c+dx)\,dx = \int (-b-a\cos(c+dx))^2\tan^2(c+dx)\,dx$$
$$= \int \left(a^2\sin^2(c+dx)+2ab\sin(c+dx)\tan(c+dx)+b^2\tan^2(c+dx)\right.$$
$$= a^2\int \sin^2(c+dx)\,dx+(2ab)\int \sin(c+dx)\tan(c+dx)\,dx+b^2\int$$
$$= -\frac{a^2\cos(c+dx)\sin(c+dx)}{2d}+\frac{b^2\tan(c+dx)}{d}+\frac{1}{2}a^2\int 1\,dx-b^2\int$$
$$= \frac{a^2x}{2}-b^2x-\frac{2ab\sin(c+dx)}{d}-\frac{a^2\cos(c+dx)\sin(c+dx)}{2d}+\frac{b^2\tan}{}$$
$$= \frac{a^2x}{2}-b^2x+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}-\frac{2ab\sin(c+dx)}{d}-\frac{a^2\cos(c}{}$$

Mathematica [A]
time = 0.40, size = 121, normalized size = 1.57

$$-\frac{-2a^2c+4b^2c-2a^2dx+4b^2dx+8ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-8ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+8ab\sin(c+dx)+a^2\sin(2(c+dx))-4b^2\tan(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Sin[c + d*x]^2,x]
```

```
[Out] -1/4*(-2*a^2*c + 4*b^2*c - 2*a^2*d*x + 4*b^2*d*x + 8*a*b*Log[Cos[(c + d*x)/
2] - Sin[(c + d*x)/2]] - 8*a*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 8
*a*b*Sin[c + d*x] + a^2*Sin[2*(c + d*x)] - 4*b^2*Tan[c + d*x])/d
```

Maple [A]
time = 0.09, size = 77, normalized size = 1.00

method	result
derivativedivides	$\frac{b^2(\tan(dx+c)-dx-c)+2ba(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+a^2\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
default	$\frac{b^2(\tan(dx+c)-dx-c)+2ba(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+a^2\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
risch	$\frac{a^2x}{2}-b^2x+\frac{ia^2e^{2i(dx+c)}}{8d}+\frac{iba\,e^{i(dx+c)}}{d}-\frac{iba\,e^{-i(dx+c)}}{d}-\frac{ia^2e^{-2i(dx+c)}}{8d}+\frac{2ib^2}{d(e^{2i(dx+c)}+1)}-\frac{2ba\ln(e^{i(dx+\cdots}{d}$
norman	$\frac{\left(-\frac{a^2}{2}+b^2\right)x+\left(-\frac{a^2}{2}+b^2\right)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{a^2}{2}-b^2\right)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{a^2}{2}-b^2\right)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{\left(a^2-4ba-2b^2\right)\left(\cdots\right.}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*sin(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^2*(tan(d*x+c)-d*x-c)+2*b*a*(-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+a
^2*(-1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.49, size = 80, normalized size = 1.04

$$\frac{(2\,dx+2\,c-\sin(2\,dx+2\,c))a^2-4\,(dx+c-\tan(dx+c))b^2+4\,ab(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)-2\,\sin(dx+c))}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c - sin(2*d*x + 2*c))*a^2 - 4*(d*x + c - tan(d*x + c))*b^2
+ 4*a*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x + c)))/d
```

Fricas [A]
time = 2.89, size = 108, normalized size = 1.40

$$\frac{(a^2-2\,b^2)dx\cos(dx+c)+2\,ab\cos(dx+c)\log(\sin(dx+c)+1)-2\,ab\cos(dx+c)\log(-\sin(dx+c)+1)-\left(a^2\cos(dx+c)^2+4\,ab\cos(dx+c)-2\,b^2\right)\sin(dx+c)}{2\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((a^2 - 2*b^2)*d*x*cos(d*x + c) + 2*a*b*cos(d*x + c)*log(sin(d*x + c) +
 1) - 2*a*b*cos(d*x + c)*log(-sin(d*x + c) + 1) - (a^2*cos(d*x + c)^2 + 4*a
*b*cos(d*x + c) - 2*b^2)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec (c+dx))^2 \sin^2 (c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*sin(d*x+c)**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*sin(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 159 vs. 2(73) = 146.
time = 0.52, size = 159, normalized size = 2.06

$$\frac{4\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-4\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+(a^2-2\,b^2)(dx+c)-\frac{4\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}+\frac{2\left(a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 4*a*b*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) + (a^2 - 2*b^2)*(d*x + c) - 4*b^2*tan(1/2*d*x + 1/2*c)/(tan(1
/2*d*x + 1/2*c)^2 - 1) + 2*(a^2*tan(1/2*d*x + 1/2*c)^3 - 4*a*b*tan(1/2*d*x
+ 1/2*c)^3 - a^2*tan(1/2*d*x + 1/2*c) - 4*a*b*tan(1/2*d*x + 1/2*c))/(tan(1/
2*d*x + 1/2*c)^2 + 1)^2)/d
```

Mupad [B]
time = 1.17, size = 143, normalized size = 1.86

$$\frac{a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{2\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{b^2\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}-\frac{2\,a\,b\,\sin\left(c+d\,x\right)}{d}+\frac{4\,a\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{a^2\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (2*b^2*atan(sin(c/2 +
 (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (b^2*sin(c + d*x))/(d*cos(c + d*x)) - (2
*a*b*sin(c + d*x))/d + (4*a*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))
/d - (a^2*cos(c + d*x)*sin(c + d*x))/(2*d)
```

3.182 $\int \csc^2(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=59

$$\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}-\frac{(a^2+b^2)\cot(c+dx)}{d}-\frac{2ab\csc(c+dx)}{d}+\frac{b^2\tan(c+dx)}{d}$$

```
[Out] 2*a*b*arctanh(sin(d*x+c))/d-(a^2+b^2)*cot(d*x+c)/d-2*a*b*csc(d*x+c)/d+b^2*t
an(d*x+c)/d
```

Rubi [A]
time = 0.30, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3957, 2990, 2701, 327, 213, 14}

$$-\frac{(a^2+b^2)\cot(c+dx)}{d}-\frac{2ab\csc(c+dx)}{d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (2*a*b*ArcTanh[Sin[c + d*x]])/d - ((a^2 + b^2)*Cot[c + d*x])/d - (2*a*b*Csc
[c + d*x])/d + (b^2*Tan[c + d*x])/d
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
```

```
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2990

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[2*a*(b/d), I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \csc^2(c+dx)(a+b\sec(c+dx))^2\,dx &= \int (-b-a\cos(c+dx))^2\csc^2(c+dx)\sec^2(c+dx)\,dx \\
&= (2ab)\int \csc^2(c+dx)\sec(c+dx)\,dx + \int \left(b^2+a^2\cos^2(c+dx)\right)\csc \\
&= \frac{\operatorname{Subst}\left(\int \frac{a^2+b^2+b^2x^2}{x^2}\,dx, x, \tan(c+dx)\right)}{d} - \frac{(2ab)\operatorname{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx,\right.}{d} \\
&= -\frac{2ab\csc(c+dx)}{d} + \frac{\operatorname{Subst}\left(\int \left(b^2+\frac{a^2+b^2}{x^2}\right)\,dx, x, \tan(c+dx)\right)}{d} - \\
&= \frac{2ab\tanh^{-1}(\sin(c+dx))}{d} - \frac{\left(a^2+b^2\right)\cot(c+dx)}{d} - \frac{2ab\csc(c+dx}{d}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 138 vs. $2(59) = 118$.
time = 0.32, size = 138, normalized size = 2.34

$$
-\frac{\csc^3\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(4ab\cos(c+dx)+\left(a^2+2b^2\right)\cos(2(c+dx))+a\left(a+2b\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sin(2(c+dx))\right)\right)}{4d\left(-1+\cot^2\left(\frac{1}{2}(c+dx)\right)\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -1/4*(Csc[(c + d*x)/2]^3*Sec[(c + d*x)/2]*(4*a*b*Cos[c + d*x] + (a^2 + 2*b^
2)*Cos[2*(c + d*x)] + a*(a + 2*b*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
```

```
- Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])*Sin[2*(c + d*x)])))/(d*(-1 + Co
t[(c + d*x)/2]^2))
```

Maple [A]
time = 0.07, size = 76, normalized size = 1.29

method	result
derivativedivides	$\frac{b^{2}\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)+2ba\left(-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)-a^{2}\cot(dx+c)}{d}$
default	$\frac{b^{2}\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)+2ba\left(-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c))\right)-a^{2}\cot(dx+c)}{d}$
risch	$-\frac{2i\left(2ab\,\mathrm{e}^{3i(dx+c)}+a^{2}\mathrm{e}^{2i(dx+c)}+2ba\,\mathrm{e}^{i(dx+c)}+a^{2}+2b^{2}\right)}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)\left(\mathrm{e}^{2i(dx+c)}-1\right)}-\frac{2ba\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{d}+\frac{2ba\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{d}$
norman	$\frac{\frac{a^{2}+2ba+b^{2}}{2d}-\frac{\left(a^{2}+3b^{2}\right)\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{\left(a^{2}-2ba+b^{2}\right)\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2ba\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}+\frac{2ba\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^2*(1/sin(d*x+c)/cos(d*x+c)-2*cot(d*x+c))+2*b*a*(-1/sin(d*x+c)+ln(sec
(d*x+c)+tan(d*x+c)))-a^2*cot(d*x+c))
```

Maxima [A]
time = 0.26, size = 73, normalized size = 1.24

$$-\frac{ab\left(\frac{2}{\sin(dx+c)}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+b^{2}\left(\frac{1}{\tan(dx+c)}-\tan\left(dx+c\right)\right)+\frac{a^{2}}{\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(a*b*(2/sin(d*x + c) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + b^
2*(1/tan(d*x + c) - tan(d*x + c)) + a^2/tan(d*x + c))/d
```

Fricas [A]
time = 3.59, size = 104, normalized size = 1.76

$$\frac{ab\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)-ab\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)-2\,ab\cos\left(dx+c\right)-\left(a^{2}+2\,b^{2}\right)\cos\left(dx+c\right)^{2}+b^{2}}{d\cos\left(dx+c\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (a*b*cos(d*x + c)*log(sin(d*x + c) + 1)*sin(d*x + c) - a*b*cos(d*x + c)*log
(-sin(d*x + c) + 1)*sin(d*x + c) - 2*a*b*cos(d*x + c) - (a^2 + 2*b^2)*cos(d
*x + c)^2 + b^2)/(d*cos(d*x + c)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^2\csc^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*csc(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 167 vs. 2(59) = 118.
time = 0.51, size = 167, normalized size = 2.83

$$\frac{4\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-4\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+5\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a^2-2\,ab-b^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 4*a*b*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) + a^2*tan(1/2*d*x + 1/2*c) - 2*a*b*tan(1/2*d*x + 1/2*c) + b^2
*tan(1/2*d*x + 1/2*c) - (a^2*tan(1/2*d*x + 1/2*c)^2 + 2*a*b*tan(1/2*d*x + 1
/2*c)^2 + 5*b^2*tan(1/2*d*x + 1/2*c)^2 - a^2 - 2*a*b - b^2)/(tan(1/2*d*x +
1/2*c)^3 - tan(1/2*d*x + 1/2*c)))/d
```

Mupad [B]
time = 1.07, size = 108, normalized size = 1.83

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,{\left(a-b\right)}^2}{2\,d}-\frac{2\,a\,b+a^2+b^2-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(a^2+2\,a\,b+5\,b^2\right)}{d\,\left(2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\right)}+\frac{4\,a\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^2/sin(c + d*x)^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(a - b)^2)/(2*d) - (2*a*b + a^2 + b^2 - tan(c/2 + (d*x)
/2)^2*(2*a*b + a^2 + 5*b^2))/(d*(2*tan(c/2 + (d*x)/2) - 2*tan(c/2 + (d*x)/2
)^3)) + (4*a*b*atanh(tan(c/2 + (d*x)/2)))/d
```

3.183 $\int \csc^4(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=100

$$\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}-\frac{(a^2+2b^2)\cot(c+dx)}{d}-\frac{(a^2+b^2)\cot^3(c+dx)}{3d}-\frac{2ab\csc(c+dx)}{d}-\frac{2ab\csc^3(c+dx)}{3d}$$

```
[Out] 2*a*b*arctanh(sin(d*x+c))/d-(a^2+2*b^2)*cot(d*x+c)/d-1/3*(a^2+b^2)*cot(d*x+
c)^3/d-2*a*b*csc(d*x+c)/d-2/3*a*b*csc(d*x+c)^3/d+b^2*tan(d*x+c)/d
```

Rubi [A]
time = 0.23, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3957, 2990, 2701, 308, 213, 459}

$$-\frac{(a^2+b^2)\cot^3(c+dx)}{3d}-\frac{(a^2+2b^2)\cot(c+dx)}{d}-\frac{2ab\csc^3(c+dx)}{3d}-\frac{2ab\csc(c+dx)}{d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (2*a*b*ArcTanh[Sin[c + d*x]])/d - ((a^2 + 2*b^2)*Cot[c + d*x])/d - ((a^2 +
b^2)*Cot[c + d*x]^3)/(3*d) - (2*a*b*Csc[c + d*x])/d - (2*a*b*Csc[c + d*x]^3
)/(3*d) + (b^2*Tan[c + d*x])/d
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
```

```
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2990

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[2*a*(b/d), I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \csc^4(c+dx)(a+b\sec(c+dx))^2\,dx &= \int (-b-a\cos(c+dx))^2\csc^4(c+dx)\sec^2(c+dx)\,dx \\
&= (2ab)\int \csc^4(c+dx)\sec(c+dx)\,dx + \int \left(b^2+a^2\cos^2(c+dx)\right)\csc\cdots \\
&= \frac{\operatorname{Subst}\left(\int \frac{(1+x^2)\left(a^2+b^2+b^2x^2\right)}{x^4}\,dx, x, \tan(c+dx)\right)}{d} - \frac{(2ab)\operatorname{Subst}\left(\int \cdots\right.}{\cdots} \\
&= \frac{\operatorname{Subst}\left(\int \left(b^2+\frac{a^2+b^2}{x^4}+\frac{a^2+2b^2}{x^2}\right)dx, x, \tan(c+dx)\right)}{d} - \frac{(2ab)\operatorname{Subst}\left(\cdots\right.}{\cdots} \\
&= -\frac{\left(a^2+2b^2\right)\cot(c+dx)}{d} - \frac{\left(a^2+b^2\right)\cot^3(c+dx)}{3d} - \frac{2ab\csc(c+d\cdots}{d} \\
&= \frac{2ab\tanh^{-1}(\sin(c+dx))}{d} - \frac{\left(a^2+2b^2\right)\cot(c+dx)}{d} - \frac{\left(a^2+b^2\right)\cot^3\cdots}{3d}
\end{aligned}
$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 259 vs. 2(100) = 200.
time = 0.42, size = 259, normalized size = 2.59

$$
\frac{\csc^5\left(\frac{1}{2}(c+dx)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(-3a^2-14ab\cos(c+dx)-2\left(a^2+4b^2\right)\cos(2(c+dx))+6ab\cos(3(c+dx))+a^2\cos(4(c+dx))+4b^2\cos(4(c+dx))-6ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(2(c+dx))+6ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(2(c+dx))+3ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(4(c+dx))-3ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(4(c+dx))\right)}{96d\left(-1+\cot^2\left(\frac{1}{2}(c+dx)\right)\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (Csc[(c + d*x)/2]^5*Sec[(c + d*x)/2]^3*(-3*a^2 - 14*a*b*Cos[c + d*x] - 2*(a
^2 + 4*b^2)*Cos[2*(c + d*x)] + 6*a*b*Cos[3*(c + d*x)] + a^2*Cos[4*(c + d*x)
] + 4*b^2*Cos[4*(c + d*x)] - 6*a*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
*Sin[2*(c + d*x)] + 6*a*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[2*(c
 + d*x)] + 3*a*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[4*(c + d*x)]
- 3*a*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[4*(c + d*x)]))/(96*d*(
-1 + Cot[(c + d*x)/2]^2))
```

Maple [A]
time = 0.10, size = 116, normalized size = 1.16

method	result
derivativedivides	$\frac{b^2\left(-\frac{1}{3\sin(dx+c)^3\cos(dx+c)}+\frac{4}{3\sin(dx+c)\cos(dx+c)}-\frac{8\cot(dx+c)}{3}\right)+2ba\left(-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c\right.}{d}$
default	$\frac{b^2\left(-\frac{1}{3\sin(dx+c)^3\cos(dx+c)}+\frac{4}{3\sin(dx+c)\cos(dx+c)}-\frac{8\cot(dx+c)}{3}\right)+2ba\left(-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\ln(\sec(dx+c)+\tan(dx+c\right.}{d}$
risch	$-\frac{4i\left(3ba\,e^{7i(dx+c)}-7ba\,e^{5i(dx+c)}-3a^2e^{4i(dx+c)}-7ab\,e^{3i(dx+c)}-2a^2e^{2i(dx+c)}-8b^2e^{2i(dx+c)}+3ba\,e^{i(dx+c)}+a^2+4b^2\right)}{3d\left(e^{2i(dx+c)}+1\right)\left(e^{2i(dx+c)}-1\right)^3}+$
norman	$\frac{\frac{a^2+2ba+b^2}{24d}-\frac{3\left(a^2+5b^2\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{\left(a^2-2ba+b^2\right)\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}+\frac{\left(2a^2-7ba+5b^2\right)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}+\frac{\left(2a^2+7ba+5b^2\right)\left(\mathrm{ta}\right.}{6d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^2*(-1/3/sin(d*x+c)^3/cos(d*x+c)+4/3/sin(d*x+c)/cos(d*x+c)-8/3*cot(d*
x+c))+2*b*a*(-1/3/sin(d*x+c)^3-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+a^2*
(-2/3-1/3*csc(d*x+c)^2)*cot(d*x+c))
```

Maxima [A]
time = 0.28, size = 112, normalized size = 1.12

$$-\frac{ab\left(\frac{2\left(3\sin(dx+c)^2+1\right)}{\sin(dx+c)^3}-3\log\left(\sin\left(dx+c\right)+1\right)+3\log\left(\sin\left(dx+c\right)-1\right)\right)+b^2\left(\frac{6\tan(dx+c)^2+1}{\tan(dx+c)^3}-3\tan\left(dx+c\right)\right)+\frac{\left(3\tan(dx+c)^2+1\right)a^2}{\tan(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3*(a*b*(2*(3*sin(d*x + c)^2 + 1)/sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1
) + 3*log(sin(d*x + c) - 1)) + b^2*((6*tan(d*x + c)^2 + 1)/tan(d*x + c)^3 -
 3*tan(d*x + c)) + (3*tan(d*x + c)^2 + 1)*a^2/tan(d*x + c)^3)/d
```

Fricas [A]
time = 3.29, size = 178, normalized size = 1.78

$$-\frac{6\,ab\cos\left(dx+c\right)^3+2\left(a^2+4\,b^2\right)\cos\left(dx+c\right)^4-8\,ab\cos\left(dx+c\right)-3\left(a^2+4\,b^2\right)\cos\left(dx+c\right)^2-3\left(ab\cos\left(dx+c\right)^3-ab\cos\left(dx+c\right)\right)\log\left(\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)+3\left(ab\cos\left(dx+c\right)^3-ab\cos\left(dx+c\right)\right)\log\left(-\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)+3\,b^2}{3\left(d\cos\left(dx+c\right)^3-d\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(6*a*b*cos(d*x + c)^3 + 2*(a^2 + 4*b^2)*cos(d*x + c)^4 - 8*a*b*cos(d*x
 + c) - 3*(a^2 + 4*b^2)*cos(d*x + c)^2 - 3*(a*b*cos(d*x + c)^3 - a*b*cos(d*
x + c))*log(sin(d*x + c) + 1)*sin(d*x + c) + 3*(a*b*cos(d*x + c)^3 - a*b*co
s(d*x + c))*log(-sin(d*x + c) + 1)*sin(d*x + c) + 3*b^2)/((d*cos(d*x + c)^3
 - d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec (c + dx))^2 \csc^4 (c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*csc(c + d*x)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(96) = 192.
time = 0.53, size = 226, normalized size = 2.26

$$\frac{a^2 \tan \left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 - 2\,ab\tan \left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 + b^2 \tan \left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 + 48\,ab\log \left(\left|\tan \left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 48\,ab\log \left(\left|\tan \left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + 9\,a^2 \tan \left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 30\,ab\tan \left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 21\,b^2 \tan \left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - \frac{48\,b^2 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1} - \frac{9\,a^2 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+30\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+21\,b^2 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a^2+2\,ab+b^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(a^2*tan(1/2*d*x + 1/2*c)^3 - 2*a*b*tan(1/2*d*x + 1/2*c)^3 + b^2*tan(1
/2*d*x + 1/2*c)^3 + 48*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 48*a*b*log(
abs(tan(1/2*d*x + 1/2*c) - 1)) + 9*a^2*tan(1/2*d*x + 1/2*c) - 30*a*b*tan(1/
2*d*x + 1/2*c) + 21*b^2*tan(1/2*d*x + 1/2*c) - 48*b^2*tan(1/2*d*x + 1/2*c)/
(tan(1/2*d*x + 1/2*c)^2 - 1) - (9*a^2*tan(1/2*d*x + 1/2*c)^2 + 30*a*b*tan(1
/2*d*x + 1/2*c)^2 + 21*b^2*tan(1/2*d*x + 1/2*c)^2 + a^2 + 2*a*b + b^2)/tan(
1/2*d*x + 1/2*c)^3)/d
```

Mupad [B]
time = 1.10, size = 182, normalized size = 1.82

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,{\left(a-b\right)}^2}{24\,d}-\frac{\frac{2\,a\,b}{3}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(3\,a^2+10\,a\,b+23\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{8\,a^2}{3}+\frac{28\,a\,b}{3}+\frac{20\,b^2}{3}\right)+\frac{a^2}{3}+\frac{b^2}{3}}{d\,\left(8\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3-8\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\right)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{a^2}{8}-\frac{3\,a\,b}{4}+\frac{5\,b^2}{8}+\frac{{\left(a-b\right)}^2}{4}\right)}{d}+\frac{4\,a\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^2/sin(c + d*x)^4,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3*(a - b)^2)/(24*d) - ((2*a*b)/3 - tan(c/2 + (d*x)/2)^4
*(10*a*b + 3*a^2 + 23*b^2) + tan(c/2 + (d*x)/2)^2*((28*a*b)/3 + (8*a^2)/3 +
 (20*b^2)/3) + a^2/3 + b^2/3)/(d*(8*tan(c/2 + (d*x)/2)^3 - 8*tan(c/2 + (d*x
)/2)^5)) + (tan(c/2 + (d*x)/2)*(a^2/8 - (3*a*b)/4 + (5*b^2)/8 + (a - b)^2/4
))/d + (4*a*b*atanh(tan(c/2 + (d*x)/2)))/d
```

3.184 $\int \csc^6(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=143

$$\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}-\frac{(a^2+3b^2)\cot(c+dx)}{d}-\frac{(2a^2+3b^2)\cot^3(c+dx)}{3d}-\frac{(a^2+b^2)\cot^5(c+dx)}{5d}-2ab\,\mathrm{cs}$$

```
[Out] 2*a*b*arctanh(sin(d*x+c))/d-(a^2+3*b^2)*cot(d*x+c)/d-1/3*(2*a^2+3*b^2)*cot(
d*x+c)^3/d-1/5*(a^2+b^2)*cot(d*x+c)^5/d-2*a*b*csc(d*x+c)/d-2/3*a*b*csc(d*x+
c)^3/d-2/5*a*b*csc(d*x+c)^5/d+b^2*tan(d*x+c)/d
```

Rubi [A]
time = 0.29, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3957, 2990, 2701, 308, 213, 459}

$$-\frac{(a^2+b^2)\cot^5(c+dx)}{5d}-\frac{(2a^2+3b^2)\cot^3(c+dx)}{3d}-\frac{(a^2+3b^2)\cot(c+dx)}{d}-\frac{2ab\csc^5(c+dx)}{5d}-\frac{2ab\csc^3(c+dx)}{3d}-\frac{2ab\csc(c+dx)}{d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (2*a*b*ArcTanh[Sin[c + d*x]])/d - ((a^2 + 3*b^2)*Cot[c + d*x])/d - ((2*a^2
+ 3*b^2)*Cot[c + d*x]^3)/(3*d) - ((a^2 + b^2)*Cot[c + d*x]^5)/(5*d) - (2*a*
b*Csc[c + d*x])/d - (2*a*b*Csc[c + d*x]^3)/(3*d) - (2*a*b*Csc[c + d*x]^5)/(
5*d) + (b^2*Tan[c + d*x])/d
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2990

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[2*a*(b/d), I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \csc^6(c+dx)(a+b\sec(c+dx))^2\,dx &= \int (-b-a\cos(c+dx))^2\csc^6(c+dx)\sec^2(c+dx)\,dx \\
&= (2ab)\int \csc^6(c+dx)\sec(c+dx)\,dx+\int \left(b^2+a^2\cos^2(c+dx)\right)\csc^6 \\
&= \frac{\text{Subst}\left(\int \frac{(1+x^2)^2(a^2+b^2+b^2x^2)}{x^6}\,dx,x,\tan(c+dx)\right)}{d}-\frac{(2ab)\text{Subst}\left(\int \frac{x^6}{-1+}\right.}{} \\
&= \frac{\text{Subst}\left(\int \left(b^2+\frac{a^2+b^2}{x^6}+\frac{2a^2+3b^2}{x^4}+\frac{a^2+3b^2}{x^2}\right)dx,x,\tan(c+dx)\right)}{d}-\frac{(2a}{} \\
&= -\frac{(a^2+3b^2)\cot(c+dx)}{d}-\frac{(2a^2+3b^2)\cot^3(c+dx)}{3d}-\frac{(a^2+b^2)\cot^5}{5d} \\
&= \frac{2ab\tanh^{-1}(\sin(c+dx))}{d}-\frac{(a^2+3b^2)\cot(c+dx)}{d}-\frac{(2a^2+3b^2)\cot^3}{3d}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 368 vs. 2(143) = 286.
time = 0.47, size = 368, normalized size = 2.57

$$
-\frac{\csc^7\left(\frac{1}{2}(c+dx)\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\left(40a^2+196ab\cos(c+dx)+20(a^2+6b^2)\cos(2(c+dx))-130ab\cos(3(c+dx))-16a^2\cos(4(c+dx))-96b^2\cos(4(c+dx))+30ab\cos(5(c+dx))+4a^2\cos(6(c+dx))+24b^2\cos(6(c+dx))+75ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(2(c+dx))-75ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(2(c+dx))-60ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(4(c+dx))+60ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(4(c+dx))+15ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(6(c+dx))-15ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(6(c+dx))\right)}{7680d\left(-1+\cot^2\left(\frac{1}{2}(c+dx)\right)\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -1/7680*(Csc[(c + d*x)/2]^7*Sec[(c + d*x)/2]^5*(40*a^2 + 196*a*b*Cos[c + d*
x] + 20*(a^2 + 6*b^2)*Cos[2*(c + d*x)] - 130*a*b*Cos[3*(c + d*x)] - 16*a^2*
Cos[4*(c + d*x)] - 96*b^2*Cos[4*(c + d*x)] + 30*a*b*Cos[5*(c + d*x)] + 4*a^
2*Cos[6*(c + d*x)] + 24*b^2*Cos[6*(c + d*x)] + 75*a*b*Log[Cos[(c + d*x)/2]
- Sin[(c + d*x)/2]]*Sin[2*(c + d*x)] - 75*a*b*Log[Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2]]*Sin[2*(c + d*x)] - 60*a*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/
2]]*Sin[4*(c + d*x)] + 60*a*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[
4*(c + d*x)] + 15*a*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[6*(c + d
*x)] - 15*a*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[6*(c + d*x)]))/(
d*(-1 + Cot[(c + d*x)/2]^2))
```

Maple [A]

time = 0.12, size = 154, normalized size = 1.08

method	result
derivativedivides	$\frac{b^2\left(-\frac{1}{5\sin(dx+c)^5\cos(dx+c)}-\frac{2}{5\sin(dx+c)^3\cos(dx+c)}+\frac{8}{5\sin(dx+c)\cos(dx+c)}-\frac{16\cot(dx+c)}{5}\right)+2ba\left(-\frac{1}{5\sin(dx+c)^5}-\frac{1}{3\sin(dx+c}\right.}{d}$
default	$\frac{b^2\left(-\frac{1}{5\sin(dx+c)^5\cos(dx+c)}-\frac{2}{5\sin(dx+c)^3\cos(dx+c)}+\frac{8}{5\sin(dx+c)\cos(dx+c)}-\frac{16\cot(dx+c)}{5}\right)+2ba\left(-\frac{1}{5\sin(dx+c)^5}-\frac{1}{3\sin(dx+c}\right.}{d}$
risch	$-\frac{4i\left(15ab\,\mathrm{e}^{11i(dx+c)}-65ab\,\mathrm{e}^{9i(dx+c)}+98ba\,\mathrm{e}^{7i(dx+c)}+40a^2\mathrm{e}^{6i(dx+c)}+98ba\,\mathrm{e}^{5i(dx+c)}+20a^2\mathrm{e}^{4i(dx+c)}+120b^2\mathrm{e}^{4i(dx+c)}-\right.}{15d\left(\mathrm{e}^{2i(dx+c)}+1\right)\left(\mathrm{e}^{2i(dx+c)}-1\right)^5}$
norman	$\frac{\frac{a^2+2ba+b^2}{160d}-\frac{5\left(a^2+7b^2\right)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}+\frac{\left(a^2-2ba+b^2\right)\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{160d}+\frac{\left(11a^2-32ba+21b^2\right)\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{240d}+\frac{\left(11a^2+32ba\right.}{}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6*(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^2*(-1/5/sin(d*x+c)^5/cos(d*x+c)-2/5/sin(d*x+c)^3/cos(d*x+c)+8/5/sin(
d*x+c)/cos(d*x+c)-16/5*cot(d*x+c))+2*b*a*(-1/5/sin(d*x+c)^5-1/3/sin(d*x+c)^
3-1/sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+a^2*(-8/15-1/5*csc(d*x+c)^4-4/15*
csc(d*x+c)^2)*cot(d*x+c))
```

Maxima [A]

time = 0.27, size = 143, normalized size = 1.00

$$-\frac{ab\left(\frac{2\left(15\sin(dx+c)^4+5\sin(dx+c)^2+3\right)}{\sin(dx+c)^5}-15\log\left(\sin\left(dx+c\right)+1\right)+15\log\left(\sin\left(dx+c\right)-1\right)\right)+3b^2\left(\frac{15\tan(dx+c)^4+5\tan(dx+c)^2+1}{\tan(dx+c)^5}-5\tan\left(dx+c\right)\right)+\frac{\left(15\tan(dx+c)^4+10\tan(dx+c)^2+3\right)a^2}{\tan(dx+c)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/15*(a*b*(2*(15*sin(d*x + c)^4 + 5*sin(d*x + c)^2 + 3)/sin(d*x + c)^5 - 1
5*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) + 3*b^2*((15*tan(d*x +
```

```
c)^4 + 5*tan(d*x + c)^2 + 1)/tan(d*x + c)^5 - 5*tan(d*x + c)) + (15*tan(d*x
 + c)^4 + 10*tan(d*x + c)^2 + 3)*a^2/tan(d*x + c)^5)/d
```

Fricas [A]
time = 3.90, size = 241, normalized size = 1.69

$$-\frac{30\,ab\cos\left(dx+c\right)^{5}+8\,\left(a^{2}+6\,b^{2}\right)\cos\left(dx+c\right)^{6}-70\,ab\cos\left(dx+c\right)^{3}-20\,\left(a^{2}+6\,b^{2}\right)\cos\left(dx+c\right)^{4}+46\,ab\cos\left(dx+c\right)+15\,\left(a^{2}+6\,b^{2}\right)\cos\left(dx+c\right)^{2}-15\,\left(ab\cos\left(dx+c\right)^{5}-2\,ab\cos\left(dx+c\right)^{3}+ab\cos\left(dx+c\right)\right)\log\left(\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)+15\,\left(ab\cos\left(dx+c\right)^{5}-2\,ab\cos\left(dx+c\right)^{3}+ab\cos\left(dx+c\right)\right)\log\left(-\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)-15\,b^{2}}{15\,\left(d\cos\left(dx+c\right)^{5}-2\,d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/15*(30*a*b*cos(d*x + c)^5 + 8*(a^2 + 6*b^2)*cos(d*x + c)^6 - 70*a*b*cos(
d*x + c)^3 - 20*(a^2 + 6*b^2)*cos(d*x + c)^4 + 46*a*b*cos(d*x + c) + 15*(a^
2 + 6*b^2)*cos(d*x + c)^2 - 15*(a*b*cos(d*x + c)^5 - 2*a*b*cos(d*x + c)^3 +
 a*b*cos(d*x + c))*log(sin(d*x + c) + 1)*sin(d*x + c) + 15*(a*b*cos(d*x + c
)^5 - 2*a*b*cos(d*x + c)^3 + a*b*cos(d*x + c))*log(-sin(d*x + c) + 1)*sin(d
*x + c) - 15*b^2)/((d*cos(d*x + c)^5 - 2*d*cos(d*x + c)^3 + d*cos(d*x + c))
*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 326 vs. 2(135) = 270.
time = 0.54, size = 326, normalized size = 2.28

$$\frac{3\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-6\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+3\,b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+25\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-70\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+45\,b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+960\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-960\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+150\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-660\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+570\,b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{960\,b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1}-\frac{150\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+660\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+570\,b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+25\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+70\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+45\,b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+3\,a^{2}+6\,ab+3\,b^{2}}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/480*(3*a^2*tan(1/2*d*x + 1/2*c)^5 - 6*a*b*tan(1/2*d*x + 1/2*c)^5 + 3*b^2*
tan(1/2*d*x + 1/2*c)^5 + 25*a^2*tan(1/2*d*x + 1/2*c)^3 - 70*a*b*tan(1/2*d*x
 + 1/2*c)^3 + 45*b^2*tan(1/2*d*x + 1/2*c)^3 + 960*a*b*log(abs(tan(1/2*d*x +
 1/2*c) + 1)) - 960*a*b*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 150*a^2*tan(1/
2*d*x + 1/2*c) - 660*a*b*tan(1/2*d*x + 1/2*c) + 570*b^2*tan(1/2*d*x + 1/2*c
) - 960*b^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - (150*a^2*ta
n(1/2*d*x + 1/2*c)^4 + 660*a*b*tan(1/2*d*x + 1/2*c)^4 + 570*b^2*tan(1/2*d*x
 + 1/2*c)^4 + 25*a^2*tan(1/2*d*x + 1/2*c)^2 + 70*a*b*tan(1/2*d*x + 1/2*c)^2
```

```
 + 45*b^2*tan(1/2*d*x + 1/2*c)^2 + 3*a^2 + 6*a*b + 3*b^2)/tan(1/2*d*x + 1/2
*c)^5)/d
```

Mupad [B]
time = 1.04, size = 248, normalized size = 1.73

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5 (a-b)^2}{160\,d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3 \left(\frac{a^2}{32}-\frac{5\,a\,b}{48}+\frac{7\,b^2}{96}+\frac{(a-b)^2}{48}\right)}{d}-\frac{\frac{2\,a\,b}{5}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 \left(\frac{22\,a^2}{15}+\frac{64\,a\,b}{15}+\frac{14\,b^2}{5}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 (10\,a^2+44\,a\,b+102\,b^2)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 \left(\frac{25\,a^2}{3}+\frac{118\,a\,b}{3}+35\,b^2\right)+\frac{a^2}{5}+\frac{b^2}{5}}{d\,\left(32\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5-32\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7\right)}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(\frac{7\,a^2}{32}-\frac{19\,a\,b}{16}+\frac{35\,b^2}{32}+\frac{3\,(a-b)^2}{32}\right)}{d}+\frac{4\,a\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^2/sin(c + d*x)^6,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^5*(a - b)^2)/(160*d) + (tan(c/2 + (d*x)/2)^3*(a^2/32 -
(5*a*b)/48 + (7*b^2)/96 + (a - b)^2/48))/d - ((2*a*b)/5 + tan(c/2 + (d*x)/2
)^2*((64*a*b)/15 + (22*a^2)/15 + (14*b^2)/5) - tan(c/2 + (d*x)/2)^6*(44*a*b
 + 10*a^2 + 102*b^2) + tan(c/2 + (d*x)/2)^4*((118*a*b)/3 + (25*a^2)/3 + 35*
b^2) + a^2/5 + b^2/5)/(d*(32*tan(c/2 + (d*x)/2)^5 - 32*tan(c/2 + (d*x)/2)^7
)) + (tan(c/2 + (d*x)/2)*((7*a^2)/32 - (19*a*b)/16 + (35*b^2)/32 + (3*(a -
b)^2)/32))/d + (4*a*b*atanh(tan(c/2 + (d*x)/2)))/d
```

3.185 $\int (a+b\sec(c+dx))^3 \sin^5(c+dx)\,dx$

Optimal. Leaf size=170

$$-\frac{a(a^2-6b^2)\cos(c+dx)}{d}+\frac{b(6a^2-b^2)\cos^2(c+dx)}{2d}+\frac{a(2a^2-3b^2)\cos^3(c+dx)}{3d}-\frac{3a^2b\cos^4(c+dx)}{4d}-\frac{a^3\cos^5(\cdots}{5\cdots}$$

```
[Out] -a*(a^2-6*b^2)*cos(d*x+c)/d+1/2*b*(6*a^2-b^2)*cos(d*x+c)^2/d+1/3*a*(2*a^2-3
*b^2)*cos(d*x+c)^3/d-3/4*a^2*b*cos(d*x+c)^4/d-1/5*a^3*cos(d*x+c)^5/d-b*(3*a
^2-2*b^2)*ln(cos(d*x+c))/d+3*a*b^2*sec(d*x+c)/d+1/2*b^3*sec(d*x+c)^2/d
```

Rubi [A]
time = 0.19, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3957, 2916, 12, 962}

$$-\frac{a^3\cos^5(c+dx)}{5d}+\frac{a(2a^2-3b^2)\cos^3(c+dx)}{3d}+\frac{b(6a^2-b^2)\cos^2(c+dx)}{2d}-\frac{a(a^2-6b^2)\cos(c+dx)}{d}-\frac{b(3a^2-2b^2)\log(\cos(c+dx))}{d}-\frac{3a^2b\cos^4(c+dx)}{4d}+\frac{3ab^2\sec(c+dx)}{d}+\frac{b^3\sec^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^3*Sin[c + d*x]^5,x]
```

```
[Out] -((a*(a^2 - 6*b^2)*Cos[c + d*x])/d) + (b*(6*a^2 - b^2)*Cos[c + d*x]^2)/(2*d
) + (a*(2*a^2 - 3*b^2)*Cos[c + d*x]^3)/(3*d) - (3*a^2*b*Cos[c + d*x]^4)/(4*
d) - (a^3*Cos[c + d*x]^5)/(5*d) - (b*(3*a^2 - 2*b^2)*Log[Cos[c + d*x]])/d +
 (3*a*b^2*Sec[c + d*x])/d + (b^3*Sec[c + d*x]^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 962

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+b\sec(c+dx))^3\sin^5(c+dx)\,dx = -\int (-b-a\cos(c+dx))^3\sin^2(c+dx)\tan^3(c+dx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{a^3(-b+x)^3\left(a^2-x^2\right)^2}{x^3}\,dx, x, -a\cos(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int \frac{(-b+x)^3\left(a^2-x^2\right)^2}{x^3}\,dx, x, -a\cos(c+dx)\right)}{a^2d}$$
$$= \frac{\text{Subst}\left(\int \left(a^4\left(1-\frac{6b^2}{a^2}\right)-\frac{a^4b^3}{x^3}+\frac{3a^4b^2}{x^2}+\frac{-3a^4b+2a^2b^3}{x}-b(-6a^2+b^2)\right.\right.}{a^2d}$$
$$= -\frac{a(a^2-6b^2)\cos(c+dx)}{d}+\frac{b(6a^2-b^2)\cos^2(c+dx)}{2d}+\frac{a(2a^2-3b}{}$$

Mathematica [A]
time = 0.41, size = 154, normalized size = 0.91

$$\frac{-60a(5a^2-42b^2)\cos(c+dx)+60(9a^2b-2b^3)\cos(2(c+dx))+50a^3\cos(3(c+dx))-120ab^2\cos(3(c+dx))-45a^2b\cos(4(c+dx))-6a^3\cos(5(c+dx))-1440a^2b\log(\cos(c+dx))+960b^3\log(\cos(c+dx))+1440ab^2\sec(c+dx)+240b^3\sec^2(c+dx)}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^3*Sin[c + d*x]^5,x]
```

```
[Out] (-60*a*(5*a^2 - 42*b^2)*Cos[c + d*x] + 60*(9*a^2*b - 2*b^3)*Cos[2*(c + d*x)
] + 50*a^3*Cos[3*(c + d*x)] - 120*a*b^2*Cos[3*(c + d*x)] - 45*a^2*b*Cos[4*(
c + d*x)] - 6*a^3*Cos[5*(c + d*x)] - 1440*a^2*b*Log[Cos[c + d*x]] + 960*b^3
*Log[Cos[c + d*x]] + 1440*a*b^2*Sec[c + d*x] + 240*b^3*Sec[c + d*x]^2)/(480
*d)
```

Maple [A]
time = 0.14, size = 174, normalized size = 1.02

method	result
derivativedivides	$\frac{b^3\left(\frac{\sin^6(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^4(dx+c)\right)}{2}+\sin^2(dx+c)+2\ln(\cos(dx+c))\right)+3b^2a\left(\frac{\sin^6(dx+c)}{\cos(dx+c)}+\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\right.}{d}$

default	$\frac{b^3\left(\frac{\sin^6(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^4(dx+c)\right)}{2}+\sin^2(dx+c)+2\ln(\cos(dx+c))\right)+3b^2a\left(\frac{\sin^6(dx+c)}{\cos(dx+c)}+\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\text{co}}{d}$
norman	$\frac{\frac{\left(16a^3+24b\,a^2-48b^2a+16b^3\right)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{16a^3-240b^2a}{15d}-\frac{\left(6b\,a^2-4b^3\right)\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{\left(18b\,a^2-12b^3\right)\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}$
risch	$3ia^2bx-2ib^3x+\frac{5a^3\mathrm{e}^{3i(dx+c)}}{96d}-\frac{\mathrm{e}^{3i(dx+c)}b^2a}{8d}+\frac{9\,\mathrm{e}^{2i(dx+c)}b\,a^2}{16d}-\frac{\mathrm{e}^{2i(dx+c)}b^3}{8d}-\frac{5a^3\mathrm{e}^{i(dx+c)}}{16d}+\frac{21\,\mathrm{e}^{i(dx+c}}{8d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3*sin(d*x+c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^3*(1/2*sin(d*x+c)^6/cos(d*x+c)^2+1/2*sin(d*x+c)^4+sin(d*x+c)^2+2*ln(
cos(d*x+c)))+3*b^2*a*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin(d*x+c)^4+4/3*sin(d*x
+c)^2)*cos(d*x+c))+3*b*a^2*(-1/4*sin(d*x+c)^4-1/2*sin(d*x+c)^2-ln(cos(d*x+c
)))-1/5*a^3*(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))
```

Maxima [A]
time = 0.29, size = 142, normalized size = 0.84

$$-\frac{12\,a^3\cos(dx+c)^5+45\,a^2b\cos(dx+c)^4-20\left(2\,a^3-3\,ab^2\right)\cos(dx+c)^3-30\left(6\,a^2b-b^3\right)\cos(dx+c)^2+60\left(a^3-6\,ab^2\right)\cos(dx+c)+60\left(3\,a^2b-2\,b^3\right)\log(\cos(dx+c))-\frac{30\left(6\,ab^2\cos(dx+c)+b^3\right)}{\cos(dx+c)^2}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/60*(12*a^3*cos(d*x + c)^5 + 45*a^2*b*cos(d*x + c)^4 - 20*(2*a^3 - 3*a*b^
2)*cos(d*x + c)^3 - 30*(6*a^2*b - b^3)*cos(d*x + c)^2 + 60*(a^3 - 6*a*b^2)*
cos(d*x + c) + 60*(3*a^2*b - 2*b^3)*log(cos(d*x + c)) - 30*(6*a*b^2*cos(d*x
 + c) + b^3)/cos(d*x + c)^2)/d
```

Fricas [A]
time = 3.22, size = 175, normalized size = 1.03

$$-\frac{96\,a^3\cos(dx+c)^7+360\,a^2b\cos(dx+c)^6-160\left(2\,a^3-3\,ab^2\right)\cos(dx+c)^5-240\left(6\,a^2b-b^3\right)\cos(dx+c)^4-1440\,ab^2\cos(dx+c)+480\left(a^3-6\,ab^2\right)\cos(dx+c)^3+480\left(3\,a^2b-2\,b^3\right)\cos(dx+c)^2\log(-\cos(dx+c))-240\,b^3+15\left(39\,a^2b-8\,b^3\right)\cos(dx+c)^2}{480\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/480*(96*a^3*cos(d*x + c)^7 + 360*a^2*b*cos(d*x + c)^6 - 160*(2*a^3 - 3*a
*b^2)*cos(d*x + c)^5 - 240*(6*a^2*b - b^3)*cos(d*x + c)^4 - 1440*a*b^2*cos(
d*x + c) + 480*(a^3 - 6*a*b^2)*cos(d*x + c)^3 + 480*(3*a^2*b - 2*b^3)*cos(d
*x + c)^2*log(-cos(d*x + c)) - 240*b^3 + 15*(39*a^2*b - 8*b^3)*cos(d*x + c)
^2)/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3*sin(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 695 vs. $2(160) = 320$.

time = 0.60, size = 695, normalized size = 4.09

$$\frac{60\,(3\,a^2b-2\,b^3)\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-60\,(3\,a^2b-2\,b^3)\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{30\left(9\,a^2b+12\,ab^2-6\,b^3+\frac{18\,a^2b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{12\,ab^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{16\,b^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{9\,a^2b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{6\,b^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^2}+\frac{64\,a^3+411\,a^2b-600\,ab^2-274\,b^3-\frac{320\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{2415\,a^2b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{2640\,ab^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{1490\,b^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{640\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{5910\,a^2b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{3840\,ab^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{3100\,b^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{5910\,a^2b(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{2160\,ab^2(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{3100\,b^3(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{2415\,a^2b(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}-\frac{360\,ab^2(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}-\frac{1490\,b^3(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}-\frac{411\,a^2b(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5}+\frac{274\,b^3(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/60*(60*(3*a^2*b - 2*b^3)*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) +
 1)) - 60*(3*a^2*b - 2*b^3)*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
- 1)) + 30*(9*a^2*b + 12*a*b^2 - 6*b^3 + 18*a^2*b*(cos(d*x + c) - 1)/(cos(d
*x + c) + 1) + 12*a*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 16*b^3*(cos
(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*a^2*b*(cos(d*x + c) - 1)^2/(cos(d*x +
 c) + 1)^2 - 6*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/((cos(d*x + c
) - 1)/(cos(d*x + c) + 1) + 1)^2 + (64*a^3 + 411*a^2*b - 600*a*b^2 - 274*b^
3 - 320*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2415*a^2*b*(cos(d*x + c
) - 1)/(cos(d*x + c) + 1) + 2640*a*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1
) + 1490*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 640*a^3*(cos(d*x + c)
- 1)^2/(cos(d*x + c) + 1)^2 + 5910*a^2*b*(cos(d*x + c) - 1)^2/(cos(d*x + c)
 + 1)^2 - 3840*a*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 3100*b^3*(
cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 5910*a^2*b*(cos(d*x + c) - 1)^3/
(cos(d*x + c) + 1)^3 + 2160*a*b^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3
 + 3100*b^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 2415*a^2*b*(cos(d*x
 + c) - 1)^4/(cos(d*x + c) + 1)^4 - 360*a*b^2*(cos(d*x + c) - 1)^4/(cos(d*x
 + c) + 1)^4 - 1490*b^3*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 411*a^2
*b*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 274*b^3*(cos(d*x + c) - 1)^5
/(cos(d*x + c) + 1)^5)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^5)/d
```

Mupad [B]

time = 0.96, size = 143, normalized size = 0.84

$$-\frac{\cos(c+d\,x)^3\left(a\,b^2-\frac{2\,a^3}{3}\right)-\cos(c+d\,x)^2\left(3\,a^2\,b-\frac{b^3}{2}\right)+\ln(\cos(c+d\,x))\,(3\,a^2\,b-2\,b^3)-\frac{\frac{b^3}{2}+3\,a\,\cos(c+d\,x)\,b^2}{\cos(c+d\,x)^2}-\cos(c+d\,x)\,(6\,a\,b^2-a^3)+\frac{a^3\cos(c+d\,x)^5}{5}+\frac{3\,a^2\,b\cos(c+d\,x)^4}{4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5*(a + b/cos(c + d*x))^3,x)
```

```
[Out] -(cos(c + d*x)^3*(a*b^2 - (2*a^3)/3) - cos(c + d*x)^2*(3*a^2*b - b^3/2) + l
og(cos(c + d*x))*(3*a^2*b - 2*b^3) - (b^3/2 + 3*a*b^2*cos(c + d*x))/cos(c +
 d*x)^2 - cos(c + d*x)*(6*a*b^2 - a^3) + (a^3*cos(c + d*x)^5)/5 + (3*a^2*b*
cos(c + d*x)^4)/4)/d
```

3.186 $\int (a+b\sec(c+dx))^3\sin^3(c+dx)\,dx$

Optimal. Leaf size=116

$$-\frac{a(a^2-3b^2)\cos(c+dx)}{d}+\frac{3a^2b\cos^2(c+dx)}{2d}+\frac{a^3\cos^3(c+dx)}{3d}-\frac{b(3a^2-b^2)\log(\cos(c+dx))}{d}+\frac{3ab^2\sec(c+d}{d}$$

```
[Out] -a*(a^2-3*b^2)*cos(d*x+c)/d+3/2*a^2*b*cos(d*x+c)^2/d+1/3*a^3*cos(d*x+c)^3/d
-b*(3*a^2-b^2)*ln(cos(d*x+c))/d+3*a*b^2*sec(d*x+c)/d+1/2*b^3*sec(d*x+c)^2/d
```

Rubi [A]
time = 0.09, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3957, 2800, 908}

$$\frac{a^3\cos^3(c+dx)}{3d}-\frac{a(a^2-3b^2)\cos(c+dx)}{d}-\frac{b(3a^2-b^2)\log(\cos(c+dx))}{d}+\frac{3a^2b\cos^2(c+dx)}{2d}+\frac{3ab^2\sec(c+dx)}{d}+\frac{b^3\sec^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^3*Sin[c + d*x]^3,x]
```

```
[Out] -((a*(a^2 - 3*b^2)*Cos[c + d*x])/d) + (3*a^2*b*Cos[c + d*x]^2)/(2*d) + (a^3
*Cos[c + d*x]^3)/(3*d) - (b*(3*a^2 - b^2)*Log[Cos[c + d*x]])/d + (3*a*b^2*S
ec[c + d*x])/d + (b^3*Sec[c + d*x]^2)/(2*d)
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+b\sec(c+dx))^3\sin^3(c+dx)\,dx = -\int (-b-a\cos(c+dx))^3\tan^3(c+dx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{(-b+x)^3(a^2-x^2)}{x^3}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(a^2\left(1-\frac{3b^2}{a^2}\right)-\frac{a^2b^3}{x^3}+\frac{3a^2b^2}{x^2}+\frac{-3a^2b+b^3}{x}+3bx-x^2\right)dx, x\right.}{d}$$
$$= -\frac{a(a^2-3b^2)\cos(c+dx)}{d}+\frac{3a^2b\cos^2(c+dx)}{2d}+\frac{a^3\cos^3(c+dx)}{3d}-$$

Mathematica [A]
time = 0.22, size = 102, normalized size = 0.88

$$\frac{-9a(a^2-4b^2)\cos(c+dx)+9a^2b\cos(2(c+dx))+a^3\cos(3(c+dx))-36a^2b\log(\cos(c+dx))+12b^3\log(\cos(c+dx))+36ab^2\sec(c+dx)+6b^3\sec^2(c+dx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^3*Sin[c + d*x]^3,x]
```

```
[Out] (-9*a*(a^2 - 4*b^2)*Cos[c + d*x] + 9*a^2*b*Cos[2*(c + d*x)] + a^3*Cos[3*(c
+ d*x)] - 36*a^2*b*Log[Cos[c + d*x]] + 12*b^3*Log[Cos[c + d*x]] + 36*a*b^2*
Sec[c + d*x] + 6*b^3*Sec[c + d*x]^2)/(12*d)
```

Maple [A]
time = 0.11, size = 116, normalized size = 1.00

method	result
derivativedivides	$\frac{b^3\left(\frac{\left(\tan^2(dx+c)\right)}{2}+\ln(\cos(dx+c))\right)+3b^2a\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+\left(2+\sin^2(dx+c)\right)\cos(dx+c)\right)+3b\,a^2\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx\right.}{d}$
default	$\frac{b^3\left(\frac{\left(\tan^2(dx+c)\right)}{2}+\ln(\cos(dx+c))\right)+3b^2a\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+\left(2+\sin^2(dx+c)\right)\cos(dx+c)\right)+3b\,a^2\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx\right.}{d}$
risch	$-ib^3x-\frac{2ib^3c}{d}+\frac{a^3e^{3i(dx+c)}}{24d}+\frac{3\,e^{2i(dx+c)}b\,a^2}{8d}-\frac{3a^3e^{i(dx+c)}}{8d}+\frac{3\,e^{i(dx+c)}b^2a}{2d}-\frac{3a^3e^{-i(dx+c)}}{8d}+\frac{3\,e^{-i(dx-}}{2d}$
norman	$\frac{-\frac{4a^3-36b^2a}{3d}-\frac{\left(6b\,a^2-2b^3\right)\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2\left(2a^3-3b\,a^2+6b^2a-3b^3\right)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{\left(4a^3+18b\,a^2-36b^2a-6b^3\right)\left(\tan^2\left(\frac{dx}{2}+\right.\right.}{3d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3*sin(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^3*(1/2*tan(d*x+c)^2+ln(cos(d*x+c)))+3*b^2*a*(sin(d*x+c)^4/cos(d*x+c)
+(2+sin(d*x+c)^2)*cos(d*x+c))+3*b*a^2*(-1/2*sin(d*x+c)^2-ln(cos(d*x+c)))-1/
3*a^3*(2+sin(d*x+c)^2)*cos(d*x+c))
```

Maxima [A]
time = 0.27, size = 98, normalized size = 0.84

$$\frac{2\,a^3\cos\left(dx+c\right)^3+9\,a^2b\cos\left(dx+c\right)^2-6\left(a^3-3\,ab^2\right)\cos\left(dx+c\right)-6\left(3\,a^2b-b^3\right)\log\left(\cos\left(dx+c\right)\right)+\frac{3\left(6\,ab^2\cos(dx+c)+b^3\right)}{\cos(dx+c)^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(2*a^3*cos(d*x + c)^3 + 9*a^2*b*cos(d*x + c)^2 - 6*(a^3 - 3*a*b^2)*cos(
d*x + c) - 6*(3*a^2*b - b^3)*log(cos(d*x + c)) + 3*(6*a*b^2*cos(d*x + c) +
b^3)/cos(d*x + c)^2)/d
```

Fricas [A]
time = 3.73, size = 123, normalized size = 1.06

$$\frac{4\,a^3\cos\left(dx+c\right)^5+18\,a^2b\cos\left(dx+c\right)^4-9\,a^2b\cos\left(dx+c\right)^2+36\,ab^2\cos\left(dx+c\right)-12\left(a^3-3\,ab^2\right)\cos\left(dx+c\right)^3-12\left(3\,a^2b-b^3\right)\cos\left(dx+c\right)^2\log\left(-\cos\left(dx+c\right)\right)+6\,b^3}{12\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(4*a^3*cos(d*x + c)^5 + 18*a^2*b*cos(d*x + c)^4 - 9*a^2*b*cos(d*x + c)
^2 + 36*a*b^2*cos(d*x + c) - 12*(a^3 - 3*a*b^2)*cos(d*x + c)^3 - 12*(3*a^2*
b - b^3)*cos(d*x + c)^2*log(-cos(d*x + c)) + 6*b^3)/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec\left(c+dx\right)\right)^3\sin^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3*sin(d*x+c)**3,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3*sin(c + d*x)**3, x)
```

Giac [A]
time = 0.54, size = 128, normalized size = 1.10

$$-\frac{\left(3\,a^2b-b^3\right)\log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d}+\frac{6\,ab^2\cos\left(dx+c\right)+b^3}{2\,d\cos\left(dx+c\right)^2}+\frac{2\,a^3d^8\cos\left(dx+c\right)^3+9\,a^2bd^8\cos\left(dx+c\right)^2-6\,a^3d^8\cos\left(dx+c\right)+18\,ab^2d^8\cos\left(dx+c\right)}{6\,d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -(3*a^2*b - b^3)*log(abs(cos(d*x + c))/abs(d))/d + 1/2*(6*a*b^2*cos(d*x + c
) + b^3)/(d*cos(d*x + c)^2) + 1/6*(2*a^3*d^8*cos(d*x + c)^3 + 9*a^2*b*d^8*c
os(d*x + c)^2 - 6*a^3*d^8*cos(d*x + c) + 18*a*b^2*d^8*cos(d*x + c))/d^9
```

Mupad [B]
time = 0.07, size = 99, normalized size = 0.85

$$\frac{\frac{\frac{b^3}{2}+3\,a\,\cos(c+d\,x)\,b^2}{\cos(c+d\,x)^2}-\ln\left(\cos\left(c+d\,x\right)\right)\,\left(3\,a^2\,b-b^3\right)+\cos\left(c+d\,x\right)\,\left(3\,a\,b^2-a^3\right)+\frac{a^3\cos(c+d\,x)^3}{3}+\frac{3\,a^2\,b\cos(c+d\,x)^2}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3*(a + b/cos(c + d*x))^3,x)
```

```
[Out] ((b^3/2 + 3*a*b^2*cos(c + d*x))/cos(c + d*x)^2 - log(cos(c + d*x))*(3*a^2*b
 - b^3) + cos(c + d*x)*(3*a*b^2 - a^3) + (a^3*cos(c + d*x)^3)/3 + (3*a^2*b*
cos(c + d*x)^2)/2)/d
```

3.187 $\int (a+b\sec(c+dx))^3\sin(c+dx)\,dx$

Optimal. Leaf size=64

$$-\frac{a^3\cos(c+dx)}{d}-\frac{3a^2b\log(\cos(c+dx))}{d}+\frac{3ab^2\sec(c+dx)}{d}+\frac{b^3\sec^2(c+dx)}{2d}$$

```
[Out] -a^3*cos(d*x+c)/d-3*a^2*b*ln(cos(d*x+c))/d+3*a*b^2*sec(d*x+c)/d+1/2*b^3*sec
(d*x+c)^2/d
```

Rubi [A]
time = 0.07, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.210$, Rules used = {3957, 2912, 12, 45}

$$-\frac{a^3\cos(c+dx)}{d}-\frac{3a^2b\log(\cos(c+dx))}{d}+\frac{3ab^2\sec(c+dx)}{d}+\frac{b^3\sec^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^3*Sin[c + d*x],x]
```

```
[Out] -((a^3*Cos[c + d*x])/d) - (3*a^2*b*Log[Cos[c + d*x]])/d + (3*a*b^2*Sec[c +
d*x])/d + (b^3*Sec[c + d*x]^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
```

```
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int (a+b\sec(c+dx))^3\sin(c+dx)\,dx &= -\int (-b-a\cos(c+dx))^3\sec^2(c+dx)\tan(c+dx)\,dx\\ &= \frac{\text{Subst}\left(\int \frac{a^3(-b+x)^3}{x^3}\,dx, x, -a\cos(c+dx)\right)}{ad}\\ &= \frac{a^2\text{Subst}\left(\int \frac{(-b+x)^3}{x^3}\,dx, x, -a\cos(c+dx)\right)}{d}\\ &= \frac{a^2\text{Subst}\left(\int \left(1-\frac{b^3}{x^3}+\frac{3b^2}{x^2}-\frac{3b}{x}\right)\,dx, x, -a\cos(c+dx)\right)}{d}\\ &= -\frac{a^3\cos(c+dx)}{d}-\frac{3a^2b\log(\cos(c+dx))}{d}+\frac{3ab^2\sec(c+dx)}{d}+\frac{b^3\text{se}}{}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 56, normalized size = 0.88

$$\frac{-2a^3\cos(c+dx)+b(-6a^2\log(\cos(c+dx))+6ab\sec(c+dx)+b^2\sec^2(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^3*Sin[c + d*x],x]
[Out] (-2*a^3*Cos[c + d*x] + b*(-6*a^2*Log[Cos[c + d*x]] + 6*a*b*Sec[c + d*x] + b
^2*Sec[c + d*x]^2))/(2*d)
```

Maple [A]
time = 0.06, size = 57, normalized size = 0.89

method	result
derivativedivides	$\frac{\frac{\left(\sec^2(dx+c)\right)b^3}{2}+3b^2a\sec(dx+c)-\frac{a^3}{\sec(dx+c)}+3b\,a^2\ln(\sec(dx+c))}{d}$
default	$\frac{\frac{\left(\sec^2(dx+c)\right)b^3}{2}+3b^2a\sec(dx+c)-\frac{a^3}{\sec(dx+c)}+3b\,a^2\ln(\sec(dx+c))}{d}$
risch	$3ia^2bx-\frac{a^3e^{i(dx+c)}}{2d}-\frac{a^3e^{-i(dx+c)}}{2d}+\frac{6ib\,a^2c}{d}+\frac{2b^2\left(3a\,e^{3i(dx+c)}+b\,e^{2i(dx+c)}+3\,e^{i(dx+c)}a\right)}{d\left(e^{2i(dx+c)}+1\right)^2}-\frac{3b\ln\left(e^{2i(dx+c)}\right.}{d}$
norman	$\frac{\frac{\left(4a^3+2b^3\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2a^3-6b^2a}{d}-\frac{\left(2a^3+6b^2a-2b^3\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{3b\,a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{3b\,a^2\ln\left(\tan\left(\right.\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3*sin(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*sec(d*x+c)^2*b^3+3*b^2*a*sec(d*x+c)-a^3/sec(d*x+c)+3*b*a^2*ln(sec(
d*x+c)))
```

Maxima [A]
time = 0.26, size = 57, normalized size = 0.89

$$-\frac{2\,a^3\cos\left(dx+c\right)+6\,a^2b\log\left(\cos\left(dx+c\right)\right)-\frac{6\,ab^2}{\cos\left(dx+c\right)}-\frac{b^3}{\cos\left(dx+c\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*(2*a^3*cos(d*x + c) + 6*a^2*b*log(cos(d*x + c)) - 6*a*b^2/cos(d*x + c)
 - b^3/cos(d*x + c)^2)/d
```

Fricas [A]
time = 4.68, size = 67, normalized size = 1.05

$$-\frac{2\,a^3\cos\left(dx+c\right)^3+6\,a^2b\cos\left(dx+c\right)^2\log\left(-\cos\left(dx+c\right)\right)-6\,ab^2\cos\left(dx+c\right)-b^3}{2\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^3*cos(d*x + c)^3 + 6*a^2*b*cos(d*x + c)^2*log(-cos(d*x + c)) - 6*
a*b^2*cos(d*x + c) - b^3)/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec\left(c+dx\right)\right)^3\sin\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3*sin(d*x+c),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3*sin(c + d*x), x)
```

Giac [A]
time = 0.50, size = 66, normalized size = 1.03

$$-\frac{a^3\cos\left(dx+c\right)}{d}-\frac{3\,a^2b\log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d}+\frac{6\,ab^2\cos\left(dx+c\right)+b^3}{2\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -a^3*cos(d*x + c)/d - 3*a^2*b*log(abs(cos(d*x + c))/abs(d))/d + 1/2*(6*a*b^
2*cos(d*x + c) + b^3)/(d*cos(d*x + c)^2)
```

Mupad [B]
time = 0.93, size = 57, normalized size = 0.89

$$-\frac{a^3\,\cos\left(c+d\,x\right)-\frac{\frac{b^3}{2}+3\,a\,\cos(c+d\,x)\,b^2}{\cos(c+d\,x)^2}+3\,a^2\,b\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a + b/cos(c + d*x))^3,x)
```

```
[Out] -(a^3*cos(c + d*x) - (b^3/2 + 3*a*b^2*cos(c + d*x))/cos(c + d*x)^2 + 3*a^2*
b*log(cos(c + d*x)))/d
```

3.188 $\int \csc(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=102

$$\frac{(a+b)^3\log(1-\cos(c+dx))}{2d}-\frac{b(3a^2+b^2)\log(\cos(c+dx))}{d}-\frac{(a-b)^3\log(1+\cos(c+dx))}{2d}+\frac{3ab^2\sec(c+dx)}{d}$$

```
[Out] 1/2*(a+b)^3*ln(1-cos(d*x+c))/d-b*(3*a^2+b^2)*ln(cos(d*x+c))/d-1/2*(a-b)^3*l
n(1+cos(d*x+c))/d+3*a*b^2*sec(d*x+c)/d+1/2*b^3*sec(d*x+c)^2/d
```

Rubi [A]

time = 0.16, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3957, 2916, 12, 1816}

$$-\frac{b(3a^2+b^2)\log(\cos(c+dx))}{d}+\frac{3ab^2\sec(c+dx)}{d}-\frac{(a-b)^3\log(\cos(c+dx)+1)}{2d}+\frac{(a+b)^3\log(1-\cos(c+dx))}{2d}+\frac{b^3\sec^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((a + b)^3*Log[1 - Cos[c + d*x]])/(2*d) - (b*(3*a^2 + b^2)*Log[Cos[c + d*x]
])/d - ((a - b)^3*Log[1 + Cos[c + d*x]])/(2*d) + (3*a*b^2*Sec[c + d*x])/d +
 (b^3*Sec[c + d*x]^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
```

```
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc(c+dx)(a+b\sec(c+dx))^3\,dx = -\int(-b-a\cos(c+dx))^3\csc(c+dx)\sec^3(c+dx)\,dx$$
$$= \frac{a\text{Subst}\left(\int \frac{a^3(-b+x)^3}{x^3(a^2-x^2)}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^4\text{Subst}\left(\int \frac{(-b+x)^3}{x^3(a^2-x^2)}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^4\text{Subst}\left(\int \left(\frac{(a-b)^3}{2a^4(a-x)} - \frac{b^3}{a^2x^3} + \frac{3b^2}{a^2x^2} + \frac{b(-3a^2-b^2)}{a^4x} + \frac{(a+b)^3}{2a^4(a+x)}\right)\,dx, x, -\right.}{d}$$
$$= \frac{(a+b)^3\log(1-\cos(c+dx))}{2d} - \frac{b(3a^2+b^2)\log(\cos(c+dx))}{d} - \frac{(a-}{}$$

Mathematica [A]
time = 0.20, size = 89, normalized size = 0.87

$$\frac{-2(a-b)^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) - 2b(3a^2+b^2)\log(\cos(c+dx)) + 2(a+b)^3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + 6ab^2\sec(c+dx) + b^3\sec^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*(a + b*Sec[c + d*x])^3,x]
[Out] (-2*(a - b)^3*Log[Cos[(c + d*x)/2]] - 2*b*(3*a^2 + b^2)*Log[Cos[c + d*x]] +
 2*(a + b)^3*Log[Sin[(c + d*x)/2]] + 6*a*b^2*Sec[c + d*x] + b^3*Sec[c + d*x
]^2)/(2*d)
```

Maple [A]
time = 0.10, size = 92, normalized size = 0.90

method	result
derivativedivides	$\frac{b^3\left(\frac{1}{2\cos(dx+c)^2}+\ln(\tan(dx+c))\right)+3b^2a\left(\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c))\right)+3b\,a^2\ln(\tan(dx+c))+a^3\ln(\csc(dx+c}{d}$
default	$\frac{b^3\left(\frac{1}{2\cos(dx+c)^2}+\ln(\tan(dx+c))\right)+3b^2a\left(\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c))\right)+3b\,a^2\ln(\tan(dx+c))+a^3\ln(\csc(dx+c}{d}$
norman	$\frac{\frac{6b^2a}{d}-\frac{2(3b^2a-b^3)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{(a^3+3b\,a^2+3b^2a+b^3)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{b(3a^2+b^2)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-$
risch	$\frac{2b^2(3a\,e^{3i(dx+c)}+b\,e^{2i(dx+c)}+3\,e^{i(dx+c)}a)}{d(e^{2i(dx+c)}+1)^2}-\frac{a^3\ln(e^{i(dx+c)}+1)}{d}+\frac{3\ln(e^{i(dx+c)}+1)b\,a^2}{d}-\frac{3\ln(e^{i(dx+c)}+1)b^2a}{d}+$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(b^3*(1/2/cos(d*x+c)^2+ln(tan(d*x+c)))+3*b^2*a*(1/cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))+3*b*a^2*ln(tan(d*x+c))+a^3*ln(csc(d*x+c)-cot(d*x+c)))

Maxima [A]
time = 0.27, size = 112, normalized size = 1.10

$$-\frac{\left(a^{3}-3\,a^{2}b+3\,ab^{2}-b^{3}\right)\log\left(\cos\left(dx+c\right)+1\right)-\left(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}\right)\log\left(\cos\left(dx+c\right)-1\right)+2\left(3\,a^{2}b+b^{3}\right)\log\left(\cos\left(dx+c\right)\right)-\frac{6\,ab^{2}\cos(dx+c)+b^{3}}{\cos(dx+c)^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c))^3,x, algorithm="maxima")

[Out] -1/2*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*log(cos(d*x + c) + 1) - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*log(cos(d*x + c) - 1) + 2*(3*a^2*b + b^3)*log(cos(d*x + c)) - (6*a*b^2*cos(d*x + c) + b^3)/cos(d*x + c)^2)/d

Fricas [A]
time = 7.58, size = 139, normalized size = 1.36

$$\frac{6\,ab^{2}\cos\left(dx+c\right)-2\left(3\,a^{2}b+b^{3}\right)\cos\left(dx+c\right)^{2}\log\left(-\cos\left(dx+c\right)\right)-\left(a^{3}-3\,a^{2}b+3\,ab^{2}-b^{3}\right)\cos\left(dx+c\right)^{2}\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+\left(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}\right)\cos\left(dx+c\right)^{2}\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+b^{3}}{2\,d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")

[Out] 1/2*(6*a*b^2*cos(d*x + c) - 2*(3*a^2*b + b^3)*cos(d*x + c)^2*log(-cos(d*x + c)) - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(d*x + c)^2*log(1/2*cos(d*x + c) + 1/2) + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(d*x + c)^2*log(-1/2*cos(d*x + c) + 1/2) + b^3)/(d*cos(d*x + c)^2)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec\left(c+dx\right)\right)^{3}\csc\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c))**3,x)

[Out] Integral((a + b*sec(c + d*x))**3*csc(c + d*x), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(96) = 192.

time = 0.52, size = 250, normalized size = 2.45

$$\frac{(a^3+3a^2b+3ab^2+b^3)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-2\,(3a^2b+b^3)\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{9a^2b+12ab^2+3b^3+\frac{18a^2b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{12ab^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{2b^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{9a^2b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{3b^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x
 + c) + 1)) - 2*(3*a^2*b + b^3)*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) +
 1) - 1)) + (9*a^2*b + 12*a*b^2 + 3*b^3 + 18*a^2*b*(cos(d*x + c) - 1)/(cos(
d*x + c) + 1) + 12*a*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2*b^3*(cos
(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*a^2*b*(cos(d*x + c) - 1)^2/(cos(d*x +
 c) + 1)^2 + 3*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/((cos(d*x + c
) - 1)/(cos(d*x + c) + 1) + 1)^2)/d
```

Mupad [B]

time = 0.13, size = 85, normalized size = 0.83

$$\frac{\frac{\ln(\cos(c+d\,x)-1)\,(a+b)^3}{2}-\frac{\ln(\cos(c+d\,x)+1)\,(a-b)^3}{2}+\frac{\frac{b^3}{2}+3\,a\,\cos(c+d\,x)\,b^2}{\cos(c+d\,x)^2}-\ln\left(\cos\left(c+d\,x\right)\right)\,\left(3\,a^2\,b+b^3\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^3/sin(c + d*x),x)
```

```
[Out] ((log(cos(c + d*x) - 1)*(a + b)^3)/2 - (log(cos(c + d*x) + 1)*(a - b)^3)/2
+ (b^3/2 + 3*a*b^2*cos(c + d*x))/cos(c + d*x)^2 - log(cos(c + d*x))*(3*a^2*
b + b^3))/d
```

3.189 $\int \csc^3(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=162

$$-\frac{a^2\left(b\left(3+\frac{b^2}{a^2}\right)+a\left(1+\frac{3b^2}{a^2}\right)\cos(c+dx)\right)\csc^2(c+dx)}{2d}+\frac{(a+b)^2(a+4b)\log(1-\cos(c+dx))}{4d}-\frac{b(3a^2+2b^2}{}$$

```
[Out] -1/2*a^2*(b*(3+b^2/a^2)+a*(1+3*b^2/a^2)*cos(d*x+c))*csc(d*x+c)^2/d+1/4*(a+b
)^2*(a+4*b)*ln(1-cos(d*x+c))/d-b*(3*a^2+2*b^2)*ln(cos(d*x+c))/d-1/4*(a-4*b)
*(a-b)^2*ln(1+cos(d*x+c))/d+3*a*b^2*sec(d*x+c)/d+1/2*b^3*sec(d*x+c)^2/d
```

Rubi [A]
time = 0.25, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3957, 2916, 12, 1819, 1816}

$$-\frac{b(3a^2+2b^2)\log(\cos(c+dx))}{d}-\frac{a^2\csc^2(c+dx)\left(a\left(\frac{3b^2}{a^2}+1\right)\cos(c+dx)+b\left(\frac{b^2}{a^2}+3\right)\right)}{2d}+\frac{3ab^2\sec(c+dx)}{d}+\frac{(a+b)^2(a+4b)\log(1-\cos(c+dx))}{4d}-\frac{(a-4b)(a-b)^2\log(\cos(c+dx)+1)}{4d}+\frac{b^3\sec^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -1/2*(a^2*(b*(3 + b^2/a^2) + a*(1 + (3*b^2)/a^2)*Cos[c + d*x])*Csc[c + d*x]
^2)/d + ((a + b)^2*(a + 4*b)*Log[1 - Cos[c + d*x]])/(4*d) - (b*(3*a^2 + 2*b
^2)*Log[Cos[c + d*x]])/d - ((a - 4*b)*(a - b)^2*Log[1 + Cos[c + d*x]])/(4*d
) + (3*a*b^2*Sec[c + d*x])/d + (b^3*Sec[c + d*x]^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \csc^3(c+dx)(a+b\sec(c+dx))^3\,dx &= -\int(-b-a\cos(c+dx))^3\csc^3(c+dx)\sec^3(c+dx)\,dx \\
&= \frac{a^3\text{Subst}\left(\int\frac{a^3(-b+x)^3}{x^3(a^2-x^2)^2}\,dx,x,-a\cos(c+dx)\right)}{d} \\
&= \frac{a^6\text{Subst}\left(\int\frac{(-b+x)^3}{x^3(a^2-x^2)^2}\,dx,x,-a\cos(c+dx)\right)}{d} \\
&= -\frac{a^2\left(b\left(3+\frac{b^2}{a^2}\right)+a\left(1+\frac{3b^2}{a^2}\right)\cos(c+dx)\right)\csc^2(c+dx)}{2d}-\frac{a^4\text{Subs}}{} \\
&= -\frac{a^2\left(b\left(3+\frac{b^2}{a^2}\right)+a\left(1+\frac{3b^2}{a^2}\right)\cos(c+dx)\right)\csc^2(c+dx)}{2d}-\frac{a^4\text{Subs}}{} \\
&= -\frac{a^2\left(b\left(3+\frac{b^2}{a^2}\right)+a\left(1+\frac{3b^2}{a^2}\right)\cos(c+dx)\right)\csc^2(c+dx)}{2d}+\frac{(a+b)}{}
\end{aligned}$$

Mathematica [A]
time = 4.69, size = 260, normalized size = 1.60

$$\frac{24ab^2-(a+b)^3\csc^2\left(\frac{1}{2}(c+dx)\right)-4(a-4b)(a-b)^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-8b(3a^2+2b^2)\log(\cos(c+dx))+4(a+b)^2(a+4b)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+(a-b)^3\sec^2\left(\frac{1}{2}(c+dx)\right)+\frac{2b^3}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{24ab^2\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}+\frac{2b^3}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}-\frac{24ab^2\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)}}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*(a + b*Sec[c + d*x])^3,x]
[Out] (24*a*b^2 - (a + b)^3*Csc[(c + d*x)/2]^2 - 4*(a - 4*b)*(a - b)^2*Log[Cos[(c
 + d*x)/2]] - 8*b*(3*a^2 + 2*b^2)*Log[Cos[c + d*x]] + 4*(a + b)^2*(a + 4*b)
```

```
*Log[Sin[(c + d*x)/2]] + (a - b)^3*Sec[(c + d*x)/2]^2 + (2*b^3)/(Cos[(c + d
*x)/2] - Sin[(c + d*x)/2])^2 + (24*a*b^2*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2
] - Sin[(c + d*x)/2]) + (2*b^3)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 - (
24*a*b^2*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))/(8*d)
```

Maple [A]
time = 0.14, size = 162, normalized size = 1.00

method	result
derivativedivides	$\frac{b^3\left(\frac{1}{2\sin(dx+c)^2\cos(dx+c)^2}-\frac{1}{\sin(dx+c)^2}+2\ln(\tan(dx+c))\right)+3b^2a\left(-\frac{1}{2\sin(dx+c)^2\cos(dx+c)}+\frac{3}{2\cos(dx+c)}+\frac{3\ln(\csc(dx+c)-\cot}{2}\right.}{d}$
default	$\frac{b^3\left(\frac{1}{2\sin(dx+c)^2\cos(dx+c)^2}-\frac{1}{\sin(dx+c)^2}+2\ln(\tan(dx+c))\right)+3b^2a\left(-\frac{1}{2\sin(dx+c)^2\cos(dx+c)}+\frac{3}{2\cos(dx+c)}+\frac{3\ln(\csc(dx+c)-\cot}{2}\right.}{d}$
norman	$\frac{-\frac{a^3+3b\,a^2+3b^2a+b^3}{8d}+\frac{\left(a^3+15b^2a\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{\left(a^3-3b\,a^2+3b^2a-b^3\right)\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{\left(2a^3-3b\,a^2+30b^2a-9b^3\right)\left(\tan^4\left(\frac{dx}{2}\right.\right.}{4d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}$
risch	$\frac{a^3e^{7i(dx+c)}+9b^2a\,e^{7i(dx+c)}+6b\,a^2e^{6i(dx+c)}+4b^3e^{6i(dx+c)}+3a^3e^{5i(dx+c)}+3b^2a\,e^{5i(dx+c)}+12b\,a^2e^{4i(dx+c)}+3a^3e^{3i(dx+c)}}{d\left(e^{2i(dx+c)}+1\right)^2\left(e^{2i(dx+c)}-1\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^3*(1/2/sin(d*x+c)^2/cos(d*x+c)^2-1/sin(d*x+c)^2+2*ln(tan(d*x+c)))+3*
b^2*a*(-1/2/sin(d*x+c)^2/cos(d*x+c)+3/2/cos(d*x+c)+3/2*ln(csc(d*x+c)-cot(d*
x+c)))+3*b*a^2*(-1/2/sin(d*x+c)^2+ln(tan(d*x+c)))+a^3*(-1/2*csc(d*x+c)*cot(
d*x+c)+1/2*ln(csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]
time = 0.27, size = 171, normalized size = 1.06

$$-\frac{(a^3-6a^2b+9ab^2-4b^3)\log(\cos(dx+c)+1)-(a^3+6a^2b+9ab^2+4b^3)\log(\cos(dx+c)-1)+4(3a^2b+2b^3)\log(\cos(dx+c))+\frac{2\left(6ab^2\cos(dx+c)-(a^3+9ab^2)\cos(dx+c)^3+b^3-(3a^2b+2b^3)\cos(dx+c)^2\right)}{\cos(dx+c)^4-\cos(dx+c)^2}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*((a^3 - 6*a^2*b + 9*a*b^2 - 4*b^3)*log(cos(d*x + c) + 1) - (a^3 + 6*a^
2*b + 9*a*b^2 + 4*b^3)*log(cos(d*x + c) - 1) + 4*(3*a^2*b + 2*b^3)*log(cos(
d*x + c)) + 2*(6*a*b^2*cos(d*x + c) - (a^3 + 9*a*b^2)*cos(d*x + c)^3 + b^3
- (3*a^2*b + 2*b^3)*cos(d*x + c)^2)/(cos(d*x + c)^4 - cos(d*x + c)^2))/d
```

Fricas [A]
time = 4.67, size = 290, normalized size = 1.79

$$-\frac{12ab^2\cos(dx+c)-2(a^3+9ab^2)\cos(dx+c)^3+2b^3-2(3a^2b+2b^3)\cos(dx+c)^2+4\left((3a^2b+2b^3)\cos(dx+c)^4-(3a^2b+2b^3)\cos(dx+c)^2\right)\log(-\cos(dx+c))+\left((a^3-6a^2b+9ab^2-4b^3)\cos(dx+c)^4-(a^3-6a^2b+9ab^2-4b^3)\cos(dx+c)^2\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-\left((a^3+6a^2b+9ab^2+4b^3)\cos(dx+c)^4-(a^3+6a^2b+9ab^2+4b^3)\cos(dx+c)^2\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)}{4\left(d\cos(dx+c)^4-d\cos(dx+c)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(12*a*b^2*cos(d*x + c) - 2*(a^3 + 9*a*b^2)*cos(d*x + c)^3 + 2*b^3 - 2*
(3*a^2*b + 2*b^3)*cos(d*x + c)^2 + 4*((3*a^2*b + 2*b^3)*cos(d*x + c)^4 - (3
*a^2*b + 2*b^3)*cos(d*x + c)^2)*log(-cos(d*x + c)) + ((a^3 - 6*a^2*b + 9*a*
b^2 - 4*b^3)*cos(d*x + c)^4 - (a^3 - 6*a^2*b + 9*a*b^2 - 4*b^3)*cos(d*x + c
)^2)*log(1/2*cos(d*x + c) + 1/2) - ((a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cos(d
*x + c)^4 - (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cos(d*x + c)^2)*log(-1/2*cos(
d*x + c) + 1/2))/(d*cos(d*x + c)^4 - d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec (c + dx))^3 \csc^3 (c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3*csc(c + d*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 482 vs. 2(154) = 308.
time = 0.54, size = 482, normalized size = 2.98

$$-\frac{\frac{a^3(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{3a^2b(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{3ab^2(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{b^3(\cos(dx+c)-1)}{\cos(dx+c)+1} - 2\left(a^3 + 6a^2b + 9ab^2 + 4b^3\right)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) + 8\left(3a^2b + 2b^3\right)\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1} - 1\right|\right) - \frac{\left(a^3+3a^2b+3ab^2+b^3 - \frac{2a^3(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{12a^2b(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{18ab^2(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{8b^3(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{\cos(dx+c)-1} - \frac{4\left(9a^2b+12ab^2+6b^3 + \frac{18a^2b(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{12ab^2(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{8b^3(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{9a^2b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} + \frac{6b^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/8*(a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 3*a^2*b*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) + 3*a*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b^3*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)
*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) + 8*(3*a^2*b + 2*b^3)*lo
g(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - (a^3 + 3*a^2*b + 3*a*b
^2 + b^3 - 2*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 12*a^2*b*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) - 18*a*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) +
1) - 8*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))*(cos(d*x + c) + 1)/(cos(d
*x + c) - 1) - 4*(9*a^2*b + 12*a*b^2 + 6*b^3 + 18*a^2*b*(cos(d*x + c) - 1)/
(cos(d*x + c) + 1) + 12*a*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 8*b^3
*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*a^2*b*(cos(d*x + c) - 1)^2/(cos(
d*x + c) + 1)^2 + 6*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/((cos(d*
x + c) - 1)/(cos(d*x + c) + 1) + 1)^2)/d
```

Mupad [B]
time = 1.01, size = 159, normalized size = 0.98

$$\frac{\ln\left(\cos\left(c+d\,x\right)-1\right)\,{\left(a+b\right)}^2\,\left(a+4\,b\right)}{4\,d}-\frac{\ln\left(\cos\left(c+d\,x\right)\right)\,\left(3\,a^2\,b+2\,b^3\right)}{d}-\frac{{\cos\left(c+d\,x\right)}^3\,\left(\frac{a^3}{2}+\frac{9\,a\,b^2}{2}\right)-\frac{b^3}{2}+{\cos\left(c+d\,x\right)}^2\,\left(\frac{3\,a^2\,b}{2}+b^3\right)-3\,a\,b^2\,\cos\left(c+d\,x\right)}{d\,\left({\cos\left(c+d\,x\right)}^2-{\cos\left(c+d\,x\right)}^4\right)}-\frac{\ln\left(\cos\left(c+d\,x\right)+1\right)\,{\left(a-b\right)}^2\,\left(a-4\,b\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^3/sin(c + d*x)^3,x)
```

```
[Out] (log(cos(c + d*x) - 1)*(a + b)^2*(a + 4*b))/(4*d) - (log(cos(c + d*x))*(3*a
^2*b + 2*b^3))/d - (cos(c + d*x)^3*((9*a*b^2)/2 + a^3/2) - b^3/2 + cos(c +
d*x)^2*((3*a^2*b)/2 + b^3) - 3*a*b^2*cos(c + d*x))/(d*(cos(c + d*x)^2 - cos
(c + d*x)^4)) - (log(cos(c + d*x) + 1)*(a - b)^2*(a - 4*b))/(4*d)
```

3.190 $\int (a+b\sec(c+dx))^3\sin^6(c+dx)\,dx$

Optimal. Leaf size=299

$$\frac{5a^3x}{16}-\frac{45}{8}ab^2x+\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}-\frac{5b^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{3a^2b\sin(c+dx)}{d}+\frac{5b^3\sin(c+dx)}{2d}-\frac{5a$$

```
[Out] 5/16*a^3*x-45/8*a*b^2*x+3*a^2*b*arctanh(sin(d*x+c))/d-5/2*b^3*arctanh(sin(d
*x+c))/d-3*a^2*b*sin(d*x+c)/d+5/2*b^3*sin(d*x+c)/d-5/16*a^3*cos(d*x+c)*sin(
d*x+c)/d-a^2*b*sin(d*x+c)^3/d+5/6*b^3*sin(d*x+c)^3/d-5/24*a^3*cos(d*x+c)*si
n(d*x+c)^3/d-3/5*a^2*b*sin(d*x+c)^5/d-1/6*a^3*cos(d*x+c)*sin(d*x+c)^5/d+45/
8*a*b^2*tan(d*x+c)/d-15/8*a*b^2*sin(d*x+c)^2*tan(d*x+c)/d-3/4*a*b^2*sin(d*x
+c)^4*tan(d*x+c)/d+1/2*b^3*sin(d*x+c)^3*tan(d*x+c)^2/d
```

Rubi [A]
time = 0.25, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3957, 2991, 2715, 8, 2672, 308, 212, 2671, 294, 327, 209}

$$-\frac{a^3\sin^5(c+dx)\cos(c+dx)}{6d}-\frac{5a^3\sin^3(c+dx)\cos(c+dx)}{24d}-\frac{5a^3\sin(c+dx)\cos(c+dx)}{16d}+\frac{5a^3x}{16}-\frac{3a^2b\sin^5(c+dx)}{5d}-\frac{a^2b\sin^3(c+dx)}{d}-\frac{3a^2b\sin(c+dx)}{d}+\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}+\frac{45ab^2\tan(c+dx)}{8d}-\frac{3ab^2\sin^4(c+dx)\tan(c+dx)}{4d}-\frac{15ab^2\sin^2(c+dx)\tan(c+dx)}{8d}-\frac{45}{8}ab^2x+\frac{5b^3\sin^3(c+dx)}{6d}+\frac{5b^3\sin(c+dx)}{2d}+\frac{b^3\sin^3(c+dx)\tan^2(c+dx)}{2d}-\frac{5b^3\tanh^{-1}(\sin(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^3*Sin[c + d*x]^6,x]
```

```
[Out] (5*a^3*x)/16 - (45*a*b^2*x)/8 + (3*a^2*b*ArcTanh[Sin[c + d*x]])/d - (5*b^3*
ArcTanh[Sin[c + d*x]])/(2*d) - (3*a^2*b*Sin[c + d*x])/d + (5*b^3*Sin[c + d*
x])/(2*d) - (5*a^3*Cos[c + d*x]*Sin[c + d*x])/(16*d) - (a^2*b*Sin[c + d*x]^
3)/d + (5*b^3*Sin[c + d*x]^3)/(6*d) - (5*a^3*Cos[c + d*x]*Sin[c + d*x]^3)/(
24*d) - (3*a^2*b*Sin[c + d*x]^5)/(5*d) - (a^3*Cos[c + d*x]*Sin[c + d*x]^5)/
(6*d) + (45*a*b^2*Tan[c + d*x])/(8*d) - (15*a*b^2*Sin[c + d*x]^2*Tan[c + d*
x])/(8*d) - (3*a*b^2*Sin[c + d*x]^4*Tan[c + d*x])/(4*d) + (b^3*Sin[c + d*x]
^3*Tan[c + d*x]^2)/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2991

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+b\sec(c+dx))^3\sin^6(c+dx)\,dx = -\int (-b-a\cos(c+dx))^3\sin^3(c+dx)\tan^3(c+dx)\,dx$$

$$= -\int \left(-a^3\sin^6(c+dx) - 3a^2b\sin^5(c+dx)\tan(c+dx) - 3ab^2\sin^4\right.$$

$$= a^3\int \sin^6(c+dx)\,dx + \left(3a^2b\right)\int \sin^5(c+dx)\tan(c+dx)\,dx + (3a$$

$$= -\frac{a^3\cos(c+dx)\sin^5(c+dx)}{6d} + \frac{1}{6}\left(5a^3\right)\int \sin^4(c+dx)\,dx + \frac{(3a^2b)}{}$$

$$= -\frac{5a^3\cos(c+dx)\sin^3(c+dx)}{24d} - \frac{a^3\cos(c+dx)\sin^5(c+dx)}{6d} - \frac{3ab}{}$$

$$= -\frac{3a^2b\sin(c+dx)}{d} - \frac{5a^3\cos(c+dx)\sin(c+dx)}{16d} - \frac{a^2b\sin^3(c+dx}{d}$$

$$= \frac{5a^3x}{16} + \frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d} - \frac{3a^2b\sin(c+dx)}{d} + \frac{5b^3\sin(c+}{2d}$$

$$= \frac{5a^3x}{16} - \frac{45}{8}ab^2x + \frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d} - \frac{5b^3\tanh^{-1}(\sin(c+}{2d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 818 vs. 2(299) = 598.
time = 6.18, size = 818, normalized size = 2.74

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^3*Sin[c + d*x]^6,x]
```

```
[Out] (5*a*(a^2 - 18*b^2)*(c + d*x)*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3)/(16*d*
(b + a*Cos[c + d*x])^3) + ((-6*a^2*b + 5*b^3)*Cos[c + d*x]^3*Log[Cos[(c + d
*x)/2] - Sin[(c + d*x)/2]]*(a + b*Sec[c + d*x])^3)/(2*d*(b + a*Cos[c + d*x]
)^3) + ((6*a^2*b - 5*b^3)*Cos[c + d*x]^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*
x)/2]]*(a + b*Sec[c + d*x])^3)/(2*d*(b + a*Cos[c + d*x])^3) + (b^3*Cos[c +
d*x]^3*(a + b*Sec[c + d*x])^3)/(4*d*(b + a*Cos[c + d*x])^3*(Cos[(c + d*x)/2
] - Sin[(c + d*x)/2])^2) + (3*a*b^2*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3*S
in[(c + d*x)/2])/(d*(b + a*Cos[c + d*x])^3*(Cos[(c + d*x)/2] - Sin[(c + d*x
)/2])) - (b^3*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3)/(4*d*(b + a*Cos[c + d*
x])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (3*a*b^2*Cos[c + d*x]^3*(a
 + b*Sec[c + d*x])^3*Sin[(c + d*x)/2])/(d*(b + a*Cos[c + d*x])^3*(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2])) + (3*b*(-11*a^2 + 6*b^2)*Cos[c + d*x]^3*(a + b
*Sec[c + d*x])^3*Sin[c + d*x])/(8*d*(b + a*Cos[c + d*x])^3) - (3*a*(5*a^2 -
 32*b^2)*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3*Sin[2*(c + d*x)])/(64*d*(b +
 a*Cos[c + d*x])^3) - (b*(-21*a^2 + 4*b^2)*Cos[c + d*x]^3*(a + b*Sec[c + d*
x])^3*Sin[3*(c + d*x)])/(48*d*(b + a*Cos[c + d*x])^3) + (3*a*(a^2 - 2*b^2)*
Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3*Sin[4*(c + d*x)])/(64*d*(b + a*Cos[c
+ d*x])^3) - (3*a^2*b*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3*Sin[5*(c + d*x)
])/(80*d*(b + a*Cos[c + d*x])^3) - (a^3*Cos[c + d*x]^3*(a + b*Sec[c + d*x])
^3*Sin[6*(c + d*x)])/(192*d*(b + a*Cos[c + d*x])^3)
```

Maple [A]
time = 0.16, size = 234, normalized size = 0.78 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3*sin(d*x+c)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^3*(1/2*sin(d*x+c)^7/cos(d*x+c)^2+1/2*sin(d*x+c)^5+5/6*sin(d*x+c)^3+5
/2*sin(d*x+c)-5/2*ln(sec(d*x+c)+tan(d*x+c)))+3*b^2*a*(sin(d*x+c)^7/cos(d*x+
c)+(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)-15/8*d*x-15/8
*c)+3*b*a^2*(-1/5*sin(d*x+c)^5-1/3*sin(d*x+c)^3-sin(d*x+c)+ln(sec(d*x+c)+ta
n(d*x+c)))+a^3*(-1/6*(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*
x+c)+5/16*d*x+5/16*c))
```

Maxima [A]
time = 0.49, size = 242, normalized size = 0.81

$$\frac{5\left(4\sin\left(2dx+2c\right)^3+60dx+60c+9\sin\left(4dx+4c\right)-48\sin\left(2dx+2c\right)\right)a^3-96\left(6\sin\left(dx+c\right)^5+10\sin\left(dx+c\right)^3-15\log\left(\sin\left(dx+c\right)+1\right)+15\log\left(\sin\left(dx+c\right)-1\right)+30\sin\left(dx+c\right)\right)a^2b-360\left(15dx+15c-\frac{9\tan(dx+c)^3+7\tan(dx+c)}{\tan(dx+c)^4+2\tan(dx+c)^2+1}-8\tan\left(dx+c\right)\right)ab^2+80\left(4\sin\left(dx+c\right)^3-\frac{6\sin(dx+c)}{\sin(dx+c)^2-1}-15\log\left(\sin\left(dx+c\right)+1\right)+15\log\left(\sin\left(dx+c\right)-1\right)+24\sin\left(dx+c\right)\right)b^3}{960d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/960*(5*(4*sin(2*d*x + 2*c)^3 + 60*d*x + 60*c + 9*sin(4*d*x + 4*c) - 48*si
n(2*d*x + 2*c))*a^3 - 96*(6*sin(d*x + c)^5 + 10*sin(d*x + c)^3 - 15*log(sin
(d*x + c) + 1) + 15*log(sin(d*x + c) - 1) + 30*sin(d*x + c))*a^2*b - 360*(1
5*d*x + 15*c - (9*tan(d*x + c)^3 + 7*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(
```

```
d*x + c)^2 + 1) - 8*tan(d*x + c))*a*b^2 + 80*(4*sin(d*x + c)^3 - 6*sin(d*x
+ c)/(sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c)
- 1) + 24*sin(d*x + c))*b^3)/d
```

Fricas [**A**]
time = 4.58, size = 241, normalized size = 0.81

$$\frac{75\left(a^{3}-18\,ab^{2}\right)dx\cos\left(dx+c\right)^{2}+60\left(6\,a^{2}b-5\,b^{3}\right)\cos\left(dx+c\right)^{2}\log\left(\sin\left(dx+c\right)+1\right)-60\left(6\,a^{2}b-5\,b^{3}\right)\cos\left(dx+c\right)^{2}\log\left(-\sin\left(dx+c\right)+1\right)-\left(40\,a^{3}\cos\left(dx+c\right)^{7}+144\,a^{2}b\cos\left(dx+c\right)^{6}-10\left(13\,a^{3}-18\,ab^{2}\right)\cos\left(dx+c\right)^{5}-16\left(33\,a^{2}b-5\,b^{3}\right)\cos\left(dx+c\right)^{4}-720\,ab^{2}\cos\left(dx+c\right)+15\left(11\,a^{3}-54\,ab^{2}\right)\cos\left(dx+c\right)^{3}-120\,b^{3}+16\left(69\,a^{2}b-35\,b^{3}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{240\,d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] 1/240*(75*(a^3 - 18*a*b^2)*d*x*cos(d*x + c)^2 + 60*(6*a^2*b - 5*b^3)*cos(d*
x + c)^2*log(sin(d*x + c) + 1) - 60*(6*a^2*b - 5*b^3)*cos(d*x + c)^2*log(-s
in(d*x + c) + 1) - (40*a^3*cos(d*x + c)^7 + 144*a^2*b*cos(d*x + c)^6 - 10*(
13*a^3 - 18*a*b^2)*cos(d*x + c)^5 - 16*(33*a^2*b - 5*b^3)*cos(d*x + c)^4 -
720*a*b^2*cos(d*x + c) + 15*(11*a^3 - 54*a*b^2)*cos(d*x + c)^3 - 120*b^3 +
16*(69*a^2*b - 35*b^3)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3*sin(d*x+c)**6,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 563 vs. 2(273) = 546.
time = 0.57, size = 563, normalized size = 1.88

$$75\left(a^{3}-18\,ab^{2}\right)\left(dx+c\right)+120\left(6\,a^{2}b-5\,b^{3}\right)\log\left(\left|\tan\left(\tfrac{1}{2}\,dx+\tfrac{1}{2}\,c\right)+1\right|\right)-120\left(6\,a^{2}b-5\,b^{3}\right)\log\left(\left|\tan\left(\tfrac{1}{2}\,dx+\tfrac{1}{2}\,c\right)-1\right|\right)-\frac{240\left(6\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-6\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{2}}+\frac{2\left(75\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-720\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11} \ldots\right)}{\ldots}$$ [illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^6,x, algorithm="giac")
```

```
[Out] 1/240*(75*(a^3 - 18*a*b^2)*(d*x + c) + 120*(6*a^2*b - 5*b^3)*log(abs(tan(1/
2*d*x + 1/2*c) + 1)) - 120*(6*a^2*b - 5*b^3)*log(abs(tan(1/2*d*x + 1/2*c) -
 1)) - 240*(6*a*b^2*tan(1/2*d*x + 1/2*c)^3 - b^3*tan(1/2*d*x + 1/2*c)^3 - 6
*a*b^2*tan(1/2*d*x + 1/2*c) - b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*
c)^2 - 1)^2 + 2*(75*a^3*tan(1/2*d*x + 1/2*c)^11 - 720*a^2*b*tan(1/2*d*x + 1
/2*c)^11 - 630*a*b^2*tan(1/2*d*x + 1/2*c)^11 + 480*b^3*tan(1/2*d*x + 1/2*c)
^11 + 425*a^3*tan(1/2*d*x + 1/2*c)^9 - 4560*a^2*b*tan(1/2*d*x + 1/2*c)^9 -
2610*a*b^2*tan(1/2*d*x + 1/2*c)^9 + 2720*b^3*tan(1/2*d*x + 1/2*c)^9 + 990*a
```

```
^3*tan(1/2*d*x + 1/2*c)^7 - 12384*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 1980*a*b^2
*tan(1/2*d*x + 1/2*c)^7 + 5760*b^3*tan(1/2*d*x + 1/2*c)^7 - 990*a^3*tan(1/2
*d*x + 1/2*c)^5 - 12384*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 1980*a*b^2*tan(1/2*d
*x + 1/2*c)^5 + 5760*b^3*tan(1/2*d*x + 1/2*c)^5 - 425*a^3*tan(1/2*d*x + 1/2
*c)^3 - 4560*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 2610*a*b^2*tan(1/2*d*x + 1/2*c)
^3 + 2720*b^3*tan(1/2*d*x + 1/2*c)^3 - 75*a^3*tan(1/2*d*x + 1/2*c) - 720*a^
2*b*tan(1/2*d*x + 1/2*c) + 630*a*b^2*tan(1/2*d*x + 1/2*c) + 480*b^3*tan(1/2
*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

Mupad [B]
time = 1.69, size = 373, normalized size = 1.25

$$\frac{7\,b^3\,\sin\left(c+d\,x\right)}{3\,d}+\frac{5\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{8\,d}-\frac{5\,b^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{13\,a^3\,\cos\left(c+d\,x\right)^3\,\sin\left(c+d\,x\right)}{24\,d}-\frac{a^3\,\cos\left(c+d\,x\right)^5\,\sin\left(c+d\,x\right)}{6\,d}+\frac{b^3\,\sin\left(c+d\,x\right)}{2\,d\,\cos\left(c+d\,x\right)^2}-\frac{b^3\,\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{3\,d}-\frac{11\,a^3\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{16\,d}-\frac{23\,a^2\,b\,\sin\left(c+d\,x\right)}{5\,d}-\frac{45\,a\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{4\,d}+\frac{6\,a^2\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{27\,a\,b^2\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{8\,d}+\frac{3\,a\,b^2\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}+\frac{11\,a^2\,b\,\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{5\,d}-\frac{3\,a\,b^2\,\cos\left(c+d\,x\right)^3\,\sin\left(c+d\,x\right)}{4\,d}-\frac{3\,a^2\,b\,\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6*(a + b/cos(c + d*x))^3,x)
```

```
[Out] (7*b^3*sin(c + d*x))/(3*d) + (5*a^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)
/2)))/(8*d) - (5*b^3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (13*
a^3*cos(c + d*x)^3*sin(c + d*x))/(24*d) - (a^3*cos(c + d*x)^5*sin(c + d*x))
/(6*d) + (b^3*sin(c + d*x))/(2*d*cos(c + d*x)^2) - (b^3*cos(c + d*x)^2*sin(
c + d*x))/(3*d) - (11*a^3*cos(c + d*x)*sin(c + d*x))/(16*d) - (23*a^2*b*sin
(c + d*x))/(5*d) - (45*a*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(
4*d) + (6*a^2*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (27*a*b^2
*cos(c + d*x)*sin(c + d*x))/(8*d) + (3*a*b^2*sin(c + d*x))/(d*cos(c + d*x))
 + (11*a^2*b*cos(c + d*x)^2*sin(c + d*x))/(5*d) - (3*a*b^2*cos(c + d*x)^3*s
in(c + d*x))/(4*d) - (3*a^2*b*cos(c + d*x)^4*sin(c + d*x))/(5*d)
```

3.191 $\int (a+b\sec(c+dx))^3\sin^4(c+dx)\,dx$

Optimal. Leaf size=236

$$\frac{3}{8}a\left(a^2-12b^2\right)x+\frac{3b(2a^2-b^2)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{b(17a^2-b^2)\sin(c+dx)}{2d}-\frac{a(21a^2-2b^2)\cos(c+dx)\text{si}}{8d}$$

```
[Out] 3/8*a*(a^2-12*b^2)*x+3/2*b*(2*a^2-b^2)*arctanh(sin(d*x+c))/d-1/2*b*(17*a^2-
b^2)*sin(d*x+c)/d-1/8*a*(21*a^2-2*b^2)*cos(d*x+c)*sin(d*x+c)/d-1/4*(6*a^2-b
^2)*(b+a*cos(d*x+c))^2*sin(d*x+c)/b/d-1/4*(4*a^2-b^2)*(b+a*cos(d*x+c))^3*si
n(d*x+c)/b^2/d+a*(b+a*cos(d*x+c))^4*tan(d*x+c)/b^2/d+1/2*(b+a*cos(d*x+c))^4
*sec(d*x+c)*tan(d*x+c)/b/d
```

Rubi [A]
time = 0.52, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3957, 2972, 3128, 3112, 3102, 2814, 3855}

$$-\frac{b(17a^2-b^2)\sin(c+dx)}{2d}+\frac{3b(2a^2-b^2)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{(4a^2-b^2)\sin(c+dx)(a\cos(c+dx)+b)^3}{4b^2d}-\frac{(6a^2-b^2)\sin(c+dx)(a\cos(c+dx)+b)^2}{4bd}-\frac{a(21a^2-2b^2)\sin(c+dx)\cos(c+dx)}{8d}+\frac{3}{8}ax\left(a^2-12b^2\right)+\frac{a\tan(c+dx)(a\cos(c+dx)+b)^4}{b^2d}+\frac{\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+b)^4}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^3*Sin[c + d*x]^4,x]
```

```
[Out] (3*a*(a^2 - 12*b^2)*x)/8 + (3*b*(2*a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(2*d)
- (b*(17*a^2 - b^2)*Sin[c + d*x])/(2*d) - (a*(21*a^2 - 2*b^2)*Cos[c + d*x]*
Sin[c + d*x])/(8*d) - ((6*a^2 - b^2)*(b + a*Cos[c + d*x])^2*Sin[c + d*x])/(
4*b*d) - ((4*a^2 - b^2)*(b + a*Cos[c + d*x])^3*Sin[c + d*x])/(4*b^2*d) + (a
*(b + a*Cos[c + d*x])^4*Tan[c + d*x])/(b^2*d) + ((b + a*Cos[c + d*x])^4*Sec
[c + d*x]*Tan[c + d*x])/(2*b*d)
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2972

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*d*f*(n + 1))), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((
d*Sin[e + f*x])^(n + 2)/(a^2*d^2*f*(n + 1)*(n + 2))), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

```
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3112

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin
[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A
*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int (a+b\sec(c+dx))^3\sin^4(c+dx)\,dx &= -\int (-b-a\cos(c+dx))^3\sin(c+dx)\tan^3(c+dx)\,dx\\ &= \frac{a(b+a\cos(c+dx))^4\tan(c+dx)}{b^2d} + \frac{(b+a\cos(c+dx))^4\sec(c+d\\ &= -\frac{(4a^2-b^2)\,(b+a\cos(c+dx))^3\sin(c+dx)}{4b^2d} + \frac{a(b+a\cos(c+dx)}{b^2d}\\ &= -\frac{(6a^2-b^2)\,(b+a\cos(c+dx))^2\sin(c+dx)}{4bd} - \frac{(4a^2-b^2)\,(b+a\,\mathrm{co}}{4}\\ &= -\frac{a(21a^2-2b^2)\cos(c+dx)\sin(c+dx)}{8d} - \frac{(6a^2-b^2)\,(b+a\cos(c+}{4bd}\\ &= -\frac{b(17a^2-b^2)\sin(c+dx)}{2d} - \frac{a(21a^2-2b^2)\cos(c+dx)\sin(c+dx)}{8d}\\ &= \frac{3}{8}a\left(a^2-12b^2\right)x - \frac{b(17a^2-b^2)\sin(c+dx)}{2d} - \frac{a(21a^2-2b^2)\cos(c}{8d}\\ &= \frac{3}{8}a\left(a^2-12b^2\right)x + \frac{3b(2a^2-b^2)\tanh^{-1}(\sin(c+dx))}{2d} - \frac{b(17a^2-b^2}{2}\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 696 vs. $2(236) = 472$.
time = 6.11, size = 696, normalized size = 2.95

$$\frac{3a(a^2-12b^2)(c+dx)\cos^3(c+dx)(a+b\sec(c+dx))^3}{8d(b+a\cos(c+dx))^3}+\frac{3(-2a^2b+b^3)\cos^3(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\sec(c+dx))^3}{2d(b+a\cos(c+dx))^3}-\frac{3(-2a^2b+b^3)\cos^3(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\sec(c+dx))^3}{2d(b+a\cos(c+dx))^3}+\frac{b^3\cos^3(c+dx)(a+b\sec(c+dx))^3}{4d(b+a\cos(c+dx))^3\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{3ab^2\cos^3(c+dx)(a+b\sec(c+dx))^3\sin\left(\frac{1}{2}(c+dx)\right)}{d(b+a\cos(c+dx))^3\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}-\frac{b^3\cos^3(c+dx)(a+b\sec(c+dx))^3}{4d(b+a\cos(c+dx))^3\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{3ab^2\cos^3(c+dx)(a+b\sec(c+dx))^3\sin\left(\frac{1}{2}(c+dx)\right)}{d(b+a\cos(c+dx))^3\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{b(-15a^2+4b^2)\cos^3(c+dx)(a+b\sec(c+dx))^3\sin(c+dx)}{4d(b+a\cos(c+dx))^3}-\frac{a(a^2-3b^2)\cos^3(c+dx)(a+b\sec(c+dx))^3\sin(2(c+dx))}{4d(b+a\cos(c+dx))^3}+\frac{a^2b\cos^3(c+dx)(a+b\sec(c+dx))^3\sin(3(c+dx))}{4d(b+a\cos(c+dx))^3}+\frac{a^3\cos^3(c+dx)(a+b\sec(c+dx))^3\sin(4(c+dx))}{32d(b+a\cos(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^3*Sin[c + d*x]^4,x]
```

```
[Out] (3*a*(a^2 - 12*b^2)*(c + d*x)*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3)/(8*d*(
b + a*Cos[c + d*x])^3) + (3*(-2*a^2*b + b^3)*Cos[c + d*x]^3*Log[Cos[(c + d*
x)/2] - Sin[(c + d*x)/2]]*(a + b*Sec[c + d*x])^3)/(2*d*(b + a*Cos[c + d*x])
^3) - (3*(-2*a^2*b + b^3)*Cos[c + d*x]^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*
x)/2]]*(a + b*Sec[c + d*x])^3)/(2*d*(b + a*Cos[c + d*x])^3) + (b^3*Cos[c +
d*x]^3*(a + b*Sec[c + d*x])^3)/(4*d*(b + a*Cos[c + d*x])^3*(Cos[(c + d*x)/2
] - Sin[(c + d*x)/2])^2) + (3*a*b^2*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3*S
in[(c + d*x)/2])/(d*(b + a*Cos[c + d*x])^3*(Cos[(c + d*x)/2] - Sin[(c + d*x
)/2])) - (b^3*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3)/(4*d*(b + a*Cos[c + d*
x])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (3*a*b^2*Cos[c + d*x]^3*(a
 + b*Sec[c + d*x])^3*Sin[(c + d*x)/2])/(d*(b + a*Cos[c + d*x])^3*(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2])) + (b*(-15*a^2 + 4*b^2)*Cos[c + d*x]^3*(a + b*S
ec[c + d*x])^3*Sin[c + d*x])/(4*d*(b + a*Cos[c + d*x])^3) - (a*(a^2 - 3*b^2
)*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3*Sin[2*(c + d*x)])/(4*d*(b + a*Cos[c
```

```
 + d*x])^3) + (a^2*b*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3*Sin[3*(c + d*x)]
)/(4*d*(b + a*Cos[c + d*x])^3) + (a^3*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3
*Sin[4*(c + d*x)])/(32*d*(b + a*Cos[c + d*x])^3)
```

Maple [A]
time = 0.12, size = 194, normalized size = 0.82

method	result
derivativedivides	$b^3\left(\frac{\sin^5(dx+c)}{2\cos(dx+c)^2}+\frac{(\sin^3(dx+c))}{2}+\frac{3\sin(dx+c)}{2}-\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3b^2a\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)c\cdots\right.$
default	$b^3\left(\frac{\sin^5(dx+c)}{2\cos(dx+c)^2}+\frac{(\sin^3(dx+c))}{2}+\frac{3\sin(dx+c)}{2}-\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3b^2a\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)c\cdots\right.$
risch	$\frac{3a^3x}{8}-\frac{9a\,b^2x}{2}-\frac{ib^2\left(b\,e^{3i(dx+c)}-6a\,e^{2i(dx+c)}-b\,e^{i(dx+c)}-6a\right)}{d\left(e^{2i(dx+c)}+1\right)^2}-\frac{ie^{-2i(dx+c)}a^3}{8d}+\frac{ie^{-i(dx+c)}b^3}{2d}+\frac{ie^{-3i(dx+c)}b\,a\cdots}{8d}$
norman	$\left(\frac{3}{8}a^3-\frac{9}{2}b^2a\right)x+\left(-\frac{3}{2}a^3+18b^2a\right)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{3}{8}a^3+\frac{9}{2}b^2a\right)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{3}{8}a^3+\frac{9}{2}b^2a\right)x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3*sin(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^3*(1/2*sin(d*x+c)^5/cos(d*x+c)^2+1/2*sin(d*x+c)^3+3/2*sin(d*x+c)-3/2
*ln(sec(d*x+c)+tan(d*x+c)))+3*b^2*a*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+
3/2*sin(d*x+c))*cos(d*x+c)-3/2*d*x-3/2*c)+3*b*a^2*(-1/3*sin(d*x+c)^3-sin(d*
x+c)+ln(sec(d*x+c)+tan(d*x+c)))+a^3*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos
(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A]
time = 0.49, size = 183, normalized size = 0.78

$$\frac{(12\,dx+12\,c+\sin(4\,dx+4\,c)-8\,\sin(2\,dx+2\,c))a^3-16\left(2\,\sin(dx+c)^3-3\,\log(\sin(dx+c)+1)+3\,\log(\sin(dx+c)-1)+6\,\sin(dx+c)\right)a^2b-48\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^2+1}-2\,\tan(dx+c)\right)ab^2-8\,b^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+3\,\log(\sin(dx+c)+1)-3\,\log(\sin(dx+c)-1)-4\,\sin(dx+c)\right)}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/32*((12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*d*x + 2*c))*a^3 - 16*(2*s
in(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1) + 6*sin(d
*x + c))*a^2*b - 48*(3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*ta
n(d*x + c))*a*b^2 - 8*b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + 3*log(sin(
d*x + c) + 1) - 3*log(sin(d*x + c) - 1) - 4*sin(d*x + c)))/d
```

Fricas [A]
time = 3.26, size = 196, normalized size = 0.83

$$\frac{3\,(a^3-12\,ab^2)dx\cos(dx+c)^2+6\,(2\,a^2b-b^3)\cos(dx+c)^2\log(\sin(dx+c)+1)-6\,(2\,a^2b-b^3)\cos(dx+c)^2\log(-\sin(dx+c)+1)+\left(2\,a^3\cos(dx+c)^5+8\,a^2b\cos(dx+c)^4+24\,ab^2\cos(dx+c)-(5\,a^3-12\,ab^2)\cos(dx+c)^3+4\,b^3-8\,(4\,a^2b-b^3)\cos(dx+c)^2\right)\sin(dx+c)}{8\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/8*(3*(a^3 - 12*a*b^2)*d*x*cos(d*x + c)^2 + 6*(2*a^2*b - b^3)*cos(d*x + c)
^2*log(sin(d*x + c) + 1) - 6*(2*a^2*b - b^3)*cos(d*x + c)^2*log(-sin(d*x +
c) + 1) + (2*a^3*cos(d*x + c)^5 + 8*a^2*b*cos(d*x + c)^4 + 24*a*b^2*cos(d*x
 + c) - (5*a^3 - 12*a*b^2)*cos(d*x + c)^3 + 4*b^3 - 8*(4*a^2*b - b^3)*cos(d
*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^3\sin^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3*sin(d*x+c)**4,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3*sin(c + d*x)**4, x)
```

Giac [A]
time = 0.59, size = 431, normalized size = 1.83

$$\frac{3\left(a^3-12\,ab^2\right)(dx+c)+12\left(2\,a^2b-b^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-12\left(2\,a^2b-b^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{8\left(6\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}+\frac{2\left(3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-24\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-12\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+8\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+11\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-104\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-12\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+24\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-11\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-104\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+12\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+24\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-24\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+12\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+8\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/8*(3*(a^3 - 12*a*b^2)*(d*x + c) + 12*(2*a^2*b - b^3)*log(abs(tan(1/2*d*x
+ 1/2*c) + 1)) - 12*(2*a^2*b - b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 8*
(6*a*b^2*tan(1/2*d*x + 1/2*c)^3 - b^3*tan(1/2*d*x + 1/2*c)^3 - 6*a*b^2*tan(
1/2*d*x + 1/2*c) - b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2
 + 2*(3*a^3*tan(1/2*d*x + 1/2*c)^7 - 24*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 12*a
*b^2*tan(1/2*d*x + 1/2*c)^7 + 8*b^3*tan(1/2*d*x + 1/2*c)^7 + 11*a^3*tan(1/2
*d*x + 1/2*c)^5 - 104*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 12*a*b^2*tan(1/2*d*x +
 1/2*c)^5 + 24*b^3*tan(1/2*d*x + 1/2*c)^5 - 11*a^3*tan(1/2*d*x + 1/2*c)^3 -
 104*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 12*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 24*b^
3*tan(1/2*d*x + 1/2*c)^3 - 3*a^3*tan(1/2*d*x + 1/2*c) - 24*a^2*b*tan(1/2*d*
x + 1/2*c) + 12*a*b^2*tan(1/2*d*x + 1/2*c) + 8*b^3*tan(1/2*d*x + 1/2*c))/(t
an(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

Mupad [B]
time = 1.47, size = 281, normalized size = 1.19

$$\frac{b^3\,\sin\left(c+d\,x\right)}{d}+\frac{3\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{4\,d}-\frac{3\,b^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{a^3\cos\left(c+d\,x\right)^3\,\sin\left(c+d\,x\right)}{4\,d}+\frac{b^3\,\sin\left(c+d\,x\right)}{2\,d\cos\left(c+d\,x\right)^2}-\frac{5\,a^3\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{8\,d}-\frac{4\,a^2\,b\,\sin\left(c+d\,x\right)}{d}-\frac{9\,a\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{6\,a^2\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{3\,a\,b^2\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{2\,d}+\frac{3\,a\,b^2\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}+\frac{a^2\,b\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4*(a + b/cos(c + d*x))^3,x)
```

```
[Out] (b^3*sin(c + d*x))/d + (3*a^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/
(4*d) - (3*b^3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (a^3*cos(c
 + d*x)^3*sin(c + d*x))/(4*d) + (b^3*sin(c + d*x))/(2*d*cos(c + d*x)^2) - (
5*a^3*cos(c + d*x)*sin(c + d*x))/(8*d) - (4*a^2*b*sin(c + d*x))/d - (9*a*b^
2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (6*a^2*b*atanh(sin(c/2 +
 (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (3*a*b^2*cos(c + d*x)*sin(c + d*x))/(2*d
) + (3*a*b^2*sin(c + d*x))/(d*cos(c + d*x)) + (a^2*b*cos(c + d*x)^2*sin(c +
 d*x))/d
```

3.192 $\int (a+b\sec(c+dx))^3\sin^2(c+dx)\,dx$

Optimal. Leaf size=138

$$\frac{1}{2}a\left(a^2-6b^2\right)x+\frac{b\left(6a^2-b^2\right)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{15a^2b\sin(c+dx)}{2d}-\frac{5a^3\cos(c+dx)\sin(c+dx)}{2d}+\frac{3a(b+\cdots}{}$$

```
[Out] 1/2*a*(a^2-6*b^2)*x+1/2*b*(6*a^2-b^2)*arctanh(sin(d*x+c))/d-15/2*a^2*b*sin(
d*x+c)/d-5/2*a^3*cos(d*x+c)*sin(d*x+c)/d+3/2*a*(b+a*cos(d*x+c))^2*tan(d*x+c
)/d+1/2*(b+a*cos(d*x+c))^3*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A]
time = 0.35, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.381$, Rules used = {3957, 2968, 3127, 3126, 3112, 3102, 2814, 3855}

$$-\frac{5a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{b\left(6a^2-b^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{1}{2}ax\left(a^2-6b^2\right)-\frac{15a^2b\sin(c+dx)}{2d}+\frac{3a\tan(c+dx)(a\cos(c+dx)+b)^2}{2d}+\frac{\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+b)^3}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^3*Sin[c + d*x]^2,x]
```

```
[Out] (a*(a^2 - 6*b^2)*x)/2 + (b*(6*a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(2*d) - (15
*a^2*b*Sin[c + d*x])/(2*d) - (5*a^3*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (3*a
*(b + a*Cos[c + d*x])^2*Tan[c + d*x])/(2*d) + ((b + a*Cos[c + d*x])^3*Sec[c
 + d*x]*Tan[c + d*x])/(2*d)
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
```

```
&& !LtQ[m, -1]
```

Rule 3112

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin
[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A
*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && !LtQ[m, -1]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3127

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d
*(b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*
c*(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A
*d^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int (a+b\sec(c+dx))^3\sin^2(c+dx)\,dx &= -\int (-b-a\cos(c+dx))^3\sec(c+dx)\tan^2(c+dx)\,dx \\
&= -\int (-b-a\cos(c+dx))^3\left(1-\cos^2(c+dx)\right)\sec^3(c+dx)\,dx \\
&= \frac{(b+a\cos(c+dx))^3\sec(c+dx)\tan(c+dx)}{2d}-\frac{1}{2}\int (-b-a\cos(c+ \\
&= \frac{3a(b+a\cos(c+dx))^2\tan(c+dx)}{2d}+\frac{(b+a\cos(c+dx))^3\sec(c+ \\
&= -\frac{5a^3\cos(c+dx)\sin(c+dx)}{2d}+\frac{3a(b+a\cos(c+dx))^2\tan(c+dx)}{2d} \\
&= -\frac{15a^2b\sin(c+dx)}{2d}-\frac{5a^3\cos(c+dx)\sin(c+dx)}{2d}+\frac{3a(b+a\cos(\\
&= \frac{1}{2}a\left(a^2-6b^2\right)x-\frac{15a^2b\sin(c+dx)}{2d}-\frac{5a^3\cos(c+dx)\sin(c+dx)}{2d} \\
&= \frac{1}{2}a\left(a^2-6b^2\right)x+\frac{b(6a^2-b^2)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{15a^2b\sin(c+}{2d}
\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 327 vs. 2(138) = 276.
time = 0.58, size = 327, normalized size = 2.37

$$\frac{\sec^2(c+dx)\left(a^3c-6ab^2c+a^3dx-6ab^2dx-6a^2b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+b^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+6a^2b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-b^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+\cos(2(c+dx))\left(a(a^2-6b^2)(c+dx)+(-6a^2b+b^3)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-b(-6a^2+b^2)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+(-3a^2b+2b^3)\sin(c+dx)-\frac{1}{2}a^3\sin(2(c+dx))+6ab^2\sin(2(c+dx))-3a^2b\sin(3(c+dx))-\frac{1}{4}a^3\sin(4(c+dx))\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^3*Sin[c + d*x]^2,x]
```

```
[Out] (Sec[c + d*x]^2*(a^3*c - 6*a*b^2*c + a^3*d*x - 6*a*b^2*d*x - 6*a^2*b*Log[Co
s[(c + d*x)/2] - Sin[(c + d*x)/2]] + b^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]] + 6*a^2*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - b^3*Log[Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2]] + Cos[2*(c + d*x)]*(a*(a^2 - 6*b^2)*(c + d*x)
 + (-6*a^2*b + b^3)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - b*(-6*a^2 +
b^2)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (-3*a^2*b + 2*b^3)*Sin[c +
 d*x] - (a^3*Sin[2*(c + d*x)])/2 + 6*a*b^2*Sin[2*(c + d*x)] - 3*a^2*b*Sin[3
*(c + d*x)] - (a^3*Sin[4*(c + d*x)])/4))/(4*d)
```

Maple [A]
time = 0.10, size = 128, normalized size = 0.93

method	result
derivativedivides	$\frac{b^3\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3b^2a(\tan(dx+c)-dx-c)+3b\,a^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan}{d}$
default	$\frac{b^3\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3b^2a(\tan(dx+c)-dx-c)+3b\,a^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan}{d}$
risch	$\frac{a^3x}{2}-3a\,b^2x+\frac{i e^{2i(dx+c)}a^3}{8d}+\frac{3i e^{i(dx+c)}b\,a^2}{2d}-\frac{3i e^{-i(dx+c)}b\,a^2}{2d}-\frac{i e^{-2i(dx+c)}a^3}{8d}-\frac{ib^2\left(b\,e^{3i(dx+c)}-6a\,e^{2i(dx}{d(e^{2i(dx+c}}$
norman	$\frac{\left(\frac{1}{2}a^3-3b^2a\right)x+\left(\frac{1}{2}a^3-3b^2a\right)x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-a^3+6b^2a\right)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{\left(a^3-6b\,a^2-6b^2a+b^3\right)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{3(}{}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2\left(1+\tan\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3*sin(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^3*(1/2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*sin(d*x+c)-1/2*ln(sec(d*x+c)+ta
n(d*x+c)))+3*b^2*a*(tan(d*x+c)-d*x-c)+3*b*a^2*(-sin(d*x+c)+ln(sec(d*x+c)+ta
n(d*x+c)))+a^3*(-1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.49, size = 129, normalized size = 0.93

$$\frac{(2\,dx+2\,c-\sin(2\,dx+2\,c))a^3-12\,(dx+c-\tan(dx+c))ab^2-b^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)\right)+6\,a^2b(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)-2\,\sin(dx+c))}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c - sin(2*d*x + 2*c))*a^3 - 12*(d*x + c - tan(d*x + c))*a*b
^2 - b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - log
(sin(d*x + c) - 1)) + 6*a^2*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1
) - 2*sin(d*x + c)))/d
```

Fricas [A]
time = 3.90, size = 151, normalized size = 1.09

$$\frac{2\,(a^3-6\,ab^2)dx\cos(dx+c)^2+(6\,a^2b-b^3)\cos(dx+c)^2\log(\sin(dx+c)+1)-(6\,a^2b-b^3)\cos(dx+c)^2\log(-\sin(dx+c)+1)-2\,(a^3\cos(dx+c)^3+6\,a^2b\cos(dx+c)^2-6\,ab^2\cos(dx+c)-b^3)\sin(dx+c)}{4\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(a^3 - 6*a*b^2)*d*x*cos(d*x + c)^2 + (6*a^2*b - b^3)*cos(d*x + c)^2*
log(sin(d*x + c) + 1) - (6*a^2*b - b^3)*cos(d*x + c)^2*log(-sin(d*x + c) +
```

```
1) - 2*(a^3*cos(d*x + c)^3 + 6*a^2*b*cos(d*x + c)^2 - 6*a*b^2*cos(d*x + c)
- b^3)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^3\sin^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3*sin(d*x+c)**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3*sin(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 346 vs. 2(126) = 252.
time = 0.52, size = 346, normalized size = 2.51

$$\frac{(a^3-6ab^2)(dx+c)+(6a^2b-b^3)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-(6a^2b-b^3)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)+\frac{2\left(a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-6a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-6ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-3a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+6a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-6ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+3b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+3a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+6a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+6ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+3b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-6a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+6ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-1\right)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*((a^3 - 6*a*b^2)*(d*x + c) + (6*a^2*b - b^3)*log(abs(tan(1/2*d*x + 1/2*
c) + 1)) - (6*a^2*b - b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(a^3*tan(
1/2*d*x + 1/2*c)^7 - 6*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 6*a*b^2*tan(1/2*d*x +
 1/2*c)^7 + b^3*tan(1/2*d*x + 1/2*c)^7 - 3*a^3*tan(1/2*d*x + 1/2*c)^5 + 6*a
^2*b*tan(1/2*d*x + 1/2*c)^5 - 6*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 3*b^3*tan(1/
2*d*x + 1/2*c)^5 + 3*a^3*tan(1/2*d*x + 1/2*c)^3 + 6*a^2*b*tan(1/2*d*x + 1/2
*c)^3 + 6*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 3*b^3*tan(1/2*d*x + 1/2*c)^3 - a^3
*tan(1/2*d*x + 1/2*c) - 6*a^2*b*tan(1/2*d*x + 1/2*c) + 6*a*b^2*tan(1/2*d*x
+ 1/2*c) + b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^4 - 1)^2)/d
```

Mupad [B]
time = 1.27, size = 202, normalized size = 1.46

$$\frac{a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{b^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{b^3\sin\left(c+d\,x\right)}{2\,d\cos\left(c+d\,x\right)^2}-\frac{a^3\cos\left(c+d\,x\right)\sin\left(c+d\,x\right)}{2\,d}-\frac{3\,a^2\,b\sin\left(c+d\,x\right)}{d}-\frac{6\,a\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{6\,a^2\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{3\,a\,b^2\sin\left(c+d\,x\right)}{d\cos\left(c+d\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2*(a + b/cos(c + d*x))^3,x)
```

```
[Out] (a^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (b^3*atanh(sin(c/2 +
(d*x)/2)/cos(c/2 + (d*x)/2)))/d + (b^3*sin(c + d*x))/(2*d*cos(c + d*x)^2) -
 (a^3*cos(c + d*x)*sin(c + d*x))/(2*d) - (3*a^2*b*sin(c + d*x))/d - (6*a*b^
2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (6*a^2*b*atanh(sin(c/2 +
 (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (3*a*b^2*sin(c + d*x))/(d*cos(c + d*x))
```

3.193 $\int \csc^2(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=133

$$\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}+\frac{3b^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{a^3\cot(c+dx)}{d}-\frac{3ab^2\cot(c+dx)}{d}-\frac{3a^2b\csc(c+dx)}{d}-3b$$

```
[Out] 3*a^2*b*arctanh(sin(d*x+c))/d+3/2*b^3*arctanh(sin(d*x+c))/d-a^3*cot(d*x+c)/
d-3*a*b^2*cot(d*x+c)/d-3*a^2*b*csc(d*x+c)/d-3/2*b^3*csc(d*x+c)/d+1/2*b^3*cs
c(d*x+c)*sec(d*x+c)^2/d+3*a*b^2*tan(d*x+c)/d
```

Rubi [A]
time = 0.20, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3957, 2991, 3852, 8, 2701, 327, 213, 2700, 14, 294}

$$-\frac{a^3\cot(c+dx)}{d}-\frac{3a^2b\csc(c+dx)}{d}+\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}+\frac{3ab^2\tan(c+dx)}{d}-\frac{3ab^2\cot(c+dx)}{d}-\frac{3b^3\csc(c+dx)}{2d}+\frac{3b^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b^3\csc(c+dx)\sec^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (3*a^2*b*ArcTanh[Sin[c + d*x]])/d + (3*b^3*ArcTanh[Sin[c + d*x]])/(2*d) - (
a^3*Cot[c + d*x])/d - (3*a*b^2*Cot[c + d*x])/d - (3*a^2*b*Csc[c + d*x])/d -
 (3*b^3*Csc[c + d*x])/(2*d) + (b^3*Csc[c + d*x]*Sec[c + d*x]^2)/(2*d) + (3*
a*b^2*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
/; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2991

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \csc^2(c+dx)(a+b\sec(c+dx))^3\,dx &= -\int (-b-a\cos(c+dx))^3\csc^2(c+dx)\sec^3(c+dx)\,dx \\
&= \int \left(a^3\csc^2(c+dx)+3a^2b\csc^2(c+dx)\sec(c+dx)+3ab^2\csc^2(c+a\right. \\
&= a^3\int \csc^2(c+dx)\,dx+\left(3a^2b\right)\int \csc^2(c+dx)\sec(c+dx)\,dx+\left(3ab\right. \\
&= -\frac{a^3\text{Subst}\left(\int 1\,dx,x,\cot(c+dx)\right)}{d}-\frac{\left(3a^2b\right)\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx,x,\csc(\right.}{d} \\
&= -\frac{a^3\cot(c+dx)}{d}-\frac{3a^2b\csc(c+dx)}{d}+\frac{b^3\csc(c+dx)\sec^2(c+dx)}{2d}- \\
&= \frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^3\cot(c+dx)}{d}-\frac{3ab^2\cot(c+dx)}{d}-\frac{3a}{} \\
&= \frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}+\frac{3b^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{a^3\cot(c+dx}{d}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 406 vs. $2(133) = 266$.
time = 0.44, size = 406, normalized size = 3.05

$$
-\frac{\csc^5\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(12a^2b+2b^3+6a\left(a^2+2b^2\right)\cos(c+dx)+6\left(2a^2b+b^3\right)\cos(2(c+dx))+2a^3\cos(3(c+dx))+12ab^2\cos(3(c+dx))+6a^2b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)+3b^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)-6a^2b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)-3b^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)+6a^2b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))+3b^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))-6a^2b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))-3b^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))\right)}{16d\left(-1+\cot^2\left(\frac{1}{2}(c+dx)\right)\right)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -1/16*(Csc[(c + d*x)/2]^5*Sec[(c + d*x)/2]*(12*a^2*b + 2*b^3 + 6*a*(a^2 + 2
*b^2)*Cos[c + d*x] + 6*(2*a^2*b + b^3)*Cos[2*(c + d*x)] + 2*a^3*Cos[3*(c +
d*x)] + 12*a*b^2*Cos[3*(c + d*x)] + 6*a^2*b*Log[Cos[(c + d*x)/2] - Sin[(c +
 d*x)/2]]*Sin[c + d*x] + 3*b^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin
[c + d*x] - 6*a^2*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] -
 3*b^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] + 6*a^2*b*Log[
Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] + 3*b^3*Log[Cos[(c +
d*x)/2] - Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 6*a^2*b*Log[Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 3*b^3*Log[Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2]]*Sin[3*(c + d*x)]))/(d*(-1 + Cot[(c + d*x)/2]^2)^2)
```

Maple [A]
time = 0.09, size = 129, normalized size = 0.97

method	result

derivativedivides	$\frac{b^3\left(\frac{1}{2\sin(dx+c)\cos(dx+c)^2}-\frac{3}{2\sin(dx+c)}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3b^2a\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)+3ba^2\left(-\frac{1}{s\cdots}\right.}{d}$
default	$\frac{b^3\left(\frac{1}{2\sin(dx+c)\cos(dx+c)^2}-\frac{3}{2\sin(dx+c)}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3b^2a\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)+3ba^2\left(-\frac{1}{s\cdots}\right.}{d}$
norman	$\frac{-\frac{a^3+3ba^2+3b^2a+b^3}{2d}+\frac{(a^3-3ba^2+3b^2a-b^3)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{3(a^3-ba^2+7b^2a-b^3)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{3(a^3+ba^2+7b^2a+b^3)(\cdots}{2d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}$
risch	$-\frac{i\left(6a^2b\,e^{5i(dx+c)}+3b^3e^{5i(dx+c)}+2a^3e^{4i(dx+c)}+12ba^2e^{3i(dx+c)}+2b^3e^{3i(dx+c)}+4a^3e^{2i(dx+c)}+12ab^2e^{2i(dx+c)}+6a^2b\,e\cdots\right.}{d\left(e^{2i(dx+c)}-1\right)\left(e^{2i(dx+c)}+1\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^3*(1/2/sin(d*x+c)/cos(d*x+c)^2-3/2/sin(d*x+c)+3/2*ln(sec(d*x+c)+tan(
d*x+c)))+3*b^2*a*(1/sin(d*x+c)/cos(d*x+c)-2*cot(d*x+c))+3*b*a^2*(-1/sin(d*x
+c)+ln(sec(d*x+c)+tan(d*x+c)))-a^3*cot(d*x+c))
```

Maxima [A]

time = 0.27, size = 139, normalized size = 1.05

$$-\frac{b^3\left(\frac{2\left(3\sin(dx+c)^2-2\right)}{\sin(dx+c)^3-\sin(dx+c)}-3\log(\sin(dx+c)+1)+3\log(\sin(dx+c)-1)\right)+6a^2b\left(\frac{2}{\sin(dx+c)}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+12ab^2\left(\frac{1}{\tan(dx+c)}-\tan(dx+c)\right)+\frac{4a^3}{\tan(dx+c)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b^3*(2*(3*sin(d*x + c)^2 - 2)/(sin(d*x + c)^3 - sin(d*x + c)) - 3*log
(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) + 6*a^2*b*(2/sin(d*x + c) - l
og(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 12*a*b^2*(1/tan(d*x + c) -
tan(d*x + c)) + 4*a^3/tan(d*x + c))/d
```

Fricas [A]

time = 4.85, size = 151, normalized size = 1.14

$$\frac{3(2a^2b+b^3)\cos(dx+c)^2\log(\sin(dx+c)+1)\sin(dx+c)-3(2a^2b+b^3)\cos(dx+c)^2\log(-\sin(dx+c)+1)\sin(dx+c)+12ab^2\cos(dx+c)-4(a^3+6ab^2)\cos(dx+c)^3+2b^3-6(2a^2b+b^3)\cos(dx+c)^2}{4d\cos(dx+c)^2\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(3*(2*a^2*b + b^3)*cos(d*x + c)^2*log(sin(d*x + c) + 1)*sin(d*x + c) -
3*(2*a^2*b + b^3)*cos(d*x + c)^2*log(-sin(d*x + c) + 1)*sin(d*x + c) + 12*a
*b^2*cos(d*x + c) - 4*(a^3 + 6*a*b^2)*cos(d*x + c)^3 + 2*b^3 - 6*(2*a^2*b +
 b^3)*cos(d*x + c)^2)/(d*cos(d*x + c)^2*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^3\csc^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3*csc(c + d*x)**2, x)
```

Giac [A]
time = 0.52, size = 225, normalized size = 1.69

$$\frac{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-3a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3\left(2a^2b+b^3\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-3\left(2a^2b+b^3\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)-\frac{a^3+3a^2b+3ab^2+b^3}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}-\frac{2\left(6ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-6ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(a^3*tan(1/2*d*x + 1/2*c) - 3*a^2*b*tan(1/2*d*x + 1/2*c) + 3*a*b^2*tan(
1/2*d*x + 1/2*c) - b^3*tan(1/2*d*x + 1/2*c) + 3*(2*a^2*b + b^3)*log(abs(tan
(1/2*d*x + 1/2*c) + 1)) - 3*(2*a^2*b + b^3)*log(abs(tan(1/2*d*x + 1/2*c) -
1)) - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)/tan(1/2*d*x + 1/2*c) - 2*(6*a*b^2*tan
(1/2*d*x + 1/2*c)^3 - b^3*tan(1/2*d*x + 1/2*c)^3 - 6*a*b^2*tan(1/2*d*x + 1/
2*c) - b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

Mupad [B]
time = 1.47, size = 181, normalized size = 1.36

$$\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\,\left(6a^2b+3b^3\right)}{d}-\frac{3ab^2-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(2a^3+6a^2b+18ab^2+4b^3\right)+3a^2b+a^3+b^3+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(a^3+3a^2b+15ab^2-b^3\right)}{d\,\left(2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,{(a-b)}^3}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^3/sin(c + d*x)^2,x)
```

```
[Out] (atanh(tan(c/2 + (d*x)/2))*(6*a^2*b + 3*b^3))/d - (3*a*b^2 - tan(c/2 + (d*x
)/2)^2*(18*a*b^2 + 6*a^2*b + 2*a^3 + 4*b^3) + 3*a^2*b + a^3 + b^3 + tan(c/2
 + (d*x)/2)^4*(15*a*b^2 + 3*a^2*b + a^3 - b^3))/(d*(2*tan(c/2 + (d*x)/2) -
4*tan(c/2 + (d*x)/2)^3 + 2*tan(c/2 + (d*x)/2)^5)) + (tan(c/2 + (d*x)/2)*(a
- b)^3)/(2*d)
```

3.194 $\int \csc^4(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=205

$$\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}+\frac{5b^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{a^3\cot(c+dx)}{d}-\frac{6ab^2\cot(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{3d}-\frac{ab}{}$$

[Out] 3*a^2*b*arctanh(sin(d*x+c))/d+5/2*b^3*arctanh(sin(d*x+c))/d-a^3*cot(d*x+c)/d-6*a*b^2*cot(d*x+c)/d-1/3*a^3*cot(d*x+c)^3/d-a*b^2*cot(d*x+c)^3/d-3*a^2*b*csc(d*x+c)/d-5/2*b^3*csc(d*x+c)/d-a^2*b*csc(d*x+c)^3/d-5/6*b^3*csc(d*x+c)^3/d+1/2*b^3*csc(d*x+c)^3*sec(d*x+c)^2/d+3*a*b^2*tan(d*x+c)/d

Rubi [A]
time = 0.21, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3957, 2991, 3852, 2701, 308, 213, 2700, 276, 294}

$$-\frac{a^3\cot^3(c+dx)}{3d}-\frac{a^3\cot(c+dx)}{d}-\frac{a^2b\csc^3(c+dx)}{d}-\frac{3a^2b\csc(c+dx)}{d}+\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}+\frac{3ab^2\tan(c+dx)}{d}-\frac{ab^2\cot^3(c+dx)}{d}-\frac{6ab^2\cot(c+dx)}{d}-\frac{5b^3\csc^3(c+dx)}{6d}-\frac{5b^3\csc(c+dx)}{2d}+\frac{5b^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b^3\csc^3(c+dx)\sec^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

[In] Int[Csc[c + d*x]^4*(a + b*Sec[c + d*x])^3,x]

[Out] (3*a^2*b*ArcTanh[Sin[c + d*x]])/d + (5*b^3*ArcTanh[Sin[c + d*x]])/(2*d) - (a^3*Cot[c + d*x])/d - (6*a*b^2*Cot[c + d*x])/d - (a^3*Cot[c + d*x]^3)/(3*d) - (a*b^2*Cot[c + d*x]^3)/d - (3*a^2*b*Csc[c + d*x])/d - (5*b^3*Csc[c + d*x])/(2*d) - (a^2*b*Csc[c + d*x]^3)/d - (5*b^3*Csc[c + d*x]^3)/(6*d) + (b^3*Csc[c + d*x]^3*Sec[c + d*x]^2)/(2*d) + (3*a*b^2*Tan[c + d*x])/d

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2991

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \csc^4(c+dx)(a+b\sec(c+dx))^3\,dx &= -\int (-b-a\cos(c+dx))^3\csc^4(c+dx)\sec^3(c+dx)\,dx \\ &= \int \left(a^3\csc^4(c+dx)+3a^2b\csc^4(c+dx)\sec(c+dx)+3ab^2\csc^4(c+\right. \\ &= a^3\int \csc^4(c+dx)\,dx+\left(3a^2b\right)\int \csc^4(c+dx)\sec(c+dx)\,dx+\left(3a\right. \\ &= -\frac{a^3\text{Subst}\left(\int \left(1+x^2\right)\,dx,x,\cot(c+dx)\right)}{d}-\frac{\left(3a^2b\right)\text{Subst}\left(\int \frac{x^4}{-1+x^2}\right.}{d} \\ &= -\frac{a^3\cot(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{3d}+\frac{b^3\csc^3(c+dx)\sec^2(c+dx)}{2d} \\ &= -\frac{a^3\cot(c+dx)}{d}-\frac{6ab^2\cot(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{3d}-\frac{ab^2\cot^3(}{d} \\ &= \frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^3\cot(c+dx)}{d}-\frac{6ab^2\cot(c+dx)}{d}- \\ &= \frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}+\frac{5b^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{a^3\cot(c+}{d}\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 610 vs. 2(205) = 410.
time = 0.60, size = 610, normalized size = 2.98

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -1/768*(Csc[(c + d*x)/2]^7*Sec[(c + d*x)/2]^3*(84*a^2*b + 22*b^3 + 32*a*(a^
2 + 3*b^2)*Cos[c + d*x] + 8*(6*a^2*b + 5*b^3)*Cos[2*(c + d*x)] + 4*a^3*Cos[
3*(c + d*x)] + 48*a*b^2*Cos[3*(c + d*x)] - 36*a^2*b*Cos[4*(c + d*x)] - 30*b
^3*Cos[4*(c + d*x)] - 4*a^3*Cos[5*(c + d*x)] - 48*a*b^2*Cos[5*(c + d*x)] +
36*a^2*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] + 30*b^3*Log
[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] - 36*a^2*b*Log[Cos[(c +
d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] - 30*b^3*Log[Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]]*Sin[c + d*x] + 18*a^2*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]]*Sin[3*(c + d*x)] + 15*b^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Si
n[3*(c + d*x)] - 18*a^2*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[3*(c
 + d*x)] - 15*b^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[3*(c + d*x)]
 - 18*a^2*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] - 15*
b^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] + 18*a^2*b*Lo
g[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] + 15*b^3*Log[Cos[(c
```

```
 + d*x)/2] + Sin[(c + d*x)/2]]*Sin[5*(c + d*x)]))/(d*(-1 + Cot[(c + d*x)/2]
^2)^2)
```

Maple [A]
time = 0.14, size = 187, normalized size = 0.91

method	result
derivativedivides	$\frac{b^3\left(-\frac{1}{3\sin(dx+c)^3\cos(dx+c)^2}+\frac{5}{6\sin(dx+c)\cos(dx+c)^2}-\frac{5}{2\sin(dx+c)}+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3b^2a\left(-\frac{1}{3\sin(dx+c)^3\cos(dx+}\right.}{}$
default	$\frac{b^3\left(-\frac{1}{3\sin(dx+c)^3\cos(dx+c)^2}+\frac{5}{6\sin(dx+c)\cos(dx+c)^2}-\frac{5}{2\sin(dx+c)}+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3b^2a\left(-\frac{1}{3\sin(dx+c)^3\cos(dx+}\right.}{}$
norman	$\frac{-\frac{a^3+3b\,a^2+3b^2a+b^3}{24d}+\frac{\left(a^3-3b\,a^2+3b^2a-b^3\right)\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}+\frac{\left(7a^3-39b\,a^2+57b^2a-25b^3\right)\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}-\frac{\left(7a^3+39b\,a^2+57b^2\right.}{}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3\left(\tan^2\left(\right.\right.}$
risch	$-\frac{i\left(18b\,a^2e^{9i(dx+c)}+15b^3e^{9i(dx+c)}-24b\,a^2e^{7i(dx+c)}-20b^3e^{7i(dx+c)}-12a^3e^{6i(dx+c)}-84a^2b\,e^{5i(dx+c)}-22b^3e^{5i(dx+c)}-20\right.}{3d\left(e^{2i}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^3*(-1/3/sin(d*x+c)^3/cos(d*x+c)^2+5/6/sin(d*x+c)/cos(d*x+c)^2-5/2/si
n(d*x+c)+5/2*ln(sec(d*x+c)+tan(d*x+c)))+3*b^2*a*(-1/3/sin(d*x+c)^3/cos(d*x+
c)+4/3/sin(d*x+c)/cos(d*x+c)-8/3*cot(d*x+c))+3*b*a^2*(-1/3/sin(d*x+c)^3-1/s
in(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+a^3*(-2/3-1/3*csc(d*x+c)^2)*cot(d*x+c)
)
```

Maxima [A]
time = 0.27, size = 190, normalized size = 0.93

$$-\frac{b^3\left(\frac{2\left(15\sin(dx+c)^4-10\sin(dx+c)^2-2\right)}{\sin(dx+c)^5-\sin(dx+c)^3}-15\log\left(\sin\left(dx+c\right)+1\right)+15\log\left(\sin\left(dx+c\right)-1\right)\right)+6a^2b\left(\frac{2\left(3\sin(dx+c)^2+1\right)}{\sin(dx+c)^3}-3\log\left(\sin\left(dx+c\right)+1\right)+3\log\left(\sin\left(dx+c\right)-1\right)\right)+12ab^2\left(\frac{6\tan(dx+c)^2+1}{\tan(dx+c)^3}-3\tan\left(dx+c\right)\right)+\frac{4\left(3\tan(dx+c)^2+1\right)a^3}{\tan(dx+c)^3}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/12*(b^3*(2*(15*sin(d*x + c)^4 - 10*sin(d*x + c)^2 - 2)/(sin(d*x + c)^5 -
 sin(d*x + c)^3) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) + 6
*a^2*b*(2*(3*sin(d*x + c)^2 + 1)/sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) +
 3*log(sin(d*x + c) - 1)) + 12*a*b^2*((6*tan(d*x + c)^2 + 1)/tan(d*x + c)^3
 - 3*tan(d*x + c)) + 4*(3*tan(d*x + c)^2 + 1)*a^3/tan(d*x + c)^3)/d
```

Fricas [A]
time = 3.41, size = 260, normalized size = 1.27

$$-\frac{8\left(a^3+12ab^2\right)\cos\left(dx+c\right)^5+6\left(6a^2b+5b^3\right)\cos\left(dx+c\right)^4+36ab^2\cos\left(dx+c\right)-12\left(a^3+12ab^2\right)\cos\left(dx+c\right)^3+6b^3-8\left(6a^2b+5b^3\right)\cos\left(dx+c\right)^2-3\left(\left(6a^2b+5b^3\right)\cos\left(dx+c\right)^4-\left(6a^2b+5b^3\right)\cos\left(dx+c\right)^2\right)\log\left(\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)+3\left(\left(6a^2b+5b^3\right)\cos\left(dx+c\right)^4-\left(6a^2b+5b^3\right)\cos\left(dx+c\right)^2\right)\log\left(-\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)}{12\left(d\cos\left(dx+c\right)^4-d\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/12*(8*(a^3 + 12*a*b^2)*cos(d*x + c)^5 + 6*(6*a^2*b + 5*b^3)*cos(d*x + c)
^4 + 36*a*b^2*cos(d*x + c) - 12*(a^3 + 12*a*b^2)*cos(d*x + c)^3 + 6*b^3 - 8
*(6*a^2*b + 5*b^3)*cos(d*x + c)^2 - 3*((6*a^2*b + 5*b^3)*cos(d*x + c)^4 - (
6*a^2*b + 5*b^3)*cos(d*x + c)^2)*log(sin(d*x + c) + 1)*sin(d*x + c) + 3*((6
*a^2*b + 5*b^3)*cos(d*x + c)^4 - (6*a^2*b + 5*b^3)*cos(d*x + c)^2)*log(-sin
(d*x + c) + 1)*sin(d*x + c))/((d*cos(d*x + c)^4 - d*cos(d*x + c)^2)*sin(d*x
 + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec{(c + dx)})^3 \csc^4{(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3*csc(c + d*x)**4, x)
```

Giac [A]
time = 0.54, size = 361, normalized size = 1.76

$$\frac{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-45\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+63\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-27\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+12\,(6\,a^2b+5\,b^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-12\,(6\,a^2b+5\,b^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{24\left(6\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}-\frac{9\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+45\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+63\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+27\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a^3+3\,a^2b+3\,ab^2+b^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/24*(a^3*tan(1/2*d*x + 1/2*c)^3 - 3*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 3*a*b^2
*tan(1/2*d*x + 1/2*c)^3 - b^3*tan(1/2*d*x + 1/2*c)^3 + 9*a^3*tan(1/2*d*x +
1/2*c) - 45*a^2*b*tan(1/2*d*x + 1/2*c) + 63*a*b^2*tan(1/2*d*x + 1/2*c) - 27
*b^3*tan(1/2*d*x + 1/2*c) + 12*(6*a^2*b + 5*b^3)*log(abs(tan(1/2*d*x + 1/2*
c) + 1)) - 12*(6*a^2*b + 5*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 24*(6*
a*b^2*tan(1/2*d*x + 1/2*c)^3 - b^3*tan(1/2*d*x + 1/2*c)^3 - 6*a*b^2*tan(1/2
*d*x + 1/2*c) - b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2 -
(9*a^3*tan(1/2*d*x + 1/2*c)^2 + 45*a^2*b*tan(1/2*d*x + 1/2*c)^2 + 63*a*b^2*
tan(1/2*d*x + 1/2*c)^2 + 27*b^3*tan(1/2*d*x + 1/2*c)^2 + a^3 + 3*a^2*b + 3*
a*b^2 + b^3)/tan(1/2*d*x + 1/2*c)^3)/d
```

Mupad [B]
time = 1.17, size = 260, normalized size = 1.27

$$\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,{\left(a-b\right)}^3}{24\,d}-\frac{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{3\,b\,{\left(a-b\right)}^2}{4}-\frac{3\,{\left(a-b\right)}^3}{8}\right)}{d}-\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(\frac{7\,a^3}{3}+13\,a^2\,b+19\,a\,b^2+\frac{25\,b^3}{3}\right)-{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(\frac{17\,a^3}{3}+29\,a^2\,b+89\,a\,b^2+\frac{77\,b^3}{3}\right)+a\,b^2+a^2\,b+\frac{a^3}{3}+\frac{b^3}{3}+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(3\,a^3+15\,a^2\,b+69\,a\,b^2+b^3\right)}{d\,\left(8\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7-16\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+8\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\right)}-\frac{\mathrm{atanh}\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(a^2\,b\,6{}\mathrm{i}+b^3\,5{}\mathrm{i}\right)\,1{}\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^3/sin(c + d*x)^4,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3*(a - b)^3)/(24*d) - (tan(c/2 + (d*x)/2)*((3*b*(a - b)
^2)/4 - (3*(a - b)^3)/8))/d - (atanh(tan(c/2 + (d*x)/2))*(a^2*b*6i + b^3*5i
)*1i)/d - (tan(c/2 + (d*x)/2)^2*(19*a*b^2 + 13*a^2*b + (7*a^3)/3 + (25*b^3)
/3) - tan(c/2 + (d*x)/2)^4*(89*a*b^2 + 29*a^2*b + (17*a^3)/3 + (77*b^3)/3)
+ a*b^2 + a^2*b + a^3/3 + b^3/3 + tan(c/2 + (d*x)/2)^6*(69*a*b^2 + 15*a^2*b
 + 3*a^3 + b^3))/(d*(8*tan(c/2 + (d*x)/2)^3 - 16*tan(c/2 + (d*x)/2)^5 + 8*t
an(c/2 + (d*x)/2)^7))
```

3.195 $\int \csc^6(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=279

$$\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}+\frac{7b^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{a^3\cot(c+dx)}{d}-\frac{9ab^2\cot(c+dx)}{d}-\frac{2a^3\cot^3(c+dx)}{3d}-\cdots$$

```
[Out] 3*a^2*b*arctanh(sin(d*x+c))/d+7/2*b^3*arctanh(sin(d*x+c))/d-a^3*cot(d*x+c)/
d-9*a*b^2*cot(d*x+c)/d-2/3*a^3*cot(d*x+c)^3/d-3*a*b^2*cot(d*x+c)^3/d-1/5*a^
3*cot(d*x+c)^5/d-3/5*a*b^2*cot(d*x+c)^5/d-3*a^2*b*csc(d*x+c)/d-7/2*b^3*csc(
d*x+c)/d-a^2*b*csc(d*x+c)^3/d-7/6*b^3*csc(d*x+c)^3/d-3/5*a^2*b*csc(d*x+c)^5
/d-7/10*b^3*csc(d*x+c)^5/d+1/2*b^3*csc(d*x+c)^5*sec(d*x+c)^2/d+3*a*b^2*tan(
d*x+c)/d
```

Rubi [A]
time = 0.22, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3957, 2991, 3852, 2701, 308, 213, 2700, 276, 294}

$$-\frac{a^3\cot^5(c+dx)}{5d}-\frac{2a^3\cot^3(c+dx)}{3d}-\frac{a^3\cot(c+dx)}{d}-\frac{3a^2b\csc^5(c+dx)}{5d}-\frac{a^2b\csc^3(c+dx)}{d}-\frac{3a^2b\csc(c+dx)}{d}+\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}+\frac{3ab^2\tan(c+dx)}{d}-\frac{3ab^2\cot^5(c+dx)}{5d}-\frac{3ab^2\cot^3(c+dx)}{d}-\frac{9ab^2\cot(c+dx)}{d}-\frac{7b^3\csc^5(c+dx)}{10d}-\frac{7b^3\csc^3(c+dx)}{6d}-\frac{7b^3\csc(c+dx)}{2d}+\frac{7b^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b^3\csc^5(c+dx)\sec^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (3*a^2*b*ArcTanh[Sin[c + d*x]])/d + (7*b^3*ArcTanh[Sin[c + d*x]])/(2*d) - (
a^3*Cot[c + d*x])/d - (9*a*b^2*Cot[c + d*x])/d - (2*a^3*Cot[c + d*x]^3)/(3*
d) - (3*a*b^2*Cot[c + d*x]^3)/d - (a^3*Cot[c + d*x]^5)/(5*d) - (3*a*b^2*Cot
[c + d*x]^5)/(5*d) - (3*a^2*b*Csc[c + d*x])/d - (7*b^3*Csc[c + d*x])/(2*d)
- (a^2*b*Csc[c + d*x]^3)/d - (7*b^3*Csc[c + d*x]^3)/(6*d) - (3*a^2*b*Csc[c
+ d*x]^5)/(5*d) - (7*b^3*Csc[c + d*x]^5)/(10*d) + (b^3*Csc[c + d*x]^5*Sec[c
 + d*x]^2)/(2*d) + (3*a*b^2*Tan[c + d*x])/d
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
```

```
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2991

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc^6(c+dx)(a+b\sec(c+dx))^3\,dx = -\int (-b-a\cos(c+dx))^3\csc^6(c+dx)\sec^3(c+dx)\,dx$$
$$= \int \left(a^3\csc^6(c+dx)+3a^2b\csc^6(c+dx)\sec(c+dx)+3ab^2\csc^6(c+\right.$$
$$= a^3\int \csc^6(c+dx)\,dx + \left(3a^2b\right)\int \csc^6(c+dx)\sec(c+dx)\,dx + \left(3a\right.$$
$$= -\frac{a^3\mathrm{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx, x, \cot(c+dx)\right)}{d} - \frac{\left(3a^2b\right)\mathrm{Subst}\left(\int\right.}{}$$
$$= -\frac{a^3\cot(c+dx)}{d} - \frac{2a^3\cot^3(c+dx)}{3d} - \frac{a^3\cot^5(c+dx)}{5d} + \frac{b^3\csc^5(c}{}$$
$$= -\frac{a^3\cot(c+dx)}{d} - \frac{9ab^2\cot(c+dx)}{d} - \frac{2a^3\cot^3(c+dx)}{3d} - \frac{3ab^2\cot}{}$$
$$= \frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d} - \frac{a^3\cot(c+dx)}{d} - \frac{9ab^2\cot(c+dx)}{d} - \frac{2}{}$$
$$= \frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d} + \frac{7b^3\tanh^{-1}(\sin(c+dx))}{2d} - \frac{a^3\cot(c+}{d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 812 vs. 2(279) = 558.
time = 0.93, size = 812, normalized size = 2.91

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -1/61440*(Csc[(c + d*x)/2]^9*Sec[(c + d*x)/2]^5*(1176*a^2*b + 412*b^3 + 80*
a*(5*a^2 + 18*b^2)*Cos[c + d*x] + 66*(6*a^2*b + 7*b^3)*Cos[2*(c + d*x)] + 1
6*a^3*Cos[3*(c + d*x)] + 288*a*b^2*Cos[3*(c + d*x)] - 600*a^2*b*Cos[4*(c +
d*x)] - 700*b^3*Cos[4*(c + d*x)] - 48*a^3*Cos[5*(c + d*x)] - 864*a*b^2*Cos[
5*(c + d*x)] + 180*a^2*b*Cos[6*(c + d*x)] + 210*b^3*Cos[6*(c + d*x)] + 16*a
^3*Cos[7*(c + d*x)] + 288*a*b^2*Cos[7*(c + d*x)] + 450*a^2*b*Log[Cos[(c + d
*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] + 525*b^3*Log[Cos[(c + d*x)/2] - Si
n[(c + d*x)/2]]*Sin[c + d*x] - 450*a^2*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*
x)/2]]*Sin[c + d*x] - 525*b^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[
c + d*x] + 90*a^2*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[3*(c + d*x
)] + 105*b^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 90
*a^2*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 105*b^3*
Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 270*a^2*b*Log[C
os[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] - 315*b^3*Log[Cos[(c +
```

```
 d*x)/2] - Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] + 270*a^2*b*Log[Cos[(c + d*x)
/2] + Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] + 315*b^3*Log[Cos[(c + d*x)/2] + S
in[(c + d*x)/2]]*Sin[5*(c + d*x)] + 90*a^2*b*Log[Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]]*Sin[7*(c + d*x)] + 105*b^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/
2]]*Sin[7*(c + d*x)] - 90*a^2*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Si
n[7*(c + d*x)] - 105*b^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[7*(c
+ d*x)]))/(d*(-1 + Cot[(c + d*x)/2]^2)^2)
```

Maple [A]
time = 0.15, size = 243, normalized size = 0.87 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6*(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^3*(-1/5/sin(d*x+c)^5/cos(d*x+c)^2-7/15/sin(d*x+c)^3/cos(d*x+c)^2+7/6
/sin(d*x+c)/cos(d*x+c)^2-7/2/sin(d*x+c)+7/2*ln(sec(d*x+c)+tan(d*x+c)))+3*b^
2*a*(-1/5/sin(d*x+c)^5/cos(d*x+c)-2/5/sin(d*x+c)^3/cos(d*x+c)+8/5/sin(d*x+c
)/cos(d*x+c)-16/5*cot(d*x+c))+3*b*a^2*(-1/5/sin(d*x+c)^5-1/3/sin(d*x+c)^3-1
/sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+a^3*(-8/15-1/5*csc(d*x+c)^4-4/15*csc
(d*x+c)^2)*cot(d*x+c))
```

Maxima [A]
time = 0.28, size = 230, normalized size = 0.82

$$-\frac{b^3\left(\frac{2\left(105\sin(dx+c)^6-70\sin(dx+c)^4-14\sin(dx+c)^2-6\right)}{\sin(dx+c)^7-\sin(dx+c)^5}-105\log\left(\sin\left(dx+c\right)+1\right)+105\log\left(\sin\left(dx+c\right)-1\right)\right)+6a^2b\left(\frac{2\left(15\sin(dx+c)^4+5\sin(dx+c)^2+3\right)}{\sin(dx+c)^5}-15\log\left(\sin\left(dx+c\right)+1\right)+15\log\left(\sin\left(dx+c\right)-1\right)\right)+36ab^2\left(\frac{15\tan(dx+c)^4+5\tan(dx+c)^2+1}{\tan(dx+c)^5}-5\tan\left(dx+c\right)\right)+\frac{4\left(15\tan(dx+c)^4+10\tan(dx+c)^2+3\right)a^3}{\tan(dx+c)^5}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/60*(b^3*(2*(105*sin(d*x + c)^6 - 70*sin(d*x + c)^4 - 14*sin(d*x + c)^2 -
 6)/(sin(d*x + c)^7 - sin(d*x + c)^5) - 105*log(sin(d*x + c) + 1) + 105*log
(sin(d*x + c) - 1)) + 6*a^2*b*(2*(15*sin(d*x + c)^4 + 5*sin(d*x + c)^2 + 3)
/sin(d*x + c)^5 - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) + 36
*a*b^2*((15*tan(d*x + c)^4 + 5*tan(d*x + c)^2 + 1)/tan(d*x + c)^5 - 5*tan(d
*x + c)) + 4*(15*tan(d*x + c)^4 + 10*tan(d*x + c)^2 + 3)*a^3/tan(d*x + c)^5
)/d
```

Fricas [A]
time = 4.15, size = 354, normalized size = 1.27

$$-\frac{32\left(a^3+18ab^2\right)\cos\left(dx+c\right)^7+30\left(6a^2b+7b^3\right)\cos\left(dx+c\right)^6-80\left(a^3+18ab^2\right)\cos\left(dx+c\right)^5-70\left(6a^2b+7b^3\right)\cos\left(dx+c\right)^4-180ab^2\cos\left(dx+c\right)+60\left(a^3+18ab^2\right)\cos\left(dx+c\right)^3-30b^3+46\left(6a^2b+7b^3\right)\cos\left(dx+c\right)^2-15\left(\left(6a^2b+7b^3\right)\cos\left(dx+c\right)^6-2\left(6a^2b+7b^3\right)\cos\left(dx+c\right)^4+\left(6a^2b+7b^3\right)\cos\left(dx+c\right)^2\right)\log\left(\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)+15\left(\left(6a^2b+7b^3\right)\cos\left(dx+c\right)^6-2\left(6a^2b+7b^3\right)\cos\left(dx+c\right)^4+\left(6a^2b+7b^3\right)\cos\left(dx+c\right)^2\right)\log\left(-\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)}{60\left(d\cos\left(dx+c\right)^6-2d\cos\left(dx+c\right)^4+d\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/60*(32*(a^3 + 18*a*b^2)*cos(d*x + c)^7 + 30*(6*a^2*b + 7*b^3)*cos(d*x +
c)^6 - 80*(a^3 + 18*a*b^2)*cos(d*x + c)^5 - 70*(6*a^2*b + 7*b^3)*cos(d*x +
```

```
c)^4 - 180*a*b^2*cos(d*x + c) + 60*(a^3 + 18*a*b^2)*cos(d*x + c)^3 - 30*b^3
 + 46*(6*a^2*b + 7*b^3)*cos(d*x + c)^2 - 15*((6*a^2*b + 7*b^3)*cos(d*x + c)
^6 - 2*(6*a^2*b + 7*b^3)*cos(d*x + c)^4 + (6*a^2*b + 7*b^3)*cos(d*x + c)^2)
*log(sin(d*x + c) + 1)*sin(d*x + c) + 15*((6*a^2*b + 7*b^3)*cos(d*x + c)^6
- 2*(6*a^2*b + 7*b^3)*cos(d*x + c)^4 + (6*a^2*b + 7*b^3)*cos(d*x + c)^2)*lo
g(-sin(d*x + c) + 1)*sin(d*x + c))/((d*cos(d*x + c)^6 - 2*d*cos(d*x + c)^4
+ d*cos(d*x + c)^2)*sin(d*x + c))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**A**]
time = 0.55, size = 498, normalized size = 1.78

$$\frac{3a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-9a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+9ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-3b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+25a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-105a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+135ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-55b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+150a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-990a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1710ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-870b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+240(6a^2b+7b^3)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-240(6a^2b+7b^3)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)-\frac{480\left(6ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-6ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2}-\frac{150a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+990a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+1710ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+870b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+25a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+105a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+135ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+55b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+3a^3+9a^2b+9ab^2+3b^3}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5}}{480d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/480*(3*a^3*tan(1/2*d*x + 1/2*c)^5 - 9*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 9*a*
b^2*tan(1/2*d*x + 1/2*c)^5 - 3*b^3*tan(1/2*d*x + 1/2*c)^5 + 25*a^3*tan(1/2*
d*x + 1/2*c)^3 - 105*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 135*a*b^2*tan(1/2*d*x +
 1/2*c)^3 - 55*b^3*tan(1/2*d*x + 1/2*c)^3 + 150*a^3*tan(1/2*d*x + 1/2*c) -
990*a^2*b*tan(1/2*d*x + 1/2*c) + 1710*a*b^2*tan(1/2*d*x + 1/2*c) - 870*b^3*
tan(1/2*d*x + 1/2*c) + 240*(6*a^2*b + 7*b^3)*log(abs(tan(1/2*d*x + 1/2*c) +
 1)) - 240*(6*a^2*b + 7*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 480*(6*a*
b^2*tan(1/2*d*x + 1/2*c)^3 - b^3*tan(1/2*d*x + 1/2*c)^3 - 6*a*b^2*tan(1/2*d
*x + 1/2*c) - b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2 - (1
50*a^3*tan(1/2*d*x + 1/2*c)^4 + 990*a^2*b*tan(1/2*d*x + 1/2*c)^4 + 1710*a*b
^2*tan(1/2*d*x + 1/2*c)^4 + 870*b^3*tan(1/2*d*x + 1/2*c)^4 + 25*a^3*tan(1/2
*d*x + 1/2*c)^2 + 105*a^2*b*tan(1/2*d*x + 1/2*c)^2 + 135*a*b^2*tan(1/2*d*x
+ 1/2*c)^2 + 55*b^3*tan(1/2*d*x + 1/2*c)^2 + 3*a^3 + 9*a^2*b + 9*a*b^2 + 3*
b^3)/tan(1/2*d*x + 1/2*c)^5)/d
```

Mupad [**B**]
time = 1.22, size = 363, normalized size = 1.30

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5(a-b)^3}{160d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{19a^3}{15}+\frac{29a^2b}{5}+\frac{39ab^2}{5}+\frac{49b^3}{15}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8(10a^3+66a^2b+306ab^2+26b^3)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(\frac{55a^3}{3}+125a^2b+411ab^2+\frac{433b^3}{3}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(\frac{103a^3}{15}+\frac{263a^2b}{5}+\frac{483ab^2}{5}+\frac{763b^3}{15}\right)+\frac{3ab^2}{5}+\frac{3a^2b}{5}+\frac{a^3}{5}+\frac{b^3}{5}}{d\left(32\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9-64\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+32\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\right)}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{21ab^2}{16}-\frac{3a^2b}{8}-\frac{a^3}{16}-\frac{7b^3}{8}+\frac{3(a-b)^2(a-4b)}{16}+\frac{3(a-b)^3}{16}\right)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{(a-b)^2(a-4b)}{48}+\frac{(a-b)^3}{32}\right)}{d}-\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)(a^2b\,6i+b^3\,7i)\,1i}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^3/sin(c + d*x)^6,x)

[Out] (tan(c/2 + (d*x)/2)^5*(a - b)^3)/(160*d) - (tan(c/2 + (d*x)/2)^2*((39*a*b^2
)/5 + (29*a^2*b)/5 + (19*a^3)/15 + (49*b^3)/15) + tan(c/2 + (d*x)/2)^8*(306
*a*b^2 + 66*a^2*b + 10*a^3 + 26*b^3) - tan(c/2 + (d*x)/2)^6*(411*a*b^2 + 12
5*a^2*b + (55*a^3)/3 + (433*b^3)/3) + tan(c/2 + (d*x)/2)^4*((483*a*b^2)/5 +
 (263*a^2*b)/5 + (103*a^3)/15 + (763*b^3)/15) + (3*a*b^2)/5 + (3*a^2*b)/5 +
 a^3/5 + b^3/5)/(d*(32*tan(c/2 + (d*x)/2)^5 - 64*tan(c/2 + (d*x)/2)^7 + 32*
tan(c/2 + (d*x)/2)^9)) - (atanh(tan(c/2 + (d*x)/2))*(a^2*b*6i + b^3*7i)*1i)
/d + (tan(c/2 + (d*x)/2)*((21*a*b^2)/16 - (3*a^2*b)/8 - a^3/16 - (7*b^3)/8
+ (3*(a - b)^2*(a - 4*b))/16 + (3*(a - b)^3)/16))/d + (tan(c/2 + (d*x)/2)^3
*(((a - b)^2*(a - 4*b))/48 + (a - b)^3/32))/d
```

3.196 $\int \frac{\sin^7(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=223

$$-\frac{(a^2-b^2)^3\cos(c+dx)}{a^7d}-\frac{b(3a^4-3a^2b^2+b^4)\cos^2(c+dx)}{2a^6d}+\frac{(3a^4-3a^2b^2+b^4)\cos^3(c+dx)}{3a^5d}+\frac{b(3a^2-b^2)\cos\ldots}{4a^4\ldots}$$

```
[Out] -(a^2-b^2)^3*cos(d*x+c)/a^7/d-1/2*b*(3*a^4-3*a^2*b^2+b^4)*cos(d*x+c)^2/a^6/
d+1/3*(3*a^4-3*a^2*b^2+b^4)*cos(d*x+c)^3/a^5/d+1/4*b*(3*a^2-b^2)*cos(d*x+c)
^4/a^4/d-1/5*(3*a^2-b^2)*cos(d*x+c)^5/a^3/d-1/6*b*cos(d*x+c)^6/a^2/d+1/7*co
s(d*x+c)^7/a/d+b*(a^2-b^2)^3*ln(b+a*cos(d*x+c))/a^8/d
```

Rubi [A]
time = 0.18, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3957, 2916, 12, 786}

$$-\frac{b\cos^6(c+dx)}{6a^2d}+\frac{b(a^2-b^2)^3\log(a\cos(c+dx)+b)}{a^8d}-\frac{(a^2-b^2)^3\cos(c+dx)}{a^7d}+\frac{b(3a^2-b^2)\cos^4(c+dx)}{4a^4d}-\frac{(3a^2-b^2)\cos^5(c+dx)}{5a^3d}-\frac{b(3a^4-3a^2b^2+b^4)\cos^2(c+dx)}{2a^6d}+\frac{(3a^4-3a^2b^2+b^4)\cos^3(c+dx)}{3a^5d}+\frac{\cos^7(c+dx)}{7ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^7/(a + b*Sec[c + d*x]),x]
```

```
[Out] -(((a^2 - b^2)^3*Cos[c + d*x])/(a^7*d)) - (b*(3*a^4 - 3*a^2*b^2 + b^4)*Cos[
c + d*x]^2)/(2*a^6*d) + ((3*a^4 - 3*a^2*b^2 + b^4)*Cos[c + d*x]^3)/(3*a^5*d
) + (b*(3*a^2 - b^2)*Cos[c + d*x]^4)/(4*a^4*d) - ((3*a^2 - b^2)*Cos[c + d*x
]^5)/(5*a^3*d) - (b*Cos[c + d*x]^6)/(6*a^2*d) + Cos[c + d*x]^7/(7*a*d) + (b
*(a^2 - b^2)^3*Log[b + a*Cos[c + d*x]])/(a^8*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^7(c+dx)}{a+b\sec(c+dx)}\,dx &= -\int \frac{\cos(c+dx)\sin^7(c+dx)}{-b-a\cos(c+dx)}\,dx \\
&= \frac{\text{Subst}\left(\int \frac{x\left(a^2-x^2\right)^3}{a(-b+x)}\,dx, x, -a\cos(c+dx)\right)}{a^7 d} \\
&= \frac{\text{Subst}\left(\int \frac{x\left(a^2-x^2\right)^3}{-b+x}\,dx, x, -a\cos(c+dx)\right)}{a^8 d} \\
&= \frac{\text{Subst}\left(\int \left(\left(a^2-b^2\right)^3+\frac{b\left(-a^2+b^2\right)^3}{b-x}-b\left(3a^4-3a^2b^2+b^4\right)x-\left(3a^4-3a^2b^2+b^4\right)x^2\right.\right.}{a^8 d} \\
&= -\frac{\left(a^2-b^2\right)^3\cos(c+dx)}{a^7 d}-\frac{b\left(3a^4-3a^2b^2+b^4\right)\cos^2(c+dx)}{2a^6 d}+\frac{\left(3a^4-3a^2b^2+b^4\right)c}{3a^5 d}
\end{aligned}$$

Mathematica [A]
time = 0.96, size = 282, normalized size = 1.26

$$\frac{-105a\left(35a^6-152a^4b^2+176a^2b^4-64b^6\right)\cos(c+dx)-105\left(29a^6b-40a^4b^3+16a^2b^5\right)\cos(2(c+dx))+735a^7\cos(3(c+dx))-1260a^5b^2\cos(3(c+dx))+560a^3b^4\cos(3(c+dx))+420a^6b\cos(4(c+dx))-210a^4b^3\cos(4(c+dx))-147a^7\cos(5(c+dx))+84a^5b^2\cos(5(c+dx))-35a^6b\cos(6(c+dx))+15a^7\cos(7(c+dx))+6720a^6b\log(b+a\cos(c+dx))-20160a^4b^3\log(b+a\cos(c+dx))+20160a^2b^5\log(b+a\cos(c+dx))-6720b^7\log(b+a\cos(c+dx))}{6720a^8 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^7/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-105*a*(35*a^6 - 152*a^4*b^2 + 176*a^2*b^4 - 64*b^6)*Cos[c + d*x] - 105*(2
9*a^6*b - 40*a^4*b^3 + 16*a^2*b^5)*Cos[2*(c + d*x)] + 735*a^7*Cos[3*(c + d*
x)] - 1260*a^5*b^2*Cos[3*(c + d*x)] + 560*a^3*b^4*Cos[3*(c + d*x)] + 420*a^
6*b*Cos[4*(c + d*x)] - 210*a^4*b^3*Cos[4*(c + d*x)] - 147*a^7*Cos[5*(c + d*
x)] + 84*a^5*b^2*Cos[5*(c + d*x)] - 35*a^6*b*Cos[6*(c + d*x)] + 15*a^7*Cos[
7*(c + d*x)] + 6720*a^6*b*Log[b + a*Cos[c + d*x]] - 20160*a^4*b^3*Log[b + a
*Cos[c + d*x]] + 20160*a^2*b^5*Log[b + a*Cos[c + d*x]] - 6720*b^7*Log[b + a
*Cos[c + d*x]])/(6720*a^8*d)
```

Maple [A]
time = 0.20, size = 275, normalized size = 1.23 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^7/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/a^7*(1/7*cos(d*x+c)^7*a^6-1/6*b*cos(d*x+c)^6*a^5-3/5*a^6*cos(d*x+c)^
5+1/5*a^4*b^2*cos(d*x+c)^5+3/4*a^5*b*cos(d*x+c)^4-1/4*a^3*b^3*cos(d*x+c)^4+
a^6*cos(d*x+c)^3-a^4*b^2*cos(d*x+c)^3+1/3*a^2*b^4*cos(d*x+c)^3-3/2*a^5*b*co
s(d*x+c)^2+3/2*a^3*b^3*cos(d*x+c)^2-1/2*a*b^5*cos(d*x+c)^2-a^6*cos(d*x+c)+3
*a^4*b^2*cos(d*x+c)-3*a^2*b^4*cos(d*x+c)+b^6*cos(d*x+c))+b*(a^6-3*a^4*b^2+3
*a^2*b^4-b^6)/a^8*ln(b+a*cos(d*x+c)))
```

Maxima [A]
time = 0.26, size = 224, normalized size = 1.00

$$\frac{\frac{60\,a^6\cos(dx+c)^7-70\,a^5b\cos(dx+c)^6-84\,(3\,a^6-a^4b^2)\cos(dx+c)^5+105\,(3\,a^5b-a^3b^3)\cos(dx+c)^4+140\,(3\,a^6-3\,a^4b^2+a^2b^4)\cos(dx+c)^3-210\,(3\,a^5b-3\,a^3b^3+ab^5)\cos(dx+c)^2-420\,(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\cos(dx+c)}{a^7}+\frac{420\,(a^6b-3\,a^4b^3+3\,a^2b^5-b^7)\log(a\cos(dx+c)+b)}{a^8}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/420*((60*a^6*cos(d*x + c)^7 - 70*a^5*b*cos(d*x + c)^6 - 84*(3*a^6 - a^4*b
^2)*cos(d*x + c)^5 + 105*(3*a^5*b - a^3*b^3)*cos(d*x + c)^4 + 140*(3*a^6 -
3*a^4*b^2 + a^2*b^4)*cos(d*x + c)^3 - 210*(3*a^5*b - 3*a^3*b^3 + a*b^5)*cos
(d*x + c)^2 - 420*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cos(d*x + c))/a^7 + 4
20*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*log(a*cos(d*x + c) + b)/a^8)/d
```

Fricas [A]
time = 3.50, size = 222, normalized size = 1.00

$$\frac{60\,a^7\cos(dx+c)^7-70\,a^6b\cos(dx+c)^6-84\,(3\,a^7-a^5b^2)\cos(dx+c)^5+105\,(3\,a^6b-a^4b^3)\cos(dx+c)^4+140\,(3\,a^7-3\,a^5b^2+a^3b^4)\cos(dx+c)^3-210\,(3\,a^6b-3\,a^4b^3+a^2b^5)\cos(dx+c)^2-420\,(a^7-3\,a^5b^2+3\,a^3b^4-ab^6)\cos(dx+c)+420\,(a^6b-3\,a^4b^3+3\,a^2b^5-b^7)\log(a\cos(dx+c)+b)}{420\,a^8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/420*(60*a^7*cos(d*x + c)^7 - 70*a^6*b*cos(d*x + c)^6 - 84*(3*a^7 - a^5*b^
2)*cos(d*x + c)^5 + 105*(3*a^6*b - a^4*b^3)*cos(d*x + c)^4 + 140*(3*a^7 - 3
*a^5*b^2 + a^3*b^4)*cos(d*x + c)^3 - 210*(3*a^6*b - 3*a^4*b^3 + a^2*b^5)*co
s(d*x + c)^2 - 420*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cos(d*x + c) + 420
*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*log(a*cos(d*x + c) + b))/(a^8*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**7/(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1559 vs. $2(211) = 422$.
time = 0.49, size = 1559, normalized size = 6.99

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/420*(420*(a^7*b - a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 + 3*a^3*b^5 - 3*a^2*b^6
 - a*b^7 + b^8)*log(abs(a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b
*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^9 - a^8*b) - 420*(a^6*b - 3*a^4
*b^3 + 3*a^2*b^5 - b^7)*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)
)/a^8 + (384*a^7 - 1089*a^6*b - 1848*a^5*b^2 + 3267*a^4*b^3 + 2240*a^3*b^4
- 3267*a^2*b^5 - 840*a*b^6 + 1089*b^7 - 2688*a^7*(cos(d*x + c) - 1)/(cos(d*
x + c) + 1) + 8463*a^6*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 12096*a^5*
b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 24549*a^4*b^3*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) - 14000*a^3*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
+ 23709*a^2*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 5040*a*b^6*(cos(d*x
 + c) - 1)/(cos(d*x + c) + 1) - 7623*b^7*(cos(d*x + c) - 1)/(cos(d*x + c) +
 1) + 8064*a^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 28749*a^6*b*(cos
(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 32088*a^5*b^2*(cos(d*x + c) - 1)^2/
(cos(d*x + c) + 1)^2 + 78687*a^4*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1
)^2 + 35280*a^3*b^4*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 72807*a^2*b
^5*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 12600*a*b^6*(cos(d*x + c) -
1)^2/(cos(d*x + c) + 1)^2 + 22869*b^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) +
1)^2 - 13440*a^7*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 56035*a^6*b*(c
os(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 40320*a^5*b^2*(cos(d*x + c) - 1)^
3/(cos(d*x + c) + 1)^3 - 136185*a^4*b^3*(cos(d*x + c) - 1)^3/(cos(d*x + c)
+ 1)^3 - 45920*a^3*b^4*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 122745*a
^2*b^5*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 16800*a*b^6*(cos(d*x + c
) - 1)^3/(cos(d*x + c) + 1)^3 - 38115*b^7*(cos(d*x + c) - 1)^3/(cos(d*x + c
) + 1)^3 - 56035*a^6*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 24360*a^
5*b^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 136185*a^4*b^3*(cos(d*x +
 c) - 1)^4/(cos(d*x + c) + 1)^4 + 32480*a^3*b^4*(cos(d*x + c) - 1)^4/(cos(d
*x + c) + 1)^4 - 122745*a^2*b^5*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 -
 12600*a*b^6*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 38115*b^7*(cos(d*x
 + c) - 1)^4/(cos(d*x + c) + 1)^4 + 28749*a^6*b*(cos(d*x + c) - 1)^5/(cos(d
*x + c) + 1)^5 + 6720*a^5*b^2*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 7
8687*a^4*b^3*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 11760*a^3*b^4*(cos
(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 72807*a^2*b^5*(cos(d*x + c) - 1)^5/
(cos(d*x + c) + 1)^5 + 5040*a*b^6*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5
 - 22869*b^7*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 8463*a^6*b*(cos(d*
x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 840*a^5*b^2*(cos(d*x + c) - 1)^6/(cos(
d*x + c) + 1)^6 + 24549*a^4*b^3*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 +
```

```
1680*a^3*b^4*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 23709*a^2*b^5*(co
s(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 840*a*b^6*(cos(d*x + c) - 1)^6/(co
s(d*x + c) + 1)^6 + 7623*b^7*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 10
89*a^6*b*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 - 3267*a^4*b^3*(cos(d*x
+ c) - 1)^7/(cos(d*x + c) + 1)^7 + 3267*a^2*b^5*(cos(d*x + c) - 1)^7/(cos(d
*x + c) + 1)^7 - 1089*b^7*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7)/(a^8*(
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^7))/d
```

Mupad [B]

time = 0.16, size = 249, normalized size = 1.12

$$\frac{\cos\left(c+d\,x\right)^3\,\left(\frac{1}{a}-\frac{b^2\,\left(\frac{1}{a}-\frac{b^2}{3\,a^3}\right)}{a^2}\right)-\cos\left(c+d\,x\right)^5\,\left(\frac{3}{5\,a}-\frac{b^2}{5\,a^3}\right)-\cos\left(c+d\,x\right)\,\left(\frac{1}{a}-\frac{b^2\,\left(\frac{3}{a}-\frac{b^2\,\left(\frac{3}{a}-\frac{b^2}{a^3}\right)}{a^2}\right)}{a^2}\right)+\frac{\cos(c+d\,x)^7}{7\,a}+\frac{\ln(b+a\,\cos(c+d\,x))\,\left(a^6\,b-3\,a^4\,b^3+3\,a^2\,b^5-b^7\right)}{a^8}-\frac{b\cos(c+d\,x)^6}{6\,a^2}-\frac{b\cos(c+d\,x)^2\,\left(\frac{3}{a}-\frac{b^2\,\left(\frac{3}{a}-\frac{b^2}{a^3}\right)}{a^2}\right)}{2\,a}+\frac{b\cos(c+d\,x)^4\,\left(\frac{3}{a}-\frac{b^2}{a^3}\right)}{4\,a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^7/(a + b/cos(c + d*x)),x)
```

```
[Out] (cos(c + d*x)^3*(1/a - (b^2*(1/a - b^2/(3*a^3)))/a^2) - cos(c + d*x)^5*(3/(
5*a) - b^2/(5*a^3)) - cos(c + d*x)*(1/a - (b^2*(3/a - (b^2*(3/a - b^2/a^3))
/a^2))/a^2) + cos(c + d*x)^7/(7*a) + (log(b + a*cos(c + d*x))*(a^6*b - b^7
+ 3*a^2*b^5 - 3*a^4*b^3))/a^8 - (b*cos(c + d*x)^6)/(6*a^2) - (b*cos(c + d*x
)^2*(3/a - (b^2*(3/a - b^2/a^3))/a^2))/(2*a) + (b*cos(c + d*x)^4*(3/a - b^2
/a^3))/(4*a))/d
```

3.197 $\int \frac{\sin^5(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=152

$$-\frac{(a^2-b^2)^2\cos(c+dx)}{a^5d}-\frac{b(2a^2-b^2)\cos^2(c+dx)}{2a^4d}+\frac{(2a^2-b^2)\cos^3(c+dx)}{3a^3d}+\frac{b\cos^4(c+dx)}{4a^2d}-\frac{\cos^5(c+dx)}{5ad}+$$

```
[Out] -(a^2-b^2)^2*cos(d*x+c)/a^5/d-1/2*b*(2*a^2-b^2)*cos(d*x+c)^2/a^4/d+1/3*(2*a
^2-b^2)*cos(d*x+c)^3/a^3/d+1/4*b*cos(d*x+c)^4/a^2/d-1/5*cos(d*x+c)^5/a/d+b*
(a^2-b^2)^2*ln(b+a*cos(d*x+c))/a^6/d
```

Rubi [A]
time = 0.14, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3957, 2916, 12, 786}

$$\frac{b\cos^4(c+dx)}{4a^2d}+\frac{b(a^2-b^2)^2\log(a\cos(c+dx)+b)}{a^6d}-\frac{(a^2-b^2)^2\cos(c+dx)}{a^5d}-\frac{b(2a^2-b^2)\cos^2(c+dx)}{2a^4d}+\frac{(2a^2-b^2)\cos^3(c+dx)}{3a^3d}-\frac{\cos^5(c+dx)}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^5/(a + b*Sec[c + d*x]),x]
```

```
[Out] -(((a^2 - b^2)^2*Cos[c + d*x])/(a^5*d)) - (b*(2*a^2 - b^2)*Cos[c + d*x]^2)/
(2*a^4*d) + ((2*a^2 - b^2)*Cos[c + d*x]^3)/(3*a^3*d) + (b*Cos[c + d*x]^4)/(
4*a^2*d) - Cos[c + d*x]^5/(5*a*d) + (b*(a^2 - b^2)^2*Log[b + a*Cos[c + d*x]
])/(a^6*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^5(c+dx)}{a+b\sec(c+dx)}\,dx &= -\int \frac{\cos(c+dx)\sin^5(c+dx)}{-b-a\cos(c+dx)}\,dx \\
&= \frac{\mathrm{Subst}\left(\int \frac{x(a^2-x^2)^2}{a(-b+x)}\,dx, x, -a\cos(c+dx)\right)}{a^5 d} \\
&= \frac{\mathrm{Subst}\left(\int \frac{x(a^2-x^2)^2}{-b+x}\,dx, x, -a\cos(c+dx)\right)}{a^6 d} \\
&= \frac{\mathrm{Subst}\left(\int \left((a^2-b^2)^2 - \frac{b(-a^2+b^2)^2}{b-x} + b(-2a^2+b^2)\,x - (2a^2-b^2)\,x^2 + bx^3 + x^4\right) d\right.}{a^6 d} \\
&= -\frac{(a^2-b^2)^2\cos(c+dx)}{a^5 d} - \frac{b(2a^2-b^2)\cos^2(c+dx)}{2a^4 d} + \frac{(2a^2-b^2)\cos^3(c+dx)}{3a^3 d} + \cdots
\end{aligned}$$

Mathematica [A]
time = 0.26, size = 172, normalized size = 1.13

$$\frac{-60a(5a^4-14a^2b^2+8b^4)\cos(c+dx)-60(3a^4b-2a^2b^3)\cos(2(c+dx))+50a^5\cos(3(c+dx))-40a^3b^2\cos(3(c+dx))+15a^4b\cos(4(c+dx))-6a^5\cos(5(c+dx))+480a^4b\log(b+a\cos(c+dx))-960a^2b^3\log(b+a\cos(c+dx))+480b^5\log(b+a\cos(c+dx))}{480a^6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^5/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-60*a*(5*a^4 - 14*a^2*b^2 + 8*b^4)*Cos[c + d*x] - 60*(3*a^4*b - 2*a^2*b^3)
*Cos[2*(c + d*x)] + 50*a^5*Cos[3*(c + d*x)] - 40*a^3*b^2*Cos[3*(c + d*x)] +
 15*a^4*b*Cos[4*(c + d*x)] - 6*a^5*Cos[5*(c + d*x)] + 480*a^4*b*Log[b + a*C
os[c + d*x]] - 960*a^2*b^3*Log[b + a*Cos[c + d*x]] + 480*b^5*Log[b + a*Cos[
c + d*x]])/(480*a^6*d)
```

Maple [A]
time = 0.13, size = 160, normalized size = 1.05

method	result
derivativedivides	$-\frac{\frac{\frac{\left(\cos^5(dx+c)\right)a^4}{5}-\frac{b\left(\cos^4(dx+c)\right)a^3}{4}-\frac{2a^4\left(\cos^3(dx+c)\right)}{3}+\frac{a^2b^2\left(\cos^3(dx+c)\right)}{3}+a^3b\left(\cos^2(dx+c)\right)-\frac{a\,b^3\left(\cos^2(dx+c)\right)}{2}+a^4\cos(dx-\cdots}{a^5}}{d}$
default	$-\frac{\frac{\frac{\left(\cos^5(dx+c)\right)a^4}{5}-\frac{b\left(\cos^4(dx+c)\right)a^3}{4}-\frac{2a^4\left(\cos^3(dx+c)\right)}{3}+\frac{a^2b^2\left(\cos^3(dx+c)\right)}{3}+a^3b\left(\cos^2(dx+c)\right)-\frac{a\,b^3\left(\cos^2(dx+c)\right)}{2}+a^4\cos(dx-\cdots}{a^5}}{d}$

norman	$\frac{\frac{\left(2b\,a^3+2b^2a^2-2b^3a-2b^4\right)\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^5d}+\frac{-16a^4+50b^2a^2-30b^4}{15d\,a^5}+\frac{2\left(5b\,a^3+6b^2a^2-3b^3a-4b^4\right)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^5}+\big(-16a^4+6b\,a^3+}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}$
risch	$-\frac{ibx}{a^2}-\frac{3b\,\mathrm{e}^{2i(dx+c)}}{16a^2d}+\frac{b^3\mathrm{e}^{2i(dx+c)}}{8a^4d}+\frac{7\,\mathrm{e}^{i(dx+c)}b^2}{8a^3d}-\frac{\mathrm{e}^{i(dx+c)}b^4}{2a^5d}+\frac{7\,\mathrm{e}^{-i(dx+c)}b^2}{8a^3d}-\frac{\mathrm{e}^{-i(dx+c)}b^4}{2a^5d}-\frac{3b\,\mathrm{e}^{-2i(dx+}}{16a^2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^5/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/a^5*(1/5*cos(d*x+c)^5*a^4-1/4*b*cos(d*x+c)^4*a^3-2/3*a^4*cos(d*x+c)
^3+1/3*a^2*b^2*cos(d*x+c)^3+a^3*b*cos(d*x+c)^2-1/2*a*b^3*cos(d*x+c)^2+a^4*c
os(d*x+c)-2*a^2*b^2*cos(d*x+c)+b^4*cos(d*x+c))+b*(a^4-2*a^2*b^2+b^4)/a^6*ln
(b+a*cos(d*x+c)))
```

Maxima [A]
time = 0.27, size = 141, normalized size = 0.93

$$-\frac{\frac{12\,a^4\cos(dx+c)^5-15\,a^3b\cos(dx+c)^4-20\left(2\,a^4-a^2b^2\right)\cos(dx+c)^3+30\left(2\,a^3b-ab^3\right)\cos(dx+c)^2+60\left(a^4-2\,a^2b^2+b^4\right)\cos(dx+c)}{a^5}-\frac{60\left(a^4b-2\,a^2b^3+b^5\right)\log(a\cos(dx+c)+b)}{a^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/60*((12*a^4*cos(d*x + c)^5 - 15*a^3*b*cos(d*x + c)^4 - 20*(2*a^4 - a^2*b
^2)*cos(d*x + c)^3 + 30*(2*a^3*b - a*b^3)*cos(d*x + c)^2 + 60*(a^4 - 2*a^2*
b^2 + b^4)*cos(d*x + c))/a^5 - 60*(a^4*b - 2*a^2*b^3 + b^5)*log(a*cos(d*x +
 c) + b)/a^6)/d
```

Fricas [A]
time = 2.58, size = 140, normalized size = 0.92

$$-\frac{12\,a^5\cos\left(dx+c\right)^5-15\,a^4b\cos\left(dx+c\right)^4-20\left(2\,a^5-a^3b^2\right)\cos\left(dx+c\right)^3+30\left(2\,a^4b-a^2b^3\right)\cos\left(dx+c\right)^2+60\left(a^5-2\,a^3b^2+ab^4\right)\cos\left(dx+c\right)-60\left(a^4b-2\,a^2b^3+b^5\right)\log\left(a\cos\left(dx+c\right)+b\right)}{60\,a^6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/60*(12*a^5*cos(d*x + c)^5 - 15*a^4*b*cos(d*x + c)^4 - 20*(2*a^5 - a^3*b^
2)*cos(d*x + c)^3 + 30*(2*a^4*b - a^2*b^3)*cos(d*x + c)^2 + 60*(a^5 - 2*a^3
*b^2 + a*b^4)*cos(d*x + c) - 60*(a^4*b - 2*a^2*b^3 + b^5)*log(a*cos(d*x + c
) + b))/(a^6*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**5/(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 867 vs. $2(144) = 288$.
time = 0.52, size = 867, normalized size = 5.70

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/60*(60*(a^5*b - a^4*b^2 - 2*a^3*b^3 + 2*a^2*b^4 + a*b^5 - b^6)*log(abs(a
+ b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d
*x + c) + 1)))/(a^7 - a^6*b) - 60*(a^4*b - 2*a^2*b^3 + b^5)*log(abs(-(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^6 + (64*a^5 - 137*a^4*b - 200*a^3*b
^2 + 274*a^2*b^3 + 120*a*b^4 - 137*b^5 - 320*a^5*(cos(d*x + c) - 1)/(cos(d*
x + c) + 1) + 805*a^4*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 880*a^3*b^2
*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1490*a^2*b^3*(cos(d*x + c) - 1)/(c
os(d*x + c) + 1) - 480*a*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 685*b^
5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 640*a^5*(cos(d*x + c) - 1)^2/(cos
(d*x + c) + 1)^2 - 1970*a^4*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 1
280*a^3*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 3100*a^2*b^3*(cos(d
*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 720*a*b^4*(cos(d*x + c) - 1)^2/(cos(d
*x + c) + 1)^2 - 1370*b^5*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 1970*
a^4*b*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 720*a^3*b^2*(cos(d*x + c)
 - 1)^3/(cos(d*x + c) + 1)^3 - 3100*a^2*b^3*(cos(d*x + c) - 1)^3/(cos(d*x +
 c) + 1)^3 - 480*a*b^4*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 1370*b^5
*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 805*a^4*b*(cos(d*x + c) - 1)^4
/(cos(d*x + c) + 1)^4 - 120*a^3*b^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)
^4 + 1490*a^2*b^3*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 120*a*b^4*(co
s(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 685*b^5*(cos(d*x + c) - 1)^4/(cos(
d*x + c) + 1)^4 + 137*a^4*b*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 274
*a^2*b^3*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 137*b^5*(cos(d*x + c)
- 1)^5/(cos(d*x + c) + 1)^5)/(a^6*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) -
1)^5))/d
```

Mupad [B]
time = 0.09, size = 151, normalized size = 0.99

$$-\frac{\cos\left(c+d\,x\right)\,\left(\frac{1}{a}-\frac{b^2\,\left(\frac{2}{a}-\frac{b^2}{a^3}\right)}{a^2}\right)-{\cos\left(c+d\,x\right)}^3\,\left(\frac{2}{3\,a}-\frac{b^2}{3\,a^3}\right)+\frac{{\cos\left(c+d\,x\right)}^5}{5\,a}-\frac{b\,{\cos\left(c+d\,x\right)}^4}{4\,a^2}-\frac{\ln\left(b+a\,\cos\left(c+d\,x\right)\right)\,\left(a^4\,b-2\,a^2\,b^3+b^5\right)}{a^6}+\frac{b\,{\cos\left(c+d\,x\right)}^2\,\left(\frac{2}{a}-\frac{b^2}{a^3}\right)}{2\,a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5/(a + b/cos(c + d*x)),x)
```

```
[Out] -(cos(c + d*x)*(1/a - (b^2*(2/a - b^2/a^3))/a^2) - cos(c + d*x)^3*(2/(3*a)
- b^2/(3*a^3)) + cos(c + d*x)^5/(5*a) - (b*cos(c + d*x)^4)/(4*a^2) - (log(b
 + a*cos(c + d*x))*(a^4*b + b^5 - 2*a^2*b^3))/a^6 + (b*cos(c + d*x)^2*(2/a
- b^2/a^3))/(2*a))/d
```

3.198 $\int \frac{\sin^3(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=89

$$-\frac{(a^2-b^2)\cos(c+dx)}{a^3d}-\frac{b\cos^2(c+dx)}{2a^2d}+\frac{\cos^3(c+dx)}{3ad}+\frac{b(a^2-b^2)\log(b+a\cos(c+dx))}{a^4d}$$

```
[Out] -(a^2-b^2)*cos(d*x+c)/a^3/d-1/2*b*cos(d*x+c)^2/a^2/d+1/3*cos(d*x+c)^3/a/d+b
*(a^2-b^2)*ln(b+a*cos(d*x+c))/a^4/d
```

Rubi [A]
time = 0.11, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3957, 2916, 12, 786}

$$-\frac{b\cos^2(c+dx)}{2a^2d}+\frac{b(a^2-b^2)\log(a\cos(c+dx)+b)}{a^4d}-\frac{(a^2-b^2)\cos(c+dx)}{a^3d}+\frac{\cos^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

```
[Out] -(((a^2 - b^2)*Cos[c + d*x])/(a^3*d)) - (b*Cos[c + d*x]^2)/(2*a^2*d) + Cos[
c + d*x]^3/(3*a*d) + (b*(a^2 - b^2)*Log[b + a*Cos[c + d*x]])/(a^4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
```

```
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{a+b\sec(c+dx)}\,dx = -\int \frac{\cos(c+dx)\sin^3(c+dx)}{-b-a\cos(c+dx)}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{x(a^2-x^2)}{a(-b+x)}\,dx, x, -a\cos(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \frac{x(a^2-x^2)}{-b+x}\,dx, x, -a\cos(c+dx)\right)}{a^4d}$$
$$= \frac{\text{Subst}\left(\int \left(a^2\left(1-\frac{b^2}{a^2}\right)+\frac{-a^2b+b^3}{b-x}-bx-x^2\right)\,dx, x, -a\cos(c+dx)\right)}{a^4d}$$
$$= -\frac{(a^2-b^2)\cos(c+dx)}{a^3d}-\frac{b\cos^2(c+dx)}{2a^2d}+\frac{\cos^3(c+dx)}{3ad}+\frac{b(a^2-b^2)\log(b+a\cos}{a^4d}$$

Mathematica [A]
time = 0.11, size = 89, normalized size = 1.00

$$\frac{(-9a^3+12ab^2)\cos(c+dx)-3a^2b\cos(2(c+dx))+a^3\cos(3(c+dx))+12a^2b\log(b+a\cos(c+dx))-12b^3\log(b+a\cos(c+dx))}{12a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/(a + b*Sec[c + d*x]),x]
[Out] ((-9*a^3 + 12*a*b^2)*Cos[c + d*x] - 3*a^2*b*Cos[2*(c + d*x)] + a^3*Cos[3*(c
 + d*x)] + 12*a^2*b*Log[b + a*Cos[c + d*x]] - 12*b^3*Log[b + a*Cos[c + d*x]
])/(12*a^4*d)
```

Maple [A]
time = 0.10, size = 82, normalized size = 0.92

method	result
derivativedivides	$\frac{\frac{\frac{\left(\cos^3(dx+c)\right)a^2}{3}-\frac{b\left(\cos^2(dx+c)\right)a}{2}-a^2\cos(dx+c)+b^2\cos(dx+c)}{a^3}+\frac{b\left(a^2-b^2\right)\ln(b+a\cos(dx+c))}{a^4}}{d}$
default	$\frac{\frac{\frac{\left(\cos^3(dx+c)\right)a^2}{3}-\frac{b\left(\cos^2(dx+c)\right)a}{2}-a^2\cos(dx+c)+b^2\cos(dx+c)}{a^3}+\frac{b\left(a^2-b^2\right)\ln(b+a\cos(dx+c))}{a^4}}{d}$
norman	$\frac{\frac{\left(4a^2-2b^2\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3d}+\frac{-2ba+2b^2}{3d\,a^3}+\frac{\left(4a^2-2ba-4b^2\right)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d\,a^3}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{(a+b)b(a-b)\ln\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{d}{2}\right.\right.\right.}{a^4d}$

risch	$-\frac{ibx}{a^2}+\frac{ib^3x}{a^4}-\frac{b\,\mathrm{e}^{2i(dx+c)}}{8a^2d}-\frac{3\,\mathrm{e}^{i(dx+c)}}{8ad}+\frac{\mathrm{e}^{i(dx+c)}b^2}{2a^3d}-\frac{3\,\mathrm{e}^{-i(dx+c)}}{8ad}+\frac{\mathrm{e}^{-i(dx+c)}b^2}{2a^3d}-\frac{b\,\mathrm{e}^{-2i(dx+c)}}{8a^2d}-\frac{2ibc}{a^2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/a^3*(1/3*cos(d*x+c)^3*a^2-1/2*b*cos(d*x+c)^2*a-a^2*cos(d*x+c)+b^2*co
s(d*x+c))+b*(a^2-b^2)/a^4*ln(b+a*cos(d*x+c)))
```

Maxima [A]
time = 0.27, size = 80, normalized size = 0.90

$$\frac{\frac{2\,a^2\cos(dx+c)^3-3\,ab\cos(dx+c)^2-6\,(a^2-b^2)\cos(dx+c)}{a^3}+\frac{6\,(a^2b-b^3)\log(a\cos(dx+c)+b)}{a^4}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*((2*a^2*cos(d*x + c)^3 - 3*a*b*cos(d*x + c)^2 - 6*(a^2 - b^2)*cos(d*x +
 c))/a^3 + 6*(a^2*b - b^3)*log(a*cos(d*x + c) + b)/a^4)/d
```

Fricas [A]
time = 2.41, size = 78, normalized size = 0.88

$$\frac{2\,a^3\cos\left(dx+c\right)^3-3\,a^2b\cos\left(dx+c\right)^2-6\left(a^3-ab^2\right)\cos\left(dx+c\right)+6\left(a^2b-b^3\right)\log\left(a\cos\left(dx+c\right)+b\right)}{6\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a^3*cos(d*x + c)^3 - 3*a^2*b*cos(d*x + c)^2 - 6*(a^3 - a*b^2)*cos(d*
x + c) + 6*(a^2*b - b^3)*log(a*cos(d*x + c) + b))/(a^4*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.48, size = 102, normalized size = 1.15

$$\frac{\left(a^2b-b^3\right)\log\left(\left|-a\cos\left(dx+c\right)-b\right|\right)}{a^4d}+\frac{2\,a^2d^2\cos\left(dx+c\right)^3-3\,abd^2\cos\left(dx+c\right)^2-6\,a^2d^2\cos\left(dx+c\right)+6\,b^2d^2\cos\left(dx+c\right)}{6\,a^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] (a^2*b - b^3)*log(abs(-a*cos(d*x + c) - b))/(a^4*d) + 1/6*(2*a^2*d^2*cos(d*
x + c)^3 - 3*a*b*d^2*cos(d*x + c)^2 - 6*a^2*d^2*cos(d*x + c) + 6*b^2*d^2*co
s(d*x + c))/(a^3*d^3)
```

Mupad [B]
time = 1.02, size = 79, normalized size = 0.89

$$-\frac{\cos\left(c+d\,x\right)\,\left(\frac{1}{a}-\frac{b^2}{a^3}\right)-\frac{\cos(c+d\,x)^3}{3\,a}+\frac{b\cos(c+d\,x)^2}{2\,a^2}-\frac{\ln(b+a\,\cos(c+d\,x))\,\left(a^2\,b-b^3\right)}{a^4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(a + b/cos(c + d*x)),x)
```

```
[Out] -(cos(c + d*x)*(1/a - b^2/a^3) - cos(c + d*x)^3/(3*a) + (b*cos(c + d*x)^2)/
(2*a^2) - (log(b + a*cos(c + d*x))*(a^2*b - b^3))/a^4)/d
```

3.199 $\int \frac{\sin(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=34

$$-\frac{\cos(c+dx)}{ad}+\frac{b\log(b+a\cos(c+dx))}{a^2d}$$

```
[Out] -cos(d*x+c)/a/d+b*ln(b+a*cos(d*x+c))/a^2/d
```

Rubi [A]
time = 0.05, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3957, 2912, 12, 45}

$$\frac{b\log(a\cos(c+dx)+b)}{a^2d}-\frac{\cos(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + b*Sec[c + d*x]),x]
```

```
[Out] -(Cos[c + d*x]/(a*d)) + (b*Log[b + a*Cos[c + d*x]])/(a^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(c+dx)}{a+b\sec(c+dx)}\,dx &= -\int \frac{\cos(c+dx)\sin(c+dx)}{-b-a\cos(c+dx)}\,dx \\
&= \frac{\text{Subst}\left(\int \frac{x}{a(-b+x)}\,dx, x, -a\cos(c+dx)\right)}{ad} \\
&= \frac{\text{Subst}\left(\int \frac{x}{-b+x}\,dx, x, -a\cos(c+dx)\right)}{a^2d} \\
&= \frac{\text{Subst}\left(\int \left(1-\frac{b}{b-x}\right)\,dx, x, -a\cos(c+dx)\right)}{a^2d} \\
&= -\frac{\cos(c+dx)}{ad}+\frac{b\log(b+a\cos(c+dx))}{a^2d}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 30, normalized size = 0.88

$$\frac{-a\cos(c+dx)+b\log(b+a\cos(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + b*Sec[c + d*x]),x]
[Out] (-(a*Cos[c + d*x]) + b*Log[b + a*Cos[c + d*x]])/(a^2*d)
```

Maple [A]
time = 0.05, size = 48, normalized size = 1.41

method	result	size
derivativedivides	$\frac{-\frac{1}{a\sec(dx+c)}-\frac{b\ln(\sec(dx+c))}{a^2}+\frac{b\ln(a+b\sec(dx+c))}{a^2}}{d}$	48
default	$\frac{-\frac{1}{a\sec(dx+c)}-\frac{b\ln(\sec(dx+c))}{a^2}+\frac{b\ln(a+b\sec(dx+c))}{a^2}}{d}$	48
risch	$-\frac{ibx}{a^2}-\frac{e^{i(dx+c)}}{2ad}-\frac{e^{-i(dx+c)}}{2ad}-\frac{2ibc}{a^2d}+\frac{b\ln\left(e^{2i(dx+c)}+\frac{2b\,e^{i(dx+c)}}{a}+1\right)}{a^2d}$	90
norman	$-\frac{2}{ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{b\ln\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)}{a^2d}-\frac{b\ln\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2d}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(-1/a/sec(d*x+c)-b/a^2*ln(sec(d*x+c))+b/a^2*ln(a+b*sec(d*x+c)))
```

Maxima [A]
time = 0.27, size = 33, normalized size = 0.97

$$-\frac{\frac{\cos(dx+c)}{a} - \frac{b\log(a\cos(dx+c)+b)}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="maxima")
[Out] -(cos(d*x + c)/a - b*log(a*cos(d*x + c) + b)/a^2)/d
```

Fricas [A]
time = 2.11, size = 31, normalized size = 0.91

$$-\frac{a\cos\left(dx+c\right) - b\log\left(a\cos\left(dx+c\right)+b\right)}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="fricas")
[Out] -(a*cos(d*x + c) - b*log(a*cos(d*x + c) + b))/(a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(c+dx\right)}{a+b\sec\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sec(d*x+c)),x)
[Out] Integral(sin(c + d*x)/(a + b*sec(c + d*x)), x)
```

Giac [A]
time = 0.50, size = 38, normalized size = 1.12

$$-\frac{\cos\left(dx+c\right)}{ad} + \frac{b\log\left(\left|-a\cos\left(dx+c\right)-b\right|\right)}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="giac")
[Out] -cos(d*x + c)/(a*d) + b*log(abs(-a*cos(d*x + c) - b))/(a^2*d)
```

Mupad [B]
time = 0.06, size = 30, normalized size = 0.88

$$\frac{b\,\ln\left(b+a\,\cos\left(c+d\,x\right)\right)-a\,\cos\left(c+d\,x\right)}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(a + b/cos(c + d*x)),x)
[Out] (b*log(b + a*cos(c + d*x)) - a*cos(c + d*x))/(a^2*d)
```

3.200 $\int \frac{\csc(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=74

$$\frac{\log(1-\cos(c+dx))}{2(a+b)d}-\frac{\log(1+\cos(c+dx))}{2(a-b)d}+\frac{b\log(b+a\cos(c+dx))}{(a^2-b^2)\,d}$$

```
[Out] 1/2*ln(1-cos(d*x+c))/(a+b)/d-1/2*ln(1+cos(d*x+c))/(a-b)/d+b*ln(b+a*cos(d*x+
c))/(a^2-b^2)/d
```

Rubi [A]
time = 0.07, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3957, 2800, 815}

$$\frac{b\log(a\cos(c+dx)+b)}{d\,(a^2-b^2)}+\frac{\log(1-\cos(c+dx))}{2d(a+b)}-\frac{\log(\cos(c+dx)+1)}{2d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a + b*Sec[c + d*x]),x]
```

```
[Out] Log[1 - Cos[c + d*x]]/(2*(a + b)*d) - Log[1 + Cos[c + d*x]]/(2*(a - b)*d) +
 (b*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)*d)
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\csc(c+dx)}{a+b\sec(c+dx)}\,dx = -\int \frac{\cot(c+dx)}{-b-a\cos(c+dx)}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{x}{(-b+x)(a^2-x^2)}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{2(a-b)(a-x)} - \frac{b}{(a-b)(a+b)(b-x)} + \frac{1}{2(a+b)(a+x)}\right)\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{\log(1-\cos(c+dx))}{2(a+b)d} - \frac{\log(1+\cos(c+dx))}{2(a-b)d} + \frac{b\log(b+a\cos(c+dx))}{(a^2-b^2)\,d}$$

Mathematica [A]
time = 0.07, size = 63, normalized size = 0.85

$$\frac{-\left((a+b)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)+b\log(b+a\cos(c+dx))+(a-b)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{(a-b)(a+b)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-((a + b)*Log[Cos[(c + d*x)/2]]) + b*Log[b + a*Cos[c + d*x]] + (a - b)*Log
[Sin[(c + d*x)/2]])/((a - b)*(a + b)*d)
```

Maple [A]
time = 0.10, size = 70, normalized size = 0.95

method	result
derivativedivides	$\frac{-\frac{\ln(1+\cos(dx+c))}{2a-2b}+\frac{\ln(-1+\cos(dx+c))}{2a+2b}+\frac{b\ln(b+a\cos(dx+c))}{(a+b)(a-b)}}{d}$
default	$\frac{-\frac{\ln(1+\cos(dx+c))}{2a-2b}+\frac{\ln(-1+\cos(dx+c))}{2a+2b}+\frac{b\ln(b+a\cos(dx+c))}{(a+b)(a-b)}}{d}$
norman	$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d(a+b)}+\frac{b\ln\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)}{d(a^2-b^2)}$
risch	$-\frac{ix}{a+b}-\frac{ic}{d(a+b)}+\frac{ix}{a-b}+\frac{ic}{d(a-b)}-\frac{2ibx}{a^2-b^2}-\frac{2ibc}{d(a^2-b^2)}+\frac{\ln(e^{i(dx+c)}-1)}{d(a+b)}-\frac{\ln(e^{i(dx+c)}+1)}{d(a-b)}+\frac{b\ln\left(e^{2i(d\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/(2*a-2*b)*ln(1+cos(d*x+c))+1/(2*a+2*b)*ln(-1+cos(d*x+c))+b/(a+b)/(a
-b)*ln(b+a*cos(d*x+c)))
```

Maxima [A]
time = 0.28, size = 64, normalized size = 0.86

$$\frac{\frac{2\,b\log\left(a\cos\left(dx+c\right)+b\right)}{a^2-b^2}-\frac{\log\left(\cos\left(dx+c\right)+1\right)}{a-b}+\frac{\log\left(\cos\left(dx+c\right)-1\right)}{a+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*log(a*cos(d*x + c) + b)/(a^2 - b^2) - log(cos(d*x + c) + 1)/(a - b
) + log(cos(d*x + c) - 1)/(a + b))/d
```

Fricas [A]
time = 2.76, size = 64, normalized size = 0.86

$$\frac{2\,b\log\left(a\cos\left(dx+c\right)+b\right)-\left(a+b\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+\left(a-b\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\left(a^2-b^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*log(a*cos(d*x + c) + b) - (a + b)*log(1/2*cos(d*x + c) + 1/2) + (a
 - b)*log(-1/2*cos(d*x + c) + 1/2))/((a^2 - b^2)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc\left(c+dx\right)}{a+b\sec\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)/(a + b*sec(c + d*x)), x)
```

Giac [A]
time = 0.48, size = 100, normalized size = 1.35

$$\frac{\frac{2\,b\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^2-b^2}+\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*b*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(
d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^2 - b^2) + log(abs(-cos(d*x + c) + 1)
/abs(cos(d*x + c) + 1))/(a + b))/d
```

Mupad [B]
time = 1.15, size = 68, normalized size = 0.92

$$\frac{\ln\left(\cos\left(c+d\,x\right)-1\right)}{2\,d\,\left(a+b\right)}-\frac{\ln\left(\cos\left(c+d\,x\right)+1\right)}{2\,d\,\left(a-b\right)}+\frac{b\,\ln\left(b+a\,\cos\left(c+d\,x\right)\right)}{d\,\left(a^2-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)*(a + b/cos(c + d*x))),x)
```

```
[Out] log(cos(c + d*x) - 1)/(2*d*(a + b)) - log(cos(c + d*x) + 1)/(2*d*(a - b)) +
 (b*log(b + a*cos(c + d*x)))/(d*(a^2 - b^2))
```

3.201 $\int \frac{\csc^3(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=116

$$\frac{(b-a\cos(c+dx))\csc^2(c+dx)}{2(a^2-b^2)\,d}+\frac{a\log(1-\cos(c+dx))}{4(a+b)^2d}-\frac{a\log(1+\cos(c+dx))}{4(a-b)^2d}+\frac{a^2b\log(b+a\cos(c+dx))}{(a^2-b^2)^2\,d}$$

```
[Out] 1/2*(b-a*cos(d*x+c))*csc(d*x+c)^2/(a^2-b^2)/d+1/4*a*ln(1-cos(d*x+c))/(a+b)^
2/d-1/4*a*ln(1+cos(d*x+c))/(a-b)^2/d+a^2*b*ln(b+a*cos(d*x+c))/(a^2-b^2)^2/d
```

Rubi [A]
time = 0.15, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3957, 2916, 12, 837, 815}

$$\frac{a^2b\log(a\cos(c+dx)+b)}{d\,(a^2-b^2)^2}+\frac{\csc^2(c+dx)(b-a\cos(c+dx))}{2d\,(a^2-b^2)}+\frac{a\log(1-\cos(c+dx))}{4d(a+b)^2}-\frac{a\log(\cos(c+dx)+1)}{4d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((b - a*Cos[c + d*x])*Csc[c + d*x]^2)/(2*(a^2 - b^2)*d) + (a*Log[1 - Cos[c
+ d*x]])/(4*(a + b)^2*d) - (a*Log[1 + Cos[c + d*x]])/(4*(a - b)^2*d) + (a^2
*b*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^3(c+dx)}{a+b\sec(c+dx)}\,dx &= -\int \frac{\cot(c+dx)\csc^2(c+dx)}{-b-a\cos(c+dx)}\,dx \\
&= \frac{a^3\text{Subst}\left(\int \frac{x}{a(-b+x)(a^2-x^2)^2}\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= \frac{a^2\text{Subst}\left(\int \frac{x}{(-b+x)(a^2-x^2)^2}\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= \frac{(b-a\cos(c+dx))\csc^2(c+dx)}{2(a^2-b^2)\,d} + \frac{\text{Subst}\left(\int \frac{a^2b+a^2x}{(-b+x)(a^2-x^2)}\,dx, x, -a\cos(c+dx)\right)}{2(a^2-b^2)\,d} \\
&= \frac{(b-a\cos(c+dx))\csc^2(c+dx)}{2(a^2-b^2)\,d} + \frac{\text{Subst}\left(\int \left(\frac{a(a+b)}{2(a-b)(a-x)} - \frac{2a^2b}{(a-b)(a+b)(b-x)} + \frac{a(a-}{2(a+b)(}\right.\right.}{2(a^2-b^2)\,d} \\
&= \frac{(b-a\cos(c+dx))\csc^2(c+dx)}{2(a^2-b^2)\,d} + \frac{a\log(1-\cos(c+dx))}{4(a+b)^2d} - \frac{a\log(1+\cos(c+dx}{4(a-b)^2d}
\end{aligned}$$

Mathematica [A]
time = 0.40, size = 123, normalized size = 1.06

$$\frac{-(a-b)^2(a+b)\csc^2\left(\frac{1}{2}(c+dx)\right)-4a\left((a+b)^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-2ab\log(b+a\cos(c+dx))-(a-b)^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+(a-b)(a+b)^2\sec^2\left(\frac{1}{2}(c+dx)\right)}{8(a-b)^2(a+b)^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-((a - b)^2*(a + b)*Csc[(c + d*x)/2]^2) - 4*a*((a + b)^2*Log[Cos[(c + d*x)
/2]] - 2*a*b*Log[b + a*Cos[c + d*x]] - (a - b)^2*Log[Sin[(c + d*x)/2]]) + (
a - b)*(a + b)^2*Sec[(c + d*x)/2]^2)/(8*(a - b)^2*(a + b)^2*d)
```

Maple [A]
time = 0.15, size = 110, normalized size = 0.95

method	result
derivativedivides	$\frac{\frac{1}{(4a-4b)(1+\cos(dx+c))}-\frac{a\ln(1+\cos(dx+c))}{4(a-b)^2}+\frac{1}{(4a+4b)(-1+\cos(dx+c))}+\frac{a\ln(-1+\cos(dx+c))}{4(a+b)^2}+\frac{b\,a^2\ln(b+a\cos(dx+c))}{(a+b)^2(a-b)^2}}{d}$
default	$\frac{\frac{1}{(4a-4b)(1+\cos(dx+c))}-\frac{a\ln(1+\cos(dx+c))}{4(a-b)^2}+\frac{1}{(4a+4b)(-1+\cos(dx+c))}+\frac{a\ln(-1+\cos(dx+c))}{4(a+b)^2}+\frac{b\,a^2\ln(b+a\cos(dx+c))}{(a+b)^2(a-b)^2}}{d}$
norman	$\frac{-\frac{1}{8d(a+b)}+\frac{\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d(a-b)}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{b\,a^2\ln\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)}{d(a^4-2b^2a^2+b^4)}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d(a^2+2ba+b^2)}$
risch	$-\frac{iax}{2(a^2+2ba+b^2)}-\frac{iac}{2d(a^2+2ba+b^2)}+\frac{iax}{2a^2-4ba+2b^2}+\frac{iac}{2d(a^2-2ba+b^2)}-\frac{2ib\,a^2x}{a^4-2b^2a^2+b^4}-\frac{2ib\,a^2c}{d(a^4-2b^2a^2+b^4)}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/(4*a-4*b)/(1+cos(d*x+c))-1/4*a/(a-b)^2*ln(1+cos(d*x+c))+1/(4*a+4*b)/
(-1+cos(d*x+c))+1/4*a/(a+b)^2*ln(-1+cos(d*x+c))+b*a^2/(a+b)^2/(a-b)^2*ln(b+
a*cos(d*x+c)))
```

Maxima [A]
time = 0.27, size = 132, normalized size = 1.14

$$\frac{\frac{4\,a^2b\log(a\cos(dx+c)+b)}{a^4-2\,a^2b^2+b^4}-\frac{a\log(\cos(dx+c)+1)}{a^2-2\,ab+b^2}+\frac{a\log(\cos(dx+c)-1)}{a^2+2\,ab+b^2}+\frac{2\,(a\cos(dx+c)-b)}{(a^2-b^2)\cos(dx+c)^2-a^2+b^2}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*a^2*b*log(a*cos(d*x + c) + b)/(a^4 - 2*a^2*b^2 + b^4) - a*log(cos(d*
x + c) + 1)/(a^2 - 2*a*b + b^2) + a*log(cos(d*x + c) - 1)/(a^2 + 2*a*b + b^
2) + 2*(a*cos(d*x + c) - b)/((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2))/d
```

Fricas [A]
time = 2.50, size = 216, normalized size = 1.86

$$-\frac{2\,a^2b-2\,b^3-2\,(a^3-ab^2)\cos(dx+c)-4\,\left(a^2b\cos(dx+c)^2-a^2b\right)\log(a\cos(dx+c)+b)-\left(a^3+2\,a^2b+ab^2-(a^3+2\,a^2b+ab^2)\cos(dx+c)^2\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+\left(a^3-2\,a^2b+ab^2-(a^3-2\,a^2b+ab^2)\cos(dx+c)^2\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)}{4\,\left((a^4-2\,a^2b^2+b^4)d\cos(dx+c)^2-(a^4-2\,a^2b^2+b^4)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^2*b - 2*b^3 - 2*(a^3 - a*b^2)*cos(d*x + c) - 4*(a^2*b*cos(d*x + c
)^2 - a^2*b)*log(a*cos(d*x + c) + b) - (a^3 + 2*a^2*b + a*b^2 - (a^3 + 2*a^
```

```
2*b + a*b^2)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) + (a^3 - 2*a^2*b +
 a*b^2 - (a^3 - 2*a^2*b + a*b^2)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/
2))/((a^4 - 2*a^2*b^2 + b^4)*d*cos(d*x + c)^2 - (a^4 - 2*a^2*b^2 + b^4)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**3/(a + b*sec(c + d*x)), x)
```

Giac [A]
time = 0.48, size = 202, normalized size = 1.74

$$\frac{\frac{8\,a^2b\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^4-2\,a^2b^2+b^4}+\frac{2\,a\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^2+2\,ab+b^2}+\frac{\left(a+b-\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{(a^2+2\,ab+b^2)(\cos(dx+c)-1)}-\frac{\cos(dx+c)-1}{(a-b)(\cos(dx+c)+1)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(8*a^2*b*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(
cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^4 - 2*a^2*b^2 + b^4) + 2*a*log(ab
s(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^2 + 2*a*b + b^2) + (a + b -
2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))*(cos(d*x + c) + 1)/((a^2 + 2*a*b
 + b^2)*(cos(d*x + c) - 1)) - (cos(d*x + c) - 1)/((a - b)*(cos(d*x + c) + 1
)))/d
```

Mupad [B]
time = 0.30, size = 133, normalized size = 1.15

$$\frac{a\,\ln\left(\cos\left(c+d\,x\right)-1\right)}{4\,d\,{\left(a+b\right)}^2}-\frac{\ln\left(b+a\,\cos\left(c+d\,x\right)\right)\,\left(\frac{a}{4\,{\left(a+b\right)}^2}-\frac{a}{4\,{\left(a-b\right)}^2}\right)}{d}-\frac{\frac{b}{2\,\left(a^2-b^2\right)}-\frac{a\,\cos\left(c+d\,x\right)}{2\,\left(a^2-b^2\right)}}{d\,\left({\cos\left(c+d\,x\right)}^2-1\right)}-\frac{a\,\ln\left(\cos\left(c+d\,x\right)+1\right)}{4\,d\,{\left(a-b\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^3*(a + b/cos(c + d*x))),x)
```

```
[Out] (a*log(cos(c + d*x) - 1))/(4*d*(a + b)^2) - (log(b + a*cos(c + d*x))*(a/(4*
(a + b)^2) - a/(4*(a - b)^2)))/d - (b/(2*(a^2 - b^2)) - (a*cos(c + d*x))/(2
*(a^2 - b^2)))/(d*(cos(c + d*x)^2 - 1)) - (a*log(cos(c + d*x) + 1))/(4*d*(a
 - b)^2)
```

3.202 $\int \frac{\csc^5(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=179

$$\frac{(4a^2b-a(3a^2+b^2)\cos(c+dx))\csc^2(c+dx)}{8(a^2-b^2)^2 d}+\frac{(b-a\cos(c+dx))\csc^4(c+dx)}{4(a^2-b^2)d}+\frac{a(3a+b)\log(1-\cos(c+d}{16(a+b)^3 d}$$

```
[Out] 1/8*(4*a^2*b-a*(3*a^2+b^2)*cos(d*x+c))*csc(d*x+c)^2/(a^2-b^2)^2/d+1/4*(b-a*
cos(d*x+c))*csc(d*x+c)^4/(a^2-b^2)/d+1/16*a*(3*a+b)*ln(1-cos(d*x+c))/(a+b)^
3/d-1/16*a*(3*a-b)*ln(1+cos(d*x+c))/(a-b)^3/d+a^4*b*ln(b+a*cos(d*x+c))/(a^2
-b^2)^3/d
```

Rubi [A]
time = 0.22, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3957, 2916, 12, 837, 815}

$$\frac{\csc^4(c+dx)(b-a\cos(c+dx))}{4d(a^2-b^2)}+\frac{\csc^2(c+dx)\left(4a^2b-a(3a^2+b^2)\cos(c+dx)\right)}{8d(a^2-b^2)^2}+\frac{a^4b\log(a\cos(c+dx)+b)}{d(a^2-b^2)^3}+\frac{a(3a+b)\log(1-\cos(c+dx))}{16d(a+b)^3}-\frac{a(3a-b)\log(\cos(c+dx)+1)}{16d(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((4*a^2*b - a*(3*a^2 + b^2)*Cos[c + d*x])*Csc[c + d*x]^2)/(8*(a^2 - b^2)^2*
d) + ((b - a*Cos[c + d*x])*Csc[c + d*x]^4)/(4*(a^2 - b^2)*d) + (a*(3*a + b)
*Log[1 - Cos[c + d*x]])/(16*(a + b)^3*d) - (a*(3*a - b)*Log[1 + Cos[c + d*x
]])/(16*(a - b)^3*d) + (a^4*b*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
```

```
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc^5(c+dx)}{a+b\sec(c+dx)}\,dx &= -\int \frac{\cot(c+dx)\csc^4(c+dx)}{-b-a\cos(c+dx)}\,dx\\
&= \frac{a^5\text{Subst}\left(\int \frac{x}{a(-b+x)(a^2-x^2)^3}\,dx, x, -a\cos(c+dx)\right)}{d}\\
&= \frac{a^4\text{Subst}\left(\int \frac{x}{(-b+x)(a^2-x^2)^3}\,dx, x, -a\cos(c+dx)\right)}{d}\\
&= \frac{(b-a\cos(c+dx))\csc^4(c+dx)}{4(a^2-b^2)d} + \frac{a^2\text{Subst}\left(\int \frac{a^2b+3a^2x}{(-b+x)(a^2-x^2)^2}\,dx, x, -a\cos(c+dx)\right)}{4(a^2-b^2)d}\\
&= \frac{(4a^2b-a(3a^2+b^2)\cos(c+dx))\csc^2(c+dx)}{8(a^2-b^2)^2d} + \frac{(b-a\cos(c+dx))\csc^4(c+dx)}{4(a^2-b^2)d}\\
&= \frac{(4a^2b-a(3a^2+b^2)\cos(c+dx))\csc^2(c+dx)}{8(a^2-b^2)^2d} + \frac{(b-a\cos(c+dx))\csc^4(c+dx)}{4(a^2-b^2)d}\\
&= \frac{(4a^2b-a(3a^2+b^2)\cos(c+dx))\csc^2(c+dx)}{8(a^2-b^2)^2d} + \frac{(b-a\cos(c+dx))\csc^4(c+dx)}{4(a^2-b^2)d}
\end{aligned}
$$

Mathematica [A]
time = 3.06, size = 207, normalized size = 1.16

$$
\frac{-2(a-b)^3\left(3a^2+4ab+b^2\right)\csc^2\left(\frac{1}{2}(c+dx)\right)-(a-b)^3(a+b)^2\csc^4\left(\frac{1}{2}(c+dx)\right)+8a\left(-\left((3a-b)(a+b)^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)+8a^3b\log(b+a\cos(c+dx))+(a-b)^3(3a+b)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+2(a+b)^3\left(3a^2-4ab+b^2\right)\sec^2\left(\frac{1}{2}(c+dx)\right)+(a-b)^2(a+b)^3\sec^4\left(\frac{1}{2}(c+dx)\right)}{64(a-b)^3(a+b)^3d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^5/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-2*(a - b)^3*(3*a^2 + 4*a*b + b^2)*Csc[(c + d*x)/2]^2 - (a - b)^3*(a + b)^
2*Csc[(c + d*x)/2]^4 + 8*a*(-((3*a - b)*(a + b)^3*Log[Cos[(c + d*x)/2]]) +
8*a^3*b*Log[b + a*Cos[c + d*x]] + (a - b)^3*(3*a + b)*Log[Sin[(c + d*x)/2]]
) + 2*(a + b)^3*(3*a^2 - 4*a*b + b^2)*Sec[(c + d*x)/2]^2 + (a - b)^2*(a + b
)^3*Sec[(c + d*x)/2]^4)/(64*(a - b)^3*(a + b)^3*d)
```

Maple [A]

time = 0.19, size = 172, normalized size = 0.96

method	result
derivativedivides	$\frac{\frac{1}{2(8a-8b)(1+\cos(dx+c))^2}-\frac{-3a+b}{16(a-b)^2(1+\cos(dx+c))}-\frac{(3a-b)a\ln(1+\cos(dx+c))}{16(a-b)^3}-\frac{1}{2(8a+8b)(-1+\cos(dx+c))^2}-\frac{-3a-b}{16(a+b)^2(-1+\cos(dx}}{d}$
default	$\frac{\frac{1}{2(8a-8b)(1+\cos(dx+c))^2}-\frac{-3a+b}{16(a-b)^2(1+\cos(dx+c))}-\frac{(3a-b)a\ln(1+\cos(dx+c))}{16(a-b)^3}-\frac{1}{2(8a+8b)(-1+\cos(dx+c))^2}-\frac{-3a-b}{16(a+b)^2(-1+\cos(dx}}{d}$
norman	$\frac{-\frac{1}{64d(a+b)}+\frac{\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)}{64d(a-b)}+\frac{(2a-b)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16d\left(a^2-2ba+b^2\right)}-\frac{(2a+b)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16d\left(a^2+2ba+b^2\right)}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}+\frac{a^4b\ln\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{d(a^6-3a^4b^2+3a^2b^4-b^6)}$
risch	$\frac{3ia^2c}{8d(a^3-3b\,a^2+3b^2a-b^3)}-\frac{iabx}{8(a^3+3b\,a^2+3b^2a+b^3)}-\frac{2ia^4bx}{a^6-3a^4b^2+3a^2b^4-b^6}-\frac{iabx}{8(a^3-3b\,a^2+3b^2a-b^3)}-\frac{3ia}{8(a^3+3b\,a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2/(8*a-8*b)/(1+cos(d*x+c))^2-1/16*(-3*a+b)/(a-b)^2/(1+cos(d*x+c))-1/
16*(3*a-b)*a/(a-b)^3*ln(1+cos(d*x+c))-1/2/(8*a+8*b)/(-1+cos(d*x+c))^2-1/16*
(-3*a-b)/(a+b)^2/(-1+cos(d*x+c))+1/16*(3*a+b)/(a+b)^3*a*ln(-1+cos(d*x+c))+b
*a^4/(a+b)^3/(a-b)^3*ln(b+a*cos(d*x+c)))
```

Maxima [A]

time = 0.27, size = 268, normalized size = 1.50

$$\frac{\frac{16\,a^4b\log(a\cos(dx+c)+b)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}-\frac{(3\,a^2-ab)\log(\cos(dx+c)+1)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{(3\,a^2+ab)\log(\cos(dx+c)-1)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{2\left(4\,a^2b\cos(dx+c)^2-(3\,a^3+ab^2)\cos(dx+c)^3-6\,a^2b+2\,b^3+(5\,a^3-ab^2)\cos(dx+c)\right)}{(a^4-2\,a^2b^2+b^4)\cos(dx+c)^4+a^4-2\,a^2b^2+b^4-2\,(a^4-2\,a^2b^2+b^4)\cos(dx+c)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(16*a^4*b*log(a*cos(d*x + c) + b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)
- (3*a^2 - a*b)*log(cos(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + (3*
a^2 + a*b)*log(cos(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 2*(4*a^2
*b*cos(d*x + c)^2 - (3*a^3 + a*b^2)*cos(d*x + c)^3 - 6*a^2*b + 2*b^3 + (5*a
```

```
^3 - a*b^2)*cos(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^4 + a^4 - 2
*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2))/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 469 vs. $2(172) = 344$.
time = 3.25, size = 469, normalized size = 2.62

$$\frac{12\,a^{4}b - 16\,a^{2}b^{3} + 4\,b^{5} + 2\,(3\,a^{5} - 2\,a^{3}b^{2} - ab^{4})\cos\left(dx+c\right)^{3} - 8\,(a^{4}b - a^{2}b^{3})\cos\left(dx+c\right)^{2} - 2\,(5\,a^{5} - 6\,a^{3}b^{2} + ab^{4})\cos\left(dx+c\right) + 16\,(a^{4}b\cos\left(dx+c\right)^{4} - 2\,a^{4}b\cos\left(dx+c\right)^{2} + a^{4}b)\log\left(a\cos\left(dx+c\right)+b\right) - (3\,a^{5} + 8\,a^{4}b + 6\,a^{3}b^{2} - ab^{4} + (3\,a^{5} + 8\,a^{4}b + 6\,a^{3}b^{2} - ab^{4})\cos\left(dx+c\right)^{4} - 2\,(3\,a^{5} + 8\,a^{4}b + 6\,a^{3}b^{2} - ab^{4})\cos\left(dx+c\right)^{2})\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right) + (3\,a^{5} - 8\,a^{4}b + 6\,a^{3}b^{2} - ab^{4} + (3\,a^{5} - 8\,a^{4}b + 6\,a^{3}b^{2} - ab^{4})\cos\left(dx+c\right)^{4} - 2\,(3\,a^{5} - 8\,a^{4}b + 6\,a^{3}b^{2} - ab^{4})\cos\left(dx+c\right)^{2})\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{16\,((a^{6} - 3\,a^{4}b^{2} + 3\,a^{2}b^{4} - b^{6})d\cos\left(dx+c\right)^{4} - 2\,(a^{6} - 3\,a^{4}b^{2} + 3\,a^{2}b^{4} - b^{6})d\cos\left(dx+c\right)^{2} + (a^{6} - 3\,a^{4}b^{2} + 3\,a^{2}b^{4} - b^{6})d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(12*a^4*b - 16*a^2*b^3 + 4*b^5 + 2*(3*a^5 - 2*a^3*b^2 - a*b^4)*cos(d*x
 + c)^3 - 8*(a^4*b - a^2*b^3)*cos(d*x + c)^2 - 2*(5*a^5 - 6*a^3*b^2 + a*b^4
)*cos(d*x + c) + 16*(a^4*b*cos(d*x + c)^4 - 2*a^4*b*cos(d*x + c)^2 + a^4*b)
*log(a*cos(d*x + c) + b) - (3*a^5 + 8*a^4*b + 6*a^3*b^2 - a*b^4 + (3*a^5 +
8*a^4*b + 6*a^3*b^2 - a*b^4)*cos(d*x + c)^4 - 2*(3*a^5 + 8*a^4*b + 6*a^3*b^
2 - a*b^4)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) + (3*a^5 - 8*a^4*b +
 6*a^3*b^2 - a*b^4 + (3*a^5 - 8*a^4*b + 6*a^3*b^2 - a*b^4)*cos(d*x + c)^4 -
 2*(3*a^5 - 8*a^4*b + 6*a^3*b^2 - a*b^4)*cos(d*x + c)^2)*log(-1/2*cos(d*x +
 c) + 1/2))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x + c)^4 - 2*(a^6
- 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x + c)^2 + (a^6 - 3*a^4*b^2 + 3*a^2*
b^4 - b^6)*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{5}\left(c+dx\right)}{a+b\sec\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**5/(a + b*sec(c + d*x)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 419 vs. $2(172) = 344$.
time = 0.48, size = 419, normalized size = 2.34

$$\frac{\frac{64\,a^{4}b\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^{6}-3\,a^{4}b^{2}+3\,a^{2}b^{4}-b^{6}} + \frac{4\,(3\,a^{2}+ab)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}} - \frac{\frac{8\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{4\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{a(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{b(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}}{a^{2}-2\,ab+b^{2}} - \frac{\left(a^{2}+2\,ab+b^{2}-\frac{8\,a^{2}(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{12\,ab(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{4\,b^{2}(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{18\,a^{2}(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{6\,ab(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}\right)(\cos(dx+c)+1)^{2}}{(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3})(\cos(dx+c)-1)^{2}}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/64*(64*a^4*b*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b
*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6
```

```
) + 4*(3*a^2 + a*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^3
+ 3*a^2*b + 3*a*b^2 + b^3) - (8*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 4
*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - a*(cos(d*x + c) - 1)^2/(cos(d*x
+ c) + 1)^2 + b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/(a^2 - 2*a*b + b
^2) - (a^2 + 2*a*b + b^2 - 8*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 12
*a*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 4*b^2*(cos(d*x + c) - 1)/(cos(
d*x + c) + 1) + 18*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 6*a*b*(c
os(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)^2/((a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*(cos(d*x + c) - 1)^2))/d
```

Mupad [B]
time = 1.61, size = 297, normalized size = 1.66

$$\frac{\frac{3\,a^2\,b-b^3}{4\,{\left(a^2-b^2\right)}^2}+\frac{{\cos\left(c+d\,x\right)}^3\,\left(3\,a^3+a\,b^2\right)}{8\,\left(a^4-2\,a^2\,b^2+b^4\right)}-\frac{a^2\,b\,{\cos\left(c+d\,x\right)}^2}{2\,{\left(a^2-b^2\right)}^2}-\frac{a\,\cos\left(c+d\,x\right)\,\left(5\,a^2-b^2\right)}{8\,\left(a^4-2\,a^2\,b^2+b^4\right)}}{d\,\left({\cos\left(c+d\,x\right)}^4-2\,{\cos\left(c+d\,x\right)}^2+1\right)}+\frac{\ln\left(\cos\left(c+d\,x\right)-1\right)\,\left(\frac{3}{16\,\left(a+b\right)}-\frac{5\,b}{16\,{\left(a+b\right)}^2}+\frac{b^2}{8\,{\left(a+b\right)}^3}\right)}{d}-\frac{\ln\left(\cos\left(c+d\,x\right)+1\right)\,\left(\frac{b^2}{8\,{\left(a-b\right)}^3}+\frac{5\,b}{16\,{\left(a-b\right)}^2}+\frac{3}{16\,\left(a-b\right)}\right)}{d}+\frac{a^4\,b\,\ln\left(b+a\,\cos\left(c+d\,x\right)\right)}{d\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^5*(a + b/cos(c + d*x))),x)
```

```
[Out] ((3*a^2*b - b^3)/(4*(a^2 - b^2)^2) + (cos(c + d*x)^3*(a*b^2 + 3*a^3))/(8*(a
^4 + b^4 - 2*a^2*b^2)) - (a^2*b*cos(c + d*x)^2)/(2*(a^2 - b^2)^2) - (a*cos(
c + d*x)*(5*a^2 - b^2))/(8*(a^4 + b^4 - 2*a^2*b^2)))/(d*(cos(c + d*x)^4 - 2
*cos(c + d*x)^2 + 1)) + (log(cos(c + d*x) - 1)*(3/(16*(a + b)) - (5*b)/(16*
(a + b)^2) + b^2/(8*(a + b)^3)))/d - (log(cos(c + d*x) + 1)*(b^2/(8*(a - b)
^3) + (5*b)/(16*(a - b)^2) + 3/(16*(a - b))))/d + (a^4*b*log(b + a*cos(c +
d*x)))/(d*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))
```

3.203 $\int \frac{\sin^6(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=230

$$\frac{(5a^6-30a^4b^2+40a^2b^4-16b^6)\,x}{16a^7}-\frac{2(a-b)^{5/2}b(a+b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^7d}+\frac{\left(16b(a^2-b^2)^2-\right.}{}$$

```
[Out] 1/16*(5*a^6-30*a^4*b^2+40*a^2*b^4-16*b^6)*x/a^7-2*(a-b)^(5/2)*b*(a+b)^(5/2)
*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^7/d+1/16*(16*b*(a^2-
b^2)^2-a*(5*a^4-14*a^2*b^2+8*b^4)*cos(d*x+c))*sin(d*x+c)/a^6/d+1/24*(8*b*(a
^2-b^2)-a*(5*a^2-6*b^2)*cos(d*x+c))*sin(d*x+c)^3/a^4/d+1/30*(6*b-5*a*cos(d*
x+c))*sin(d*x+c)^5/a^2/d
```

Rubi [A]

time = 0.41, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3957, 2944, 2814, 2738, 214}

$$-\frac{2b(a-b)^{5/2}(a+b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^7d}+\frac{\sin^5(c+dx)(6b-5a\cos(c+dx))}{30a^2d}+\frac{\sin^3(c+dx)\left(8b(a^2-b^2)-a(5a^2-6b^2)\cos(c+dx)\right)}{24a^4d}+\frac{\sin(c+dx)\left(16b(a^2-b^2)^2-a(5a^4-14a^2b^2+8b^4)\cos(c+dx)\right)}{16a^6d}+\frac{x(5a^6-30a^4b^2+40a^2b^4-16b^6)}{16a^7}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^6/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((5*a^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*x)/(16*a^7) - (2*(a - b)^(5/2)*
b*(a + b)^(5/2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^7*d
) + ((16*b*(a^2 - b^2)^2 - a*(5*a^4 - 14*a^2*b^2 + 8*b^4)*Cos[c + d*x])*Sin
[c + d*x])/(16*a^6*d) + ((8*b*(a^2 - b^2) - a*(5*a^2 - 6*b^2)*Cos[c + d*x])
*Sin[c + d*x]^3)/(24*a^4*d) + ((6*b - 5*a*Cos[c + d*x])*Sin[c + d*x]^5)/(30
*a^2*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
```

```
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin^6(c+dx)}{a+b\sec(c+dx)}\,dx = -\int \frac{\cos(c+dx)\sin^6(c+dx)}{-b-a\cos(c+dx)}\,dx$$

$$= \frac{(6b-5a\cos(c+dx))\sin^5(c+dx)}{30a^2d} - \frac{\int \frac{(-ab+(5a^2-6b^2)\cos(c+dx))\sin^4(c+dx)}{-b-a\cos(c+dx)}\,dx}{6a^2}$$

$$= \frac{(8b(a^2-b^2)-a(5a^2-6b^2)\cos(c+dx))\sin^3(c+dx)}{24a^4d} + \frac{(6b-5a\cos(c+dx))\sin^5(}{30a^2d}$$

$$= \frac{\left(16b(a^2-b^2)^2-a(5a^4-14a^2b^2+8b^4)\cos(c+dx)\right)\sin(c+dx)}{16a^6d} + \frac{(8b(a^2-b^2)-}{}$$

$$= \frac{(5a^6-30a^4b^2+40a^2b^4-16b^6)\,x}{16a^7} + \frac{\left(16b(a^2-b^2)^2-a(5a^4-14a^2b^2+8b^4)\cos(c\right.}{16a^6d}$$

$$= \frac{(5a^6-30a^4b^2+40a^2b^4-16b^6)\,x}{16a^7} + \frac{\left(16b(a^2-b^2)^2-a(5a^4-14a^2b^2+8b^4)\cos(c\right.}{16a^6d}$$

$$= \frac{(5a^6-30a^4b^2+40a^2b^4-16b^6)\,x}{16a^7} - \frac{2(a-b)^{5/2}b(a+b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tan(\frac{1}{2}(c}{\sqrt{a+b}}\right.}{a^7d}$$

Mathematica [A]

time = 1.57, size = 268, normalized size = 1.17

$$\frac{300a^6c - 1800a^4b^2c + 2400a^2b^4c - 960b^6c + 300a^6dx - 1800a^4b^2dx + 2400a^2b^4dx - 960b^6dx + 1920b(a^2-b^2)^{5/2}\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right) + 120ab(11a^4 - 18a^2b^2 + 8b^4)\sin(c+dx) - 15(15a^6 - 32a^4b^2 + 16a^2b^4)\sin(2(c+dx)) - 140a^5b\sin(3(c+dx)) + 80a^3b^3\sin(3(c+dx)) + 45a^6\sin(4(c+dx)) - 30a^4b^2\sin(4(c+dx)) + 12a^5b\sin(5(c+dx)) - 5a^6\sin(6(c+dx))}{960a^7d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^6/(a + b*Sec[c + d*x]),x]
```

```
[Out] (300*a^6*c - 1800*a^4*b^2*c + 2400*a^2*b^4*c - 960*b^6*c + 300*a^6*d*x - 18
00*a^4*b^2*d*x + 2400*a^2*b^4*d*x - 960*b^6*d*x + 1920*b*(a^2 - b^2)^(5/2)*
ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] + 120*a*b*(11*a^4 - 18
*a^2*b^2 + 8*b^4)*Sin[c + d*x] - 15*(15*a^6 - 32*a^4*b^2 + 16*a^2*b^4)*Sin[
2*(c + d*x)] - 140*a^5*b*Sin[3*(c + d*x)] + 80*a^3*b^3*Sin[3*(c + d*x)] + 4
5*a^6*Sin[4*(c + d*x)] - 30*a^4*b^2*Sin[4*(c + d*x)] + 12*a^5*b*Sin[5*(c +
d*x)] - 5*a^6*Sin[6*(c + d*x)])/(960*a^7*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 440 vs. $2(213) = 426$.
time = 0.19, size = 441, normalized size = 1.92 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^6/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(a-b)^3*(a+b)^3*b/a^7/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x
+1/2*c)/((a+b)*(a-b))^(1/2))+2/a^7*(((5/16*a^6+a^5*b-7/8*a^4*b^2-2*a^3*b^3+
1/2*a^2*b^4+a*b^5)*tan(1/2*d*x+1/2*c)^11+(19/3*a^5*b-29/8*a^4*b^2+3/2*a^2*b
^4+5*a*b^5+85/48*a^6-34/3*a^3*b^3)*tan(1/2*d*x+1/2*c)^9+(86/5*a^5*b-11/4*a^
4*b^2-24*a^3*b^3+a^2*b^4+10*a*b^5+33/8*a^6)*tan(1/2*d*x+1/2*c)^7+(-33/8*a^6
+11/4*a^4*b^2-a^2*b^4+86/5*a^5*b-24*a^3*b^3+10*a*b^5)*tan(1/2*d*x+1/2*c)^5+
(19/3*a^5*b+29/8*a^4*b^2-34/3*a^3*b^3-3/2*a^2*b^4+5*a*b^5-85/48*a^6)*tan(1/
2*d*x+1/2*c)^3+(a^5*b-2*a^3*b^3+a*b^5-5/16*a^6+7/8*a^4*b^2-1/2*a^2*b^4)*tan
(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^6+1/16*(5*a^6-30*a^4*b^2+40*a^2*b
^4-16*b^6)*arctan(tan(1/2*d*x+1/2*c))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 3.15, size = 553, normalized size = 2.40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/240*(15*(5*a^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*d*x + 120*(a^4*b - 2*
a^2*b^3 + b^5)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(
d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 -
b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - (40*a^6*cos(d*x + c
)^5 - 48*a^5*b*cos(d*x + c)^4 - 368*a^5*b + 560*a^3*b^3 - 240*a*b^5 - 10*(1
3*a^6 - 6*a^4*b^2)*cos(d*x + c)^3 + 16*(11*a^5*b - 5*a^3*b^3)*cos(d*x + c)^
2 + 15*(11*a^6 - 18*a^4*b^2 + 8*a^2*b^4)*cos(d*x + c))*sin(d*x + c))/(a^7*d
), 1/240*(15*(5*a^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*d*x - 240*(a^4*b -
2*a^2*b^3 + b^5)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c)
+ a)/((a^2 - b^2)*sin(d*x + c))) - (40*a^6*cos(d*x + c)^5 - 48*a^5*b*cos(d*
x + c)^4 - 368*a^5*b + 560*a^3*b^3 - 240*a*b^5 - 10*(13*a^6 - 6*a^4*b^2)*co
s(d*x + c)^3 + 16*(11*a^5*b - 5*a^3*b^3)*cos(d*x + c)^2 + 15*(11*a^6 - 18*a
^4*b^2 + 8*a^2*b^4)*cos(d*x + c))*sin(d*x + c))/(a^7*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^6(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**6/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)**6/(a + b*sec(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 781 vs. 2(212) = 424.
time = 0.49, size = 781, normalized size = 3.40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/240*(15*(5*a^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*(d*x + c)/a^7 - 480*(a
^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-
2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt
```

```
(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*a^7) + 2*(75*a^5*tan(1/2*d*x + 1/2*c)^11 +
 240*a^4*b*tan(1/2*d*x + 1/2*c)^11 - 210*a^3*b^2*tan(1/2*d*x + 1/2*c)^11 -
480*a^2*b^3*tan(1/2*d*x + 1/2*c)^11 + 120*a*b^4*tan(1/2*d*x + 1/2*c)^11 + 2
40*b^5*tan(1/2*d*x + 1/2*c)^11 + 425*a^5*tan(1/2*d*x + 1/2*c)^9 + 1520*a^4*
b*tan(1/2*d*x + 1/2*c)^9 - 870*a^3*b^2*tan(1/2*d*x + 1/2*c)^9 - 2720*a^2*b^
3*tan(1/2*d*x + 1/2*c)^9 + 360*a*b^4*tan(1/2*d*x + 1/2*c)^9 + 1200*b^5*tan(
1/2*d*x + 1/2*c)^9 + 990*a^5*tan(1/2*d*x + 1/2*c)^7 + 4128*a^4*b*tan(1/2*d*
x + 1/2*c)^7 - 660*a^3*b^2*tan(1/2*d*x + 1/2*c)^7 - 5760*a^2*b^3*tan(1/2*d*
x + 1/2*c)^7 + 240*a*b^4*tan(1/2*d*x + 1/2*c)^7 + 2400*b^5*tan(1/2*d*x + 1/
2*c)^7 - 990*a^5*tan(1/2*d*x + 1/2*c)^5 + 4128*a^4*b*tan(1/2*d*x + 1/2*c)^5
 + 660*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 - 5760*a^2*b^3*tan(1/2*d*x + 1/2*c)^5
 - 240*a*b^4*tan(1/2*d*x + 1/2*c)^5 + 2400*b^5*tan(1/2*d*x + 1/2*c)^5 - 425
*a^5*tan(1/2*d*x + 1/2*c)^3 + 1520*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 870*a^3*b
^2*tan(1/2*d*x + 1/2*c)^3 - 2720*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 - 360*a*b^4
*tan(1/2*d*x + 1/2*c)^3 + 1200*b^5*tan(1/2*d*x + 1/2*c)^3 - 75*a^5*tan(1/2*
d*x + 1/2*c) + 240*a^4*b*tan(1/2*d*x + 1/2*c) + 210*a^3*b^2*tan(1/2*d*x + 1
/2*c) - 480*a^2*b^3*tan(1/2*d*x + 1/2*c) - 120*a*b^4*tan(1/2*d*x + 1/2*c) +
 240*b^5*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^6*a^6))/d
```

Mupad [B]
time = 3.88, size = 2500, normalized size = 10.87

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6/(a + b/cos(c + d*x)),x)
```

```
[Out] (atan(((((((42*a^21*b - 10*a^22 + 32*a^14*b^8 - 48*a^15*b^7 - 80*a^16*b^6 +
 140*a^17*b^5 + 52*a^18*b^4 - 134*a^19*b^3 + 6*a^20*b^2)/a^18 - (tan(c/2 +
(d*x)/2)*(512*a^16*b + 512*a^14*b^3 - 1024*a^15*b^2)*(a^6*5i - b^6*16i + a^
2*b^4*40i - a^4*b^2*30i))/(128*a^19))*(a^6*5i - b^6*16i + a^2*b^4*40i - a^4
*b^2*30i))/(16*a^7) + (tan(c/2 + (d*x)/2)*(1024*a*b^14 - 75*a^14*b + 25*a^1
5 - 512*b^15 + 2048*a^2*b^13 - 5120*a^3*b^12 - 2560*a^4*b^11 + 10240*a^5*b^
10 - 10240*a^7*b^8 + 2540*a^8*b^7 + 5180*a^9*b^6 - 2064*a^10*b^5 - 1136*a^1
1*b^4 + 619*a^12*b^3 + 31*a^13*b^2))/(8*a^12))*(a^6*5i - b^6*16i + a^2*b^4*
40i - a^4*b^2*30i)*1i)/(16*a^7) - (((((42*a^21*b - 10*a^22 + 32*a^14*b^8 -
48*a^15*b^7 - 80*a^16*b^6 + 140*a^17*b^5 + 52*a^18*b^4 - 134*a^19*b^3 + 6*a
^20*b^2)/a^18 + (tan(c/2 + (d*x)/2)*(512*a^16*b + 512*a^14*b^3 - 1024*a^15*
b^2)*(a^6*5i - b^6*16i + a^2*b^4*40i - a^4*b^2*30i))/(128*a^19))*(a^6*5i -
b^6*16i + a^2*b^4*40i - a^4*b^2*30i))/(16*a^7) - (tan(c/2 + (d*x)/2)*(1024*
a*b^14 - 75*a^14*b + 25*a^15 - 512*b^15 + 2048*a^2*b^13 - 5120*a^3*b^12 - 2
560*a^4*b^11 + 10240*a^5*b^10 - 10240*a^7*b^8 + 2540*a^8*b^7 + 5180*a^9*b^6
 - 2064*a^10*b^5 - 1136*a^11*b^4 + 619*a^12*b^3 + 31*a^13*b^2))/(8*a^12))*(
a^6*5i - b^6*16i + a^2*b^4*40i - a^4*b^2*30i)*1i)/(16*a^7))/(((25*a^19*b)/4
 - 96*a*b^19 + 64*b^20 - 480*a^2*b^18 + 760*a^3*b^17 + 1544*a^4*b^16 - 2628
```

*a^5*b^15 - 2748*a^6*b^14 + 5179*a^7*b^13 + 2890*a^8*b^12 - 6359*a^9*b^11 - 1736*a^10*b^10 + (19951*a^11*b^9)/4 + (937*a^12*b^8)/2 - (4915*a^13*b^7)/2 + (85*a^14*b^6)/2 + 715*a^15*b^5 - (105*a^16*b^4)/2 - (215*a^17*b^3)/2 + (15*a^18*b^2)/2)/a^18 + (((((42*a^21*b - 10*a^22 + 32*a^14*b^8 - 48*a^15*b^7 - 80*a^16*b^6 + 140*a^17*b^5 + 52*a^18*b^4 - 134*a^19*b^3 + 6*a^20*b^2)/a^18 - (tan(c/2 + (d*x)/2)*(512*a^16*b + 512*a^14*b^3 - 1024*a^15*b^2)*(a^6*5i - b^6*16i + a^2*b^4*40i - a^4*b^2*30i))/(128*a^19))*(a^6*5i - b^6*16i + a^2*b^4*40i - a^4*b^2*30i))/(16*a^7) + (tan(c/2 + (d*x)/2)*(1024*a*b^14 - 75*a^14*b + 25*a^15 - 512*b^15 + 2048*a^2*b^13 - 5120*a^3*b^12 - 2560*a^4*b^11 + 10240*a^5*b^10 - 10240*a^7*b^8 + 2540*a^8*b^7 + 5180*a^9*b^6 - 2064*a^10*b^5 - 1136*a^11*b^4 + 619*a^12*b^3 + 31*a^13*b^2))/(8*a^12))*(a^6*5i - b^6*16i + a^2*b^4*40i - a^4*b^2*30i))/(16*a^7) + (((((42*a^21*b - 10*a^22 + 32*a^14*b^8 - 48*a^15*b^7 - 80*a^16*b^6 + 140*a^17*b^5 + 52*a^18*b^4 - 134*a^19*b^3 + 6*a^20*b^2)/a^18 + (tan(c/2 + (d*x)/2)*(512*a^16*b + 512*a^14*b^3 - 1024*a^15*b^2)*(a^6*5i - b^6*16i + a^2*b^4*40i - a^4*b^2*30i))/(128*a^19))*(a^6*5i - b^6*16i + a^2*b^4*40i - a^4*b^2*30i))/(16*a^7) - (tan(c/2 + (d*x)/2)*(1024*a*b^14 - 75*a^14*b + 25*a^15 - 512*b^15 + 2048*a^2*b^13 - 5120*a^3*b^12 - 2560*a^4*b^11 + 10240*a^5*b^10 - 10240*a^7*b^8 + 2540*a^8*b^7 + 5180*a^9*b^6 - 2064*a^10*b^5 - 1136*a^11*b^4 + 619*a^12*b^3 + 31*a^13*b^2))/(8*a^12))*(a^6*5i - b^6*16i + a^2*b^4*40i - a^4*b^2*30i))/(16*a^7)))*(a^6*5i - b^6*16i + a^2*b^4*40i - a^4*b^2*30i)*1i)/(8*a^7*d) - ((tan(c/2 + (d*x)/2)*(8*a*b^4 - 16*a^4*b + 5*a^5 - 16*b^5 + 32*a^2*b^3 - 14*a^3*b^2))/(8*a^6) - (tan(c/2 + (d*x)/2)^11*(8*a*b^4 + 16*a^4*b + 5*a^5 + 16*b^5 - 32*a^2*b^3 - 14*a^3*b^2))/(8*a^6) + (tan(c/2 + (d*x)/2)^3*(72*a*b^4 - 304*a^4*b + 85*a^5 - 240*b^5 + 544*a^2*b^3 - 174*a^3*b^2))/(24*a^6) - (tan(c/2 + (d*x)/2)^9*(72*a*b^4 + 304*a^4*b + 85*a^5 + 240*b^5 - 544*a^2*b^3 - 174*a^3*b^2))/(24*a^6) + (tan(c/2 + (d*x)/2)^5*(40*a*b^4 - 688*a^4*b + 165*a^5 - 400*b^5 + 960*a^2*b^3 - 110*a^3*b^2))/(20*a^6) - (tan(c/2 + (d*x)/2)^7*(40*a*b^4 + 688*a^4*b + 165*a^5 + 400*b^5 - 960*a^2*b^3 - 110*a^3*b^2))/(20*a^6))/(d*(6*tan(c/2 + (d*x)/2)^2 + 15*tan(c/2 + (d*x)/2)^4 + 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 + 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1)) + (b*atan(((b*((tan(c/2 + (d*x)/2)*(1024*a*b^14 - 75*a^14*b + 25*a^15 - 512*b^15 + 2048*a^2*b^13 - 5120*a^3*b^12 - 2560*a^4*b^11 + 10240*a^5*b^10 - 10240*a^7*b^8 + 2540*a^8*b^7 + 5180*a^9*b^6 - 2064*a^10*b^5 - 1136*a^11*b^4 + 619*a^12*b^3 + 31*a^13*b^2))/(8*a^12) + (b*((a + b)^5*(a - b)^5)^(1/2)*((42*a^21*b - 10*a^22 + 32*a^14*b^8 - 48*a^15*b^7 - 80*a^16*b^6 + 140*a^17*b^5 + 52*a^18*b^4 - 134*a^19*b^3 + 6*a^20*b^2)/a^18 - (b*tan(c/2 + (d*x)/2)*((a + b)^5*(a - b)^5)^(1/2)*(512*a^16*b + 512*a^14*b^3 - 1024*a^15*b^2))/(8*a^19)))/a^7)*((a + b)^5*(a - b)^5)^(1/2)*1i)/a^7 + (b*((tan(c/2 + (d*x)/2)*(1024*a*b^14 - 75*a^14*b + 25*a^15 - 512*b^15 + 2048*a^2*b^13 - 5120*a^3*b^12 - 2560*a^4*b^11 + 10240*a^5*b^10 - 10240*a^7*b^8 + 2540*a^8*b^7 + 5180*a^9*b^6 - 2064*a^10*b^5 - 1136*a^11*b^4 + 619*a^12*b^3 + 31*a^13*b^2))/(8*a^12) - (b*((a + b)^5*(a - b)^5)^(1/2)*((42*a^21*b - 10*a^22 + 32*a^14*b^8 - 48*a^15*b^7 - 80*a^16*b^6 + 140*a^17*b^5 + 52*a^18*b^4 - 134*a^19*b^3 + 6*a^20*b^2)/a^18 + (b*tan(c/2 + (d*x)/2)*((a + b)^5*(a - b)^5)^(1/2)*(512

```
*a^16*b + 512*a^14*b^3 - 1024*a^15*b^2))/(8*a^19)))/a^7)*((a + b)^5*(a - b)
^5)^(1/2)*1i)/a^7)/(((25*a^19*b)/4 - 96*a*b^19 + 64*b^20 - 480*a^2*b^18 + 7
60*a^3*b^17 + 1544*a^4*b^16 - 2628*a^5*b^15 - 2748*a^6*b^14 + 5179*a^7*b^13
 + 2890*a^8*b^12 - 6359*a^9*b^11 - 1736*a^10*b^...
```

3.204 $\int \frac{\sin^4(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=161

$$\frac{(3a^4-12a^2b^2+8b^4)\,x}{8a^5}-\frac{2(a-b)^{3/2}b(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d}+\frac{(8b(a^2-b^2)-a(3a^2-4b^2)\,\mathrm{co}}{8a^4d}$$

```
[Out] 1/8*(3*a^4-12*a^2*b^2+8*b^4)*x/a^5-2*(a-b)^(3/2)*b*(a+b)^(3/2)*arctanh((a-b
)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^5/d+1/8*(8*b*(a^2-b^2)-a*(3*a^2-4
*b^2)*cos(d*x+c))*sin(d*x+c)/a^4/d+1/12*(4*b-3*a*cos(d*x+c))*sin(d*x+c)^3/a
^2/d
```

Rubi [A]
time = 0.27, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3957, 2944, 2814, 2738, 214}

$$-\frac{2b(a-b)^{3/2}(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d}+\frac{\sin^3(c+dx)(4b-3a\cos(c+dx))}{12a^2d}+\frac{\sin(c+dx)\,(8b(a^2-b^2)-a(3a^2-4b^2)\cos(c+dx))}{8a^4d}+\frac{x(3a^4-12a^2b^2+8b^4)}{8a^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((3*a^4 - 12*a^2*b^2 + 8*b^4)*x)/(8*a^5) - (2*(a - b)^(3/2)*b*(a + b)^(3/2)
*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^5*d) + ((8*b*(a^2
- b^2) - a*(3*a^2 - 4*b^2)*Cos[c + d*x])*Sin[c + d*x])/(8*a^4*d) + ((4*b -
3*a*Cos[c + d*x])*Sin[c + d*x]^3)/(12*a^2*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin^4(c+dx)}{a+b\sec(c+dx)}\,dx = -\int \frac{\cos(c+dx)\sin^4(c+dx)}{-b-a\cos(c+dx)}\,dx$$

$$= \frac{(4b-3a\cos(c+dx))\sin^3(c+dx)}{12a^2d} - \frac{\int \frac{\left(-ab+(3a^2-4b^2)\cos(c+dx)\right)\sin^2(c+dx)}{-b-a\cos(c+dx)}\,dx}{4a^2}$$

$$= \frac{(8b(a^2-b^2)-a(3a^2-4b^2)\cos(c+dx))\sin(c+dx)}{8a^4d} + \frac{(4b-3a\cos(c+dx))\sin^3}{12a^2d}$$

$$= \frac{(3a^4-12a^2b^2+8b^4)\,x}{8a^5} + \frac{(8b(a^2-b^2)-a(3a^2-4b^2)\cos(c+dx))\sin(c+dx)}{8a^4d} +$$

$$= \frac{(3a^4-12a^2b^2+8b^4)\,x}{8a^5} + \frac{(8b(a^2-b^2)-a(3a^2-4b^2)\cos(c+dx))\sin(c+dx)}{8a^4d} +$$

$$= \frac{(3a^4-12a^2b^2+8b^4)\,x}{8a^5} - \frac{2(a-b)^{3/2}b(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d} +$$

Mathematica [A]

time = 0.47, size = 172, normalized size = 1.07

$$\frac{36a^4c-144a^2b^2c+96b^4c+36a^4dx-144a^2b^2dx+96b^4dx+192b(a^2-b^2)^{3/2}\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)+24ab(5a^2-4b^2)\sin(c+dx)-24(a^4-a^2b^2)\sin(2(c+dx))-8a^3b\sin(3(c+dx))+3a^4\sin(4(c+dx))}{96a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

```
[Out] (36*a^4*c - 144*a^2*b^2*c + 96*b^4*c + 36*a^4*d*x - 144*a^2*b^2*d*x + 96*b^
4*d*x + 192*b*(a^2 - b^2)^(3/2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^
2 - b^2]] + 24*a*b*(5*a^2 - 4*b^2)*Sin[c + d*x] - 24*(a^4 - a^2*b^2)*Sin[2*
(c + d*x)] - 8*a^3*b*Sin[3*(c + d*x)] + 3*a^4*Sin[4*(c + d*x)])/(96*a^5*d)
```

Maple [A]
time = 0.15, size = 264, normalized size = 1.64

method	result
derivativedivides	$\frac{-\frac{2(a-b)^2(a+b)^2 b \operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a^5\sqrt{(a+b)(a-b)}}+\frac{2\left(\left(\frac{3}{8}a^4+b\,a^3-\frac{1}{2}b^2a^2-b^3a\right)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{13}{3}b\,a^3-\frac{1}{2}b^2a^2-3b^3a+\ldots\right.}{\ldots}}{\ldots}$
default	$\frac{-\frac{2(a-b)^2(a+b)^2 b \operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a^5\sqrt{(a+b)(a-b)}}+\frac{2\left(\left(\frac{3}{8}a^4+b\,a^3-\frac{1}{2}b^2a^2-b^3a\right)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{13}{3}b\,a^3-\frac{1}{2}b^2a^2-3b^3a+\ldots\right.}{\ldots}}{\ldots}$
risch	$\frac{3x}{8a}-\frac{3x\,b^2}{2a^3}+\frac{x\,b^4}{a^5}-\frac{5ib\,e^{i(dx+c)}}{8a^2d}+\frac{ib^3e^{i(dx+c)}}{2a^4d}+\frac{5ib\,e^{-i(dx+c)}}{8a^2d}-\frac{ib^3e^{-i(dx+c)}}{2a^4d}+\frac{\sqrt{a^2-b^2}\,b\ln\left(e^{i(dx+c)}\ldots\right.}{d\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(a-b)^2*(a+b)^2*b/a^5/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x
+1/2*c)/((a+b)*(a-b))^(1/2))+2/a^5*(((3/8*a^4+b*a^3-1/2*b^2*a^2-b^3*a)*tan(
1/2*d*x+1/2*c)^7+(13/3*b*a^3-1/2*b^2*a^2-3*b^3*a+11/8*a^4)*tan(1/2*d*x+1/2*
c)^5+(-11/8*a^4+1/2*b^2*a^2+13/3*b*a^3-3*b^3*a)*tan(1/2*d*x+1/2*c)^3+(b*a^3
-b^3*a-3/8*a^4+1/2*b^2*a^2)*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^4+
1/8*(3*a^4-12*a^2*b^2+8*b^4)*arctan(tan(1/2*d*x+1/2*c))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 3.22, size = 393, normalized size = 2.44

$$\left[\frac{3\left(3a^4-12a^2b^2+8b^4\right)dx-12\left(a^2b-b^3\right)\sqrt{a^2-b^2}\log\left(\frac{2ab\cos(dx+c)-(a^2-2b^2)\cos(dx+c)^2+2\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2a^2-b^2}{a^2\cos(dx+c)^2+2ab\cos(dx+c)+b^2}\right)+\left(6a^4\cos(dx+c)^3-8a^3b\cos(dx+c)^2+32a^3b-24ab^3-3\left(5a^4-4a^2b^2\right)\cos(dx+c)\right)\sin(dx+c)}{24a^5d}, \frac{3\left(3a^4-12a^2b^2+8b^4\right)dx-24\left(a^2b-b^3\right)\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)+\left(6a^4\cos(dx+c)^3-8a^3b\cos(dx+c)^2+32a^3b-24ab^3-3\left(5a^4-4a^2b^2\right)\cos(dx+c)\right)\sin(dx+c)}{24a^5d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/24*(3*(3*a^4 - 12*a^2*b^2 + 8*b^4)*d*x - 12*(a^2*b - b^3)*sqrt(a^2 - b^2
)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2
)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*
a*b*cos(d*x + c) + b^2)) + (6*a^4*cos(d*x + c)^3 - 8*a^3*b*cos(d*x + c)^2 +
 32*a^3*b - 24*a*b^3 - 3*(5*a^4 - 4*a^2*b^2)*cos(d*x + c))*sin(d*x + c))/(a
^5*d), 1/24*(3*(3*a^4 - 12*a^2*b^2 + 8*b^4)*d*x - 24*(a^2*b - b^3)*sqrt(-a^
2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x
 + c))) + (6*a^4*cos(d*x + c)^3 - 8*a^3*b*cos(d*x + c)^2 + 32*a^3*b - 24*a*
b^3 - 3*(5*a^4 - 4*a^2*b^2)*cos(d*x + c))*sin(d*x + c))/(a^5*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^4(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)**4/(a + b*sec(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 407 vs. 2(145) = 290.
time = 0.44, size = 407, normalized size = 2.53

$$\frac{\frac{3\left(3a^4-12a^2b^2+8b^4\right)(dx+c)}{a^5}-\frac{48\left(a^4b-2a^2b^3+b^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\sqrt{-a^2+b^2}\,a^5}+\frac{2\left(9a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+24a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-12ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-24b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+33a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+104a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-12ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-72b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-33a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+104a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+12ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-72b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-9a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+24a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+12ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-24b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^4a^4}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(3*(3*a^4 - 12*a^2*b^2 + 8*b^4)*(d*x + c)/a^5 - 48*(a^4*b - 2*a^2*b^3
+ b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1
/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b
^2)*a^5) + 2*(9*a^3*tan(1/2*d*x + 1/2*c)^7 + 24*a^2*b*tan(1/2*d*x + 1/2*c)^
7 - 12*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 24*b^3*tan(1/2*d*x + 1/2*c)^7 + 33*a^
3*tan(1/2*d*x + 1/2*c)^5 + 104*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 12*a*b^2*tan(
1/2*d*x + 1/2*c)^5 - 72*b^3*tan(1/2*d*x + 1/2*c)^5 - 33*a^3*tan(1/2*d*x + 1
```

```
/2*c)^3 + 104*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 12*a*b^2*tan(1/2*d*x + 1/2*c)^
3 - 72*b^3*tan(1/2*d*x + 1/2*c)^3 - 9*a^3*tan(1/2*d*x + 1/2*c) + 24*a^2*b*t
an(1/2*d*x + 1/2*c) + 12*a*b^2*tan(1/2*d*x + 1/2*c) - 24*b^3*tan(1/2*d*x +
1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*a^4))/d
```

Mupad [B]
time = 2.29, size = 317, normalized size = 1.97

$$\frac{\frac{5\,b\,\sin(c+d\,x)}{4}-\frac{b\,\sin(3\,c+3\,d\,x)}{12}}{a^2\,d}-\frac{3\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)-\frac{b^2\,\sin(2\,c+2\,d\,x)}{4}}{a^3\,d}+\frac{\frac{3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{4}-\frac{\sin(2\,c+2\,d\,x)}{4}+\frac{\sin(4\,c+4\,d\,x)}{32}}{a\,d}-\frac{b^3\,\sin\left(c+d\,x\right)}{a^4\,d}+\frac{2\,b^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{a^5\,d}-\frac{2\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^3+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b-\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b^2-\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^3}\right)\,\sqrt{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}}{a^5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(a + b/cos(c + d*x)),x)
```

```
[Out] ((5*b*sin(c + d*x))/4 - (b*sin(3*c + 3*d*x))/12)/(a^2*d) - (3*b^2*atan(sin(
c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) - (b^2*sin(2*c + 2*d*x))/4)/(a^3*d) + ((
3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/4 - sin(2*c + 2*d*x)/4 + sin
(4*c + 4*d*x)/32)/(a*d) - (b^3*sin(c + d*x))/(a^4*d) + (2*b^4*atan(sin(c/2
+ (d*x)/2)/cos(c/2 + (d*x)/2)))/(a^5*d) - (2*b*atanh((sin(c/2 + (d*x)/2)*(a
^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2))/(a^3*cos(c/2 + (d*x)/2) - b^3*cos(
c/2 + (d*x)/2) - a*b^2*cos(c/2 + (d*x)/2) + a^2*b*cos(c/2 + (d*x)/2)))*(a^6
 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2))/(a^5*d)
```

3.205 $\int \frac{\sin^2(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=100

$$\frac{(a^2-2b^2)\,x}{2a^3}-\frac{2\sqrt{a-b}\,b\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d}+\frac{(2b-a\cos(c+dx))\sin(c+dx)}{2a^2d}$$

```
[Out] 1/2*(a^2-2*b^2)*x/a^3+1/2*(2*b-a*cos(d*x+c))*sin(d*x+c)/a^2/d-2*b*arctanh((
a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))*(a-b)^(1/2)*(a+b)^(1/2)/a^3/d
```

Rubi [A]
time = 0.15, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3957, 2944, 2814, 2738, 214}

$$-\frac{2b\sqrt{a-b}\,\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d}+\frac{\sin(c+dx)(2b-a\cos(c+dx))}{2a^2d}+\frac{x(a^2-2b^2)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((a^2 - 2*b^2)*x)/(2*a^3) - (2*Sqrt[a - b]*b*Sqrt[a + b]*ArcTanh[(Sqrt[a -
b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*d) + ((2*b - a*Cos[c + d*x])*Sin[c
+ d*x])/(2*a^2*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^2(c+dx)}{a+b\sec(c+dx)}\,dx &= -\int \frac{\cos(c+dx)\sin^2(c+dx)}{-b-a\cos(c+dx)}\,dx \\
&= \frac{(2b-a\cos(c+dx))\sin(c+dx)}{2a^2d} - \frac{\int \frac{-ab+\left(a^2-2b^2\right)\cos(c+dx)}{-b-a\cos(c+dx)}\,dx}{2a^2} \\
&= \frac{\left(a^2-2b^2\right)x}{2a^3} + \frac{(2b-a\cos(c+dx))\sin(c+dx)}{2a^2d} + \frac{\left(b\left(a^2-b^2\right)\right)\int \frac{1}{-b-a\cos(c+dx)}\,dx}{a^3} \\
&= \frac{\left(a^2-2b^2\right)x}{2a^3} + \frac{(2b-a\cos(c+dx))\sin(c+dx)}{2a^2d} + \frac{\left(2b\left(a^2-b^2\right)\right)\text{Subst}\left(\int \frac{1}{-a-b+(a-b \cdots}\right.}{a^3d} \\
&= \frac{\left(a^2-2b^2\right)x}{2a^3} - \frac{2\sqrt{a-b}\,b\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d} + \frac{(2b-a\cos(c+ \cdots}{2 \cdots}
\end{aligned}
$$

Mathematica [A]
time = 0.22, size = 96, normalized size = 0.96

$$
\frac{2\left(a^2-2b^2\right)(c+dx)+8b\sqrt{a^2-b^2}\,\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)+4ab\sin(c+dx)-a^2\sin(2(c+dx))}{4a^3d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*(a^2 - 2*b^2)*(c + d*x) + 8*b*Sqrt[a^2 - b^2]*ArcTanh[((-a + b)*Tan[(c +
 d*x)/2])/Sqrt[a^2 - b^2]] + 4*a*b*Sin[c + d*x] - a^2*Sin[2*(c + d*x)])/(4*
a^3*d)
```

Maple [A]
time = 0.11, size = 142, normalized size = 1.42

method	result
derivativedivides	$\frac{-\frac{2(a-b)b(a+b)\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a^3\sqrt{(a+b)(a-b)}}+\frac{\frac{2\left(\left(\frac{1}{2}a^2+ba\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(ba-\frac{1}{2}a^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\left(a^2-2b^2\right)\mathrm{arct}}{a^3}}{d}$
default	$\frac{-\frac{2(a-b)b(a+b)\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a^3\sqrt{(a+b)(a-b)}}+\frac{\frac{2\left(\left(\frac{1}{2}a^2+ba\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(ba-\frac{1}{2}a^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\left(a^2-2b^2\right)\mathrm{arct}}{a^3}}{d}$
risch	$\frac{x}{2a}-\frac{x\,b^2}{a^3}-\frac{ib\,e^{i(dx+c)}}{2a^2d}+\frac{ib\,e^{-i(dx+c)}}{2a^2d}-\frac{\sqrt{a^2-b^2}\,b\ln\left(e^{i(dx+c)}+\frac{i\sqrt{a^2-b^2}+b}{a}\right)}{d\,a^3}+\frac{\sqrt{a^2-b^2}\,b\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(a-b)*b*(a+b)/a^3/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2
*c)/((a+b)*(a-b))^(1/2))+2/a^3*(((1/2*a^2+b*a)*tan(1/2*d*x+1/2*c)^3+(b*a-1/
2*a^2)*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^2+1/2*(a^2-2*b^2)*arcta
n(tan(1/2*d*x+1/2*c))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 3.90, size = 258, normalized size = 2.58

$$\left[\frac{(a^2-2b^2)dx+\sqrt{a^2-b^2}\,b\log\left(\frac{2ab\cos(dx+c)-(a^2-2b^2)\cos(dx+c)^2-2\sqrt{a^2-b^2}\,(b\cos(dx+c)+a)\sin(dx+c)+2a^2-b^2}{a^2\cos(dx+c)^2+2ab\cos(dx+c)+b^2}\right)-(a^2\cos(dx+c)-2ab)\sin(dx+c)}{2a^3d}, \frac{(a^2-2b^2)dx-2\sqrt{-a^2+b^2}\,b\arctan\left(-\frac{\sqrt{-a^2+b^2}\,(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)-(a^2\cos(dx+c)-2ab)\sin(dx+c)}{2a^3d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*((a^2 - 2*b^2)*d*x + sqrt(a^2 - b^2)*b*log((2*a*b*cos(d*x + c) - (a^2
 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x +
c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - (a^2*c
os(d*x + c) - 2*a*b)*sin(d*x + c))/(a^3*d), 1/2*((a^2 - 2*b^2)*d*x - 2*sqrt
(-a^2 + b^2)*b*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*s
in(d*x + c))) - (a^2*cos(d*x + c) - 2*a*b)*sin(d*x + c))/(a^3*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)**2/(a + b*sec(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(86) = 172.
time = 0.43, size = 185, normalized size = 1.85

$$\frac{\frac{(a^2-2b^2)(dx+c)}{a^3}-\frac{4(a^2b-b^3)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\sqrt{-a^2+b^2}\,a^3}+\frac{2\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2a^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*((a^2 - 2*b^2)*(d*x + c)/a^3 - 4*(a^2*b - b^3)*(pi*floor(1/2*(d*x + c)/
pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x
 + 1/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*a^3) + 2*(a*tan(1/2*d*x + 1
/2*c)^3 + 2*b*tan(1/2*d*x + 1/2*c)^3 - a*tan(1/2*d*x + 1/2*c) + 2*b*tan(1/2
*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2))/d
```

Mupad [B]
time = 1.48, size = 147, normalized size = 1.47

$$\frac{\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)-\frac{\sin(2\,c+2\,d\,x)}{4}}{a\,d}-\frac{2\,b^2\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{a^3\,d}+\frac{b\,\sin\left(c+d\,x\right)}{a^2\,d}-\frac{2\,b\,\operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{a^2-b^2}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)(a+b)}\right)\sqrt{a^2-b^2}}{a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(a + b/cos(c + d*x)),x)
```

```
[Out] (atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) - sin(2*c + 2*d*x)/4)/(a*d) -
(2*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(a^3*d) + (b*sin(c + d*
x))/(a^2*d) - (2*b*atanh((sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2))/(cos(c/2 +
(d*x)/2)*(a + b)))*(a^2 - b^2)^(1/2))/(a^3*d)
```

3.206 $\int \frac{\csc^2(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=84

$$-\frac{2ab\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}d}+\frac{(b-a\cos(c+dx))\csc(c+dx)}{(a^2-b^2)\,d}$$

```
[Out] -2*a*b*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(3/2)/(a+b
)^(3/2)/d+(b-a*cos(d*x+c))*csc(d*x+c)/(a^2-b^2)/d
```

Rubi [A]
time = 0.11, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3957, 2945, 12, 2738, 214}

$$\frac{\csc(c+dx)(b-a\cos(c+dx))}{d\,(a^2-b^2)}-\frac{2ab\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{3/2}(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-2*a*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)
*(a + b)^(3/2)*d) + ((b - a*Cos[c + d*x])*Csc[c + d*x])/((a^2 - b^2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
```

```
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^2(c+dx)}{a+b\sec(c+dx)}\,dx &= -\int \frac{\cot(c+dx)\csc(c+dx)}{-b-a\cos(c+dx)}\,dx \\
&= \frac{(b-a\cos(c+dx))\csc(c+dx)}{(a^2-b^2)\,d} + \frac{\int \frac{ab}{-b-a\cos(c+dx)}\,dx}{a^2-b^2} \\
&= \frac{(b-a\cos(c+dx))\csc(c+dx)}{(a^2-b^2)\,d} + \frac{(ab)\int \frac{1}{-b-a\cos(c+dx)}\,dx}{a^2-b^2} \\
&= \frac{(b-a\cos(c+dx))\csc(c+dx)}{(a^2-b^2)\,d} + \frac{(2ab)\text{Subst}\left(\int \frac{1}{-a-b+(a-b)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx\right.\right.}{(a^2-b^2)\,d} \\
&= -\frac{2ab\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}d} + \frac{(b-a\cos(c+dx))\csc(c+dx)}{(a^2-b^2)\,d}
\end{aligned}$$

Mathematica [A]
time = 0.14, size = 118, normalized size = 1.40

$$\frac{\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{a^2-b^2}\,(b-a\cos(c+dx))+2ab\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)\sin(c+dx)\right)}{2(a-b)(a+b)\sqrt{a^2-b^2}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

```
[Out] (Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(Sqrt[a^2 - b^2]*(b - a*Cos[c + d*x]) +
2*a*b*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*Sin[c + d*x]))/(
2*(a - b)*(a + b)*Sqrt[a^2 - b^2]*d)
```

Maple [A]
time = 0.12, size = 96, normalized size = 1.14

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a-2b}-\frac{2ba\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}-\frac{1}{2(a+b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{d}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a-2b}-\frac{2ba\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}-\frac{1}{2(a+b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{d}$
risch	$-\frac{2i\left(-b\,\mathrm{e}^{i(dx+c)}+a\right)}{d(a^2-b^2)\left(\mathrm{e}^{2i(dx+c)}-1\right)}-\frac{ab\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia^2-ib^2+b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,(a+b)(a-b)d}+\frac{ab\ln\left(\mathrm{e}^{i(dx+c)}-\frac{ia^2-ib^2-b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,(a+b)(a-b)d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2/(a-b)*tan(1/2*d*x+1/2*c)-2*b/(a+b)/(a-b)*a/((a+b)*(a-b))^(1/2)*arc
tanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))-1/2/(a+b)/tan(1/2*d*x+1/
2*c))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 2.68, size = 300, normalized size = 3.57

$$\left[-\frac{\sqrt{a^2-b^2}\,ab\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2+2\,\sqrt{a^2-b^2}\,(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)\sin(dx+c)-2\,a^2b+2\,b^3+2\,(a^3-ab^2)\cos(dx+c)}{2\,(a^4-2\,a^2b^2+b^4)d\sin(dx+c)},-\frac{\sqrt{-a^2+b^2}\,ab\arctan\left(-\frac{\sqrt{-a^2+b^2}\,(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)\sin(dx+c)-a^2b+b^3+(a^3-ab^2)\cos(dx+c)}{(a^4-2\,a^2b^2+b^4)d\sin(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(a^2 - b^2)*a*b*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x
+ c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)
/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) - 2*a^2*b +
2*b^3 + 2*(a^3 - a*b^2)*cos(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*d*sin(d*x +
c)), -(sqrt(-a^2 + b^2)*a*b*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/(
(a^2 - b^2)*sin(d*x + c)))*sin(d*x + c) - a^2*b + b^3 + (a^3 - a*b^2)*cos(d
*x + c))/((a^4 - 2*a^2*b^2 + b^4)*d*sin(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**2/(a + b*sec(c + d*x)), x)
```

Giac [A]
time = 0.44, size = 129, normalized size = 1.54

$$-\frac{\frac{4\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)ab}{(a^2-b^2)\sqrt{-a^2+b^2}}-\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a-b}+\frac{1}{(a+b)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(4*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(
1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*a*b/((a^2 - b
^2)*sqrt(-a^2 + b^2)) - tan(1/2*d*x + 1/2*c)/(a - b) + 1/((a + b)*tan(1/2*d
*x + 1/2*c)))/d
```

Mupad [B]
time = 1.30, size = 109, normalized size = 1.30

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\,(2a-2b)}-\frac{a-b}{d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,(a+b)\,(2a-2b)}-\frac{2ab\,\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,(a^2-b^2)}{(a+b)^{3/2}\sqrt{a-b}}\right)}{d\,(a+b)^{3/2}\,(a-b)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^2*(a + b/cos(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)/(d*(2*a - 2*b)) - (a - b)/(d*tan(c/2 + (d*x)/2)*(a + b)*
(2*a - 2*b)) - (2*a*b*atanh((tan(c/2 + (d*x)/2)*(a^2 - b^2))/((a + b)^(3/2)
*(a - b)^(1/2))))/(d*(a + b)^(3/2)*(a - b)^(3/2))
```

3.207 $\int \frac{\csc^4(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=140

$$-\frac{2a^3b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d}+\frac{(3a^2b-a(2a^2+b^2)\cos(c+dx))\csc(c+dx)}{3\left(a^2-b^2\right)^2d}+\frac{(b-a\cos(c+dx))\csc^3}{3\left(a^2-b^2\right)d}$$

```
[Out] -2*a^3*b*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)/(a
+b)^(5/2)/d+1/3*(3*a^2*b-a*(2*a^2+b^2)*cos(d*x+c))*csc(d*x+c)/(a^2-b^2)^2/d
+1/3*(b-a*cos(d*x+c))*csc(d*x+c)^3/(a^2-b^2)/d
```

Rubi [A]
time = 0.22, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3957, 2945, 12, 2738, 214}

$$-\frac{2a^3b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}+\frac{\csc^3(c+dx)(b-a\cos(c+dx))}{3d\left(a^2-b^2\right)}+\frac{\csc(c+dx)\left(3a^2b-a(2a^2+b^2)\cos(c+dx)\right)}{3d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-2*a^3*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/
2)*(a + b)^(5/2)*d) + ((3*a^2*b - a*(2*a^2 + b^2)*Cos[c + d*x])*Csc[c + d*x
])/(3*(a^2 - b^2)^2*d) + ((b - a*Cos[c + d*x])*Csc[c + d*x]^3)/(3*(a^2 - b^
2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\csc^4(c+dx)}{a+b\sec(c+dx)}\,dx = -\int \frac{\cot(c+dx)\csc^3(c+dx)}{-b-a\cos(c+dx)}\,dx$$

$$= \frac{(b-a\cos(c+dx))\csc^3(c+dx)}{3(a^2-b^2)d} + \frac{\int \frac{(ab-2a^2\cos(c+dx))\csc^2(c+dx)}{-b-a\cos(c+dx)}\,dx}{3(a^2-b^2)}$$

$$= \frac{(3a^2b-a(2a^2+b^2)\cos(c+dx))\csc(c+dx)}{3(a^2-b^2)^2 d} + \frac{(b-a\cos(c+dx))\csc^3(c+dx)}{3(a^2-b^2)d} +$$

$$= \frac{(3a^2b-a(2a^2+b^2)\cos(c+dx))\csc(c+dx)}{3(a^2-b^2)^2 d} + \frac{(b-a\cos(c+dx))\csc^3(c+dx)}{3(a^2-b^2)d} +$$

$$= \frac{(3a^2b-a(2a^2+b^2)\cos(c+dx))\csc(c+dx)}{3(a^2-b^2)^2 d} + \frac{(b-a\cos(c+dx))\csc^3(c+dx)}{3(a^2-b^2)d} +$$

$$= -\frac{2a^3b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} + \frac{(3a^2b-a(2a^2+b^2)\cos(c+dx))\csc(c+}{3(a^2-b^2)^2 d}$$

Mathematica [A]
time = 0.58, size = 162, normalized size = 1.16

$$\frac{24a^3b\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)+\sqrt{a^2-b^2}\,(10a^2b-4b^3+(-6a^3+3ab^2)\cos(c+dx)-6a^2b\cos(2(c+dx))+2a^3\cos(3(c+dx))+ab^2\cos(3(c+dx)))\csc^3(c+dx)}{12(a-b)^2(a+b)^2\sqrt{a^2-b^2}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

```
[Out] (24*a^3*b*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] + Sqrt[a^2 -
 b^2]*(10*a^2*b - 4*b^3 + (-6*a^3 + 3*a*b^2)*Cos[c + d*x] - 6*a^2*b*Cos[2*(
c + d*x)] + 2*a^3*Cos[3*(c + d*x)] + a*b^2*Cos[3*(c + d*x)])*Csc[c + d*x]^3
)/(12*(a - b)^2*(a + b)^2*Sqrt[a^2 - b^2]*d)
```

Maple [A]
time = 0.17, size = 165, normalized size = 1.18

method	result
derivativedivides	$\frac{\frac{\frac{a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+3a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8(a-b)^2}-\frac{1}{24(a+b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{3a+b}{8(a+b)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2b\,a^3\,\mathrm{arcta}}{(a+b)^2(a}}{d}$
default	$\frac{\frac{\frac{a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+3a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8(a-b)^2}-\frac{1}{24(a+b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{3a+b}{8(a+b)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2b\,a^3\,\mathrm{arcta}}{(a+b)^2(a}}{d}$
risch	$\frac{2i\left(3a^2b\,\mathrm{e}^{5i(dx+c)}-3a\,b^2\mathrm{e}^{4i(dx+c)}-10b\,a^2\mathrm{e}^{3i(dx+c)}+4b^3\mathrm{e}^{3i(dx+c)}+6a^3\mathrm{e}^{2i(dx+c)}+3a^2b\,\mathrm{e}^{i(dx+c)}-2a^3-b^2a\right)}{3d\left(-a^2+b^2\right)^2\left(\mathrm{e}^{2i(dx+c)}-1\right)^3}+\frac{a^3b\ln\left(\mathrm{e}^{i}\right.}{\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/8/(a-b)^2*(1/3*a*tan(1/2*d*x+1/2*c)^3-1/3*b*tan(1/2*d*x+1/2*c)^3+3*a
*tan(1/2*d*x+1/2*c)-b*tan(1/2*d*x+1/2*c))-1/24/(a+b)/tan(1/2*d*x+1/2*c)^3-1
/8*(3*a+b)/(a+b)^2/tan(1/2*d*x+1/2*c)-2*b/(a+b)^2/(a-b)^2*a^3/((a+b)*(a-b))
^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 2.48, size = 558, normalized size = 3.99

$$\left[-\frac{8\,a^4b-10\,a^2b^3+2\,b^5+2\,(2\,a^5-a^3b^2-ab^4)\cos(dx+c)^3-3\,(a^3b\cos(dx+c)^2-a^3b)\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2-2\,\sqrt{a^2-b^2}\,(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)\sin(dx+c)-6\,(a^4b-a^2b^3)\cos(dx+c)^2-6\,(a^5-a^3b^2)\cos(dx+c)}{6\left((a^6-3\,a^4b^2+3\,a^2b^4-b^6)d\cos(dx+c)^2-(a^6-3\,a^4b^2+3\,a^2b^4-b^6)d\right)\sin(dx+c)},\ -\frac{4\,a^4b-5\,a^2b^3+b^5+(2\,a^5-a^3b^2-ab^4)\cos(dx+c)^3+3\,(a^3b\cos(dx+c)^2-a^3b)\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}\,(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)\sin(dx+c)-3\,(a^4b-a^2b^3)\cos(dx+c)^2-3\,(a^5-a^3b^2)\cos(dx+c)}{3\left((a^6-3\,a^4b^2+3\,a^2b^4-b^6)d\cos(dx+c)^2-(a^6-3\,a^4b^2+3\,a^2b^4-b^6)d\right)\sin(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/6*(8*a^4*b - 10*a^2*b^3 + 2*b^5 + 2*(2*a^5 - a^3*b^2 - a*b^4)*cos(d*x +
 c)^3 - 3*(a^3*b*cos(d*x + c)^2 - a^3*b)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x
 + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) +
a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b
^2))*sin(d*x + c) - 6*(a^4*b - a^2*b^3)*cos(d*x + c)^2 - 6*(a^5 - a^3*b^2)*
cos(d*x + c))/(((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x + c)^2 - (a^6
 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d)*sin(d*x + c)), -1/3*(4*a^4*b - 5*a^2*b^3
 + b^5 + (2*a^5 - a^3*b^2 - a*b^4)*cos(d*x + c)^3 + 3*(a^3*b*cos(d*x + c)^2
 - a^3*b)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((
a^2 - b^2)*sin(d*x + c)))*sin(d*x + c) - 3*(a^4*b - a^2*b^3)*cos(d*x + c)^2
 - 3*(a^5 - a^3*b^2)*cos(d*x + c))/(((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*
cos(d*x + c)^2 - (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d)*sin(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^4(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**4/(a + b*sec(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 269 vs. 2(128) = 256.
time = 0.49, size = 269, normalized size = 1.92

$$\frac{\frac{48\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)a^3b}{(a^4-2a^2b^2+b^4)\sqrt{-a^2+b^2}}+\frac{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+9a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-12ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^3-3a^2b+3ab^2-b^3}-\frac{9a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a+b}{(a^2+2ab+b^2)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(48*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1
/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*a^3*b/((a^4 -
2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) + (a^2*tan(1/2*d*x + 1/2*c)^3 - 2*a*b*ta
n(1/2*d*x + 1/2*c)^3 + b^2*tan(1/2*d*x + 1/2*c)^3 + 9*a^2*tan(1/2*d*x + 1/2
*c) - 12*a*b*tan(1/2*d*x + 1/2*c) + 3*b^2*tan(1/2*d*x + 1/2*c))/(a^3 - 3*a^
2*b + 3*a*b^2 - b^3) - (9*a*tan(1/2*d*x + 1/2*c)^2 + 3*b*tan(1/2*d*x + 1/2*
c)^2 + a + b)/((a^2 + 2*a*b + b^2)*tan(1/2*d*x + 1/2*c)^3))/d
```

Mupad [B]
time = 1.38, size = 219, normalized size = 1.56

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{2}{8\,a-8\,b}+\frac{8\,a+8\,b}{{\left(8\,a-8\,b\right)}^2}\right)}{d}+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{3\,d\,\left(8\,a-8\,b\right)}-\frac{\frac{a^2-2\,a\,b+b^2}{3\,\left(a+b\right)}+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(3\,a^3-5\,a^2\,b+a\,b^2+b^3\right)}{{\left(a+b\right)}^2}}{d\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(8\,a^2-16\,a\,b+8\,b^2\right)}-\frac{2\,a^3\,b\,\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^4-2\,a^2\,b^2+b^4\right)}{{\left(a+b\right)}^{5/2}\,{\left(a-b\right)}^{3/2}}\right)}{d\,{\left(a+b\right)}^{5/2}\,{\left(a-b\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^4*(a + b/cos(c + d*x))),x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(2/(8*a - 8*b) + (8*a + 8*b)/(8*a - 8*b)^2))/d + tan(c/
2 + (d*x)/2)^3/(3*d*(8*a - 8*b)) - ((a^2 - 2*a*b + b^2)/(3*(a + b)) + (tan(
c/2 + (d*x)/2)^2*(a*b^2 - 5*a^2*b + 3*a^3 + b^3))/(a + b)^2)/(d*tan(c/2 + (
d*x)/2)^3*(8*a^2 - 16*a*b + 8*b^2)) - (2*a^3*b*atanh((tan(c/2 + (d*x)/2)*(a
^4 + b^4 - 2*a^2*b^2))/((a + b)^(5/2)*(a - b)^(3/2))))/(d*(a + b)^(5/2)*(a
- b)^(5/2))
```

3.208 $\int \frac{\csc^6(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=201

$$-\frac{2a^5b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d}+\frac{(15a^4b-a(8a^4+9a^2b^2-2b^4)\cos(c+dx))\csc(c+dx)}{15\left(a^2-b^2\right)^3d}+\frac{(5a^2b-a(4}{}$$

```
[Out] -2*a^5*b*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(7/2)/(a
+b)^(7/2)/d+1/15*(15*a^4*b-a*(8*a^4+9*a^2*b^2-2*b^4)*cos(d*x+c))*csc(d*x+c)
/(a^2-b^2)^3/d+1/15*(5*a^2*b-a*(4*a^2+b^2)*cos(d*x+c))*csc(d*x+c)^3/(a^2-b^
2)^2/d+1/5*(b-a*cos(d*x+c))*csc(d*x+c)^5/(a^2-b^2)/d
```

Rubi [A]
time = 0.36, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$,
Rules used = {3957, 2945, 12, 2738, 214}

$$-\frac{2a^5b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}+\frac{\csc^5(c+dx)(b-a\cos(c+dx))}{5d\left(a^2-b^2\right)}+\frac{\csc^3(c+dx)\left(5a^2b-a\left(4a^2+b^2\right)\cos(c+dx)\right)}{15d\left(a^2-b^2\right)^2}+\frac{\csc(c+dx)\left(15a^4b-a\left(8a^4+9a^2b^2-2b^4\right)\cos(c+dx)\right)}{15d\left(a^2-b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-2*a^5*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(7/
2)*(a + b)^(7/2)*d) + ((15*a^4*b - a*(8*a^4 + 9*a^2*b^2 - 2*b^4)*Cos[c + d*
x])*Csc[c + d*x])/(15*(a^2 - b^2)^3*d) + ((5*a^2*b - a*(4*a^2 + b^2)*Cos[c
+ d*x])*Csc[c + d*x]^3)/(15*(a^2 - b^2)^2*d) + ((b - a*Cos[c + d*x])*Csc[c
+ d*x]^5)/(5*(a^2 - b^2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\csc^6(c+dx)}{a+b\sec(c+dx)}\,dx = -\int \frac{\cot(c+dx)\csc^5(c+dx)}{-b-a\cos(c+dx)}\,dx$$

$$= \frac{(b-a\cos(c+dx))\csc^5(c+dx)}{5\left(a^2-b^2\right)d} + \frac{\int \frac{\left(ab-4a^2\cos(c+dx)\right)\csc^4(c+dx)}{-b-a\cos(c+dx)}\,dx}{5\left(a^2-b^2\right)}$$

$$= \frac{\left(5a^2b-a(4a^2+b^2)\cos(c+dx)\right)\csc^3(c+dx)}{15\left(a^2-b^2\right)^2 d} + \frac{(b-a\cos(c+dx))\csc^5(c+dx)}{5\left(a^2-b^2\right)d} +$$

$$= \frac{\left(15a^4b-a(8a^4+9a^2b^2-2b^4)\cos(c+dx)\right)\csc(c+dx)}{15\left(a^2-b^2\right)^3 d} + \frac{\left(5a^2b-a(4a^2+b^2)\cos(\right.}{15\left(a^2-\right.}$$

$$= \frac{\left(15a^4b-a(8a^4+9a^2b^2-2b^4)\cos(c+dx)\right)\csc(c+dx)}{15\left(a^2-b^2\right)^3 d} + \frac{\left(5a^2b-a(4a^2+b^2)\cos(\right.}{15\left(a^2-\right.}$$

$$= \frac{\left(15a^4b-a(8a^4+9a^2b^2-2b^4)\cos(c+dx)\right)\csc(c+dx)}{15\left(a^2-b^2\right)^3 d} + \frac{\left(5a^2b-a(4a^2+b^2)\cos(\right.}{15\left(a^2-\right.}$$

$$= -\frac{2a^5b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} + \frac{\left(15a^4b-a(8a^4+9a^2b^2-2b^4)\cos(c+dx)\right.}{15\left(a^2-b^2\right)^3 d}$$

Mathematica [A]
time = 0.84, size = 277, normalized size = 1.38

$$\frac{(b+a\cos(c+dx))\sec(c+dx)\left(\frac{960a^5b\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}} - \frac{2\left(64a^2+43ab+9b^2\right)\cot\left(\frac{1}{2}(c+dx)\right)}{(a+b)^3} + \frac{8(19a-9b)\csc^3(c+dx)\sin^4\left(\frac{1}{2}(c+dx)\right)}{(a-b)^2} + \frac{96\csc^5(c+dx)\sin^6\left(\frac{1}{2}(c+dx)\right)}{a-b} - \frac{(19a+9b)\csc^4\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)}{2(a+b)^2} - \frac{3\csc^6\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)}{2(a+b)} + \frac{2\left(64a^2-43ab+9b^2\right)\tan\left(\frac{1}{2}(c+dx)\right)}{(a-b)^3}\right)}{480d(a+b\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]*((960*a^5*b*ArcTanh[((-a + b)*Tan[(c + d
*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(7/2) - (2*(64*a^2 + 43*a*b + 9*b^2)*
Cot[(c + d*x)/2])/(a + b)^3 + (8*(19*a - 9*b)*Csc[c + d*x]^3*Sin[(c + d*x)/
2]^4)/(a - b)^2 + (96*Csc[c + d*x]^5*Sin[(c + d*x)/2]^6)/(a - b) - ((19*a +
 9*b)*Csc[(c + d*x)/2]^4*Sin[c + d*x])/(2*(a + b)^2) - (3*Csc[(c + d*x)/2]^
6*Sin[c + d*x])/(2*(a + b)) + (2*(64*a^2 - 43*a*b + 9*b^2)*Tan[(c + d*x)/2]
)/(a - b)^3))/(480*d*(a + b*Sec[c + d*x]))
```

Maple [A]
time = 0.23, size = 282, normalized size = 1.40

method	result
derivativedivides	$\frac{\frac{\frac{a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{2ab\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{b^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{5a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{8ab\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}{32(a-b)^3}}{}$
default	$\frac{\frac{\frac{a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{2ab\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{b^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{5a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{8ab\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}{32(a-b)^3}}{}$
risch	$-\frac{2i\left(15a^4b\,e^{9i(dx+c)}-15a^3b^2e^{8i(dx+c)}-80a^4b\,e^{7i(dx+c)}+20a^2b^3e^{7i(dx+c)}+90a^3b^2e^{6i(dx+c)}-30a\,b^4e^{6i(dx+c)}+178a^4b\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/32/(a-b)^3*(1/5*a^2*tan(1/2*d*x+1/2*c)^5-2/5*a*b*tan(1/2*d*x+1/2*c)^
5+1/5*b^2*tan(1/2*d*x+1/2*c)^5+5/3*a^2*tan(1/2*d*x+1/2*c)^3-8/3*a*b*tan(1/2
*d*x+1/2*c)^3+b^2*tan(1/2*d*x+1/2*c)^3+10*a^2*tan(1/2*d*x+1/2*c)-8*a*b*tan(
1/2*d*x+1/2*c)+2*b^2*tan(1/2*d*x+1/2*c))-1/160/(a+b)/tan(1/2*d*x+1/2*c)^5-1
/96*(5*a+3*b)/(a+b)^2/tan(1/2*d*x+1/2*c)^3-1/32/(a+b)^3*(10*a^2+8*a*b+2*b^2
)/tan(1/2*d*x+1/2*c)-2*b/(a+b)^3/(a-b)^3*a^5/((a+b)*(a-b))^(1/2)*arctanh((a
-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 402 vs. 2(187) = 374.
time = 3.21, size = 861, normalized size = 4.28

$$\left[\frac{46\,a^{6}b-68\,a^{4}b^{3}+28\,a^{2}b^{5}-6\,b^{7}-2\left(8\,a^{7}+a^{5}b^{2}-11\,a^{3}b^{4}+2\,ab^{6}\right)\cos\left(dx+c\right)^{5}+30\left(a^{6}b-a^{4}b^{3}\right)\cos\left(dx+c\right)^{4}+10\left(4\,a^{7}-a^{5}b^{2}-4\,a^{3}b^{4}+ab^{6}\right)\cos\left(dx+c\right)^{3}-15\left(a^{5}b\cos\left(dx+c\right)^{4}-2\,a^{5}b\cos\left(dx+c\right)^{2}+a^{5}b\right)\sqrt{a^{2}-b^{2}}\log\left(\frac{2\,ab\cos\left(dx+c\right)-\left(a^{2}-2\,b^{2}\right)\cos\left(dx+c\right)^{2}+2\sqrt{a^{2}-b^{2}}\left(b\cos\left(dx+c\right)+a\right)\sin\left(dx+c\right)+2\,a^{2}-b^{2}}{a^{2}\cos\left(dx+c\right)^{2}+2\,ab\cos\left(dx+c\right)+b^{2}}\right)\sin\left(dx+c\right)-10\left(7\,a^{6}b-8\,a^{4}b^{3}+a^{2}b^{5}\right)\cos\left(dx+c\right)^{2}-30\left(a^{7}-a^{5}b^{2}\right)\cos\left(dx+c\right)}{30\left(\left(a^{8}-4\,a^{6}b^{2}+6\,a^{4}b^{4}-4\,a^{2}b^{6}+b^{8}\right)d\cos\left(dx+c\right)^{4}-2\left(a^{8}-4\,a^{6}b^{2}+6\,a^{4}b^{4}-4\,a^{2}b^{6}+b^{8}\right)d\cos\left(dx+c\right)^{2}+\left(a^{8}-4\,a^{6}b^{2}+6\,a^{4}b^{4}-4\,a^{2}b^{6}+b^{8}\right)d\right)\sin\left(dx+c\right)},\ \frac{23\,a^{6}b-34\,a^{4}b^{3}+14\,a^{2}b^{5}-3\,b^{7}-\left(8\,a^{7}+a^{5}b^{2}-11\,a^{3}b^{4}+2\,ab^{6}\right)\cos\left(dx+c\right)^{5}+15\left(a^{6}b-a^{4}b^{3}\right)\cos\left(dx+c\right)^{4}+5\left(4\,a^{7}-a^{5}b^{2}-4\,a^{3}b^{4}+ab^{6}\right)\cos\left(dx+c\right)^{3}-15\left(a^{5}b\cos\left(dx+c\right)^{4}-2\,a^{5}b\cos\left(dx+c\right)^{2}+a^{5}b\right)\sqrt{-a^{2}+b^{2}}\arctan\left(-\frac{\sqrt{-a^{2}+b^{2}}\left(b\cos\left(dx+c\right)+a\right)}{\left(a^{2}-b^{2}\right)\sin\left(dx+c\right)}\right)\sin\left(dx+c\right)-5\left(7\,a^{6}b-8\,a^{4}b^{3}+a^{2}b^{5}\right)\cos\left(dx+c\right)^{2}-15\left(a^{7}-a^{5}b^{2}\right)\cos\left(dx+c\right)}{15\left(\left(a^{8}-4\,a^{6}b^{2}+6\,a^{4}b^{4}-4\,a^{2}b^{6}+b^{8}\right)d\cos\left(dx+c\right)^{4}-2\left(a^{8}-4\,a^{6}b^{2}+6\,a^{4}b^{4}-4\,a^{2}b^{6}+b^{8}\right)d\cos\left(dx+c\right)^{2}+\left(a^{8}-4\,a^{6}b^{2}+6\,a^{4}b^{4}-4\,a^{2}b^{6}+b^{8}\right)d\right)\sin\left(dx+c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^6/(a+b*sec(d*x+c)),x, algorithm="fricas")

[Out] [1/30*(46*a^6*b - 68*a^4*b^3 + 28*a^2*b^5 - 6*b^7 - 2*(8*a^7 + a^5*b^2 - 11
*a^3*b^4 + 2*a*b^6)*cos(d*x + c)^5 + 30*(a^6*b - a^4*b^3)*cos(d*x + c)^4 +
10*(4*a^7 - a^5*b^2 - 4*a^3*b^4 + a*b^6)*cos(d*x + c)^3 - 15*(a^5*b*cos(d*x
 + c)^4 - 2*a^5*b*cos(d*x + c)^2 + a^5*b)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*
x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) +
 a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) +
b^2))*sin(d*x + c) - 10*(7*a^6*b - 8*a^4*b^3 + a^2*b^5)*cos(d*x + c)^2 - 30
*(a^7 - a^5*b^2)*cos(d*x + c))/(((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 +
 b^8)*d*cos(d*x + c)^4 - 2*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*
d*cos(d*x + c)^2 + (a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*d)*sin(d
x + c)), 1/15(23*a^6*b - 34*a^4*b^3 + 14*a^2*b^5 - 3*b^7 - (8*a^7 + a^5*b
^2 - 11*a^3*b^4 + 2*a*b^6)*cos(d*x + c)^5 + 15*(a^6*b - a^4*b^3)*cos(d*x +
c)^4 + 5*(4*a^7 - a^5*b^2 - 4*a^3*b^4 + a*b^6)*cos(d*x + c)^3 - 15*(a^5*b*c
os(d*x + c)^4 - 2*a^5*b*cos(d*x + c)^2 + a^5*b)*sqrt(-a^2 + b^2)*arctan(-sq
rt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))*sin(d*x + c
) - 5*(7*a^6*b - 8*a^4*b^3 + a^2*b^5)*cos(d*x + c)^2 - 15*(a^7 - a^5*b^2)*c
os(d*x + c))/(((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*d*cos(d*x +
c)^4 - 2*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*d*cos(d*x + c)^2 +
 (a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*d)*sin(d*x + c))]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{6}\left(c+dx\right)}{a+b\sec\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)**6/(a+b*sec(d*x+c)),x)

[Out] Integral(csc(c + d*x)**6/(a + b*sec(c + d*x)), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 541 vs. 2(187) = 374.

time = 0.52, size = 541, normalized size = 2.69

$$-\frac{\frac{960\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)a^5b}{(a^6-3a^4b^2+3a^2b^4-b^6)\sqrt{-a^2+b^2}}-\frac{3a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-12a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+18a^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-12ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+3b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+25a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-90a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+120a^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-70ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+15b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+150a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-420a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+420a^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-180ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+30b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^5-5a^4b+10a^3b^2-10a^2b^3+5ab^4-b^5}+\frac{150a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+120ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+30b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+25a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+40ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+15b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+3a^2+6ab+3b^2}{(a^3+3a^2b+3ab^2+b^3)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5}}{480d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/480*(960*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*
tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*a^5*b/((a
^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) - (3*a^4*tan(1/2*d*x +
1/2*c)^5 - 12*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 18*a^2*b^2*tan(1/2*d*x + 1/2*c
)^5 - 12*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 3*b^4*tan(1/2*d*x + 1/2*c)^5 + 25*a
^4*tan(1/2*d*x + 1/2*c)^3 - 90*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 120*a^2*b^2*t
an(1/2*d*x + 1/2*c)^3 - 70*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 15*b^4*tan(1/2*d*
x + 1/2*c)^3 + 150*a^4*tan(1/2*d*x + 1/2*c) - 420*a^3*b*tan(1/2*d*x + 1/2*c
) + 420*a^2*b^2*tan(1/2*d*x + 1/2*c) - 180*a*b^3*tan(1/2*d*x + 1/2*c) + 30*
b^4*tan(1/2*d*x + 1/2*c))/(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^
4 - b^5) + (150*a^2*tan(1/2*d*x + 1/2*c)^4 + 120*a*b*tan(1/2*d*x + 1/2*c)^4
 + 30*b^2*tan(1/2*d*x + 1/2*c)^4 + 25*a^2*tan(1/2*d*x + 1/2*c)^2 + 40*a*b*t
an(1/2*d*x + 1/2*c)^2 + 15*b^2*tan(1/2*d*x + 1/2*c)^2 + 3*a^2 + 6*a*b + 3*b
^2)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*tan(1/2*d*x + 1/2*c)^5))/d
```

Mupad [B]
time = 1.69, size = 387, normalized size = 1.93

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{5d\,(32a-32b)}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{4}{3(32a-32b)}+\frac{32a+32b}{3(32a-32b)^2}\right)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{5}{32a-32b}+\frac{\left(\frac{4}{32a-32b}+\frac{32a+32b}{(32a-32b)^2}\right)(32a+32b)}{32a-32b}\right)}{d}-\frac{\frac{a^3-3a^2b+3ab^2-b^3}{5(a+b)}-\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4(-5a^5+11a^4b-4a^3b^2-4a^2b^3+ab^4+b^5)}{(a+b)^3}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(5a^4-12a^3b+6a^2b^2+4ab^3-3b^4)}{3(a+b)^2}}{d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5(32a^3-96a^2b+96ab^2-32b^3)}-\frac{2a^5b\,\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(a^6-3a^4b^2+3a^2b^4-b^6)}{(a+b)^{7/2}(a-b)^{5/2}}\right)}{d\,(a+b)^{7/2}(a-b)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^6*(a + b/cos(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)^5/(5*d*(32*a - 32*b)) + (tan(c/2 + (d*x)/2)^3*(4/(3*(32*
a - 32*b)) + (32*a + 32*b)/(3*(32*a - 32*b)^2)))/d + (tan(c/2 + (d*x)/2)*(5
/(32*a - 32*b) + ((4/(32*a - 32*b) + (32*a + 32*b)/(32*a - 32*b)^2)*(32*a +
 32*b))/(32*a - 32*b)))/d - ((3*a*b^2 - 3*a^2*b + a^3 - b^3)/(5*(a + b)) -
(2*tan(c/2 + (d*x)/2)^4*(a*b^4 + 11*a^4*b - 5*a^5 + b^5 - 4*a^2*b^3 - 4*a^3
*b^2))/(a + b)^3 + (tan(c/2 + (d*x)/2)^2*(4*a*b^3 - 12*a^3*b + 5*a^4 - 3*b^
4 + 6*a^2*b^2))/(3*(a + b)^2))/(d*tan(c/2 + (d*x)/2)^5*(96*a*b^2 - 96*a^2*b
 + 32*a^3 - 32*b^3)) - (2*a^5*b*atanh((tan(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^
2*b^4 - 3*a^4*b^2))/((a + b)^(7/2)*(a - b)^(5/2))))/(d*(a + b)^(7/2)*(a - b
)^(7/2))
```

3.209 $\int \frac{\sin^7(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=267

$$-\frac{(a^2-7b^2)(a^2-b^2)^2\cos(c+dx)}{a^8d}-\frac{3b(a^2-b^2)^2\cos^2(c+dx)}{a^7d}+\frac{(3a^4-9a^2b^2+5b^4)\cos^3(c+dx)}{3a^6d}+\frac{b(3a^2-2b$$

```
[Out] -(a^2-7*b^2)*(a^2-b^2)^2*cos(d*x+c)/a^8/d-3*b*(a^2-b^2)^2*cos(d*x+c)^2/a^7/
d+1/3*(3*a^4-9*a^2*b^2+5*b^4)*cos(d*x+c)^3/a^6/d+1/2*b*(3*a^2-2*b^2)*cos(d*
x+c)^4/a^5/d-3/5*(a^2-b^2)*cos(d*x+c)^5/a^4/d-1/3*b*cos(d*x+c)^6/a^3/d+1/7*
cos(d*x+c)^7/a^2/d+b^2*(a^2-b^2)^3/a^9/d/(b+a*cos(d*x+c))+2*b*(a^2-4*b^2)*(
a^2-b^2)^2*ln(b+a*cos(d*x+c))/a^9/d
```

Rubi [A]
time = 0.26, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3957, 2916, 12, 962}

$$-\frac{b\cos^6(c+dx)}{3a^3d}+\frac{\cos^7(c+dx)}{7a^2d}+\frac{b^2(a^2-b^2)^3}{a^9d(a\cos(c+dx)+b)}+\frac{2b(a^2-4b^2)(a^2-b^2)^2\log(a\cos(c+dx)+b)}{a^9d}-\frac{(a^2-7b^2)(a^2-b^2)^2\cos(c+dx)}{a^8d}-\frac{3b(a^2-b^2)^2\cos^2(c+dx)}{a^7d}+\frac{b(3a^2-2b^2)\cos^4(c+dx)}{2a^5d}-\frac{3(a^2-b^2)\cos^5(c+dx)}{5a^4d}+\frac{(3a^4-9a^2b^2+5b^4)\cos^3(c+dx)}{3a^6d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^7/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(((a^2 - 7*b^2)*(a^2 - b^2)^2*Cos[c + d*x])/(a^8*d)) - (3*b*(a^2 - b^2)^2*
Cos[c + d*x]^2)/(a^7*d) + ((3*a^4 - 9*a^2*b^2 + 5*b^4)*Cos[c + d*x]^3)/(3*a
^6*d) + (b*(3*a^2 - 2*b^2)*Cos[c + d*x]^4)/(2*a^5*d) - (3*(a^2 - b^2)*Cos[c
 + d*x]^5)/(5*a^4*d) - (b*Cos[c + d*x]^6)/(3*a^3*d) + Cos[c + d*x]^7/(7*a^2
*d) + (b^2*(a^2 - b^2)^3)/(a^9*d*(b + a*Cos[c + d*x])) + (2*b*(a^2 - 4*b^2)
*(a^2 - b^2)^2*Log[b + a*Cos[c + d*x]])/(a^9*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 962

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
```

```
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^7(c+dx)}{(a+b\sec(c+dx))^2}\,dx &= \int \frac{\cos^2(c+dx)\sin^7(c+dx)}{(-b-a\cos(c+dx))^2}\,dx \\
&= \frac{\text{Subst}\left(\int \frac{x^2\left(a^2-x^2\right)^3}{a^2(-b+x)^2}\,dx, x, -a\cos(c+dx)\right)}{a^7 d} \\
&= \frac{\text{Subst}\left(\int \frac{x^2\left(a^2-x^2\right)^3}{(-b+x)^2}\,dx, x, -a\cos(c+dx)\right)}{a^9 d} \\
&= \frac{\text{Subst}\left(\int \left(\left(a^2-7b^2\right)\left(a^2-b^2\right)^2 - \frac{b^2\left(-a^2+b^2\right)^3}{(b-x)^2} + \frac{2b\left(-a^2+b^2\right)^2\left(-a^2+4b^2\right)}{b-x} - 6b(-a^2 + \right.\right.}{} \\
&= -\frac{\left(a^2-7b^2\right)\left(a^2-b^2\right)^2\cos(c+dx)}{a^8 d} - \frac{3b\left(a^2-b^2\right)^2\cos^2(c+dx)}{a^7 d} + \frac{\left(3a^4-9a^2b^2\right.}{}
\end{aligned}
$$

Mathematica [A]
time = 2.34, size = 417, normalized size = 1.56

$$
\frac{-3675a^8+61320a^6b^2-132720a^4b^4+87360a^2b^6-13440b^8-140\left(21a^8-228a^6b^2+400a^4b^4-192a^2b^6\right)\cos(2(c+dx))-3780a^7b\cos(3(c+dx))+8400a^5b^3\cos(3(c+dx))-4480a^3b^5\cos(3(c+dx))+588a^8\cos(4(c+dx))-1848a^6b^2\cos(4(c+dx))+1120a^4b^4\cos(4(c+dx))+476a^7b\cos(5(c+dx))-336a^5b^3\cos(5(c+dx))-132a^8\cos(6(c+dx))+112a^6b^2\cos(6(c+dx))-40a^7b\cos(7(c+dx))+15a^8\cos(8(c+dx))+26880a^6b^2\log(b+a\cos(c+dx))-161280a^4b^4\log(b+a\cos(c+dx))+241920a^2b^6\log(b+a\cos(c+dx))-107520b^8\log(b+a\cos(c+dx))+1680ab\cos(c+dx)\left(-8a^6+67a^4b^2-116a^2b^4+56b^6+16\left(a^2-4b^2\right)\left(a^2-b^2\right)^2\log(b+a\cos(c+dx))\right)}{13440a^9d(b+a\cos(c+dx))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^7/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-3675*a^8 + 61320*a^6*b^2 - 132720*a^4*b^4 + 87360*a^2*b^6 - 13440*b^8 - 1
40*(21*a^8 - 228*a^6*b^2 + 400*a^4*b^4 - 192*a^2*b^6)*Cos[2*(c + d*x)] - 37
80*a^7*b*Cos[3*(c + d*x)] + 8400*a^5*b^3*Cos[3*(c + d*x)] - 4480*a^3*b^5*Co
s[3*(c + d*x)] + 588*a^8*Cos[4*(c + d*x)] - 1848*a^6*b^2*Cos[4*(c + d*x)] +
 1120*a^4*b^4*Cos[4*(c + d*x)] + 476*a^7*b*Cos[5*(c + d*x)] - 336*a^5*b^3*C
os[5*(c + d*x)] - 132*a^8*Cos[6*(c + d*x)] + 112*a^6*b^2*Cos[6*(c + d*x)] -
 40*a^7*b*Cos[7*(c + d*x)] + 15*a^8*Cos[8*(c + d*x)] + 26880*a^6*b^2*Log[b
+ a*Cos[c + d*x]] - 161280*a^4*b^4*Log[b + a*Cos[c + d*x]] + 241920*a^2*b^6
```

```
*Log[b + a*Cos[c + d*x]] - 107520*b^8*Log[b + a*Cos[c + d*x]] + 1680*a*b*Co
s[c + d*x]*(-8*a^6 + 67*a^4*b^2 - 116*a^2*b^4 + 56*b^6 + 16*(a^2 - 4*b^2)*(
a^2 - b^2)^2*Log[b + a*Cos[c + d*x]]))/(13440*a^9*d*(b + a*Cos[c + d*x]))
```

Maple [A]
time = 0.29, size = 321, normalized size = 1.20 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^7/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/a^8*(1/7*cos(d*x+c)^7*a^6-1/3*b*cos(d*x+c)^6*a^5-3/5*a^6*cos(d*x+c)^
5+3/5*a^4*b^2*cos(d*x+c)^5+3/2*a^5*b*cos(d*x+c)^4-a^3*b^3*cos(d*x+c)^4+a^6*
cos(d*x+c)^3-3*a^4*b^2*cos(d*x+c)^3+5/3*a^2*b^4*cos(d*x+c)^3-3*a^5*b*cos(d*
x+c)^2+6*a^3*b^3*cos(d*x+c)^2-3*a*b^5*cos(d*x+c)^2-a^6*cos(d*x+c)+9*a^4*b^2
*cos(d*x+c)-15*a^2*b^4*cos(d*x+c)+7*b^6*cos(d*x+c))+b^2*(a^6-3*a^4*b^2+3*a^
2*b^4-b^6)/a^9/(b+a*cos(d*x+c))+2*b/a^9*(a^6-6*a^4*b^2+9*a^2*b^4-4*b^6)*ln(
b+a*cos(d*x+c)))
```

Maxima [A]
time = 0.27, size = 271, normalized size = 1.01

$$\frac{\frac{210\,\left(a^{6}b^{2}-3\,a^{4}b^{4}+3\,a^{2}b^{6}-b^{8}\right)}{a^{10}\cos\left(dx+c\right)+a^{9}b}+\frac{30\,a^{6}\cos\left(dx+c\right)^{7}-70\,a^{5}b\cos\left(dx+c\right)^{6}-126\,\left(a^{6}-a^{4}b^{2}\right)\cos\left(dx+c\right)^{5}+105\,\left(3\,a^{5}b-2\,a^{3}b^{3}\right)\cos\left(dx+c\right)^{4}+70\,\left(3\,a^{6}-9\,a^{4}b^{2}+5\,a^{2}b^{4}\right)\cos\left(dx+c\right)^{3}-630\,\left(a^{5}b-2\,a^{3}b^{3}+ab^{5}\right)\cos\left(dx+c\right)^{2}-210\,\left(a^{6}-9\,a^{4}b^{2}+15\,a^{2}b^{4}-7\,b^{6}\right)\cos\left(dx+c\right)}{a^{8}}+\frac{420\,\left(a^{6}b-6\,a^{4}b^{3}+9\,a^{2}b^{5}-4\,b^{7}\right)\log\left(a\cos\left(dx+c\right)+b\right)}{a^{9}}}{210\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/210*(210*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)/(a^10*cos(d*x + c) + a^9
*b) + (30*a^6*cos(d*x + c)^7 - 70*a^5*b*cos(d*x + c)^6 - 126*(a^6 - a^4*b^2
)*cos(d*x + c)^5 + 105*(3*a^5*b - 2*a^3*b^3)*cos(d*x + c)^4 + 70*(3*a^6 - 9
*a^4*b^2 + 5*a^2*b^4)*cos(d*x + c)^3 - 630*(a^5*b - 2*a^3*b^3 + a*b^5)*cos(
d*x + c)^2 - 210*(a^6 - 9*a^4*b^2 + 15*a^2*b^4 - 7*b^6)*cos(d*x + c))/a^8 +
 420*(a^6*b - 6*a^4*b^3 + 9*a^2*b^5 - 4*b^7)*log(a*cos(d*x + c) + b)/a^9)/d
```

Fricas [A]
time = 2.70, size = 344, normalized size = 1.29

$$\frac{120\,a^{8}\cos\left(dx+c\right)^{8}-160\,a^{7}b\cos\left(dx+c\right)^{7}+1715\,a^{6}b^{2}-4725\,a^{4}b^{4}+3780\,a^{2}b^{6}-840\,b^{8}-56\,\left(9\,a^{8}-4\,a^{6}b^{2}\right)\cos\left(dx+c\right)^{6}+84\,\left(9\,a^{7}b-4\,a^{5}b^{3}\right)\cos\left(dx+c\right)^{5}+140\,\left(6\,a^{8}-9\,a^{6}b^{2}+4\,a^{4}b^{4}\right)\cos\left(dx+c\right)^{4}-280\,\left(6\,a^{7}b-9\,a^{5}b^{3}+4\,a^{3}b^{5}\right)\cos\left(dx+c\right)^{3}-840\,\left(a^{8}-6\,a^{6}b^{2}+9\,a^{4}b^{4}-4\,a^{2}b^{6}\right)\cos\left(dx+c\right)^{2}+35\,\left(a^{7}b+153\,a^{5}b^{3}-324\,a^{3}b^{5}+168\,ab^{7}\right)\cos\left(dx+c\right)+1680\,\left(a^{6}b^{2}-6\,a^{4}b^{4}+9\,a^{2}b^{6}-4\,b^{8}+\left(a^{7}b-6\,a^{5}b^{3}+9\,a^{3}b^{5}-4\,ab^{7}\right)\cos\left(dx+c\right)\right)\log\left(a\cos\left(dx+c\right)+b\right)}{840\,\left(a^{10}d\cos\left(dx+c\right)+a^{9}bd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/840*(120*a^8*cos(d*x + c)^8 - 160*a^7*b*cos(d*x + c)^7 + 1715*a^6*b^2 - 4
725*a^4*b^4 + 3780*a^2*b^6 - 840*b^8 - 56*(9*a^8 - 4*a^6*b^2)*cos(d*x + c)^
6 + 84*(9*a^7*b - 4*a^5*b^3)*cos(d*x + c)^5 + 140*(6*a^8 - 9*a^6*b^2 + 4*a^
4*b^4)*cos(d*x + c)^4 - 280*(6*a^7*b - 9*a^5*b^3 + 4*a^3*b^5)*cos(d*x + c)^
3 - 840*(a^8 - 6*a^6*b^2 + 9*a^4*b^4 - 4*a^2*b^6)*cos(d*x + c)^2 + 35*(a^7*
```

```
b + 153*a^5*b^3 - 324*a^3*b^5 + 168*a*b^7)*cos(d*x + c) + 1680*(a^6*b^2 - 6
*a^4*b^4 + 9*a^2*b^6 - 4*b^8 + (a^7*b - 6*a^5*b^3 + 9*a^3*b^5 - 4*a*b^7)*co
s(d*x + c))*log(a*cos(d*x + c) + b))/(a^10*d*cos(d*x + c) + a^9*b*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**7/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1861 vs. $2(257) = 514$.
time = 0.52, size = 1861, normalized size = 6.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/210*(420*(a^7*b - a^6*b^2 - 6*a^5*b^3 + 6*a^4*b^4 + 9*a^3*b^5 - 9*a^2*b^6
 - 4*a*b^7 + 4*b^8)*log(abs(a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
 - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^10 - a^9*b) - 420*(a^6*b -
6*a^4*b^3 + 9*a^2*b^5 - 4*b^7)*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) +
1) + 1))/a^9 - 420*(a^7*b - 7*a^5*b^3 - 4*a^4*b^4 + 11*a^3*b^5 + 8*a^2*b^6
- 5*a*b^7 - 4*b^8 + a^7*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - a^6*b^2*(
cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 6*a^5*b^3*(cos(d*x + c) - 1)/(cos(d*
x + c) + 1) + 6*a^4*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*a^3*b^5*(
cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 9*a^2*b^6*(cos(d*x + c) - 1)/(cos(d*
x + c) + 1) - 4*a*b^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 4*b^8*(cos(d*
x + c) - 1)/(cos(d*x + c) + 1))/((a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))*a^9) + (192*a^7 - 1089*a^
6*b - 2772*a^5*b^2 + 6534*a^4*b^3 + 5600*a^3*b^4 - 9801*a^2*b^5 - 2940*a*b^
6 + 4356*b^7 - 1344*a^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 8463*a^6*b*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 18144*a^5*b^2*(cos(d*x + c) - 1)/(c
os(d*x + c) + 1) - 49098*a^4*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 35
000*a^3*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 71127*a^2*b^5*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) + 17640*a*b^6*(cos(d*x + c) - 1)/(cos(d*x + c)
 + 1) - 30492*b^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 4032*a^7*(cos(d*x
 + c) - 1)^2/(cos(d*x + c) + 1)^2 - 28749*a^6*b*(cos(d*x + c) - 1)^2/(cos(d
*x + c) + 1)^2 - 48132*a^5*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 +
157374*a^4*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 88200*a^3*b^4*(c
os(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 218421*a^2*b^5*(cos(d*x + c) - 1)
```

```
^2/(cos(d*x + c) + 1)^2 - 44100*a*b^6*(cos(d*x + c) - 1)^2/(cos(d*x + c) +
1)^2 + 91476*b^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 6720*a^7*(cos(
d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 56035*a^6*b*(cos(d*x + c) - 1)^3/(co
s(d*x + c) + 1)^3 + 60480*a^5*b^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3
 - 272370*a^4*b^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 114800*a^3*b^
4*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 368235*a^2*b^5*(cos(d*x + c)
- 1)^3/(cos(d*x + c) + 1)^3 + 58800*a*b^6*(cos(d*x + c) - 1)^3/(cos(d*x + c
) + 1)^3 - 152460*b^7*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 56035*a^6
*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 36540*a^5*b^2*(cos(d*x + c)
- 1)^4/(cos(d*x + c) + 1)^4 + 272370*a^4*b^3*(cos(d*x + c) - 1)^4/(cos(d*x
+ c) + 1)^4 + 81200*a^3*b^4*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 368
235*a^2*b^5*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 44100*a*b^6*(cos(d*
x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 152460*b^7*(cos(d*x + c) - 1)^4/(cos(d
*x + c) + 1)^4 + 28749*a^6*b*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 10
080*a^5*b^2*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 157374*a^4*b^3*(cos
(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 29400*a^3*b^4*(cos(d*x + c) - 1)^5/
(cos(d*x + c) + 1)^5 + 218421*a^2*b^5*(cos(d*x + c) - 1)^5/(cos(d*x + c) +
1)^5 + 17640*a*b^6*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 91476*b^7*(c
os(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 8463*a^6*b*(cos(d*x + c) - 1)^6/(
cos(d*x + c) + 1)^6 - 1260*a^5*b^2*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^
6 + 49098*a^4*b^3*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 4200*a^3*b^4*
(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 71127*a^2*b^5*(cos(d*x + c) - 1
)^6/(cos(d*x + c) + 1)^6 - 2940*a*b^6*(cos(d*x + c) - 1)^6/(cos(d*x + c) +
1)^6 + 30492*b^7*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 1089*a^6*b*(co
s(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 - 6534*a^4*b^3*(cos(d*x + c) - 1)^7/
(cos(d*x + c) + 1)^7 + 9801*a^2*b^5*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)
^7 - 4356*b^7*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7)/(a^9*((cos(d*x + c
) - 1)/(cos(d*x + c) + 1) - 1)^7))/d
```

Mupad [B]
time = 0.19, size = 588, normalized size = 2.20

$$\frac{\cos(c+dx)^4\left(\frac{b^3}{2a^5}+\frac{b\left(\frac{3}{a^2}-\frac{3b^2}{a^4}\right)}{2a}\right)}{d}-\frac{\cos(c+dx)^2\left(\frac{b^2\left(\frac{2b^3}{a^5}+\frac{2b\left(\frac{3}{a^2}-\frac{3b^2}{a^4}\right)}{a}\right)}{2a^2}+\frac{b\left(\frac{3}{a^2}+\frac{b^2\left(\frac{3}{a^2}-\frac{3b^2}{a^4}\right)}{a^2}-\frac{2b\left(\frac{2b^3}{a^5}+\frac{2b\left(\frac{3}{a^2}-\frac{3b^2}{a^4}\right)}{a}\right)}{a}\right)}{a}\right)}{d}-\frac{\cos(c+dx)^5\left(\frac{3}{5a^2}-\frac{3b^2}{5a^4}\right)}{d}+\frac{\cos(c+dx)^7}{7a^2d}-\frac{\cos(c+dx)\left(\frac{1}{a^2}+\frac{b^2\left(\frac{3}{a^2}+\frac{b^2\left(\frac{3}{a^2}-\frac{3b^2}{a^4}\right)}{a^2}-\frac{2b\left(\frac{2b^3}{a^5}+\frac{2b\left(\frac{3}{a^2}-\frac{3b^2}{a^4}\right)}{a}\right)}{a}\right)}{a^2}-\frac{2b\left(\frac{b^2\left(\frac{2b^3}{a^5}+\frac{2b\left(\frac{3}{a^2}-\frac{3b^2}{a^4}\right)}{a}\right)}{a^2}+\frac{2b\left(\frac{3}{a^2}+\frac{b^2\left(\frac{3}{a^2}-\frac{3b^2}{a^4}\right)}{a^2}-\frac{2b\left(\frac{2b^3}{a^5}+\frac{2b\left(\frac{3}{a^2}-\frac{3b^2}{a^4}\right)}{a}\right)}{a}\right)}{a}\right)}{a}\right)}{d}+\frac{\cos(c+dx)^3\left(\frac{1}{a^2}+\frac{b^2\left(\frac{3}{a^2}-\frac{3b^2}{a^4}\right)}{3a^2}-\frac{2b\left(\frac{2b^3}{a^5}+\frac{2b\left(\frac{3}{a^2}-\frac{3b^2}{a^4}\right)}{a}\right)}{3a}\right)}{d}-\frac{b\cos(c+dx)^6}{3a^3d}-\frac{-a^6b^2+3a^4b^4-3a^2b^6+b^8}{a\,d\,(\cos(c+dx)\,a^9+b\,a^8)}+\frac{\ln(b+a\cos(c+dx))\,(2a^6b-12a^4b^3+18a^2b^5-8b^7)}{a^9d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^7/(a + b/cos(c + d*x))^2,x)
```

```
[Out] (cos(c + d*x)^4*(b^3/(2*a^5) + (b*(3/a^2 - (3*b^2)/a^4))/(2*a)))/d - (cos(c
 + d*x)^2*((b^2*((2*b^3)/a^5 + (2*b*(3/a^2 - (3*b^2)/a^4))/a))/(2*a^2) + (b
*(3/a^2 + (b^2*(3/a^2 - (3*b^2)/a^4))/a^2 - (2*b*((2*b^3)/a^5 + (2*b*(3/a^2
 - (3*b^2)/a^4))/a))/a))/a))/d - (cos(c + d*x)^5*(3/(5*a^2) - (3*b^2)/(5*a^
4)))/d + cos(c + d*x)^7/(7*a^2*d) - (cos(c + d*x)*(1/a^2 + (b^2*(3/a^2 + (b
^2*(3/a^2 - (3*b^2)/a^4))/a^2 - (2*b*((2*b^3)/a^5 + (2*b*(3/a^2 - (3*b^2)/a
```

```
^4))/a))/a))/a^2 - (2*b*((b^2*((2*b^3)/a^5 + (2*b*(3/a^2 - (3*b^2)/a^4))/a)
)/a^2 + (2*b*(3/a^2 + (b^2*(3/a^2 - (3*b^2)/a^4))/a^2 - (2*b*((2*b^3)/a^5 +
 (2*b*(3/a^2 - (3*b^2)/a^4))/a))/a))/a))/a))/d + (cos(c + d*x)^3*(1/a^2 + (
b^2*(3/a^2 - (3*b^2)/a^4))/(3*a^2) - (2*b*((2*b^3)/a^5 + (2*b*(3/a^2 - (3*b
^2)/a^4))/a))/(3*a)))/d - (b*cos(c + d*x)^6)/(3*a^3*d) - (b^8 - 3*a^2*b^6 +
 3*a^4*b^4 - a^6*b^2)/(a*d*(a^9*cos(c + d*x) + a^8*b)) + (log(b + a*cos(c +
 d*x))*(2*a^6*b - 8*b^7 + 18*a^2*b^5 - 12*a^4*b^3))/(a^9*d)
```

3.210 $\int \frac{\sin^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=194

$$-\frac{(a^4-6a^2b^2+5b^4)\cos(c+dx)}{a^6d}-\frac{2b(a^2-b^2)\cos^2(c+dx)}{a^5d}+\frac{(2a^2-3b^2)\cos^3(c+dx)}{3a^4d}+\frac{b\cos^4(c+dx)}{2a^3d}-\frac{\cos^5(}{5}$$

```
[Out] -(a^4-6*a^2*b^2+5*b^4)*cos(d*x+c)/a^6/d-2*b*(a^2-b^2)*cos(d*x+c)^2/a^5/d+1/
3*(2*a^2-3*b^2)*cos(d*x+c)^3/a^4/d+1/2*b*cos(d*x+c)^4/a^3/d-1/5*cos(d*x+c)^
5/a^2/d+b^2*(a^2-b^2)^2/a^7/d/(b+a*cos(d*x+c))+2*b*(a^4-4*a^2*b^2+3*b^4)*ln
(b+a*cos(d*x+c))/a^7/d
```

Rubi [A]
time = 0.21, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3957, 2916, 12, 962}

$$\frac{b\cos^4(c+dx)}{2a^3d}-\frac{\cos^5(c+dx)}{5a^2d}+\frac{b^2(a^2-b^2)^2}{a^7d(a\cos(c+dx)+b)}-\frac{2b(a^2-b^2)\cos^2(c+dx)}{a^5d}+\frac{(2a^2-3b^2)\cos^3(c+dx)}{3a^4d}+\frac{2b(a^4-4a^2b^2+3b^4)\log(a\cos(c+dx)+b)}{a^7d}-\frac{(a^4-6a^2b^2+5b^4)\cos(c+dx)}{a^6d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^5/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(((a^4 - 6*a^2*b^2 + 5*b^4)*Cos[c + d*x])/(a^6*d)) - (2*b*(a^2 - b^2)*Cos[
c + d*x]^2)/(a^5*d) + ((2*a^2 - 3*b^2)*Cos[c + d*x]^3)/(3*a^4*d) + (b*Cos[c
 + d*x]^4)/(2*a^3*d) - Cos[c + d*x]^5/(5*a^2*d) + (b^2*(a^2 - b^2)^2)/(a^7*
d*(b + a*Cos[c + d*x])) + (2*b*(a^4 - 4*a^2*b^2 + 3*b^4)*Log[b + a*Cos[c +
d*x]])/(a^7*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 962

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
```

```
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx &= \int \frac{\cos^2(c+dx)\sin^5(c+dx)}{(-b-a\cos(c+dx))^2}\,dx \\
&= \frac{\text{Subst}\left(\int \frac{x^2\left(a^2-x^2\right)^2}{a^2(-b+x)^2}\,dx, x, -a\cos(c+dx)\right)}{a^5d} \\
&= \frac{\text{Subst}\left(\int \frac{x^2\left(a^2-x^2\right)^2}{(-b+x)^2}\,dx, x, -a\cos(c+dx)\right)}{a^7d} \\
&= \frac{\text{Subst}\left(\int \left(a^4\left(1+\frac{-6a^2b^2+5b^4}{a^4}\right)+\frac{b^2\left(a^2-b^2\right)^2}{(b-x)^2}-\frac{2b\left(a^4-4a^2b^2+3b^4\right)}{b-x}+4b\left(-a^2+b^2\right)x -\right.\right.}{a^7d} \\
&= -\frac{\left(a^4-6a^2b^2+5b^4\right)\cos(c+dx)}{a^6d}-\frac{2b\left(a^2-b^2\right)\cos^2(c+dx)}{a^5d}+\frac{\left(2a^2-3b^2\right)\cos^3}{3a^4d}
\end{aligned}$$

Mathematica [A]
time = 0.79, size = 280, normalized size = 1.44

$$\frac{-150a^6+1740a^4b^2-2160a^2b^4+480b^6-5\left(25a^6-168a^4b^2+144a^2b^4\right)\cos(2(c+dx))-115a^5b\cos(3(c+dx))+120a^3b^3\cos(3(c+dx))+22a^6\cos(4(c+dx))-30a^4b^2\cos(4(c+dx))+9a^5b\cos(5(c+dx))-3a^6\cos(6(c+dx))+960a^4b^2\log(b+a\cos(c+dx))-3840a^2b^4\log(b+a\cos(c+dx))+2880b^6\log(b+a\cos(c+dx))+120ab\cos(c+dx)\left(-4a^4+23a^2b^2-20b^4+8\left(a^4-4a^2b^2+3b^4\right)\log(b+a\cos(c+dx))\right)}{480a^7d(b+a\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^5/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-150*a^6 + 1740*a^4*b^2 - 2160*a^2*b^4 + 480*b^6 - 5*(25*a^6 - 168*a^4*b^2
 + 144*a^2*b^4)*Cos[2*(c + d*x)] - 115*a^5*b*Cos[3*(c + d*x)] + 120*a^3*b^3
*Cos[3*(c + d*x)] + 22*a^6*Cos[4*(c + d*x)] - 30*a^4*b^2*Cos[4*(c + d*x)] +
 9*a^5*b*Cos[5*(c + d*x)] - 3*a^6*Cos[6*(c + d*x)] + 960*a^4*b^2*Log[b + a*
Cos[c + d*x]] - 3840*a^2*b^4*Log[b + a*Cos[c + d*x]] + 2880*b^6*Log[b + a*C
os[c + d*x]] + 120*a*b*Cos[c + d*x]*(-4*a^4 + 23*a^2*b^2 - 20*b^4 + 8*(a^4
- 4*a^2*b^2 + 3*b^4)*Log[b + a*Cos[c + d*x]]))/(480*a^7*d*(b + a*Cos[c + d*
x]))
```

Maple [A]
time = 0.20, size = 198, normalized size = 1.02

method	result
derivativedivides	$-\frac{\frac{\left(\cos^5(dx+c)\right)a^4}{5}-\frac{b\left(\cos^4(dx+c)\right)a^3}{2}-\frac{2a^4\left(\cos^3(dx+c)\right)}{3}+a^2b^2\left(\cos^3(dx+c)\right)+2a^3b\left(\cos^2(dx+c)\right)-2a\,b^3\left(\cos^2(dx+c)\right)+a^4\cos(dx+c}{a^6}}{d}$
default	$-\frac{\frac{\left(\cos^5(dx+c)\right)a^4}{5}-\frac{b\left(\cos^4(dx+c)\right)a^3}{2}-\frac{2a^4\left(\cos^3(dx+c)\right)}{3}+a^2b^2\left(\cos^3(dx+c)\right)+2a^3b\left(\cos^2(dx+c)\right)-2a\,b^3\left(\cos^2(dx+c)\right)+a^4\cos(dx+c}{a^6}}{d}$
risch	$\frac{2b^2(a^4-2b^2a^2+b^4)\,e^{i(dx+c)}}{a^7d\left(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a\right)}+\frac{8ib^3x}{a^5}-\frac{6ib^5x}{a^7}-\frac{3b\,e^{2i(dx+c)}}{8a^3d}+\frac{b^3e^{2i(dx+c)}}{2a^5d}+\frac{21\,e^{i(dx+c)}b^2}{8a^4d}-\frac{5\,e^{i(dx+c)}b^4}{2a^6d}$
norman	$\frac{(32a^5+32a^4b+96a^3b^2-360a^2b^3-144a\,b^4+360b^5)(a+b)}{60a^6bd}-\frac{(32a^6-128a^5b+120a^3b^3-408a^2b^4-72a\,b^5+360b^6)\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12a^6bd}+\frac{(32a}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^5/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/a^6*(1/5*cos(d*x+c)^5*a^4-1/2*b*cos(d*x+c)^4*a^3-2/3*a^4*cos(d*x+c)
^3+a^2*b^2*cos(d*x+c)^3+2*a^3*b*cos(d*x+c)^2-2*a*b^3*cos(d*x+c)^2+a^4*cos(d
*x+c)-6*a^2*b^2*cos(d*x+c)+5*b^4*cos(d*x+c))+b^2*(a^4-2*a^2*b^2+b^4)/a^7/(b
+a*cos(d*x+c))+2*b/a^7*(a^4-4*a^2*b^2+3*b^4)*ln(b+a*cos(d*x+c)))
```

Maxima [A]
time = 0.28, size = 184, normalized size = 0.95

$$\frac{\frac{30\,(a^4b^2-2\,a^2b^4+b^6)}{a^8\cos(dx+c)+a^7b}-\frac{6\,a^4\cos(dx+c)^5-15\,a^3b\cos(dx+c)^4-10\,(2\,a^4-3\,a^2b^2)\cos(dx+c)^3+60\,(a^3b-ab^3)\cos(dx+c)^2+30\,(a^4-6\,a^2b^2+5\,b^4)\cos(dx+c)}{a^6}+\frac{60\,(a^4b-4\,a^2b^3+3\,b^5)\log(a\cos(dx+c)+b)}{a^7}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/30*(30*(a^4*b^2 - 2*a^2*b^4 + b^6)/(a^8*cos(d*x + c) + a^7*b) - (6*a^4*co
s(d*x + c)^5 - 15*a^3*b*cos(d*x + c)^4 - 10*(2*a^4 - 3*a^2*b^2)*cos(d*x + c
)^3 + 60*(a^3*b - a*b^3)*cos(d*x + c)^2 + 30*(a^4 - 6*a^2*b^2 + 5*b^4)*cos(
d*x + c))/a^6 + 60*(a^4*b - 4*a^2*b^3 + 3*b^5)*log(a*cos(d*x + c) + b)/a^7)
/d
```

Fricas [A]
time = 2.61, size = 240, normalized size = 1.24

$$-\frac{48\,a^6\cos(dx+c)^6-72\,a^5b\cos(dx+c)^5-435\,a^4b^2+720\,a^2b^4-240\,b^6-40\,(4\,a^6-3\,a^4b^2)\cos(dx+c)^4+80\,(4\,a^5b-3\,a^3b^3)\cos(dx+c)^3+240\,(a^6-4\,a^4b^2+3\,a^2b^4)\cos(dx+c)^2+15\,(3\,a^5b-80\,a^3b^3+80\,ab^5)\cos(dx+c)-480\,(a^4b^2-4\,a^2b^4+3\,b^6+(a^5b-4\,a^3b^3+3\,ab^5)\cos(dx+c))\log(a\cos(dx+c)+b)}{240\,(a^8d\cos(dx+c)+a^7bd)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/240*(48*a^6*cos(d*x + c)^6 - 72*a^5*b*cos(d*x + c)^5 - 435*a^4*b^2 + 720
*a^2*b^4 - 240*b^6 - 40*(4*a^6 - 3*a^4*b^2)*cos(d*x + c)^4 + 80*(4*a^5*b -
```

```
3*a^3*b^3)*cos(d*x + c)^3 + 240*(a^6 - 4*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c)^
2 + 15*(3*a^5*b - 80*a^3*b^3 + 80*a*b^5)*cos(d*x + c) - 480*(a^4*b^2 - 4*a^
2*b^4 + 3*b^6 + (a^5*b - 4*a^3*b^3 + 3*a*b^5)*cos(d*x + c))*log(a*cos(d*x +
 c) + b))/(a^8*d*cos(d*x + c) + a^7*b*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**5/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1102 vs. 2(188) = 376.
time = 0.48, size = 1102, normalized size = 5.68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/30*(60*(a^5*b - a^4*b^2 - 4*a^3*b^3 + 4*a^2*b^4 + 3*a*b^5 - 3*b^6)*log(ab
s(a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(c
os(d*x + c) + 1)))/(a^8 - a^7*b) - 60*(a^4*b - 4*a^2*b^3 + 3*b^5)*log(abs(-
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^7 - 60*(a^5*b - 5*a^3*b^3 - 3
*a^2*b^4 + 4*a*b^5 + 3*b^6 + a^5*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) -
a^4*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 4*a^3*b^3*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) + 4*a^2*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 3*
a*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 3*b^6*(cos(d*x + c) - 1)/(cos
(d*x + c) + 1))/((a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(
d*x + c) - 1)/(cos(d*x + c) + 1))*a^7) + (32*a^5 - 137*a^4*b - 300*a^3*b^2
+ 548*a^2*b^3 + 300*a*b^4 - 411*b^5 - 160*a^5*(cos(d*x + c) - 1)/(cos(d*x +
 c) + 1) + 805*a^4*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1320*a^3*b^2*(
cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2980*a^2*b^3*(cos(d*x + c) - 1)/(cos
(d*x + c) + 1) - 1200*a*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2055*b^
5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 320*a^5*(cos(d*x + c) - 1)^2/(cos
(d*x + c) + 1)^2 - 1970*a^4*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 1
920*a^3*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 6200*a^2*b^3*(cos(d
*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 1800*a*b^4*(cos(d*x + c) - 1)^2/(cos(
d*x + c) + 1)^2 - 4110*b^5*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 1970
*a^4*b*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 1080*a^3*b^2*(cos(d*x +
c) - 1)^3/(cos(d*x + c) + 1)^3 - 6200*a^2*b^3*(cos(d*x + c) - 1)^3/(cos(d*x
 + c) + 1)^3 - 1200*a*b^4*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 4110*
```

```
b^5*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 805*a^4*b*(cos(d*x + c) - 1
)^4/(cos(d*x + c) + 1)^4 - 180*a^3*b^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) +
 1)^4 + 2980*a^2*b^3*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 300*a*b^4*
(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 2055*b^5*(cos(d*x + c) - 1)^4/(
cos(d*x + c) + 1)^4 + 137*a^4*b*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 -
 548*a^2*b^3*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 411*b^5*(cos(d*x +
 c) - 1)^5/(cos(d*x + c) + 1)^5)/(a^7*((cos(d*x + c) - 1)/(cos(d*x + c) + 1
) - 1)^5))/d
```

Mupad [B]

time = 0.12, size = 253, normalized size = 1.30

$$\frac{\cos\left(c+d\,x\right)^3\,\left(\frac{2}{3\,a^2}-\frac{b^2}{a^4}\right)}{d}-\frac{\cos\left(c+d\,x\right)^2\,\left(\frac{b^3}{a^5}+\frac{b\,\left(\frac{2}{a^2}-\frac{3\,b^2}{a^4}\right)}{a}\right)}{d}-\frac{\cos\left(c+d\,x\right)\,\left(\frac{1}{a^2}+\frac{b^2\,\left(\frac{2}{a^2}-\frac{3\,b^2}{a^4}\right)}{a^2}-\frac{2\,b\,\left(\frac{2\,b^3}{a^5}+\frac{2\,b\,\left(\frac{2}{a^2}-\frac{3\,b^2}{a^4}\right)}{a}\right)}{a}\right)}{d}-\frac{\cos\left(c+d\,x\right)^5}{5\,a^2\,d}+\frac{b\cos\left(c+d\,x\right)^4}{2\,a^3\,d}+\frac{\ln\left(b+a\,\cos\left(c+d\,x\right)\right)\,\left(2\,a^4\,b-8\,a^2\,b^3+6\,b^5\right)}{a^7\,d}+\frac{a^4\,b^2-2\,a^2\,b^4+b^6}{a\,d\,\left(\cos\left(c+d\,x\right)\,a^7+b\,a^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5/(a + b/cos(c + d*x))^2,x)
```

```
[Out] (cos(c + d*x)^3*(2/(3*a^2) - b^2/a^4))/d - (cos(c + d*x)^2*(b^3/a^5 + (b*(2
/a^2 - (3*b^2)/a^4))/a))/d - (cos(c + d*x)*(1/a^2 + (b^2*(2/a^2 - (3*b^2)/a
^4))/a^2 - (2*b*((2*b^3)/a^5 + (2*b*(2/a^2 - (3*b^2)/a^4))/a))/a))/d - cos(
c + d*x)^5/(5*a^2*d) + (b*cos(c + d*x)^4)/(2*a^3*d) + (log(b + a*cos(c + d*
x))*(2*a^4*b + 6*b^5 - 8*a^2*b^3))/(a^7*d) + (b^6 - 2*a^2*b^4 + a^4*b^2)/(a
*d*(a^7*cos(c + d*x) + a^6*b))
```

3.211 $\int \frac{\sin^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=119

$$-\frac{(a^2-3b^2)\cos(c+dx)}{a^4d}-\frac{b\cos^2(c+dx)}{a^3d}+\frac{\cos^3(c+dx)}{3a^2d}+\frac{b^2(a^2-b^2)}{a^5d(b+a\cos(c+dx))}+\frac{2b(a^2-2b^2)\log(b+a\cos$$

```
[Out] -(a^2-3*b^2)*cos(d*x+c)/a^4/d-b*cos(d*x+c)^2/a^3/d+1/3*cos(d*x+c)^3/a^2/d+b
^2*(a^2-b^2)/a^5/d/(b+a*cos(d*x+c))+2*b*(a^2-2*b^2)*ln(b+a*cos(d*x+c))/a^5/
d
```

Rubi [A]
time = 0.16, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3957, 2916, 12, 908}

$$-\frac{b\cos^2(c+dx)}{a^3d}+\frac{\cos^3(c+dx)}{3a^2d}+\frac{b^2(a^2-b^2)}{a^5d(a\cos(c+dx)+b)}+\frac{2b(a^2-2b^2)\log(a\cos(c+dx)+b)}{a^5d}-\frac{(a^2-3b^2)\cos(c+dx)}{a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(((a^2 - 3*b^2)*Cos[c + d*x])/(a^4*d)) - (b*Cos[c + d*x]^2)/(a^3*d) + Cos[
c + d*x]^3/(3*a^2*d) + (b^2*(a^2 - b^2))/(a^5*d*(b + a*Cos[c + d*x])) + (2*
b*(a^2 - 2*b^2)*Log[b + a*Cos[c + d*x]])/(a^5*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx &= \int \frac{\cos^2(c+dx)\sin^3(c+dx)}{(-b-a\cos(c+dx))^2}\,dx\\ &= \frac{\text{Subst}\left(\int \frac{x^2(a^2-x^2)}{a^2(-b+x)^2}\,dx, x, -a\cos(c+dx)\right)}{a^3d}\\ &= \frac{\text{Subst}\left(\int \frac{x^2(a^2-x^2)}{(-b+x)^2}\,dx, x, -a\cos(c+dx)\right)}{a^5d}\\ &= \frac{\text{Subst}\left(\int \left(a^2\left(1-\frac{3b^2}{a^2}\right)-\frac{b^2(-a^2+b^2)}{(b-x)^2}+\frac{2b(-a^2+2b^2)}{b-x}-2bx-x^2\right)\,dx, x, -a\cos(c+\right.}{a^5d}\\ &= -\frac{(a^2-3b^2)\cos(c+dx)}{a^4d}-\frac{b\cos^2(c+dx)}{a^3d}+\frac{\cos^3(c+dx)}{3a^2d}+\frac{b^2(a^2-b^2)}{a^5d(b+a\cos(c+d}\end{aligned}$$

Mathematica [A]
time = 0.33, size = 167, normalized size = 1.40

$$\frac{-9a^4+60a^2b^2-24b^4-8(a^4-3a^2b^2)\cos(2(c+dx))-4a^3b\cos(3(c+dx))+a^4\cos(4(c+dx))+48a^2b^2\log(b+a\cos(c+dx))-96b^4\log(b+a\cos(c+dx))+24ab\cos(c+dx)\left(-a^2+3b^2+2(a^2-2b^2)\log(b+a\cos(c+dx))\right)}{24a^5d(b+a\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-9*a^4 + 60*a^2*b^2 - 24*b^4 - 8*(a^4 - 3*a^2*b^2)*Cos[2*(c + d*x)] - 4*a^
3*b*Cos[3*(c + d*x)] + a^4*Cos[4*(c + d*x)] + 48*a^2*b^2*Log[b + a*Cos[c +
d*x]] - 96*b^4*Log[b + a*Cos[c + d*x]] + 24*a*b*Cos[c + d*x]*(-a^2 + 3*b^2
+ 2*(a^2 - 2*b^2)*Log[b + a*Cos[c + d*x]]))/(24*a^5*d*(b + a*Cos[c + d*x]))
```

Maple [A]
time = 0.13, size = 112, normalized size = 0.94

method	result
derivativedivides	$\frac{\frac{\frac{\left(\cos^3(dx+c)\right)a^2}{3}-b\left(\cos^2(dx+c)\right)a-a^2\cos(dx+c)+3b^2\cos(dx+c)}{a^4}+\frac{b^2\left(a^2-b^2\right)}{a^5(b+a\cos(dx+c))}+\frac{2b\left(a^2-2b^2\right)\ln(b+a\cos(dx+c))}{a^5}}{d}$

default	$\frac{\frac{\frac{\left(\cos^{3}(dx+c)\right)a^{2}}{3}-b\left(\cos^{2}(dx+c)\right)a-a^{2}\cos(dx+c)+3b^{2}\cos(dx+c)}{a^{4}}+\frac{b^{2}\left(a^{2}-b^{2}\right)}{a^{5}(b+a\cos(dx+c))}+\frac{2b\left(a^{2}-2b^{2}\right)\ln(b+a\cos(dx+c))}{a^{5}}}{d}$
risch	$\frac{8ib^{3}c}{a^{5}d}+\frac{4ib^{3}x}{a^{5}}-\frac{b\,\mathrm{e}^{2i(dx+c)}}{4a^{3}d}-\frac{3\,\mathrm{e}^{i(dx+c)}}{8a^{2}d}+\frac{3\,\mathrm{e}^{i(dx+c)}b^{2}}{2a^{4}d}-\frac{3\,\mathrm{e}^{-i(dx+c)}}{8a^{2}d}+\frac{3\,\mathrm{e}^{-i(dx+c)}b^{2}}{2a^{4}d}-\frac{b\,\mathrm{e}^{-2i(dx+c)}}{4a^{3}d}-$
norman	$\frac{-\frac{\left(4a^{4}-8b\,a^{3}+12b^{2}a^{2}+16b^{3}a-24b^{4}\right)\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6a^{4}db}+\frac{\left(4a^{3}+4b\,a^{2}+8b^{2}a-24b^{3}\right)(a+b)}{6a^{4}bd}-\frac{\left(4a^{4}-12b\,a^{3}+4b^{2}a^{2}+8b^{3}a-24b^{4}\right)\left(\tan^{6}\right.}{3a^{4}bd}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}\left(a\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/a^4*(1/3*cos(d*x+c)^3*a^2-b*cos(d*x+c)^2*a-a^2*cos(d*x+c)+3*b^2*cos(
d*x+c))+b^2*(a^2-b^2)/a^5/(b+a*cos(d*x+c))+2*b/a^5*(a^2-2*b^2)*ln(b+a*cos(d
*x+c)))
```

Maxima [A]
time = 0.26, size = 112, normalized size = 0.94

$$\frac{\frac{3\,(a^2b^2-b^4)}{a^6\cos(dx+c)+a^5b}+\frac{a^2\cos(dx+c)^3-3\,ab\cos(dx+c)^2-3\,(a^2-3\,b^2)\cos(dx+c)}{a^4}+\frac{6\,(a^2b-2\,b^3)\log(a\cos(dx+c)+b)}{a^5}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(3*(a^2*b^2 - b^4)/(a^6*cos(d*x + c) + a^5*b) + (a^2*cos(d*x + c)^3 - 3
*a*b*cos(d*x + c)^2 - 3*(a^2 - 3*b^2)*cos(d*x + c))/a^4 + 6*(a^2*b - 2*b^3)
*log(a*cos(d*x + c) + b)/a^5)/d
```

Fricas [A]
time = 2.70, size = 150, normalized size = 1.26

$$\frac{2\,a^4\cos(dx+c)^4-4\,a^3b\cos(dx+c)^3+9\,a^2b^2-6\,b^4-6\,(a^4-2\,a^2b^2)\cos(dx+c)^2-3\,(a^3b-6\,ab^3)\cos(dx+c)+12\,(a^2b^2-2\,b^4+(a^3b-2\,ab^3)\cos(dx+c))\log(a\cos(dx+c)+b)}{6\,(a^6d\cos(dx+c)+a^5bd)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a^4*cos(d*x + c)^4 - 4*a^3*b*cos(d*x + c)^3 + 9*a^2*b^2 - 6*b^4 - 6*
(a^4 - 2*a^2*b^2)*cos(d*x + c)^2 - 3*(a^3*b - 6*a*b^3)*cos(d*x + c) + 12*(a
^2*b^2 - 2*b^4 + (a^3*b - 2*a*b^3)*cos(d*x + c))*log(a*cos(d*x + c) + b))/(
a^6*d*cos(d*x + c) + a^5*b*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.46, size = 139, normalized size = 1.17

$$\frac{2\,(a^2b-2\,b^3)\log\left(|-a\cos\left(dx+c\right)-b|\right)}{a^5d}+\frac{a^2b^2-b^4}{(a\cos\left(dx+c\right)+b)a^5d}+\frac{a^4d^5\cos\left(dx+c\right)^3-3\,a^3bd^5\cos\left(dx+c\right)^2-3\,a^4d^5\cos\left(dx+c\right)+9\,a^2b^2d^5\cos\left(dx+c\right)}{3\,a^6d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2*(a^2*b - 2*b^3)*log(abs(-a*cos(d*x + c) - b))/(a^5*d) + (a^2*b^2 - b^4)/(
(a*cos(d*x + c) + b)*a^5*d) + 1/3*(a^4*d^5*cos(d*x + c)^3 - 3*a^3*b*d^5*cos
(d*x + c)^2 - 3*a^4*d^5*cos(d*x + c) + 9*a^2*b^2*d^5*cos(d*x + c))/(a^6*d^6
)
```

Mupad [B]
time = 0.09, size = 113, normalized size = 0.95

$$-\frac{\cos\left(c+d\,x\right)\,\left(\frac{1}{a^2}-\frac{3\,b^2}{a^4}\right)-\frac{{\cos\left(c+d\,x\right)}^3}{3\,a^2}+\frac{b\,{\cos\left(c+d\,x\right)}^2}{a^3}-\frac{\ln\left(b+a\,\cos\left(c+d\,x\right)\right)\,\left(2\,a^2\,b-4\,b^3\right)}{a^5}+\frac{b^4-a^2\,b^2}{a\,\left(\cos\left(c+d\,x\right)\,a^5+b\,a^4\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(a + b/cos(c + d*x))^2,x)
```

```
[Out] -(cos(c + d*x)*(1/a^2 - (3*b^2)/a^4) - cos(c + d*x)^3/(3*a^2) + (b*cos(c +
d*x)^2)/a^3 - (log(b + a*cos(c + d*x))*(2*a^2*b - 4*b^3))/a^5 + (b^4 - a^2*
b^2)/(a*(a^5*cos(c + d*x) + a^4*b)))/d
```

3.212 $\int \frac{\sin(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=57

$$-\frac{\cos(c+dx)}{a^2d}+\frac{b^2}{a^3d(b+a\cos(c+dx))}+\frac{2b\log(b+a\cos(c+dx))}{a^3d}$$

```
[Out] -cos(d*x+c)/a^2/d+b^2/a^3/d/(b+a*cos(d*x+c))+2*b*ln(b+a*cos(d*x+c))/a^3/d
```

Rubi [A]
time = 0.08, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3957, 2912, 12, 45}

$$\frac{b^2}{a^3d(a\cos(c+dx)+b)}+\frac{2b\log(a\cos(c+dx)+b)}{a^3d}-\frac{\cos(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(Cos[c + d*x]/(a^2*d)) + b^2/(a^3*d*(b + a*Cos[c + d*x])) + (2*b*Log[b + a
*Cos[c + d*x]])/(a^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
```

```
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)\sin(c+dx)}{(-b-a\cos(c+dx))^2}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{a^2(-b+x)^2}\,dx, x, -a\cos(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{(-b+x)^2}\,dx, x, -a\cos(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{b^2}{(b-x)^2}-\frac{2b}{b-x}\right)dx, x, -a\cos(c+dx)\right)}{a^3d}$$
$$= -\frac{\cos(c+dx)}{a^2d}+\frac{b^2}{a^3d(b+a\cos(c+dx))}+\frac{2b\log(b+a\cos(c+dx))}{a^3d}$$

Mathematica [A]
time = 0.11, size = 76, normalized size = 1.33

$$\frac{-a^2\cos^2(c+dx)+ab\cos(c+dx)(-1+2\log(b+a\cos(c+dx)))+b^2(1+2\log(b+a\cos(c+dx)))}{a^3d(b+a\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + b*Sec[c + d*x])^2,x]
[Out] (-(a^2*Cos[c + d*x]^2) + a*b*Cos[c + d*x]*(-1 + 2*Log[b + a*Cos[c + d*x]])
 + b^2*(1 + 2*Log[b + a*Cos[c + d*x]]))/(a^3*d*(b + a*Cos[c + d*x]))
```

Maple [A]
time = 0.06, size = 67, normalized size = 1.18

method	result
derivativedivides	$\frac{-\frac{1}{a^2\sec(dx+c)}-\frac{2b\ln(\sec(dx+c))}{a^3}-\frac{b}{a^2(a+b\sec(dx+c))}+\frac{2b\ln(a+b\sec(dx+c))}{a^3}}{d}$
default	$\frac{-\frac{1}{a^2\sec(dx+c)}-\frac{2b\ln(\sec(dx+c))}{a^3}-\frac{b}{a^2(a+b\sec(dx+c))}+\frac{2b\ln(a+b\sec(dx+c))}{a^3}}{d}$
risch	$-\frac{2ibx}{a^3}-\frac{e^{i(dx+c)}}{2a^2d}-\frac{e^{-i(dx+c)}}{2a^2d}-\frac{4ibc}{a^3d}+\frac{2b^2e^{i(dx+c)}}{a^3d(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a)}+\frac{2b\ln\left(e^{2i(dx+c)}+\frac{2b\,e^{i(dx+c)}}{a}+1\right)}{a^3d}$
norman	$\frac{\frac{2a^2+4ba+4b^2}{2a^2db}-\frac{(2a^2-4ba+4b^2)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2db}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)}-\frac{2b\ln\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3d}+\frac{2b\ln\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/a^2/sec(d*x+c)-2/a^3*b*ln(sec(d*x+c))-b/a^2/(a+b*sec(d*x+c))+2/a^3*
b*ln(a+b*sec(d*x+c)))
```

Maxima [**A**]
time = 0.27, size = 55, normalized size = 0.96

$$\frac{\frac{b^2}{a^4\cos(dx+c)+a^3b} - \frac{\cos(dx+c)}{a^2} + \frac{2\,b\log(a\cos(dx+c)+b)}{a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (b^2/(a^4*cos(d*x + c) + a^3*b) - cos(d*x + c)/a^2 + 2*b*log(a*cos(d*x + c)
 + b)/a^3)/d
```

Fricas [**A**]
time = 2.69, size = 75, normalized size = 1.32

$$-\frac{a^2\cos(dx+c)^2 + ab\cos(dx+c) - b^2 - 2\left(ab\cos(dx+c) + b^2\right)\log(a\cos(dx+c) + b)}{a^4d\cos(dx+c) + a^3bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(a^2*cos(d*x + c)^2 + a*b*cos(d*x + c) - b^2 - 2*(a*b*cos(d*x + c) + b^2)*
log(a*cos(d*x + c) + b))/(a^4*d*cos(d*x + c) + a^3*b*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(sin(c + d*x)/(a + b*sec(c + d*x))**2, x)
```

Giac [**A**]
time = 0.47, size = 61, normalized size = 1.07

$$-\frac{\cos(dx+c)}{a^2d} + \frac{2\,b\log(|-a\cos(dx+c) - b|)}{a^3d} + \frac{b^2}{(a\cos(dx+c)+b)a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -cos(d*x + c)/(a^2*d) + 2*b*log(abs(-a*cos(d*x + c) - b))/(a^3*d) + b^2/((a
*cos(d*x + c) + b)*a^3*d)
```

Mupad [B]
time = 1.02, size = 60, normalized size = 1.05

$$\frac{b^2}{d\,\left(\cos\left(c+d\,x\right)\,a^4+b\,a^3\right)}-\frac{\cos\left(c+d\,x\right)}{a^2\,d}+\frac{2\,b\,\ln\left(b+a\,\cos\left(c+d\,x\right)\right)}{a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(a + b/cos(c + d*x))^2,x)
```

```
[Out] b^2/(d*(a^4*cos(c + d*x) + a^3*b)) - cos(c + d*x)/(a^2*d) + (2*b*log(b + a*
cos(c + d*x)))/(a^3*d)
```

3.213 $\int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=109

$$\frac{b^2}{a(a^2-b^2)\,d(b+a\cos(c+dx))}+\frac{\log(1-\cos(c+dx))}{2(a+b)^2d}-\frac{\log(1+\cos(c+dx))}{2(a-b)^2d}+\frac{2ab\log(b+a\cos(c+dx))}{(a^2-b^2)^2\,d}$$

```
[Out] b^2/a/(a^2-b^2)/d/(b+a*cos(d*x+c))+1/2*ln(1-cos(d*x+c))/(a+b)^2/d-1/2*ln(1+
cos(d*x+c))/(a-b)^2/d+2*a*b*ln(b+a*cos(d*x+c))/(a^2-b^2)^2/d
```

Rubi [A]
time = 0.16, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3957, 2916, 12, 1643}

$$\frac{b^2}{ad(a^2-b^2)(a\cos(c+dx)+b)}+\frac{2ab\log(a\cos(c+dx)+b)}{d(a^2-b^2)^2}+\frac{\log(1-\cos(c+dx))}{2d(a+b)^2}-\frac{\log(\cos(c+dx)+1)}{2d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] b^2/(a*(a^2 - b^2)*d*(b + a*Cos[c + d*x])) + Log[1 - Cos[c + d*x]]/(2*(a +
b)^2*d) - Log[1 + Cos[c + d*x]]/(2*(a - b)^2*d) + (2*a*b*Log[b + a*Cos[c +
d*x]])/((a^2 - b^2)^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^2}\,dx &= \int \frac{\cos(c+dx)\cot(c+dx)}{(-b-a\cos(c+dx))^2}\,dx \\
&= \frac{a\text{Subst}\left(\int \frac{x^2}{a^2(-b+x)^2(a^2-x^2)}\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{x^2}{(-b+x)^2(a^2-x^2)}\,dx, x, -a\cos(c+dx)\right)}{ad} \\
&= \frac{\text{Subst}\left(\int \left(\frac{a}{2(a-b)^2(a-x)} + \frac{b^2}{(a-b)(a+b)(b-x)^2} - \frac{2a^2b}{(a-b)^2(a+b)^2(b-x)} + \frac{a}{2(a+b)^2(a+x)}\right)\,dx, x, -\right.}{ad} \\
&= \frac{b^2}{a(a^2-b^2)\,d(b+a\cos(c+dx))} + \frac{\log(1-\cos(c+dx))}{2(a+b)^2d} - \frac{\log(1+\cos(c+dx))}{2(a-b)^2d}
\end{aligned}$$

Mathematica [A]
time = 0.20, size = 165, normalized size = 1.51

$$\frac{-a^2\cos(c+dx)\left((a+b)^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-2ab\log(b+a\cos(c+dx))-(a-b)^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+b\left(-a(a+b)^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+2a^2b\log(b+a\cos(c+dx))+(a-b)\left(b(a+b)+a(a-b)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\right)}{a(a-b)^2(a+b)^2d(b+a\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-(a^2*Cos[c + d*x]*((a + b)^2*Log[Cos[(c + d*x)/2]] - 2*a*b*Log[b + a*Cos[
c + d*x]] - (a - b)^2*Log[Sin[(c + d*x)/2]])) + b*(-(a*(a + b)^2*Log[Cos[(c
 + d*x)/2]]) + 2*a^2*b*Log[b + a*Cos[c + d*x]] + (a - b)*(b*(a + b) + a*(a
- b)*Log[Sin[(c + d*x)/2]])))/(a*(a - b)^2*(a + b)^2*d*(b + a*Cos[c + d*x])
)
```

Maple [A]
time = 0.14, size = 98, normalized size = 0.90

method	result
derivativedivides	$\frac{-\frac{\ln(1+\cos(dx+c))}{2(a-b)^2}+\frac{\ln(-1+\cos(dx+c))}{2(a+b)^2}+\frac{b^2}{(a+b)(a-b)a(b+a\cos(dx+c))}+\frac{2ab\ln(b+a\cos(dx+c))}{(a+b)^2(a-b)^2}}{d}$
default	$\frac{-\frac{\ln(1+\cos(dx+c))}{2(a-b)^2}+\frac{\ln(-1+\cos(dx+c))}{2(a+b)^2}+\frac{b^2}{(a+b)(a-b)a(b+a\cos(dx+c))}+\frac{2ab\ln(b+a\cos(dx+c))}{(a+b)^2(a-b)^2}}{d}$

norman	$-\frac{2b^2}{d(a^3-b\,a^2-b^2a+b^3)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d(a^2+2ba+b^2)}+\frac{2ba\ln\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{d(a^4-}$
risch	$\frac{ix}{a^2-2ba+b^2}+\frac{ic}{d(a^2-2ba+b^2)}-\frac{ix}{a^2+2ba+b^2}-\frac{ic}{d(a^2+2ba+b^2)}-\frac{4iabx}{a^4-2b^2a^2+b^4}-\frac{4iabc}{d(a^4-2b^2a^2+b^4)}-\frac{}{ad(-a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/(a-b)^2*ln(1+cos(d*x+c))+1/2/(a+b)^2*ln(-1+cos(d*x+c))+b^2/(a+b)/
(a-b)/a/(b+a*cos(d*x+c))+2*a*b/(a+b)^2/(a-b)^2*ln(b+a*cos(d*x+c)))
```

Maxima [A]
time = 0.27, size = 123, normalized size = 1.13

$$\frac{\frac{4\,ab\log\left(a\cos\left(dx+c\right)+b\right)}{a^4-2\,a^2b^2+b^4}+\frac{2\,b^2}{a^3b-ab^3+\left(a^4-a^2b^2\right)\cos\left(dx+c\right)}-\frac{\log\left(\cos\left(dx+c\right)+1\right)}{a^2-2\,ab+b^2}+\frac{\log\left(\cos\left(dx+c\right)-1\right)}{a^2+2\,ab+b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(4*a*b*log(a*cos(d*x + c) + b)/(a^4 - 2*a^2*b^2 + b^4) + 2*b^2/(a^3*b -
 a*b^3 + (a^4 - a^2*b^2)*cos(d*x + c)) - log(cos(d*x + c) + 1)/(a^2 - 2*a*b
 + b^2) + log(cos(d*x + c) - 1)/(a^2 + 2*a*b + b^2))/d
```

Fricas [A]
time = 3.11, size = 210, normalized size = 1.93

$$\frac{2\,a^2b^2-2\,b^4+4\,\left(a^3b\cos\left(dx+c\right)+a^2b^2\right)\log\left(a\cos\left(dx+c\right)+b\right)-\left(a^3b+2\,a^2b^2+ab^3+\left(a^4+2\,a^3b+a^2b^2\right)\cos\left(dx+c\right)\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+\left(a^3b-2\,a^2b^2+ab^3+\left(a^4-2\,a^3b+a^2b^2\right)\cos\left(dx+c\right)\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\,\left(\left(a^6-2\,a^4b^2+a^2b^4\right)d\cos\left(dx+c\right)+\left(a^5b-2\,a^3b^3+ab^5\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^2*b^2 - 2*b^4 + 4*(a^3*b*cos(d*x + c) + a^2*b^2)*log(a*cos(d*x + c
) + b) - (a^3*b + 2*a^2*b^2 + a*b^3 + (a^4 + 2*a^3*b + a^2*b^2)*cos(d*x + c
))*log(1/2*cos(d*x + c) + 1/2) + (a^3*b - 2*a^2*b^2 + a*b^3 + (a^4 - 2*a^3*
b + a^2*b^2)*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/((a^6 - 2*a^4*b^2
+ a^2*b^4)*d*cos(d*x + c) + (a^5*b - 2*a^3*b^3 + a*b^5)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(csc(c + d*x)/(a + b*sec(c + d*x))**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(105) = 210$.
time = 0.50, size = 213, normalized size = 1.95

$$\frac{\frac{4\,ab\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^4-2\,a^2b^2+b^4}+\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^2+2\,ab+b^2}-\frac{4\left(ab+b^2+\frac{ab(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{(a^3+a^2b-ab^2-b^3)\left(a+b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*a*b*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(co
s(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^4 - 2*a^2*b^2 + b^4) + log(abs(-cos
(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^2 + 2*a*b + b^2) - 4*(a*b + b^2 +
a*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((a^3 + a^2*b - a*b^2 - b^3)*(a
+ b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d
*x + c) + 1))))/d
```

Mupad [B]
time = 0.23, size = 103, normalized size = 0.94

$$\frac{\ln\left(\cos\left(c+d\,x\right)-1\right)}{2\,d\,{\left(a+b\right)}^2}-\frac{\ln\left(\cos\left(c+d\,x\right)+1\right)}{2\,d\,{\left(a-b\right)}^2}+\frac{b^2}{a\,d\,\left(a^2-b^2\right)\,\left(b+a\,\cos\left(c+d\,x\right)\right)}+\frac{2\,a\,b\,\ln\left(b+a\,\cos\left(c+d\,x\right)\right)}{d\,{\left(a^2-b^2\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)*(a + b/cos(c + d*x))^2),x)
```

```
[Out] log(cos(c + d*x) - 1)/(2*d*(a + b)^2) - log(cos(c + d*x) + 1)/(2*d*(a - b)^
2) + b^2/(a*d*(a^2 - b^2)*(b + a*cos(c + d*x))) + (2*a*b*log(b + a*cos(c +
d*x)))/(d*(a^2 - b^2)^2)
```

3.214 $\int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=168

$$\frac{ab^2}{(a^2-b^2)^2\,d(b+a\cos(c+dx))}+\frac{(2ab-(a^2+b^2)\cos(c+dx))\csc^2(c+dx)}{2\,(a^2-b^2)^2\,d}+\frac{(a-b)\log(1-\cos(c+dx))}{4(a+b)^3d}-$$

```
[Out] a*b^2/(a^2-b^2)^2/d/(b+a*cos(d*x+c))+1/2*(2*a*b-(a^2+b^2)*cos(d*x+c))*csc(d
*x+c)^2/(a^2-b^2)^2/d+1/4*(a-b)*ln(1-cos(d*x+c))/(a+b)^3/d-1/4*(a+b)*ln(1+c
os(d*x+c))/(a-b)^3/d+2*a*b*(a^2+b^2)*ln(b+a*cos(d*x+c))/(a^2-b^2)^3/d
```

Rubi [A]
time = 0.33, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3957, 2916, 12, 1661, 1643}

$$\frac{ab^2}{d\,(a^2-b^2)^2\,(a\cos(c+dx)+b)}+\frac{2ab(a^2+b^2)\log(a\cos(c+dx)+b)}{d\,(a^2-b^2)^3}+\frac{\csc^2(c+dx)\,(2ab-(a^2+b^2)\cos(c+dx))}{2d\,(a^2-b^2)^2}+\frac{(a-b)\log(1-\cos(c+dx))}{4d(a+b)^3}-\frac{(a+b)\log(\cos(c+dx)+1)}{4d(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (a*b^2)/((a^2 - b^2)^2*d*(b + a*Cos[c + d*x])) + ((2*a*b - (a^2 + b^2)*Cos[
c + d*x])*Csc[c + d*x]^2)/(2*(a^2 - b^2)^2*d) + ((a - b)*Log[1 - Cos[c + d*
x]])/(4*(a + b)^3*d) - ((a + b)*Log[1 + Cos[c + d*x]])/(4*(a - b)^3*d) + (2
*a*b*(a^2 + b^2)*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
```

```
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cot^2(c+dx)\csc(c+dx)}{(-b-a\cos(c+dx))^2}\,dx$$

$$= \frac{a^3\text{Subst}\left(\int \frac{x^2}{a^2(-b+x)^2(a^2-x^2)^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$

$$= \frac{a\text{Subst}\left(\int \frac{x^2}{(-b+x)^2(a^2-x^2)^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$

$$= \frac{(2ab-(a^2+b^2)\cos(c+dx))\csc^2(c+dx)}{2(a^2-b^2)^2 d} + \frac{\text{Subst}\left(\int \frac{-\frac{a^2b^2(a^2+b^2)}{(a^2-b^2)^2}+\frac{2a^2bx}{a^2-b^2}+\frac{a^2(a^2+b}{(a^2-b}}{(-b+x)^2(a^2-x^2)}\right.}{2ad}$$

$$= \frac{(2ab-(a^2+b^2)\cos(c+dx))\csc^2(c+dx)}{2(a^2-b^2)^2 d} + \frac{\text{Subst}\left(\int\left(\frac{a(a+b)}{2(a-b)^3(a-x)}+\frac{2a^2b^2}{(a-b)^2(a+b)}\right.\right.}{}$$

$$= \frac{ab^2}{(a^2-b^2)^2 d(b+a\cos(c+dx))} + \frac{(2ab-(a^2+b^2)\cos(c+dx))\csc^2(c+dx)}{2(a^2-b^2)^2 d} + \ldots$$

Mathematica [A]
time = 0.82, size = 224, normalized size = 1.33

$$\frac{(b+a\cos(c+dx))\left(\frac{8ab^2}{(a-b)^2(a+b)^2}-\frac{(b+a\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2}+\frac{4(a+b)(b+a\cos(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{(-a+b)^3}+\frac{16ab(a^2+b^2)(b+a\cos(c+dx))\log(b+a\cos(c+dx))}{(a^2-b^2)^3}+\frac{4(a-b)(b+a\cos(c+dx))\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{(a+b)^3}+\frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{(a-b)^2}\right)\sec^2(c+dx)}{8d(a+b\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])*((8*a*b^2)/((a - b)^2*(a + b)^2) - ((b + a*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/(a + b)^2 + (4*(a + b)*(b + a*Cos[c + d*x])*Log[Cos
[(c + d*x)/2]])/(-a + b)^3 + (16*a*b*(a^2 + b^2)*(b + a*Cos[c + d*x])*Log[b
 + a*Cos[c + d*x]])/(a^2 - b^2)^3 + (4*(a - b)*(b + a*Cos[c + d*x])*Log[Sin
[(c + d*x)/2]])/(a + b)^3 + ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a -
b)^2)*Sec[c + d*x]^2)/(8*d*(a + b*Sec[c + d*x])^2)
```

Maple [A]
time = 0.20, size = 151, normalized size = 0.90

method	result
derivativedivides	$\frac{\frac{1}{4(a-b)^2(1+\cos(dx+c))}+\frac{(-a-b)\ln(1+\cos(dx+c))}{4(a-b)^3}+\frac{1}{4(a+b)^2(-1+\cos(dx+c))}+\frac{(a-b)\ln(-1+\cos(dx+c))}{4(a+b)^3}+\frac{b^2 a}{(a+b)^2(a-b)^2(b+a\cos(}}{d}$
default	$\frac{\frac{1}{4(a-b)^2(1+\cos(dx+c))}+\frac{(-a-b)\ln(1+\cos(dx+c))}{4(a-b)^3}+\frac{1}{4(a+b)^2(-1+\cos(dx+c))}+\frac{(a-b)\ln(-1+\cos(dx+c))}{4(a+b)^3}+\frac{b^2 a}{(a+b)^2(a-b)^2(b+a\cos(}}{d}$
norman	$\frac{\frac{1}{8d(a+b)}+\frac{\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d(a-b)}-\frac{\left(a^4+14b^2a^2+b^4\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d\left(a^5-a^4b-2a^3b^2+2a^2b^3+a\,b^4-b^5\right)}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)}+\frac{(a-b)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d(a^3+3b\,a^2+3b^2a+b^3)}+\frac{2ab(a^2+b^2)\ln\left(a\left(\tan^2\left(\frac{d}{2}\right.\right.\right.}{d(a^6-3a}$
risch	$\frac{ibx}{2a^3+6b\,a^2+6b^2a+2b^3}-\frac{4ia\,b^3x}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{iax}{2a^3-6b\,a^2+6b^2a-2b^3}-\frac{iax}{2(a^3+3b\,a^2+3b^2a+b^3)}+\frac{ia}{2d(a^3-3b\,a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4/(a-b)^2/(1+cos(d*x+c))+1/4/(a-b)^3*(-a-b)*ln(1+cos(d*x+c))+1/4/(a+
b)^2/(-1+cos(d*x+c))+1/4*(a-b)/(a+b)^3*ln(-1+cos(d*x+c))+b^2/(a+b)^2*a/(a-b
)^2/(b+a*cos(d*x+c))+2*b*a*(a^2+b^2)/(a+b)^3/(a-b)^3*ln(b+a*cos(d*x+c)))
```

Maxima [A]
time = 0.29, size = 274, normalized size = 1.63

$$\frac{\frac{8\left(a^3b+ab^3\right)\log(a\cos(dx+c)+b)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}-\frac{(a+b)\log(\cos(dx+c)+1)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{(a-b)\log(\cos(dx+c)-1)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{2\left(4\,ab^2-\left(a^3+3\,ab^2\right)\cos(dx+c)^2+\left(a^2b-b^3\right)\cos(dx+c)\right)}{a^4b-2\,a^2b^3+b^5-\left(a^5-2\,a^3b^2+ab^4\right)\cos(dx+c)^3-\left(a^4b-2\,a^2b^3+b^5\right)\cos(dx+c)^2+\left(a^5-2\,a^3b^2+ab^4\right)\cos(dx+c)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*(8*(a^3*b + a*b^3)*log(a*cos(d*x + c) + b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4
 - b^6) - (a + b)*log(cos(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + (
a - b)*log(cos(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 2*(4*a*b^2 -
 (a^3 + 3*a*b^2)*cos(d*x + c)^2 + (a^2*b - b^3)*cos(d*x + c))/(a^4*b - 2*a^
2*b^3 + b^5 - (a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x + c)^3 - (a^4*b - 2*a^2*b^3
 + b^5)*cos(d*x + c)^2 + (a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x + c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 630 vs. $2(162) = 324$.
time = 3.47, size = 630, normalized size = 3.75

$$-\frac{8\,a^{3}b^{2} - 8\,ab^{4} - 2\,(a^{5} + 2\,a^{3}b^{2} - 3\,ab^{4})\cos(dx+c)^{2} + 2\,(a^{4}b - 2\,a^{2}b^{3} + b^{5})\cos(dx+c) + 8\,(a^{3}b^{2} + ab^{4} - (a^{4}b + a^{2}b^{3})\cos(dx+c)^{3} - (a^{3}b^{2} + ab^{4})\cos(dx+c)^{2} + (a^{4}b + a^{2}b^{3})\cos(dx+c))\log(a\cos(dx+c) + b) - (a^{4}b + 4\,a^{3}b^{2} + 6\,a^{2}b^{3} + 4\,ab^{4} + b^{5} - (a^{5} + 4\,a^{4}b + 6\,a^{3}b^{2} + 4\,a^{2}b^{3} + ab^{4})\cos(dx+c)^{3} - (a^{4}b + 4\,a^{3}b^{2} + 6\,a^{2}b^{3} + 4\,ab^{4} + b^{5})\cos(dx+c)^{2} + (a^{5} + 4\,a^{4}b + 6\,a^{3}b^{2} + 4\,a^{2}b^{3} + ab^{4})\cos(dx+c))\log\left(\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right) + (a^{4}b - 4\,a^{3}b^{2} + 6\,a^{2}b^{3} - 4\,ab^{4} + b^{5} - (a^{5} - 4\,a^{4}b + 6\,a^{3}b^{2} - 4\,a^{2}b^{3} + ab^{4})\cos(dx+c)^{3} - (a^{4}b - 4\,a^{3}b^{2} + 6\,a^{2}b^{3} - 4\,ab^{4} + b^{5})\cos(dx+c)^{2} + (a^{5} - 4\,a^{4}b + 6\,a^{3}b^{2} - 4\,a^{2}b^{3} + ab^{4})\cos(dx+c))\log\left(-\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right)}{4\,((a^{7} - 3\,a^{5}b^{2} + 3\,a^{3}b^{4} - ab^{6})d\cos(dx+c)^{3} + (a^{6}b - 3\,a^{4}b^{3} + 3\,a^{2}b^{5} - b^{7})d\cos(dx+c)^{2} - (a^{7} - 3\,a^{5}b^{2} + 3\,a^{3}b^{4} - ab^{6})d\cos(dx+c) - (a^{6}b - 3\,a^{4}b^{3} + 3\,a^{2}b^{5} - b^{7})d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/4*(8*a^3*b^2 - 8*a*b^4 - 2*(a^5 + 2*a^3*b^2 - 3*a*b^4)*cos(d*x + c)^2 +
2*(a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c) + 8*(a^3*b^2 + a*b^4 - (a^4*b + a^
2*b^3)*cos(d*x + c)^3 - (a^3*b^2 + a*b^4)*cos(d*x + c)^2 + (a^4*b + a^2*b^3
)*cos(d*x + c))*log(a*cos(d*x + c) + b) - (a^4*b + 4*a^3*b^2 + 6*a^2*b^3 +
4*a*b^4 + b^5 - (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cos(d*x + c
)^3 - (a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*cos(d*x + c)^2 + (a^5
 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cos(d*x + c))*log(1/2*cos(d*x +
 c) + 1/2) + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5 - (a^5 - 4*a^4*
b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*cos(d*x + c)^3 - (a^4*b - 4*a^3*b^2 + 6*
a^2*b^3 - 4*a*b^4 + b^5)*cos(d*x + c)^2 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^
2*b^3 + a*b^4)*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/((a^7 - 3*a^5*b^
2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c)^3 + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 -
b^7)*d*cos(d*x + c)^2 - (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c
) - (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{3}(c + dx)}{(a + b\sec(c + dx))^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(csc(c + d*x)**3/(a + b*sec(c + d*x))**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 456 vs. $2(162) = 324$.
time = 0.54, size = 456, normalized size = 2.71

$$\frac{\frac{2\,(a-b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}} + \frac{16\,(a^{3}b+ab^{3})\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^{6}-3\,a^{4}b^{2}+3\,a^{2}b^{4}-b^{6}} + \frac{a^{3}-a^{2}b-ab^{2}+b^{3}-\frac{8\,a^{2}b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{8\,ab^{2}(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{a^{3}(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{3\,a^{2}b(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{3\,ab^{2}(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{b^{3}(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}}{(a^{4}-2\,a^{2}b^{2}+b^{4})\left(\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{b(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}\right)} - \frac{\cos(dx+c)-1}{(a^{2}-2\,ab+b^{2})(\cos(dx+c)+1)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(2*(a - b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^3 + 3*a
^2*b + 3*a*b^2 + b^3) + 16*(a^3*b + a*b^3)*log(abs(-a - b - a*(cos(d*x + c)
```

```
 - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^6 -
 3*a^4*b^2 + 3*a^2*b^4 - b^6) + (a^3 - a^2*b - a*b^2 + b^3 - 8*a^2*b*(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) + 8*a*b^2*(cos(d*x + c) - 1)/(cos(d*x + c)
+ 1) - a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 3*a^2*b*(cos(d*x + c
) - 1)^2/(cos(d*x + c) + 1)^2 - 3*a*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c)
+ 1)^2 - b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/((a^4 - 2*a^2*b^2 +
 b^4)*(a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c) - 1)/(cos(
d*x + c) + 1) + a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - b*(cos(d*x +
c) - 1)^2/(cos(d*x + c) + 1)^2)) - (cos(d*x + c) - 1)/((a^2 - 2*a*b + b^2)*
(cos(d*x + c) + 1)))/d
```

Mupad [B]
time = 1.47, size = 228, normalized size = 1.36

$$\frac{\frac{2\,a\,b^2}{{\left(a^2-b^2\right)}^2}+\frac{b\,\cos(c+d\,x)}{2\,\left(a^2-b^2\right)}-\frac{{\cos(c+d\,x)}^2\,\left(a^3+3\,a\,b^2\right)}{2\,\left(a^4-2\,a^2\,b^2+b^4\right)}}{d\,\left(-a\,{\cos\left(c+d\,x\right)}^3-b\,{\cos\left(c+d\,x\right)}^2+a\,\cos\left(c+d\,x\right)+b\right)}-\frac{\ln\left(\cos\left(c+d\,x\right)-1\right)\,\left(\frac{b}{2\,{\left(a+b\right)}^3}-\frac{1}{4\,{\left(a+b\right)}^2}\right)}{d}+\frac{\ln\left(b+a\,\cos\left(c+d\,x\right)\right)\,\left(2\,a^3\,b+2\,a\,b^3\right)}{d\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}-\frac{\ln\left(\cos\left(c+d\,x\right)+1\right)\,\left(a+b\right)}{4\,d\,{\left(a-b\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^3*(a + b/cos(c + d*x))^2),x)
```

```
[Out] ((2*a*b^2)/(a^2 - b^2)^2 + (b*cos(c + d*x))/(2*(a^2 - b^2)) - (cos(c + d*x)
^2*(3*a*b^2 + a^3))/(2*(a^4 + b^4 - 2*a^2*b^2)))/(d*(b + a*cos(c + d*x) - a
*cos(c + d*x)^3 - b*cos(c + d*x)^2)) - (log(cos(c + d*x) - 1)*(b/(2*(a + b)
^3) - 1/(4*(a + b)^2)))/d + (log(b + a*cos(c + d*x))*(2*a*b^3 + 2*a^3*b))/(
d*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - (log(cos(c + d*x) + 1)*(a + b))/(4
*d*(a - b)^3)
```

3.215 $\int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=259

$$\frac{a^3b^2}{(a^2-b^2)^3\,d(b+a\cos(c+dx))}+\frac{(8ab(a^2+b^2)-(3a^4+12a^2b^2+b^4)\cos(c+dx))\csc^2(c+dx)}{8\,(a^2-b^2)^3\,d}+\frac{(2ab-(a^2+$$

```
[Out] a^3*b^2/(a^2-b^2)^3/d/(b+a*cos(d*x+c))+1/8*(8*a*b*(a^2+b^2)-(3*a^4+12*a^2*b
^2+b^4)*cos(d*x+c))*csc(d*x+c)^2/(a^2-b^2)^3/d+1/4*(2*a*b-(a^2+b^2)*cos(d*x
+c))*csc(d*x+c)^4/(a^2-b^2)^2/d+1/16*(3*a^2-4*a*b-b^2)*ln(1-cos(d*x+c))/(a+
b)^4/d-1/16*(3*a^2+4*a*b-b^2)*ln(1+cos(d*x+c))/(a-b)^4/d+2*a^3*b*(a^2+2*b^2
)*ln(b+a*cos(d*x+c))/(a^2-b^2)^4/d
```

Rubi [A]
time = 0.59, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3957, 2916, 12, 1661, 1643}

$$\frac{(3a^2-4ab-b^2)\log(1-\cos(c+dx))}{16d(a+b)^4}-\frac{(3a^2+4ab-b^2)\log(\cos(c+dx)+1)}{16d(a-b)^4}+\frac{\csc^4(c+dx)\,(2ab-(a^2+b^2)\cos(c+dx))}{4d\,(a^2-b^2)^2}+\frac{\csc^2(c+dx)\,(8ab(a^2+b^2)-(3a^4+12a^2b^2+b^4)\cos(c+dx))}{8d\,(a^2-b^2)^3}+\frac{a^3b^2}{d\,(a^2-b^2)^3\,(a\cos(c+dx)+b)}+\frac{2a^3b(a^2+2b^2)\log(a\cos(c+dx)+b)}{d\,(a^2-b^2)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (a^3*b^2)/((a^2 - b^2)^3*d*(b + a*Cos[c + d*x])) + ((8*a*b*(a^2 + b^2) - (3
*a^4 + 12*a^2*b^2 + b^4)*Cos[c + d*x])*Csc[c + d*x]^2)/(8*(a^2 - b^2)^3*d)
+ ((2*a*b - (a^2 + b^2)*Cos[c + d*x])*Csc[c + d*x]^4)/(4*(a^2 - b^2)^2*d) +
 ((3*a^2 - 4*a*b - b^2)*Log[1 - Cos[c + d*x]])/(16*(a + b)^4*d) - ((3*a^2 +
 4*a*b - b^2)*Log[1 + Cos[c + d*x]])/(16*(a - b)^4*d) + (2*a^3*b*(a^2 + 2*b
^2)*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)^4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
```

```
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cot^2(c+dx)\csc^3(c+dx)}{(-b-a\cos(c+dx))^2}\,dx$$

$$= \frac{a^5 \text{Subst}\left(\int \frac{x^2}{a^2(-b+x)^2(a^2-x^2)^3}\,dx, x, -a\cos(c+dx)\right)}{d}$$

$$= \frac{a^3 \text{Subst}\left(\int \frac{x^2}{(-b+x)^2(a^2-x^2)^3}\,dx, x, -a\cos(c+dx)\right)}{d}$$

$$= \frac{(2ab-(a^2+b^2)\cos(c+dx))\csc^4(c+dx)}{4(a^2-b^2)^2 d} + \frac{a\text{Subst}\left(\int \frac{-\frac{a^2b^2(a^2+b^2)}{(a^2-b^2)^2}+\frac{2a^2b(a^2-3b^2)x}{(a^2-b^2)^2}}{(-b+x)^2(a^2-x^2)}\right.}{4}$$

$$= \frac{(8ab(a^2+b^2)-(3a^4+12a^2b^2+b^4)\cos(c+dx))\csc^2(c+dx)}{8(a^2-b^2)^3 d} + \frac{(2ab-(a^2+b^2}{4}$$

$$= \frac{(8ab(a^2+b^2)-(3a^4+12a^2b^2+b^4)\cos(c+dx))\csc^2(c+dx)}{8(a^2-b^2)^3 d} + \frac{(2ab-(a^2+b^2}{4}$$

$$= \frac{a^3b^2}{(a^2-b^2)^3 d(b+a\cos(c+dx))} + \frac{(8ab(a^2+b^2)-(3a^4+12a^2b^2+b^4)\cos(c+d}{8(a^2-b^2)^3 d}$$

Mathematica [A]
time = 0.88, size = 320, normalized size = 1.24

$$\frac{(b+a\cos(c+dx))\left(\frac{64a^3b^2}{(a-b)^3(a+b)^3}+\frac{2(-3a+b)(b+a\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)}{(a+b)^3}-\frac{(b+a\cos(c+dx))\csc^4\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2}+\frac{8(-3a^2-4ab+b^2)(b+a\cos(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{(a-b)^4}+\frac{128a^3b(a^2+2b^2)(b+a\cos(c+dx))\log(b+a\cos(c+dx))}{(a^2-b^2)^4}+\frac{8(3a^2-4ab-b^2)(b+a\cos(c+dx))\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{(a+b)^4}+\frac{2(3a+b)(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{(a-b)^3}+\frac{(b+a\cos(c+dx))\sec^4\left(\frac{1}{2}(c+dx)\right)}{(a-b)^2}\right)\sec^2(c+dx)}{64d(a+b\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^5/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])*((64*a^3*b^2)/((a - b)^3*(a + b)^3) + (2*(-3*a + b)*(
b + a*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/(a + b)^3 - ((b + a*Cos[c + d*x])*C
sc[(c + d*x)/2]^4)/(a + b)^2 + (8*(-3*a^2 - 4*a*b + b^2)*(b + a*Cos[c + d*x
])*Log[Cos[(c + d*x)/2]])/(a - b)^4 + (128*a^3*b*(a^2 + 2*b^2)*(b + a*Cos[c
 + d*x])*Log[b + a*Cos[c + d*x]])/(a^2 - b^2)^4 + (8*(3*a^2 - 4*a*b - b^2)*
(b + a*Cos[c + d*x])*Log[Sin[(c + d*x)/2]])/(a + b)^4 + (2*(3*a + b)*(b + a
*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a - b)^3 + ((b + a*Cos[c + d*x])*Sec[(c
 + d*x)/2]^4)/(a - b)^2)*Sec[c + d*x]^2)/(64*d*(a + b*Sec[c + d*x])^2)
```

Maple [A]
time = 0.26, size = 221, normalized size = 0.85

method	result
derivativedivides	$\frac{\frac{1}{16(a-b)^2(1+\cos(dx+c))^2}-\frac{-3a-b}{16(a-b)^3(1+\cos(dx+c))}+\frac{(-3a^2-4ba+b^2)\ln(1+\cos(dx+c))}{16(a-b)^4}-\frac{1}{16(a+b)^2(-1+\cos(dx+c))^2}-\frac{\ldots}{16(a+b)^3(\ldots}}{d}$
default	$\frac{\frac{1}{16(a-b)^2(1+\cos(dx+c))^2}-\frac{-3a-b}{16(a-b)^3(1+\cos(dx+c))}+\frac{(-3a^2-4ba+b^2)\ln(1+\cos(dx+c))}{16(a-b)^4}-\frac{1}{16(a+b)^2(-1+\cos(dx+c))^2}-\frac{\ldots}{16(a+b)^3(\ldots}}{d}$
norman	$\frac{\frac{1}{64d(a+b)}+\frac{\tan^{10}(\frac{dx}{2}+\frac{c}{2})}{64d(a-b)}+\frac{(7a-b)(\tan^8(\frac{dx}{2}+\frac{c}{2}))}{64d(a^2-2ba+b^2)}+\frac{(7a+b)(\tan^2(\frac{dx}{2}+\frac{c}{2}))}{64d(a^2+2ba+b^2)}-\frac{(a^6+18a^4b^2+5a^2b^4)(\tan^4(\frac{dx}{2}+\frac{c}{2}))}{4d(a^7-a^6b-3a^5b^2+3a^4b^3+3a^3b^4-3a^2b^5-a\,b^6+\ldots}}{\tan(\frac{dx}{2}+\frac{c}{2})^4(a(\tan^2(\frac{dx}{2}+\frac{c}{2}))-b(\tan^2(\frac{dx}{2}+\frac{c}{2}))-a-b)}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/16/(a-b)^2/(1+cos(d*x+c))^2-1/16*(-3*a-b)/(a-b)^3/(1+cos(d*x+c))+1/1
6/(a-b)^4*(-3*a^2-4*a*b+b^2)*ln(1+cos(d*x+c))-1/16/(a+b)^2/(-1+cos(d*x+c))^
2-1/16*(-3*a+b)/(a+b)^3/(-1+cos(d*x+c))+1/16*(3*a^2-4*a*b-b^2)/(a+b)^4*ln(-
1+cos(d*x+c))+a^3*b^2/(a+b)^3/(a-b)^3/(b+a*cos(d*x+c))+2*a^3*b*(a^2+2*b^2)/
(a+b)^4/(a-b)^4*ln(b+a*cos(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 511 vs. $2(251) = 502$.
time = 0.28, size = 511, normalized size = 1.97

$$\frac{\frac{32\,(a^5b+2\,a^3b^3)\log(a\cos(dx+c)+b)}{a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8}-\frac{(3\,a^2+4\,ab-b^2)\log(\cos(dx+c)+1)}{a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4}+\frac{(3\,a^2-4\,ab-b^2)\log(\cos(dx+c)-1)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}+\frac{2\left(20\,a^3b^2+4\,ab^4+(3\,a^5+20\,a^3b^2+ab^4)\cos(dx+c)^4-(5\,a^4b-4\,a^2b^3-b^5)\cos(dx+c)^3-(5\,a^5+36\,a^3b^2+7\,ab^4)\cos(dx+c)^2+(7\,a^4b-8\,a^2b^3+b^5)\cos(dx+c)\right)}{a^6b-3\,a^4b^3+3\,a^2b^5-b^7+(a^7-3\,a^5b^2+3\,a^3b^4-ab^6)\cos(dx+c)^5+(a^6b-3\,a^4b^3+3\,a^2b^5-b^7)\cos(dx+c)^4-2\,(a^7-3\,a^5b^2+3\,a^3b^4-ab^6)\cos(dx+c)^3-2\,(a^6b-3\,a^4b^3+3\,a^2b^5-b^7)\cos(dx+c)^2+(a^7-3\,a^5b^2+3\,a^3b^4-ab^6)\cos(dx+c)}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/16*(32*(a^5*b + 2*a^3*b^3)*log(a*cos(d*x + c) + b)/(a^8 - 4*a^6*b^2 + 6*a
^4*b^4 - 4*a^2*b^6 + b^8) - (3*a^2 + 4*a*b - b^2)*log(cos(d*x + c) + 1)/(a^
4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) + (3*a^2 - 4*a*b - b^2)*log(cos(d*
x + c) - 1)/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + 2*(20*a^3*b^2 + 4
*a*b^4 + (3*a^5 + 20*a^3*b^2 + a*b^4)*cos(d*x + c)^4 - (5*a^4*b - 4*a^2*b^3
 - b^5)*cos(d*x + c)^3 - (5*a^5 + 36*a^3*b^2 + 7*a*b^4)*cos(d*x + c)^2 + (7
*a^4*b - 8*a^2*b^3 + b^5)*cos(d*x + c))/(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^
7 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cos(d*x + c)^5 + (a^6*b - 3*a^4*b
^3 + 3*a^2*b^5 - b^7)*cos(d*x + c)^4 - 2*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b
^6)*cos(d*x + c)^3 - 2*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*cos(d*x + c)^2
 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cos(d*x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1205 vs. $2(251) = 502$.
time = 4.58, size = 1205, normalized size = 4.65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/16*(40*a^5*b^2 - 32*a^3*b^4 - 8*a*b^6 + 2*(3*a^7 + 17*a^5*b^2 - 19*a^3*b^
4 - a*b^6)*cos(d*x + c)^4 - 2*(5*a^6*b - 9*a^4*b^3 + 3*a^2*b^5 + b^7)*cos(d
*x + c)^3 - 2*(5*a^7 + 31*a^5*b^2 - 29*a^3*b^4 - 7*a*b^6)*cos(d*x + c)^2 +
2*(7*a^6*b - 15*a^4*b^3 + 9*a^2*b^5 - b^7)*cos(d*x + c) + 32*(a^5*b^2 + 2*a
^3*b^4 + (a^6*b + 2*a^4*b^3)*cos(d*x + c)^5 + (a^5*b^2 + 2*a^3*b^4)*cos(d*x
 + c)^4 - 2*(a^6*b + 2*a^4*b^3)*cos(d*x + c)^3 - 2*(a^5*b^2 + 2*a^3*b^4)*co
s(d*x + c)^2 + (a^6*b + 2*a^4*b^3)*cos(d*x + c))*log(a*cos(d*x + c) + b) -
(3*a^6*b + 16*a^5*b^2 + 33*a^4*b^3 + 32*a^3*b^4 + 13*a^2*b^5 - b^7 + (3*a^7
 + 16*a^6*b + 33*a^5*b^2 + 32*a^4*b^3 + 13*a^3*b^4 - a*b^6)*cos(d*x + c)^5
+ (3*a^6*b + 16*a^5*b^2 + 33*a^4*b^3 + 32*a^3*b^4 + 13*a^2*b^5 - b^7)*cos(d
*x + c)^4 - 2*(3*a^7 + 16*a^6*b + 33*a^5*b^2 + 32*a^4*b^3 + 13*a^3*b^4 - a*
b^6)*cos(d*x + c)^3 - 2*(3*a^6*b + 16*a^5*b^2 + 33*a^4*b^3 + 32*a^3*b^4 + 1
3*a^2*b^5 - b^7)*cos(d*x + c)^2 + (3*a^7 + 16*a^6*b + 33*a^5*b^2 + 32*a^4*b
^3 + 13*a^3*b^4 - a*b^6)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + (3*a^6
*b - 16*a^5*b^2 + 33*a^4*b^3 - 32*a^3*b^4 + 13*a^2*b^5 - b^7 + (3*a^7 - 16*
a^6*b + 33*a^5*b^2 - 32*a^4*b^3 + 13*a^3*b^4 - a*b^6)*cos(d*x + c)^5 + (3*a
^6*b - 16*a^5*b^2 + 33*a^4*b^3 - 32*a^3*b^4 + 13*a^2*b^5 - b^7)*cos(d*x + c
)^4 - 2*(3*a^7 - 16*a^6*b + 33*a^5*b^2 - 32*a^4*b^3 + 13*a^3*b^4 - a*b^6)*c
os(d*x + c)^3 - 2*(3*a^6*b - 16*a^5*b^2 + 33*a^4*b^3 - 32*a^3*b^4 + 13*a^2*
b^5 - b^7)*cos(d*x + c)^2 + (3*a^7 - 16*a^6*b + 33*a^5*b^2 - 32*a^4*b^3 + 1
3*a^3*b^4 - a*b^6)*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/((a^9 - 4*a^
7*b^2 + 6*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*cos(d*x + c)^5 + (a^8*b - 4*a^6*b^
3 + 6*a^4*b^5 - 4*a^2*b^7 + b^9)*d*cos(d*x + c)^4 - 2*(a^9 - 4*a^7*b^2 + 6*
a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*cos(d*x + c)^3 - 2*(a^8*b - 4*a^6*b^3 + 6*a^
4*b^5 - 4*a^2*b^7 + b^9)*d*cos(d*x + c)^2 + (a^9 - 4*a^7*b^2 + 6*a^5*b^4 -
4*a^3*b^6 + a*b^8)*d*cos(d*x + c) + (a^8*b - 4*a^6*b^3 + 6*a^4*b^5 - 4*a^2*
b^7 + b^9)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(csc(c + d*x)**5/(a + b*sec(c + d*x))**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 710 vs. 2(251) = 502.
time = 0.57, size = 710, normalized size = 2.74

$$\frac{\frac{4\,(3\,a^2-4\,ab-b^2)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}+\frac{128\,(a^5b+2\,a^3b^3)\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8}-\frac{\frac{8\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{8\,ab(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{2\,ab(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{b^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4}-\frac{\left(a^2+2\,ab+b^2-\frac{8\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{8\,ab(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{18\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{24\,ab(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{6\,b^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)^2}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)(\cos(dx+c)-1)^2}-\frac{128\left(a^6b+a^4b^3+2\,a^3b^4+\frac{a^6b(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{a^5b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{2\,a^4b^3(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{2\,a^3b^4(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{(a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8)\left(a+b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/64*(4*(3*a^2 - 4*a*b - b^2)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) +
 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + 128*(a^5*b + 2*a^3*b^3)*
log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c)
- 1)/(cos(d*x + c) + 1)))/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) -
 (8*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 8*a*b*(cos(d*x + c) - 1)/(c
os(d*x + c) + 1) - a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 2*a*b*(c
os(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - b^2*(cos(d*x + c) - 1)^2/(cos(d*x
 + c) + 1)^2)/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) - (a^2 + 2*a*b +
b^2 - 8*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 8*a*b*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) + 18*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 2
4*a*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 6*b^2*(cos(d*x + c) - 1)^
2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)^2/((a^4 + 4*a^3*b + 6*a^2*b^2 +
4*a*b^3 + b^4)*(cos(d*x + c) - 1)^2) - 128*(a^6*b + a^4*b^3 + 2*a^3*b^4 + a
^6*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - a^5*b^2*(cos(d*x + c) - 1)/(co
s(d*x + c) + 1) + 2*a^4*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*a^3*b
^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4
*a^2*b^6 + b^8)*(a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d
*x + c) - 1)/(cos(d*x + c) + 1))))/d
```

Mupad [B]
time = 1.91, size = 447, normalized size = 1.73

$$\frac{\ln\left(\cos\left(c+d\,x\right)-1\right)\,\left(\frac{3}{16\,{\left(a+b\right)}^2}-\frac{5\,b}{8\,{\left(a+b\right)}^3}+\frac{3\,b^2}{8\,{\left(a+b\right)}^4}\right)}{d}-\frac{\ln\left(\cos\left(c+d\,x\right)+1\right)\,\left(\frac{3\,b^2}{8\,{\left(a-b\right)}^4}+\frac{5\,b}{8\,{\left(a-b\right)}^3}+\frac{3}{16\,{\left(a-b\right)}^2}\right)}{d}+\frac{\frac{{\cos(c+d\,x)}^4\,\left(3\,a^5+20\,a^3\,b^2+a\,b^4\right)}{8\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}+\frac{\cos(c+d\,x)\,\left(7\,a^2\,b-b^3\right)}{8\,\left(a^4-2\,a^2\,b^2+b^4\right)}-\frac{{\cos(c+d\,x)}^3\,\left(5\,a^2\,b+b^3\right)}{8\,\left(a^4-2\,a^2\,b^2+b^4\right)}-\frac{{\cos(c+d\,x)}^2\,\left(5\,a^5+36\,a^3\,b^2+7\,a\,b^4\right)}{8\,\left(a^2-b^2\right)\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{b\,\left(5\,a^3\,b+a\,b^3\right)}{2\,\left(a^2-b^2\right)\,\left(a^4-2\,a^2\,b^2+b^4\right)}}{d\,\left(a\,{\cos\left(c+d\,x\right)}^5+b\,{\cos\left(c+d\,x\right)}^4-2\,a\,{\cos\left(c+d\,x\right)}^3-2\,b\,{\cos\left(c+d\,x\right)}^2+a\,\cos\left(c+d\,x\right)+b\right)}+\frac{\ln\left(b+a\,\cos\left(c+d\,x\right)\right)\,\left(2\,a^5\,b+4\,a^3\,b^3\right)}{d\,\left(a^8-4\,a^6\,b^2+6\,a^4\,b^4-4\,a^2\,b^6+b^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^5*(a + b/cos(c + d*x))^2),x)
```

```
[Out] (log(cos(c + d*x) - 1)*(3/(16*(a + b)^2) - (5*b)/(8*(a + b)^3) + (3*b^2)/(8
*(a + b)^4)))/d - (log(cos(c + d*x) + 1)*((3*b^2)/(8*(a - b)^4) + (5*b)/(8*
(a - b)^3) + 3/(16*(a - b)^2)))/d + ((cos(c + d*x)^4*(a*b^4 + 3*a^5 + 20*a^
3*b^2))/(8*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (cos(c + d*x)*(7*a^2*b -
b^3))/(8*(a^4 + b^4 - 2*a^2*b^2)) - (cos(c + d*x)^3*(5*a^2*b + b^3))/(8*(a^
4 + b^4 - 2*a^2*b^2)) - (cos(c + d*x)^2*(7*a*b^4 + 5*a^5 + 36*a^3*b^2))/(8*
(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)) + (b*(a*b^3 + 5*a^3*b))/(2*(a^2 - b^2)
*(a^4 + b^4 - 2*a^2*b^2)))/(d*(b + a*cos(c + d*x) - 2*a*cos(c + d*x)^3 + a*
cos(c + d*x)^5 - 2*b*cos(c + d*x)^2 + b*cos(c + d*x)^4)) + (log(b + a*cos(c
 + d*x))*(2*a^5*b + 4*a^3*b^3))/(d*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a
^6*b^2))
```

3.216 $\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=473

$$\frac{(5a^6-90a^4b^2+200a^2b^4-112b^6)\,x}{16a^8}-\frac{2(a-b)^{3/2}b(a+b)^{3/2}\,(2a^2-7b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^8d}+\frac{b(61a}{}$$

[Out] 1/16*(5*a^6-90*a^4*b^2+200*a^2*b^4-112*b^6)*x/a^8-2*(a-b)^(3/2)*b*(a+b)^(3/2)*(2*a^2-7*b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^8/d+1/15*b*(61*a^4-170*a^2*b^2+105*b^4)*sin(d*x+c)/a^7/d-1/16*(27*a^4-86*a^2*b^2+56*b^4)*cos(d*x+c)*sin(d*x+c)/a^6/d+1/15*(15*a^4-52*a^2*b^2+35*b^4)*cos(d*x+c)^2*sin(d*x+c)/a^5/b/d-1/24*(16*a^4-61*a^2*b^2+42*b^4)*cos(d*x+c)^3*sin(d*x+c)/a^4/b^2/d-1/3*cos(d*x+c)^3*sin(d*x+c)/b/d/(b+a*cos(d*x+c))+1/6*a*cos(d*x+c)^4*sin(d*x+c)/b^2/d/(b+a*cos(d*x+c))+1/10*(5*a^4-20*a^2*b^2+14*b^4)*cos(d*x+c)^4*sin(d*x+c)/a^3/b^2/d/(b+a*cos(d*x+c))+7/30*b*cos(d*x+c)^5*sin(d*x+c)/a^2/d/(b+a*cos(d*x+c))-1/6*cos(d*x+c)^6*sin(d*x+c)/a/d/(b+a*cos(d*x+c))

Rubi [A]
time = 1.17, antiderivative size = 473, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3957, 2975, 3126, 3128, 3102, 2814, 2738, 214}

$$\frac{7b\sin(c+dx)\cos^5(c+dx)}{30a^2d(a\cos(c+dx)+b)}-\frac{2b(a-b)^{3/2}(a+b)^{3/2}\,(2a^2-7b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^8d}-\frac{(16a^4-61a^2b^2+42b^4)\sin(c+dx)\cos^3(c+dx)}{24a^4b^2d}+\frac{b(61a^4-170a^2b^2+105b^4)\sin(c+dx)}{15a^7d}-\frac{(27a^4-86a^2b^2+56b^4)\sin(c+dx)\cos(c+dx)}{16a^6d}+\frac{(15a^4-52a^2b^2+35b^4)\sin(c+dx)\cos^2(c+dx)}{15a^5bd}+\frac{(5a^4-20a^2b^2+14b^4)\sin(c+dx)\cos^4(c+dx)}{10a^3b^2d(a\cos(c+dx)+b)}+\frac{x(5a^6-90a^4b^2+200a^2b^4-112b^6)}{16a^8}+\frac{a\sin(c+dx)\cos^4(c+dx)}{6b^2d(a\cos(c+dx)+b)}-\frac{\sin(c+dx)\cos^6(c+dx)}{6ad(a\cos(c+dx)+b)}-\frac{\sin(c+dx)\cos^3(c+dx)}{3bd(a\cos(c+dx)+b)}$$

Antiderivative was successfully verified.

[In] Int[Sin[c + d*x]^6/(a + b*Sec[c + d*x])^2,x]

[Out] ((5*a^6 - 90*a^4*b^2 + 200*a^2*b^4 - 112*b^6)*x)/(16*a^8) - (2*(a - b)^(3/2)*b*(a + b)^(3/2)*(2*a^2 - 7*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^8*d) + (b*(61*a^4 - 170*a^2*b^2 + 105*b^4)*Sin[c + d*x])/(15*a^7*d) - ((27*a^4 - 86*a^2*b^2 + 56*b^4)*Cos[c + d*x]*Sin[c + d*x])/(16*a^6*d) + ((15*a^4 - 52*a^2*b^2 + 35*b^4)*Cos[c + d*x]^2*Sin[c + d*x])/(15*a^5*b*d) - ((16*a^4 - 61*a^2*b^2 + 42*b^4)*Cos[c + d*x]^3*Sin[c + d*x])/(24*a^4*b^2*d) - (Cos[c + d*x]^3*Sin[c + d*x])/(3*b*d*(b + a*Cos[c + d*x])) + (a*Cos[c + d*x]^4*Sin[c + d*x])/(6*b^2*d*(b + a*Cos[c + d*x])) + ((5*a^4 - 20*a^2*b^2 + 14*b^4)*Cos[c + d*x]^4*Sin[c + d*x])/(10*a^3*b^2*d*(b + a*Cos[c + d*x])) + (7*b*Cos[c + d*x]^5*Sin[c + d*x])/(30*a^2*d*(b + a*Cos[c + d*x])) - (Cos[c + d*x]^6*Sin[c + d*x])/(6*a*d*(b + a*Cos[c + d*x]))

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2975

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(d*Sin[
e + f*x])^(n + 1)*((a + b*Sin[e + f*x])^(m + 1)/(a*d*f*(n + 1))), x] + (Dis
t[1/(a^2*b^2*d^2*(n + 1)*(n + 2)*(m + n + 5)*(m + n + 6)), Int[(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5
) - a^2*b^2*(n + 2)*(2*n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m
+ n + 3)*(m + n + 5)*(m + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n
+ 5)*(m + n + 6))*Sin[e + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4
*(m + n + 2)*(m + n + 4)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*
(m + n + 5)*(2*n + 2*m + 13))*Sin[e + f*x]^2, x], x], x] - Simp[b*(m + n +
2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*((a + b*Sin[e + f*x])^(m + 1)/(a^2
*d^2*f*(n + 1)*(n + 2))), x] - Simp[a*(n + 5)*Cos[e + f*x]*(d*Sin[e + f*x])
^(n + 3)*((a + b*Sin[e + f*x])^(m + 1)/(b^2*d^3*f*(m + n + 5)*(m + n + 6)))
, x] + Simp[Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*((a + b*Sin[e + f*x])^(m
+ 1)/(b*d^4*f*(m + n + 6))), x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m +
 n + 5, 0] && NeQ[m + n + 6, 0] &&  !IGtQ[m, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)\sin^6(c+dx)}{(-b-a\cos(c+dx))^2}\,dx$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{3bd(b+a\cos(c+dx))} + \frac{a\cos^4(c+dx)\sin(c+dx)}{6b^2d(b+a\cos(c+dx))} + \frac{7b\cos^5(c+dx)\sin(c}{30a^2d(b+a\cos(c-}$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{3bd(b+a\cos(c+dx))} + \frac{a\cos^4(c+dx)\sin(c+dx)}{6b^2d(b+a\cos(c+dx))} + \frac{(5a^4-20a^2b^2+14b}{10a^3b^2d(b}$$

$$= -\frac{(16a^4-61a^2b^2+42b^4)\cos^3(c+dx)\sin(c+dx)}{24a^4b^2d} - \frac{\cos^3(c+dx)\sin(c+dx)}{3bd(b+a\cos(c+dx))}$$

$$= \frac{(15a^4-52a^2b^2+35b^4)\cos^2(c+dx)\sin(c+dx)}{15a^5bd} - \frac{(16a^4-61a^2b^2+42b^4)\cos^3}{24a^4b^2d}$$

$$= -\frac{(27a^4-86a^2b^2+56b^4)\cos(c+dx)\sin(c+dx)}{16a^6d} + \frac{(15a^4-52a^2b^2+35b^4)\cos}{15a^5b}$$

$$= \frac{b(61a^4-170a^2b^2+105b^4)\sin(c+dx)}{15a^7d} - \frac{(27a^4-86a^2b^2+56b^4)\cos(c+dx)\sin}{16a^6d}$$

$$= \frac{(5a^6-90a^4b^2+200a^2b^4-112b^6)\,x}{16a^8} + \frac{b(61a^4-170a^2b^2+105b^4)\sin(c+dx)}{15a^7d}$$

$$= \frac{(5a^6-90a^4b^2+200a^2b^4-112b^6)\,x}{16a^8} + \frac{b(61a^4-170a^2b^2+105b^4)\sin(c+dx)}{15a^7d}$$

$$= \frac{(5a^6-90a^4b^2+200a^2b^4-112b^6)\,x}{16a^8} - \frac{2(a-b)^{3/2}b(a+b)^{3/2}(2a^2-7b^2)\tanh^{-1}}{a^8d}$$

Mathematica [A]
time = 4.58, size = 402, normalized size = 0.85

$$\frac{3840b(2a^2-7b^2)(a^2-b^2)^{3/2}\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)+\frac{600a^6bc-10800a^4b^3c+24000a^2b^5c-13440b^7c+600a^6bdx-10800a^4b^3dx+24000a^2b^5dx-13440b^7dx+120a(5a^6-90a^4b^2+200a^2b^4-112b^6)(c+dx)\cos(c+dx)-15a(15a^6-576a^4b^2+1488a^2b^4-896b^6)\sin(c+dx)+1910a^6b\sin(2(c+dx))-5440a^4b^3\sin(2(c+dx))+3360a^2b^5\sin(2(c+dx))-180a^7\sin(3(c+dx))+790a^5b^2\sin(3(c+dx))-560a^3b^4\sin(3(c+dx))-166a^6b\sin(4(c+dx))+140a^4b^3\sin(4(c+dx))+40a^7\sin(5(c+dx))-42a^5b^2\sin(5(c+dx))+14a^6b\sin(6(c+dx))-5a^7\sin(7(c+dx))}{b+a\cos(c+dx)}}{1920a^8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^6/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (3840*b*(2*a^2 - 7*b^2)*(a^2 - b^2)^(3/2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2
])/Sqrt[a^2 - b^2]] + (600*a^6*b*c - 10800*a^4*b^3*c + 24000*a^2*b^5*c - 13
440*b^7*c + 600*a^6*b*d*x - 10800*a^4*b^3*d*x + 24000*a^2*b^5*d*x - 13440*b
^7*d*x + 120*a*(5*a^6 - 90*a^4*b^2 + 200*a^2*b^4 - 112*b^6)*(c + d*x)*Cos[c
 + d*x] - 15*a*(15*a^6 - 576*a^4*b^2 + 1488*a^2*b^4 - 896*b^6)*Sin[c + d*x]
```

```
 + 1910*a^6*b*Sin[2*(c + d*x)] - 5440*a^4*b^3*Sin[2*(c + d*x)] + 3360*a^2*b
^5*Sin[2*(c + d*x)] - 180*a^7*Sin[3*(c + d*x)] + 790*a^5*b^2*Sin[3*(c + d*x
)] - 560*a^3*b^4*Sin[3*(c + d*x)] - 166*a^6*b*Sin[4*(c + d*x)] + 140*a^4*b^
3*Sin[4*(c + d*x)] + 40*a^7*Sin[5*(c + d*x)] - 42*a^5*b^2*Sin[5*(c + d*x)]
+ 14*a^6*b*Sin[6*(c + d*x)] - 5*a^7*Sin[7*(c + d*x)])/(b + a*Cos[c + d*x]))
/(1920*a^8*d)
```

Maple [A]
time = 0.26, size = 509, normalized size = 1.08 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^6/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*(a-b)^2*(a+b)^2*b/a^8*(-a*b*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)
^2-b*tan(1/2*d*x+1/2*c)^2-a-b)-(2*a^2-7*b^2)/((a+b)*(a-b))^(1/2)*arctanh((a
-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))+2/a^8*(((5/16*a^6+2*a^5*b-21/8
*a^4*b^2-8*a^3*b^3+5/2*a^2*b^4+6*a*b^5)*tan(1/2*d*x+1/2*c)^11+(38/3*a^5*b-8
7/8*a^4*b^2+15/2*a^2*b^4+30*a*b^5+85/48*a^6-136/3*a^3*b^3)*tan(1/2*d*x+1/2*
c)^9+(172/5*a^5*b-33/4*a^4*b^2-96*a^3*b^3+5*a^2*b^4+60*a*b^5+33/8*a^6)*tan(
1/2*d*x+1/2*c)^7+(-33/8*a^6+33/4*a^4*b^2-5*a^2*b^4+172/5*a^5*b-96*a^3*b^3+6
0*a*b^5)*tan(1/2*d*x+1/2*c)^5+(38/3*a^5*b+87/8*a^4*b^2-136/3*a^3*b^3-15/2*a
^2*b^4+30*a*b^5-85/48*a^6)*tan(1/2*d*x+1/2*c)^3+(2*a^5*b-8*a^3*b^3+6*a*b^5-
5/16*a^6+21/8*a^4*b^2-5/2*a^2*b^4)*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c
)^2)^6+1/16*(5*a^6-90*a^4*b^2+200*a^2*b^4-112*b^6)*arctan(tan(1/2*d*x+1/2*c
))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 5.24, size = 793, normalized size = 1.68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/240*(15*(5*a^7 - 90*a^5*b^2 + 200*a^3*b^4 - 112*a*b^6)*d*x*cos(d*x + c)
+ 15*(5*a^6*b - 90*a^4*b^3 + 200*a^2*b^5 - 112*b^7)*d*x + 120*(2*a^4*b^2 -
9*a^2*b^4 + 7*b^6 + (2*a^5*b - 9*a^3*b^3 + 7*a*b^5)*cos(d*x + c))*sqrt(a^2
- b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2
- b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2
 + 2*a*b*cos(d*x + c) + b^2)) - (40*a^7*cos(d*x + c)^6 - 56*a^6*b*cos(d*x +
 c)^5 - 976*a^5*b^2 + 2720*a^3*b^4 - 1680*a*b^6 - 2*(65*a^7 - 42*a^5*b^2)*c
os(d*x + c)^4 + 2*(111*a^6*b - 70*a^4*b^3)*cos(d*x + c)^3 + (165*a^7 - 458*
a^5*b^2 + 280*a^3*b^4)*cos(d*x + c)^2 - (571*a^6*b - 1430*a^4*b^3 + 840*a^2
*b^5)*cos(d*x + c))*sin(d*x + c))/(a^9*d*cos(d*x + c) + a^8*b*d), 1/240*(15
*(5*a^7 - 90*a^5*b^2 + 200*a^3*b^4 - 112*a*b^6)*d*x*cos(d*x + c) + 15*(5*a^
6*b - 90*a^4*b^3 + 200*a^2*b^5 - 112*b^7)*d*x - 240*(2*a^4*b^2 - 9*a^2*b^4
+ 7*b^6 + (2*a^5*b - 9*a^3*b^3 + 7*a*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*ar
ctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (
40*a^7*cos(d*x + c)^6 - 56*a^6*b*cos(d*x + c)^5 - 976*a^5*b^2 + 2720*a^3*b^
4 - 1680*a*b^6 - 2*(65*a^7 - 42*a^5*b^2)*cos(d*x + c)^4 + 2*(111*a^6*b - 70
*a^4*b^3)*cos(d*x + c)^3 + (165*a^7 - 458*a^5*b^2 + 280*a^3*b^4)*cos(d*x +
c)^2 - (571*a^6*b - 1430*a^4*b^3 + 840*a^2*b^5)*cos(d*x + c))*sin(d*x + c))
/(a^9*d*cos(d*x + c) + a^8*b*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**6/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(sin(c + d*x)**6/(a + b*sec(c + d*x))**2, x)
```

Giac [A]
time = 0.49, size = 870, normalized size = 1.84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/240*(15*(5*a^6 - 90*a^4*b^2 + 200*a^2*b^4 - 112*b^6)*(d*x + c)/a^8 - 480*
(2*a^6*b - 11*a^4*b^3 + 16*a^2*b^5 - 7*b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/
2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*
c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*a^8) - 480*(a^4*b^2*tan(1/2*d*x +
1/2*c) - 2*a^2*b^4*tan(1/2*d*x + 1/2*c) + b^6*tan(1/2*d*x + 1/2*c))/((a*tan
(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)*a^7) + 2*(75*a^5*ta
n(1/2*d*x + 1/2*c)^11 + 480*a^4*b*tan(1/2*d*x + 1/2*c)^11 - 630*a^3*b^2*tan
```

```
(1/2*d*x + 1/2*c)^11 - 1920*a^2*b^3*tan(1/2*d*x + 1/2*c)^11 + 600*a*b^4*tan
(1/2*d*x + 1/2*c)^11 + 1440*b^5*tan(1/2*d*x + 1/2*c)^11 + 425*a^5*tan(1/2*d
*x + 1/2*c)^9 + 3040*a^4*b*tan(1/2*d*x + 1/2*c)^9 - 2610*a^3*b^2*tan(1/2*d*
x + 1/2*c)^9 - 10880*a^2*b^3*tan(1/2*d*x + 1/2*c)^9 + 1800*a*b^4*tan(1/2*d*
x + 1/2*c)^9 + 7200*b^5*tan(1/2*d*x + 1/2*c)^9 + 990*a^5*tan(1/2*d*x + 1/2*
c)^7 + 8256*a^4*b*tan(1/2*d*x + 1/2*c)^7 - 1980*a^3*b^2*tan(1/2*d*x + 1/2*c
)^7 - 23040*a^2*b^3*tan(1/2*d*x + 1/2*c)^7 + 1200*a*b^4*tan(1/2*d*x + 1/2*c
)^7 + 14400*b^5*tan(1/2*d*x + 1/2*c)^7 - 990*a^5*tan(1/2*d*x + 1/2*c)^5 + 8
256*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 1980*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 - 23
040*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 - 1200*a*b^4*tan(1/2*d*x + 1/2*c)^5 + 14
400*b^5*tan(1/2*d*x + 1/2*c)^5 - 425*a^5*tan(1/2*d*x + 1/2*c)^3 + 3040*a^4*
b*tan(1/2*d*x + 1/2*c)^3 + 2610*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 10880*a^2*
b^3*tan(1/2*d*x + 1/2*c)^3 - 1800*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 7200*b^5*t
an(1/2*d*x + 1/2*c)^3 - 75*a^5*tan(1/2*d*x + 1/2*c) + 480*a^4*b*tan(1/2*d*x
 + 1/2*c) + 630*a^3*b^2*tan(1/2*d*x + 1/2*c) - 1920*a^2*b^3*tan(1/2*d*x + 1
/2*c) - 600*a*b^4*tan(1/2*d*x + 1/2*c) + 1440*b^5*tan(1/2*d*x + 1/2*c))/((t
an(1/2*d*x + 1/2*c)^2 + 1)^6*a^7))/d
```

Mupad [B]
time = 4.87, size = 2500, normalized size = 5.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6/(a + b/cos(c + d*x))^2,x)
```

```
[Out] (atan(((((((74*a^23*b - 10*a^24 + 224*a^16*b^8 - 336*a^17*b^7 - 400*a^18*b^
6 + 740*a^19*b^5 + 124*a^20*b^4 - 478*a^21*b^3 + 62*a^22*b^2)/a^21 - (tan(c
/2 + (d*x)/2)*(512*a^18*b + 512*a^16*b^3 - 1024*a^17*b^2)*(a^6*5i - b^6*112
i + a^2*b^4*200i - a^4*b^2*90i))/(128*a^22))*(a^6*5i - b^6*112i + a^2*b^4*2
00i - a^4*b^2*90i))/(16*a^8) + (tan(c/2 + (d*x)/2)*(50176*a*b^14 - 75*a^14*
b + 25*a^15 - 25088*b^15 + 64512*a^2*b^13 - 179200*a^3*b^12 - 30720*a^4*b^1
1 + 240640*a^5*b^10 - 46080*a^6*b^9 - 148480*a^7*b^8 + 53900*a^8*b^7 + 4054
0*a^9*b^6 - 18136*a^10*b^5 - 3864*a^11*b^4 + 1651*a^12*b^3 + 199*a^13*b^2))
/(8*a^14))*(a^6*5i - b^6*112i + a^2*b^4*200i - a^4*b^2*90i)*1i)/(16*a^8) -
(((((74*a^23*b - 10*a^24 + 224*a^16*b^8 - 336*a^17*b^7 - 400*a^18*b^6 + 740
*a^19*b^5 + 124*a^20*b^4 - 478*a^21*b^3 + 62*a^22*b^2)/a^21 + (tan(c/2 + (d
*x)/2)*(512*a^18*b + 512*a^16*b^3 - 1024*a^17*b^2)*(a^6*5i - b^6*112i + a^2
*b^4*200i - a^4*b^2*90i))/(128*a^22))*(a^6*5i - b^6*112i + a^2*b^4*200i - a
^4*b^2*90i))/(16*a^8) - (tan(c/2 + (d*x)/2)*(50176*a*b^14 - 75*a^14*b + 25*
a^15 - 25088*b^15 + 64512*a^2*b^13 - 179200*a^3*b^12 - 30720*a^4*b^11 + 240
640*a^5*b^10 - 46080*a^6*b^9 - 148480*a^7*b^8 + 53900*a^8*b^7 + 40540*a^9*b
^6 - 18136*a^10*b^5 - 3864*a^11*b^4 + 1651*a^12*b^3 + 199*a^13*b^2))/(8*a^1
4))*(a^6*5i - b^6*112i + a^2*b^4*200i - a^4*b^2*90i)*1i)/(16*a^8))/((((((74
*a^23*b - 10*a^24 + 224*a^16*b^8 - 336*a^17*b^7 - 400*a^18*b^6 + 740*a^19*b
```

^5 + 124*a^20*b^4 - 478*a^21*b^3 + 62*a^22*b^2)/a^21 - (tan(c/2 + (d*x)/2)*
(512*a^18*b + 512*a^16*b^3 - 1024*a^17*b^2)*(a^6*5i - b^6*112i + a^2*b^4*20
0i - a^4*b^2*90i))/(128*a^22))*(a^6*5i - b^6*112i + a^2*b^4*200i - a^4*b^2*
90i))/(16*a^8) + (tan(c/2 + (d*x)/2)*(50176*a*b^14 - 75*a^14*b + 25*a^15 -
25088*b^15 + 64512*a^2*b^13 - 179200*a^3*b^12 - 30720*a^4*b^11 + 240640*a^5
*b^10 - 46080*a^6*b^9 - 148480*a^7*b^8 + 53900*a^8*b^7 + 40540*a^9*b^6 - 18
136*a^10*b^5 - 3864*a^11*b^4 + 1651*a^12*b^3 + 199*a^13*b^2))/(8*a^14))*(a^
6*5i - b^6*112i + a^2*b^4*200i - a^4*b^2*90i))/(16*a^8) - (32928*a*b^19 - (
25*a^19*b)/2 - 21952*b^20 + 117600*a^2*b^18 - 190120*a^3*b^17 - 257432*a^4*
b^16 + 463764*a^5*b^15 + 290284*a^6*b^14 - 620037*a^7*b^13 - 169030*a^8*b^1
2 + 492572*a^9*b^11 + 35558*a^10*b^10 - (941393*a^11*b^9)/4 + (22469*a^12*b
^8)/2 + (260375*a^13*b^7)/4 - 7490*a^14*b^6 - (37705*a^15*b^5)/4 + (2565*a^
16*b^4)/2 + (2345*a^17*b^3)/4 - 55*a^18*b^2)/a^21 + (((((74*a^23*b - 10*a^2
4 + 224*a^16*b^8 - 336*a^17*b^7 - 400*a^18*b^6 + 740*a^19*b^5 + 124*a^20*b^
4 - 478*a^21*b^3 + 62*a^22*b^2)/a^21 + (tan(c/2 + (d*x)/2)*(512*a^18*b + 51
2*a^16*b^3 - 1024*a^17*b^2)*(a^6*5i - b^6*112i + a^2*b^4*200i - a^4*b^2*90i
))/(128*a^22))*(a^6*5i - b^6*112i + a^2*b^4*200i - a^4*b^2*90i))/(16*a^8) -
(tan(c/2 + (d*x)/2)*(50176*a*b^14 - 75*a^14*b + 25*a^15 - 25088*b^15 + 645
12*a^2*b^13 - 179200*a^3*b^12 - 30720*a^4*b^11 + 240640*a^5*b^10 - 46080*a^
6*b^9 - 148480*a^7*b^8 + 53900*a^8*b^7 + 40540*a^9*b^6 - 18136*a^10*b^5 - 3
864*a^11*b^4 + 1651*a^12*b^3 + 199*a^13*b^2))/(8*a^14))*(a^6*5i - b^6*112i
+ a^2*b^4*200i - a^4*b^2*90i))/(16*a^8)))*(a^6*5i - b^6*112i + a^2*b^4*200i
- a^4*b^2*90i)*1i)/(8*a^8*d) - ((tan(c/2 + (d*x)/2)^11*(336*a*b^5 + 206*a^
5*b + 35*a^6 - 1008*b^6 + 1688*a^2*b^4 - 572*a^3*b^3 - 694*a^4*b^2))/(12*a^
7) - (tan(c/2 + (d*x)/2)^3*(336*a*b^5 + 206*a^5*b - 35*a^6 + 1008*b^6 - 168
8*a^2*b^4 - 572*a^3*b^3 + 694*a^4*b^2))/(12*a^7) - (tan(c/2 + (d*x)/2)^5*(4
200*a*b^5 + 3801*a^5*b - 565*a^6 + 25200*b^6 - 40520*a^2*b^4 - 7570*a^3*b^3
+ 14266*a^4*b^2))/(120*a^7) + (tan(c/2 + (d*x)/2)^9*(4200*a*b^5 + 3801*a^5
*b + 565*a^6 - 25200*b^6 + 40520*a^2*b^4 - 7570*a^3*b^3 - 14266*a^4*b^2))/(
120*a^7) - (tan(c/2 + (d*x)/2)^7*(165*a^6 + 2800*b^6 - 4440*a^2*b^4 + 1446*
a^4*b^2))/(10*a^7) + (tan(c/2 + (d*x)/2)^13*(a - b)*(56*a*b^4 + 32*a^4*b +
5*a^5 + 112*b^5 - 144*a^2*b^3 - 58*a^3*b^2))/(8*a^7) + (tan(c/2 + (d*x)/2)*
(a + b)*(56*a*b^4 - 32*a^4*b + 5*a^5 - 112*b^5 + 144*a^2*b^3 - 58*a^3*b^2))
/(8*a^7))/(d*(a + b - tan(c/2 + (d*x)/2)^14*(a - b) + tan(c/2 + (d*x)/2)^2*
(5*a + 7*b) - tan(c/2 + (d*x)/2)^12*(5*a - 7*b) + tan(c/2 + (d*x)/2)^4*(9*a
+ 21*b) - tan(c/2 + (d*x)/2)^10*(9*a - 21*b) + tan(c/2 + (d*x)/2)^6*(5*a +
35*b) - tan(c/2 + (d*x)/2)^8*(5*a - 35*b))) - (b*atan(((b*((tan(c/2 + (d*x
)/2)*(50176*a*b^14 - 75*a^14*b + 25*a^15 - 25088*b^15 + 64512*a^2*b^13 - 17
9200*a^3*b^12 - 30720*a^4*b^11 + 240640*a^5*b^10 - 46080*a^6*b^9 - 148480*a
^7*b^8 + 53900*a^8*b^7 + 40540*a^9*b^6 - 18136*a^10*b^5 - 3864*a^11*b^4 + 1
651*a^12*b^3 + 199*a^13*b^2))/(8*a^14) + (b*((74*a^23*b - 10*a^24 + 224*a^1
6*b^8 - 336*a^17*b^7 - 400*a^18*b^6 + 740*a^19*b^5 + 124*a^20*b^4 - 478*a^2
1*b^3 + 62*a^22*b^2)/a^21 - (b*tan(c/2 + (d*x)/2)*(2*a^2 - 7*b^2)*((a + b)^
3*(a - b)^3)^(1/2)*(512*a^18*b + 512*a^16*b^3 - 1024*a^17*b^2))/(8*a^22))*(
2*a^2 - 7*b^2)*((a + b)^3*(a - b)^3)^(1/2))/a^8)*(2*a^2 - 7*b^2)*((a + b)^3

```
*(a - b)^3)^(1/2)*1i)/a^8 + (b*((tan(c/2 + (d*x)/2)*(50176*a*b^14 - 75*a^14
*b + 25*a^15 - 25088*b^15 + 64512*a^2*b^13 - 179200*a^3*b^12 - 30720*a^4*b^
11 + 240640*a^5*b^10 - 46080*a^6*b^9 - 148480*a...
```

3.217 $\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=261

$$\frac{(3a^4-36a^2b^2+40b^4)\,x}{8a^6}-\frac{2\sqrt{a-b}\,b\sqrt{a+b}\,(2a^2-5b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^6d}+\frac{b(11a^2-15b^2)\sin(\ldots}{3a^5d}$$

```
[Out] 1/8*(3*a^4-36*a^2*b^2+40*b^4)*x/a^6+1/3*b*(11*a^2-15*b^2)*sin(d*x+c)/a^5/d-
1/8*(13*a^2-20*b^2)*cos(d*x+c)*sin(d*x+c)/a^4/d+1/3*(3*a^2-5*b^2)*cos(d*x+c
)^2*sin(d*x+c)/a^3/b/d+1/4*cos(d*x+c)^3*sin(d*x+c)/a^2/d-(a^2-b^2)*cos(d*x+
c)^3*sin(d*x+c)/a^2/b/d/(b+a*cos(d*x+c))-2*b*(2*a^2-5*b^2)*arctanh((a-b)^(1
/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))*(a-b)^(1/2)*(a+b)^(1/2)/a^6/d
```

Rubi [A]
time = 0.57, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3957, 2971, 3128, 3102, 2814, 2738, 214}

$$-\frac{(a^2-b^2)\sin(c+dx)\cos^3(c+dx)}{a^2bd(a\cos(c+dx)+b)}+\frac{\sin(c+dx)\cos^3(c+dx)}{4a^2d}-\frac{2b\sqrt{a-b}\,\sqrt{a+b}\,(2a^2-5b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^6d}+\frac{b(11a^2-15b^2)\sin(c+dx)}{3a^5d}-\frac{(13a^2-20b^2)\sin(c+dx)\cos(c+dx)}{8a^4d}+\frac{(3a^2-5b^2)\sin(c+dx)\cos^2(c+dx)}{3a^3bd}+\frac{x(3a^4-36a^2b^2+40b^4)}{8a^6}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((3*a^4 - 36*a^2*b^2 + 40*b^4)*x)/(8*a^6) - (2*Sqrt[a - b]*b*Sqrt[a + b]*(2
*a^2 - 5*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^6*d)
+ (b*(11*a^2 - 15*b^2)*Sin[c + d*x])/(3*a^5*d) - ((13*a^2 - 20*b^2)*Cos[c +
 d*x]*Sin[c + d*x])/(8*a^4*d) + ((3*a^2 - 5*b^2)*Cos[c + d*x]^2*Sin[c + d*x
])/(3*a^3*b*d) + (Cos[c + d*x]^3*Sin[c + d*x])/(4*a^2*d) - ((a^2 - b^2)*Cos
[c + d*x]^3*Sin[c + d*x])/(a^2*b*d*(b + a*Cos[c + d*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2971

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a^2 - b^2)*Cos[e +
f*x]*(a + b*Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*b^2*d*f*(m +
 1))), x] + (-Dist[1/(a*b^2*(m + 1)*(m + n + 4)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(d*Sin[e + f*x])^n*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m +
n + 4) + a*b*(m + 1)*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3)*
(m + n + 4))*Sin[e + f*x]^2, x], x], x] - Simp[Cos[e + f*x]*(a + b*Sin[e +
f*x])^(m + 2)*((d*Sin[e + f*x])^(n + 1)/(b^2*d*f*(m + n + 4))), x]) /; Free
Q[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && LtQ
[m, -1] &&  !LtQ[n, -1] && NeQ[m + n + 4, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)\sin^4(c+dx)}{(-b-a\cos(c+dx))^2}\,dx$$

$$= \frac{\cos^3(c+dx)\sin(c+dx)}{4a^2d} - \frac{(a^2-b^2)\cos^3(c+dx)\sin(c+dx)}{a^2bd(b+a\cos(c+dx))} - \frac{\int \frac{\cos^2(c+dx)(-8}{}}{}$$

$$= \frac{(3a^2-5b^2)\cos^2(c+dx)\sin(c+dx)}{3a^3bd} + \frac{\cos^3(c+dx)\sin(c+dx)}{4a^2d} - \frac{(a^2-b^2)\,\mathrm{co}}{a^2bd(b}$$

$$= -\frac{(13a^2-20b^2)\cos(c+dx)\sin(c+dx)}{8a^4d} + \frac{(3a^2-5b^2)\cos^2(c+dx)\sin(c+dx)}{3a^3bd}$$

$$= \frac{b(11a^2-15b^2)\sin(c+dx)}{3a^5d} - \frac{(13a^2-20b^2)\cos(c+dx)\sin(c+dx)}{8a^4d} + \frac{(3a^2-5}{}$$

$$= \frac{(3a^4-36a^2b^2+40b^4)\,x}{8a^6} + \frac{b(11a^2-15b^2)\sin(c+dx)}{3a^5d} - \frac{(13a^2-20b^2)\cos(c+}{8a^4d}$$

$$= \frac{(3a^4-36a^2b^2+40b^4)\,x}{8a^6} + \frac{b(11a^2-15b^2)\sin(c+dx)}{3a^5d} - \frac{(13a^2-20b^2)\cos(c+}{8a^4d}$$

$$= \frac{(3a^4-36a^2b^2+40b^4)\,x}{8a^6} - \frac{2\sqrt{a-b}\,b\sqrt{a+b}\,(2a^2-5b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan(}{\sqrt{a+}}\right.}{a^6d}$$

Mathematica [A]
time = 2.13, size = 282, normalized size = 1.08

$$\frac{\frac{384b(2a^4-7a^2b^2+5b^4)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{72a^4bc-864a^2b^3c+960b^5c+72a^4bdx-864a^2b^3dx+960b^5dx+24a(3a^4-36a^2b^2+40b^4)(c+dx)\cos(c+dx)-24a(a^4-31a^2b^2+40b^4)\sin(c+dx)+176a^4b\sin(2(c+dx))-240a^2b^3\sin(2(c+dx))-21a^5\sin(3(c+dx))+40a^3b^2\sin(3(c+dx))-10a^4b\sin(4(c+dx))+3a^5\sin(5(c+dx))}{b+a\cos(c+dx)}}{192a^6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((384*b*(2*a^4 - 7*a^2*b^2 + 5*b^4)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqr
t[a^2 - b^2]])/Sqrt[a^2 - b^2] + (72*a^4*b*c - 864*a^2*b^3*c + 960*b^5*c +
72*a^4*b*d*x - 864*a^2*b^3*d*x + 960*b^5*d*x + 24*a*(3*a^4 - 36*a^2*b^2 + 4
0*b^4)*(c + d*x)*Cos[c + d*x] - 24*a*(a^4 - 31*a^2*b^2 + 40*b^4)*Sin[c + d*
x] + 176*a^4*b*Sin[2*(c + d*x)] - 240*a^2*b^3*Sin[2*(c + d*x)] - 21*a^5*Sin
[3*(c + d*x)] + 40*a^3*b^2*Sin[3*(c + d*x)] - 10*a^4*b*Sin[4*(c + d*x)] + 3
*a^5*Sin[5*(c + d*x)])/(b + a*Cos[c + d*x]))/(192*a^6*d)
```

Maple [A]
time = 0.20, size = 325, normalized size = 1.25

method	result
derivativedivides	$\frac{2(a-b)(a+b)b\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b}-\frac{\left(2a^2-5b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a^6}+\frac{2\left(\left(\frac{3}{8}a^4+2b\,a^3\right.\right.}{}$
default	$\frac{2(a-b)(a+b)b\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b}-\frac{\left(2a^2-5b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a^6}+\frac{2\left(\left(\frac{3}{8}a^4+2b\,a^3\right.\right.}{}$
risch	$\frac{3x}{8a^2}-\frac{9x\,b^2}{2a^4}+\frac{5x\,b^4}{a^6}-\frac{i\mathrm{e}^{-2i(dx+c)}}{8a^2d}+\frac{i\mathrm{e}^{2i(dx+c)}}{8a^2d}-\frac{3i\mathrm{e}^{2i(dx+c)}b^2}{8a^4d}+\frac{5ib\,\mathrm{e}^{-i(dx+c)}}{4a^3d}-\frac{5ib\,\mathrm{e}^{i(dx+c)}}{4a^3d}-\frac{2ib^3\mathrm{e}^{-i(d}}{a^5d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*(a-b)*(a+b)*b/a^6*(-a*b*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-b
*tan(1/2*d*x+1/2*c)^2-a-b)-(2*a^2-5*b^2)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*
tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))+2/a^6*(((3/8*a^4+2*b*a^3-3/2*b^2*a
^2-4*b^3*a)*tan(1/2*d*x+1/2*c)^7+(26/3*b*a^3-3/2*b^2*a^2-12*b^3*a+11/8*a^4)
*tan(1/2*d*x+1/2*c)^5+(-11/8*a^4+3/2*b^2*a^2+26/3*b*a^3-12*b^3*a)*tan(1/2*d
*x+1/2*c)^3+(2*b*a^3-4*b^3*a-3/8*a^4+3/2*b^2*a^2)*tan(1/2*d*x+1/2*c))/(1+ta
n(1/2*d*x+1/2*c)^2)^4+1/8*(3*a^4-36*a^2*b^2+40*b^4)*arctan(tan(1/2*d*x+1/2*
c))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 5.01, size = 581, normalized size = 2.23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/24*(3*(3*a^5 - 36*a^3*b^2 + 40*a*b^4)*d*x*cos(d*x + c) + 3*(3*a^4*b - 36
*a^2*b^3 + 40*b^5)*d*x - 12*(2*a^2*b^2 - 5*b^4 + (2*a^3*b - 5*a*b^3)*cos(d*
x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c
)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a
^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + (6*a^5*cos(d*x + c)^4 - 10
*a^4*b*cos(d*x + c)^3 + 88*a^3*b^2 - 120*a*b^4 - 5*(3*a^5 - 4*a^3*b^2)*cos(
d*x + c)^2 + (49*a^4*b - 60*a^2*b^3)*cos(d*x + c))*sin(d*x + c))/(a^7*d*cos
(d*x + c) + a^6*b*d), 1/24*(3*(3*a^5 - 36*a^3*b^2 + 40*a*b^4)*d*x*cos(d*x +
 c) + 3*(3*a^4*b - 36*a^2*b^3 + 40*b^5)*d*x - 24*(2*a^2*b^2 - 5*b^4 + (2*a^
3*b - 5*a*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*c
os(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (6*a^5*cos(d*x + c)^4 - 10*a
^4*b*cos(d*x + c)^3 + 88*a^3*b^2 - 120*a*b^4 - 5*(3*a^5 - 4*a^3*b^2)*cos(d*
x + c)^2 + (49*a^4*b - 60*a^2*b^3)*cos(d*x + c))*sin(d*x + c))/(a^7*d*cos(d
*x + c) + a^6*b*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(sin(c + d*x)**4/(a + b*sec(c + d*x))**2, x)
```

Giac [A]
time = 0.47, size = 482, normalized size = 1.85

$$\frac{\frac{3\,(3\,a^4-36\,a^2b^2+40\,b^4)(dx+c)}{a^6}-\frac{48\,(2\,a^4b-7\,a^2b^3+5\,b^5)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac12\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan(\frac12\,dx+\frac12\,c)-b\tan(\frac12\,dx+\frac12\,c)}{\sqrt{-a^2+b^2}}\right)\right)}{\sqrt{-a^2+b^2}\,a^6}-\frac{48\,(a^2b^2\tan(\frac12\,dx+\frac12\,c)-b^4\tan(\frac12\,dx+\frac12\,c))}{\left(a\tan(\frac12\,dx+\frac12\,c)^2-b\tan(\frac12\,dx+\frac12\,c)^2-a-b\right)a^5}+\frac{2\left(9\,a^3\tan(\frac12\,dx+\frac12\,c)^7+48\,a^2b\tan(\frac12\,dx+\frac12\,c)^7-36\,ab^2\tan(\frac12\,dx+\frac12\,c)^7-96\,b^3\tan(\frac12\,dx+\frac12\,c)^7+33\,a^3\tan(\frac12\,dx+\frac12\,c)^5+208\,a^2b\tan(\frac12\,dx+\frac12\,c)^5-36\,ab^2\tan(\frac12\,dx+\frac12\,c)^5-288\,b^3\tan(\frac12\,dx+\frac12\,c)^5-33\,a^3\tan(\frac12\,dx+\frac12\,c)^3+208\,a^2b\tan(\frac12\,dx+\frac12\,c)^3+36\,ab^2\tan(\frac12\,dx+\frac12\,c)^3-288\,b^3\tan(\frac12\,dx+\frac12\,c)^3-9\,a^3\tan(\frac12\,dx+\frac12\,c)+48\,a^2b\tan(\frac12\,dx+\frac12\,c)+36\,ab^2\tan(\frac12\,dx+\frac12\,c)-96\,b^3\tan(\frac12\,dx+\frac12\,c)\right)}{\left(\tan(\frac12\,dx+\frac12\,c)^2+1\right)^4a^5}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(3*(3*a^4 - 36*a^2*b^2 + 40*b^4)*(d*x + c)/a^6 - 48*(2*a^4*b - 7*a^2*b
^3 + 5*b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*
tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^
2 + b^2)*a^6) - 48*(a^2*b^2*tan(1/2*d*x + 1/2*c) - b^4*tan(1/2*d*x + 1/2*c)
)/((a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)*a^5) + 2*(
9*a^3*tan(1/2*d*x + 1/2*c)^7 + 48*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 36*a*b^2*t
an(1/2*d*x + 1/2*c)^7 - 96*b^3*tan(1/2*d*x + 1/2*c)^7 + 33*a^3*tan(1/2*d*x
+ 1/2*c)^5 + 208*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 36*a*b^2*tan(1/2*d*x + 1/2*
```

```
c)^5 - 288*b^3*tan(1/2*d*x + 1/2*c)^5 - 33*a^3*tan(1/2*d*x + 1/2*c)^3 + 208
*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 36*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 288*b^3*t
an(1/2*d*x + 1/2*c)^3 - 9*a^3*tan(1/2*d*x + 1/2*c) + 48*a^2*b*tan(1/2*d*x +
 1/2*c) + 36*a*b^2*tan(1/2*d*x + 1/2*c) - 96*b^3*tan(1/2*d*x + 1/2*c))/((ta
n(1/2*d*x + 1/2*c)^2 + 1)^4*a^5))/d
```

Mupad [B]
time = 3.86, size = 2500, normalized size = 9.58

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(a + b/cos(c + d*x))^2,x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^5*(33*a^4 - 360*b^4 + 244*a^2*b^2))/(6*a^5) - (tan(c/2
 + (d*x)/2)^3*(60*a*b^3 - 59*a^3*b + 12*a^4 + 240*b^4 - 176*a^2*b^2))/(6*a^
5) - (tan(c/2 + (d*x)/2)^7*(59*a^3*b - 60*a*b^3 + 12*a^4 + 240*b^4 - 176*a^
2*b^2))/(6*a^5) + (tan(c/2 + (d*x)/2)^9*(a - b)*(20*a*b^2 - 16*a^2*b - 3*a^
3 + 40*b^3))/(4*a^5) + (tan(c/2 + (d*x)/2)*(a + b)*(20*a*b^2 + 16*a^2*b - 3
*a^3 - 40*b^3))/(4*a^5))/(d*(a + b - tan(c/2 + (d*x)/2)^10*(a - b) + tan(c/
2 + (d*x)/2)^2*(3*a + 5*b) + tan(c/2 + (d*x)/2)^4*(2*a + 10*b) - tan(c/2 +
(d*x)/2)^8*(3*a - 5*b) - tan(c/2 + (d*x)/2)^6*(2*a - 10*b))) + (atan((((((
76*a^17*b - 12*a^18 - 160*a^12*b^6 + 240*a^13*b^5 + 144*a^14*b^4 - 316*a^15
*b^3 + 28*a^16*b^2)/a^15 - (tan(c/2 + (d*x)/2)*(a^4*3i + b^4*40i - a^2*b^2*
36i)*(128*a^14*b + 128*a^12*b^3 - 256*a^13*b^2))/(16*a^16))*(a^4*3i + b^4*4
0i - a^2*b^2*36i))/(8*a^6) + (tan(c/2 + (d*x)/2)*(6400*a*b^10 - 27*a^10*b +
 9*a^11 - 3200*b^11 + 2560*a^2*b^9 - 11520*a^3*b^8 + 2688*a^4*b^7 + 6144*a^
5*b^6 - 2600*a^6*b^5 - 904*a^7*b^4 + 383*a^8*b^3 + 67*a^9*b^2))/(2*a^10))*(
a^4*3i + b^4*40i - a^2*b^2*36i)*1i)/(8*a^6) - (((((76*a^17*b - 12*a^18 - 16
0*a^12*b^6 + 240*a^13*b^5 + 144*a^14*b^4 - 316*a^15*b^3 + 28*a^16*b^2)/a^15
 + (tan(c/2 + (d*x)/2)*(a^4*3i + b^4*40i - a^2*b^2*36i)*(128*a^14*b + 128*a
^12*b^3 - 256*a^13*b^2))/(16*a^16))*(a^4*3i + b^4*40i - a^2*b^2*36i))/(8*a^
6) - (tan(c/2 + (d*x)/2)*(6400*a*b^10 - 27*a^10*b + 9*a^11 - 3200*b^11 + 25
60*a^2*b^9 - 11520*a^3*b^8 + 2688*a^4*b^7 + 6144*a^5*b^6 - 2600*a^6*b^5 - 9
04*a^7*b^4 + 383*a^8*b^3 + 67*a^9*b^2))/(2*a^10))*(a^4*3i + b^4*40i - a^2*b
^2*36i)*1i)/(8*a^6))/((12000*a*b^13 + 18*a^13*b - 8000*b^14 + 21600*a^2*b^1
2 - 37400*a^3*b^11 - 19240*a^4*b^10 + 43960*a^5*b^9 + 4672*a^6*b^8 - 23963*
a^7*b^7 + 1742*a^8*b^6 + 5958*a^9*b^5 - 834*a^10*b^4 - 573*a^11*b^3 + 60*a^
12*b^2)/a^15 + (((((76*a^17*b - 12*a^18 - 160*a^12*b^6 + 240*a^13*b^5 + 144
*a^14*b^4 - 316*a^15*b^3 + 28*a^16*b^2)/a^15 - (tan(c/2 + (d*x)/2)*(a^4*3i
+ b^4*40i - a^2*b^2*36i)*(128*a^14*b + 128*a^12*b^3 - 256*a^13*b^2))/(16*a^
16))*(a^4*3i + b^4*40i - a^2*b^2*36i))/(8*a^6) + (tan(c/2 + (d*x)/2)*(6400*
a*b^10 - 27*a^10*b + 9*a^11 - 3200*b^11 + 2560*a^2*b^9 - 11520*a^3*b^8 + 26
88*a^4*b^7 + 6144*a^5*b^6 - 2600*a^6*b^5 - 904*a^7*b^4 + 383*a^8*b^3 + 67*a
^9*b^2))/(2*a^10))*(a^4*3i + b^4*40i - a^2*b^2*36i))/(8*a^6) + (((((76*a^17
```

```
*b - 12*a^18 - 160*a^12*b^6 + 240*a^13*b^5 + 144*a^14*b^4 - 316*a^15*b^3 +
28*a^16*b^2)/a^15 + (tan(c/2 + (d*x)/2)*(a^4*3i + b^4*40i - a^2*b^2*36i)*(1
28*a^14*b + 128*a^12*b^3 - 256*a^13*b^2))/(16*a^16))*(a^4*3i + b^4*40i - a^
2*b^2*36i))/(8*a^6) - (tan(c/2 + (d*x)/2)*(6400*a*b^10 - 27*a^10*b + 9*a^11
 - 3200*b^11 + 2560*a^2*b^9 - 11520*a^3*b^8 + 2688*a^4*b^7 + 6144*a^5*b^6 -
 2600*a^6*b^5 - 904*a^7*b^4 + 383*a^8*b^3 + 67*a^9*b^2))/(2*a^10))*(a^4*3i
+ b^4*40i - a^2*b^2*36i))/(8*a^6)))*(a^4*3i + b^4*40i - a^2*b^2*36i)*1i)/(4
*a^6*d) + (b*atan(((b*(a^2 - b^2)^(1/2)*(2*a^2 - 5*b^2)*((tan(c/2 + (d*x)/2
)*(6400*a*b^10 - 27*a^10*b + 9*a^11 - 3200*b^11 + 2560*a^2*b^9 - 11520*a^3*
b^8 + 2688*a^4*b^7 + 6144*a^5*b^6 - 2600*a^6*b^5 - 904*a^7*b^4 + 383*a^8*b^
3 + 67*a^9*b^2))/(2*a^10) + (b*((76*a^17*b - 12*a^18 - 160*a^12*b^6 + 240*a
^13*b^5 + 144*a^14*b^4 - 316*a^15*b^3 + 28*a^16*b^2)/a^15 - (b*tan(c/2 + (d
*x)/2)*(a^2 - b^2)^(1/2)*(2*a^2 - 5*b^2)*(128*a^14*b + 128*a^12*b^3 - 256*a
^13*b^2))/(2*a^16))*(a^2 - b^2)^(1/2)*(2*a^2 - 5*b^2))/a^6)*1i)/a^6 + (b*(a
^2 - b^2)^(1/2)*(2*a^2 - 5*b^2)*((tan(c/2 + (d*x)/2)*(6400*a*b^10 - 27*a^10
*b + 9*a^11 - 3200*b^11 + 2560*a^2*b^9 - 11520*a^3*b^8 + 2688*a^4*b^7 + 614
4*a^5*b^6 - 2600*a^6*b^5 - 904*a^7*b^4 + 383*a^8*b^3 + 67*a^9*b^2))/(2*a^10
) - (b*((76*a^17*b - 12*a^18 - 160*a^12*b^6 + 240*a^13*b^5 + 144*a^14*b^4 -
 316*a^15*b^3 + 28*a^16*b^2)/a^15 + (b*tan(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)
*(2*a^2 - 5*b^2)*(128*a^14*b + 128*a^12*b^3 - 256*a^13*b^2))/(2*a^16))*(a^2
 - b^2)^(1/2)*(2*a^2 - 5*b^2))/a^6)*1i)/a^6)/((12000*a*b^13 + 18*a^13*b - 8
000*b^14 + 21600*a^2*b^12 - 37400*a^3*b^11 - 19240*a^4*b^10 + 43960*a^5*b^9
 + 4672*a^6*b^8 - 23963*a^7*b^7 + 1742*a^8*b^6 + 5958*a^9*b^5 - 834*a^10*b^
4 - 573*a^11*b^3 + 60*a^12*b^2)/a^15 + (b*(a^2 - b^2)^(1/2)*(2*a^2 - 5*b^2)
*((tan(c/2 + (d*x)/2)*(6400*a*b^10 - 27*a^10*b + 9*a^11 - 3200*b^11 + 2560*
a^2*b^9 - 11520*a^3*b^8 + 2688*a^4*b^7 + 6144*a^5*b^6 - 2600*a^6*b^5 - 904*
a^7*b^4 + 383*a^8*b^3 + 67*a^9*b^2))/(2*a^10) + (b*((76*a^17*b - 12*a^18 -
160*a^12*b^6 + 240*a^13*b^5 + 144*a^14*b^4 - 316*a^15*b^3 + 28*a^16*b^2)/a^
15 - (b*tan(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)*(2*a^2 - 5*b^2)*(128*a^14*b +
128*a^12*b^3 - 256*a^13*b^2))/(2*a^16))*(a^2 - b^2)^(1/2)*(2*a^2 - 5*b^2))/
a^6))/a^6 - (b*(a^2 - b^2)^(1/2)*(2*a^2 - 5*b^2)*((tan(c/2 + (d*x)/2)*(6400
*a*b^10 - 27*a^10*b + 9*a^11 - 3200*b^11 + 2560*a^2*b^9 - 11520*a^3*b^8 + 2
688*a^4*b^7 + 6144*a^5*b^6 - 2600*a^6*b^5 - 904*a^7*b^4 + 383*a^8*b^3 + 67*
a^9*b^2))/(2*a^10) - (b*((76*a^17*b - 12*a^18 - 160*a^12*b^6 + 240*a^13*b^5
 + 144*a^14*b^4 - 316*a^15*b^3 + 28*a^16*b^2)/a^15 + (b*tan(c/2 + (d*x)/2)*
(a^2 - b^2)^(1/2)*(2*a^2 - 5*b^2)*(128*a^14*b +...
```

3.218 $\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=152

$$\frac{(a^2-6b^2)\,x}{2a^4}-\frac{2b(2a^2-3b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4\sqrt{a-b}\,\sqrt{a+b}\,d}+\frac{3b\sin(c+dx)}{a^3d}-\frac{3\cos(c+dx)\sin(c+dx)}{2a^2d}+\frac{\cos^2(\ldots}{ad(\ldots}$$

```
[Out] 1/2*(a^2-6*b^2)*x/a^4+3*b*sin(d*x+c)/a^3/d-3/2*cos(d*x+c)*sin(d*x+c)/a^2/d+
cos(d*x+c)^2*sin(d*x+c)/a/d/(b+a*cos(d*x+c))-2*b*(2*a^2-3*b^2)*arctanh((a-b
)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^4/d/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A]
time = 0.39, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3957, 2968, 3127, 3129, 3102, 2814, 2738, 214}

$$\frac{3b\sin(c+dx)}{a^3d}-\frac{3\sin(c+dx)\cos(c+dx)}{2a^2d}-\frac{2b(2a^2-3b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d\sqrt{a-b}\,\sqrt{a+b}}+\frac{x(a^2-6b^2)}{2a^4}+\frac{\sin(c+dx)\cos^2(c+dx)}{ad(a\cos(c+dx)+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((a^2 - 6*b^2)*x)/(2*a^4) - (2*b*(2*a^2 - 3*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(
c + d*x)/2])/Sqrt[a + b]])/(a^4*Sqrt[a - b]*Sqrt[a + b]*d) + (3*b*Sin[c + d
*x])/(a^3*d) - (3*Cos[c + d*x]*Sin[c + d*x])/(2*a^2*d) + (Cos[c + d*x]^2*Si
n[c + d*x])/(a*d*(b + a*Cos[c + d*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3127

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d
*(b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*
c*(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A
*d^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3129

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n +
1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x]
)^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n
 + 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(
a*d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0
])))
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
```

```
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)\sin^2(c+dx)}{(-b-a\cos(c+dx))^2}\,dx$$
$$= \int \frac{\cos^2(c+dx)\left(1-\cos^2(c+dx)\right)}{(-b-a\cos(c+dx))^2}\,dx$$
$$= \frac{\cos^2(c+dx)\sin(c+dx)}{ad(b+a\cos(c+dx))} - \frac{\int \frac{\cos(c+dx)\left(2(a^2-b^2)-3(a^2-b^2)\cos^2(c+dx)\right)}{-b-a\cos(c+dx)}\,dx}{a\left(a^2-b^2\right)}$$
$$= -\frac{3\cos(c+dx)\sin(c+dx)}{2a^2d} + \frac{\cos^2(c+dx)\sin(c+dx)}{ad(b+a\cos(c+dx))} + \frac{\int \frac{3b(a^2-b^2)-a(a^2-b^2)\cos(c}{-b-a\,c}}{2a^2\,(}$$
$$= \frac{3b\sin(c+dx)}{a^3d} - \frac{3\cos(c+dx)\sin(c+dx)}{2a^2d} + \frac{\cos^2(c+dx)\sin(c+dx)}{ad(b+a\cos(c+dx))} - \int \frac{-3ab(}{}$$
$$= \frac{\left(a^2-6b^2\right)x}{2a^4} + \frac{3b\sin(c+dx)}{a^3d} - \frac{3\cos(c+dx)\sin(c+dx)}{2a^2d} + \frac{\cos^2(c+dx)\sin(c-}{ad(b+a\cos(c+a}$$
$$= \frac{\left(a^2-6b^2\right)x}{2a^4} + \frac{3b\sin(c+dx)}{a^3d} - \frac{3\cos(c+dx)\sin(c+dx)}{2a^2d} + \frac{\cos^2(c+dx)\sin(c-}{ad(b+a\cos(c+a}$$
$$= \frac{\left(a^2-6b^2\right)x}{2a^4} - \frac{2b(2a^2-3b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4\sqrt{a-b}\,\sqrt{a+b}\,d} + \frac{3b\sin(c+dx)}{a^3d} -$$

Mathematica [A]
time = 0.55, size = 178, normalized size = 1.17

$$\frac{\frac{16b(2a^2-3b^2)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{4a^2bc-24b^3c+4a^2bdx-24b^3dx+4a\left(a^2-6b^2\right)(c+dx)\cos(c+dx)-a\left(a^2-24b^2\right)\sin(c+dx)+6a^2b\sin(2(c+dx))-a^3\sin(3(c+dx))}{b+a\cos(c+dx)}}{8a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((16*b*(2*a^2 - 3*b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]
)/Sqrt[a^2 - b^2] + (4*a^2*b*c - 24*b^3*c + 4*a^2*b*d*x - 24*b^3*d*x + 4*a*
(a^2 - 6*b^2)*(c + d*x)*Cos[c + d*x] - a*(a^2 - 24*b^2)*Sin[c + d*x] + 6*a^
2*b*Sin[2*(c + d*x)] - a^3*Sin[3*(c + d*x)])/(b + a*Cos[c + d*x]))/(8*a^4*d
)
```

Maple [A]
time = 0.15, size = 199, normalized size = 1.31

method	result
derivativedivides	$\frac{\frac{2b\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b}-\frac{\left(2a^2-3b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a^4}+\frac{2\left(\left(\frac{1}{2}a^2+2ba\right)\left(\tan^3\left(\frac{dx}{2}\right.\right.\right.}{\left(1+\mathrm{t}\right.}}{d}$
default	$\frac{\frac{2b\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b}-\frac{\left(2a^2-3b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a^4}+\frac{2\left(\left(\frac{1}{2}a^2+2ba\right)\left(\tan^3\left(\frac{dx}{2}\right.\right.\right.}{\left(1+\mathrm{t}\right.}}{d}$
risch	$\frac{x}{2a^2}-\frac{3xb^2}{a^4}+\frac{i\mathrm{e}^{2i(dx+c)}}{8a^2d}-\frac{ib\,\mathrm{e}^{i(dx+c)}}{a^3d}+\frac{ib\,\mathrm{e}^{-i(dx+c)}}{a^3d}-\frac{i\mathrm{e}^{-2i(dx+c)}}{8a^2d}+\frac{2ib^2\left(b\,\mathrm{e}^{i(dx+c)}+a\right)}{a^4d\left(a\,\mathrm{e}^{2i(dx+c)}+2b\,\mathrm{e}^{i(dx+c)}+a\right)}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*b/a^4*(-a*b*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-b*tan(1/2*d*x
+1/2*c)^2-a-b)-(2*a^2-3*b^2)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+
1/2*c)/((a+b)*(a-b))^(1/2)))+2/a^4*(((1/2*a^2+2*b*a)*tan(1/2*d*x+1/2*c)^3+(
2*b*a-1/2*a^2)*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^2+1/2*(a^2-6*b^
2)*arctan(tan(1/2*d*x+1/2*c))))
```

<u>Maxima</u> [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

<u>Fricas</u> [A]
time = 4.62, size = 551, normalized size = 3.62

$$\left[\frac{(a^5-7a^3b^2+6ab^4)dx\cos(dx+c)+(a^4b-7a^2b^3+6b^5)dx-(2a^2b^2-3b^4+(2a^3b-3ab^3)\cos(dx+c))\sqrt{a^2-b^2}\log\left(\frac{2ab\cos(dx+c)-(a^2-2b^2)\cos(dx+c)^2+2\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2a^2-b^2}{a^2\cos(dx+c)^2+2ab\cos(dx+c)+b^2}\right)+(6a^3b^2-6ab^4-(a^5-a^3b^2)\cos(dx+c)^2+3(a^4b-a^2b^3)\cos(dx+c))\sin(dx+c)}{2((a^7-a^5b^2)d\cos(dx+c)+(a^6b-a^4b^3)d)}, \frac{(a^5-7a^3b^2+6ab^4)dx\cos(dx+c)+(a^4b-7a^2b^3+6b^5)dx-2(2a^2b^2-3b^4+(2a^3b-3ab^3)\cos(dx+c))\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)+(6a^3b^2-6ab^4-(a^5-a^3b^2)\cos(dx+c)^2+3(a^4b-a^2b^3)\cos(dx+c))\sin(dx+c)}{2((a^7-a^5b^2)d\cos(dx+c)+(a^6b-a^4b^3)d)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*((a^5 - 7*a^3*b^2 + 6*a*b^4)*d*x*cos(d*x + c) + (a^4*b - 7*a^2*b^3 + 6
*b^5)*d*x - (2*a^2*b^2 - 3*b^4 + (2*a^3*b - 3*a*b^3)*cos(d*x + c))*sqrt(a^2
 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2
 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^
2 + 2*a*b*cos(d*x + c) + b^2)) + (6*a^3*b^2 - 6*a*b^4 - (a^5 - a^3*b^2)*cos
(d*x + c)^2 + 3*(a^4*b - a^2*b^3)*cos(d*x + c))*sin(d*x + c))/((a^7 - a^5*b
^2)*d*cos(d*x + c) + (a^6*b - a^4*b^3)*d), 1/2*((a^5 - 7*a^3*b^2 + 6*a*b^4)
*d*x*cos(d*x + c) + (a^4*b - 7*a^2*b^3 + 6*b^5)*d*x - 2*(2*a^2*b^2 - 3*b^4
+ (2*a^3*b - 3*a*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^
2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (6*a^3*b^2 - 6*a*b^4
- (a^5 - a^3*b^2)*cos(d*x + c)^2 + 3*(a^4*b - a^2*b^3)*cos(d*x + c))*sin(d*
x + c))/((a^7 - a^5*b^2)*d*cos(d*x + c) + (a^6*b - a^4*b^3)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(sin(c + d*x)**2/(a + b*sec(c + d*x))**2, x)
```

Giac [A]
time = 0.47, size = 240, normalized size = 1.58

$$-\frac{\frac{4\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)a^3}-\frac{\left(a^2-6\,b^2\right)(dx+c)}{a^4}+\frac{4\left(2\,a^2b-3\,b^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\sqrt{-a^2+b^2}\,a^4}-\frac{2\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*b^2*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x
 + 1/2*c)^2 - a - b)*a^3) - (a^2 - 6*b^2)*(d*x + c)/a^4 + 4*(2*a^2*b - 3*b^
3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d
*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*
a^4) - 2*(a*tan(1/2*d*x + 1/2*c)^3 + 4*b*tan(1/2*d*x + 1/2*c)^3 - a*tan(1/2
*d*x + 1/2*c) + 4*b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a
^3))/d
```

Mupad [B]
time = 3.57, size = 1655, normalized size = 10.89

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(a + b/cos(c + d*x))^2,x)
```

```
[Out] ((2*tan(c/2 + (d*x)/2)^3*(a^2 + 6*b^2))/a^3 + (tan(c/2 + (d*x)/2)*(3*a*b -
a^2 + 6*b^2))/a^3 - (tan(c/2 + (d*x)/2)^5*(3*a*b + a^2 - 6*b^2))/a^3)/(d*(a
 + b + tan(c/2 + (d*x)/2)^2*(a + 3*b) - tan(c/2 + (d*x)/2)^4*(a - 3*b) - ta
n(c/2 + (d*x)/2)^6*(a - b))) + (atan((8*tan(c/2 + (d*x)/2))/((8*b)/a + (24*
b^2)/a^2 - (24*b^3)/a^3 + (144*b^4)/a^4 - (144*b^5)/a^5 - 8) + (8*b*tan(c/2
 + (d*x)/2))/(8*a - 8*b - (24*b^2)/a + (24*b^3)/a^2 - (144*b^4)/a^3 + (144*
b^5)/a^4) - (24*b^2*tan(c/2 + (d*x)/2))/(8*a*b - 8*a^2 + 24*b^2 - (24*b^3)/
a + (144*b^4)/a^2 - (144*b^5)/a^3) + (24*b^3*tan(c/2 + (d*x)/2))/(24*a*b^2
+ 8*a^2*b - 8*a^3 - 24*b^3 + (144*b^4)/a - (144*b^5)/a^2) + (144*b^4*tan(c/
2 + (d*x)/2))/(24*a*b^3 - 8*a^3*b + 8*a^4 - 144*b^4 - 24*a^2*b^2 + (144*b^5
)/a) + (144*b^5*tan(c/2 + (d*x)/2))/(144*a*b^4 + 8*a^4*b - 8*a^5 - 144*b^5
- 24*a^2*b^3 + 24*a^3*b^2))*(a^2*1i - b^2*6i)*1i)/(a^4*d) - (b*atan(((b*((a
 + b)*(a - b))^(1/2)*(2*a^2 - 3*b^2)*((8*tan(c/2 + (d*x)/2)*(144*a*b^6 - 3*
a^6*b + a^7 - 72*b^7 - 48*a^2*b^5 - 48*a^3*b^4 + 19*a^4*b^3 + 7*a^5*b^2))/a
^6 + (b*((a + b)*(a - b))^(1/2)*((8*(2*a^12 - 10*a^11*b - 12*a^8*b^4 + 18*a
^9*b^3 + 2*a^10*b^2))/a^9 - (8*b*tan(c/2 + (d*x)/2)*((a + b)*(a - b))^(1/2)
*(2*a^2 - 3*b^2)*(8*a^10*b + 8*a^8*b^3 - 16*a^9*b^2))/(a^6*(a^6 - a^4*b^2))
)*(2*a^2 - 3*b^2))/(a^6 - a^4*b^2))*1i)/(a^6 - a^4*b^2) + (b*((a + b)*(a -
b))^(1/2)*(2*a^2 - 3*b^2)*((8*tan(c/2 + (d*x)/2)*(144*a*b^6 - 3*a^6*b + a^7
 - 72*b^7 - 48*a^2*b^5 - 48*a^3*b^4 + 19*a^4*b^3 + 7*a^5*b^2))/a^6 - (b*((a
 + b)*(a - b))^(1/2)*((8*(2*a^12 - 10*a^11*b - 12*a^8*b^4 + 18*a^9*b^3 + 2*
a^10*b^2))/a^9 + (8*b*tan(c/2 + (d*x)/2)*((a + b)*(a - b))^(1/2)*(2*a^2 - 3
*b^2)*(8*a^10*b + 8*a^8*b^3 - 16*a^9*b^2))/(a^6*(a^6 - a^4*b^2)))*(2*a^2 -
3*b^2))/(a^6 - a^4*b^2))*1i)/(a^6 - a^4*b^2))/((16*(162*a*b^7 - 2*a^7*b - 1
08*b^8 + 54*a^2*b^6 - 153*a^3*b^5 + 18*a^4*b^4 + 33*a^5*b^3 - 4*a^6*b^2))/a
^9 + (b*((a + b)*(a - b))^(1/2)*(2*a^2 - 3*b^2)*((8*tan(c/2 + (d*x)/2)*(144
*a*b^6 - 3*a^6*b + a^7 - 72*b^7 - 48*a^2*b^5 - 48*a^3*b^4 + 19*a^4*b^3 + 7*
a^5*b^2))/a^6 + (b*((a + b)*(a - b))^(1/2)*((8*(2*a^12 - 10*a^11*b - 12*a^8
*b^4 + 18*a^9*b^3 + 2*a^10*b^2))/a^9 - (8*b*tan(c/2 + (d*x)/2)*((a + b)*(a
- b))^(1/2)*(2*a^2 - 3*b^2)*(8*a^10*b + 8*a^8*b^3 - 16*a^9*b^2))/(a^6*(a^6
- a^4*b^2)))*(2*a^2 - 3*b^2))/(a^6 - a^4*b^2)))/(a^6 - a^4*b^2) - (b*((a +
b)*(a - b))^(1/2)*(2*a^2 - 3*b^2)*((8*tan(c/2 + (d*x)/2)*(144*a*b^6 - 3*a^6
*b + a^7 - 72*b^7 - 48*a^2*b^5 - 48*a^3*b^4 + 19*a^4*b^3 + 7*a^5*b^2))/a^6
- (b*((a + b)*(a - b))^(1/2)*((8*(2*a^12 - 10*a^11*b - 12*a^8*b^4 + 18*a^9*
b^3 + 2*a^10*b^2))/a^9 + (8*b*tan(c/2 + (d*x)/2)*((a + b)*(a - b))^(1/2)*(2
*a^2 - 3*b^2)*(8*a^10*b + 8*a^8*b^3 - 16*a^9*b^2))/(a^6*(a^6 - a^4*b^2)))*(
2*a^2 - 3*b^2))/(a^6 - a^4*b^2)))/(a^6 - a^4*b^2)))*((a + b)*(a - b))^(1/2)
*(2*a^2 - 3*b^2)*2i)/(d*(a^6 - a^4*b^2))
```

3.219 $\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=203

$$-\frac{4a^2b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d}-\frac{2b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d}-\frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))}+\frac{}{2(a}$$

```
[Out] -4*a^2*b*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)/(a
+b)^(5/2)/d-2*b^3*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)
^(5/2)/(a+b)^(5/2)/d-1/2*sin(d*x+c)/(a+b)^2/d/(1-cos(d*x+c))+1/2*sin(d*x+c)
/(a-b)^2/d/(1+cos(d*x+c))+a*b^2*sin(d*x+c)/(a^2-b^2)^2/d/(b+a*cos(d*x+c))
```

Rubi [A]
time = 0.34, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$,
Rules used = {3957, 2810, 2727, 2743, 12, 2738, 214}

$$\frac{ab^2\sin(c+dx)}{d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}-\frac{4a^2b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{2b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{\sin(c+dx)}{2d(a+b)^2(1-\cos(c+dx))}+\frac{\sin(c+dx)}{2d(a-b)^2(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-4*a^2*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/
2)*(a + b)^(5/2)*d) - (2*b^3*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a
+ b]])/((a - b)^(5/2)*(a + b)^(5/2)*d) - Sin[c + d*x]/(2*(a + b)^2*d*(1 - C
os[c + d*x])) + Sin[c + d*x]/(2*(a - b)^2*d*(1 + Cos[c + d*x])) + (a*b^2*Si
n[c + d*x])/((a^2 - b^2)^2*d*(b + a*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2810

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e + f*x])^m/
(1 - Sin[e + f*x]^2)^(p/2)), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 -
b^2, 0] && IntegersQ[m, p/2]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cot^2(c+dx)}{(-b-a\cos(c+dx))^2}\,dx$$
$$= \int \left(-\frac{1}{2(a+b)^2(-1+\cos(c+dx))} + \frac{1}{2(a-b)^2(1+\cos(c+dx))} - \frac{}{(-a^2+b^2)}\right.$$
$$= \frac{\int \frac{1}{1+\cos(c+dx)}\,dx}{2(a-b)^2} - \frac{\int \frac{1}{-1+\cos(c+dx)}\,dx}{2(a+b)^2} - \frac{(2a^2b)\int \frac{1}{b+a\cos(c+dx)}\,dx}{(a^2-b^2)^2} + \frac{b^2\int \frac{1}{(b+a\cos(c+d}}{a^2-b^2}$$
$$= -\frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))} + \frac{\sin(c+dx)}{2(a-b)^2d(1+\cos(c+dx))} + \frac{ab^2\sin(}{(a^2-b^2)^2\,d(b+}$$
$$= -\frac{4a^2b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))} + \frac{}{2(a-b)}$$
$$= -\frac{4a^2b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))} + \frac{}{2(a-b)}$$
$$= -\frac{4a^2b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{2b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{}{2(}$$

Mathematica [A]
time = 0.59, size = 128, normalized size = 0.63

$$\frac{\frac{4b(2a^2+b^2)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} - \frac{\cot\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2} + \frac{\frac{2ab^2\sin(c+dx)}{(a+b)^2(b+a\cos(c+dx))}+\tan\left(\frac{1}{2}(c+dx)\right)}{(a-b)^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((4*b*(2*a^2 + b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(
a^2 - b^2)^(5/2) - Cot[(c + d*x)/2]/(a + b)^2 + ((2*a*b^2*Sin[c + d*x])/((a
 + b)^2*(b + a*Cos[c + d*x])) + Tan[(c + d*x)/2])/(a - b)^2)/(2*d)
```

Maple [A]
time = 0.18, size = 162, normalized size = 0.80

method	result

derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^{2}-4ba+2b^{2}}-\frac{1}{2(a+b)^{2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2b\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b}-\frac{\left(2a^{2}+b^{2}\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\ldots\right.}{\sqrt{(a+b)(a\ldots}}\right.}{\sqrt{(a+b)(a-b)}}\right.}{(a-b)^{2}(a+b)^{2}}}{d}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^{2}-4ba+2b^{2}}-\frac{1}{2(a+b)^{2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2b\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b}-\frac{\left(2a^{2}+b^{2}\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\ldots\right.}{\sqrt{(a+b)(a\ldots}}\right.}{\sqrt{(a+b)(a-b)}}\right.}{(a-b)^{2}(a+b)^{2}}}{d}$
risch	$-\frac{2i\left(-2b\,a^{2}\mathrm{e}^{3i(dx+c)}-b^{3}\mathrm{e}^{3i(dx+c)}+a^{3}\mathrm{e}^{2i(dx+c)}-4a\,b^{2}\mathrm{e}^{2i(dx+c)}+3b^{3}\mathrm{e}^{i(dx+c)}+a^{3}+2b^{2}a\right)}{\left(a\,\mathrm{e}^{2i(dx+c)}+2b\,\mathrm{e}^{i(dx+c)}+a\right)\left(a^{2}-b^{2}\right)^{2}\left(\mathrm{e}^{2i(dx+c)}-1\right)d}-\frac{2b\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia^{2}-ib^{2}\ldots}{a\sqrt{\ldots}}\right.}{\sqrt{a^{2}-b^{2}}\,(a\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)-1/2/(a+b)^2/tan(1/2*d*x+1/2*c)+
2*b/(a-b)^2/(a+b)^2*(-a*b*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-b*tan(
1/2*d*x+1/2*c)^2-a-b)-(2*a^2+b^2)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2
*d*x+1/2*c)/((a+b)*(a-b))^(1/2))))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]

time = 3.53, size = 526, normalized size = 2.59

$$\left[\frac{6\,a^{3}b^{2}-6\,ab^{4}+\left(2\,a^{2}b^{2}+b^{4}+\left(2\,a^{3}b+ab^{3}\right)\cos\left(dx+c\right)\right)\sqrt{a^{2}-b^{2}}\log\left(\frac{2\,ab\cos\left(dx+c\right)-\left(a^{2}-2\,b^{2}\right)\cos\left(dx+c\right)^{2}-2\,\sqrt{a^{2}-b^{2}}\left(b\cos\left(dx+c\right)+a\right)\sin\left(dx+c\right)+2\,a^{2}-b^{2}}{a^{2}\cos\left(dx+c\right)^{2}+2\,ab\cos\left(dx+c\right)+b^{2}}\right)\sin\left(dx+c\right)-2\left(a^{5}+a^{3}b^{2}-2\,ab^{4}\right)\cos\left(dx+c\right)^{2}+2\left(a^{4}b-2\,a^{2}b^{3}+b^{5}\right)\cos\left(dx+c\right)}{2\left(\left(a^{7}-3\,a^{5}b^{2}+3\,a^{3}b^{4}-ab^{6}\right)d\cos\left(dx+c\right)+\left(a^{6}b-3\,a^{4}b^{3}+3\,a^{2}b^{5}-b^{7}\right)d\right)\sin\left(dx+c\right)},\ \frac{3\,a^{3}b^{2}-3\,ab^{4}-\left(2\,a^{2}b^{2}+b^{4}+\left(2\,a^{3}b+ab^{3}\right)\cos\left(dx+c\right)\right)\sqrt{-a^{2}+b^{2}}\arctan\left(-\frac{\sqrt{-a^{2}+b^{2}}\left(b\cos\left(dx+c\right)+a\right)}{\left(a^{2}-b^{2}\right)\sin\left(dx+c\right)}\right)\sin\left(dx+c\right)-\left(a^{5}+a^{3}b^{2}-2\,ab^{4}\right)\cos\left(dx+c\right)^{2}+\left(a^{4}b-2\,a^{2}b^{3}+b^{5}\right)\cos\left(dx+c\right)}{\left(\left(a^{7}-3\,a^{5}b^{2}+3\,a^{3}b^{4}-ab^{6}\right)d\cos\left(dx+c\right)+\left(a^{6}b-3\,a^{4}b^{3}+3\,a^{2}b^{5}-b^{7}\right)d\right)\sin\left(dx+c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(6*a^3*b^2 - 6*a*b^4 + (2*a^2*b^2 + b^4 + (2*a^3*b + a*b^3)*cos(d*x +
c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2
- 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*c
os(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) - 2*(a^5 + a^3*b^2
- 2*a*b^4)*cos(d*x + c)^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c))/(((a^
7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c) + (a^6*b - 3*a^4*b^3 + 3*
a^2*b^5 - b^7)*d)*sin(d*x + c)), (3*a^3*b^2 - 3*a*b^4 - (2*a^2*b^2 + b^4 +
(2*a^3*b + a*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(
b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))*sin(d*x + c) - (a^5 + a^3*b
^2 - 2*a*b^4)*cos(d*x + c)^2 + (a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c))/(((a
^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c) + (a^6*b - 3*a^4*b^3 + 3
*a^2*b^5 - b^7)*d)*sin(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(csc(c + d*x)**2/(a + b*sec(c + d*x))**2, x)
```

Giac [A]
time = 0.49, size = 289, normalized size = 1.42

$$\frac{\frac{4\,(2\,a^2b+b^3)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^4-2\,a^2b^2+b^4)\sqrt{-a^2+b^2}}+\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2-2\,ab+b^2}-\frac{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+7\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a^3+a^2b+ab^2-b^3}{(a^4-2\,a^2b^2+b^4)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*(2*a^2*b + b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + a
rctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/
((a^4 - 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) + tan(1/2*d*x + 1/2*c)/(a^2 - 2*
a*b + b^2) - (a^3*tan(1/2*d*x + 1/2*c)^2 - 3*a^2*b*tan(1/2*d*x + 1/2*c)^2 +
 7*a*b^2*tan(1/2*d*x + 1/2*c)^2 - b^3*tan(1/2*d*x + 1/2*c)^2 - a^3 + a^2*b
+ a*b^2 - b^3)/((a^4 - 2*a^2*b^2 + b^4)*(a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1
/2*d*x + 1/2*c)^3 - a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))))/d
```

Mupad [B]
time = 1.79, size = 245, normalized size = 1.21

$$\frac{\frac{a^2-2\,a\,b+b^2}{a+b}-\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(a^3-3\,a^2\,b+7\,a\,b^2-b^3\right)}{{\left(a+b\right)}^2}}{d\,\left(\left(2\,a^3-6\,a^2\,b+6\,a\,b^2-2\,b^3\right)\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+\left(-2\,a^3+2\,a^2\,b+2\,a\,b^2-2\,b^3\right)\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}+\frac{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d\,{\left(a-b\right)}^2}+\frac{b\,\mathrm{atan}\left(\frac{1{}\mathrm{i}\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^4-2{}\mathrm{i}\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b^2+1{}\mathrm{i}\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^4}{{\left(a+b\right)}^{5/2}\,{\left(a-b\right)}^{3/2}}\right)\,\left(2\,a^2+b^2\right)\,2{}\mathrm{i}}{d\,{\left(a+b\right)}^{5/2}\,{\left(a-b\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^2*(a + b/cos(c + d*x))^2),x)
```

```
[Out] ((a^2 - 2*a*b + b^2)/(a + b) - (tan(c/2 + (d*x)/2)^2*(7*a*b^2 - 3*a^2*b + a
^3 - b^3))/(a + b)^2)/(d*(tan(c/2 + (d*x)/2)^3*(6*a*b^2 - 6*a^2*b + 2*a^3 -
 2*b^3) + tan(c/2 + (d*x)/2)*(2*a*b^2 + 2*a^2*b - 2*a^3 - 2*b^3))) + tan(c/
2 + (d*x)/2)/(2*d*(a - b)^2) + (b*atan((a^4*tan(c/2 + (d*x)/2)*1i + b^4*tan
(c/2 + (d*x)/2)*1i - a^2*b^2*tan(c/2 + (d*x)/2)*2i)/((a + b)^(5/2)*(a - b)^
(3/2)))*(2*a^2 + b^2)*2i)/(d*(a + b)^(5/2)*(a - b)^(5/2))
```

3.220 $\int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=343

$$-\frac{2a^2b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d}-\frac{4a^2b(a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d}-\frac{\sin(c+dx)}{12(a+b)^2d(1-\cos(c$$

```
[Out] -2*a^2*b^3*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(7/2)/
(a+b)^(7/2)/d-4*a^2*b*(a^2+b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b
)^(1/2))/(a-b)^(7/2)/(a+b)^(7/2)/d-1/12*sin(d*x+c)/(a+b)^2/d/(1-cos(d*x+c))
^2-1/4*(a-b)*sin(d*x+c)/(a+b)^3/d/(1-cos(d*x+c))-1/12*sin(d*x+c)/(a+b)^2/d/
(1-cos(d*x+c))+1/12*sin(d*x+c)/(a-b)^2/d/(1+cos(d*x+c))^2+1/12*sin(d*x+c)/(
a-b)^2/d/(1+cos(d*x+c))+1/4*(a+b)*sin(d*x+c)/(a-b)^3/d/(1+cos(d*x+c))+a^3*b
^2*sin(d*x+c)/(a^2-b^2)^3/d/(b+a*cos(d*x+c))
```

Rubi [A]
time = 0.39, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3957, 2976, 2729, 2727, 2743, 12, 2738, 214}

$$-\frac{2a^2b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{4a^2b(a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}+\frac{a^3b^2\sin(c+dx)}{d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}-\frac{\sin(c+dx)}{12d(a+b)^2(1-\cos(c+dx))}-\frac{(a-b)\sin(c+dx)}{4d(a+b)^3(1-\cos(c+dx))}+\frac{(a+b)\sin(c+dx)}{4d(a-b)^3(\cos(c+dx)+1)}+\frac{\sin(c+dx)}{12d(a-b)^2(\cos(c+dx)+1)}-\frac{\sin(c+dx)}{12d(a+b)^2(1-\cos(c+dx))^2}+\frac{\sin(c+dx)}{12d(a-b)^2(\cos(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-2*a^2*b^3*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(
7/2)*(a + b)^(7/2)*d) - (4*a^2*b*(a^2 + b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c +
d*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d) - Sin[c + d*x]/(12*(
a + b)^2*d*(1 - Cos[c + d*x])^2) - ((a - b)*Sin[c + d*x])/(4*(a + b)^3*d*(1
 - Cos[c + d*x])) - Sin[c + d*x]/(12*(a + b)^2*d*(1 - Cos[c + d*x])) + Sin[
c + d*x]/(12*(a - b)^2*d*(1 + Cos[c + d*x])^2) + Sin[c + d*x]/(12*(a - b)^2
*d*(1 + Cos[c + d*x])) + ((a + b)*Sin[c + d*x])/(4*(a - b)^3*d*(1 + Cos[c +
 d*x])) + (a^3*b^2*Sin[c + d*x])/((a^2 - b^2)^3*d*(b + a*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2976

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx &= \int \frac{\cot^2(c+dx)\csc^2(c+dx)}{(-b-a\cos(c+dx))^2}\,dx \\
&= \int \left(\frac{1}{4(a-b)^2(-1-\cos(c+dx))^2} + \frac{-a-b}{4(a-b)^3(-1-\cos(c+dx))} + \frac{}{4(a+b)^2(}\right. \\
&= \frac{\int \frac{1}{(-1-\cos(c+dx))^2}\,dx}{4(a-b)^2} + \frac{(a-b)\int \frac{1}{1-\cos(c+dx)}\,dx}{4(a+b)^3} + \frac{\int \frac{1}{(1-\cos(c+dx))^2}\,dx}{4(a+b)^2} - \frac{(a+b)\int}{4(a} \\
&= -\frac{\sin(c+dx)}{12(a+b)^2d(1-\cos(c+dx))^2} - \frac{(a-b)\sin(c+dx)}{4(a+b)^3d(1-\cos(c+dx))} + \frac{\sin(}{12(a-b)^2d(1} \\
&= -\frac{4a^2b(a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{\sin(c+dx)}{12(a+b)^2d(1-\cos(c+dx))^2} \\
&= -\frac{4a^2b(a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{\sin(c+dx)}{12(a+b)^2d(1-\cos(c+dx))^2} \\
&= -\frac{2a^2b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{4a^2b(a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+\right.}{\sqrt{a+b}}\right.}{(a-b)^{7/2}(a+b)^{7/2}d}
\end{aligned}$$

Mathematica [A]
time = 0.75, size = 281, normalized size = 0.82

$$\frac{(b+a\cos(c+dx))\sec^2(c+dx)\left(\frac{48a^2b(2a^2+3b^2)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)(b+a\cos(c+dx))}{(a^2-b^2)^{7/2}} - \frac{4(2a-b)(b+a\cos(c+dx))\cot\left(\frac{1}{2}(c+dx)\right)}{(a+b)^3} - \frac{(b+a\cos(c+dx))\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2} + \frac{24a^3b^2\sin(c+dx)}{(a-b)^3(a+b)^3} + \frac{4(2a+b)(b+a\cos(c+dx))\tan\left(\frac{1}{2}(c+dx)\right)}{(a-b)^3} + \frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{(a-b)^2}\right)}{24d(a+b\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^2*((48*a^2*b*(2*a^2 + 3*b^2)*ArcTanh[((-
a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x]))/(a^2 - b^2)
^(7/2) - (4*(2*a - b)*(b + a*Cos[c + d*x])*Cot[(c + d*x)/2])/(a + b)^3 - ((
b + a*Cos[c + d*x])*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(a + b)^2 + (24*a^
3*b^2*Sin[c + d*x])/((a - b)^3*(a + b)^3) + (4*(2*a + b)*(b + a*Cos[c + d*x
])*Tan[(c + d*x)/2])/(a - b)^3 + ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*T
an[(c + d*x)/2])/(a - b)^2))/(24*d*(a + b*Sec[c + d*x])^2)
```

Maple [A]
time = 0.25, size = 242, normalized size = 0.71

method	result
derivativedivides	$\frac{\frac{\frac{a\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{b\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+3a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8\left(a^{2}-2ba+b^{2}\right)(a-b)}-\frac{1}{24(a+b)^{2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}-\frac{3a-b}{8(a+b)^{3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2b\,a^{2}\left(\cdots\right)}{\cdots}}{d}$
default	$\frac{\frac{\frac{a\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{b\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+3a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8\left(a^{2}-2ba+b^{2}\right)(a-b)}-\frac{1}{24(a+b)^{2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}-\frac{3a-b}{8(a+b)^{3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2b\,a^{2}\left(\cdots\right)}{\cdots}}{d}$
risch	$\frac{2i\left(-6a^{4}b\,e^{7i(dx+c)}-9a^{2}b^{3}e^{7i(dx+c)}-6a^{3}b^{2}e^{6i(dx+c)}-9a\,b^{4}e^{6i(dx+c)}+14a^{4}b\,e^{5i(dx+c)}+25a^{2}b^{3}e^{5i(dx+c)}+6b^{5}e^{5i(dx+c)}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/8/(a^2-2*a*b+b^2)/(a-b)*(1/3*a*tan(1/2*d*x+1/2*c)^3-1/3*b*tan(1/2*d*
x+1/2*c)^3+3*a*tan(1/2*d*x+1/2*c)+b*tan(1/2*d*x+1/2*c))-1/24/(a+b)^2/tan(1/
2*d*x+1/2*c)^3-1/8*(3*a-b)/(a+b)^3/tan(1/2*d*x+1/2*c)+2*b*a^2/(a+b)^3/(a-b)
^3*(-a*b*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-b*tan(1/2*d*x+1/2*c)^2-
a-b)-(2*a^2+3*b^2)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a
+b)*(a-b))^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 4.16, size = 1040, normalized size = 3.03

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(22*a^5*b^2 - 14*a^3*b^4 - 8*a*b^6 + 2*(2*a^7 + 10*a^5*b^2 - 11*a^3*b
^4 - a*b^6)*cos(d*x + c)^4 - 2*(4*a^6*b - 7*a^4*b^3 + 2*a^2*b^5 + b^7)*cos(
d*x + c)^3 - 3*(2*a^4*b^2 + 3*a^2*b^4 - (2*a^5*b + 3*a^3*b^3)*cos(d*x + c)^
3 - (2*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c)^2 + (2*a^5*b + 3*a^3*b^3)*cos(d*x
+ c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^
2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2
*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) - 6*(a^7 + 6*a^5*
b^2 - 5*a^3*b^4 - 2*a*b^6)*cos(d*x + c)^2 + 10*(a^6*b - 2*a^4*b^3 + a^2*b^5
)*cos(d*x + c))/(((a^9 - 4*a^7*b^2 + 6*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*cos(d
*x + c)^3 + (a^8*b - 4*a^6*b^3 + 6*a^4*b^5 - 4*a^2*b^7 + b^9)*d*cos(d*x + c
)^2 - (a^9 - 4*a^7*b^2 + 6*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*cos(d*x + c) - (a
^8*b - 4*a^6*b^3 + 6*a^4*b^5 - 4*a^2*b^7 + b^9)*d)*sin(d*x + c)), -1/3*(11*
a^5*b^2 - 7*a^3*b^4 - 4*a*b^6 + (2*a^7 + 10*a^5*b^2 - 11*a^3*b^4 - a*b^6)*c
os(d*x + c)^4 - (4*a^6*b - 7*a^4*b^3 + 2*a^2*b^5 + b^7)*cos(d*x + c)^3 - 3*
(2*a^4*b^2 + 3*a^2*b^4 - (2*a^5*b + 3*a^3*b^3)*cos(d*x + c)^3 - (2*a^4*b^2
+ 3*a^2*b^4)*cos(d*x + c)^2 + (2*a^5*b + 3*a^3*b^3)*cos(d*x + c))*sqrt(-a^2
 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x
+ c)))*sin(d*x + c) - 3*(a^7 + 6*a^5*b^2 - 5*a^3*b^4 - 2*a*b^6)*cos(d*x + c
)^2 + 5*(a^6*b - 2*a^4*b^3 + a^2*b^5)*cos(d*x + c))/(((a^9 - 4*a^7*b^2 + 6*
a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*cos(d*x + c)^3 + (a^8*b - 4*a^6*b^3 + 6*a^4*
b^5 - 4*a^2*b^7 + b^9)*d*cos(d*x + c)^2 - (a^9 - 4*a^7*b^2 + 6*a^5*b^4 - 4*
a^3*b^6 + a*b^8)*d*cos(d*x + c) - (a^8*b - 4*a^6*b^3 + 6*a^4*b^5 - 4*a^2*b^
7 + b^9)*d)*sin(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(csc(c + d*x)**4/(a + b*sec(c + d*x))**2, x)
```

Giac [A]
time = 0.52, size = 457, normalized size = 1.33

$$-\frac{\frac{48\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)}-\frac{48\,(2\,a^4b+3\,a^2b^3)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\sqrt{-a^2+b^2}}-\frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-24\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+18\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6-6\,a^5b+15\,a^4b^2-20\,a^3b^3+15\,a^2b^4-6\,ab^5+b^6}+\frac{9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a+b}{(a^3+3\,a^2b+3\,ab^2+b^3)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/24*(48*a^3*b^2*tan(1/2*d*x + 1/2*c)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)
*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)) - 48*(2*a^4
*b + 3*a^2*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((
a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^6 -
 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) - (a^4*tan(1/2*d*x + 1/2*c)
^3 - 4*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 6*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 4*
a*b^3*tan(1/2*d*x + 1/2*c)^3 + b^4*tan(1/2*d*x + 1/2*c)^3 + 9*a^4*tan(1/2*d
*x + 1/2*c) - 24*a^3*b*tan(1/2*d*x + 1/2*c) + 18*a^2*b^2*tan(1/2*d*x + 1/2*
c) - 3*b^4*tan(1/2*d*x + 1/2*c))/(a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 +
 15*a^2*b^4 - 6*a*b^5 + b^6) + (9*a*tan(1/2*d*x + 1/2*c)^2 - 3*b*tan(1/2*d*
x + 1/2*c)^2 + a + b)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*tan(1/2*d*x + 1/2*c)
^3))/d
```

Mupad [B]

time = 1.62, size = 403, normalized size = 1.17

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d\,{\left(a-b\right)}^2}+\frac{\frac{a^3-3\,a^2\,b+3\,a\,b^2-b^3}{3\,\left(a+b\right)}+\frac{2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(4\,a^4-13\,a^3\,b+15\,a^2\,b^2-7\,a\,b^3+b^4\right)}{3\,{\left(a+b\right)}^2}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(3\,a^5-13\,a^4\,b+38\,a^3\,b^2-18\,a^2\,b^3+7\,a\,b^4-b^5\right)}{{\left(a+b\right)}^3}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(8\,a^4-32\,a^3\,b+48\,a^2\,b^2-32\,a\,b^3+8\,b^4\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(8\,a^4-16\,a^3\,b+16\,a\,b^3-8\,b^4\right)\right)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{16\,a^2-16\,b^2}{64\,{\left(a-b\right)}^4}+\frac{1}{8\,{\left(a-b\right)}^2}\right)}{d}+\frac{a^2\,b\,\mathrm{atan}\left(\frac{1\mathrm{i}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^6-3\mathrm{i}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^4\,b^2+3\mathrm{i}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b^4-1\mathrm{i}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^6}{{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{5/2}}\right)\,\left(2\,a^2+3\,b^2\right)\,2\mathrm{i}}{d\,{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^4*(a + b/cos(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(24*d*(a - b)^2) + ((3*a*b^2 - 3*a^2*b + a^3 - b^3)/(3
*(a + b)) + (2*tan(c/2 + (d*x)/2)^2*(4*a^4 - 13*a^3*b - 7*a*b^3 + b^4 + 15*
a^2*b^2))/(3*(a + b)^2) - (tan(c/2 + (d*x)/2)^4*(7*a*b^4 - 13*a^4*b + 3*a^5
 - b^5 - 18*a^2*b^3 + 38*a^3*b^2))/(a + b)^3)/(d*(tan(c/2 + (d*x)/2)^5*(8*a
^4 - 32*a^3*b - 32*a*b^3 + 8*b^4 + 48*a^2*b^2) - tan(c/2 + (d*x)/2)^3*(16*a
*b^3 - 16*a^3*b + 8*a^4 - 8*b^4))) + (tan(c/2 + (d*x)/2)*((16*a^2 - 16*b^2)
/(64*(a - b)^4) + 1/(8*(a - b)^2)))/d + (a^2*b*atan((a^6*tan(c/2 + (d*x)/2)
*1i - b^6*tan(c/2 + (d*x)/2)*1i + a^2*b^4*tan(c/2 + (d*x)/2)*3i - a^4*b^2*t
an(c/2 + (d*x)/2)*3i)/((a + b)^(7/2)*(a - b)^(5/2)))*(2*a^2 + 3*b^2)*2i)/(d
*(a + b)^(7/2)*(a - b)^(7/2))
```

3.221 $\int \frac{\sin^7(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=329

$$-\frac{(a^6-18a^4b^2+45a^2b^4-28b^6)\cos(c+dx)}{a^9d}-\frac{3b(3a^4-10a^2b^2+7b^4)\cos^2(c+dx)}{2a^8d}+\frac{(a^4-6a^2b^2+5b^4)\cos^3(c+dx)}{a^7d}$$

```
[Out] -(a^6-18*a^4*b^2+45*a^2*b^4-28*b^6)*cos(d*x+c)/a^9/d-3/2*b*(3*a^4-10*a^2*b^
2+7*b^4)*cos(d*x+c)^2/a^8/d+(a^4-6*a^2*b^2+5*b^4)*cos(d*x+c)^3/a^7/d+1/4*b*
(9*a^2-10*b^2)*cos(d*x+c)^4/a^6/d-3/5*(a^2-2*b^2)*cos(d*x+c)^5/a^5/d-1/2*b*
cos(d*x+c)^6/a^4/d+1/7*cos(d*x+c)^7/a^3/d-1/2*b^3*(a^2-b^2)^3/a^10/d/(b+a*c
os(d*x+c))^2+3*b^2*(a^2-3*b^2)*(a^2-b^2)^2/a^10/d/(b+a*cos(d*x+c))+3*b*(a^2
-b^2)*(a^4-9*a^2*b^2+12*b^4)*ln(b+a*cos(d*x+c))/a^10/d
```

Rubi [A]
time = 0.36, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3957, 2916, 12, 962}

$$-\frac{b\cos^6(c+dx)}{2a^4d}+\frac{\cos^7(c+dx)}{7a^3d}+\frac{3b^2(a^2-3b^2)(a^2-b^2)^2}{a^{10}d(a\cos(c+dx)+b)}-\frac{b^3(a^2-b^2)^3}{2a^{10}d(a\cos(c+dx)+b)^2}+\frac{b(9a^2-10b^2)\cos^4(c+dx)}{4a^6d}-\frac{3(a^2-2b^2)\cos^5(c+dx)}{5a^5d}+\frac{3b(a^2-b^2)(a^4-9a^2b^2+12b^4)\log(a\cos(c+dx)+b)}{a^{10}d}-\frac{3b(3a^4-10a^2b^2+7b^4)\cos^2(c+dx)}{2a^8d}+\frac{(a^4-6a^2b^2+5b^4)\cos^3(c+dx)}{a^7d}-\frac{(a^6-18a^4b^2+45a^2b^4-28b^6)\cos(c+dx)}{a^9d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^7/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -(((a^6 - 18*a^4*b^2 + 45*a^2*b^4 - 28*b^6)*Cos[c + d*x])/(a^9*d)) - (3*b*(
3*a^4 - 10*a^2*b^2 + 7*b^4)*Cos[c + d*x]^2)/(2*a^8*d) + ((a^4 - 6*a^2*b^2 +
 5*b^4)*Cos[c + d*x]^3)/(a^7*d) + (b*(9*a^2 - 10*b^2)*Cos[c + d*x]^4)/(4*a^
6*d) - (3*(a^2 - 2*b^2)*Cos[c + d*x]^5)/(5*a^5*d) - (b*Cos[c + d*x]^6)/(2*a
^4*d) + Cos[c + d*x]^7/(7*a^3*d) - (b^3*(a^2 - b^2)^3)/(2*a^10*d*(b + a*Cos
[c + d*x])^2) + (3*b^2*(a^2 - 3*b^2)*(a^2 - b^2)^2)/(a^10*d*(b + a*Cos[c +
d*x])) + (3*b*(a^2 - b^2)*(a^4 - 9*a^2*b^2 + 12*b^4)*Log[b + a*Cos[c + d*x]
])/(a^10*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 962

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin^7(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\int \frac{\cos^3(c+dx)\sin^7(c+dx)}{(-b-a\cos(c+dx))^3}\,dx$$

$$= \frac{\text{Subst}\left(\int \frac{x^3\left(a^2-x^2\right)^3}{a^3(-b+x)^3}\,dx, x, -a\cos(c+dx)\right)}{a^7 d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^3\left(a^2-x^2\right)^3}{(-b+x)^3}\,dx, x, -a\cos(c+dx)\right)}{a^{10} d}$$

$$= \frac{\text{Subst}\left(\int \left(a^6\left(1+\frac{-18a^4b^2+45a^2b^4-28b^6}{a^6}\right)+\frac{b^3\left(-a^2+b^2\right)^3}{(b-x)^3}+\frac{3b^2\left(a^2-3b^2\right)\left(a^2-b^2\right)^2}{(b-x)^2}+\frac{3b(-\cdots}{\cdots}\right.\right.}{\cdots}$$

$$= -\frac{\left(a^6-18a^4b^2+45a^2b^4-28b^6\right)\cos(c+dx)}{a^9 d} - \frac{3b\left(3a^4-10a^2b^2+7b^4\right)\cos^2(c\cdots}{2a^8 d}$$

Mathematica [A]
time = 3.19, size = 550, normalized size = 1.67

$$\frac{-7945a^8b+164080a^6b^3-502320a^4b^5+425600a^2b^7-76160b^9-784a^9\cos(3(c+dx))+17528a^7b^2\cos(3(c+dx))-43680a^5b^4\cos(3(c+dx))+26880a^3b^6\cos(3(c+dx))-1456a^8b\cos(4(c+dx))+4872a^6b^3\cos(4(c+dx))-3360a^4b^5\cos(4(c+dx))+152a^9\cos(5(c+dx))-840a^7b^2\cos(5(c+dx))+672a^5b^4\cos(5(c+dx))+174a^8b\cos(6(c+dx))-168a^6b^3\cos(6(c+dx))-39a^9\cos(7(c+dx))+48a^7b^2\cos(7(c+dx))-15a^8b\cos(8(c+dx))+5a^9\cos(9(c+dx))+13440a^8b\log(b+a\cos(c+dx))-107520a^6b^3\log(b+a\cos(c+dx))+13440a^4b^5\log(b+a\cos(c+dx))+403200a^2b^7\log(b+a\cos(c+dx))-322560b^9\log(b+a\cos(c+dx))+70a^2b\cos(2(c+dx))\left(-137a^6+1896a^4b^2-4656a^2b^4+2912b^6+192\left(a^6-10a^4b^2+21a^2b^4-12b^6\right)\log(b+a\cos(c+dx))\right)-70a\cos(c+dx)\left(49a^8-1472a^6b^2+3216a^4b^4+576a^2b^6-2432b^8-768b^2\left(a^6-10a^4b^2+21a^2b^4-12b^6\right)\log(b+a\cos(c+dx))\right)}{8960a^{10}d(b+a\cos(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^7/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (-7945*a^8*b + 164080*a^6*b^3 - 502320*a^4*b^5 + 425600*a^2*b^7 - 76160*b^9
 - 784*a^9*Cos[3*(c + d*x)] + 17528*a^7*b^2*Cos[3*(c + d*x)] - 43680*a^5*b^
4*Cos[3*(c + d*x)] + 26880*a^3*b^6*Cos[3*(c + d*x)] - 1456*a^8*b*Cos[4*(c +
 d*x)] + 4872*a^6*b^3*Cos[4*(c + d*x)] - 3360*a^4*b^5*Cos[4*(c + d*x)] + 15
2*a^9*Cos[5*(c + d*x)] - 840*a^7*b^2*Cos[5*(c + d*x)] + 672*a^5*b^4*Cos[5*(
```

```
c + d*x)] + 174*a^8*b*Cos[6*(c + d*x)] - 168*a^6*b^3*Cos[6*(c + d*x)] - 39*
a^9*Cos[7*(c + d*x)] + 48*a^7*b^2*Cos[7*(c + d*x)] - 15*a^8*b*Cos[8*(c + d*
x)] + 5*a^9*Cos[9*(c + d*x)] + 13440*a^8*b*Log[b + a*Cos[c + d*x]] - 107520
*a^6*b^3*Log[b + a*Cos[c + d*x]] + 13440*a^4*b^5*Log[b + a*Cos[c + d*x]] +
403200*a^2*b^7*Log[b + a*Cos[c + d*x]] - 322560*b^9*Log[b + a*Cos[c + d*x]]
 + 70*a^2*b*Cos[2*(c + d*x)]*(-137*a^6 + 1896*a^4*b^2 - 4656*a^2*b^4 + 2912
*b^6 + 192*(a^6 - 10*a^4*b^2 + 21*a^2*b^4 - 12*b^6)*Log[b + a*Cos[c + d*x]]
) - 70*a*Cos[c + d*x]*(49*a^8 - 1472*a^6*b^2 + 3216*a^4*b^4 + 576*a^2*b^6 -
 2432*b^8 - 768*b^2*(a^6 - 10*a^4*b^2 + 21*a^2*b^4 - 12*b^6)*Log[b + a*Cos[
c + d*x]]))/(8960*a^10*d*(b + a*Cos[c + d*x])^2)
```

Maple [A]
time = 0.15, size = 367, normalized size = 1.12 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^7/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/a^9*(1/7*cos(d*x+c)^7*a^6-1/2*b*cos(d*x+c)^6*a^5-3/5*a^6*cos(d*x+c)^
5+6/5*a^4*b^2*cos(d*x+c)^5+9/4*a^5*b*cos(d*x+c)^4-5/2*a^3*b^3*cos(d*x+c)^4+
a^6*cos(d*x+c)^3-6*a^4*b^2*cos(d*x+c)^3+5*a^2*b^4*cos(d*x+c)^3-9/2*a^5*b*co
s(d*x+c)^2+15*a^3*b^3*cos(d*x+c)^2-21/2*a*b^5*cos(d*x+c)^2-a^6*cos(d*x+c)+1
8*a^4*b^2*cos(d*x+c)-45*a^2*b^4*cos(d*x+c)+28*b^6*cos(d*x+c))+3*b^2/a^10*(a
^6-5*a^4*b^2+7*a^2*b^4-3*b^6)/(b+a*cos(d*x+c))+3*b/a^10*(a^6-10*a^4*b^2+21*
a^2*b^4-12*b^6)*ln(b+a*cos(d*x+c))-1/2*b^3*(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/a^
10/(b+a*cos(d*x+c))^2)
```

Maxima [A]
time = 0.27, size = 326, normalized size = 0.99

$$\frac{\frac{70\left(5a^6b^3-27a^4b^5+39a^2b^7-17b^9+6\left(a^7b^2-5a^5b^4+7a^3b^6-3ab^8\right)\cos(dx+c)\right)}{a^{12}\cos(dx+c)^2+2a^{11}b\cos(dx+c)+a^{10}b^2}+\frac{20a^6\cos(dx+c)^7-70a^5b\cos(dx+c)^6-84\left(a^6-2a^4b^2\right)\cos(dx+c)^5+35\left(9a^5b-10a^3b^3\right)\cos(dx+c)^4+140\left(a^6-6a^4b^2+5a^2b^4\right)\cos(dx+c)^3-210\left(3a^5b-10a^3b^3+7ab^5\right)\cos(dx+c)^2-140\left(a^6-18a^4b^2+45a^2b^4-28b^6\right)\cos(dx+c)}{a^9}+\frac{420\left(a^6b-10a^4b^3+21a^2b^5-12b^7\right)\log(a\cos(dx+c)+b)}{a^{10}}}{140d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/140*(70*(5*a^6*b^3 - 27*a^4*b^5 + 39*a^2*b^7 - 17*b^9 + 6*(a^7*b^2 - 5*a^
5*b^4 + 7*a^3*b^6 - 3*a*b^8)*cos(d*x + c))/(a^12*cos(d*x + c)^2 + 2*a^11*b*
cos(d*x + c) + a^10*b^2) + (20*a^6*cos(d*x + c)^7 - 70*a^5*b*cos(d*x + c)^6
 - 84*(a^6 - 2*a^4*b^2)*cos(d*x + c)^5 + 35*(9*a^5*b - 10*a^3*b^3)*cos(d*x
+ c)^4 + 140*(a^6 - 6*a^4*b^2 + 5*a^2*b^4)*cos(d*x + c)^3 - 210*(3*a^5*b -
10*a^3*b^3 + 7*a*b^5)*cos(d*x + c)^2 - 140*(a^6 - 18*a^4*b^2 + 45*a^2*b^4 -
 28*b^6)*cos(d*x + c))/a^9 + 420*(a^6*b - 10*a^4*b^3 + 21*a^2*b^5 - 12*b^7)
*log(a*cos(d*x + c) + b)/a^10)/d
```

Fricas [A]
time = 4.56, size = 447, normalized size = 1.36

$$\frac{80a^9\cos(dx+c)^9-120a^8b\cos(dx+c)^8+2275a^6b^3-11235a^4b^5+13860a^2b^7-4760b^9-48\left(7a^9-4a^7b^2\right)\cos(dx+c)^7+84\left(7a^8b-4a^6b^3\right)\cos(dx+c)^6+56\left(10a^9-21a^7b^2+12a^5b^4\right)\cos(dx+c)^5-140\left(10a^8b-21a^6b^3+12a^4b^5\right)\cos(dx+c)^4-560\left(a^9-10a^7b^2+21a^5b^4-12a^3b^6\right)\cos(dx+c)^3-35\left(7a^8b-399a^6b^3+1116a^4b^5-728a^2b^7\right)\cos(dx+c)^2+70\left(41a^7b^2-81a^5b^4-108a^3b^6+152ab^8\right)\cos(dx+c)+1680\left(a^6b^3-10a^4b^5+21a^2b^7-12b^9+\left(a^8b-10a^6b^3+21a^4b^5-12a^2b^7\right)\cos(dx+c)^2+2\left(a^7b^2-10a^5b^4+21a^3b^6-12ab^8\right)\cos(dx+c)\right)\log(a\cos(dx+c)+b)}{560\left(a^{12}d\cos(dx+c)^2+2a^{11}bd\cos(dx+c)+a^{10}b^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^7/(a+b*sec(d*x+c))^3,x, algorithm="fricas")

[Out] 1/560*(80*a^9*cos(d*x + c)^9 - 120*a^8*b*cos(d*x + c)^8 + 2275*a^6*b^3 - 11235*a^4*b^5 + 13860*a^2*b^7 - 4760*b^9 - 48*(7*a^9 - 4*a^7*b^2)*cos(d*x + c)^7 + 84*(7*a^8*b - 4*a^6*b^3)*cos(d*x + c)^6 + 56*(10*a^9 - 21*a^7*b^2 + 12*a^5*b^4)*cos(d*x + c)^5 - 140*(10*a^8*b - 21*a^6*b^3 + 12*a^4*b^5)*cos(d*x + c)^4 - 560*(a^9 - 10*a^7*b^2 + 21*a^5*b^4 - 12*a^3*b^6)*cos(d*x + c)^3 - 35*(7*a^8*b - 399*a^6*b^3 + 1116*a^4*b^5 - 728*a^2*b^7)*cos(d*x + c)^2 + 70*(41*a^7*b^2 - 81*a^5*b^4 - 108*a^3*b^6 + 152*a*b^8)*cos(d*x + c) + 1680*(a^6*b^3 - 10*a^4*b^5 + 21*a^2*b^7 - 12*b^9 + (a^8*b - 10*a^6*b^3 + 21*a^4*b^5 - 12*a^2*b^7)*cos(d*x + c)^2 + 2*(a^7*b^2 - 10*a^5*b^4 + 21*a^3*b^6 - 12*a*b^8)*cos(d*x + c))*log(a*cos(d*x + c) + b))/(a^12*d*cos(d*x + c)^2 + 2*a^11*b*d*cos(d*x + c) + a^10*b^2*d)

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)**7/(a+b*sec(d*x+c))**3,x)

[Out] Timed out

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2150 vs. $2(317) = 634$.
time = 0.62, size = 2150, normalized size = 6.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^7/(a+b*sec(d*x+c))^3,x, algorithm="giac")

[Out] 1/140*(420*(a^7*b - a^6*b^2 - 10*a^5*b^3 + 10*a^4*b^4 + 21*a^3*b^5 - 21*a^2*b^6 - 12*a*b^7 + 12*b^8)*log(abs(a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^11 - a^10*b) - 420*(a^6*b - 10*a^4*b^3 + 21*a^2*b^5 - 12*b^7)*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^10 - 70*(9*a^8*b + 6*a^7*b^2 - 105*a^6*b^3 - 148*a^5*b^4 + 187*a^4*b^5 + 390*a^3*b^6 + 17*a^2*b^7 - 248*a*b^8 - 108*b^9 + 18*a^8*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 12*a^7*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 202*a^6*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 56*a^5*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 566*a^4*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 76*a^3*b^6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 598*a^2*b^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 32*a*b^8*(cos(d*x + c) -

1)/(cos(d*x + c) + 1) + 216*b^9*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*
a^8*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 18*a^7*b^2*(cos(d*x + c)
- 1)^2/(cos(d*x + c) + 1)^2 - 81*a^6*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c)
+ 1)^2 + 180*a^5*b^4*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 99*a^4*b^
5*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 378*a^3*b^6*(cos(d*x + c) - 1
)^2/(cos(d*x + c) + 1)^2 + 81*a^2*b^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) +
1)^2 + 216*a*b^8*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 108*b^9*(cos(d
*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/((a + b + a*(cos(d*x + c) - 1)/(cos(d*
x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))^2*a^10) + (128*a^7 -
1089*a^6*b - 3696*a^5*b^2 + 10890*a^4*b^3 + 11200*a^3*b^4 - 22869*a^2*b^5
- 7840*a*b^6 + 13068*b^7 - 896*a^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) +
8463*a^6*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 24192*a^5*b^2*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) - 81830*a^4*b^3*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1) - 70000*a^3*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 165963*a^2*b
^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 47040*a*b^6*(cos(d*x + c) - 1)/(
cos(d*x + c) + 1) - 91476*b^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2688*
a^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 28749*a^6*b*(cos(d*x + c) -
1)^2/(cos(d*x + c) + 1)^2 - 64176*a^5*b^2*(cos(d*x + c) - 1)^2/(cos(d*x +
c) + 1)^2 + 262290*a^4*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 1764
00*a^3*b^4*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 509649*a^2*b^5*(cos(
d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 117600*a*b^6*(cos(d*x + c) - 1)^2/(c
os(d*x + c) + 1)^2 + 274428*b^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 -
4480*a^7*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 56035*a^6*b*(cos(d*x
+ c) - 1)^3/(cos(d*x + c) + 1)^3 + 80640*a^5*b^2*(cos(d*x + c) - 1)^3/(cos(
d*x + c) + 1)^3 - 453950*a^4*b^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3
- 229600*a^3*b^4*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 859215*a^2*b^5
*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 156800*a*b^6*(cos(d*x + c) - 1
)^3/(cos(d*x + c) + 1)^3 - 457380*b^7*(cos(d*x + c) - 1)^3/(cos(d*x + c) +
1)^3 - 56035*a^6*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 48720*a^5*b^
2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 453950*a^4*b^3*(cos(d*x + c)
- 1)^4/(cos(d*x + c) + 1)^4 + 162400*a^3*b^4*(cos(d*x + c) - 1)^4/(cos(d*x
+ c) + 1)^4 - 859215*a^2*b^5*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 11
7600*a*b^6*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 457380*b^7*(cos(d*x
+ c) - 1)^4/(cos(d*x + c) + 1)^4 + 28749*a^6*b*(cos(d*x + c) - 1)^5/(cos(d*
x + c) + 1)^5 + 13440*a^5*b^2*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 2
62290*a^4*b^3*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 58800*a^3*b^4*(co
s(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 509649*a^2*b^5*(cos(d*x + c) - 1)^
5/(cos(d*x + c) + 1)^5 + 47040*a*b^6*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1
)^5 - 274428*b^7*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 8463*a^6*b*(co
s(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 1680*a^5*b^2*(cos(d*x + c) - 1)^6/
(cos(d*x + c) + 1)^6 + 81830*a^4*b^3*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1
)^6 + 8400*a^3*b^4*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 165963*a^2*b
^5*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 7840*a*b^6*(cos(d*x + c) - 1
)^6/(cos(d*x + c) + 1)^6 + 91476*b^7*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1
)^6 + 1089*a^6*b*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 - 10890*a^4*b^3*

```
(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 + 22869*a^2*b^5*(cos(d*x + c) - 1
)^7/(cos(d*x + c) + 1)^7 - 13068*b^7*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1
)^7)/(a^10*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^7))/d
```

Mupad [B]
time = 0.24, size = 762, normalized size = 2.32

$$\frac{\cos(c+dx)^4\left(\frac{2b^3}{a^6}+\frac{3b\left(\frac{3}{a^3}-\frac{6b^2}{a^5}\right)}{4a}\right)}{d}-\frac{\frac{-5a^6b^3+27a^4b^5-39a^2b^7+17b^9}{2a}+\cos(c+dx)\left(-3a^6b^2+15a^4b^4-21a^2b^6+9b^8\right)}{d\left(a^{11}\cos(c+dx)^2+2a^{10}b\cos(c+dx)+a^9b^2\right)}-\frac{\cos(c+dx)^5\left(\frac{3}{5a^3}-\frac{6b^2}{5a^5}\right)}{d}-\frac{\cos(c+dx)^2\left(\frac{3b^2\left(\frac{8b^3}{a^6}+\frac{3b\left(\frac{3}{a^3}-\frac{6b^2}{a^5}\right)}{a}\right)}{2a^2}-\frac{b^3\left(\frac{3}{a^3}-\frac{6b^2}{a^5}\right)}{2a^3}+\frac{3b\left(\frac{3}{a^3}+\frac{3b^4}{a^7}+\frac{3b^2\left(\frac{3}{a^3}-\frac{6b^2}{a^5}\right)}{a^2}-\frac{3b\left(\frac{8b^3}{a^6}+\frac{3b\left(\frac{3}{a^3}-\frac{6b^2}{a^5}\right)}{a}\right)}{a}\right)}{2a}\right)}{d}+\frac{\cos(c+dx)^7}{7a^3d}-\frac{\cos(c+dx)\left(\frac{1}{a^3}+\frac{b^3\left(\frac{8b^3}{a^6}+\frac{3b\left(\frac{3}{a^3}-\frac{6b^2}{a^5}\right)}{a}\right)}{a^3}+\frac{3b^2\left(\frac{3}{a^3}+\frac{3b^4}{a^7}+\frac{3b^2\left(\frac{3}{a^3}-\frac{6b^2}{a^5}\right)}{a^2}-\frac{3b\left(\frac{8b^3}{a^6}+\frac{3b\left(\frac{3}{a^3}-\frac{6b^2}{a^5}\right)}{a}\right)}{a}\right)}{a^2}-\frac{3b\left(\frac{3b^2\left(\frac{8b^3}{a^6}+\frac{3b\left(\frac{3}{a^3}-\frac{6b^2}{a^5}\right)}{a}\right)}{a^2}-\frac{b^3\left(\frac{3}{a^3}-\frac{6b^2}{a^5}\right)}{a^3}+\frac{3b\left(\frac{3}{a^3}+\frac{3b^4}{a^7}+\frac{3b^2\left(\frac{3}{a^3}-\frac{6b^2}{a^5}\right)}{a^2}-\frac{3b\left(\frac{8b^3}{a^6}+\frac{3b\left(\frac{3}{a^3}-\frac{6b^2}{a^5}\right)}{a}\right)}{a}\right)}{a}\right)}{a}\right)}{d}+\frac{\cos(c+dx)^3\left(\frac{1}{a^3}+\frac{b^4}{a^7}+\frac{b^2\left(\frac{3}{a^3}-\frac{6b^2}{a^5}\right)}{a^2}-\frac{b\left(\frac{8b^3}{a^6}+\frac{3b\left(\frac{3}{a^3}-\frac{6b^2}{a^5}\right)}{a}\right)}{a}\right)}{d}-\frac{b\cos(c+dx)^6}{2a^4d}+\frac{\ln(b+a\cos(c+dx))\left(3a^6b-30a^4b^3+63a^2b^5-36b^7\right)}{a^{10}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^7/(a + b/cos(c + d*x))^3,x)
```

```
[Out] (cos(c + d*x)^4*((2*b^3)/a^6 + (3*b*(3/a^3 - (6*b^2)/a^5))/(4*a)))/d - ((17
*b^9 - 39*a^2*b^7 + 27*a^4*b^5 - 5*a^6*b^3)/(2*a) + cos(c + d*x)*(9*b^8 - 2
1*a^2*b^6 + 15*a^4*b^4 - 3*a^6*b^2))/(d*(a^11*cos(c + d*x)^2 + a^9*b^2 + 2*
a^10*b*cos(c + d*x))) - (cos(c + d*x)^5*(3/(5*a^3) - (6*b^2)/(5*a^5)))/d -
(cos(c + d*x)^2*((3*b^2*((8*b^3)/a^6 + (3*b*(3/a^3 - (6*b^2)/a^5))/a))/(2*a
^2) - (b^3*(3/a^3 - (6*b^2)/a^5))/(2*a^3) + (3*b*(3/a^3 + (3*b^4)/a^7 + (3*
b^2*(3/a^3 - (6*b^2)/a^5))/a^2 - (3*b*((8*b^3)/a^6 + (3*b*(3/a^3 - (6*b^2)/
a^5))/a))/a))/(2*a)))/d + cos(c + d*x)^7/(7*a^3*d) - (cos(c + d*x)*(1/a^3 +
 (b^3*((8*b^3)/a^6 + (3*b*(3/a^3 - (6*b^2)/a^5))/a))/a^3 + (3*b^2*(3/a^3 +
(3*b^4)/a^7 + (3*b^2*(3/a^3 - (6*b^2)/a^5))/a^2 - (3*b*((8*b^3)/a^6 + (3*b*
(3/a^3 - (6*b^2)/a^5))/a))/a))/a^2 - (3*b*((3*b^2*((8*b^3)/a^6 + (3*b*(3/a^
3 - (6*b^2)/a^5))/a))/a^2 - (b^3*(3/a^3 - (6*b^2)/a^5))/a^3 + (3*b*(3/a^3 +
 (3*b^4)/a^7 + (3*b^2*(3/a^3 - (6*b^2)/a^5))/a^2 - (3*b*((8*b^3)/a^6 + (3*b
*(3/a^3 - (6*b^2)/a^5))/a))/a))/a))/a))/d + (cos(c + d*x)^3*(1/a^3 + b^4/a^
7 + (b^2*(3/a^3 - (6*b^2)/a^5))/a^2 - (b*((8*b^3)/a^6 + (3*b*(3/a^3 - (6*b^
2)/a^5))/a))/a))/d - (b*cos(c + d*x)^6)/(2*a^4*d) + (log(b + a*cos(c + d*x)
)*(3*a^6*b - 36*b^7 + 63*a^2*b^5 - 30*a^4*b^3))/(a^10*d)
```

3.222 $\int \frac{\sin^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=239

$$-\frac{(a^4-12a^2b^2+15b^4)\cos(c+dx)}{a^7d}-\frac{b(3a^2-5b^2)\cos^2(c+dx)}{a^6d}+\frac{2(a^2-3b^2)\cos^3(c+dx)}{3a^5d}+\frac{3b\cos^4(c+dx)}{4a^4d}-$$

```
[Out] -(a^4-12*a^2*b^2+15*b^4)*cos(d*x+c)/a^7/d-b*(3*a^2-5*b^2)*cos(d*x+c)^2/a^6/
d+2/3*(a^2-3*b^2)*cos(d*x+c)^3/a^5/d+3/4*b*cos(d*x+c)^4/a^4/d-1/5*cos(d*x+c
)^5/a^3/d-1/2*b^3*(a^2-b^2)^2/a^8/d/(b+a*cos(d*x+c))^2+b^2*(3*a^4-10*a^2*b^
2+7*b^4)/a^8/d/(b+a*cos(d*x+c))+b*(3*a^4-20*a^2*b^2+21*b^4)*ln(b+a*cos(d*x+
c))/a^8/d
```

Rubi [A]
time = 0.25, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3957, 2916, 12, 962}

$$\frac{3b\cos^4(c+dx)}{4a^4d}-\frac{\cos^5(c+dx)}{5a^3d}-\frac{b^3(a^2-b^2)^2}{2a^8d(a\cos(c+dx)+b)^2}-\frac{b(3a^2-5b^2)\cos^2(c+dx)}{a^6d}+\frac{2(a^2-3b^2)\cos^3(c+dx)}{3a^5d}+\frac{b^2(3a^4-10a^2b^2+7b^4)}{a^8d(a\cos(c+dx)+b)}+\frac{b(3a^4-20a^2b^2+21b^4)\log(a\cos(c+dx)+b)}{a^8d}-\frac{(a^4-12a^2b^2+15b^4)\cos(c+dx)}{a^7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^5/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -(((a^4 - 12*a^2*b^2 + 15*b^4)*Cos[c + d*x])/(a^7*d)) - (b*(3*a^2 - 5*b^2)*
Cos[c + d*x]^2)/(a^6*d) + (2*(a^2 - 3*b^2)*Cos[c + d*x]^3)/(3*a^5*d) + (3*b
*Cos[c + d*x]^4)/(4*a^4*d) - Cos[c + d*x]^5/(5*a^3*d) - (b^3*(a^2 - b^2)^2)
/(2*a^8*d*(b + a*Cos[c + d*x])^2) + (b^2*(3*a^4 - 10*a^2*b^2 + 7*b^4))/(a^8
*d*(b + a*Cos[c + d*x])) + (b*(3*a^4 - 20*a^2*b^2 + 21*b^4)*Log[b + a*Cos[c
 + d*x]])/(a^8*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 962

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
```

```
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx &= -\int \frac{\cos^3(c+dx)\sin^5(c+dx)}{(-b-a\cos(c+dx))^3}\,dx \\
&= \frac{\operatorname{Subst}\left(\int \frac{x^3(a^2-x^2)^2}{a^3(-b+x)^3}\,dx, x, -a\cos(c+dx)\right)}{a^5 d} \\
&= \frac{\operatorname{Subst}\left(\int \frac{x^3(a^2-x^2)^2}{(-b+x)^3}\,dx, x, -a\cos(c+dx)\right)}{a^8 d} \\
&= \frac{\operatorname{Subst}\left(\int \left(a^4\left(1+\frac{3b^2(-4a^2+5b^2)}{a^4}\right)-\frac{b^3(-a^2+b^2)^2}{(b-x)^3}+\frac{3a^4b^2-10a^2b^4+7b^6}{(b-x)^2}+\frac{-3a^4b+20a^2b^3-}{b-x}\right.\right.}{a^8} \\
&= -\frac{(a^4-12a^2b^2+15b^4)\cos(c+dx)}{a^7 d}-\frac{b(3a^2-5b^2)\cos^2(c+dx)}{a^6 d}+\frac{2(a^2-3b^2)}{3a}
\end{aligned}
$$

Mathematica [A]
time = 1.87, size = 388, normalized size = 1.62

$$\frac{-1740a^6b+26160a^4b^3-46080a^2b^5+12480b^7-206a^7\cos(3(c+dx))+2780a^5b^2\cos(3(c+dx))-3360a^3b^4\cos(3(c+dx))-274a^6b\cos(4(c+dx))+420a^4b^3\cos(4(c+dx))+38a^7\cos(5(c+dx))-84a^5b^2\cos(5(c+dx))+21a^6b\cos(6(c+dx))-6a^7\cos(7(c+dx))+2880a^6b\log(b+a\cos(c+dx))-13440a^4b^3\log(b+a\cos(c+dx))-18240a^2b^5\log(b+a\cos(c+dx))+40320b^7\log(b+a\cos(c+dx))+5a^2b\cos(2(c+dx))\left(-407a^4+3888a^2b^2-4800b^4+192(3a^4-20a^2b^2+21b^4)\log(b+a\cos(c+dx))\right)-10a\cos(c+dx)\left(85a^6-1728a^4b^2+1584a^2b^4+1536b^6-384b^2(3a^4-20a^2b^2+21b^4)\log(b+a\cos(c+dx))\right)}{1920a^8d(b+a\cos(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^5/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (-1740*a^6*b + 26160*a^4*b^3 - 46080*a^2*b^5 + 12480*b^7 - 206*a^7*Cos[3*(c
 + d*x)] + 2780*a^5*b^2*Cos[3*(c + d*x)] - 3360*a^3*b^4*Cos[3*(c + d*x)] -
274*a^6*b*Cos[4*(c + d*x)] + 420*a^4*b^3*Cos[4*(c + d*x)] + 38*a^7*Cos[5*(c
 + d*x)] - 84*a^5*b^2*Cos[5*(c + d*x)] + 21*a^6*b*Cos[6*(c + d*x)] - 6*a^7*
Cos[7*(c + d*x)] + 2880*a^6*b*Log[b + a*Cos[c + d*x]] - 13440*a^4*b^3*Log[b
 + a*Cos[c + d*x]] - 18240*a^2*b^5*Log[b + a*Cos[c + d*x]] + 40320*b^7*Log[
b + a*Cos[c + d*x]] + 5*a^2*b*Cos[2*(c + d*x)]*(-407*a^4 + 3888*a^2*b^2 - 4
800*b^4 + 192*(3*a^4 - 20*a^2*b^2 + 21*b^4)*Log[b + a*Cos[c + d*x]]) - 10*a
```

```
*Cos[c + d*x]*(85*a^6 - 1728*a^4*b^2 + 1584*a^2*b^4 + 1536*b^6 - 384*b^2*(3
*a^4 - 20*a^2*b^2 + 21*b^4)*Log[b + a*Cos[c + d*x]]))/(1920*a^8*d*(b + a*Co
s[c + d*x])^2)
```

Maple [A]
time = 0.33, size = 239, normalized size = 1.00 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^5/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/a^7*(1/5*cos(d*x+c)^5*a^4-3/4*b*cos(d*x+c)^4*a^3-2/3*a^4*cos(d*x+c)
^3+2*a^2*b^2*cos(d*x+c)^3+3*a^3*b*cos(d*x+c)^2-5*a*b^3*cos(d*x+c)^2+a^4*cos
(d*x+c)-12*a^2*b^2*cos(d*x+c)+15*b^4*cos(d*x+c))+b^2/a^8*(3*a^4-10*a^2*b^2+
7*b^4)/(b+a*cos(d*x+c))+b/a^8*(3*a^4-20*a^2*b^2+21*b^4)*ln(b+a*cos(d*x+c))-
1/2*b^3*(a^4-2*a^2*b^2+b^4)/a^8/(b+a*cos(d*x+c))^2)
```

Maxima [A]
time = 0.29, size = 234, normalized size = 0.98

$$\frac{\frac{30\left(5a^4b^3-18a^2b^5+13b^7+2\left(3a^5b^2-10a^3b^4+7ab^6\right)\cos\left(dx+c\right)\right)}{a^{10}\cos\left(dx+c\right)^2+2a^9b\cos\left(dx+c\right)+a^8b^2}-\frac{12a^4\cos\left(dx+c\right)^5-45a^3b\cos\left(dx+c\right)^4-40\left(a^4-3a^2b^2\right)\cos\left(dx+c\right)^3+60\left(3a^3b-5ab^3\right)\cos\left(dx+c\right)^2+60\left(a^4-12a^2b^2+15b^4\right)\cos\left(dx+c\right)}{a^7}+\frac{60\left(3a^4b-20a^2b^3+21b^5\right)\log\left(a\cos\left(dx+c\right)+b\right)}{a^8}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/60*(30*(5*a^4*b^3 - 18*a^2*b^5 + 13*b^7 + 2*(3*a^5*b^2 - 10*a^3*b^4 + 7*a
*b^6)*cos(d*x + c))/(a^10*cos(d*x + c)^2 + 2*a^9*b*cos(d*x + c) + a^8*b^2)
- (12*a^4*cos(d*x + c)^5 - 45*a^3*b*cos(d*x + c)^4 - 40*(a^4 - 3*a^2*b^2)*c
os(d*x + c)^3 + 60*(3*a^3*b - 5*a*b^3)*cos(d*x + c)^2 + 60*(a^4 - 12*a^2*b^
2 + 15*b^4)*cos(d*x + c))/a^7 + 60*(3*a^4*b - 20*a^2*b^3 + 21*b^5)*log(a*co
s(d*x + c) + b)/a^8)/d
```

Fricas [A]
time = 3.99, size = 331, normalized size = 1.38

$$-\frac{96a^7\cos\left(dx+c\right)^7-168a^6b\cos\left(dx+c\right)^6-1785a^4b^3+5520a^2b^5-3120b^7-16\left(20a^7-21a^5b^2\right)\cos\left(dx+c\right)^5+40\left(20a^6b-21a^4b^3\right)\cos\left(dx+c\right)^4+160\left(3a^7-20a^5b^2+21a^3b^4\right)\cos\left(dx+c\right)^3+15\left(25a^6b-592a^4b^3+800a^2b^5\right)\cos\left(dx+c\right)^2-30\left(71a^5b^2-48a^3b^4-128ab^6\right)\cos\left(dx+c\right)-480\left(3a^4b^3-20a^2b^5+21b^7+\left(3a^6b-20a^4b^3+21a^2b^5\right)\cos\left(dx+c\right)^2+2\left(3a^5b^2-20a^3b^4+21ab^6\right)\cos\left(dx+c\right)\right)\log\left(a\cos\left(dx+c\right)+b\right)}{480\left(a^{10}d\cos\left(dx+c\right)^2+2a^9bd\cos\left(dx+c\right)+a^8b^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/480*(96*a^7*cos(d*x + c)^7 - 168*a^6*b*cos(d*x + c)^6 - 1785*a^4*b^3 + 5
520*a^2*b^5 - 3120*b^7 - 16*(20*a^7 - 21*a^5*b^2)*cos(d*x + c)^5 + 40*(20*a
^6*b - 21*a^4*b^3)*cos(d*x + c)^4 + 160*(3*a^7 - 20*a^5*b^2 + 21*a^3*b^4)*c
os(d*x + c)^3 + 15*(25*a^6*b - 592*a^4*b^3 + 800*a^2*b^5)*cos(d*x + c)^2 -
30*(71*a^5*b^2 - 48*a^3*b^4 - 128*a*b^6)*cos(d*x + c) - 480*(3*a^4*b^3 - 20
*a^2*b^5 + 21*b^7 + (3*a^6*b - 20*a^4*b^3 + 21*a^2*b^5)*cos(d*x + c)^2 + 2*
(3*a^5*b^2 - 20*a^3*b^4 + 21*a*b^6)*cos(d*x + c))*log(a*cos(d*x + c) + b))/
(a^10*d*cos(d*x + c)^2 + 2*a^9*b*d*cos(d*x + c) + a^8*b^2*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**5/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1337 vs. 2(231) = 462.
time = 0.56, size = 1337, normalized size = 5.59

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(60*(3*a^5*b - 3*a^4*b^2 - 20*a^3*b^3 + 20*a^2*b^4 + 21*a*b^5 - 21*b^6
)*log(abs(a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1)))/(a^9 - a^8*b) - 60*(3*a^4*b - 20*a^2*b^3 + 21*b^
5)*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^8 - 30*(9*a^6*b +
 6*a^5*b^2 - 75*a^4*b^3 - 108*a^3*b^4 + 51*a^2*b^5 + 150*a*b^6 + 63*b^7 + 1
8*a^6*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 12*a^5*b^2*(cos(d*x + c) -
1)/(cos(d*x + c) + 1) - 142*a^4*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) +
 36*a^3*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 250*a^2*b^5*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) - 24*a*b^6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
 - 126*b^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*a^6*b*(cos(d*x + c) -
1)^2/(cos(d*x + c) + 1)^2 - 18*a^5*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) +
 1)^2 - 51*a^4*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 120*a^3*b^4*
(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 3*a^2*b^5*(cos(d*x + c) - 1)^2/
(cos(d*x + c) + 1)^2 - 126*a*b^6*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2
+ 63*b^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/((a + b + a*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))^2*a^8
) + (64*a^5 - 411*a^4*b - 1200*a^3*b^2 + 2740*a^2*b^3 + 1800*a*b^4 - 2877*b
^5 - 320*a^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2415*a^4*b*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) + 5280*a^3*b^2*(cos(d*x + c) - 1)/(cos(d*x + c)
+ 1) - 14900*a^2*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 7200*a*b^4*(co
s(d*x + c) - 1)/(cos(d*x + c) + 1) + 14385*b^5*(cos(d*x + c) - 1)/(cos(d*x
+ c) + 1) + 640*a^5*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 5910*a^4*b*
(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 7680*a^3*b^2*(cos(d*x + c) - 1)
^2/(cos(d*x + c) + 1)^2 + 31000*a^2*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c)
+ 1)^2 + 10800*a*b^4*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 28770*b^5*
(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 5910*a^4*b*(cos(d*x + c) - 1)^3
```

```
/(cos(d*x + c) + 1)^3 + 4320*a^3*b^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1
)^3 - 31000*a^2*b^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 7200*a*b^4*
(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 28770*b^5*(cos(d*x + c) - 1)^3/
(cos(d*x + c) + 1)^3 - 2415*a^4*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4
 - 720*a^3*b^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 14900*a^2*b^3*(c
os(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 1800*a*b^4*(cos(d*x + c) - 1)^4/(
cos(d*x + c) + 1)^4 - 14385*b^5*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 +
 411*a^4*b*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 2740*a^2*b^3*(cos(d*
x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 2877*b^5*(cos(d*x + c) - 1)^5/(cos(d*x
 + c) + 1)^5)/(a^8*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^5))/d
```

Mupad [B]
time = 1.11, size = 315, normalized size = 1.32

$$\frac{\cos(c+dx)^3\left(\frac{2}{3a^3}-\frac{2b^2}{a^5}\right)}{d}-\frac{\cos(c+dx)^2\left(\frac{4b^3}{a^6}+\frac{3b\left(\frac{2}{a^3}-\frac{6b^2}{a^5}\right)}{2a}\right)}{d}+\frac{\cos(c+dx)\,(3a^4b^2-10a^2b^4+7b^6)+\frac{5a^4b^3-18a^2b^5+13b^7}{2a}}{d\,\left(a^9\cos(c+dx)^2+2a^8b\cos(c+dx)+a^7b^2\right)}-\frac{\cos(c+dx)\left(\frac{1}{a^3}+\frac{3b^4}{a^7}+\frac{3b^2\left(\frac{2}{a^3}-\frac{6b^2}{a^5}\right)}{a^2}-\frac{3b\left(\frac{8b^3}{a^6}+\frac{3b\left(\frac{2}{a^3}-\frac{6b^2}{a^5}\right)}{a}\right)}{a}\right)}{d}-\frac{\cos(c+dx)^5}{5a^3d}+\frac{3b\cos(c+dx)^4}{4a^4d}+\frac{\ln(b+a\cos(c+dx))\,(3a^4b-20a^2b^3+21b^5)}{a^8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5/(a + b/cos(c + d*x))^3,x)
```

```
[Out] (cos(c + d*x)^3*(2/(3*a^3) - (2*b^2)/a^5))/d - (cos(c + d*x)^2*((4*b^3)/a^6
 + (3*b*(2/a^3 - (6*b^2)/a^5))/(2*a)))/d + (cos(c + d*x)*(7*b^6 - 10*a^2*b^
4 + 3*a^4*b^2) + (13*b^7 - 18*a^2*b^5 + 5*a^4*b^3)/(2*a))/(d*(a^9*cos(c + d
*x)^2 + a^7*b^2 + 2*a^8*b*cos(c + d*x))) - (cos(c + d*x)*(1/a^3 + (3*b^4)/a
^7 + (3*b^2*(2/a^3 - (6*b^2)/a^5))/a^2 - (3*b*((8*b^3)/a^6 + (3*b*(2/a^3 -
(6*b^2)/a^5))/a))/a))/d - cos(c + d*x)^5/(5*a^3*d) + (3*b*cos(c + d*x)^4)/(
4*a^4*d) + (log(b + a*cos(c + d*x))*(3*a^4*b + 21*b^5 - 20*a^2*b^3))/(a^8*d
)
```

3.223 $\int \frac{\sin^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=158

$$-\frac{(a^2-6b^2)\cos(c+dx)}{a^5d}-\frac{3b\cos^2(c+dx)}{2a^4d}+\frac{\cos^3(c+dx)}{3a^3d}-\frac{b^3(a^2-b^2)}{2a^6d(b+a\cos(c+dx))^2}+\frac{b^2(3a^2-5b^2)}{a^6d(b+a\cos(c+dx))}$$

```
[Out] -(a^2-6*b^2)*cos(d*x+c)/a^5/d-3/2*b*cos(d*x+c)^2/a^4/d+1/3*cos(d*x+c)^3/a^3
/d-1/2*b^3*(a^2-b^2)/a^6/d/(b+a*cos(d*x+c))^2+b^2*(3*a^2-5*b^2)/a^6/d/(b+a*
cos(d*x+c))+b*(3*a^2-10*b^2)*ln(b+a*cos(d*x+c))/a^6/d
```

Rubi [A]
time = 0.20, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3957, 2916, 12, 908}

$$-\frac{3b\cos^2(c+dx)}{2a^4d}+\frac{\cos^3(c+dx)}{3a^3d}+\frac{b^2(3a^2-5b^2)}{a^6d(a\cos(c+dx)+b)}+\frac{b(3a^2-10b^2)\log(a\cos(c+dx)+b)}{a^6d}-\frac{b^3(a^2-b^2)}{2a^6d(a\cos(c+dx)+b)^2}-\frac{(a^2-6b^2)\cos(c+dx)}{a^5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -(((a^2 - 6*b^2)*Cos[c + d*x])/(a^5*d)) - (3*b*Cos[c + d*x]^2)/(2*a^4*d) +
Cos[c + d*x]^3/(3*a^3*d) - (b^3*(a^2 - b^2))/(2*a^6*d*(b + a*Cos[c + d*x])^
2) + (b^2*(3*a^2 - 5*b^2))/(a^6*d*(b + a*Cos[c + d*x])) + (b*(3*a^2 - 10*b^
2)*Log[b + a*Cos[c + d*x]])/(a^6*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
```

```
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx &= -\int \frac{\cos^3(c+dx)\sin^3(c+dx)}{(-b-a\cos(c+dx))^3}\,dx \\
&= \frac{\text{Subst}\left(\int \frac{x^3(a^2-x^2)}{a^3(-b+x)^3}\,dx, x, -a\cos(c+dx)\right)}{a^3d} \\
&= \frac{\text{Subst}\left(\int \frac{x^3(a^2-x^2)}{(-b+x)^3}\,dx, x, -a\cos(c+dx)\right)}{a^6d} \\
&= \frac{\text{Subst}\left(\int \left(a^2\left(1-\frac{6b^2}{a^2}\right)+\frac{-a^2b^3+b^5}{(b-x)^3}+\frac{3a^2b^2-5b^4}{(b-x)^2}+\frac{-3a^2b+10b^3}{b-x}-3bx-x^2\right)\,dx, x, -\right.}{a^6d} \\
&= -\frac{(a^2-6b^2)\cos(c+dx)}{a^5d}-\frac{3b\cos^2(c+dx)}{2a^4d}+\frac{\cos^3(c+dx)}{3a^3d}-\frac{b^3(a^2-b^2)}{2a^6d(b+a\cos(c+}
\end{aligned}$$

Mathematica [A]
time = 0.52, size = 208, normalized size = 1.32

$$\frac{(b+a\cos(c+dx))\left(9a^4(b+2a\cos(c+dx))-(b+a\cos(c+dx))^2\left(72a(a^2-8b^2)\cos(c+dx)+\frac{-9a^4b+48a^2b^3-48b^5}{(b+a\cos(c+dx))^2}+\frac{6(3a^4-48a^2b^2+80b^4)}{b+a\cos(c+dx)}+72a^2b\cos(2(c+dx))-8a^3\cos(3(c+dx))+96(-3a^2b+10b^3)\log(b+a\cos(c+dx))\right)\right)\sec^3(c+dx)}{96a^6d(a+b\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((b + a*Cos[c + d*x])*(9*a^4*(b + 2*a*Cos[c + d*x]) - (b + a*Cos[c + d*x])^
2*(72*a*(a^2 - 8*b^2)*Cos[c + d*x] + (-9*a^4*b + 48*a^2*b^3 - 48*b^5)/(b +
a*Cos[c + d*x])^2 + (6*(3*a^4 - 48*a^2*b^2 + 80*b^4))/(b + a*Cos[c + d*x])
+ 72*a^2*b*Cos[2*(c + d*x)] - 8*a^3*Cos[3*(c + d*x)] + 96*(-3*a^2*b + 10*b^
3)*Log[b + a*Cos[c + d*x]]))*Sec[c + d*x]^3)/(96*a^6*d*(a + b*Sec[c + d*x])
^3)
```

Maple [A]
time = 0.22, size = 144, normalized size = 0.91

method	result

derivativedivides	$\frac{\frac{\left(\cos^3(dx+c)\right)a^2}{3}-\frac{3b\left(\cos^2(dx+c)\right)a}{2}-a^2\cos(dx+c)+6b^2\cos(dx+c)}{a^5}+\frac{b^2\left(3a^2-5b^2\right)}{a^6(b+a\cos(dx+c))}+\frac{b\left(3a^2-10b^2\right)\ln(b+a\cos(dx+c))}{a^6}-\frac{b}{2a^6(b}$ ... $\over d$
default	$\frac{\frac{\left(\cos^3(dx+c)\right)a^2}{3}-\frac{3b\left(\cos^2(dx+c)\right)a}{2}-a^2\cos(dx+c)+6b^2\cos(dx+c)}{a^5}+\frac{b^2\left(3a^2-5b^2\right)}{a^6(b+a\cos(dx+c))}+\frac{b\left(3a^2-10b^2\right)\ln(b+a\cos(dx+c))}{a^6}-\frac{b}{2a^6(b}$ ... $\over d$
risch	$-\frac{3ibx}{a^4}+\frac{10ib^3x}{a^6}+\frac{e^{3i(dx+c)}}{24a^3d}-\frac{3b\,e^{2i(dx+c)}}{8a^4d}-\frac{3\,e^{i(dx+c)}}{8a^3d}+\frac{3\,e^{i(dx+c)}b^2}{a^5d}-\frac{3\,e^{-i(dx+c)}}{8a^3d}+\frac{3\,e^{-i(dx+c)}b^2}{a^5d}-$
norman	$\frac{\frac{\left(6a^4b-12a^3b^2-14a^2b^3+40a\,b^4-20b^5\right)\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^5d(a-b)}+\frac{-4a^6+62a^4b^2-118a^2b^4+60b^6}{3a^5d\left(a^2-2ba+b^2\right)}-\frac{2\left(2a^6-5a^5b+18a^4b^2-17a^3b^3-48a^2b^4+\right.}{a^5d\left(a^2-2ba+\right.}}{\left(1+\tan\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/a^5*(1/3*cos(d*x+c)^3*a^2-3/2*b*cos(d*x+c)^2*a-a^2*cos(d*x+c)+6*b^2*
cos(d*x+c))+b^2/a^6*(3*a^2-5*b^2)/(b+a*cos(d*x+c))+b/a^6*(3*a^2-10*b^2)*ln(
b+a*cos(d*x+c))-1/2*b^3*(a^2-b^2)/a^6/(b+a*cos(d*x+c))^2)
```

Maxima [A]

time = 0.26, size = 154, normalized size = 0.97

$$\frac{\frac{3\left(5a^2b^3-9b^5+2\left(3a^3b^2-5ab^4\right)\cos(dx+c)\right)}{a^8\cos(dx+c)^2+2a^7b\cos(dx+c)+a^6b^2}+\frac{2a^2\cos(dx+c)^3-9ab\cos(dx+c)^2-6\left(a^2-6b^2\right)\cos(dx+c)}{a^5}+\frac{6\left(3a^2b-10b^3\right)\log(a\cos(dx+c)+b)}{a^6}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/6*(3*(5*a^2*b^3 - 9*b^5 + 2*(3*a^3*b^2 - 5*a*b^4)*cos(d*x + c))/(a^8*cos(
d*x + c)^2 + 2*a^7*b*cos(d*x + c) + a^6*b^2) + (2*a^2*cos(d*x + c)^3 - 9*a*
b*cos(d*x + c)^2 - 6*(a^2 - 6*b^2)*cos(d*x + c))/a^5 + 6*(3*a^2*b - 10*b^3)
*log(a*cos(d*x + c) + b)/a^6)/d
```

Fricas [A]

time = 4.48, size = 226, normalized size = 1.43

$$\frac{4a^5\cos(dx+c)^5-10a^4b\cos(dx+c)^4+39a^2b^3-54b^5-4\left(3a^5-10a^3b^2\right)\cos(dx+c)^3-3\left(5a^4b-42a^2b^3\right)\cos(dx+c)^2+6\left(7a^3b^2+2ab^4\right)\cos(dx+c)+12\left(3a^2b^3-10b^5+\left(3a^4b-10a^2b^3\right)\cos(dx+c)^2+2\left(3a^3b^2-10ab^4\right)\cos(dx+c)\right)\log(a\cos(dx+c)+b)}{12\left(a^8d\cos(dx+c)^2+2a^7bd\cos(dx+c)+a^6b^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(4*a^5*cos(d*x + c)^5 - 10*a^4*b*cos(d*x + c)^4 + 39*a^2*b^3 - 54*b^5
- 4*(3*a^5 - 10*a^3*b^2)*cos(d*x + c)^3 - 3*(5*a^4*b - 42*a^2*b^3)*cos(d*x
+ c)^2 + 6*(7*a^3*b^2 + 2*a*b^4)*cos(d*x + c) + 12*(3*a^2*b^3 - 10*b^5 + (3
```

```
*a^4*b - 10*a^2*b^3)*cos(d*x + c)^2 + 2*(3*a^3*b^2 - 10*a*b^4)*cos(d*x + c)
)*log(a*cos(d*x + c) + b))/(a^8*d*cos(d*x + c)^2 + 2*a^7*b*d*cos(d*x + c) +
 a^6*b^2*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.55, size = 170, normalized size = 1.08

$$\frac{(3\,a^2b-10\,b^3)\log\left(|-a\cos\left(dx+c\right)-b|\right)}{a^6d}+\frac{5\,a^2b^3-9\,b^5+\frac{2\,(3\,a^3b^2d-5\,ab^4d)\cos(dx+c)}{d}}{2\,(a\cos\left(dx+c\right)+b)^2a^6d}+\frac{2\,a^6d^8\cos\left(dx+c\right)^3-9\,a^5bd^8\cos\left(dx+c\right)^2-6\,a^6d^8\cos\left(dx+c\right)+36\,a^4b^2d^8\cos\left(dx+c\right)}{6\,a^9d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] (3*a^2*b - 10*b^3)*log(abs(-a*cos(d*x + c) - b))/(a^6*d) + 1/2*(5*a^2*b^3 -
 9*b^5 + 2*(3*a^3*b^2*d - 5*a*b^4*d)*cos(d*x + c)/d)/((a*cos(d*x + c) + b)^
2*a^6*d) + 1/6*(2*a^6*d^8*cos(d*x + c)^3 - 9*a^5*b*d^8*cos(d*x + c)^2 - 6*a
^6*d^8*cos(d*x + c) + 36*a^4*b^2*d^8*cos(d*x + c))/(a^9*d^9)
```

Mupad [**B**]
time = 0.10, size = 167, normalized size = 1.06

$$\frac{\cos\left(c+d\,x\right)^3}{3\,a^3\,d}-\frac{\cos\left(c+d\,x\right)\,\left(5\,b^4-3\,a^2\,b^2\right)+\frac{9\,b^5-5\,a^2\,b^3}{2\,a}}{d\,\left(a^7\cos\left(c+d\,x\right)^2+2\,a^6\,b\,\cos\left(c+d\,x\right)+a^5\,b^2\right)}-\frac{\cos\left(c+d\,x\right)\,\left(\frac{1}{a^3}-\frac{6\,b^2}{a^5}\right)}{d}-\frac{3\,b\cos\left(c+d\,x\right)^2}{2\,a^4\,d}+\frac{\ln\left(b+a\,\cos\left(c+d\,x\right)\right)\,\left(3\,a^2\,b-10\,b^3\right)}{a^6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(a + b/cos(c + d*x))^3,x)
```

```
[Out] cos(c + d*x)^3/(3*a^3*d) - (cos(c + d*x)*(5*b^4 - 3*a^2*b^2) + (9*b^5 - 5*a
^2*b^3)/(2*a))/(d*(a^7*cos(c + d*x)^2 + a^5*b^2 + 2*a^6*b*cos(c + d*x))) -
(cos(c + d*x)*(1/a^3 - (6*b^2)/a^5))/d - (3*b*cos(c + d*x)^2)/(2*a^4*d) + (
log(b + a*cos(c + d*x))*(3*a^2*b - 10*b^3))/(a^6*d)
```

3.224 $\int \frac{\sin(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=83

$$-\frac{\cos(c+dx)}{a^3d}-\frac{b^3}{2a^4d(b+a\cos(c+dx))^2}+\frac{3b^2}{a^4d(b+a\cos(c+dx))}+\frac{3b\log(b+a\cos(c+dx))}{a^4d}$$

```
[Out] -cos(d*x+c)/a^3/d-1/2*b^3/a^4/d/(b+a*cos(d*x+c))^2+3*b^2/a^4/d/(b+a*cos(d*x
+c))+3*b*ln(b+a*cos(d*x+c))/a^4/d
```

Rubi [A]
time = 0.09, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3957, 2912, 12, 45}

$$-\frac{b^3}{2a^4d(a\cos(c+dx)+b)^2}+\frac{3b^2}{a^4d(a\cos(c+dx)+b)}+\frac{3b\log(a\cos(c+dx)+b)}{a^4d}-\frac{\cos(c+dx)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -(Cos[c + d*x]/(a^3*d)) - b^3/(2*a^4*d*(b + a*Cos[c + d*x])^2) + (3*b^2)/(a
^4*d*(b + a*Cos[c + d*x])) + (3*b*Log[b + a*Cos[c + d*x]])/(a^4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
```

```
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(c+dx)}{(a+b\sec(c+dx))^3}\,dx &= -\int \frac{\cos^3(c+dx)\sin(c+dx)}{(-b-a\cos(c+dx))^3}\,dx\\ &= \frac{\text{Subst}\left(\int \frac{x^3}{a^3(-b+x)^3}\,dx, x, -a\cos(c+dx)\right)}{ad}\\ &= \frac{\text{Subst}\left(\int \frac{x^3}{(-b+x)^3}\,dx, x, -a\cos(c+dx)\right)}{a^4d}\\ &= \frac{\text{Subst}\left(\int \left(1-\frac{b^3}{(b-x)^3}+\frac{3b^2}{(b-x)^2}-\frac{3b}{b-x}\right)\,dx, x, -a\cos(c+dx)\right)}{a^4d}\\ &= -\frac{\cos(c+dx)}{a^3d}-\frac{b^3}{2a^4d(b+a\cos(c+dx))^2}+\frac{3b^2}{a^4d(b+a\cos(c+dx))}+\frac{3b\log(b+\end{aligned}$$

Mathematica [A]
time = 0.27, size = 111, normalized size = 1.34

$$\frac{-2a^3\cos^3(c+dx)+2a^2b\cos^2(c+dx)(-2+3\log(b+a\cos(c+dx)))+4ab^2\cos(c+dx)(1+3\log(b+a\cos(c+dx)))+b^3(5+6\log(b+a\cos(c+dx)))}{2a^4d(b+a\cos(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (-2*a^3*Cos[c + d*x]^3 + 2*a^2*b*Cos[c + d*x]^2*(-2 + 3*Log[b + a*Cos[c + d
*x]]) + 4*a*b^2*Cos[c + d*x]*(1 + 3*Log[b + a*Cos[c + d*x]]) + b^3*(5 + 6*L
og[b + a*Cos[c + d*x]]))/(2*a^4*d*(b + a*Cos[c + d*x])^2)
```

Maple [A]
time = 0.12, size = 85, normalized size = 1.02

method	result
derivativedivides	$\frac{-\frac{1}{a^3\sec(dx+c)}-\frac{3b\ln(\sec(dx+c))}{a^4}-\frac{b}{2a^2(a+b\sec(dx+c))^2}+\frac{3b\ln(a+b\sec(dx+c))}{a^4}-\frac{2b}{a^3(a+b\sec(dx+c))}}{d}$
default	$\frac{-\frac{1}{a^3\sec(dx+c)}-\frac{3b\ln(\sec(dx+c))}{a^4}-\frac{b}{2a^2(a+b\sec(dx+c))^2}+\frac{3b\ln(a+b\sec(dx+c))}{a^4}-\frac{2b}{a^3(a+b\sec(dx+c))}}{d}$
risch	$-\frac{3ibx}{a^4}-\frac{e^{i(dx+c)}}{2a^3d}-\frac{e^{-i(dx+c)}}{2a^3d}-\frac{6ibc}{a^4d}+\frac{2b^2\left(3a\,e^{3i(dx+c)}+5b\,e^{2i(dx+c)}+3\,e^{i(dx+c)}a\right)}{a^4\left(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a\right)^2d}+\frac{3b\ln\left(e^{2i(dx+c)}+\frac{2b\,e^{i(dx}}{a}\right.}{a^4d}$

norman	$\frac{\frac{-2a^4+10b^2a^2-6b^4}{a^3d\left(a^2-2ba+b^2\right)}-\frac{\left(2a^3-6b\,a^2+12b^2a-6b^3\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3d(a-b)}-\frac{\left(-4a^4+8b\,a^3-18b^3a+12b^4\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3d\left(a^2-2ba+b^2\right)}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)^2}-\frac{3b\ln\left(1+\tan\right.}{d\,c}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(d*x+c)/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(-1/a^3/sec(d*x+c)-3/a^4*b*ln(sec(d*x+c))-1/2*b/a^2/(a+b*sec(d*x+c))^2+3/a^4*b*ln(a+b*sec(d*x+c))-2/a^3*b/(a+b*sec(d*x+c)))

Maxima [A]
time = 0.26, size = 87, normalized size = 1.05

$$\frac{\frac{6\,ab^2\cos(dx+c)+5\,b^3}{a^6\cos(dx+c)^2+2\,a^5b\cos(dx+c)+a^4b^2}-\frac{2\,\cos(dx+c)}{a^3}+\frac{6\,b\log(a\cos(dx+c)+b)}{a^4}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")

[Out] 1/2*((6*a*b^2*cos(d*x + c) + 5*b^3)/(a^6*cos(d*x + c)^2 + 2*a^5*b*cos(d*x + c) + a^4*b^2) - 2*cos(d*x + c)/a^3 + 6*b*log(a*cos(d*x + c) + b)/a^4)/d

Fricas [A]
time = 2.97, size = 126, normalized size = 1.52

$$-\frac{2\,a^3\cos\left(dx+c\right)^3+4\,a^2b\cos\left(dx+c\right)^2-4\,ab^2\cos\left(dx+c\right)-5\,b^3-6\,\left(a^2b\cos\left(dx+c\right)^2+2\,ab^2\cos\left(dx+c\right)+b^3\right)\log\left(a\cos\left(dx+c\right)+b\right)}{2\,\left(a^6d\cos\left(dx+c\right)^2+2\,a^5bd\cos\left(dx+c\right)+a^4b^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")

[Out] -1/2*(2*a^3*cos(d*x + c)^3 + 4*a^2*b*cos(d*x + c)^2 - 4*a*b^2*cos(d*x + c) - 5*b^3 - 6*(a^2*b*cos(d*x + c)^2 + 2*a*b^2*cos(d*x + c) + b^3)*log(a*cos(d*x + c) + b))/(a^6*d*cos(d*x + c)^2 + 2*a^5*b*d*cos(d*x + c) + a^4*b^2*d)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)/(a+b*sec(d*x+c))**3,x)

[Out] Integral(sin(c + d*x)/(a + b*sec(c + d*x))**3, x)

Giac [A]
time = 0.52, size = 77, normalized size = 0.93

$$-\frac{\cos\left(dx+c\right)}{a^{3}d}+\frac{3\,b\log\left(\left|-a\cos\left(dx+c\right)-b\right|\right)}{a^{4}d}+\frac{6\,ab^{2}\cos\left(dx+c\right)+5\,b^{3}}{2\left(a\cos\left(dx+c\right)+b\right)^{2}a^{4}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -cos(d*x + c)/(a^3*d) + 3*b*log(abs(-a*cos(d*x + c) - b))/(a^4*d) + 1/2*(6*
a*b^2*cos(d*x + c) + 5*b^3)/((a*cos(d*x + c) + b)^2*a^4*d)
```

Mupad [B]
time = 1.07, size = 93, normalized size = 1.12

$$\frac{3\,b^{2}\,\cos\left(c+d\,x\right)+\frac{5\,b^{3}}{2\,a}}{d\,\left(a^{5}\cos\left(c+d\,x\right)^{2}+2\,a^{4}\,b\,\cos\left(c+d\,x\right)+a^{3}\,b^{2}\right)}-\frac{\cos\left(c+d\,x\right)}{a^{3}\,d}+\frac{3\,b\,\ln\left(b+a\,\cos\left(c+d\,x\right)\right)}{a^{4}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(a + b/cos(c + d*x))^3,x)
```

```
[Out] (3*b^2*cos(c + d*x) + (5*b^3)/(2*a))/(d*(a^5*cos(c + d*x)^2 + a^3*b^2 + 2*a
^4*b*cos(c + d*x))) - cos(c + d*x)/(a^3*d) + (3*b*log(b + a*cos(c + d*x)))/
(a^4*d)
```

3.225 $\int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=163

$$-\frac{b^3}{2a^2\left(a^2-b^2\right)d(b+a\cos(c+dx))^2}+\frac{b^2(3a^2-b^2)}{a^2\left(a^2-b^2\right)^2d(b+a\cos(c+dx))}+\frac{\log(1-\cos(c+dx))}{2(a+b)^3d}-\frac{\log(1+\cos}{2(a-}$$

```
[Out] -1/2*b^3/a^2/(a^2-b^2)/d/(b+a*cos(d*x+c))^2+b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2
/d/(b+a*cos(d*x+c))+1/2*ln(1-cos(d*x+c))/(a+b)^3/d-1/2*ln(1+cos(d*x+c))/(a-
b)^3/d+b*(3*a^2+b^2)*ln(b+a*cos(d*x+c))/(a^2-b^2)^3/d
```

Rubi [A]

time = 0.24, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3957, 2916, 12, 1643}

$$\frac{b^2(3a^2-b^2)}{a^2d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}+\frac{b(3a^2+b^2)\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)^3}-\frac{b^3}{2a^2d\left(a^2-b^2\right)(a\cos(c+dx)+b)^2}+\frac{\log(1-\cos(c+dx))}{2d(a+b)^3}-\frac{\log(\cos(c+dx)+1)}{2d(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -1/2*b^3/(a^2*(a^2 - b^2)*d*(b + a*Cos[c + d*x])^2) + (b^2*(3*a^2 - b^2))/(
a^2*(a^2 - b^2)^2*d*(b + a*Cos[c + d*x])) + Log[1 - Cos[c + d*x]]/(2*(a + b
)^3*d) - Log[1 + Cos[c + d*x]]/(2*(a - b)^3*d) + (b*(3*a^2 + b^2)*Log[b + a
*Cos[c + d*x]])/((a^2 - b^2)^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\int \frac{\cos^2(c+dx)\cot(c+dx)}{(-b-a\cos(c+dx))^3}\,dx$$

$$= \frac{a\text{Subst}\left(\int \frac{x^3}{a^3(-b+x)^3(a^2-x^2)}\,dx, x, -a\cos(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^3}{(-b+x)^3(a^2-x^2)}\,dx, x, -a\cos(c+dx)\right)}{a^2d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a^2}{2(a-b)^3(a-x)} - \frac{b^3}{(a-b)(a+b)(b-x)^3} + \frac{3a^2b^2-b^4}{(a-b)^2(a+b)^2(b-x)^2} - \frac{a^2b(3a^2+b^2)}{(a-b)^3(a+b)^3(b-x)} + \cdots\right.\right.}{a^2d}$$

$$= -\frac{b^3}{2a^2\left(a^2-b^2\right)d(b+a\cos(c+dx))^2} + \frac{b^2(3a^2-b^2)}{a^2\left(a^2-b^2\right)^2d(b+a\cos(c+dx))} + \frac{\log(\cdots}{\cdots}$$

Mathematica [A]
time = 0.38, size = 203, normalized size = 1.25

$$\frac{(b+a\cos(c+dx))\left(\frac{b^3}{a^2(-a^2+b^2)} - \frac{2b^2(-3a^2+b^2)(b+a\cos(c+dx))}{a^2(a-b)^2(a+b)^2} + \frac{2(b+a\cos(c+dx))^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{(-a+b)^3} + \frac{2b(3a^2+b^2)(b+a\cos(c+dx))^2\log(b+a\cos(c+dx))}{(a^2-b^2)^3} + \frac{2(b+a\cos(c+dx))^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{(a+b)^3}\right)\sec^3(c+dx)}{2d(a+b\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((b + a*Cos[c + d*x])*(b^3/(a^2*(-a^2 + b^2)) - (2*b^2*(-3*a^2 + b^2)*(b +
a*Cos[c + d*x]))/(a^2*(a - b)^2*(a + b)^2) + (2*(b + a*Cos[c + d*x])^2*Log[
Cos[(c + d*x)/2]])/(-a + b)^3 + (2*b*(3*a^2 + b^2)*(b + a*Cos[c + d*x])^2*L
og[b + a*Cos[c + d*x]])/(a^2 - b^2)^3 + (2*(b + a*Cos[c + d*x])^2*Log[Sin[(
c + d*x)/2]])/(a + b)^3)*Sec[c + d*x]^3)/(2*d*(a + b*Sec[c + d*x])^3)
```

Maple [A]
time = 0.18, size = 148, normalized size = 0.91

method	result
derivativedivides	$\frac{-\frac{\ln(1+\cos(dx+c))}{2(a-b)^3}+\frac{\ln(-1+\cos(dx+c))}{2(a+b)^3}-\frac{b^3}{2a^2(a+b)(a-b)(b+a\cos(dx+c))^2}+\frac{b\left(3a^2+b^2\right)\ln(b+a\cos(dx+c))}{(a+b)^3(a-b)^3}+\frac{b^2\left(3a^2-b^2\right)}{(a+b)^2(a-b)^2a^2(b+a\cos\cdots}}{d}$
default	$\frac{-\frac{\ln(1+\cos(dx+c))}{2(a-b)^3}+\frac{\ln(-1+\cos(dx+c))}{2(a+b)^3}-\frac{b^3}{2a^2(a+b)(a-b)(b+a\cos(dx+c))^2}+\frac{b\left(3a^2+b^2\right)\ln(b+a\cos(dx+c))}{(a+b)^3(a-b)^3}+\frac{b^2\left(3a^2-b^2\right)}{(a+b)^2(a-b)^2a^2(b+a\cos\cdots}}{d}$

norman	$\frac{\frac{6b^2a}{d\left(a^4-2b\,a^3+2b^3a-b^4\right)}-\frac{2\left(3b^2a+b^3\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\left(a^4-2b^2a^2+b^4\right)}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)^2}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d(a^3+3b\,a^2+3b^2a+b^3)}+\frac{b(3a^2+b^2)\ln\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\right.\right.}{d(a^6-3a^4b^2+3a^2b^4}$
risch	$\frac{ix}{a^3-3b\,a^2+3b^2a-b^3}+\frac{ic}{d(a^3-3b\,a^2+3b^2a-b^3)}-\frac{ix}{a^3+3b\,a^2+3b^2a+b^3}-\frac{ic}{d(a^3+3b\,a^2+3b^2a+b^3)}-\frac{6ib\,a^2x}{a^6-3a^4b^2+3a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/(a-b)^3*ln(1+cos(d*x+c))+1/2/(a+b)^3*ln(-1+cos(d*x+c))-1/2/a^2*b^
3/(a+b)/(a-b)/(b+a*cos(d*x+c))^2+b*(3*a^2+b^2)/(a+b)^3/(a-b)^3*ln(b+a*cos(d
*x+c))+b^2*(3*a^2-b^2)/(a+b)^2/(a-b)^2/a^2/(b+a*cos(d*x+c)))
```

Maxima [A]
time = 0.28, size = 241, normalized size = 1.48

$$\frac{\frac{2\,(3\,a^2b+b^3)\log\left(a\cos\left(dx+c\right)+b\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}+\frac{5\,a^2b^3-b^5+2\,(3\,a^3b^2-ab^4)\cos\left(dx+c\right)}{a^6b^2-2\,a^4b^4+a^2b^6+(a^8-2\,a^6b^2+a^4b^4)\cos\left(dx+c\right)^2+2\,(a^7b-2\,a^5b^3+a^3b^5)\cos\left(dx+c\right)}-\frac{\log\left(\cos\left(dx+c\right)+1\right)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{\log\left(\cos\left(dx+c\right)-1\right)}{a^3+3\,a^2b+3\,ab^2+b^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(3*a^2*b + b^3)*log(a*cos(d*x + c) + b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4
 - b^6) + (5*a^2*b^3 - b^5 + 2*(3*a^3*b^2 - a*b^4)*cos(d*x + c))/(a^6*b^2 -
 2*a^4*b^4 + a^2*b^6 + (a^8 - 2*a^6*b^2 + a^4*b^4)*cos(d*x + c)^2 + 2*(a^7*
b - 2*a^5*b^3 + a^3*b^5)*cos(d*x + c)) - log(cos(d*x + c) + 1)/(a^3 - 3*a^2
*b + 3*a*b^2 - b^3) + log(cos(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 474 vs. 2(157) = 314.
time = 3.54, size = 474, normalized size = 2.91

$$\frac{5\,a^4b^3-6\,a^2b^5+b^7+2\,(3\,a^5b^2-4\,a^3b^4+ab^6)\cos\left(dx+c\right)+2\,(3\,a^4b^3+a^2b^5+(3\,a^6b+a^4b^3)\cos\left(dx+c\right)^2+2\,(3\,a^5b^2+a^3b^4)\cos\left(dx+c\right))\log\left(a\cos\left(dx+c\right)+b\right)-(a^5b^2+3\,a^4b^3+3\,a^3b^4+a^2b^5+(a^7+3\,a^6b+3\,a^5b^2+a^4b^3)\cos\left(dx+c\right)^2+2\,(a^6b+3\,a^5b^2+3\,a^4b^3+a^3b^4)\cos\left(dx+c\right))\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+(a^5b^2-3\,a^4b^3+3\,a^3b^4-a^2b^5+(a^7-3\,a^6b+3\,a^5b^2-a^4b^3)\cos\left(dx+c\right)^2+2\,(a^6b-3\,a^5b^2+3\,a^4b^3-a^3b^4)\cos\left(dx+c\right))\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\left((a^{10}-3\,a^8b^2+3\,a^6b^4-a^4b^6)d\cos\left(dx+c\right)^2+2\,(a^9b-3\,a^7b^3+3\,a^5b^5-a^3b^7)d\cos\left(dx+c\right)+(a^8b^2-3\,a^6b^4+3\,a^4b^6-a^2b^8)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(5*a^4*b^3 - 6*a^2*b^5 + b^7 + 2*(3*a^5*b^2 - 4*a^3*b^4 + a*b^6)*cos(d*
x + c) + 2*(3*a^4*b^3 + a^2*b^5 + (3*a^6*b + a^4*b^3)*cos(d*x + c)^2 + 2*(3
*a^5*b^2 + a^3*b^4)*cos(d*x + c))*log(a*cos(d*x + c) + b) - (a^5*b^2 + 3*a^
4*b^3 + 3*a^3*b^4 + a^2*b^5 + (a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*cos(d*x
 + c)^2 + 2*(a^6*b + 3*a^5*b^2 + 3*a^4*b^3 + a^3*b^4)*cos(d*x + c))*log(1/2
*cos(d*x + c) + 1/2) + (a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5 + (a^7 -
3*a^6*b + 3*a^5*b^2 - a^4*b^3)*cos(d*x + c)^2 + 2*(a^6*b - 3*a^5*b^2 + 3*a^
```

```
4*b^3 - a^3*b^4)*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/((a^10 - 3*a^8
*b^2 + 3*a^6*b^4 - a^4*b^6)*d*cos(d*x + c)^2 + 2*(a^9*b - 3*a^7*b^3 + 3*a^5
*b^5 - a^3*b^7)*d*cos(d*x + c) + (a^8*b^2 - 3*a^6*b^4 + 3*a^4*b^6 - a^2*b^8
)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral(csc(c + d*x)/(a + b*sec(c + d*x))**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 452 vs. 2(157) = 314.
time = 0.55, size = 452, normalized size = 2.77

$$\frac{\frac{2\,(3\,a^2b+b^3)\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}+\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{9\,a^3b+15\,a^2b^2+3\,ab^3-3\,b^4+\frac{18\,a^3b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{6\,a^2b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{10\,ab^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{2\,b^4(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{9\,a^3b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{9\,a^2b^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{3\,ab^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{3\,b^4(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{(a^5+a^4b-2\,a^3b^2-2\,a^2b^3+ab^4+b^5)\left(a+b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(3*a^2*b + b^3)*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c)
+ 1) + b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^6 - 3*a^4*b^2 + 3*a^2*b
^4 - b^6) + log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^3 + 3*a^2*
b + 3*a*b^2 + b^3) - (9*a^3*b + 15*a^2*b^2 + 3*a*b^3 - 3*b^4 + 18*a^3*b*(co
s(d*x + c) - 1)/(cos(d*x + c) + 1) + 6*a^2*b^2*(cos(d*x + c) - 1)/(cos(d*x
+ c) + 1) - 10*a*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2*b^4*(cos(d*x
 + c) - 1)/(cos(d*x + c) + 1) + 9*a^3*b*(cos(d*x + c) - 1)^2/(cos(d*x + c)
+ 1)^2 - 9*a^2*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 3*a*b^3*(cos
(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 3*b^4*(cos(d*x + c) - 1)^2/(cos(d*x
 + c) + 1)^2)/((a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*(a + b +
 a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x +
c) + 1))^2))/d
```

Mupad [B]
time = 1.39, size = 182, normalized size = 1.12

$$\frac{\ln\left(\cos\left(c+d\,x\right)-1\right)}{2\,d\,{\left(a+b\right)}^3}-\frac{\frac{\cos(c+d\,x)\,\left(b^4-3\,a^2\,b^2\right)}{a\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{b\,\left(b^4-5\,a^2\,b^2\right)}{2\,a^2\,\left(a^4-2\,a^2\,b^2+b^4\right)}}{d\,\left(a^2\cos\left(c+d\,x\right)^2+2\,a\,b\,\cos\left(c+d\,x\right)+b^2\right)}-\frac{\ln\left(\cos\left(c+d\,x\right)+1\right)}{2\,d\,{\left(a-b\right)}^3}-\frac{\ln\left(b+a\,\cos\left(c+d\,x\right)\right)\,\left(\frac{1}{2\,{\left(a+b\right)}^3}-\frac{1}{2\,{\left(a-b\right)}^3}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)*(a + b/cos(c + d*x))^3),x)
```

```
[Out] log(cos(c + d*x) - 1)/(2*d*(a + b)^3) - ((cos(c + d*x)*(b^4 - 3*a^2*b^2))/(
a*(a^4 + b^4 - 2*a^2*b^2)) + (b*(b^4 - 5*a^2*b^2))/(2*a^2*(a^4 + b^4 - 2*a^
2*b^2)))/(d*(b^2 + a^2*cos(c + d*x)^2 + 2*a*b*cos(c + d*x))) - log(cos(c +
d*x) + 1)/(2*d*(a - b)^3) - (log(b + a*cos(c + d*x))*(1/(2*(a + b)^3) - 1/(
2*(a - b)^3)))/d
```

3.226 $\int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=229

$$-\frac{b^3}{2\left(a^2-b^2\right)^2 d(b+a\cos(c+dx))^2}+\frac{b^2(3a^2+b^2)}{\left(a^2-b^2\right)^3 d(b+a\cos(c+dx))}+\frac{(b(3a^2+b^2)-a(a^2+3b^2)\cos(c+dx))}{2\left(a^2-b^2\right)^3 d}$$

```
[Out] -1/2*b^3/(a^2-b^2)^2/d/(b+a*cos(d*x+c))^2+b^2*(3*a^2+b^2)/(a^2-b^2)^3/d/(b+
a*cos(d*x+c))+1/2*(b*(3*a^2+b^2)-a*(a^2+3*b^2)*cos(d*x+c))*csc(d*x+c)^2/(a^
2-b^2)^3/d+1/4*(a-2*b)*ln(1-cos(d*x+c))/(a+b)^4/d-1/4*(a+2*b)*ln(1+cos(d*x+
c))/(a-b)^4/d+b*(3*a^4+8*a^2*b^2+b^4)*ln(b+a*cos(d*x+c))/(a^2-b^2)^4/d
```

Rubi [A]
time = 0.43, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3957, 2800, 1661, 1643}

$$\frac{b^2(3a^2+b^2)}{d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}+\frac{\csc^2(c+dx)\left(b(3a^2+b^2)-a(a^2+3b^2)\cos(c+dx)\right)}{2d\left(a^2-b^2\right)^3}-\frac{b^3}{2d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)^2}+\frac{b(3a^4+8a^2b^2+b^4)\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)^4}+\frac{(a-2b)\log(1-\cos(c+dx))}{4d(a+b)^4}-\frac{(a+2b)\log(\cos(c+dx)+1)}{4d(a-b)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -1/2*b^3/((a^2 - b^2)^2*d*(b + a*Cos[c + d*x])^2) + (b^2*(3*a^2 + b^2))/((a
^2 - b^2)^3*d*(b + a*Cos[c + d*x])) + ((b*(3*a^2 + b^2) - a*(a^2 + 3*b^2)*C
os[c + d*x])*Csc[c + d*x]^2)/(2*(a^2 - b^2)^3*d) + ((a - 2*b)*Log[1 - Cos[c
 + d*x]])/(4*(a + b)^4*d) - ((a + 2*b)*Log[1 + Cos[c + d*x]])/(4*(a - b)^4*
d) + (b*(3*a^4 + 8*a^2*b^2 + b^4)*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)^4*d
)
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx &= -\int \frac{\cot^3(c+dx)}{(-b-a\cos(c+dx))^3}\,dx \\
&= \frac{\text{Subst}\left(\int \frac{x^3}{(-b+x)^3(a^2-x^2)^2}\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= \frac{(b(3a^2+b^2)-a(a^2+3b^2)\cos(c+dx))\csc^2(c+dx)}{2(a^2-b^2)^3 d} + \frac{\text{Subst}\left(\int \frac{\frac{a^4b^3(a^2+3b^2)}{(a^2-b^2)^3} - \cdots}{\cdots}\right.}{\cdots} \\
&= \frac{(b(3a^2+b^2)-a(a^2+3b^2)\cos(c+dx))\csc^2(c+dx)}{2(a^2-b^2)^3 d} + \frac{\text{Subst}\left(\int \left(\frac{a^2(a+2b)}{2(a-b)^4(a-x)} - \cdots\right.\right.}{\cdots} \\
&= -\frac{b^3}{2(a^2-b^2)^2 d(b+a\cos(c+dx))^2} + \frac{b^2(3a^2+b^2)}{(a^2-b^2)^3 d(b+a\cos(c+dx))} + \frac{(b(3a^2 \cdots}{\cdots}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.21, size = 332, normalized size = 1.45

$$-\frac{2i(3a^4b+8a^2b^3+b^5)(c+dx)}{(a-b)^4(a+b)^4d} - \frac{i(-a-2b)\text{ArcTan}(\tan(c+dx))}{2(-a+b)^4d} - \frac{i(a-2b)\text{ArcTan}(\tan(c+dx))}{2(a+b)^4d} - \frac{b^3}{2(-a+b)^2(a+b)^2d(b+a\cos(c+dx))^2} - \frac{b^2(3a^2+b^2)}{(-a+b)^3(a+b)^3d(b+a\cos(c+dx))} - \frac{\csc^2\left(\frac{1}{2}(c+dx)\right)}{8(a+b)^3d} + \frac{(-a-2b)\log\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\right)}{4(-a+b)^4d} + \frac{(3a^4b+8a^2b^3+b^5)\log(b+a\cos(c+dx))}{(-a^2+b^2)^4 d} + \frac{(a-2b)\log\left(\sin^2\left(\frac{1}{2}(c+dx)\right)\right)}{4(a+b)^4d} - \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)}{8(-a+b)^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((-2*I)*(3*a^4*b + 8*a^2*b^3 + b^5)*(c + d*x))/((a - b)^4*(a + b)^4*d) - ((
I/2)*(-a - 2*b)*ArcTan[Tan[c + d*x]])/((-a + b)^4*d) - ((I/2)*(a - 2*b)*Arc
Tan[Tan[c + d*x]])/((a + b)^4*d) - b^3/(2*(-a + b)^2*(a + b)^2*d*(b + a*Cos
[c + d*x])^2) - (b^2*(3*a^2 + b^2))/((-a + b)^3*(a + b)^3*d*(b + a*Cos[c +
```

```
d*x])) - Csc[(c + d*x)/2]^2/(8*(a + b)^3*d) + ((-a - 2*b)*Log[Cos[(c + d*x)
/2]^2])/(4*(-a + b)^4*d) + ((3*a^4*b + 8*a^2*b^3 + b^5)*Log[b + a*Cos[c + d
*x]])/((-a^2 + b^2)^4*d) + ((a - 2*b)*Log[Sin[(c + d*x)/2]^2])/(4*(a + b)^4
*d) - Sec[(c + d*x)/2]^2/(8*(-a + b)^3*d)
```

Maple [A]
time = 0.31, size = 196, normalized size = 0.86

method	result
derivativedivides	$\frac{\frac{1}{4(a-b)^{3}(1+\cos(dx+c))}+\frac{(-a-2b)\ln(1+\cos(dx+c))}{4(a-b)^{4}}+\frac{1}{4(a+b)^{3}(-1+\cos(dx+c))}+\frac{(a-2b)\ln(-1+\cos(dx+c))}{4(a+b)^{4}}-\frac{b^{3}}{2(a+b)^{2}(a-b)^{2}(b+a\cos}}{d}$
default	$\frac{\frac{1}{4(a-b)^{3}(1+\cos(dx+c))}+\frac{(-a-2b)\ln(1+\cos(dx+c))}{4(a-b)^{4}}+\frac{1}{4(a+b)^{3}(-1+\cos(dx+c))}+\frac{(a-2b)\ln(-1+\cos(dx+c))}{4(a+b)^{4}}-\frac{b^{3}}{2(a+b)^{2}(a-b)^{2}(b+a\cos}}{d}$
norman	$\frac{-\frac{1}{8d(a+b)}+\frac{\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d(a-b)}+\frac{\left(a^{5}+22a^{3}b^{2}+13a\,b^{4}\right)\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\left(a^{6}-2a^{5}b-a^{4}b^{2}+4a^{3}b^{3}-a^{2}b^{4}-2a\,b^{5}+b^{6}\right)}-\frac{\left(2a^{5}+5a^{4}b+44a^{3}b^{2}+18a^{2}b^{3}+26a\,b^{4}+b^{5}\right)\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{4d\left(a^{6}-3a^{4}b^{2}+3a^{2}b^{4}-b^{6}\right)}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\left(a\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)^{2}}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4/(a-b)^3/(1+cos(d*x+c))+1/4/(a-b)^4*(-a-2*b)*ln(1+cos(d*x+c))+1/4/(
a+b)^3/(-1+cos(d*x+c))+1/4*(a-2*b)/(a+b)^4*ln(-1+cos(d*x+c))-1/2*b^3/(a+b)^
2/(a-b)^2/(b+a*cos(d*x+c))^2+b*(3*a^4+8*a^2*b^2+b^4)/(a+b)^4/(a-b)^4*ln(b+a
*cos(d*x+c))+b^2*(3*a^2+b^2)/(a+b)^3/(a-b)^3/(b+a*cos(d*x+c)))
```

Maxima [A]
time = 0.29, size = 435, normalized size = 1.90

$$\frac{\frac{4\left(3a^{4}b+8a^{2}b^{3}+b^{5}\right)\log\left(a\cos\left(dx+c\right)+b\right)}{a^{8}-4a^{6}b^{2}+6a^{4}b^{4}-4a^{2}b^{6}+b^{8}}-\frac{(a+2b)\log\left(\cos\left(dx+c\right)+1\right)}{a^{4}-4a^{3}b+6a^{2}b^{2}-4ab^{3}+b^{4}}+\frac{(a-2b)\log\left(\cos\left(dx+c\right)-1\right)}{a^{4}+4a^{3}b+6a^{2}b^{2}+4ab^{3}+b^{4}}+\frac{2\left(8a^{2}b^{3}+4b^{5}-\left(a^{5}+9a^{3}b^{2}+2ab^{4}\right)\cos\left(dx+c\right)^{3}+\left(a^{4}b-10a^{2}b^{3}-3b^{5}\right)\cos\left(dx+c\right)^{2}+\left(11a^{3}b^{2}+ab^{4}\right)\cos\left(dx+c\right)\right)}{a^{6}b^{2}-3a^{4}b^{4}+3a^{2}b^{6}-b^{8}-\left(a^{8}-3a^{6}b^{2}+3a^{4}b^{4}-a^{2}b^{6}\right)\cos\left(dx+c\right)^{4}-2\left(a^{7}b-3a^{5}b^{3}+3a^{3}b^{5}-ab^{7}\right)\cos\left(dx+c\right)^{3}+\left(a^{8}-4a^{6}b^{2}+6a^{4}b^{4}-4a^{2}b^{6}+b^{8}\right)\cos\left(dx+c\right)^{2}+2\left(a^{7}b-3a^{5}b^{3}+3a^{3}b^{5}-ab^{7}\right)\cos\left(dx+c\right)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*(4*(3*a^4*b + 8*a^2*b^3 + b^5)*log(a*cos(d*x + c) + b)/(a^8 - 4*a^6*b^2
 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) - (a + 2*b)*log(cos(d*x + c) + 1)/(a^4 - 4*
a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) + (a - 2*b)*log(cos(d*x + c) - 1)/(a^4 +
 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + 2*(8*a^2*b^3 + 4*b^5 - (a^5 + 9*a^3
*b^2 + 2*a*b^4)*cos(d*x + c)^3 + (a^4*b - 10*a^2*b^3 - 3*b^5)*cos(d*x + c)^
2 + (11*a^3*b^2 + a*b^4)*cos(d*x + c))/(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b
^8 - (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*cos(d*x + c)^4 - 2*(a^7*b - 3*
a^5*b^3 + 3*a^3*b^5 - a*b^7)*cos(d*x + c)^3 + (a^8 - 4*a^6*b^2 + 6*a^4*b^4
- 4*a^2*b^6 + b^8)*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^
7)*cos(d*x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1071 vs. $2(221) = 442$.
time = 4.10, size = 1071, normalized size = 4.68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(16*a^4*b^3 - 8*a^2*b^5 - 8*b^7 - 2*(a^7 + 8*a^5*b^2 - 7*a^3*b^4 - 2*a
*b^6)*cos(d*x + c)^3 + 2*(a^6*b - 11*a^4*b^3 + 7*a^2*b^5 + 3*b^7)*cos(d*x +
 c)^2 + 2*(11*a^5*b^2 - 10*a^3*b^4 - a*b^6)*cos(d*x + c) + 4*(3*a^4*b^3 + 8
*a^2*b^5 + b^7 - (3*a^6*b + 8*a^4*b^3 + a^2*b^5)*cos(d*x + c)^4 - 2*(3*a^5*
b^2 + 8*a^3*b^4 + a*b^6)*cos(d*x + c)^3 + (3*a^6*b + 5*a^4*b^3 - 7*a^2*b^5
- b^7)*cos(d*x + c)^2 + 2*(3*a^5*b^2 + 8*a^3*b^4 + a*b^6)*cos(d*x + c))*log
(a*cos(d*x + c) + b) - (a^5*b^2 + 6*a^4*b^3 + 14*a^3*b^4 + 16*a^2*b^5 + 9*a
*b^6 + 2*b^7 - (a^7 + 6*a^6*b + 14*a^5*b^2 + 16*a^4*b^3 + 9*a^3*b^4 + 2*a^2
*b^5)*cos(d*x + c)^4 - 2*(a^6*b + 6*a^5*b^2 + 14*a^4*b^3 + 16*a^3*b^4 + 9*a
^2*b^5 + 2*a*b^6)*cos(d*x + c)^3 + (a^7 + 6*a^6*b + 13*a^5*b^2 + 10*a^4*b^3
 - 5*a^3*b^4 - 14*a^2*b^5 - 9*a*b^6 - 2*b^7)*cos(d*x + c)^2 + 2*(a^6*b + 6*
a^5*b^2 + 14*a^4*b^3 + 16*a^3*b^4 + 9*a^2*b^5 + 2*a*b^6)*cos(d*x + c))*log(
1/2*cos(d*x + c) + 1/2) + (a^5*b^2 - 6*a^4*b^3 + 14*a^3*b^4 - 16*a^2*b^5 +
9*a*b^6 - 2*b^7 - (a^7 - 6*a^6*b + 14*a^5*b^2 - 16*a^4*b^3 + 9*a^3*b^4 - 2*
a^2*b^5)*cos(d*x + c)^4 - 2*(a^6*b - 6*a^5*b^2 + 14*a^4*b^3 - 16*a^3*b^4 +
9*a^2*b^5 - 2*a*b^6)*cos(d*x + c)^3 + (a^7 - 6*a^6*b + 13*a^5*b^2 - 10*a^4*
b^3 - 5*a^3*b^4 + 14*a^2*b^5 - 9*a*b^6 + 2*b^7)*cos(d*x + c)^2 + 2*(a^6*b -
 6*a^5*b^2 + 14*a^4*b^3 - 16*a^3*b^4 + 9*a^2*b^5 - 2*a*b^6)*cos(d*x + c))*l
og(-1/2*cos(d*x + c) + 1/2))/((a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a
^2*b^8)*d*cos(d*x + c)^4 + 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a
*b^9)*d*cos(d*x + c)^3 - (a^10 - 5*a^8*b^2 + 10*a^6*b^4 - 10*a^4*b^6 + 5*a^
2*b^8 - b^10)*d*cos(d*x + c)^2 - 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b
^7 + a*b^9)*d*cos(d*x + c) - (a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 +
 b^10)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral(csc(c + d*x)**3/(a + b*sec(c + d*x))**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 800 vs. 2(221) = 442.

time = 0.61, size = 800, normalized size = 3.49

$$\frac{\frac{2(a-2b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4}+\frac{8(3a^4b+8a^2b^3+b^5)\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8}+\frac{\left(a+b-\frac{2a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{4b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{(a^4+4a^3b+6a^2b^2+4ab^3+b^4)(\cos(dx+c)-1)}-\frac{\cos(dx+c)-1}{(a^3-3a^2b+3ab^2-b^3)(\cos(dx+c)+1)}-\frac{4\left(9a^6b+6a^5b^2+9a^4b^3+28a^3b^4+11a^2b^5-2ab^6+3b^7+\frac{18a^6b(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{12a^5b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{26a^4b^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{4a^3b^4(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{38a^2b^5(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{8ab^6(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{6b^7(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{9a^6b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{18a^5b^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{33a^4b^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{48a^3b^4(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{27a^2b^5(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{6ab^6(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{3b^7(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)}{(a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8)\left(a+b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(2*(a - 2*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^4 + 4
*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + 8*(3*a^4*b + 8*a^2*b^3 + b^5)*log(abs
(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c) - 1)/(c
os(d*x + c) + 1)))/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) + (a + b
 - 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 4*b*(cos(d*x + c) - 1)/(cos(
d*x + c) + 1))*(cos(d*x + c) + 1)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b
^4)*(cos(d*x + c) - 1)) - (cos(d*x + c) - 1)/((a^3 - 3*a^2*b + 3*a*b^2 - b^
3)*(cos(d*x + c) + 1)) - 4*(9*a^6*b + 6*a^5*b^2 + 9*a^4*b^3 + 28*a^3*b^4 +
11*a^2*b^5 - 2*a*b^6 + 3*b^7 + 18*a^6*b*(cos(d*x + c) - 1)/(cos(d*x + c) +
1) - 12*a^5*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 26*a^4*b^3*(cos(d*x
 + c) - 1)/(cos(d*x + c) + 1) + 4*a^3*b^4*(cos(d*x + c) - 1)/(cos(d*x + c)
+ 1) - 38*a^2*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 8*a*b^6*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) - 6*b^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
+ 9*a^6*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 18*a^5*b^2*(cos(d*x +
 c) - 1)^2/(cos(d*x + c) + 1)^2 + 33*a^4*b^3*(cos(d*x + c) - 1)^2/(cos(d*x
+ c) + 1)^2 - 48*a^3*b^4*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 27*a^2
*b^5*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 6*a*b^6*(cos(d*x + c) - 1)
^2/(cos(d*x + c) + 1)^2 + 3*b^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/
((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(a + b + a*(cos(d*x + c) -
 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))^2))/d
```

Mupad [B]

time = 1.70, size = 378, normalized size = 1.65

$$\frac{\ln\left(b+a\,\cos\left(c+d\,x\right)\right)\,\left(\frac{3\,b}{4\,{\left(a+b\right)}^4}-\frac{1}{4\,{\left(a+b\right)}^3}+\frac{3\,b}{4\,{\left(a-b\right)}^4}+\frac{1}{4\,{\left(a-b\right)}^3}\right)}{d}-\frac{\ln\left(\cos\left(c+d\,x\right)-1\right)\,\left(\frac{3\,b}{4\,{\left(a+b\right)}^4}-\frac{1}{4\,{\left(a+b\right)}^3}\right)}{d}-\frac{\frac{{\cos\left(c+d\,x\right)}^3\,\left(a^5+9\,a^3\,b^2+2\,a\,b^4\right)}{2\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}+\frac{{\cos\left(c+d\,x\right)}^2\,\left(-a^4\,b+10\,a^2\,b^3+3\,b^5\right)}{2\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}-\frac{2\,b\,\left(2\,a^2\,b^2+b^4\right)}{\left(a^2-b^2\right)\,\left(a^4-2\,a^2\,b^2+b^4\right)}-\frac{a\,\cos\left(c+d\,x\right)\,\left(11\,a^2\,b^2+b^4\right)}{2\,\left(a^2-b^2\right)\,\left(a^4-2\,a^2\,b^2+b^4\right)}}{d\,\left({\cos\left(c+d\,x\right)}^2\,\left(a^2-b^2\right)+b^2-a^2\,{\cos\left(c+d\,x\right)}^4+2\,a\,b\,\cos\left(c+d\,x\right)-2\,a\,b\,{\cos\left(c+d\,x\right)}^3\right)}-\frac{\ln\left(\cos\left(c+d\,x\right)+1\right)\,\left(a+2\,b\right)}{4\,d\,{\left(a-b\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^3*(a + b/cos(c + d*x))^3),x)
```

```
[Out] (log(b + a*cos(c + d*x))*((3*b)/(4*(a + b)^4) - 1/(4*(a + b)^3) + (3*b)/(4*
(a - b)^4) + 1/(4*(a - b)^3)))/d - (log(cos(c + d*x) - 1)*((3*b)/(4*(a + b)
^4) - 1/(4*(a + b)^3)))/d - ((cos(c + d*x)^3*(2*a*b^4 + a^5 + 9*a^3*b^2))/(
2*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (cos(c + d*x)^2*(3*b^5 - a^4*b + 1
0*a^2*b^3))/(2*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - (2*b*(b^4 + 2*a^2*b^2
))/((a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)) - (a*cos(c + d*x)*(b^4 + 11*a^2*b^
2))/(2*(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)))/(d*(cos(c + d*x)^2*(a^2 - b^2)
 + b^2 - a^2*cos(c + d*x)^4 + 2*a*b*cos(c + d*x) - 2*a*b*cos(c + d*x)^3)) -
 (log(cos(c + d*x) + 1)*(a + 2*b))/(4*d*(a - b)^4)
```

3.227 $\int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=313

$$-\frac{a^2b^3}{2\left(a^2-b^2\right)^3 d(b+a\cos(c+dx))^2}+\frac{3a^2b^2(a^2+b^2)}{\left(a^2-b^2\right)^4 d(b+a\cos(c+dx))}+\frac{(4b(3a^4+8a^2b^2+b^4)-3a(a^4+10a^2}{8\left(a^2-b\right.}$$

```
[Out] -1/2*a^2*b^3/(a^2-b^2)^3/d/(b+a*cos(d*x+c))^2+3*a^2*b^2*(a^2+b^2)/(a^2-b^2)
^4/d/(b+a*cos(d*x+c))+1/8*(4*b*(3*a^4+8*a^2*b^2+b^4)-3*a*(a^4+10*a^2*b^2+5*
b^4)*cos(d*x+c))*csc(d*x+c)^2/(a^2-b^2)^4/d+1/4*(b*(3*a^2+b^2)-a*(a^2+3*b^2
)*cos(d*x+c))*csc(d*x+c)^4/(a^2-b^2)^3/d+3/16*a*(a-3*b)*ln(1-cos(d*x+c))/(a
+b)^5/d-3/16*a*(a+3*b)*ln(1+cos(d*x+c))/(a-b)^5/d+3*a^2*b*(a^4+5*a^2*b^2+2*
b^4)*ln(b+a*cos(d*x+c))/(a^2-b^2)^5/d
```

Rubi [A]
time = 0.88, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3957, 2916, 12, 1661, 1643}

$$\frac{3a^2b^2(a^2+b^2)}{d\left(a^2-b^2\right)^4(a\cos(c+dx)+b)}+\frac{\csc^4(c+dx)\left(b(3a^2+b^2)-a(a^2+3b^2)\cos(c+dx)\right)}{4d\left(a^2-b^2\right)^3}-\frac{a^2b^3}{2d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)^2}+\frac{3a^2b(a^4+5a^2b^2+2b^4)\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)^5}+\frac{\csc^2(c+dx)\left(4b(3a^4+8a^2b^2+b^4)-3a(a^4+10a^2b^2+5b^4)\cos(c+dx)\right)}{8d\left(a^2-b^2\right)^4}+\frac{3a(a-3b)\log(1-\cos(c+dx))}{16d(a+b)^5}-\frac{3a(a+3b)\log(\cos(c+dx)+1)}{16d(a-b)^5}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -1/2*(a^2*b^3)/((a^2 - b^2)^3*d*(b + a*Cos[c + d*x])^2) + (3*a^2*b^2*(a^2 +
 b^2))/((a^2 - b^2)^4*d*(b + a*Cos[c + d*x])) + ((4*b*(3*a^4 + 8*a^2*b^2 +
b^4) - 3*a*(a^4 + 10*a^2*b^2 + 5*b^4)*Cos[c + d*x])*Csc[c + d*x]^2)/(8*(a^2
 - b^2)^4*d) + ((b*(3*a^2 + b^2) - a*(a^2 + 3*b^2)*Cos[c + d*x])*Csc[c + d*
x]^4)/(4*(a^2 - b^2)^3*d) + (3*a*(a - 3*b)*Log[1 - Cos[c + d*x]])/(16*(a +
b)^5*d) - (3*a*(a + 3*b)*Log[1 + Cos[c + d*x]])/(16*(a - b)^5*d) + (3*a^2*b
*(a^4 + 5*a^2*b^2 + 2*b^4)*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)^5*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
```

```
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\int \frac{\cot^3(c+dx)\csc^2(c+dx)}{(-b-a\cos(c+dx))^3}\,dx$$

$$= \frac{a^5\text{Subst}\left(\int \frac{x^3}{a^3(-b+x)^3(a^2-x^2)^3}\,dx, x, -a\cos(c+dx)\right)}{d}$$

$$= \frac{a^2\text{Subst}\left(\int \frac{x^3}{(-b+x)^3(a^2-x^2)^3}\,dx, x, -a\cos(c+dx)\right)}{d}$$

$$= \frac{(b(3a^2+b^2)-a(a^2+3b^2)\cos(c+dx))\csc^4(c+dx)}{4(a^2-b^2)^3 d} + \frac{\text{Subst}\left(\int \frac{\frac{a^4b^3(a^2+3b^2)}{(a^2-b^2)^3} - \frac{a}{}\cdots}{\cdots}\right.}{\cdots}$$

$$= \frac{(4b(3a^4+8a^2b^2+b^4)-3a(a^4+10a^2b^2+5b^4)\cos(c+dx))\csc^2(c+dx)}{8(a^2-b^2)^4 d} + \frac{(b(\cdots}{\cdots}$$

$$= \frac{(4b(3a^4+8a^2b^2+b^4)-3a(a^4+10a^2b^2+5b^4)\cos(c+dx))\csc^2(c+dx)}{8(a^2-b^2)^4 d} + \frac{(b(\cdots}{\cdots}$$

$$= -\frac{a^2b^3}{2(a^2-b^2)^3 d(b+a\cos(c+dx))^2} + \frac{3a^2b^2(a^2+b^2)}{(a^2-b^2)^4 d(b+a\cos(c+dx))} + \frac{(4b(3a\cdots}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.99, size = 496, normalized size = 1.58

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^5/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((b + a*Cos[c + d*x])*((32*a^2*b^3)/((-a + b)^3*(a + b)^3) + (192*a^2*(a -
I*b)*(a + I*b)*b^2*(b + a*Cos[c + d*x]))/((a - b)^4*(a + b)^4) - ((384*I)*a
^2*b*(a^4 + 5*a^2*b^2 + 2*b^4)*(c + d*x)*(b + a*Cos[c + d*x])^2)/((a - b)^5
*(a + b)^5) - ((24*I)*a*(a - 3*b)*ArcTan[Tan[c + d*x]]*(b + a*Cos[c + d*x])
^2)/(a + b)^5 + ((24*I)*a*(a + 3*b)*ArcTan[Tan[c + d*x]]*(b + a*Cos[c + d*x
])^2)/(a - b)^5 + (6*(-a + b)*(b + a*Cos[c + d*x])^2*Csc[(c + d*x)/2]^2)/(a
 + b)^4 - ((b + a*Cos[c + d*x])^2*Csc[(c + d*x)/2]^4)/(a + b)^3 - (12*a*(a
+ 3*b)*(b + a*Cos[c + d*x])^2*Log[Cos[(c + d*x)/2]^2])/(a - b)^5 + (192*a^2
*b*(a^4 + 5*a^2*b^2 + 2*b^4)*(b + a*Cos[c + d*x])^2*Log[b + a*Cos[c + d*x]]
)/(a^2 - b^2)^5 + (12*a*(a - 3*b)*(b + a*Cos[c + d*x])^2*Log[Sin[(c + d*x)/
```

```
2]^2])/(a + b)^5 + (6*(a + b)*(b + a*Cos[c + d*x])^2*Sec[(c + d*x)/2]^2)/(a
 - b)^4 + ((b + a*Cos[c + d*x])^2*Sec[(c + d*x)/2]^4)/(a - b)^3)*Sec[c + d*
x]^3)/(64*d*(a + b*Sec[c + d*x])^3)
```

Maple [A]
time = 0.39, size = 255, normalized size = 0.81

method	result
derivativedivides	$\frac{1}{16(a-b)^3(1+\cos(dx+c))^2}-\frac{-3a-3b}{16(a-b)^4(1+\cos(dx+c))}-\frac{3a(a+3b)\ln(1+\cos(dx+c))}{16(a-b)^5}-\frac{1}{16(a+b)^3(-1+\cos(dx+c))^2}-\frac{-3a+3b}{16(a+b)^4(-1+\cos(d$
default	$\frac{1}{16(a-b)^3(1+\cos(dx+c))^2}-\frac{-3a-3b}{16(a-b)^4(1+\cos(dx+c))}-\frac{3a(a+3b)\ln(1+\cos(dx+c))}{16(a-b)^5}-\frac{1}{16(a+b)^3(-1+\cos(dx+c))^2}-\frac{-3a+3b}{16(a+b)^4(-1+\cos(d$
norman	$\frac{-\frac{1}{64d(a+b)}+\frac{\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)}{64d(a-b)}-\frac{(3a-b)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32d(a^2+2ba+b^2)}+\frac{(3a+b)\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32d(a^2-2ba+b^2)}+\frac{(15a^7+459a^5b^2+621a^3b^4+57a\,b^6)\left(\tan^4\left(\frac{dx}{2}\right.\right.}{32(a^8-2a^7b-2a^6b^2+6a^5b^3-6b^5a^3+2b^6a^2+2a\,b}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\right.\right.\right.}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/16/(a-b)^3/(1+cos(d*x+c))^2-1/16*(-3*a-3*b)/(a-b)^4/(1+cos(d*x+c))-3
/16*a*(a+3*b)/(a-b)^5*ln(1+cos(d*x+c))-1/16/(a+b)^3/(-1+cos(d*x+c))^2-1/16*
(-3*a+3*b)/(a+b)^4/(-1+cos(d*x+c))+3/16*a*(a-3*b)/(a+b)^5*ln(-1+cos(d*x+c))
-1/2*b^3/(a+b)^3*a^2/(a-b)^3/(b+a*cos(d*x+c))^2+3*a^2*b*(a^4+5*a^2*b^2+2*b^
4)/(a+b)^5/(a-b)^5*ln(b+a*cos(d*x+c))+3*a^2*b^2*(a^2+b^2)/(a+b)^4/(a-b)^4/(
b+a*cos(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 707 vs. 2(303) = 606.
time = 0.29, size = 707, normalized size = 2.26

$$\frac{\frac{48\,(a^6b+5\,a^4b^3+2\,a^2b^5)\log(a\cos(dx+c)+b)}{a^{10}-5\,a^8b^2+10\,a^6b^4-10\,a^4b^6+5\,a^2b^8-b^{10}}-\frac{3\,(a^2+3\,ab)\log(\cos(dx+c)+1)}{a^5-5\,a^4b+10\,a^3b^2-10\,a^2b^3+5\,ab^4-b^5}+\frac{3\,(a^2-3\,ab)\log(\cos(dx+c)-1)}{a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+5\,ab^4+b^5}+\frac{2\left(38\,a^4b^3+56\,a^2b^5+2\,b^7+3\,(a^7+18\,a^5b^2+13\,a^3b^4)\cos(dx+c)^5-6\,(a^6b-8\,a^4b^3-9\,a^2b^5)\cos(dx+c)^4-(5\,a^7+103\,a^5b^2+91\,a^3b^4-7\,ab^6)\cos(dx+c)^3+4\,(2\,a^6b-23\,a^4b^3-26\,a^2b^5-b^7)\cos(dx+c)^2+(55\,a^5b^2+46\,a^3b^4-5\,ab^6)\cos(dx+c)\right)}{a^8b^2-4\,a^6b^4+6\,a^4b^6-4\,a^2b^8+b^{10}+(a^{10}-4\,a^8b^2+6\,a^6b^4-4\,a^4b^6+a^2b^8)\cos(dx+c)^6+2\,(a^9b-4\,a^7b^3+6\,a^5b^5-4\,a^3b^7+ab^9)\cos(dx+c)^5-(2\,a^{10}-9\,a^8b^2+16\,a^6b^4-14\,a^4b^6+6\,a^2b^8-b^{10})\cos(dx+c)^4-4\,(a^9b-4\,a^7b^3+6\,a^5b^5-4\,a^3b^7+ab^9)\cos(dx+c)^3+(a^{10}-6\,a^8b^2+14\,a^6b^4-16\,a^4b^6+9\,a^2b^8-2\,b^{10})\cos(dx+c)^2+2\,(a^9b-4\,a^7b^3+6\,a^5b^5-4\,a^3b^7+ab^9)\cos(dx+c)}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/16*(48*(a^6*b + 5*a^4*b^3 + 2*a^2*b^5)*log(a*cos(d*x + c) + b)/(a^10 - 5*
a^8*b^2 + 10*a^6*b^4 - 10*a^4*b^6 + 5*a^2*b^8 - b^10) - 3*(a^2 + 3*a*b)*log
(cos(d*x + c) + 1)/(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5
) + 3*(a^2 - 3*a*b)*log(cos(d*x + c) - 1)/(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*
a^2*b^3 + 5*a*b^4 + b^5) + 2*(38*a^4*b^3 + 56*a^2*b^5 + 2*b^7 + 3*(a^7 + 18
*a^5*b^2 + 13*a^3*b^4)*cos(d*x + c)^5 - 6*(a^6*b - 8*a^4*b^3 - 9*a^2*b^5)*c
os(d*x + c)^4 - (5*a^7 + 103*a^5*b^2 + 91*a^3*b^4 - 7*a*b^6)*cos(d*x + c)^3
 + 4*(2*a^6*b - 23*a^4*b^3 - 26*a^2*b^5 - b^7)*cos(d*x + c)^2 + (55*a^5*b^2
 + 46*a^3*b^4 - 5*a*b^6)*cos(d*x + c))/(a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4
```

```
*a^2*b^8 + b^10 + (a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8)*cos(
d*x + c)^6 + 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*cos(d*x
+ c)^5 - (2*a^10 - 9*a^8*b^2 + 16*a^6*b^4 - 14*a^4*b^6 + 6*a^2*b^8 - b^10)*
cos(d*x + c)^4 - 4*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*cos(
d*x + c)^3 + (a^10 - 6*a^8*b^2 + 14*a^6*b^4 - 16*a^4*b^6 + 9*a^2*b^8 - 2*b^
10)*cos(d*x + c)^2 + 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*
cos(d*x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1803 vs. $2(303) = 606$.
time = 4.47, size = 1803, normalized size = 5.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/16*(76*a^6*b^3 + 36*a^4*b^5 - 108*a^2*b^7 - 4*b^9 + 6*(a^9 + 17*a^7*b^2 -
 5*a^5*b^4 - 13*a^3*b^6)*cos(d*x + c)^5 - 12*(a^8*b - 9*a^6*b^3 - a^4*b^5 +
 9*a^2*b^7)*cos(d*x + c)^4 - 2*(5*a^9 + 98*a^7*b^2 - 12*a^5*b^4 - 98*a^3*b^
6 + 7*a*b^8)*cos(d*x + c)^3 + 8*(2*a^8*b - 25*a^6*b^3 - 3*a^4*b^5 + 25*a^2*
b^7 + b^9)*cos(d*x + c)^2 + 2*(55*a^7*b^2 - 9*a^5*b^4 - 51*a^3*b^6 + 5*a*b^
8)*cos(d*x + c) + 48*(a^6*b^3 + 5*a^4*b^5 + 2*a^2*b^7 + (a^8*b + 5*a^6*b^3
+ 2*a^4*b^5)*cos(d*x + c)^6 + 2*(a^7*b^2 + 5*a^5*b^4 + 2*a^3*b^6)*cos(d*x +
 c)^5 - (2*a^8*b + 9*a^6*b^3 - a^4*b^5 - 2*a^2*b^7)*cos(d*x + c)^4 - 4*(a^7
*b^2 + 5*a^5*b^4 + 2*a^3*b^6)*cos(d*x + c)^3 + (a^8*b + 3*a^6*b^3 - 8*a^4*b
^5 - 4*a^2*b^7)*cos(d*x + c)^2 + 2*(a^7*b^2 + 5*a^5*b^4 + 2*a^3*b^6)*cos(d*
x + c))*log(a*cos(d*x + c) + b) - 3*(a^7*b^2 + 8*a^6*b^3 + 25*a^5*b^4 + 40*
a^4*b^5 + 35*a^3*b^6 + 16*a^2*b^7 + 3*a*b^8 + (a^9 + 8*a^8*b + 25*a^7*b^2 +
 40*a^6*b^3 + 35*a^5*b^4 + 16*a^4*b^5 + 3*a^3*b^6)*cos(d*x + c)^6 + 2*(a^8*
b + 8*a^7*b^2 + 25*a^6*b^3 + 40*a^5*b^4 + 35*a^4*b^5 + 16*a^3*b^6 + 3*a^2*b
^7)*cos(d*x + c)^5 - (2*a^9 + 16*a^8*b + 49*a^7*b^2 + 72*a^6*b^3 + 45*a^5*b
^4 - 8*a^4*b^5 - 29*a^3*b^6 - 16*a^2*b^7 - 3*a*b^8)*cos(d*x + c)^4 - 4*(a^8
*b + 8*a^7*b^2 + 25*a^6*b^3 + 40*a^5*b^4 + 35*a^4*b^5 + 16*a^3*b^6 + 3*a^2*
b^7)*cos(d*x + c)^3 + (a^9 + 8*a^8*b + 23*a^7*b^2 + 24*a^6*b^3 - 15*a^5*b^4
 - 64*a^4*b^5 - 67*a^3*b^6 - 32*a^2*b^7 - 6*a*b^8)*cos(d*x + c)^2 + 2*(a^8*
b + 8*a^7*b^2 + 25*a^6*b^3 + 40*a^5*b^4 + 35*a^4*b^5 + 16*a^3*b^6 + 3*a^2*b
^7)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 3*(a^7*b^2 - 8*a^6*b^3 + 25
*a^5*b^4 - 40*a^4*b^5 + 35*a^3*b^6 - 16*a^2*b^7 + 3*a*b^8 + (a^9 - 8*a^8*b
+ 25*a^7*b^2 - 40*a^6*b^3 + 35*a^5*b^4 - 16*a^4*b^5 + 3*a^3*b^6)*cos(d*x +
c)^6 + 2*(a^8*b - 8*a^7*b^2 + 25*a^6*b^3 - 40*a^5*b^4 + 35*a^4*b^5 - 16*a^3
*b^6 + 3*a^2*b^7)*cos(d*x + c)^5 - (2*a^9 - 16*a^8*b + 49*a^7*b^2 - 72*a^6*
b^3 + 45*a^5*b^4 + 8*a^4*b^5 - 29*a^3*b^6 + 16*a^2*b^7 - 3*a*b^8)*cos(d*x +
 c)^4 - 4*(a^8*b - 8*a^7*b^2 + 25*a^6*b^3 - 40*a^5*b^4 + 35*a^4*b^5 - 16*a^
3*b^6 + 3*a^2*b^7)*cos(d*x + c)^3 + (a^9 - 8*a^8*b + 23*a^7*b^2 - 24*a^6*b^
```

3 - 15*a^5*b^4 + 64*a^4*b^5 - 67*a^3*b^6 + 32*a^2*b^7 - 6*a*b^8)*cos(d*x + c)^2 + 2*(a^8*b - 8*a^7*b^2 + 25*a^6*b^3 - 40*a^5*b^4 + 35*a^4*b^5 - 16*a^3*b^6 + 3*a^2*b^7)*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/((a^12 - 5*a^10*b^2 + 10*a^8*b^4 - 10*a^6*b^6 + 5*a^4*b^8 - a^2*b^10)*d*cos(d*x + c)^6 + 2*(a^11*b - 5*a^9*b^3 + 10*a^7*b^5 - 10*a^5*b^7 + 5*a^3*b^9 - a*b^11)*d*cos(d*x + c)^5 - (2*a^12 - 11*a^10*b^2 + 25*a^8*b^4 - 30*a^6*b^6 + 20*a^4*b^8 - 7*a^2*b^10 + b^12)*d*cos(d*x + c)^4 - 4*(a^11*b - 5*a^9*b^3 + 10*a^7*b^5 - 10*a^5*b^7 + 5*a^3*b^9 - a*b^11)*d*cos(d*x + c)^3 + (a^12 - 7*a^10*b^2 + 20*a^8*b^4 - 30*a^6*b^6 + 25*a^4*b^8 - 11*a^2*b^10 + 2*b^12)*d*cos(d*x + c)^2 + 2*(a^11*b - 5*a^9*b^3 + 10*a^7*b^5 - 10*a^5*b^7 + 5*a^3*b^9 - a*b^11)*d*cos(d*x + c) + (a^10*b^2 - 5*a^8*b^4 + 10*a^6*b^6 - 10*a^4*b^8 + 5*a^2*b^10 - b^12)*d)

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral(csc(c + d*x)**5/(a + b*sec(c + d*x))**3, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1551 vs. 2(303) = 606.
time = 0.69, size = 1551, normalized size = 4.96

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

[Out] 1/64*(12*(a^2 - 3*a*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5) + 192*(a^6*b + 5*a^4*b^3 + 2*a^2*b^5)*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^10 - 5*a^8*b^2 + 10*a^6*b^4 - 10*a^4*b^6 + 5*a^2*b^8 - b^10) - (8*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 12*a^2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 4*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 3*a^2*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 3*a*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/(a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6) - (a^8 - 2*a^7*b - 2*a^6*b^2 + 6*a^5*b^3 - 6*a^3*b^5 + 2*a^2*b^6 + 2*a*b^7 - b^8 - 6*a^8*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 20*a^7*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 12*a^6*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)

```
- 28*a^5*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 40*a^4*b^4*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) - 4*a^3*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1
) - 20*a^2*b^6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 12*a*b^7*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) - 2*b^8*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) -
6*a^8*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 163*a^7*b*(cos(d*x + c) -
 1)^2/(cos(d*x + c) + 1)^2 - 257*a^6*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c)
 + 1)^2 + 339*a^5*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 203*a^4*b
^4*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 223*a^3*b^5*(cos(d*x + c) -
1)^2/(cos(d*x + c) + 1)^2 + 309*a^2*b^6*(cos(d*x + c) - 1)^2/(cos(d*x + c)
+ 1)^2 - 23*a*b^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 7*b^8*(cos(d*
x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 10*a^8*(cos(d*x + c) - 1)^3/(cos(d*x +
 c) + 1)^3 + 186*a^7*b*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 274*a^6*
b^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 890*a^5*b^3*(cos(d*x + c) -
 1)^3/(cos(d*x + c) + 1)^3 - 894*a^4*b^4*(cos(d*x + c) - 1)^3/(cos(d*x + c)
 + 1)^3 + 478*a^3*b^5*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 374*a^2*b
^6*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 18*a*b^7*(cos(d*x + c) - 1)^
3/(cos(d*x + c) + 1)^3 - 4*b^8*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 +
9*a^8*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 45*a^7*b*(cos(d*x + c) -
1)^4/(cos(d*x + c) + 1)^4 + 45*a^6*b^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) +
 1)^4 - 63*a^5*b^3*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 117*a^4*b^4*
(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 9*a^3*b^5*(cos(d*x + c) - 1)^4/
(cos(d*x + c) + 1)^4 + 63*a^2*b^6*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4
 + 27*a*b^7*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4)/((a^9 - a^8*b - 4*a^
7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 -
 b^9)*(a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c) - 1)/(cos(
d*x + c) + 1) + a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - b*(cos(d*x +
c) - 1)^2/(cos(d*x + c) + 1)^2)^2))/d
```

Mupad [B]
time = 2.55, size = 673, normalized size = 2.15

$$\frac{\ln\left(\cos\left(c+d\,x\right)-1\right)\,\left(\frac{3}{16\,{\left(a+b\right)}^3}-\frac{15\,b}{16\,{\left(a+b\right)}^4}+\frac{3\,b^2}{4\,{\left(a+b\right)}^5}\right)}{d}+\frac{\frac{19\,a^4\,b^3+28\,a^2\,b^5+b^7}{4\,\left(a^8-4\,a^6\,b^2+6\,a^4\,b^4-4\,a^2\,b^6+b^8\right)}-\frac{{\cos\left(c+d\,x\right)}^2\,\left(-2\,a^6\,b+23\,a^4\,b^3+26\,a^2\,b^5+b^7\right)}{2\,\left(a^8-4\,a^6\,b^2+6\,a^4\,b^4-4\,a^2\,b^6+b^8\right)}+\frac{3\,{\cos\left(c+d\,x\right)}^4\,\left(-a^6\,b+8\,a^4\,b^3+9\,a^2\,b^5\right)}{4\,\left(a^8-4\,a^6\,b^2+6\,a^4\,b^4-4\,a^2\,b^6+b^8\right)}+\frac{3\,{\cos\left(c+d\,x\right)}^5\,\left(a^7+18\,a^5\,b^2+13\,a^3\,b^4\right)}{8\,\left(a^8-4\,a^6\,b^2+6\,a^4\,b^4-4\,a^2\,b^6+b^8\right)}+\frac{a\,\cos\left(c+d\,x\right)\,\left(55\,a^4\,b^2+46\,a^2\,b^4-5\,b^6\right)}{8\,\left(a^8-4\,a^6\,b^2+6\,a^4\,b^4-4\,a^2\,b^6+b^8\right)}-\frac{a\,{\cos\left(c+d\,x\right)}^3\,\left(5\,a^6+103\,a^4\,b^2+91\,a^2\,b^4-7\,b^6\right)}{8\,\left(a^8-4\,a^6\,b^2+6\,a^4\,b^4-4\,a^2\,b^6+b^8\right)}}{d\,\left({\cos\left(c+d\,x\right)}^2\,\left(a^2-2\,b^2\right)-{\cos\left(c+d\,x\right)}^4\,\left(2\,a^2-b^2\right)+b^2+a^2\,{\cos\left(c+d\,x\right)}^6+2\,a\,b\,\cos\left(c+d\,x\right)-4\,a\,b\,{\cos\left(c+d\,x\right)}^3+2\,a\,b\,{\cos\left(c+d\,x\right)}^5\right)}-\frac{\ln\left(\cos\left(c+d\,x\right)+1\right)\,\left(\frac{3\,b^2}{4\,{\left(a-b\right)}^5}+\frac{15\,b}{16\,{\left(a-b\right)}^4}+\frac{3}{16\,{\left(a-b\right)}^3}\right)}{d}+\frac{\ln\left(b+a\,\cos\left(c+d\,x\right)\right)\,\left(3\,a^6\,b+15\,a^4\,b^3+6\,a^2\,b^5\right)}{d\,\left(a^{10}-5\,a^8\,b^2+10\,a^6\,b^4-10\,a^4\,b^6+5\,a^2\,b^8-b^{10}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^5*(a + b/cos(c + d*x))^3),x)
```

```
[Out] (log(cos(c + d*x) - 1)*(3/(16*(a + b)^3) - (15*b)/(16*(a + b)^4) + (3*b^2)/
(4*(a + b)^5)))/d + ((b^7 + 28*a^2*b^5 + 19*a^4*b^3)/(4*(a^8 + b^8 - 4*a^2*
b^6 + 6*a^4*b^4 - 4*a^6*b^2)) - (cos(c + d*x)^2*(b^7 - 2*a^6*b + 26*a^2*b^5
 + 23*a^4*b^3))/(2*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)) + (3*co
s(c + d*x)^4*(9*a^2*b^5 - a^6*b + 8*a^4*b^3))/(4*(a^8 + b^8 - 4*a^2*b^6 + 6
*a^4*b^4 - 4*a^6*b^2)) + (3*cos(c + d*x)^5*(a^7 + 13*a^3*b^4 + 18*a^5*b^2))
/(8*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)) + (a*cos(c + d*x)*(46*
a^2*b^4 - 5*b^6 + 55*a^4*b^2))/(8*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^
6*b^2)) - (a*cos(c + d*x)^3*(5*a^6 - 7*b^6 + 91*a^2*b^4 + 103*a^4*b^2))/(8*
```

```
(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)))/(d*(cos(c + d*x)^2*(a^2 -
 2*b^2) - cos(c + d*x)^4*(2*a^2 - b^2) + b^2 + a^2*cos(c + d*x)^6 + 2*a*b*c
os(c + d*x) - 4*a*b*cos(c + d*x)^3 + 2*a*b*cos(c + d*x)^5)) - (log(cos(c +
d*x) + 1)*((3*b^2)/(4*(a - b)^5) + (15*b)/(16*(a - b)^4) + 3/(16*(a - b)^3)
))/d + (log(b + a*cos(c + d*x))*(3*a^6*b + 6*a^2*b^5 + 15*a^4*b^3))/(d*(a^1
0 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2))
```

3.228 $\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=539

$$\frac{(5a^6-180a^4b^2+600a^2b^4-448b^6)\,x}{16a^9}-\frac{\sqrt{a-b}\,b\sqrt{a+b}\,(6a^4-47a^2b^2+56b^4)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^9d}$$

```
[Out] 1/16*(5*a^6-180*a^4*b^2+600*a^2*b^4-448*b^6)*x/a^9+1/30*b*(213*a^4-985*a^2*
b^2+840*b^4)*sin(d*x+c)/a^8/d-1/16*(43*a^4-244*a^2*b^2+224*b^4)*cos(d*x+c)*
sin(d*x+c)/a^7/d+1/30*(45*a^4-291*a^2*b^2+280*b^4)*cos(d*x+c)^2*sin(d*x+c)/
a^6/b/d-1/24*(24*a^4-169*a^2*b^2+168*b^4)*cos(d*x+c)^3*sin(d*x+c)/a^5/b^2/d
-1/4*cos(d*x+c)^4*sin(d*x+c)/b/d/(b+a*cos(d*x+c))^2+1/10*a*cos(d*x+c)^5*sin
(d*x+c)/b^2/d/(b+a*cos(d*x+c))^2+1/60*(9*a^4-60*a^2*b^2+56*b^4)*cos(d*x+c)^
5*sin(d*x+c)/a^3/b^2/d/(b+a*cos(d*x+c))^2+4/15*b*cos(d*x+c)^6*sin(d*x+c)/a^
2/d/(b+a*cos(d*x+c))^2-1/6*cos(d*x+c)^7*sin(d*x+c)/a/d/(b+a*cos(d*x+c))^2+1
/20*(15*a^4-110*a^2*b^2+112*b^4)*cos(d*x+c)^4*sin(d*x+c)/a^4/b^2/d/(b+a*cos
(d*x+c))-b*(6*a^4-47*a^2*b^2+56*b^4)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)
/(a+b)^(1/2))*(a-b)^(1/2)*(a+b)^(1/2)/a^9/d
```

Rubi [A]
time = 1.61, antiderivative size = 539, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3957, 2975, 3126, 3128, 3102, 2814, 2738, 214}

$$\frac{4b\sin(c+dx)\cos^6(c+dx)}{15a^2d(a\cos(c+dx)+b)^2}+\frac{(15a^4-110a^2b^2+112b^4)\sin(c+dx)\cos^4(c+dx)}{20a^4b^2d(a\cos(c+dx)+b)}-\frac{b\sqrt{a-b}\,\sqrt{a+b}\,(6a^4-47a^2b^2+56b^4)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^9d}+\frac{b(213a^4-985a^2b^2+840b^4)\sin(c+dx)}{30a^8d}-\frac{(43a^4-244a^2b^2+224b^4)\sin(c+dx)\cos(c+dx)}{16a^7d}+\frac{(45a^4-291a^2b^2+280b^4)\sin(c+dx)\cos^2(c+dx)}{30a^6bd}-\frac{(24a^4-169a^2b^2+168b^4)\sin(c+dx)\cos^3(c+dx)}{24a^5b^2d}+\frac{(9a^4-60a^2b^2+56b^4)\sin(c+dx)\cos^5(c+dx)}{60a^3b^2d(a\cos(c+dx)+b)^2}+\frac{x(5a^6-180a^4b^2+600a^2b^4-448b^6)}{16a^9}+\frac{a\sin(c+dx)\cos^5(c+dx)}{10b^2d(a\cos(c+dx)+b)^2}-\frac{\sin(c+dx)\cos^7(c+dx)}{6ad(a\cos(c+dx)+b)^2}-\frac{\sin(c+dx)\cos^4(c+dx)}{4bd(a\cos(c+dx)+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^6/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((5*a^6 - 180*a^4*b^2 + 600*a^2*b^4 - 448*b^6)*x)/(16*a^9) - (Sqrt[a - b]*b
*Sqrt[a + b]*(6*a^4 - 47*a^2*b^2 + 56*b^4)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*
x)/2])/Sqrt[a + b]])/(a^9*d) + (b*(213*a^4 - 985*a^2*b^2 + 840*b^4)*Sin[c +
 d*x])/(30*a^8*d) - ((43*a^4 - 244*a^2*b^2 + 224*b^4)*Cos[c + d*x]*Sin[c +
d*x])/(16*a^7*d) + ((45*a^4 - 291*a^2*b^2 + 280*b^4)*Cos[c + d*x]^2*Sin[c +
 d*x])/(30*a^6*b*d) - ((24*a^4 - 169*a^2*b^2 + 168*b^4)*Cos[c + d*x]^3*Sin[
c + d*x])/(24*a^5*b^2*d) - (Cos[c + d*x]^4*Sin[c + d*x])/(4*b*d*(b + a*Cos[
c + d*x])^2) + (a*Cos[c + d*x]^5*Sin[c + d*x])/(10*b^2*d*(b + a*Cos[c + d*x
])^2) + ((9*a^4 - 60*a^2*b^2 + 56*b^4)*Cos[c + d*x]^5*Sin[c + d*x])/(60*a^3
*b^2*d*(b + a*Cos[c + d*x])^2) + (4*b*Cos[c + d*x]^6*Sin[c + d*x])/(15*a^2*
d*(b + a*Cos[c + d*x])^2) - (Cos[c + d*x]^7*Sin[c + d*x])/(6*a*d*(b + a*Cos
[c + d*x])^2) + ((15*a^4 - 110*a^2*b^2 + 112*b^4)*Cos[c + d*x]^4*Sin[c + d*
x])/(20*a^4*b^2*d*(b + a*Cos[c + d*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2975

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(d*Sin[
e + f*x])^(n + 1)*((a + b*Sin[e + f*x])^(m + 1)/(a*d*f*(n + 1))), x] + (Dis
t[1/(a^2*b^2*d^2*(n + 1)*(n + 2)*(m + n + 5)*(m + n + 6)), Int[(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5
) - a^2*b^2*(n + 2)*(2*n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m
+ n + 3)*(m + n + 5)*(m + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n
+ 5)*(m + n + 6))*Sin[e + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4
*(m + n + 2)*(m + n + 4)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*
(m + n + 5)*(2*n + 2*m + 13))*Sin[e + f*x]^2, x], x], x] - Simp[b*(m + n +
2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*((a + b*Sin[e + f*x])^(m + 1)/(a^2
*d^2*f*(n + 1)*(n + 2))), x] - Simp[a*(n + 5)*Cos[e + f*x]*(d*Sin[e + f*x])
^(n + 3)*((a + b*Sin[e + f*x])^(m + 1)/(b^2*d^3*f*(m + n + 5)*(m + n + 6)))
, x] + Simp[Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*((a + b*Sin[e + f*x])^(m
+ 1)/(b*d^4*f*(m + n + 6))), x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m +
 n + 5, 0] && NeQ[m + n + 6, 0] &&  !IGtQ[m, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\int \frac{\cos^3(c+dx)\sin^6(c+dx)}{(-b-a\cos(c+dx))^3}\,dx$$

$$= -\frac{\cos^4(c+dx)\sin(c+dx)}{4bd(b+a\cos(c+dx))^2} + \frac{a\cos^5(c+dx)\sin(c+dx)}{10b^2d(b+a\cos(c+dx))^2} + \frac{4b\cos^6(c+dx)\sin(c+}{15a^2d(b+a\cos(c+}$$

$$= -\frac{\cos^4(c+dx)\sin(c+dx)}{4bd(b+a\cos(c+dx))^2} + \frac{a\cos^5(c+dx)\sin(c+dx)}{10b^2d(b+a\cos(c+dx))^2} + \frac{(9a^4-60a^2b^2+56b^4}{60a^3b^2d(b+}$$

$$= -\frac{\cos^4(c+dx)\sin(c+dx)}{4bd(b+a\cos(c+dx))^2} + \frac{a\cos^5(c+dx)\sin(c+dx)}{10b^2d(b+a\cos(c+dx))^2} + \frac{(9a^4-60a^2b^2+56b^4}{60a^3b^2d(b+}$$

$$= -\frac{(24a^4-169a^2b^2+168b^4)\cos^3(c+dx)\sin(c+dx)}{24a^5b^2d} - \frac{\cos^4(c+dx)\sin(c+dx)}{4bd(b+a\cos(c+dx))^2}$$

$$= \frac{(45a^4-291a^2b^2+280b^4)\cos^2(c+dx)\sin(c+dx)}{30a^6bd} - \frac{(24a^4-169a^2b^2+168b^4)}{24a^5}$$

$$= -\frac{(43a^4-244a^2b^2+224b^4)\cos(c+dx)\sin(c+dx)}{16a^7d} + \frac{(45a^4-291a^2b^2+280b^4)}{30a}$$

$$= \frac{b(213a^4-985a^2b^2+840b^4)\sin(c+dx)}{30a^8d} - \frac{(43a^4-244a^2b^2+224b^4)\cos(c+dx}{16a^7d}$$

$$= \frac{(5a^6-180a^4b^2+600a^2b^4-448b^6)\,x}{16a^9} + \frac{b(213a^4-985a^2b^2+840b^4)\sin(c+dx)}{30a^8d}$$

$$= \frac{(5a^6-180a^4b^2+600a^2b^4-448b^6)\,x}{16a^9} + \frac{b(213a^4-985a^2b^2+840b^4)\sin(c+dx)}{30a^8d}$$

$$= \frac{(5a^6-180a^4b^2+600a^2b^4-448b^6)\,x}{16a^9} - \frac{\sqrt{a-b}\,b\sqrt{a+b}\,(6a^4-47a^2b^2+56b^4)}{a^9d}$$

Mathematica [A]
time = 7.86, size = 599, normalized size = 1.11

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^6/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (-7680*b*(-a^2 + b^2)^3*(6*a^4 - 47*a^2*b^2 + 56*b^4)*ArcTanh[((-a + b)*Tan
[(c + d*x)/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x])^2 + 2*(a^2 - b^2)^(5/2
```

```
)*(600*a^8*c - 20400*a^6*b^2*c + 28800*a^4*b^4*c + 90240*a^2*b^6*c - 107520
*b^8*c + 600*a^8*d*x - 20400*a^6*b^2*d*x + 28800*a^4*b^4*d*x + 90240*a^2*b^
6*d*x - 107520*b^8*d*x + 480*a*b*(5*a^6 - 180*a^4*b^2 + 600*a^2*b^4 - 448*b
^6)*(c + d*x)*Cos[c + d*x] + 120*a^2*(5*a^6 - 180*a^4*b^2 + 600*a^2*b^4 - 4
48*b^6)*(c + d*x)*Cos[2*(c + d*x)] + 2640*a^7*b*Sin[c + d*x] + 16160*a^5*b^
3*Sin[c + d*x] - 117120*a^3*b^5*Sin[c + d*x] + 107520*a*b^7*Sin[c + d*x] -
405*a^8*Sin[2*(c + d*x)] + 24600*a^6*b^2*Sin[2*(c + d*x)] - 99040*a^4*b^4*S
in[2*(c + d*x)] + 80640*a^2*b^6*Sin[2*(c + d*x)] + 2436*a^7*b*Sin[3*(c + d*
x)] - 10880*a^5*b^3*Sin[3*(c + d*x)] + 8960*a^3*b^5*Sin[3*(c + d*x)] - 140*
a^8*Sin[4*(c + d*x)] + 1164*a^6*b^2*Sin[4*(c + d*x)] - 1120*a^4*b^4*Sin[4*(
c + d*x)] - 188*a^7*b*Sin[5*(c + d*x)] + 224*a^5*b^3*Sin[5*(c + d*x)] + 35*
a^8*Sin[6*(c + d*x)] - 56*a^6*b^2*Sin[6*(c + d*x)] + 16*a^7*b*Sin[7*(c + d*
x)] - 5*a^8*Sin[8*(c + d*x)]))/(7680*a^9*(a - b)^2*(a + b)^2*Sqrt[a^2 - b^2
]*d*(b + a*Cos[c + d*x])^2)
```

Maple [**A**]
time = 0.48, size = 582, normalized size = 1.08 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^6/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*(a-b)*(a+b)*b/a^9*(((5/2*a^3*b^2-7*a*b^4-3*a^4*b+15/2*a^2*b^3)*tan(1
/2*d*x+1/2*c)^3+(5/2*a^3*b^2-7*a*b^4+3*a^4*b-15/2*a^2*b^3)*tan(1/2*d*x+1/2*
c))/(a*tan(1/2*d*x+1/2*c)^2-b*tan(1/2*d*x+1/2*c)^2-a-b)^2-1/2*(6*a^4-47*a^2
*b^2+56*b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a
-b))^(1/2)))+2/a^9*(((5/16*a^6+3*a^5*b-21/4*a^4*b^2-20*a^3*b^3+15/2*a^2*b^4
+21*a*b^5)*tan(1/2*d*x+1/2*c)^11+(19*a^5*b-87/4*a^4*b^2+45/2*a^2*b^4+105*a*
b^5+85/48*a^6-340/3*a^3*b^3)*tan(1/2*d*x+1/2*c)^9+(258/5*a^5*b-33/2*a^4*b^2
-240*a^3*b^3+15*a^2*b^4+210*a*b^5+33/8*a^6)*tan(1/2*d*x+1/2*c)^7+(-33/8*a^6
+33/2*a^4*b^2-15*a^2*b^4+258/5*a^5*b-240*a^3*b^3+210*a*b^5)*tan(1/2*d*x+1/2
*c)^5+(19*a^5*b+87/4*a^4*b^2-340/3*a^3*b^3-45/2*a^2*b^4+105*a*b^5-85/48*a^6
)*tan(1/2*d*x+1/2*c)^3+(3*a^5*b-20*a^3*b^3+21*a*b^5-5/16*a^6+21/4*a^4*b^2-1
5/2*a^2*b^4)*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^6+1/16*(5*a^6-180
*a^4*b^2+600*a^2*b^4-448*b^6)*arctan(tan(1/2*d*x+1/2*c))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 4.38, size = 1057, normalized size = 1.96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/240*(15*(5*a^8 - 180*a^6*b^2 + 600*a^4*b^4 - 448*a^2*b^6)*d*x*cos(d*x +
c)^2 + 30*(5*a^7*b - 180*a^5*b^3 + 600*a^3*b^5 - 448*a*b^7)*d*x*cos(d*x + c
) + 15*(5*a^6*b^2 - 180*a^4*b^4 + 600*a^2*b^6 - 448*b^8)*d*x + 60*(6*a^4*b^
3 - 47*a^2*b^5 + 56*b^7 + (6*a^6*b - 47*a^4*b^3 + 56*a^2*b^5)*cos(d*x + c)^
2 + 2*(6*a^5*b^2 - 47*a^3*b^4 + 56*a*b^6)*cos(d*x + c))*sqrt(a^2 - b^2)*log
((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*
cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*c
os(d*x + c) + b^2)) - (40*a^8*cos(d*x + c)^7 - 64*a^7*b*cos(d*x + c)^6 - 17
04*a^5*b^3 + 7880*a^3*b^5 - 6720*a*b^7 - 2*(65*a^8 - 56*a^6*b^2)*cos(d*x +
c)^5 + 4*(67*a^7*b - 56*a^5*b^3)*cos(d*x + c)^4 + (165*a^8 - 694*a^6*b^2 +
560*a^4*b^4)*cos(d*x + c)^3 - 2*(387*a^7*b - 1444*a^5*b^3 + 1120*a^3*b^5)*c
os(d*x + c)^2 - (2763*a^6*b^2 - 12100*a^4*b^4 + 10080*a^2*b^6)*cos(d*x + c)
)*sin(d*x + c))/(a^11*d*cos(d*x + c)^2 + 2*a^10*b*d*cos(d*x + c) + a^9*b^2*
d), 1/240*(15*(5*a^8 - 180*a^6*b^2 + 600*a^4*b^4 - 448*a^2*b^6)*d*x*cos(d*x
 + c)^2 + 30*(5*a^7*b - 180*a^5*b^3 + 600*a^3*b^5 - 448*a*b^7)*d*x*cos(d*x
+ c) + 15*(5*a^6*b^2 - 180*a^4*b^4 + 600*a^2*b^6 - 448*b^8)*d*x - 120*(6*a^
4*b^3 - 47*a^2*b^5 + 56*b^7 + (6*a^6*b - 47*a^4*b^3 + 56*a^2*b^5)*cos(d*x +
 c)^2 + 2*(6*a^5*b^2 - 47*a^3*b^4 + 56*a*b^6)*cos(d*x + c))*sqrt(-a^2 + b^2
)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))
 - (40*a^8*cos(d*x + c)^7 - 64*a^7*b*cos(d*x + c)^6 - 1704*a^5*b^3 + 7880*a
^3*b^5 - 6720*a*b^7 - 2*(65*a^8 - 56*a^6*b^2)*cos(d*x + c)^5 + 4*(67*a^7*b
- 56*a^5*b^3)*cos(d*x + c)^4 + (165*a^8 - 694*a^6*b^2 + 560*a^4*b^4)*cos(d*
x + c)^3 - 2*(387*a^7*b - 1444*a^5*b^3 + 1120*a^3*b^5)*cos(d*x + c)^2 - (27
63*a^6*b^2 - 12100*a^4*b^4 + 10080*a^2*b^6)*cos(d*x + c))*sin(d*x + c))/(a^
11*d*cos(d*x + c)^2 + 2*a^10*b*d*cos(d*x + c) + a^9*b^2*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**6/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral(sin(c + d*x)**6/(a + b*sec(c + d*x))**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1030 vs. $2(508) = 1016$.
time = 0.64, size = 1030, normalized size = 1.91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/240*(15*(5*a^6 - 180*a^4*b^2 + 600*a^2*b^4 - 448*b^6)*(d*x + c)/a^9 - 240
*(6*a^6*b - 53*a^4*b^3 + 103*a^2*b^5 - 56*b^7)*(pi*floor(1/2*(d*x + c)/pi +
 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1
/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*a^9) - 240*(6*a^5*b^2*tan(1/2*d
*x + 1/2*c)^3 - 5*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 - 21*a^3*b^4*tan(1/2*d*x +
 1/2*c)^3 + 19*a^2*b^5*tan(1/2*d*x + 1/2*c)^3 + 15*a*b^6*tan(1/2*d*x + 1/2*
c)^3 - 14*b^7*tan(1/2*d*x + 1/2*c)^3 - 6*a^5*b^2*tan(1/2*d*x + 1/2*c) - 5*a
^4*b^3*tan(1/2*d*x + 1/2*c) + 21*a^3*b^4*tan(1/2*d*x + 1/2*c) + 19*a^2*b^5*
tan(1/2*d*x + 1/2*c) - 15*a*b^6*tan(1/2*d*x + 1/2*c) - 14*b^7*tan(1/2*d*x +
 1/2*c))/((a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2*a
^8) + 2*(75*a^5*tan(1/2*d*x + 1/2*c)^11 + 720*a^4*b*tan(1/2*d*x + 1/2*c)^11
 - 1260*a^3*b^2*tan(1/2*d*x + 1/2*c)^11 - 4800*a^2*b^3*tan(1/2*d*x + 1/2*c)
^11 + 1800*a*b^4*tan(1/2*d*x + 1/2*c)^11 + 5040*b^5*tan(1/2*d*x + 1/2*c)^11
 + 425*a^5*tan(1/2*d*x + 1/2*c)^9 + 4560*a^4*b*tan(1/2*d*x + 1/2*c)^9 - 522
0*a^3*b^2*tan(1/2*d*x + 1/2*c)^9 - 27200*a^2*b^3*tan(1/2*d*x + 1/2*c)^9 + 5
400*a*b^4*tan(1/2*d*x + 1/2*c)^9 + 25200*b^5*tan(1/2*d*x + 1/2*c)^9 + 990*a
^5*tan(1/2*d*x + 1/2*c)^7 + 12384*a^4*b*tan(1/2*d*x + 1/2*c)^7 - 3960*a^3*b
^2*tan(1/2*d*x + 1/2*c)^7 - 57600*a^2*b^3*tan(1/2*d*x + 1/2*c)^7 + 3600*a*b
^4*tan(1/2*d*x + 1/2*c)^7 + 50400*b^5*tan(1/2*d*x + 1/2*c)^7 - 990*a^5*tan(
1/2*d*x + 1/2*c)^5 + 12384*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 3960*a^3*b^2*tan(
1/2*d*x + 1/2*c)^5 - 57600*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 - 3600*a*b^4*tan(
1/2*d*x + 1/2*c)^5 + 50400*b^5*tan(1/2*d*x + 1/2*c)^5 - 425*a^5*tan(1/2*d*x
 + 1/2*c)^3 + 4560*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 5220*a^3*b^2*tan(1/2*d*x
+ 1/2*c)^3 - 27200*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 - 5400*a*b^4*tan(1/2*d*x
+ 1/2*c)^3 + 25200*b^5*tan(1/2*d*x + 1/2*c)^3 - 75*a^5*tan(1/2*d*x + 1/2*c)
 + 720*a^4*b*tan(1/2*d*x + 1/2*c) + 1260*a^3*b^2*tan(1/2*d*x + 1/2*c) - 480
0*a^2*b^3*tan(1/2*d*x + 1/2*c) - 1800*a*b^4*tan(1/2*d*x + 1/2*c) + 5040*b^5
*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^6*a^8))/d
```

Mupad [B]
time = 5.71, size = 2500, normalized size = 4.64

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6/(a + b/cos(c + d*x))^3,x)
```

[Out] ((tan(c/2 + (d*x)/2)^3*(10080*a*b^6 + 454*a^6*b - 55*a^7 + 9408*b^7 - 9688*a^2*b^5 - 12212*a^3*b^4 + 608*a^4*b^3 + 2969*a^5*b^2))/(24*a^8) + (tan(c/2 + (d*x)/2)^13*(454*a^6*b - 10080*a*b^6 + 55*a^7 + 9408*b^7 - 9688*a^2*b^5 + 12212*a^3*b^4 + 608*a^4*b^3 - 2969*a^5*b^2))/(24*a^8) + (tan(c/2 + (d*x)/2)^5*(90720*a*b^6 + 2154*a^6*b - 215*a^7 + 141120*b^7 - 163240*a^2*b^5 - 107220*a^3*b^4 + 32224*a^4*b^3 + 22673*a^5*b^2))/(120*a^8) + (tan(c/2 + (d*x)/2)^11*(2154*a^6*b - 90720*a*b^6 + 215*a^7 + 141120*b^7 - 163240*a^2*b^5 + 107220*a^3*b^4 + 32224*a^4*b^3 - 22673*a^5*b^2))/(120*a^8) + (tan(c/2 + (d*x)/2)^7*(50400*a*b^6 - 4994*a^6*b + 2545*a^7 + 235200*b^7 - 287000*a^2*b^5 - 58820*a^3*b^4 + 74752*a^4*b^3 + 11173*a^5*b^2))/(120*a^8) - (tan(c/2 + (d*x)/2)^9*(50400*a*b^6 + 4994*a^6*b + 2545*a^7 - 235200*b^7 + 287000*a^2*b^5 - 58820*a^3*b^4 - 74752*a^4*b^3 + 11173*a^5*b^2))/(120*a^8) + (tan(c/2 + (d*x)/2)^15*(a - b)*(224*a*b^5 + 43*a^5*b + 5*a^6 - 448*b^6 + 600*a^2*b^4 - 244*a^3*b^3 - 180*a^4*b^2))/(8*a^8) - (tan(c/2 + (d*x)/2)*(2*a*b + a^2 + b^2)*(224*a*b^4 - 48*a^4*b + 5*a^5 - 448*b^5 + 376*a^2*b^3 - 132*a^3*b^2))/(8*a^8))/(d*(2*a*b - tan(c/2 + (d*x)/2)^8*(10*a^2 - 70*b^2) + tan(c/2 + (d*x)/2)^2*(12*a*b + 4*a^2 + 8*b^2) + tan(c/2 + (d*x)/2)^14*(4*a^2 - 12*a*b + 8*b^2) + tan(c/2 + (d*x)/2)^4*(28*a*b + 4*a^2 + 28*b^2) + tan(c/2 + (d*x)/2)^12*(4*a^2 - 28*a*b + 28*b^2) + tan(c/2 + (d*x)/2)^6*(28*a*b - 4*a^2 + 56*b^2) - tan(c/2 + (d*x)/2)^10*(28*a*b + 4*a^2 - 56*b^2) + tan(c/2 + (d*x)/2)^16*(a^2 - 2*a*b + b^2) + a^2 + b^2)) + (atan((((((((106*a^25*b - 10*a^26 + 896*a^18*b^8 - 1344*a^19*b^7 - 1200*a^20*b^6 + 2360*a^21*b^5 + 136*a^22*b^4 - 1122*a^23*b^3 + 178*a^24*b^2)/a^24 - (tan(c/2 + (d*x)/2)*(512*a^20*b + 512*a^18*b^3 - 1024*a^19*b^2)*(a^6*5i - b^6*448i + a^2*b^4*600i - a^4*b^2*180i))/(128*a^25))*(a^6*5i - b^6*448i + a^2*b^4*600i - a^4*b^2*180i))/(16*a^9) + (tan(c/2 + (d*x)/2)*(802816*a*b^14 - 75*a^14*b + 25*a^15 - 401408*b^15 + 673792*a^2*b^13 - 2150400*a^3*b^12 + 32640*a^4*b^11 + 2085120*a^5*b^10 - 601600*a^6*b^9 - 881920*a^7*b^8 + 364160*a^8*b^7 + 153600*a^9*b^6 - 72696*a^10*b^5 - 7704*a^11*b^4 + 3071*a^12*b^3 + 579*a^13*b^2))/(8*a^16))*(a^6*5i - b^6*448i + a^2*b^4*600i - a^4*b^2*180i)*1i)/(16*a^9) - (((((106*a^25*b - 10*a^26 + 896*a^18*b^8 - 1344*a^19*b^7 - 1200*a^20*b^6 + 2360*a^21*b^5 + 136*a^22*b^4 - 1122*a^23*b^3 + 178*a^24*b^2)/a^24 + (tan(c/2 + (d*x)/2)*(512*a^20*b + 512*a^18*b^3 - 1024*a^19*b^2)*(a^6*5i - b^6*448i + a^2*b^4*600i - a^4*b^2*180i))/(128*a^25))*(a^6*5i - b^6*448i + a^2*b^4*600i - a^4*b^2*180i))/(16*a^9) - (tan(c/2 + (d*x)/2)*(802816*a*b^14 - 75*a^14*b + 25*a^15 - 401408*b^15 + 673792*a^2*b^13 - 2150400*a^3*b^12 + 32640*a^4*b^11 + 2085120*a^5*b^10 - 601600*a^6*b^9 - 881920*a^7*b^8 + 364160*a^8*b^7 + 153600*a^9*b^6 - 72696*a^10*b^5 - 7704*a^11*b^4 + 3071*a^12*b^3 + 579*a^13*b^2))/(8*a^16))*(a^6*5i - b^6*448i + a^2*b^4*600i - a^4*b^2*180i)*1i)/(16*a^9))/(((75*a^19*b)/4 - 2107392*a*b^19 + 1404928*b^20 - 5644800*a^2*b^18 + 9345280*a^3*b^17 + 8902208*a^4*b^16 - 17144736*a^5*b^15 - 6722456*a^6*b^14 + 16804748*a^7*b^13 + 2126380*a^8*b^12 - 9486373*a^9*b^11 + 163573*a^10*b^10 + 3099308*a^11*b^9 - 297558*a^12*b^8 - (4466945*a^13*b^7)/8 + (296845*a^14*b^6)/4 + (196765*a^15*b^5)/4 - (26515*a^16*b^4)/4 - (13415*a^17*b^3)/8 + (285*a^18*b^2)/2)/a^24 + (((((106*a^25*b - 10*a^26 + 896*a^18*b^8 - 1344*a^19*b^7 - 1200*a^20

```
*b^6 + 2360*a^21*b^5 + 136*a^22*b^4 - 1122*a^23*b^3 + 178*a^24*b^2)/a^24 -
(tan(c/2 + (d*x)/2)*(512*a^20*b + 512*a^18*b^3 - 1024*a^19*b^2)*(a^6*5i - b
^6*448i + a^2*b^4*600i - a^4*b^2*180i))/(128*a^25))*(a^6*5i - b^6*448i + a^
2*b^4*600i - a^4*b^2*180i))/(16*a^9) + (tan(c/2 + (d*x)/2)*(802816*a*b^14 -
 75*a^14*b + 25*a^15 - 401408*b^15 + 673792*a^2*b^13 - 2150400*a^3*b^12 + 3
2640*a^4*b^11 + 2085120*a^5*b^10 - 601600*a^6*b^9 - 881920*a^7*b^8 + 364160
*a^8*b^7 + 153600*a^9*b^6 - 72696*a^10*b^5 - 7704*a^11*b^4 + 3071*a^12*b^3
+ 579*a^13*b^2))/(8*a^16))*(a^6*5i - b^6*448i + a^2*b^4*600i - a^4*b^2*180i
))/(16*a^9) + (((((106*a^25*b - 10*a^26 + 896*a^18*b^8 - 1344*a^19*b^7 - 12
00*a^20*b^6 + 2360*a^21*b^5 + 136*a^22*b^4 - 1122*a^23*b^3 + 178*a^24*b^2)/
a^24 + (tan(c/2 + (d*x)/2)*(512*a^20*b + 512*a^18*b^3 - 1024*a^19*b^2)*(a^6
*5i - b^6*448i + a^2*b^4*600i - a^4*b^2*180i))/(128*a^25))*(a^6*5i - b^6*44
8i + a^2*b^4*600i - a^4*b^2*180i))/(16*a^9) - (tan(c/2 + (d*x)/2)*(802816*a
*b^14 - 75*a^14*b + 25*a^15 - 401408*b^15 + 673792*a^2*b^13 - 2150400*a^3*b
^12 + 32640*a^4*b^11 + 2085120*a^5*b^10 - 601600*a^6*b^9 - 881920*a^7*b^8 +
 364160*a^8*b^7 + 153600*a^9*b^6 - 72696*a^10*b^5 - 7704*a^11*b^4 + 3071*a^
12*b^3 + 579*a^13*b^2))/(8*a^16))*(a^6*5i - b^6*448i + a^2*b^4*600i - a^4*b
^2*180i))/(16*a^9)))*(a^6*5i - b^6*448i + a^2*b^4*600i - a^4*b^2*180i)*1i)/
(8*a^9*d) + (b*atan(((b*((a + b)*(a - b))^(1/2)*((tan(c/2 + (d*x)/2)*(80281
6*a*b^14 - 75*a^14*b + 25*a^15 - 401408*b^15 + 673792*a^2*b^13 - 2150400*a^
3*b^12 + 32640*a^4*b^11 + 2085120*a^5*b^10 - 601600*a^6*b^9 - 881920*a^7*b^
8 + 364160*a^8*b^7 + 153600*a^9*b^6 - 72696*a^1...
```

3.229 $\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=333

$$\frac{3(a^4-24a^2b^2+40b^4)\,x}{8a^7}-\frac{3b(2a^4-11a^2b^2+10b^4)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^7\sqrt{a-b}\,\sqrt{a+b}\,d}+\frac{b(13a^2-30b^2)\sin(c+dx)}{2a^6d}$$

```
[Out] 3/8*(a^4-24*a^2*b^2+40*b^4)*x/a^7+1/2*b*(13*a^2-30*b^2)*sin(d*x+c)/a^6/d-3/
8*(7*a^2-20*b^2)*cos(d*x+c)*sin(d*x+c)/a^5/d+1/2*(3*a^2-10*b^2)*cos(d*x+c)^
2*sin(d*x+c)/a^4/b/d-1/4*(4*a^2-15*b^2)*cos(d*x+c)^3*sin(d*x+c)/a^3/b^2/d-1
/2*(a^2-b^2)*cos(d*x+c)^4*sin(d*x+c)/a^2/b/d/(b+a*cos(d*x+c))^2+1/2*(2*a^2-
7*b^2)*cos(d*x+c)^4*sin(d*x+c)/a^2/b^2/d/(b+a*cos(d*x+c))-3*b*(2*a^4-11*a^2
*b^2+10*b^4)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^7/d/(a-b
)^(1/2)/(a+b)^(1/2)
```

Rubi [A]
time = 0.76, antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3957, 2970, 3128, 3102, 2814, 2738, 214}

$$\frac{(2a^2-7b^2)\sin(c+dx)\cos^4(c+dx)}{2a^2b^2d(a\cos(c+dx)+b)}-\frac{(a^2-b^2)\sin(c+dx)\cos^4(c+dx)}{2a^2bd(a\cos(c+dx)+b)^2}+\frac{b(13a^2-30b^2)\sin(c+dx)}{2a^6d}-\frac{3(7a^2-20b^2)\sin(c+dx)\cos(c+dx)}{8a^5d}+\frac{(3a^2-10b^2)\sin(c+dx)\cos^2(c+dx)}{2a^4bd}-\frac{(4a^2-15b^2)\sin(c+dx)\cos^3(c+dx)}{4a^3b^2d}-\frac{3b(2a^4-11a^2b^2+10b^4)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^7d\sqrt{a-b}\,\sqrt{a+b}}+\frac{3x(a^4-24a^2b^2+40b^4)}{8a^7}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (3*(a^4 - 24*a^2*b^2 + 40*b^4)*x)/(8*a^7) - (3*b*(2*a^4 - 11*a^2*b^2 + 10*b
^4)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^7*Sqrt[a - b]*S
qrt[a + b]*d) + (b*(13*a^2 - 30*b^2)*Sin[c + d*x])/(2*a^6*d) - (3*(7*a^2 -
20*b^2)*Cos[c + d*x]*Sin[c + d*x])/(8*a^5*d) + ((3*a^2 - 10*b^2)*Cos[c + d*
x]^2*Sin[c + d*x])/(2*a^4*b*d) - ((4*a^2 - 15*b^2)*Cos[c + d*x]^3*Sin[c + d
*x])/(4*a^3*b^2*d) - ((a^2 - b^2)*Cos[c + d*x]^4*Sin[c + d*x])/(2*a^2*b*d*(
b + a*Cos[c + d*x])^2) + ((2*a^2 - 7*b^2)*Cos[c + d*x]^4*Sin[c + d*x])/(2*a
^2*b^2*d*(b + a*Cos[c + d*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2970

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a^2 - b^2)*Cos[e +
f*x]*(a + b*Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*b^2*d*f*(m +
 1))), x] + (-Dist[1/(a^2*b^2*(m + 1)*(m + 2)), Int[(a + b*Sin[e + f*x])^(m
 + 2)*(d*Sin[e + f*x])^n*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n
+ 3) + a*b*(m + 2)*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 2)*(m
 + n + 4))*Sin[e + f*x]^2, x], x], x] + Simp[(a^2*(n - m + 1) - b^2*(m + n
+ 2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 2)*((d*Sin[e + f*x])^(n + 1)/(
a^2*b^2*d*f*(m + 1)*(m + 2))), x]) /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a
^2 - b^2, 0] && IntegersQ[2*m, 2*n] && LtQ[m, -1] &&  !LtQ[n, -1] && (LtQ[m
, -2] || EqQ[m + n + 4, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx &= -\int \frac{\cos^3(c+dx)\sin^4(c+dx)}{(-b-a\cos(c+dx))^3}\,dx \\
&= -\frac{(a^2-b^2)\cos^4(c+dx)\sin(c+dx)}{2a^2bd(b+a\cos(c+dx))^2} + \frac{(2a^2-7b^2)\cos^4(c+dx)\sin(c+dx)}{2a^2b^2d(b+a\cos(c+dx))} + \cdots \\
&= -\frac{(4a^2-15b^2)\cos^3(c+dx)\sin(c+dx)}{4a^3b^2d} - \frac{(a^2-b^2)\cos^4(c+dx)\sin(c+dx)}{2a^2bd(b+a\cos(c+dx))^2} + \cdots \\
&= \frac{(3a^2-10b^2)\cos^2(c+dx)\sin(c+dx)}{2a^4bd} - \frac{(4a^2-15b^2)\cos^3(c+dx)\sin(c+dx)}{4a^3b^2d} - \cdots \\
&= -\frac{3(7a^2-20b^2)\cos(c+dx)\sin(c+dx)}{8a^5d} + \frac{(3a^2-10b^2)\cos^2(c+dx)\sin(c+dx)}{2a^4bd} \cdots \\
&= \frac{b(13a^2-30b^2)\sin(c+dx)}{2a^6d} - \frac{3(7a^2-20b^2)\cos(c+dx)\sin(c+dx)}{8a^5d} + \frac{(3a^2-10 \cdots}{\cdots} \\
&= \frac{3(a^4-24a^2b^2+40b^4)\,x}{8a^7} + \frac{b(13a^2-30b^2)\sin(c+dx)}{2a^6d} - \frac{3(7a^2-20b^2)\cos(c+ \cdots}{8a^5d} \\
&= \frac{3(a^4-24a^2b^2+40b^4)\,x}{8a^7} + \frac{b(13a^2-30b^2)\sin(c+dx)}{2a^6d} - \frac{3(7a^2-20b^2)\cos(c+ \cdots}{8a^5d} \\
&= \frac{3(a^4-24a^2b^2+40b^4)\,x}{8a^7} - \frac{3b(2a^4-11a^2b^2+10b^4)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}} \cdots\right.}{a^7\sqrt{a-b}\,\sqrt{a+b}\,d}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1178 vs. $2(333) = 666$.
time = 5.75, size = 1178, normalized size = 3.54

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((-6*(8*(c + d*x) + (2*b*(15*a^4 - 20*a^2*b^2 + 8*b^4)*ArcTanh[((-a + b)*Ta
n[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) + (a*b*(3*a^2 - 4*b^2)*
```

```
Sin[c + d*x])/((a - b)*(a + b)*(b + a*Cos[c + d*x])^2) - (3*a*(2*a^4 - 7*a^
2*b^2 + 4*b^4)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(b + a*Cos[c + d*x]))))/a
^3 + (6*((6*a*b*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[
a^2 - b^2] + ((b*(a^2 + 2*b^2) + a*(2*a^2 + b^2)*Cos[c + d*x])*Sin[c + d*x]
)/(b + a*Cos[c + d*x])^2))/((a - b)^2*(a + b)^2) - (2*(-24*(a^2 - 8*b^2)*(c
 + d*x) + (6*b*(-35*a^6 + 140*a^4*b^2 - 168*a^2*b^4 + 64*b^6)*ArcTanh[((-a
+ b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) - 96*a*b*Sin[c +
 d*x] + (a*b*(-5*a^4 + 20*a^2*b^2 - 16*b^4)*Sin[c + d*x])/((a - b)*(a + b)*
(b + a*Cos[c + d*x])^2) + (a*(10*a^6 - 115*a^4*b^2 + 220*a^2*b^4 - 112*b^6)
*Sin[c + d*x])/((a - b)^2*(a + b)^2*(b + a*Cos[c + d*x])) + 8*a^2*Sin[2*(c
+ d*x)]))/a^5 + ((12*b*(105*a^8 - 840*a^6*b^2 + 2016*a^4*b^4 - 1920*a^2*b^6
 + 640*b^8)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^
2)^(5/2) + (48*a^10*c - 960*a^8*b^2*c + 1776*a^6*b^4*c + 2976*a^4*b^6*c - 7
680*a^2*b^8*c + 3840*b^10*c + 48*a^10*d*x - 960*a^8*b^2*d*x + 1776*a^6*b^4*
d*x + 2976*a^4*b^6*d*x - 7680*a^2*b^8*d*x + 3840*b^10*d*x + 192*a*b*(a^2 -
b^2)^2*(a^4 - 20*a^2*b^2 + 40*b^4)*(c + d*x)*Cos[c + d*x] + 48*(a^3 - a*b^2
)^2*(a^4 - 20*a^2*b^2 + 40*b^4)*(c + d*x)*Cos[2*(c + d*x)] + 114*a^9*b*Sin[
c + d*x] + 788*a^7*b^3*Sin[c + d*x] - 5696*a^5*b^5*Sin[c + d*x] + 8640*a^3*
b^7*Sin[c + d*x] - 3840*a*b^9*Sin[c + d*x] - 36*a^10*Sin[2*(c + d*x)] + 122
1*a^8*b^2*Sin[2*(c + d*x)] - 5182*a^6*b^4*Sin[2*(c + d*x)] + 6880*a^4*b^6*S
in[2*(c + d*x)] - 2880*a^2*b^8*Sin[2*(c + d*x)] + 120*a^9*b*Sin[3*(c + d*x)
] - 560*a^7*b^3*Sin[3*(c + d*x)] + 760*a^5*b^5*Sin[3*(c + d*x)] - 320*a^3*b
^7*Sin[3*(c + d*x)] - 8*a^10*Sin[4*(c + d*x)] + 56*a^8*b^2*Sin[4*(c + d*x)]
 - 88*a^6*b^4*Sin[4*(c + d*x)] + 40*a^4*b^6*Sin[4*(c + d*x)] - 8*a^9*b*Sin[
5*(c + d*x)] + 16*a^7*b^3*Sin[5*(c + d*x)] - 8*a^5*b^5*Sin[5*(c + d*x)] + 2
*a^10*Sin[6*(c + d*x)] - 4*a^8*b^2*Sin[6*(c + d*x)] + 2*a^6*b^4*Sin[6*(c +
d*x)])/((a^2 - b^2)^2*(b + a*Cos[c + d*x])^2))/a^7)/(256*d)
```

Maple [A]

time = 0.32, size = 392, normalized size = 1.18 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*b/a^7*(((5/2*a^3*b^2-5*a*b^4-3*a^4*b+11/2*a^2*b^3)*tan(1/2*d*x+1/2*c
)^3+(5/2*a^3*b^2-5*a*b^4+3*a^4*b-11/2*a^2*b^3)*tan(1/2*d*x+1/2*c))/(a*tan(1
/2*d*x+1/2*c)^2-b*tan(1/2*d*x+1/2*c)^2-a-b)^2-3/2*(2*a^4-11*a^2*b^2+10*b^4)
/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
+2/a^7*(((3/8*a^4+3*b*a^3-3*b^2*a^2-10*b^3*a)*tan(1/2*d*x+1/2*c)^7+(13*b*a^
3-3*b^2*a^2-30*b^3*a+11/8*a^4)*tan(1/2*d*x+1/2*c)^5+(-11/8*a^4+3*b^2*a^2+13
*b*a^3-30*b^3*a)*tan(1/2*d*x+1/2*c)^3+(3*b*a^3-10*b^3*a-3/8*a^4+3*b^2*a^2)*
tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^4+3/8*(a^4-24*a^2*b^2+40*b^4)*
arctan(tan(1/2*d*x+1/2*c))))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 4.03, size = 1041, normalized size = 3.13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/8*(3*(a^8 - 25*a^6*b^2 + 64*a^4*b^4 - 40*a^2*b^6)*d*x*cos(d*x + c)^2 + 6
*(a^7*b - 25*a^5*b^3 + 64*a^3*b^5 - 40*a*b^7)*d*x*cos(d*x + c) + 3*(a^6*b^2
 - 25*a^4*b^4 + 64*a^2*b^6 - 40*b^8)*d*x + 6*(2*a^4*b^3 - 11*a^2*b^5 + 10*b
^7 + (2*a^6*b - 11*a^4*b^3 + 10*a^2*b^5)*cos(d*x + c)^2 + 2*(2*a^5*b^2 - 11
*a^3*b^4 + 10*a*b^6)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c)
- (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin
(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) +
 (52*a^5*b^3 - 172*a^3*b^5 + 120*a*b^7 + 2*(a^8 - a^6*b^2)*cos(d*x + c)^5 -
 4*(a^7*b - a^5*b^3)*cos(d*x + c)^4 - 5*(a^8 - 3*a^6*b^2 + 2*a^4*b^4)*cos(d
*x + c)^3 + 2*(11*a^7*b - 31*a^5*b^3 + 20*a^3*b^5)*cos(d*x + c)^2 + (83*a^6
*b^2 - 263*a^4*b^4 + 180*a^2*b^6)*cos(d*x + c))*sin(d*x + c))/((a^11 - a^9*
b^2)*d*cos(d*x + c)^2 + 2*(a^10*b - a^8*b^3)*d*cos(d*x + c) + (a^9*b^2 - a^
7*b^4)*d), 1/8*(3*(a^8 - 25*a^6*b^2 + 64*a^4*b^4 - 40*a^2*b^6)*d*x*cos(d*x
+ c)^2 + 6*(a^7*b - 25*a^5*b^3 + 64*a^3*b^5 - 40*a*b^7)*d*x*cos(d*x + c) +
3*(a^6*b^2 - 25*a^4*b^4 + 64*a^2*b^6 - 40*b^8)*d*x - 12*(2*a^4*b^3 - 11*a^2
*b^5 + 10*b^7 + (2*a^6*b - 11*a^4*b^3 + 10*a^2*b^5)*cos(d*x + c)^2 + 2*(2*a
^5*b^2 - 11*a^3*b^4 + 10*a*b^6)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt
(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (52*a^5*b^3
 - 172*a^3*b^5 + 120*a*b^7 + 2*(a^8 - a^6*b^2)*cos(d*x + c)^5 - 4*(a^7*b -
a^5*b^3)*cos(d*x + c)^4 - 5*(a^8 - 3*a^6*b^2 + 2*a^4*b^4)*cos(d*x + c)^3 +
2*(11*a^7*b - 31*a^5*b^3 + 20*a^3*b^5)*cos(d*x + c)^2 + (83*a^6*b^2 - 263*a
^4*b^4 + 180*a^2*b^6)*cos(d*x + c))*sin(d*x + c))/((a^11 - a^9*b^2)*d*cos(d
*x + c)^2 + 2*(a^10*b - a^8*b^3)*d*cos(d*x + c) + (a^9*b^2 - a^7*b^4)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral(sin(c + d*x)**4/(a + b*sec(c + d*x))**3, x)
```

Giac [A]
time = 0.58, size = 584, normalized size = 1.75

$$\frac{\frac{3\,(a^4-24\,a^2b^2+40\,b^4)(dx+c)}{a^7}-\frac{24\,(2\,a^4b-11\,a^2b^3+10\,b^5)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan(\frac{1}{2}dx+\frac{1}{2}c)-b\tan(\frac{1}{2}dx+\frac{1}{2}c)}{\sqrt{-a^2+b^2}}\right)\right)}{\sqrt{-a^2+b^2}\,a^7}-\frac{8\left(6a^3b^2\tan(\frac{1}{2}dx+\frac{1}{2}c)^3-5a^2b^3\tan(\frac{1}{2}dx+\frac{1}{2}c)^3-11ab^4\tan(\frac{1}{2}dx+\frac{1}{2}c)^3+10b^5\tan(\frac{1}{2}dx+\frac{1}{2}c)^3-6a^3b^2\tan(\frac{1}{2}dx+\frac{1}{2}c)-5a^2b^3\tan(\frac{1}{2}dx+\frac{1}{2}c)+11ab^4\tan(\frac{1}{2}dx+\frac{1}{2}c)+10b^5\tan(\frac{1}{2}dx+\frac{1}{2}c)\right)}{\left(a\tan(\frac{1}{2}dx+\frac{1}{2}c)^2-b\tan(\frac{1}{2}dx+\frac{1}{2}c)^2-a-b\right)^2a^6}+\frac{2\left(3a^3\tan(\frac{1}{2}dx+\frac{1}{2}c)^7+24a^2b\tan(\frac{1}{2}dx+\frac{1}{2}c)^7-24ab^2\tan(\frac{1}{2}dx+\frac{1}{2}c)^7-80b^3\tan(\frac{1}{2}dx+\frac{1}{2}c)^7+11a^3\tan(\frac{1}{2}dx+\frac{1}{2}c)^5+104a^2b\tan(\frac{1}{2}dx+\frac{1}{2}c)^5-24ab^2\tan(\frac{1}{2}dx+\frac{1}{2}c)^5-240b^3\tan(\frac{1}{2}dx+\frac{1}{2}c)^5-11a^3\tan(\frac{1}{2}dx+\frac{1}{2}c)^3+104a^2b\tan(\frac{1}{2}dx+\frac{1}{2}c)^3+24ab^2\tan(\frac{1}{2}dx+\frac{1}{2}c)^3-240b^3\tan(\frac{1}{2}dx+\frac{1}{2}c)^3-3a^3\tan(\frac{1}{2}dx+\frac{1}{2}c)+24a^2b\tan(\frac{1}{2}dx+\frac{1}{2}c)+24ab^2\tan(\frac{1}{2}dx+\frac{1}{2}c)-80b^3\tan(\frac{1}{2}dx+\frac{1}{2}c)\right)}{\left(\tan(\frac{1}{2}dx+\frac{1}{2}c)^2+1\right)^4a^6}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*(a^4 - 24*a^2*b^2 + 40*b^4)*(d*x + c)/a^7 - 24*(2*a^4*b - 11*a^2*b^3
 + 10*b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*t
an(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2
 + b^2)*a^7) - 8*(6*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 5*a^2*b^3*tan(1/2*d*x
+ 1/2*c)^3 - 11*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 10*b^5*tan(1/2*d*x + 1/2*c)^
3 - 6*a^3*b^2*tan(1/2*d*x + 1/2*c) - 5*a^2*b^3*tan(1/2*d*x + 1/2*c) + 11*a*
b^4*tan(1/2*d*x + 1/2*c) + 10*b^5*tan(1/2*d*x + 1/2*c))/((a*tan(1/2*d*x + 1
/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2*a^6) + 2*(3*a^3*tan(1/2*d*x +
 1/2*c)^7 + 24*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 24*a*b^2*tan(1/2*d*x + 1/2*c)
^7 - 80*b^3*tan(1/2*d*x + 1/2*c)^7 + 11*a^3*tan(1/2*d*x + 1/2*c)^5 + 104*a^
2*b*tan(1/2*d*x + 1/2*c)^5 - 24*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 240*b^3*tan(
1/2*d*x + 1/2*c)^5 - 11*a^3*tan(1/2*d*x + 1/2*c)^3 + 104*a^2*b*tan(1/2*d*x
+ 1/2*c)^3 + 24*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 240*b^3*tan(1/2*d*x + 1/2*c)
^3 - 3*a^3*tan(1/2*d*x + 1/2*c) + 24*a^2*b*tan(1/2*d*x + 1/2*c) + 24*a*b^2*
tan(1/2*d*x + 1/2*c) - 80*b^3*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^
2 + 1)^4*a^6))/d
```

Mupad [B]
time = 5.76, size = 2500, normalized size = 7.51

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(a + b/cos(c + d*x))^3,x)
```

```
[Out] (atan(((((3*((108*a^19*b - 12*a^20 - 480*a^14*b^6 + 720*a^15*b^5 + 288*a^16
*b^4 - 732*a^17*b^3 + 108*a^18*b^2)/a^18 - (3*tan(c/2 + (d*x)/2)*(a^4*1i +
```

b^4*40i - a^2*b^2*24i)*(128*a^16*b + 128*a^14*b^3 - 256*a^15*b^2))/(16*a^19
))*(a^4*1i + b^4*40i - a^2*b^2*24i))/(8*a^7) + (tan(c/2 + (d*x)/2)*(57600*a
*b^10 - 27*a^10*b + 9*a^11 - 28800*b^11 + 5760*a^2*b^9 - 69120*a^3*b^8 + 22
752*a^4*b^7 + 23616*a^5*b^6 - 10944*a^6*b^5 - 1728*a^7*b^4 + 711*a^8*b^3 +
171*a^9*b^2))/(2*a^12))*(a^4*1i + b^4*40i - a^2*b^2*24i)*3i)/(8*a^7) - (((3
*((108*a^19*b - 12*a^20 - 480*a^14*b^6 + 720*a^15*b^5 + 288*a^16*b^4 - 732*
a^17*b^3 + 108*a^18*b^2)/a^18 + (3*tan(c/2 + (d*x)/2)*(a^4*1i + b^4*40i - a
^2*b^2*24i)*(128*a^16*b + 128*a^14*b^3 - 256*a^15*b^2))/(16*a^19))*(a^4*1i
+ b^4*40i - a^2*b^2*24i))/(8*a^7) - (tan(c/2 + (d*x)/2)*(57600*a*b^10 - 27*
a^10*b + 9*a^11 - 28800*b^11 + 5760*a^2*b^9 - 69120*a^3*b^8 + 22752*a^4*b^7
 + 23616*a^5*b^6 - 10944*a^6*b^5 - 1728*a^7*b^4 + 711*a^8*b^3 + 171*a^9*b^2
))/(2*a^12))*(a^4*1i + b^4*40i - a^2*b^2*24i)*3i)/(8*a^7))/((324000*a*b^13
+ 27*a^13*b - 216000*b^14 + 388800*a^2*b^12 - 718200*a^3*b^11 - 195480*a^4*
b^10 + 576720*a^5*b^9 - 4104*a^6*b^8 - 205119*a^7*b^7 + 24408*a^8*b^6 + (62
181*a^9*b^5)/2 - 4671*a^10*b^4 - (3267*a^11*b^3)/2 + 162*a^12*b^2)/a^18 + (
3*((3*((108*a^19*b - 12*a^20 - 480*a^14*b^6 + 720*a^15*b^5 + 288*a^16*b^4 -
 732*a^17*b^3 + 108*a^18*b^2)/a^18 - (3*tan(c/2 + (d*x)/2)*(a^4*1i + b^4*40
i - a^2*b^2*24i)*(128*a^16*b + 128*a^14*b^3 - 256*a^15*b^2))/(16*a^19))*(a^
4*1i + b^4*40i - a^2*b^2*24i))/(8*a^7) + (tan(c/2 + (d*x)/2)*(57600*a*b^10
- 27*a^10*b + 9*a^11 - 28800*b^11 + 5760*a^2*b^9 - 69120*a^3*b^8 + 22752*a^
4*b^7 + 23616*a^5*b^6 - 10944*a^6*b^5 - 1728*a^7*b^4 + 711*a^8*b^3 + 171*a^
9*b^2))/(2*a^12))*(a^4*1i + b^4*40i - a^2*b^2*24i))/(8*a^7) + (3*((3*((108*
a^19*b - 12*a^20 - 480*a^14*b^6 + 720*a^15*b^5 + 288*a^16*b^4 - 732*a^17*b^
3 + 108*a^18*b^2)/a^18 + (3*tan(c/2 + (d*x)/2)*(a^4*1i + b^4*40i - a^2*b^2*
24i)*(128*a^16*b + 128*a^14*b^3 - 256*a^15*b^2))/(16*a^19))*(a^4*1i + b^4*4
0i - a^2*b^2*24i))/(8*a^7) - (tan(c/2 + (d*x)/2)*(57600*a*b^10 - 27*a^10*b
+ 9*a^11 - 28800*b^11 + 5760*a^2*b^9 - 69120*a^3*b^8 + 22752*a^4*b^7 + 2361
6*a^5*b^6 - 10944*a^6*b^5 - 1728*a^7*b^4 + 711*a^8*b^3 + 171*a^9*b^2))/(2*a
^12))*(a^4*1i + b^4*40i - a^2*b^2*24i))/(8*a^7)))*(a^4*1i + b^4*40i - a^2*b
^2*24i)*3i)/(4*a^7*d) - ((tan(c/2 + (d*x)/2)^5*(180*a*b^4 + 26*a^4*b - 15*a
^5 + 600*b^5 - 300*a^2*b^3 - 73*a^3*b^2))/(2*a^6) - (3*tan(c/2 + (d*x)/2)^1
1*(60*a*b^4 + 6*a^4*b + a^5 - 40*b^5 + 4*a^2*b^3 - 31*a^3*b^2))/(4*a^6) + (
tan(c/2 + (d*x)/2)^7*(26*a^4*b - 180*a*b^4 + 15*a^5 + 600*b^5 - 300*a^2*b^3
 + 73*a^3*b^2))/(2*a^6) + (tan(c/2 + (d*x)/2)^3*(540*a*b^4 - 34*a^4*b + 5*a
^5 + 600*b^5 - 220*a^2*b^3 - 239*a^3*b^2))/(4*a^6) - (tan(c/2 + (d*x)/2)^9*
(540*a*b^4 + 34*a^4*b + 5*a^5 - 600*b^5 + 220*a^2*b^3 - 239*a^3*b^2))/(4*a^
6) + (3*tan(c/2 + (d*x)/2)*(a + b)*(20*a*b^3 - 7*a^3*b + a^4 + 40*b^4 - 24*
a^2*b^2))/(4*a^6))/(d*(2*a*b - tan(c/2 + (d*x)/2)^6*(4*a^2 - 20*b^2) + tan(
c/2 + (d*x)/2)^2*(8*a*b + 2*a^2 + 6*b^2) + tan(c/2 + (d*x)/2)^10*(2*a^2 - 8
*a*b + 6*b^2) + tan(c/2 + (d*x)/2)^4*(10*a*b - a^2 + 15*b^2) + tan(c/2 + (d
x)/2)^12(a^2 - 2*a*b + b^2) + a^2 + b^2 - tan(c/2 + (d*x)/2)^8*(10*a*b +
a^2 - 15*b^2))) + (b*atan(((b*((a + b)*(a - b))^(1/2)*((tan(c/2 + (d*x)/2)*
(57600*a*b^10 - 27*a^10*b + 9*a^11 - 28800*b^11 + 5760*a^2*b^9 - 69120*a^3*
b^8 + 22752*a^4*b^7 + 23616*a^5*b^6 - 10944*a^6*b^5 - 1728*a^7*b^4 + 711*a^
8*b^3 + 171*a^9*b^2))/(2*a^12) + (3*b*((a + b)*(a - b))^(1/2)*((108*a^19*b

```
- 12*a^20 - 480*a^14*b^6 + 720*a^15*b^5 + 288*a^16*b^4 - 732*a^17*b^3 + 108
*a^18*b^2)/a^18 - (3*b*tan(c/2 + (d*x)/2)*((a + b)*(a - b))^(1/2)*(2*a^4 +
10*b^4 - 11*a^2*b^2)*(128*a^16*b + 128*a^14*b^3 - 256*a^15*b^2))/(4*a^12*(a
^9 - a^7*b^2)))*(2*a^4 + 10*b^4 - 11*a^2*b^2))/(2*(a^9 - a^7*b^2)))*(2*a^4
+ 10*b^4 - 11*a^2*b^2)*3i)/(2*(a^9 - a^7*b^2)) + (b*((a + b)*(a - b))^(1/2)
*((tan(c/2 + (d*x)/2)*(57600*a*b^10 - 27*a^10*b + 9*a^11 - 28800*b^11 + 576
0*a^2*b^9 - 69120*a^3*b^8 + 22752*a^4*b^7 + 23616*a^5*b^6 - 10944*a^6*b^5 -
 1728*a^7*b^4 + 711*a^8*b^3 + 171*a^9*b^2))/(2*a^12) - (3*b*((a + b)*(a - b
))^(1/2)*((108*a^19*b - 12*a^20 - 480*a^14*b^6 + 720*a^15*b^5 + 288*a^16*b^
4 - 732*a^17*b^3 + 108*a^18*b^2)/a^18 + (3*b*tan(c/2 + (d*x)/2)*((a + b)*(a
 - b))^(1/2)*(2*a^4 + 10*b^4 - 11*a^2*b^2)*(128*a^16*b + 128*a^14*b^3 - 256
*a^15*b^2))/(4*a^12*(a^9 - a^7*b^2)))*(2*a^4 + 10*b^4 - 11*a^2*b^2))/(2*(a^
9 - a^7*b^2)))*(2*a^4 + 10*b^4 - 11*a^2*b^2)*3i)/(2*(a^9 - a^7*b^2)))/((324
000*a*b^13 + 27*a^13*b - 216000*b^14 + 388800*a^2*b^12 - 718200*a^3*b^11 -
195480*a^4*b^10 + 576720*a^5*b^9 - 4104*a^6*b^8 - 205119*a^7*b^7 + 24408*a^
8*b^6 + (62181*a^9*b^5)/2 - 4671*a^10*b^4 - (3267*a^11*b^3)/2 + 162*a^12*b^
2)/a^18 + (3*b*((a + b)*(a - b))^(1/2)*((tan(c/2 + (d*x)/2)*(57600*a*b^10 -
 27*a^10*b + 9*a^11 - 28800*b^11 + 5760*a^2*b^9 - 69120*a^3*b^8 + 22752*a^4
*b^7 + 23616*a^5*b^6 - 10944*a^6*b^5 - 1728*a^7*b^4 + 711*a^8*b^3 + 171*a^9
*b^2))/(2*a^12) + (3*b*((a + b)*(a - b))^(1/2)*...
```

3.230 $\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=267

$$\frac{(a^2-12b^2)\,x}{2a^5}-\frac{b(6a^4-19a^2b^2+12b^4)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5(a-b)^{3/2}(a+b)^{3/2}d}+\frac{b(11a^2-12b^2)\sin(c+dx)}{2a^4\,(a^2-b^2)\,d}-\frac{(5a^2-6\ldots}{}$$

```
[Out] 1/2*(a^2-12*b^2)*x/a^5-b*(6*a^4-19*a^2*b^2+12*b^4)*arctanh((a-b)^(1/2)*tan(
1/2*d*x+1/2*c)/(a+b)^(1/2))/a^5/(a-b)^(3/2)/(a+b)^(3/2)/d+1/2*b*(11*a^2-12*
b^2)*sin(d*x+c)/a^4/(a^2-b^2)/d-1/2*(5*a^2-6*b^2)*cos(d*x+c)*sin(d*x+c)/a^3
/(a^2-b^2)/d+1/2*cos(d*x+c)^3*sin(d*x+c)/a/d/(b+a*cos(d*x+c))^2+1/2*(3*a^2-
4*b^2)*cos(d*x+c)^2*sin(d*x+c)/a^2/(a^2-b^2)/d/(b+a*cos(d*x+c))
```

Rubi [A]
time = 0.65, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3957, 2968, 3127, 3128, 3102, 2814, 2738, 214}

$$\frac{(3a^2-4b^2)\sin(c+dx)\cos^2(c+dx)}{2a^2d\,(a^2-b^2)\,(a\cos(c+dx)+b)}+\frac{x(a^2-12b^2)}{2a^5}+\frac{b(11a^2-12b^2)\sin(c+dx)}{2a^4d\,(a^2-b^2)}-\frac{(5a^2-6b^2)\sin(c+dx)\cos(c+dx)}{2a^3d\,(a^2-b^2)}-\frac{b(6a^4-19a^2b^2+12b^4)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d(a-b)^{3/2}(a+b)^{3/2}}+\frac{\sin(c+dx)\cos^3(c+dx)}{2ad(a\cos(c+dx)+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((a^2 - 12*b^2)*x)/(2*a^5) - (b*(6*a^4 - 19*a^2*b^2 + 12*b^4)*ArcTanh[(Sqrt
[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^5*(a - b)^(3/2)*(a + b)^(3/2)*d)
 + (b*(11*a^2 - 12*b^2)*Sin[c + d*x])/(2*a^4*(a^2 - b^2)*d) - ((5*a^2 - 6*b
^2)*Cos[c + d*x]*Sin[c + d*x])/(2*a^3*(a^2 - b^2)*d) + (Cos[c + d*x]^3*Sin[
c + d*x])/(2*a*d*(b + a*Cos[c + d*x])^2) + ((3*a^2 - 4*b^2)*Cos[c + d*x]^2*
Sin[c + d*x])/(2*a^2*(a^2 - b^2)*d*(b + a*Cos[c + d*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3127

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d
*(b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*
c*(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A
*d^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx &= -\int \frac{\cos^3(c+dx)\sin^2(c+dx)}{(-b-a\cos(c+dx))^3}\,dx\\
&= -\int \frac{\cos^3(c+dx)\left(1-\cos^2(c+dx)\right)}{(-b-a\cos(c+dx))^3}\,dx\\
&= \frac{\cos^3(c+dx)\sin(c+dx)}{2ad(b+a\cos(c+dx))^2} + \frac{\int \frac{\cos^2(c+dx)\left(3\left(a^2-b^2\right)-4\left(a^2-b^2\right)\cos^2(c+dx)\right)}{(-b-a\cos(c+dx))^2}\,dx}{2a\left(a^2-b^2\right)}\\
&= \frac{\cos^3(c+dx)\sin(c+dx)}{2ad(b+a\cos(c+dx))^2} + \frac{\left(3a^2-4b^2\right)\cos^2(c+dx)\sin(c+dx)}{2a^2\left(a^2-b^2\right)d(b+a\cos(c+dx))} - \frac{\int \frac{\cos(c+dx)(2(3}{}}{}\\
&= -\frac{\left(5a^2-6b^2\right)\cos(c+dx)\sin(c+dx)}{2a^3\left(a^2-b^2\right)d} + \frac{\cos^3(c+dx)\sin(c+dx)}{2ad(b+a\cos(c+dx))^2} + \frac{\left(3a^2-4b^2\right)}{2a^2\left(a^2-b\right.}\\
&= \frac{b\left(11a^2-12b^2\right)\sin(c+dx)}{2a^4\left(a^2-b^2\right)d} - \frac{\left(5a^2-6b^2\right)\cos(c+dx)\sin(c+dx)}{2a^3\left(a^2-b^2\right)d} + \frac{\cos^3(c+dx}{2ad(b+a\,\mathrm{c}}\\
&= \frac{\left(a^2-12b^2\right)x}{2a^5} + \frac{b\left(11a^2-12b^2\right)\sin(c+dx)}{2a^4\left(a^2-b^2\right)d} - \frac{\left(5a^2-6b^2\right)\cos(c+dx)\sin(c+dx}{2a^3\left(a^2-b^2\right)d}\\
&= \frac{\left(a^2-12b^2\right)x}{2a^5} + \frac{b\left(11a^2-12b^2\right)\sin(c+dx)}{2a^4\left(a^2-b^2\right)d} - \frac{\left(5a^2-6b^2\right)\cos(c+dx)\sin(c+dx}{2a^3\left(a^2-b^2\right)d}\\
&= \frac{\left(a^2-12b^2\right)x}{2a^5} - \frac{b\left(6a^4-19a^2b^2+12b^4\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5(a-b)^{3/2}(a+b)^{3/2}d} + \frac{b(11a^2}{}
\end{aligned}$$

Mathematica [A]
time = 2.68, size = 282, normalized size = 1.06

$$\frac{\frac{4b\left(6a^4-19a^2b^2+12b^4\right)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{4ab\left(a^4-13a^2b^2+12b^4\right)(c+dx)\cos(c+dx)-2a^4\left(a^2-b^2\right)\cos^3(c+dx)\sin(c+dx)+2a^2\left(a^2-b^2\right)\cos^2(c+dx)\left(\left(a^2-12b^2\right)(c+dx)+4ab\sin(c+dx)\right)+b^2\left(2\left(a^4-13a^2b^2+12b^4\right)(c+dx)+\left(22a^3b-24ab^3\right)\sin(c+dx)+\left(17a^4-18a^2b^2\right)\sin(2(c+dx))\right)}{(b+a\cos(c+dx))^2}}{4a^5(a-b)(a+b)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((4*b*(6*a^4 - 19*a^2*b^2 + 12*b^4)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqr
t[a^2 - b^2]])/Sqrt[a^2 - b^2] + (4*a*b*(a^4 - 13*a^2*b^2 + 12*b^4)*(c + d*
```

```
x)*Cos[c + d*x] - 2*a^4*(a^2 - b^2)*Cos[c + d*x]^3*Sin[c + d*x] + 2*a^2*(a^
2 - b^2)*Cos[c + d*x]^2*((a^2 - 12*b^2)*(c + d*x) + 4*a*b*Sin[c + d*x]) + b
^2*(2*(a^4 - 13*a^2*b^2 + 12*b^4)*(c + d*x) + (22*a^3*b - 24*a*b^3)*Sin[c +
 d*x] + (17*a^4 - 18*a^2*b^2)*Sin[2*(c + d*x)]))/(b + a*Cos[c + d*x])^2)/(4
*a^5*(a - b)*(a + b)*d)
```

Maple [A]
time = 0.25, size = 276, normalized size = 1.03

method	result
derivativedivides	$\frac{\frac{2b\left(\frac{-\frac{\left(6a^2+ba-6b^2\right)ba\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a+b)}+\frac{\left(6a^2-ba-6b^2\right)ba\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a-2b}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)^2}-\frac{\left(6a^4-19b^2a^2+12b^4\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-}\right.}{2\left(a^2-b^2\right)\sqrt{(a+b)(a-b)}}\right.}{a^5}}{d}$
default	$\frac{\frac{2b\left(\frac{-\frac{\left(6a^2+ba-6b^2\right)ba\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a+b)}+\frac{\left(6a^2-ba-6b^2\right)ba\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a-2b}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)^2}-\frac{\left(6a^4-19b^2a^2+12b^4\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-}\right.}{2\left(a^2-b^2\right)\sqrt{(a+b)(a-b)}}\right.}{a^5}}{d}$
risch	$\frac{x}{2a^3}-\frac{6xb^2}{a^5}+\frac{i\mathrm{e}^{2i(dx+c)}}{8a^3d}-\frac{3ib\,\mathrm{e}^{i(dx+c)}}{2a^4d}+\frac{3ib\,\mathrm{e}^{-i(dx+c)}}{2a^4d}-\frac{i\mathrm{e}^{-2i(dx+c)}}{8a^3d}+\frac{ib^2\left(-7b\,a^3\mathrm{e}^{3i(dx+c)}+8a\,b^3\mathrm{e}^{3i(dx+c}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*b/a^5*((-1/2*(6*a^2+a*b-6*b^2)*b*a/(a+b)*tan(1/2*d*x+1/2*c)^3+1/2*(6
*a^2-a*b-6*b^2)*b*a/(a-b)*tan(1/2*d*x+1/2*c))/(a*tan(1/2*d*x+1/2*c)^2-b*tan
(1/2*d*x+1/2*c)^2-a-b)^2-1/2*(6*a^4-19*a^2*b^2+12*b^4)/(a^2-b^2)/((a+b)*(a-
b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))+2/a^5*(((1
/2*a^2+3*b*a)*tan(1/2*d*x+1/2*c)^3+(3*b*a-1/2*a^2)*tan(1/2*d*x+1/2*c))/(1+t
an(1/2*d*x+1/2*c)^2)^2+1/2*(a^2-12*b^2)*arctan(tan(1/2*d*x+1/2*c))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 6.01, size = 984, normalized size = 3.69

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a^8 - 14*a^6*b^2 + 25*a^4*b^4 - 12*a^2*b^6)*d*x*cos(d*x + c)^2 + 4
*(a^7*b - 14*a^5*b^3 + 25*a^3*b^5 - 12*a*b^7)*d*x*cos(d*x + c) + 2*(a^6*b^2
 - 14*a^4*b^4 + 25*a^2*b^6 - 12*b^8)*d*x - (6*a^4*b^3 - 19*a^2*b^5 + 12*b^7
 + (6*a^6*b - 19*a^4*b^3 + 12*a^2*b^5)*cos(d*x + c)^2 + 2*(6*a^5*b^2 - 19*a
^3*b^4 + 12*a*b^6)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) -
(a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d
*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2
*(11*a^5*b^3 - 23*a^3*b^5 + 12*a*b^7 - (a^8 - 2*a^6*b^2 + a^4*b^4)*cos(d*x
+ c)^3 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*cos(d*x + c)^2 + (17*a^6*b^2 - 35*
a^4*b^4 + 18*a^2*b^6)*cos(d*x + c))*sin(d*x + c))/((a^11 - 2*a^9*b^2 + a^7*
b^4)*d*cos(d*x + c)^2 + 2*(a^10*b - 2*a^8*b^3 + a^6*b^5)*d*cos(d*x + c) + (
a^9*b^2 - 2*a^7*b^4 + a^5*b^6)*d), 1/2*((a^8 - 14*a^6*b^2 + 25*a^4*b^4 - 12
*a^2*b^6)*d*x*cos(d*x + c)^2 + 2*(a^7*b - 14*a^5*b^3 + 25*a^3*b^5 - 12*a*b^
7)*d*x*cos(d*x + c) + (a^6*b^2 - 14*a^4*b^4 + 25*a^2*b^6 - 12*b^8)*d*x - (6
*a^4*b^3 - 19*a^2*b^5 + 12*b^7 + (6*a^6*b - 19*a^4*b^3 + 12*a^2*b^5)*cos(d*
x + c)^2 + 2*(6*a^5*b^2 - 19*a^3*b^4 + 12*a*b^6)*cos(d*x + c))*sqrt(-a^2 +
b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c
))) + (11*a^5*b^3 - 23*a^3*b^5 + 12*a*b^7 - (a^8 - 2*a^6*b^2 + a^4*b^4)*cos
(d*x + c)^3 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*cos(d*x + c)^2 + (17*a^6*b^2
- 35*a^4*b^4 + 18*a^2*b^6)*cos(d*x + c))*sin(d*x + c))/((a^11 - 2*a^9*b^2 +
 a^7*b^4)*d*cos(d*x + c)^2 + 2*(a^10*b - 2*a^8*b^3 + a^6*b^5)*d*cos(d*x + c
) + (a^9*b^2 - 2*a^7*b^4 + a^5*b^6)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral(sin(c + d*x)**2/(a + b*sec(c + d*x))**3, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1193 vs. $2(248) = 496$.
time = 0.64, size = 1193, normalized size = 4.47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*((a^11 - 7*a^10*b - 14*a^9*b^2 + 39*a^8*b^3 + 25*a^7*b^4 - 56*a^6*b^5 -
 12*a^5*b^6 + 24*a^4*b^7 - a^4*abs(-a^7 + a^5*b^2) - 5*a^3*b*abs(-a^7 + a^5
*b^2) + 13*a^2*b^2*abs(-a^7 + a^5*b^2) + 6*a*b^3*abs(-a^7 + a^5*b^2) - 12*b
^4*abs(-a^7 + a^5*b^2))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(tan(1/2*
d*x + 1/2*c)/sqrt(-(a^6*b - a^4*b^3 + sqrt((a^7 + a^6*b - a^5*b^2 - a^4*b^3
)*(a^7 - a^6*b - a^5*b^2 + a^4*b^3) + (a^6*b - a^4*b^3)^2))/(a^7 - a^6*b -
a^5*b^2 + a^4*b^3))))/(a^6*b*abs(-a^7 + a^5*b^2) - a^4*b^3*abs(-a^7 + a^5*b
^2) + (a^7 - a^5*b^2)^2) + ((a^4 + 5*a^3*b - 13*a^2*b^2 - 6*a*b^3 + 12*b^4)
*sqrt(-a^2 + b^2)*abs(-a^7 + a^5*b^2)*abs(-a + b) + (a^11 - 7*a^10*b - 14*a
^9*b^2 + 39*a^8*b^3 + 25*a^7*b^4 - 56*a^6*b^5 - 12*a^5*b^6 + 24*a^4*b^7)*sq
rt(-a^2 + b^2)*abs(-a + b))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(tan(
1/2*d*x + 1/2*c)/sqrt(-(a^6*b - a^4*b^3 - sqrt((a^7 + a^6*b - a^5*b^2 - a^4
*b^3)*(a^7 - a^6*b - a^5*b^2 + a^4*b^3) + (a^6*b - a^4*b^3)^2))/(a^7 - a^6*
b - a^5*b^2 + a^4*b^3))))/((a^7 - a^5*b^2)^2*(a^2 - 2*a*b + b^2) - (a^8*b -
 2*a^7*b^2 + 2*a^5*b^4 - a^4*b^5)*abs(-a^7 + a^5*b^2)) + 2*(a^5*tan(1/2*d*x
 + 1/2*c)^7 + 4*a^4*b*tan(1/2*d*x + 1/2*c)^7 - 18*a^3*b^2*tan(1/2*d*x + 1/2
*c)^7 + 7*a^2*b^3*tan(1/2*d*x + 1/2*c)^7 + 18*a*b^4*tan(1/2*d*x + 1/2*c)^7
- 12*b^5*tan(1/2*d*x + 1/2*c)^7 - 3*a^5*tan(1/2*d*x + 1/2*c)^5 - 4*a^4*b*ta
n(1/2*d*x + 1/2*c)^5 - 14*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 + 37*a^2*b^3*tan(1
/2*d*x + 1/2*c)^5 + 18*a*b^4*tan(1/2*d*x + 1/2*c)^5 - 36*b^5*tan(1/2*d*x +
1/2*c)^5 + 3*a^5*tan(1/2*d*x + 1/2*c)^3 - 4*a^4*b*tan(1/2*d*x + 1/2*c)^3 +
14*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + 37*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 - 18*
a*b^4*tan(1/2*d*x + 1/2*c)^3 - 36*b^5*tan(1/2*d*x + 1/2*c)^3 - a^5*tan(1/2*
d*x + 1/2*c) + 4*a^4*b*tan(1/2*d*x + 1/2*c) + 18*a^3*b^2*tan(1/2*d*x + 1/2*
c) + 7*a^2*b^3*tan(1/2*d*x + 1/2*c) - 18*a*b^4*tan(1/2*d*x + 1/2*c) - 12*b^
5*tan(1/2*d*x + 1/2*c))/((a^6 - a^4*b^2)*(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(
1/2*d*x + 1/2*c)^4 - 2*b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2))/d
```

Mupad [**B**]
time = 9.10, size = 2500, normalized size = 9.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(a + b/cos(c + d*x))^3,x)
```

[Out] ((tan(c/2 + (d*x)/2)*(6*a*b^3 - 5*a^3*b + a^4 + 12*b^4 - 13*a^2*b^2))/(a^4*
b - a^5) + (tan(c/2 + (d*x)/2)^3*(18*a*b^4 + 4*a^4*b - 3*a^5 + 36*b^5 - 37*
a^2*b^3 - 14*a^3*b^2))/((a^4*b - a^5)*(a + b)) + (tan(c/2 + (d*x)/2)^5*(4*a
^4*b - 18*a*b^4 + 3*a^5 + 36*b^5 - 37*a^2*b^3 + 14*a^3*b^2))/((a^4*b - a^5)
*(a + b)) + (tan(c/2 + (d*x)/2)^7*(5*a^3*b - 6*a*b^3 + a^4 + 12*b^4 - 13*a^
2*b^2))/(a^4*(a + b)))/(d*(2*a*b - tan(c/2 + (d*x)/2)^4*(2*a^2 - 6*b^2) + t
an(c/2 + (d*x)/2)^2*(4*a*b + 4*b^2) - tan(c/2 + (d*x)/2)^6*(4*a*b - 4*b^2)
+ tan(c/2 + (d*x)/2)^8*(a^2 - 2*a*b + b^2) + a^2 + b^2)) + (atan((((a^2*1i
- b^2*12i)*((((4*(24*a^16*b - 4*a^17 - 48*a^10*b^7 + 24*a^11*b^6 + 124*a^12
*b^5 - 56*a^13*b^4 - 100*a^14*b^3 + 36*a^15*b^2))/(a^14*b + a^15 - a^12*b^3
 - a^13*b^2) - (4*tan(c/2 + (d*x)/2)*(a^2*1i - b^2*12i)*(8*a^15*b - 8*a^10*
b^6 + 8*a^11*b^5 + 16*a^12*b^4 - 16*a^13*b^3 - 8*a^14*b^2))/(a^5*(a^10*b +
a^11 - a^8*b^3 - a^9*b^2)))*(a^2*1i - b^2*12i))/(2*a^5) + (8*tan(c/2 + (d*x
)/2)*(a^10 - 2*a^9*b - 288*a*b^9 + 288*b^10 - 624*a^2*b^8 + 624*a^3*b^7 + 3
86*a^4*b^6 - 386*a^5*b^5 - 61*a^6*b^4 + 52*a^7*b^3 + 11*a^8*b^2))/(a^10*b +
 a^11 - a^8*b^3 - a^9*b^2))*1i)/(2*a^5) - ((a^2*1i - b^2*12i)*((((4*(24*a^1
6*b - 4*a^17 - 48*a^10*b^7 + 24*a^11*b^6 + 124*a^12*b^5 - 56*a^13*b^4 - 100
*a^14*b^3 + 36*a^15*b^2))/(a^14*b + a^15 - a^12*b^3 - a^13*b^2) + (4*tan(c/
2 + (d*x)/2)*(a^2*1i - b^2*12i)*(8*a^15*b - 8*a^10*b^6 + 8*a^11*b^5 + 16*a^
12*b^4 - 16*a^13*b^3 - 8*a^14*b^2))/(a^5*(a^10*b + a^11 - a^8*b^3 - a^9*b^2
)))*(a^2*1i - b^2*12i))/(2*a^5) - (8*tan(c/2 + (d*x)/2)*(a^10 - 2*a^9*b - 2
88*a*b^9 + 288*b^10 - 624*a^2*b^8 + 624*a^3*b^7 + 386*a^4*b^6 - 386*a^5*b^5
 - 61*a^6*b^4 + 52*a^7*b^3 + 11*a^8*b^2))/(a^10*b + a^11 - a^8*b^3 - a^9*b^
2))*1i)/(2*a^5))/((8*(864*a*b^10 + 6*a^10*b - 1728*b^11 + 4752*a^2*b^9 - 21
60*a^3*b^8 - 4356*a^4*b^7 + 1746*a^5*b^6 + 1495*a^6*b^5 - 491*a^7*b^4 - 169
*a^8*b^3 + 30*a^9*b^2))/(a^14*b + a^15 - a^12*b^3 - a^13*b^2) + ((a^2*1i -
b^2*12i)*((((4*(24*a^16*b - 4*a^17 - 48*a^10*b^7 + 24*a^11*b^6 + 124*a^12*b
^5 - 56*a^13*b^4 - 100*a^14*b^3 + 36*a^15*b^2))/(a^14*b + a^15 - a^12*b^3 -
 a^13*b^2) - (4*tan(c/2 + (d*x)/2)*(a^2*1i - b^2*12i)*(8*a^15*b - 8*a^10*b^
6 + 8*a^11*b^5 + 16*a^12*b^4 - 16*a^13*b^3 - 8*a^14*b^2))/(a^5*(a^10*b + a^
11 - a^8*b^3 - a^9*b^2)))*(a^2*1i - b^2*12i))/(2*a^5) + (8*tan(c/2 + (d*x)/
2)*(a^10 - 2*a^9*b - 288*a*b^9 + 288*b^10 - 624*a^2*b^8 + 624*a^3*b^7 + 386
*a^4*b^6 - 386*a^5*b^5 - 61*a^6*b^4 + 52*a^7*b^3 + 11*a^8*b^2))/(a^10*b + a
^11 - a^8*b^3 - a^9*b^2)))/(2*a^5) + ((a^2*1i - b^2*12i)*((((4*(24*a^16*b -
 4*a^17 - 48*a^10*b^7 + 24*a^11*b^6 + 124*a^12*b^5 - 56*a^13*b^4 - 100*a^14
*b^3 + 36*a^15*b^2))/(a^14*b + a^15 - a^12*b^3 - a^13*b^2) + (4*tan(c/2 + (
d*x)/2)*(a^2*1i - b^2*12i)*(8*a^15*b - 8*a^10*b^6 + 8*a^11*b^5 + 16*a^12*b^
4 - 16*a^13*b^3 - 8*a^14*b^2))/(a^5*(a^10*b + a^11 - a^8*b^3 - a^9*b^2)))*(
a^2*1i - b^2*12i))/(2*a^5) - (8*tan(c/2 + (d*x)/2)*(a^10 - 2*a^9*b - 288*a*
b^9 + 288*b^10 - 624*a^2*b^8 + 624*a^3*b^7 + 386*a^4*b^6 - 386*a^5*b^5 - 61
*a^6*b^4 + 52*a^7*b^3 + 11*a^8*b^2))/(a^10*b + a^11 - a^8*b^3 - a^9*b^2)))/
(2*a^5)))*(a^2*1i - b^2*12i)*1i)/(a^5*d) + (b*atan(((b*((8*tan(c/2 + (d*x)/
2)*(a^10 - 2*a^9*b - 288*a*b^9 + 288*b^10 - 624*a^2*b^8 + 624*a^3*b^7 + 386
*a^4*b^6 - 386*a^5*b^5 - 61*a^6*b^4 + 52*a^7*b^3 + 11*a^8*b^2))/(a^10*b + a
^11 - a^8*b^3 - a^9*b^2) + (b*((4*(24*a^16*b - 4*a^17 - 48*a^10*b^7 + 24*a^

```
11*b^6 + 124*a^12*b^5 - 56*a^13*b^4 - 100*a^14*b^3 + 36*a^15*b^2))/(a^14*b
+ a^15 - a^12*b^3 - a^13*b^2) - (4*b*tan(c/2 + (d*x)/2)*((a + b)^3*(a - b)^
3)^(1/2)*(6*a^4 + 12*b^4 - 19*a^2*b^2)*(8*a^15*b - 8*a^10*b^6 + 8*a^11*b^5
+ 16*a^12*b^4 - 16*a^13*b^3 - 8*a^14*b^2))/((a^10*b + a^11 - a^8*b^3 - a^9*
b^2)*(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2)))*((a + b)^3*(a - b)^3)^(1/2)
*(6*a^4 + 12*b^4 - 19*a^2*b^2))/(2*(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2)
))*((a + b)^3*(a - b)^3)^(1/2)*(6*a^4 + 12*b^4 - 19*a^2*b^2)*1i)/(2*(a^11 -
 a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2)) + (b*((8*tan(c/2 + (d*x)/2)*(a^10 - 2*a^
9*b - 288*a*b^9 + 288*b^10 - 624*a^2*b^8 + 624*a^3*b^7 + 386*a^4*b^6 - 386*
a^5*b^5 - 61*a^6*b^4 + 52*a^7*b^3 + 11*a^8*b^2))/(a^10*b + a^11 - a^8*b^3 -
 a^9*b^2) - (b*((4*(24*a^16*b - 4*a^17 - 48*a^10*b^7 + 24*a^11*b^6 + 124*a^
12*b^5 - 56*a^13*b^4 - 100*a^14*b^3 + 36*a^15*b^2))/(a^14*b + a^15 - a^12*b
^3 - a^13*b^2) + (4*b*tan(c/2 + (d*x)/2)*((a + b)^3*(a - b)^3)^(1/2)*(6*a^4
 + 12*b^4 - 19*a^2*b^2)*(8*a^15*b - 8*a^10*b^6 + 8*a^11*b^5 + 16*a^12*b^4 -
 16*a^13*b^3 - 8*a^14*b^2))/((a^10*b + a^11 - a^8*b^3 - a^9*b^2)*(a^11 - a^
5*b^6 + 3*a^7*b^4 - 3*a^9*b^2)))*((a + b)^3*(a - b)^3)^(1/2)*(6*a^4 + 12*b^
4 - 19*a^2*b^2))/(2*(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2)))*((a + b)^3*(
a - b)^3)^(1/2)*(6*a^4 + 12*b^4 - 19*a^2*b^2)*1i)/(2*(a^11 - a^5*b^6 + 3*a^
7*b^4 - 3*a^9*b^2)))/((8*(864*a*b^10 + 6*a^10*b - 1728*b^11 + 4752*a^2*b^9
- 2160*a^3*b^8 - 4356*a^4*b^7 + 1746*a^5*b^6 + 1495*a^6*b^5 - 491*a^7*b^4 -
 169*a^8*b^3 + 30*a^9*b^2))/(a^14*b + a^15 - a^12*b^3 - a^13*b^2) + (b*((8*
tan(c/2 + (d*x)/2)*(a^10 - 2*a^9*b - 288*a*b^9 ...
```

3.231 $\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=376

$$-\frac{2b^3(3a^2-b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a(a-b)^{7/2}(a+b)^{7/2}d}-\frac{2ab(3a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d}-\frac{b^3(a^2+2b^2)\,\mathrm{ta}}{a(a}$$

```
[Out] -2*b^3*(3*a^2-b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a/(a
-b)^(7/2)/(a+b)^(7/2)/d-2*a*b*(3*a^2+b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1
/2*c)/(a+b)^(1/2))/(a-b)^(7/2)/(a+b)^(7/2)/d-b^3*(a^2+2*b^2)*arctanh((a-b)^
(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a/(a-b)^(7/2)/(a+b)^(7/2)/d-1/2*sin(d
*x+c)/(a+b)^3/d/(1-cos(d*x+c))+1/2*sin(d*x+c)/(a-b)^3/d/(1+cos(d*x+c))-1/2*
b^3*sin(d*x+c)/(a^2-b^2)^2/d/(b+a*cos(d*x+c))^2+3/2*b^4*sin(d*x+c)/(a^2-b^2
)^3/d/(b+a*cos(d*x+c))+b^2*(3*a^2-b^2)*sin(d*x+c)/(a^2-b^2)^3/d/(b+a*cos(d*
x+c))
```

Rubi [A]
time = 0.49, antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3957, 2976, 2727, 2743, 12, 2738, 214, 2833}

$$\frac{b^2(3a^2-b^2)\sin(c+dx)}{d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}-\frac{2ab(3a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}+\frac{3b^4\sin(c+dx)}{2d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}-\frac{b^3\sin(c+dx)}{2d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)^2}-\frac{2b^3(3a^2-b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad(a-b)^{7/2}(a+b)^{7/2}}-\frac{b^3(a^2+2b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad(a-b)^{7/2}(a+b)^{7/2}}-\frac{\sin(c+dx)}{2d(a+b)^3(1-\cos(c+dx))}+\frac{\sin(c+dx)}{2d(a-b)^3(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (-2*b^3*(3*a^2 - b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/
(a*(a - b)^(7/2)*(a + b)^(7/2)*d) - (2*a*b*(3*a^2 + b^2)*ArcTanh[(Sqrt[a -
b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d) - (b^3*(
a^2 + 2*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*(a - b
)^(7/2)*(a + b)^(7/2)*d) - Sin[c + d*x]/(2*(a + b)^3*d*(1 - Cos[c + d*x]))
+ Sin[c + d*x]/(2*(a - b)^3*d*(1 + Cos[c + d*x])) - (b^3*Sin[c + d*x])/(2*(
a^2 - b^2)^2*d*(b + a*Cos[c + d*x])^2) + (3*b^4*Sin[c + d*x])/(2*(a^2 - b^2
)^3*d*(b + a*Cos[c + d*x])) + (b^2*(3*a^2 - b^2)*Sin[c + d*x])/((a^2 - b^2)
^3*d*(b + a*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2976

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\int \frac{\cos(c+dx)\cot^2(c+dx)}{(-b-a\cos(c+dx))^3}\,dx$$

$$= \int \left(-\frac{1}{2(a-b)^3(-1-\cos(c+dx))} + \frac{1}{2(a+b)^3(1-\cos(c+dx))} + \frac{\quad}{a\,(a^2-b^2)^2}\right.$$

$$= -\frac{\int \frac{1}{-1-\cos(c+dx)}\,dx}{2(a-b)^3} + \frac{\int \frac{1}{1-\cos(c+dx)}\,dx}{2(a+b)^3} - \frac{b^3\int \frac{1}{(b+a\cos(c+dx))^3}\,dx}{a\,(a^2-b^2)} + \frac{(b^2(3a^2-b^2))\int}{a\,(a}$$

$$= -\frac{\sin(c+dx)}{2(a+b)^3d(1-\cos(c+dx))} + \frac{\sin(c+dx)}{2(a-b)^3d(1+\cos(c+dx))} - \frac{b^3\sin(}{2\,(a^2-b^2)^2\,d(b}$$

$$= -\frac{2ab(3a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{\sin(c+dx)}{2(a+b)^3d(1-\cos(c+dx))} +$$

$$= -\frac{2ab(3a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{\sin(c+dx)}{2(a+b)^3d(1-\cos(c+dx))} +$$

$$= -\frac{2b^3(3a^2-b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a(a-b)^{7/2}(a+b)^{7/2}d} - \frac{2ab(3a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}}{\sqrt{\ }}\right.}{(a-b)^{7/2}(a+b)^{7/}}$$

$$= -\frac{2b^3(3a^2-b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a(a-b)^{7/2}(a+b)^{7/2}d} - \frac{2ab(3a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}}{\sqrt{\ }}\right.}{(a-b)^{7/2}(a+b)^{7/}}$$

Mathematica [A]
time = 0.71, size = 231, normalized size = 0.61

$$\frac{(b+a\cos(c+dx))\sec^3(c+dx)\left(\frac{6ab(2a^2+3b^2)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)(b+a\cos(c+dx))^2}{(a^2-b^2)^{7/2}} - \frac{(b+a\cos(c+dx))^2\cot\left(\frac{1}{2}(c+dx)\right)}{(a+b)^3} - \frac{b^3\sin(c+dx)}{(a-b)^2(a+b)^2} + \frac{b^2(6a^2+b^2)(b+a\cos(c+dx))\sin(c+dx)}{(a-b)^3(a+b)^3} + \frac{(b+a\cos(c+dx))^2\tan\left(\frac{1}{2}(c+dx)\right)}{(a-b)^3}\right)}{2d(a+b\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^3*((6*a*b*(2*a^2 + 3*b^2)*ArcTanh[((-a +
 b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x])^2)/(a^2 - b^2)^
(7/2) - ((b + a*Cos[c + d*x])^2*Cot[(c + d*x)/2])/(a + b)^3 - (b^3*Sin[c +
d*x])/((a - b)^2*(a + b)^2) + (b^2*(6*a^2 + b^2)*(b + a*Cos[c + d*x])*Sin[c
 + d*x])/((a - b)^3*(a + b)^3) + ((b + a*Cos[c + d*x])^2*Tan[(c + d*x)/2])/
(a - b)^3))/(2*d*(a + b*Sec[c + d*x])^3)
```

Maple [A]
time = 0.25, size = 234, normalized size = 0.62

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^3-6b\,a^2+6b^2a-2b^3}-\frac{1}{2(a+b)^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2b\left(\frac{\left(-3b\,a^3+\frac{5}{2}b^2a^2-\frac{1}{2}b^3a+b^4\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(3b\,a^3+\frac{5}{2}b^2a^2+\frac{1}{2}b^3a+b^4\right)\tan(}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)^2}\right.}{(a-b)^3(a+b)^3}}{d}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^3-6b\,a^2+6b^2a-2b^3}-\frac{1}{2(a+b)^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2b\left(\frac{\left(-3b\,a^3+\frac{5}{2}b^2a^2-\frac{1}{2}b^3a+b^4\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(3b\,a^3+\frac{5}{2}b^2a^2+\frac{1}{2}b^3a+b^4\right)\tan(}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)^2}\right.}{(a-b)^3(a+b)^3}}{d}$
risch	$-\frac{i\left(6a^5b\,e^{5i(dx+c)}+9a^3b^3e^{5i(dx+c)}-2a^6e^{4i(dx+c)}+24a^4b^2e^{4i(dx+c)}+21a^2b^4e^{4i(dx+c)}+2b^6e^{4i(dx+c)}+4a^5b\,e^{3i(dx+c)}+\right.}{a(a}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^2/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(1/2/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)-1/2/(a+b)^3/tan(1/2*d*x+1/2*c)+2*b/(a-b)^3/(a+b)^3*(((-3*b*a^3+5/2*b^2*a^2-1/2*b^3*a+b^4)*tan(1/2*d*x+1/2*c)^3+(3*b*a^3+5/2*b^2*a^2+1/2*b^3*a+b^4)*tan(1/2*d*x+1/2*c))/(a*tan(1/2*d*x+1/2*c)^2-b*tan(1/2*d*x+1/2*c)^2-a-b)^2-3/2*(2*a^2+3*b^2)*a/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))))

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more de

Fricas [A]
time = 4.49, size = 841, normalized size = 2.24

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(22*a^4*b^3 - 14*a^2*b^5 - 8*b^7 - 2*(2*a^7 + 10*a^5*b^2 - 11*a^3*b^4
- a*b^6)*cos(d*x + c)^3 - 3*(2*a^3*b^3 + 3*a*b^5 + (2*a^5*b + 3*a^3*b^3)*co
s(d*x + c)^2 + 2*(2*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c))*sqrt(a^2 - b^2)*log(
(2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*c
os(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*co
s(d*x + c) + b^2))*sin(d*x + c) + 2*(2*a^6*b - 17*a^4*b^3 + 13*a^2*b^5 + 2*
b^7)*cos(d*x + c)^2 + 2*(16*a^5*b^2 - 17*a^3*b^4 + a*b^6)*cos(d*x + c))/(((
a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8)*d*cos(d*x + c)^2 + 2*(a
^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*d*cos(d*x + c) + (a^8*b^2
 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 + b^10)*d)*sin(d*x + c)), 1/2*(11*a^4*
b^3 - 7*a^2*b^5 - 4*b^7 - (2*a^7 + 10*a^5*b^2 - 11*a^3*b^4 - a*b^6)*cos(d*x
 + c)^3 - 3*(2*a^3*b^3 + 3*a*b^5 + (2*a^5*b + 3*a^3*b^3)*cos(d*x + c)^2 + 2
*(2*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 +
 b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))*sin(d*x + c) + (2*a^
6*b - 17*a^4*b^3 + 13*a^2*b^5 + 2*b^7)*cos(d*x + c)^2 + (16*a^5*b^2 - 17*a^
3*b^4 + a*b^6)*cos(d*x + c))/(((a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 +
a^2*b^8)*d*cos(d*x + c)^2 + 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 +
a*b^9)*d*cos(d*x + c) + (a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 + b^10
)*d)*sin(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral(csc(c + d*x)**2/(a + b*sec(c + d*x))**3, x)
```

Giac [A]
time = 0.56, size = 386, normalized size = 1.03

$$\frac{\frac{6\,(2\,a^3b+3\,ab^3)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\sqrt{-a^2+b^2}}+\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{2\left(6\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5\,a^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-5\,a^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)^2}-\frac{1}{(a^3+3\,a^2b+3\,ab^2+b^3)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(6*(2*a^3*b + 3*a*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b)
 + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2
)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) + tan(1/2*d*x +
1/2*c)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - 2*(6*a^3*b^2*tan(1/2*d*x + 1/2*c)^
3 - 5*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + a*b^4*tan(1/2*d*x + 1/2*c)^3 - 2*b^5
```

```
*tan(1/2*d*x + 1/2*c)^3 - 6*a^3*b^2*tan(1/2*d*x + 1/2*c) - 5*a^2*b^3*tan(1/
2*d*x + 1/2*c) - a*b^4*tan(1/2*d*x + 1/2*c) - 2*b^5*tan(1/2*d*x + 1/2*c))/(
(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d
*x + 1/2*c)^2 - a - b)^2) - 1/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*tan(1/2*d*x
+ 1/2*c)))/d
```

Mupad [B]
time = 2.86, size = 423, normalized size = 1.12

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d\,{\left(a-b\right)}^3}-\frac{\frac{a^3-3\,a^2\,b+3\,a\,b^2-b^3}{a+b}+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(a^5-5\,a^4\,b+22\,a^3\,b^2-20\,a^2\,b^3+7\,a\,b^4-5\,b^5\right)}{{\left(a+b\right)}^3}-\frac{2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(a^4-4\,a^3\,b+12\,a^2\,b^2-5\,a\,b^3+3\,b^4\right)}{{\left(a+b\right)}^2}}{d\,\left(\left(2\,a^5-10\,a^4\,b+20\,a^3\,b^2-20\,a^2\,b^3+10\,a\,b^4-2\,b^5\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+\left(-4\,a^5+12\,a^4\,b-8\,a^3\,b^2-8\,a^2\,b^3+12\,a\,b^4-4\,b^5\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+\left(2\,a^5-2\,a^4\,b-4\,a^3\,b^2+4\,a^2\,b^3+2\,a\,b^4-2\,b^5\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}+\frac{a\,b\,\mathrm{atan}\left(\frac{1\mathrm{i}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^6-3\mathrm{i}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^4\,b^2+3\mathrm{i}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b^4-1\mathrm{i}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^6}{{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{5/2}}\right)\,\left(2\,a^2+3\,b^2\right)\,3\mathrm{i}}{d\,{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^2*(a + b/cos(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)/(2*d*(a - b)^3) - ((3*a*b^2 - 3*a^2*b + a^3 - b^3)/(a +
b) + (tan(c/2 + (d*x)/2)^4*(7*a*b^4 - 5*a^4*b + a^5 - 5*b^5 - 20*a^2*b^3 +
22*a^3*b^2))/(a + b)^3 - (2*tan(c/2 + (d*x)/2)^2*(a^4 - 4*a^3*b - 5*a*b^3 +
 3*b^4 + 12*a^2*b^2))/(a + b)^2)/(d*(tan(c/2 + (d*x)/2)*(2*a*b^4 - 2*a^4*b
+ 2*a^5 - 2*b^5 + 4*a^2*b^3 - 4*a^3*b^2) - tan(c/2 + (d*x)/2)^3*(4*a^5 - 12
*a^4*b - 12*a*b^4 + 4*b^5 + 8*a^2*b^3 + 8*a^3*b^2) + tan(c/2 + (d*x)/2)^5*(
10*a*b^4 - 10*a^4*b + 2*a^5 - 2*b^5 - 20*a^2*b^3 + 20*a^3*b^2))) + (a*b*ata
n((a^6*tan(c/2 + (d*x)/2)*1i - b^6*tan(c/2 + (d*x)/2)*1i + a^2*b^4*tan(c/2
+ (d*x)/2)*3i - a^4*b^2*tan(c/2 + (d*x)/2)*3i)/((a + b)^(7/2)*(a - b)^(5/2)
))*(2*a^2 + 3*b^2)*3i)/(d*(a + b)^(7/2)*(a - b)^(7/2))
```

3.232 $\int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=515

$$-\frac{2ab^3(3a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{9/2}(a+b)^{9/2}d}-\frac{ab^3(a^2+2b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{9/2}(a+b)^{9/2}d}-\frac{2ab(3a^4+8a}{}$$

```
[Out] -2*a*b^3*(3*a^2+b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a
-b)^(9/2)/(a+b)^(9/2)/d-a*b^3*(a^2+2*b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1
/2*c)/(a+b)^(1/2))/(a-b)^(9/2)/(a+b)^(9/2)/d-2*a*b*(3*a^4+8*a^2*b^2+b^4)*ar
ctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(9/2)/(a+b)^(9/2)/d
-1/12*sin(d*x+c)/(a+b)^3/d/(1-cos(d*x+c))^2-1/4*(a-2*b)*sin(d*x+c)/(a+b)^4/
d/(1-cos(d*x+c))-1/12*sin(d*x+c)/(a+b)^3/d/(1-cos(d*x+c))+1/12*sin(d*x+c)/(
a-b)^3/d/(1+cos(d*x+c))^2+1/12*sin(d*x+c)/(a-b)^3/d/(1+cos(d*x+c))+1/4*(a+2
*b)*sin(d*x+c)/(a-b)^4/d/(1+cos(d*x+c))-1/2*a^2*b^3*sin(d*x+c)/(a^2-b^2)^3/
d/(b+a*cos(d*x+c))^2+3/2*a^2*b^4*sin(d*x+c)/(a^2-b^2)^4/d/(b+a*cos(d*x+c))+
a^2*b^2*(3*a^2+b^2)*sin(d*x+c)/(a^2-b^2)^4/d/(b+a*cos(d*x+c))
```

Rubi [A]

time = 0.57, antiderivative size = 515, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3957, 2976, 2729, 2727, 2743, 2833, 12, 2738, 214}

$$\frac{a^2b^2(3a^2+b^2)\sin(c+dx)}{d(a^2-b^2)^4(a\cos(c+dx)+b)}+\frac{3a^2b^4\sin(c+dx)}{2d(a^2-b^2)^4(a\cos(c+dx)+b)}-\frac{a^2b^3\sin(c+dx)}{2d(a^2-b^2)^3(a\cos(c+dx)+b)^2}-\frac{2ab^3(3a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{9/2}(a+b)^{9/2}}-\frac{ab^3(a^2+2b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{9/2}(a+b)^{9/2}}-\frac{2ab(3a^4+8a^2b^2+b^4)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{9/2}(a+b)^{9/2}}-\frac{\sin(c+dx)}{12d(a+b)^3(1-\cos(c+dx))}-\frac{(a-2b)\sin(c+dx)}{4d(a+b)^4(1-\cos(c+dx))}+\frac{(a+2b)\sin(c+dx)}{4d(a-b)^4(\cos(c+dx)+1)}+\frac{\sin(c+dx)}{12d(a-b)^3(\cos(c+dx)+1)}-\frac{\sin(c+dx)}{12d(a+b)^3(1-\cos(c+dx))^2}+\frac{\sin(c+dx)}{12d(a-b)^3(\cos(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (-2*a*b^3*(3*a^2 + b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]]
)/((a - b)^(9/2)*(a + b)^(9/2)*d) - (a*b^3*(a^2 + 2*b^2)*ArcTanh[(Sqrt[a -
b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(9/2)*(a + b)^(9/2)*d) - (2*a*b
*(3*a^4 + 8*a^2*b^2 + b^4)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a +
b]])/((a - b)^(9/2)*(a + b)^(9/2)*d) - Sin[c + d*x]/(12*(a + b)^3*d*(1 - Co
s[c + d*x])^2) - ((a - 2*b)*Sin[c + d*x])/(4*(a + b)^4*d*(1 - Cos[c + d*x])
) - Sin[c + d*x]/(12*(a + b)^3*d*(1 - Cos[c + d*x])) + Sin[c + d*x]/(12*(a
- b)^3*d*(1 + Cos[c + d*x])^2) + Sin[c + d*x]/(12*(a - b)^3*d*(1 + Cos[c +
d*x])) + ((a + 2*b)*Sin[c + d*x])/(4*(a - b)^4*d*(1 + Cos[c + d*x])) - (a^2
*b^3*Sin[c + d*x])/(2*(a^2 - b^2)^3*d*(b + a*Cos[c + d*x])^2) + (3*a^2*b^4*
Sin[c + d*x])/(2*(a^2 - b^2)^4*d*(b + a*Cos[c + d*x])) + (a^2*b^2*(3*a^2 +
b^2)*Sin[c + d*x])/((a^2 - b^2)^4*d*(b + a*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2976

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
```

```
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx &= -\int \frac{\cot^3(c+dx)\csc(c+dx)}{(-b-a\cos(c+dx))^3}\,dx \\
&= \int \left(\frac{1}{4(a-b)^3(-1-\cos(c+dx))^2} + \frac{-a-2b}{4(a-b)^4(-1-\cos(c+dx))} + \frac{}{4(a+b)^3(}\right. \\
&= \frac{\int \frac{1}{(-1-\cos(c+dx))^2}\,dx}{4(a-b)^3} + \frac{(a-2b)\int \frac{1}{1-\cos(c+dx)}\,dx}{4(a+b)^4} + \frac{\int \frac{1}{(1-\cos(c+dx))^2}\,dx}{4(a+b)^3} - \frac{(a+2b)\int}{4(} \\
&= -\frac{\sin(c+dx)}{12(a+b)^3d(1-\cos(c+dx))^2} - \frac{(a-2b)\sin(c+dx)}{4(a+b)^4d(1-\cos(c+dx))} + \frac{\sin(}{12(a-b)^3d(1} \\
&= -\frac{2ab(3a^4+8a^2b^2+b^4)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{9/2}(a+b)^{9/2}d} - \frac{\sin(c+dx)}{12(a+b)^3d(1-\cos(c} \\
&= -\frac{2ab(3a^4+8a^2b^2+b^4)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{9/2}(a+b)^{9/2}d} - \frac{\sin(c+dx)}{12(a+b)^3d(1-\cos(c} \\
&= -\frac{2ab^3(3a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{9/2}(a+b)^{9/2}d} - \frac{2ab(3a^4+8a^2b^2+b^4)\tanh^{-1}}{(a-b)^{9/2}(a-} \\
&= -\frac{2ab^3(3a^2+b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{9/2}(a+b)^{9/2}d} - \frac{ab^3(a^2+2b^2)\tanh^{-1}\left(\frac{\sqrt{a-}}{}\right.}{(a-b)^{9/2}(a+b)^{9}}
\end{aligned}
$$

Mathematica [A]
time = 0.75, size = 388, normalized size = 0.75

$$\frac{(b+a\cos(c+dx))\left(\frac{96ab(6a^4+23a^2b^2+6b^4)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)(b+a\cos(c+dx))^2}{\sqrt{a^2-b^2}} + (36a^6b+154a^4b^3+424a^2b^5+16b^7-2a(16a^6-94a^4b^2-35a^2b^4+8b^6)\cos(c+dx)+8(2a^6b-45a^4b^3-56a^2b^5-6b^7)\cos(2(c+dx))-4a^7\cos(3(c+dx))-154a^5b^2\cos(3(c+dx))-205a^3b^4\cos(3(c+dx))+48ab^6\cos(3(c+dx))-20a^6b\cos(4(c+dx))+110a^4b^3\cos(4(c+dx))+120a^2b^5\cos(4(c+dx))+4a^7\cos(5(c+dx))+62a^5b^2\cos(5(c+dx))+39a^3b^4\cos(5(c+dx)))\csc^3(c+dx)\right)\sec^3(c+dx)}{96\left(a^2-b^2\right)^4d(a+b\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((b + a*Cos[c + d*x])*((96*a*b*(6*a^4 + 23*a^2*b^2 + 6*b^4)*ArcTanh[((-a +
b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x])^2)/Sqrt[a^2 - b^
2] + (36*a^6*b + 154*a^4*b^3 + 424*a^2*b^5 + 16*b^7 - 2*a*(16*a^6 - 94*a^4*
b^2 - 35*a^2*b^4 + 8*b^6)*Cos[c + d*x] + 8*(2*a^6*b - 45*a^4*b^3 - 56*a^2*b
^5 - 6*b^7)*Cos[2*(c + d*x)] - 4*a^7*Cos[3*(c + d*x)] - 154*a^5*b^2*Cos[3*(
c + d*x)] - 205*a^3*b^4*Cos[3*(c + d*x)] + 48*a*b^6*Cos[3*(c + d*x)] - 20*a
^6*b*Cos[4*(c + d*x)] + 110*a^4*b^3*Cos[4*(c + d*x)] + 120*a^2*b^5*Cos[4*(c
 + d*x)] + 4*a^7*Cos[5*(c + d*x)] + 62*a^5*b^2*Cos[5*(c + d*x)] + 39*a^3*b^
4*Cos[5*(c + d*x)])*Csc[c + d*x]^3)*Sec[c + d*x]^3)/(96*(a^2 - b^2)^4*d*(a
+ b*Sec[c + d*x])^3)
```

Maple [A]
time = 0.35, size = 328, normalized size = 0.64 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/8/(a^3-3*a^2*b+3*a*b^2-b^3)/(a-b)*(1/3*a*tan(1/2*d*x+1/2*c)^3-1/3*b*
tan(1/2*d*x+1/2*c)^3+3*a*tan(1/2*d*x+1/2*c)+3*b*tan(1/2*d*x+1/2*c))-1/24/(a
+b)^3/tan(1/2*d*x+1/2*c)^3-1/8/(a+b)^4*(3*a-3*b)/tan(1/2*d*x+1/2*c)+2*b*a/(
a-b)^4/(a+b)^4*(((5/2*a^3*b^2+3*a*b^4-3*a^4*b-5/2*a^2*b^3)*tan(1/2*d*x+1/2*
c)^3+(5/2*a^3*b^2+3*a*b^4+3*a^4*b+5/2*a^2*b^3)*tan(1/2*d*x+1/2*c))/(a*tan(1
/2*d*x+1/2*c)^2-b*tan(1/2*d*x+1/2*c)^2-a-b)^2-1/2*(6*a^4+23*a^2*b^2+6*b^4)/
((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 3.77, size = 1550, normalized size = 3.01

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/12*(78*a^6*b^3 + 46*a^4*b^5 - 116*a^2*b^7 - 8*b^9 + 2*(4*a^9 + 58*a^7*b
^2 - 23*a^5*b^4 - 39*a^3*b^6)*cos(d*x + c)^5 - 10*(2*a^8*b - 13*a^6*b^3 - a
^4*b^5 + 12*a^2*b^7)*cos(d*x + c)^4 - 4*(3*a^9 + 55*a^7*b^2 - 8*a^5*b^4 - 5
6*a^3*b^6 + 6*a*b^8)*cos(d*x + c)^3 + 3*(6*a^5*b^3 + 23*a^3*b^5 + 6*a*b^7 -
 (6*a^7*b + 23*a^5*b^3 + 6*a^3*b^5)*cos(d*x + c)^4 - 2*(6*a^6*b^2 + 23*a^4*
b^4 + 6*a^2*b^6)*cos(d*x + c)^3 + (6*a^7*b + 17*a^5*b^3 - 17*a^3*b^5 - 6*a*
b^7)*cos(d*x + c)^2 + 2*(6*a^6*b^2 + 23*a^4*b^4 + 6*a^2*b^6)*cos(d*x + c))*
sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*
sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d
*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) + 4*(6*a^8*b - 56*a^6*b
^3 - 8*a^4*b^5 + 55*a^2*b^7 + 3*b^9)*cos(d*x + c)^2 + 10*(12*a^7*b^2 - a^5*
b^4 - 13*a^3*b^6 + 2*a*b^8)*cos(d*x + c))/(((a^12 - 5*a^10*b^2 + 10*a^8*b^4
 - 10*a^6*b^6 + 5*a^4*b^8 - a^2*b^10)*d*cos(d*x + c)^4 + 2*(a^11*b - 5*a^9*
b^3 + 10*a^7*b^5 - 10*a^5*b^7 + 5*a^3*b^9 - a*b^11)*d*cos(d*x + c)^3 - (a^1
2 - 6*a^10*b^2 + 15*a^8*b^4 - 20*a^6*b^6 + 15*a^4*b^8 - 6*a^2*b^10 + b^12)*
d*cos(d*x + c)^2 - 2*(a^11*b - 5*a^9*b^3 + 10*a^7*b^5 - 10*a^5*b^7 + 5*a^3*
b^9 - a*b^11)*d*cos(d*x + c) - (a^10*b^2 - 5*a^8*b^4 + 10*a^6*b^6 - 10*a^4*
b^8 + 5*a^2*b^10 - b^12)*d)*sin(d*x + c)), -1/6*(39*a^6*b^3 + 23*a^4*b^5 -
58*a^2*b^7 - 4*b^9 + (4*a^9 + 58*a^7*b^2 - 23*a^5*b^4 - 39*a^3*b^6)*cos(d*x
 + c)^5 - 5*(2*a^8*b - 13*a^6*b^3 - a^4*b^5 + 12*a^2*b^7)*cos(d*x + c)^4 -
2*(3*a^9 + 55*a^7*b^2 - 8*a^5*b^4 - 56*a^3*b^6 + 6*a*b^8)*cos(d*x + c)^3 -
3*(6*a^5*b^3 + 23*a^3*b^5 + 6*a*b^7 - (6*a^7*b + 23*a^5*b^3 + 6*a^3*b^5)*co
s(d*x + c)^4 - 2*(6*a^6*b^2 + 23*a^4*b^4 + 6*a^2*b^6)*cos(d*x + c)^3 + (6*a
^7*b + 17*a^5*b^3 - 17*a^3*b^5 - 6*a*b^7)*cos(d*x + c)^2 + 2*(6*a^6*b^2 + 2
3*a^4*b^4 + 6*a^2*b^6)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b
^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))*sin(d*x + c) + 2*(6*a^
8*b - 56*a^6*b^3 - 8*a^4*b^5 + 55*a^2*b^7 + 3*b^9)*cos(d*x + c)^2 + 5*(12*a
^7*b^2 - a^5*b^4 - 13*a^3*b^6 + 2*a*b^8)*cos(d*x + c))/(((a^12 - 5*a^10*b^2
 + 10*a^8*b^4 - 10*a^6*b^6 + 5*a^4*b^8 - a^2*b^10)*d*cos(d*x + c)^4 + 2*(a^
11*b - 5*a^9*b^3 + 10*a^7*b^5 - 10*a^5*b^7 + 5*a^3*b^9 - a*b^11)*d*cos(d*x
+ c)^3 - (a^12 - 6*a^10*b^2 + 15*a^8*b^4 - 20*a^6*b^6 + 15*a^4*b^8 - 6*a^2*
b^10 + b^12)*d*cos(d*x + c)^2 - 2*(a^11*b - 5*a^9*b^3 + 10*a^7*b^5 - 10*a^5
*b^7 + 5*a^3*b^9 - a*b^11)*d*cos(d*x + c) - (a^10*b^2 - 5*a^8*b^4 + 10*a^6*
b^6 - 10*a^4*b^8 + 5*a^2*b^10 - b^12)*d)*sin(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral(csc(c + d*x)**4/(a + b*sec(c + d*x))**3, x)
```

Giac [A]
time = 0.58, size = 709, normalized size = 1.38

$$\frac{\frac{24\,(6\,a^5b+23\,a^3b^3+6\,ab^5)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8)\sqrt{-a^2+b^2}}+\frac{a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6\,a^5b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+15\,a^4b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-20\,a^3b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+15\,a^2b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6\,ab^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+b^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-36\,a^5b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+45\,a^4b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-45\,a^2b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+36\,ab^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-9\,b^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^9-9\,a^8b+36\,a^7b^2-84\,a^6b^3+126\,a^5b^4-126\,a^4b^5+84\,a^3b^6-36\,a^2b^7+9\,ab^8-b^9}-\frac{24\left(6\,a^5b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5\,a^4b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+5\,a^3b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6\,a^2b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6\,a^5b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-5\,a^4b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-5\,a^3b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-6\,a^2b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{(a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)^2}-\frac{9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-9\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a+b}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/24*(24*(6*a^5*b + 23*a^3*b^3 + 6*a*b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)
*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/
sqrt(-a^2 + b^2)))/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*sqrt(-a
^2 + b^2)) + (a^6*tan(1/2*d*x + 1/2*c)^3 - 6*a^5*b*tan(1/2*d*x + 1/2*c)^3 +
 15*a^4*b^2*tan(1/2*d*x + 1/2*c)^3 - 20*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 + 15
*a^2*b^4*tan(1/2*d*x + 1/2*c)^3 - 6*a*b^5*tan(1/2*d*x + 1/2*c)^3 + b^6*tan(
1/2*d*x + 1/2*c)^3 + 9*a^6*tan(1/2*d*x + 1/2*c) - 36*a^5*b*tan(1/2*d*x + 1/
2*c) + 45*a^4*b^2*tan(1/2*d*x + 1/2*c) - 45*a^2*b^4*tan(1/2*d*x + 1/2*c) +
36*a*b^5*tan(1/2*d*x + 1/2*c) - 9*b^6*tan(1/2*d*x + 1/2*c))/(a^9 - 9*a^8*b
+ 36*a^7*b^2 - 84*a^6*b^3 + 126*a^5*b^4 - 126*a^4*b^5 + 84*a^3*b^6 - 36*a^2
*b^7 + 9*a*b^8 - b^9) - 24*(6*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 - 5*a^4*b^3*ta
n(1/2*d*x + 1/2*c)^3 + 5*a^3*b^4*tan(1/2*d*x + 1/2*c)^3 - 6*a^2*b^5*tan(1/2
*d*x + 1/2*c)^3 - 6*a^5*b^2*tan(1/2*d*x + 1/2*c) - 5*a^4*b^3*tan(1/2*d*x +
1/2*c) - 5*a^3*b^4*tan(1/2*d*x + 1/2*c) - 6*a^2*b^5*tan(1/2*d*x + 1/2*c))/(
(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(a*tan(1/2*d*x + 1/2*c)^2 -
 b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2) - (9*a*tan(1/2*d*x + 1/2*c)^2 - 9*b*t
an(1/2*d*x + 1/2*c)^2 + a + b)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)
*tan(1/2*d*x + 1/2*c)^3))/d
```

Mupad [B]
time = 1.78, size = 588, normalized size = 1.14

$$\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{24\,d\,{(a-b)}^3}+\frac{\frac{a^4-4\,a^3\,b+6\,a^2\,b^2-4\,a\,b^3+b^4}{3\,(a+b)}+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,(3\,a^7-21\,a^6\,b+111\,a^5\,b^2-145\,a^4\,b^3+145\,a^3\,b^4-111\,a^2\,b^5+21\,a\,b^6-3\,b^7)}{{(a+b)}^4}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,(17\,a^6-102\,a^5\,b+399\,a^4\,b^2-364\,a^3\,b^3+399\,a^2\,b^4-102\,a\,b^5+17\,b^6)}{3\,{(a+b)}^3}+\frac{7\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,(a-b)\,(a^4-4\,a^3\,b+6\,a^2\,b^2-4\,a\,b^3+b^4)}{3\,{(a+b)}^2}}{d\,\left(\left(-8\,a^6+48\,a^5\,b-120\,a^4\,b^2+160\,a^3\,b^3-120\,a^2\,b^4+48\,a\,b^5-8\,b^6\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7+\left(16\,a^6-64\,a^5\,b+80\,a^4\,b^2-80\,a^2\,b^4+64\,a\,b^5-16\,b^6\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+\left(-8\,a^6+16\,a^5\,b+8\,a^4\,b^2-32\,a^3\,b^3+8\,a^2\,b^4+16\,a\,b^5-8\,b^6\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\right)}+\frac{3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(a+b)}{8\,d\,{(a-b)}^4}+\frac{a\,b\,\mathrm{atan}\left(\frac{1\mathrm{i}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^8-4\mathrm{i}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^6\,b^2+6\mathrm{i}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^4\,b^4-4\mathrm{i}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b^6+1\mathrm{i}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^8}{{(a+b)}^{9/2}\,{(a-b)}^{7/2}}\right)\,(6\,a^4+23\,a^2\,b^2+6\,b^4)\,1\mathrm{i}}{d\,{(a+b)}^{9/2}\,{(a-b)}^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^4*(a + b/cos(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(24*d*(a - b)^3) + ((a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6
*a^2*b^2)/(3*(a + b)) + (tan(c/2 + (d*x)/2)^6*(21*a*b^6 - 21*a^6*b + 3*a^7
- 3*b^7 - 111*a^2*b^5 + 145*a^3*b^4 - 145*a^4*b^3 + 111*a^5*b^2))/(a + b)^4
 - (tan(c/2 + (d*x)/2)^4*(17*a^6 - 102*a^5*b - 102*a*b^5 + 17*b^6 + 399*a^2
*b^4 - 364*a^3*b^3 + 399*a^4*b^2))/(3*(a + b)^3) + (7*tan(c/2 + (d*x)/2)^2*
(a - b)*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2))/(3*(a + b)^2))/(d*(tan
(c/2 + (d*x)/2)^3*(16*a*b^5 + 16*a^5*b - 8*a^6 - 8*b^6 + 8*a^2*b^4 - 32*a^3
*b^3 + 8*a^4*b^2) - tan(c/2 + (d*x)/2)^7*(8*a^6 - 48*a^5*b - 48*a*b^5 + 8*b
^6 + 120*a^2*b^4 - 160*a^3*b^3 + 120*a^4*b^2) + tan(c/2 + (d*x)/2)^5*(64*a*
b^5 - 64*a^5*b + 16*a^6 - 16*b^6 - 80*a^2*b^4 + 80*a^4*b^2))) + (3*tan(c/2
```

```
+ (d*x)/2)*(a + b))/(8*d*(a - b)^4) + (a*b*atan((a^8*tan(c/2 + (d*x)/2)*1i
+ b^8*tan(c/2 + (d*x)/2)*1i - a^2*b^6*tan(c/2 + (d*x)/2)*4i + a^4*b^4*tan(c
/2 + (d*x)/2)*6i - a^6*b^2*tan(c/2 + (d*x)/2)*4i)/((a + b)^(9/2)*(a - b)^(7
/2)))*(6*a^4 + 6*b^4 + 23*a^2*b^2)*1i)/(d*(a + b)^(9/2)*(a - b)^(9/2))
```

3.233 $\int \frac{(e\sin(c+dx))^{7/2}}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=516

$$-\frac{b(a^2-b^2)^{5/4}e^{7/2}\mathrm{ArcTan}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{9/2}d}-\frac{b(a^2-b^2)^{5/4}e^{7/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{9/2}d}+\frac{2(5}{}$$

```
[Out] -b*(a^2-b^2)^(5/4)*e^(7/2)*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1
/4)/e^(1/2))/a^(9/2)/d-b*(a^2-b^2)^(5/4)*e^(7/2)*arctanh(a^(1/2)*(e*sin(d*x
+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^(9/2)/d+2/35*e*(7*b-5*a*cos(d*x+c))*(
e*sin(d*x+c))^(5/2)/a^2/d-2/21*(5*a^4-28*a^2*b^2+21*b^4)*e^4*(sin(1/2*c+1/4
*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+
1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/a^5/d/(e*sin(d*x+c))^(1/2)-b^2*(a^2-b^2)
^2*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ellipt
icPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^
(1/2)/a^5/d/(a^2-b^2-a*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-b^2*(a^2-b^2)^
2*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ellipti
cPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(
1/2)/a^5/d/(a^2-b^2+a*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+2/21*e^3*(21*b*
(a^2-b^2)-a*(5*a^2-7*b^2)*cos(d*x+c))*(e*sin(d*x+c))^(1/2)/a^4/d
```

Rubi [A]
time = 1.18, antiderivative size = 516, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3957, 2944, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$\frac{2e(e\sin(c+dx))^{5/2}(7b-5a\cos(c+dx))}{35a^2d}-\frac{be^{7/2}(a^2-b^2)^{5/4}\mathrm{ArcTan}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{a^{9/2}d}-\frac{be^{7/2}(a^2-b^2)^{5/4}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{a^{9/2}d}+\frac{b^2e^4(a^2-b^2)^2\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{a^5d\left(-a\sqrt{a^2-b^2}+a^2-b^2\right)\sqrt{e\sin(c+dx)}}+\frac{b^2e^4(a^2-b^2)^2\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2a}{a+\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{a^5d\left(a\sqrt{a^2-b^2}+a^2-b^2\right)\sqrt{e\sin(c+dx)}}+\frac{2e^3\sqrt{e\sin(c+dx)}\,(21b(a^2-b^2)-a(5a^2-7b^2)\cos(c+dx))}{21a^4d}+\frac{2e^4(5a^4-28a^2b^2+21b^4)\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{21a^5d\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(7/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] -((b*(a^2 - b^2)^(5/4)*e^(7/2)*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2
- b^2)^(1/4)*Sqrt[e])])/(a^(9/2)*d)) - (b*(a^2 - b^2)^(5/4)*e^(7/2)*ArcTanh
[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(9/2)*d) +
 (2*(5*a^4 - 28*a^2*b^2 + 21*b^4)*e^4*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt
[Sin[c + d*x]])/(21*a^5*d*Sqrt[e*Sin[c + d*x]]) + (b^2*(a^2 - b^2)^2*e^4*El
lipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d
*x]])/(a^5*(a^2 - b^2 - a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (b^2*(
a^2 - b^2)^2*e^4*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2
, 2]*Sqrt[Sin[c + d*x]])/(a^5*(a^2 - b^2 + a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[
c + d*x]]) + (2*e^3*(21*b*(a^2 - b^2) - a*(5*a^2 - 7*b^2)*Cos[c + d*x])*Sqr
t[e*Sin[c + d*x]])/(21*a^4*d) + (2*e*(7*b - 5*a*Cos[c + d*x])*(e*Sin[c + d*
x])^(5/2))/(35*a^2*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
```

```
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^{7/2}}{a+b\sec(c+dx)}\,dx = -\int \frac{\cos(c+dx)(e\sin(c+dx))^{7/2}}{-b-a\cos(c+dx)}\,dx$$

$$= \frac{2e(7b-5a\cos(c+dx))(e\sin(c+dx))^{5/2}}{35a^2d} - \frac{(2e^2)\int \frac{\left(-ab+\frac{1}{2}(5a^2-7b^2)\cos(c+dx)\right)(e\sin(c+d}{-b-a\cos(c+dx)}}{7a^2}$$

$$= \frac{2e^3(21b(a^2-b^2)-a(5a^2-7b^2)\cos(c+dx))\sqrt{e\sin(c+dx)}}{21a^4d} + \frac{2e(7b-5a\cos(c}{}$$

$$= \frac{2e^3(21b(a^2-b^2)-a(5a^2-7b^2)\cos(c+dx))\sqrt{e\sin(c+dx)}}{21a^4d} + \frac{2e(7b-5a\cos(c}{}$$

$$= \frac{2e^3(21b(a^2-b^2)-a(5a^2-7b^2)\cos(c+dx))\sqrt{e\sin(c+dx)}}{21a^4d} + \frac{2e(7b-5a\cos(c}{}$$

$$= \frac{2(5a^4-28a^2b^2+21b^4)e^4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{21a^5d\sqrt{e\sin(c+dx)}} + \frac{2e^3(21b(a^2-b^2)}{}$$

$$= \frac{2(5a^4-28a^2b^2+21b^4)e^4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{21a^5d\sqrt{e\sin(c+dx)}} - \frac{b^2(a^2-b^2)^{3/2}e^4\mathrm{I}}{a^5}$$

$$= -\frac{b(a^2-b^2)^{5/4}e^{7/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{9/2}d} - \frac{b(a^2-b^2)^{5/4}e^{7/2}\tanh^{-1}\left(\sqrt{a}\right.}{a^{9/2}d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 45.46, size = 2049, normalized size = 3.97

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sin[c + d*x])^(7/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((b + a*Cos[c + d*x])*(-1/42*((23*a^2 - 28*b^2)*Cos[c + d*x])/a^3 - (b*Cos[
2*(c + d*x)])/(5*a^2) + Cos[3*(c + d*x)]/(14*a))*Csc[c + d*x]^3*Sec[c + d*x
]*(e*Sin[c + d*x])^(7/2))/(d*(a + b*Sec[c + d*x])) - ((b + a*Cos[c + d*x])*
Sec[c + d*x]*(e*Sin[c + d*x])^(7/2)*((2*(-100*a^3 + 98*a*b^2)*Cos[c + d*x]^
2*(b + a*Sqrt[1 - Sin[c + d*x]^2])*((b*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[a]*Sqrt
[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[a]*Sqrt[Si
```

```
n[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - Sqrt[2]*Sqrt[a]*(
-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^
2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]
]))/(4*Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(3/4)) - (5*a*(a^2 - b^2)*AppellF1[1/4,
 -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sin[c
 + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5
/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + 2*(2*a^2*AppellF1[5
/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + (-a^2
 + b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^
2 - b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x]^2)))))/((b + a*Cos
[c + d*x])*(1 - Sin[c + d*x]^2)) + (2*(89*a^2*b - 70*b^3)*Cos[c + d*x]*(b +
 a*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[a]*(2*ArcTan[1 - ((1 + I)*
Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt
[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + Log[Sqrt[a^2 - b^2] - (1 + I)*
Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]] - Log[Sqrt
[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Si
n[c + d*x]]))/(a^2 - b^2)^(3/4) + (5*b*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/
4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sin[c + d*x]])/(S
qrt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d
*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + 2*(2*a^2*AppellF1[5/4, 1/2, 2, 9
/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + (a^2 - b^2)*AppellF
1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)])*Sin[
c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x]^2)))))/((b + a*Cos[c + d*x])*Sqrt
[1 - Sin[c + d*x]^2]) + ((-231*a^2*b + 210*b^3)*Cos[c + d*x]*Cos[2*(c + d*x
)]*(b + a*Sqrt[1 - Sin[c + d*x]^2])*(((1/2 - I/2)*(a^2 - 2*b^2)*ArcTan[1 -
((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)])/(a^(3/2)*(a^2 - b^
2)^(3/4)) - ((1/2 - I/2)*(a^2 - 2*b^2)*ArcTan[1 + ((1 + I)*Sqrt[a]*Sqrt[Sin
[c + d*x]])/(a^2 - b^2)^(1/4)])/(a^(3/2)*(a^2 - b^2)^(3/4)) + ((1/4 - I/4)*
(a^2 - 2*b^2)*Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[
Sin[c + d*x]] + I*a*Sin[c + d*x]])/(a^(3/2)*(a^2 - b^2)^(3/4)) - ((1/4 - I/
4)*(a^2 - 2*b^2)*Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sq
rt[Sin[c + d*x]] + I*a*Sin[c + d*x]])/(a^(3/2)*(a^2 - b^2)^(3/4)) + (4*Sqrt
[Sin[c + d*x]])/a + (4*b*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Si
n[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(5/2))/(5*(a^2 - b^2)) + (10*b*(a^2
 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^
2 - b^2)]*Sqrt[Sin[c + d*x]])/(Sqrt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*Appe
llF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] +
2*(2*a^2*AppellF1[5/4, 1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a
^2 - b^2)] + (a^2 - b^2)*AppellF1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Si
n[c + d*x]^2)/(a^2 - b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x]^2
)))))/((b + a*Cos[c + d*x])*(1 - 2*Sin[c + d*x]^2)*Sqrt[1 - Sin[c + d*x]^2]
)))/(420*a^3*d*(a + b*Sec[c + d*x])*Sin[c + d*x]^(7/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1178 vs. $2(550) = 1100$.

time = 0.47, size = 1179, normalized size = 2.28

method	result	size
default	Expression too large to display	1179

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(7/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] (2/5*b*e/a^2*(e*sin(d*x+c))^(5/2)+2*b*e^3/a^2*(e*sin(d*x+c))^(1/2)-2*b^3*e^
3/a^4*(e*sin(d*x+c))^(1/2)+b*e^5*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^
2)*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))-2*b^3*e^5/a^2*(e^
2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*arctan((e*sin(d*x+c))^(1/2)/(e^2*
(a^2-b^2)/a^2)^(1/4))+b^5*e^5/a^4*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e
^2)*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))+1/2*b*e^5*(e^2*(
a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b
^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))-b^3*e^5/a
^2*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*ln(((e*sin(d*x+c))^(1/2)+(e
^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))+
1/2*b^5*e^5/a^4*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*ln(((e*sin(d*x
+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/
a^2)^(1/4)))+(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*a*e^4*(-1/21/a^6/(cos(d*x+c)
^2*e*sin(d*x+c))^(1/2)*(-6*a^4*cos(d*x+c)^4*sin(d*x+c)+5*(-sin(d*x+c)+1)^(1
/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2)
,1/2*2^(1/2))*a^4-28*b^2*a^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*s
in(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+21*b^4*(-sin(d
*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+
c)+1)^(1/2),1/2*2^(1/2))+16*a^4*cos(d*x+c)^2*sin(d*x+c)-14*a^2*b^2*cos(d*x+
c)^2*sin(d*x+c))-b^2*(a^4-2*a^2*b^2+b^4)/a^6*(-1/2/(a^2-b^2)^(1/2)/a*(-sin(
d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin
(d*x+c))^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-
(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+1/2/(a^2-b^2)^(1/2)/a*(-sin(d*x+c)+1)^(1/2)
*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/
(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)
/a),1/2*2^(1/2))))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] e^(7/2)*integrate(sin(d*x + c)^(7/2)/(b*sec(d*x + c) + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(7/2)/(a+b*sec(d*x+c)),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((e*sin(d*x + c))^(7/2)/(b*sec(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)\,{\left(e\,\sin\left(c+d\,x\right)\right)}^{7/2}}{b+a\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(7/2)/(a + b/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e*sin(c + d*x))^(7/2))/(b + a*cos(c + d*x)), x)
```

3.234 $\int \frac{(e\sin(c+dx))^{5/2}}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=430

$$\frac{b(a^2-b^2)^{3/4}\,e^{5/2}\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\,\sqrt{e}}\right)}{a^{7/2}d}-\frac{b(a^2-b^2)^{3/4}\,e^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\,\sqrt{e}}\right)}{a^{7/2}d}-\frac{b^2(a^2-}{}$$

```
[Out] b*(a^2-b^2)^(3/4)*e^(5/2)*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/
4)/e^(1/2))/a^(7/2)/d-b*(a^2-b^2)^(3/4)*e^(5/2)*arctanh(a^(1/2)*(e*sin(d*x+
c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^(7/2)/d+2/15*e*(5*b-3*a*cos(d*x+c))*(e
*sin(d*x+c))^(3/2)/a^2/d+b^2*(a^2-b^2)*e^3*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1
/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a-(
a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^4/d/(a-(a^2-b^2)^(1/2))/(e*sin(
d*x+c))^(1/2)+b^2*(a^2-b^2)*e^3*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2
*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1
/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^4/d/(a+(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/
2)-2/5*(3*a^2-5*b^2)*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*
Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/
2)/a^3/d/sin(d*x+c)^(1/2)
```

Rubi [A]
time = 0.79, antiderivative size = 430, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3957, 2944, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$\frac{2e(e\sin(c+dx))^{3/2}(5b-3a\cos(c+dx))}{15a^2d}+\frac{be^{5/2}(a^2-b^2)^{3/4}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{a^2-b^2}}\right)}{a^{7/2}d}-\frac{be^{5/2}(a^2-b^2)^{3/4}\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{a^2-b^2}}\right)}{a^{7/2}d}-\frac{b^2e^3(a^2-b^2)\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{a^4d\left(a-\sqrt{a^2-b^2}\right)\sqrt{e\sin(c+dx)}}-\frac{b^2e^3(a^2-b^2)\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2a}{a+\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{a^4d\left(\sqrt{a^2-b^2}+a\right)\sqrt{e\sin(c+dx)}}+\frac{2e^2(3a^2-5b^2)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5a^3d\sqrt{\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(5/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (b*(a^2 - b^2)^(3/4)*e^(5/2)*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 -
b^2)^(1/4)*Sqrt[e])])/(a^(7/2)*d) - (b*(a^2 - b^2)^(3/4)*e^(5/2)*ArcTanh[(S
qrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(7/2)*d) - (b
^2*(a^2 - b^2)*e^3*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)
/2, 2]*Sqrt[Sin[c + d*x]])/(a^4*(a - Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]
]) - (b^2*(a^2 - b^2)*e^3*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2
 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^4*(a + Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c
 + d*x]]) + (2*(3*a^2 - 5*b^2)*e^2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*
Sin[c + d*x]])/(5*a^3*d*Sqrt[Sin[c + d*x]]) + (2*e*(5*b - 3*a*Cos[c + d*x])
*(e*Sin[c + d*x])^(3/2))/(15*a^2*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
```

```
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+b\sec(c+dx)}\,dx = -\int \frac{\cos(c+dx)(e\sin(c+dx))^{5/2}}{-b-a\cos(c+dx)}\,dx$$

$$= \frac{2e(5b-3a\cos(c+dx))(e\sin(c+dx))^{3/2}}{15a^2d} - \frac{(2e^2)\int \frac{\left(-ab+\frac{1}{2}(3a^2-5b^2)\cos(c+dx)\right)\sqrt{e\sin}}{-b-a\cos(c+dx)}}{5a^2}$$

$$= \frac{2e(5b-3a\cos(c+dx))(e\sin(c+dx))^{3/2}}{15a^2d} + \frac{((3a^2-5b^2)\,e^2)\int \sqrt{e\sin(c+dx)}\,dx}{5a^3}$$

$$= \frac{2e(5b-3a\cos(c+dx))(e\sin(c+dx))^{3/2}}{15a^2d} + \frac{(b^2(a^2-b^2)\,e^3)\int \frac{}{\sqrt{e\sin(c+dx)}\,(}}{2a^4}$$

$$= \frac{2(3a^2-5b^2)\,e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{e\sin(c+dx)}}{5a^3d\sqrt{\sin(c+dx)}} + \frac{2e(5b-3a\cos(c+dx))(}{15a^2d}$$

$$= -\frac{b^2(a^2-b^2)\,e^3\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{\sin(c+dx)}}{a^4\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} - \frac{b^2(a^2-b^2)}{}$$

$$= \frac{b(a^2-b^2)^{3/4}\,e^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{7/2}d} - \frac{b(a^2-b^2)^{3/4}\,e^{5/2}\tanh^{-1}\left(\sqrt{a}\right.}{a^{7/2}d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 33.87, size = 853, normalized size = 1.98

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sin[c + d*x])^(5/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] -1/5*((b + a*Cos[c + d*x])*Sec[c + d*x]*(e*Sin[c + d*x])^(5/2)*(((-3*a^2 +
5*b^2)*Cos[c + d*x]^2*(3*Sqrt[2]*b*(-a^2 + b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2
]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*S
qrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - Sqr
t[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]] + Log[
Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] +
a*Sin[c + d*x]]) + 8*a^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c + d*x]^2, (a
^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(3/2))*(b + a*Sqrt[1 - Sin[c +
```

```
 d*x]^2]))/(12*a^(3/2)*(a^2 - b^2)*(b + a*Cos[c + d*x])*(1 - Sin[c + d*x]^2
)) + (4*a*b*Cos[c + d*x]*(((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[a]*Sqrt[
Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[a]*Sqrt[Sin[
c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[a]*(a^2
- b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] +
 (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]]))
/(Sqrt[a]*(a^2 - b^2)^(1/4)) + (b*AppellF1[3/4, 1/2, 1, 7/4, Sin[c + d*x]^2
, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(3/2))/(3*(-a^2 + b^2)))*(
b + a*Sqrt[1 - Sin[c + d*x]^2]))/((b + a*Cos[c + d*x])*Sqrt[1 - Sin[c + d*x
]^2])))/(a^2*d*(a + b*Sec[c + d*x])*Sin[c + d*x]^(5/2)) + ((b + a*Cos[c + d
*x])*Csc[c + d*x]^2*Sec[c + d*x]*(e*Sin[c + d*x])^(5/2)*((2*b*Sin[c + d*x])
/(3*a^2) - Sin[2*(c + d*x)]/(5*a)))/(d*(a + b*Sec[c + d*x]))
```

Maple [A]
time = 0.38, size = 852, normalized size = 1.98

method	result
default	$\frac{\frac{2eb(e\sin(dx+c))^{\frac{3}{2}}}{3a^2}-\frac{e^3b\ln\left(\frac{\sqrt{e\sin(dx+c)}+\left(\frac{e^2(a^2-b^2)}{a^2}\right)^{\frac{1}{4}}}{\sqrt{e\sin(dx+c)}-\left(\frac{e^2(a^2-b^2)}{a^2}\right)^{\frac{1}{4}}}\right)}{2a^2\left(\frac{e^2(a^2-b^2)}{a^2}\right)^{\frac{1}{4}}}+\frac{e^3b^3\ln\left(\frac{\sqrt{e\sin(dx+c)}+\left(\frac{e^2(a^2-b^2)}{a^2}\right)^{\frac{1}{4}}}{\sqrt{e\sin(dx+c)}-\left(\frac{e^2(a^2-b^2)}{a^2}\right)^{\frac{1}{4}}}\right)}{2a^4\left(\frac{e^2(a^2-b^2)}{a^2}\right)^{\frac{1}{4}}}+\frac{e^3b\arctan}{a}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(5/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] (2/3*e*b/a^2*(e*sin(d*x+c))^(3/2)-1/2*e^3*b/a^2/(e^2*(a^2-b^2)/a^2)^(1/4)*l
n(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e
^2*(a^2-b^2)/a^2)^(1/4)))+1/2*e^3*b^3/a^4/(e^2*(a^2-b^2)/a^2)^(1/4)*ln(((e*
sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^
2-b^2)/a^2)^(1/4)))+e^3*b/a^2/(e^2*(a^2-b^2)/a^2)^(1/4)*arctan((e*sin(d*x+c
))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))-e^3*b^3/a^4/(e^2*(a^2-b^2)/a^2)^(1/4)*a
rctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))+(cos(d*x+c)^2*e*sin(d
*x+c))^(1/2)*a*e^3*(-1/5/a^2/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*(6*(-sin(d*x
+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)
+1)^(1/2),1/2*2^(1/2))-3*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d
*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-2*cos(d*x+c)^4+2*c
os(d*x+c)^2)+b^2/a^4*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c
```

```
)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*(2*EllipticE((-sin(d*x+c)+1)^(1/2
),1/2*2^(1/2))-EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2)))-b^2*(a^2-b^2)/
a^4*(-1/2/a^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)
/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d
*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-1/2/a^2*(-sin(d*x+c)+1)
^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^
(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)
^(1/2)/a),1/2*2^(1/2)))))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
[Out] e^(5/2)*integrate(sin(d*x + c)^(5/2)/(b*sec(d*x + c) + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(5/2)/(a+b*sec(d*x+c)),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*sin(d*x + c))^(5/2)/(b*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)\,\left(e\,\sin\left(c+d\,x\right)\right)^{5/2}}{b+a\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(5/2)/(a + b/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e*sin(c + d*x))^(5/2))/(b + a*cos(c + d*x)), x)
```

3.235 $\int \frac{(e \sin(c+dx))^{3/2}}{a+b \sec(c+dx)} \, dx$

Optimal. Leaf size=444

$$-\frac{b\sqrt[4]{a^2-b^2}\, e^{3/2} \text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{e \sin(c+dx)}}{\sqrt[4]{a^2-b^2}\,\sqrt{e}}\right)}{a^{5/2} d}-\frac{b\sqrt[4]{a^2-b^2}\, e^{3/2} \tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{e \sin(c+dx)}}{\sqrt[4]{a^2-b^2}\,\sqrt{e}}\right)}{a^{5/2} d}+\frac{2(a^2-}{}$$

```
[Out] -b*(a^2-b^2)^(1/4)*e^(3/2)*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1
/4)/e^(1/2))/a^(5/2)/d-b*(a^2-b^2)^(1/4)*e^(3/2)*arctanh(a^(1/2)*(e*sin(d*x
+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^(5/2)/d-2/3*(a^2-3*b^2)*e^2*(sin(1/2*
c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/
4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/a^3/d/(e*sin(d*x+c))^(1/2)-b^2*(a^2
-b^2)*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ell
ipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+
c)^(1/2)/a^3/d/(a^2-b^2-a*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-b^2*(a^2-b^
2)*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ellipt
icPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^
(1/2)/a^3/d/(a^2-b^2+a*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+2/3*e*(3*b-a*c
os(d*x+c))*(e*sin(d*x+c))^(1/2)/a^2/d
```

Rubi [A]
time = 0.74, antiderivative size = 444, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3957, 2944, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$\frac{2e\sqrt{e \sin(c+dx)}\,(3b-a\cos(c+dx))}{3a^2 d}-\frac{be^{3/2}\sqrt[4]{a^2-b^2}\, \text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{e \sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{a^2-b^2}}\right)}{a^{5/2} d}-\frac{be^{3/2}\sqrt[4]{a^2-b^2}\, \tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{e \sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{a^2-b^2}}\right)}{a^{5/2} d}+\frac{2e^2(a^2-3b^2)\sqrt{\sin(c+dx)}\, F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{3a^3 d\sqrt{e \sin(c+dx)}}+\frac{b^2e^2(a^2-b^2)\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{a^3 d\left(-a\sqrt{a^2-b^2}+a^2-b^2\right)\sqrt{e \sin(c+dx)}}+\frac{b^2e^2(a^2-b^2)\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2a}{a+\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{a^3 d\left(a\sqrt{a^2-b^2}+a^2-b^2\right)\sqrt{e \sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(3/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] -((b*(a^2 - b^2)^(1/4)*e^(3/2)*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2
- b^2)^(1/4)*Sqrt[e])])/(a^(5/2)*d)) - (b*(a^2 - b^2)^(1/4)*e^(3/2)*ArcTanh
[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(5/2)*d) +
 (2*(a^2 - 3*b^2)*e^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/
(3*a^3*d*Sqrt[e*Sin[c + d*x]]) + (b^2*(a^2 - b^2)*e^2*EllipticPi[(2*a)/(a -
 Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^3*(a^2 - b
^2 - a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (b^2*(a^2 - b^2)*e^2*Elli
pticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x
]])/(a^3*(a^2 - b^2 + a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (2*e*(3*
b - a*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]])/(3*a^2*d)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
```

```
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^{3/2}}{a+b\sec(c+dx)}\,dx = -\int \frac{\cos(c+dx)(e\sin(c+dx))^{3/2}}{-b-a\cos(c+dx)}\,dx$$

$$= \frac{2e(3b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}{3a^2d} - \frac{(2e^2)\int \frac{-ab+\frac{1}{2}(a^2-3b^2)\cos(c+dx)}{(-b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{3a^2}$$

$$= \frac{2e(3b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}{3a^2d} + \frac{((a^2-3b^2)\,e^2)\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{3a^3} + \cdots$$

$$= \frac{2e(3b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}{3a^2d} + \frac{\left(b^2\sqrt{a^2-b^2}\,e^2\right)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a\cdots}\right.}}{2a^3}$$

$$= \frac{2(a^2-3b^2)\,e^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{3a^3d\sqrt{e\sin(c+dx)}} + \frac{2e(3b-a\cos(c+dx))\sqrt{e\sin\cdots}}{3a^2d}$$

$$= \frac{2(a^2-3b^2)\,e^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{3a^3d\sqrt{e\sin(c+dx)}} - \frac{b^2\sqrt{a^2-b^2}\,e^2\Pi\left(\frac{2a}{a-\sqrt{a^2-b\cdots}}\right.}{a^3\left(a-\sqrt{a^2-\cdots}\right.}$$

$$= -\frac{b\sqrt[4]{a^2-b^2}\,e^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{5/2}d} - \frac{b\sqrt[4]{a^2-b^2}\,e^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\cdots}}{\sqrt[4]{a\cdots}}\right.}{a^{5/2}d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 43.40, size = 1959, normalized size = 4.41

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sin[c + d*x])^(3/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-2*(b + a*Cos[c + d*x])*Csc[c + d*x]*(e*Sin[c + d*x])^(3/2))/(3*a*d*(a + b
*Sec[c + d*x])) + ((b + a*Cos[c + d*x])*Sec[c + d*x]*(e*Sin[c + d*x])^(3/2)
*((4*a*Cos[c + d*x]^2*(b + a*Sqrt[1 - Sin[c + d*x]^2])*((b*(-2*ArcTan[1 - (
Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + 2*ArcTan[1 + (Sqr
t[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2]
 - Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]]
+ Log[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*
x]] + a*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(3/4)) - (5*a*(a^2
- b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^
```

```
2 - b^2)]*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((5*(a^2 - b^2)*Appe
llF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] +
 2*(2*a^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/
(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (a^2
*Sin[c + d*x]^2)/(a^2 - b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x
]^2)))))/((b + a*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) - (2*b*Cos[c + d*x]*(b
 + a*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[a]*(2*ArcTan[1 - ((1 + I
)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sq
rt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + Log[Sqrt[a^2 - b^2] - (1 + I
)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]] - Log[Sq
rt[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*
Sin[c + d*x]]))/(a^2 - b^2)^(3/4) + (5*b*(a^2 - b^2)*AppellF1[1/4, 1/2, 1,
5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sin[c + d*x]])/
(Sqrt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c +
 d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + 2*(2*a^2*AppellF1[5/4, 1/2, 2,
 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + (a^2 - b^2)*Appel
lF1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)])*Si
n[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x]^2)))))/((b + a*Cos[c + d*x])*Sq
rt[1 - Sin[c + d*x]^2]) + (3*b*Cos[c + d*x]*Cos[2*(c + d*x)]*(b + a*Sqrt[1
- Sin[c + d*x]^2])*(((1/2 - I/2)*(a^2 - 2*b^2)*ArcTan[1 - ((1 + I)*Sqrt[a]*
Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)])/(a^(3/2)*(a^2 - b^2)^(3/4)) - ((1/2
 - I/2)*(a^2 - 2*b^2)*ArcTan[1 + ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2
- b^2)^(1/4)])/(a^(3/2)*(a^2 - b^2)^(3/4)) + ((1/4 - I/4)*(a^2 - 2*b^2)*Log
[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I
*a*Sin[c + d*x]])/(a^(3/2)*(a^2 - b^2)^(3/4)) - ((1/4 - I/4)*(a^2 - 2*b^2)*
Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]]
+ I*a*Sin[c + d*x]])/(a^(3/2)*(a^2 - b^2)^(3/4)) + (4*Sqrt[Sin[c + d*x]])/a
 + (4*b*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^
2 - b^2)]*Sin[c + d*x]^(5/2))/(5*(a^2 - b^2)) + (10*b*(a^2 - b^2)*AppellF1[
1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Si
n[c + d*x]])/(Sqrt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1,
 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + 2*(2*a^2*AppellF1
[5/4, 1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + (a^2
 - b^2)*AppellF1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^
2 - b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x]^2)))))/((b + a*Cos
[c + d*x])*(1 - 2*Sin[c + d*x]^2)*Sqrt[1 - Sin[c + d*x]^2])))/(6*a*d*(a + b
*Sec[c + d*x])*Sin[c + d*x]^(3/2))
```

Maple [A]
time = 0.38, size = 860, normalized size = 1.94

method	result

default	$\frac{\frac{2eb\sqrt{e\sin(dx+c)}}{a^2}+\frac{e^3b\left(\frac{e^2(a^2-b^2)}{a^2}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{e\sin(dx+c)}}{\left(\frac{e^2(a^2-b^2)}{a^2}\right)^{\frac{1}{4}}}\right)}{-a^2e^2+b^2e^2}-\frac{e^3b^3\left(\frac{e^2(a^2-b^2)}{a^2}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{e\sin(dx+c)}}{\left(\frac{e^2(a^2-b^2)}{a^2}\right)^{\frac{1}{4}}}\right)}{a^2(-a^2e^2+b^2e^2)}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(3/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] (2*e*b/a^2*(e*sin(d*x+c))^(1/2)+e^3*b*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b
^2*e^2)*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))-e^3*b^3/a^2*
(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*arctan((e*sin(d*x+c))^(1/2)/(e
^2*(a^2-b^2)/a^2)^(1/4))+1/2*e^3*b*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*
e^2)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1
/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))-1/2*e^3*b^3/a^2*(e^2*(a^2-b^2)/a^2)^(1/4)/(
-a^2*e^2+b^2*e^2)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*s
in(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))+(cos(d*x+c)^2*e*sin(d*x+c))^(1
/2)*a*e^2*(-1/3/a^2/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*((-sin(d*x+c)+1)^(1/2
)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1
/2*2^(1/2))+2*cos(d*x+c)^2*sin(d*x+c))+b^2/a^4*(-sin(d*x+c)+1)^(1/2)*(2*sin
(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*Ellipti
cF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-b^2*(a^2-b^2)/a^4*(-1/2/(a^2-b^2)^(1/
2)/a*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x
+c)^2*e*sin(d*x+c))^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^
(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+1/2/(a^2-b^2)^(1/2)/a*(-sin(d*x+
c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x
+c))^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2
-b^2)^(1/2)/a),1/2*2^(1/2))))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] e^(3/2)*integrate(sin(d*x + c)^(3/2)/(b*sec(d*x + c) + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e \sin (c + dx))^{\frac{3}{2}}}{a + b \sec (c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(3/2)/(a+b*sec(d*x+c)),x)
[Out] Integral((e*sin(c + d*x))**(3/2)/(a + b*sec(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((e*sin(d*x + c))^(3/2)/(b*sec(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos (c + d\, x)\ (e\ \sin (c + d\, x))^{3/2}}{b + a\ \cos (c + d\, x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(3/2)/(a + b/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e*sin(c + d*x))^(3/2))/(b + a*cos(c + d*x)), x)
```

3.236 $\int \frac{\sqrt{e\sin(c+dx)}}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=356

$$\frac{b\sqrt{e}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\,\sqrt{e}}\right)}{a^{3/2}\sqrt[4]{a^2-b^2}\,d}-\frac{b\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\,\sqrt{e}}\right)}{a^{3/2}\sqrt[4]{a^2-b^2}\,d}-\frac{b^2e\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c-\frac{\pi}{2}\right.\right.}{a^2\left(a-\sqrt{a^2-b^2}\right)d}$$

```
[Out] b*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))*e^(1/2)/a^(3
/2)/(a^2-b^2)^(1/4)/d-b*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4
)/e^(1/2))*e^(1/2)/a^(3/2)/(a^2-b^2)^(1/4)/d+b^2*e*(sin(1/2*c+1/4*Pi+1/2*d*
x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),
2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^2/d/(a-(a^2-b^2)^(1/2))
/(e*sin(d*x+c))^(1/2)+b^2*e*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1
/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1/2))
,2^(1/2))*sin(d*x+c)^(1/2)/a^2/d/(a+(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-2
*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(co
s(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/a/d/sin(d*x+c)^(1/2)
```

Rubi [A]
time = 0.58, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3957, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$-\frac{b^2e\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{a^2d\left(a-\sqrt{a^2-b^2}\right)\sqrt{e\sin(c+dx)}}-\frac{b^2e\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2a}{a+\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{a^2d\left(\sqrt{a^2-b^2}+a\right)\sqrt{e\sin(c+dx)}}+\frac{b\sqrt{e}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{a^2-b^2}}\right)}{a^{3/2}d\sqrt[4]{a^2-b^2}}-\frac{b\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{a^2-b^2}}\right)}{a^{3/2}d\sqrt[4]{a^2-b^2}}+\frac{2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{ad\sqrt{\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Sin[c + d*x]]/(a + b*Sec[c + d*x]),x]
```

```
[Out] (b*Sqrt[e]*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e]
)])/(a^(3/2)*(a^2 - b^2)^(1/4)*d) - (b*Sqrt[e]*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[
c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(3/2)*(a^2 - b^2)^(1/4)*d) - (b
^2*e*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Si
n[c + d*x]])/(a^2*(a - Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) - (b^2*e*El
lipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d
*x]])/(a^2*(a + Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (2*EllipticE[(c
- Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(a*d*Sqrt[Sin[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{e\sin(c+dx)}}{a+b\sec(c+dx)}\,dx = -\int \frac{\cos(c+dx)\sqrt{e\sin(c+dx)}}{-b-a\cos(c+dx)}\,dx$$

$$= \frac{\int \sqrt{e\sin(c+dx)}\,dx}{a} + \frac{b\int \frac{\sqrt{e\sin(c+dx)}}{-b-a\cos(c+dx)}\,dx}{a}$$

$$= \frac{(b^2e)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{2a^2} - \frac{(b^2e)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2\right.}}$$

$$= \frac{2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{ad\sqrt{\sin(c+dx)}} + \frac{(2be)\text{Subst}\left(\int \frac{x^2}{(-a^2+b^2)e^2+a^2x^4}\,dx, x, \sqrt{e\,\mathrm{s}\right.}{d}$$

$$= -\frac{b^2e\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{a^2\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} - \frac{b^2e\Pi\left(\frac{2a}{a+\sqrt{a^2-b^2}};\frac{1}{2}\right.}{a^2\left(a+\sqrt{a^2-\right.}}$$

$$= \frac{b\sqrt{e}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{3/2}\sqrt[4]{a^2-b^2}\,d} - \frac{b\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{3/2}\sqrt[4]{a^2-b^2}\,d} - \frac{b^2e}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 50.45, size = 351, normalized size = 0.99

$$\frac{\left(b+a\sqrt{\cos^2(c+dx)}\right)\sqrt{e\sin(c+dx)}\left(3\sqrt{2}\,b(-a^2+b^2)^{3/4}\left(2\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{a}\sqrt{\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}}\right)-2\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{a}\sqrt{\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}}\right)-\log\left(\sqrt{-a^2+b^2}-\sqrt{2}\sqrt{a}\sqrt[4]{-a^2+b^2}\sqrt{\sin(c+dx)}+a\sin(c+dx)\right)+\log\left(\sqrt{-a^2+b^2}+\sqrt{2}\sqrt{a}\sqrt[4]{-a^2+b^2}\sqrt{\sin(c+dx)}+a\sin(c+dx)\right)\right)+8a^{5/2}F_1\left(\frac{3}{4};-\frac{1}{2},1;\frac{7}{4};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)\sin^{\frac{3}{2}}(c+dx)\right)}{12a^{3/2}\left(a^2-b^2\right)d(b+a\cos(c+dx))\sqrt{\sin(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[e*Sin[c + d*x]]/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((b + a*Sqrt[Cos[c + d*x]^2])*Sqrt[e*Sin[c + d*x]]*(3*Sqrt[2]*b*(-a^2 + b^2
)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/
4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)]
 - Log[Sqrt[-a^2 + b^2] - Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d
*x]] + a*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2
)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]]) + 8*a^(5/2)*AppellF1[3/4, -1/
2, 1, 7/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(
3/2)))/(12*a^(3/2)*(a^2 - b^2)*d*(b + a*Cos[c + d*x])*Sqrt[Sin[c + d*x]])
```

Maple [A]
time = 0.24, size = 460, normalized size = 1.29

method	result
default	$\frac{\frac{eb\arctan\left(\frac{\sqrt{e\sin(dx+c)}}{\left(\frac{e^2(a^2-b^2)}{a^2}\right)^{\frac{1}{4}}}\right)}{a^2\left(\frac{e^2(a^2-b^2)}{a^2}\right)^{\frac{1}{4}}}-\frac{eb\ln\left(\frac{\sqrt{e\sin(dx+c)}+\left(\frac{e^2(a^2-b^2)}{a^2}\right)^{\frac{1}{4}}}{\sqrt{e\sin(dx+c)}-\left(\frac{e^2(a^2-b^2)}{a^2}\right)^{\frac{1}{4}}}\right)}{2a^2\left(\frac{e^2(a^2-b^2)}{a^2}\right)^{\frac{1}{4}}}+\frac{e\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(d}}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(1/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] (e*b/a^2/(e^2*(a^2-b^2)/a^2)^(1/4)*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^
2)/a^2)^(1/4))-1/2*e*b/a^2/(e^2*(a^2-b^2)/a^2)^(1/4)*ln(((e*sin(d*x+c))^(1/
2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/
4)))+1/2*e*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*b^
2*(4*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a-2*EllipticF((-sin(d*x+c
)+1)^(1/2),1/2*2^(1/2))*a-EllipticPi((-sin(d*x+c)+1)^(1/2),-a/((a^2-b^2)^(1
/2)-a),1/2*2^(1/2))*(a^2-b^2)^(1/2)-EllipticPi((-sin(d*x+c)+1)^(1/2),-a/((a
^2-b^2)^(1/2)-a),1/2*2^(1/2))*a+EllipticPi((-sin(d*x+c)+1)^(1/2),a/(a+(a^2-
b^2)^(1/2)),1/2*2^(1/2))*(a^2-b^2)^(1/2)-EllipticPi((-sin(d*x+c)+1)^(1/2),a
/(a+(a^2-b^2)^(1/2)),1/2*2^(1/2))*a)/a^2/((a^2-b^2)^(1/2)-a)/(a+(a^2-b^2)^(
1/2))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
[Out] e^(1/2)*integrate(sqrt(sin(d*x + c))/(b*sec(d*x + c) + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\sin{(c+dx)}}}{a+b\sec{(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(1/2)/(a+b*sec(d*x+c)),x)
[Out] Integral(sqrt(e*sin(c + d*x))/(a + b*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
[Out] integrate(sqrt(e*sin(d*x + c))/(b*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)\,\sqrt{e\,\sin\left(c+d\,x\right)}}{b+a\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(1/2)/(a + b/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e*sin(c + d*x))^(1/2))/(b + a*cos(c + d*x)), x)
```

3.237 $\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\sin(c+dx)}}\,dx$

Optimal. Leaf size=370

$$-\frac{b\mathrm{ArcTan}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{\sqrt{a}\,(a^2-b^2)^{3/4}\,d\sqrt{e}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{\sqrt{a}\,(a^2-b^2)^{3/4}\,d\sqrt{e}}+\frac{2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{ad\sqrt{e\sin(c+dx)}}$$

```
[Out] -b*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/(a^2-b^2)^(
3/4)/d/a^(1/2)/e^(1/2)-b*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/
4)/e^(1/2))/(a^2-b^2)^(3/4)/d/a^(1/2)/e^(1/2)-2*(sin(1/2*c+1/4*Pi+1/2*d*x)^
2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1
/2))*sin(d*x+c)^(1/2)/a/d/(e*sin(d*x+c))^(1/2)-b^2*(sin(1/2*c+1/4*Pi+1/2*d*
x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),
2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a/d/(a^2-b^2-a*(a^2-b^2)^
(1/2))/(e*sin(d*x+c))^(1/2)-b^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2
*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1
/2)),2^(1/2))*sin(d*x+c)^(1/2)/a/d/(a^2-b^2+a*(a^2-b^2)^(1/2))/(e*sin(d*x+c
))^(1/2)
```

Rubi [A]
time = 0.55, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {3957, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$-\frac{b\mathrm{ArcTan}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{\sqrt{a}\,d\sqrt{e}\,(a^2-b^2)^{3/4}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{\sqrt{a}\,d\sqrt{e}\,(a^2-b^2)^{3/4}}+\frac{b^2\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{ad\left(-a\sqrt{a^2-b^2}+a^2-b^2\right)\sqrt{e\sin(c+dx)}}+\frac{b^2\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2a}{a+\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{ad\left(a\sqrt{a^2-b^2}+a^2-b^2\right)\sqrt{e\sin(c+dx)}}+\frac{2\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{ad\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sec[c + d*x])*Sqrt[e*Sin[c + d*x]]),x]
```

```
[Out] -((b*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(S
qrt[a]*(a^2 - b^2)^(3/4)*d*Sqrt[e])) - (b*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d
*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(Sqrt[a]*(a^2 - b^2)^(3/4)*d*Sqrt[e]) +
 (2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a*d*Sqrt[e*Sin[c
+ d*x]]) + (b^2*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2,
 2]*Sqrt[Sin[c + d*x]])/(a*(a^2 - b^2 - a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c +
 d*x]]) + (b^2*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2,
2]*Sqrt[Sin[c + d*x]])/(a*(a^2 - b^2 + a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c +
d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\sin(c+dx)}}\,dx = -\int \frac{\cos(c+dx)}{(-b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx$$

$$= \frac{\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{a} + \frac{b\int \frac{1}{(-b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{a}$$

$$= \frac{b^2\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{2a\sqrt{a^2-b^2}} + \frac{b^2\int \frac{1}{\sqrt{e\sin(c}}}{}$$

$$= \frac{2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{ad\sqrt{e\sin(c+dx)}} + \frac{(2be)\text{Subst}\left(\int \frac{1}{(-a^2+b^2)}\right.}{}$$

$$= \frac{2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{ad\sqrt{e\sin(c+dx)}} + \frac{b^2\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\right.}{a\left(a^2-b^2-a\sqrt{a}\right.}$$

$$= -\frac{b\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{\sqrt{a}\left(a^2-b^2\right)^{3/4}d\sqrt{e}} - \frac{b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+}}{\sqrt[4]{a^2-b^2}\sqrt{}}\right.}{\sqrt{a}\left(a^2-b^2\right)^{3/4}d\sqrt{}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 14.00, size = 546, normalized size = 1.48

$$\frac{2\left(b+a\sqrt{\cos^2(c+dx)}\right)\sqrt{\sin(c+dx)}\left(\frac{b\left(-2\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{a}\sqrt{\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}}\right)+2\text{ArcTan}\left(1+\frac{\sqrt{2}\sqrt{a}\sqrt{\sin(c+dx)}}{\sqrt[4]{-a^2+b^2}}\right)-\log\left(\sqrt{-a^2+b^2}-\sqrt{2}\sqrt{a}\sqrt[4]{-a^2+b^2}\sqrt{\sin(c+dx)}+a\sin(c+dx)\right)+\log\left(\sqrt{-a^2+b^2}+\sqrt{2}\sqrt{a}\sqrt[4]{-a^2+b^2}\sqrt{\sin(c+dx)}+a\sin(c+dx)\right)\right)}{4\sqrt{2}\sqrt{a}(-a^2+b^2)^{3/4}}-\frac{5a(a^2-b^2)F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)\sqrt{\cos^2(c+dx)}\sqrt{\sin(c+dx)}}{(-a^2+b^2+a^2\sin^2(c+dx))\left(5(a^2-b^2)F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)+2\left(2a^2F_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)+(-a^2+b^2)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)\right)\sin^2(c+dx)\right)}\right)}{d(b+a\cos(c+dx))\sqrt{e\sin(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sec[c + d*x])*Sqrt[e*Sin[c + d*x]]),x]
```

```
[Out] (2*(b + a*Sqrt[Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]]*((b*(-2*ArcTan[1 - (Sqrt
[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]
*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - S
qrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]] + Lo
g[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]]
+ a*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(3/4)) - (5*a*(a^2 - b^
2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 -
b^2)]*Sqrt[Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]])/((-a^2 + b^2 + a^2*Sin[c + d
*x]^2)*(5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[
c + d*x]^2)/(a^2 - b^2)] + 2*(2*a^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x
]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[5/4, 1/2, 1,
 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)])*Sin[c + d*x]^2))))
/(d*(b + a*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]])
```

Maple [A]
time = 0.25, size = 530, normalized size = 1.43

method	result
default	$\frac{b e\left(\frac{e^2(a^2-b^2)}{a^2}\right)^{\frac{1}{4}}\ln\left(\frac{\sqrt{e\sin(dx+c)}+\left(\frac{e^2(a^2-b^2)}{a^2}\right)^{\frac{1}{4}}}{\sqrt{e\sin(dx+c)}-\left(\frac{e^2(a^2-b^2)}{a^2}\right)^{\frac{1}{4}}}\right)}{-2a^2e^2+2b^2e^2}+\frac{b e\left(\frac{e^2(a^2-b^2)}{a^2}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{e\sin(dx+c)}}{\left(\frac{e^2(a^2-b^2)}{a^2}\right)^{\frac{1}{4}}}\right)}{-a^2e^2+b^2e^2}-\frac{\sqrt{-\sin(\ldots}}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (1/2*b*e*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*ln(((e*sin(d*x+c))^(1
/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1
/4)))+b*e*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*arctan((e*sin(d*x+c)
)^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))-1/2/a*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c
)+2)^(1/2)*sin(d*x+c)^(1/2)*(2*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))
*(a^2-b^2)^(3/2)-2*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*(a^2-b^2)^(
1/2)*a^2+EllipticPi((-sin(d*x+c)+1)^(1/2),-a/((a^2-b^2)^(1/2)-a),1/2*2^(1/2
))*(a^2-b^2)^(1/2)*b^2+EllipticPi((-sin(d*x+c)+1)^(1/2),-a/((a^2-b^2)^(1/2)
```

```
-a),1/2*2^(1/2))*a*b^2+EllipticPi((-sin(d*x+c)+1)^(1/2),a/(a+(a^2-b^2)^(1/2
)),1/2*2^(1/2))*(a^2-b^2)^(1/2)*b^2-EllipticPi((-sin(d*x+c)+1)^(1/2),a/(a+(
a^2-b^2)^(1/2)),1/2*2^(1/2))*a*b^2)/(a^2-b^2)^(1/2)/((a^2-b^2)^(1/2)-a)/(a+
(a^2-b^2)^(1/2))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
[Out] e^(-1/2)*integrate(1/((b*sec(d*x + c) + a)*sqrt(sin(d*x + c))), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{e\sin(c+dx)}\,(a+b\sec(c+dx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))**(1/2),x)
[Out] Integral(1/(sqrt(e*sin(c + d*x))*(a + b*sec(c + d*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*sec(d*x + c) + a)*sqrt(e*sin(d*x + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+dx)}{\sqrt{e\sin(c+dx)}\ (b+a\cos(c+dx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*sin(c + d*x))^(1/2)*(a + b/cos(c + d*x))),x)
[Out] int(cos(c + d*x)/((e*sin(c + d*x))^(1/2)*(b + a*cos(c + d*x))), x)
```

3.238 $\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=430

$$\frac{\sqrt{a}\,b\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\,\sqrt{e}}\right)}{(a^2-b^2)^{5/4}\,de^{3/2}}-\frac{\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\,\sqrt{e}}\right)}{(a^2-b^2)^{5/4}\,de^{3/2}}+\frac{2(b-a\cos(c+dx))}{(a^2-b^2)\,de\sqrt{e\sin(c+dx)}}$$

```
[Out] b*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))*a^(1/2)/(a^2
-b^2)^(5/4)/d/e^(3/2)-b*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4
)/e^(1/2))*a^(1/2)/(a^2-b^2)^(5/4)/d/e^(3/2)+2*(b-a*cos(d*x+c))/(a^2-b^2)/d
/e/(e*sin(d*x+c))^(1/2)+b^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1
/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a-(a^2-b^2)^(1/2))
,2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)/d/e/(a-(a^2-b^2)^(1/2))/(e*sin(d*x+c))
^(1/2)+b^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*El
lipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x
+c)^(1/2)/(a^2-b^2)/d/e/(a+(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+2*a*(sin(1
/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c
+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/(a^2-b^2)/d/e^2/sin(d*x+c)^(
1/2)
```

Rubi [A]
time = 0.74, antiderivative size = 430, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3957, 2945, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$\frac{\sqrt{a}\,b\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{a^2-b^2}}\right)}{de^{3/2}\,(a^2-b^2)^{5/4}}-\frac{\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{a^2-b^2}}\right)}{de^{3/2}\,(a^2-b^2)^{5/4}}-\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{de^2\,(a^2-b^2)\sqrt{\sin(c+dx)}}+\frac{2(b-a\cos(c+dx))}{de\,(a^2-b^2)\sqrt{e\sin(c+dx)}}-\frac{b^2\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{de\,(a^2-b^2)\left(a-\sqrt{a^2-b^2}\right)\sqrt{e\sin(c+dx)}}-\frac{b^2\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2a}{a+\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{de\,(a^2-b^2)\left(\sqrt{a^2-b^2}+a\right)\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sec[c + d*x])*(e*Sin[c + d*x])^(3/2)),x]
```

```
[Out] (Sqrt[a]*b*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e]
)])/((a^2 - b^2)^(5/4)*d*e^(3/2)) - (Sqrt[a]*b*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[
c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/((a^2 - b^2)^(5/4)*d*e^(3/2)) + (2
*(b - a*Cos[c + d*x]))/((a^2 - b^2)*d*e*Sqrt[e*Sin[c + d*x]]) - (b^2*Ellipt
icPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]]
)/((a^2 - b^2)*(a - Sqrt[a^2 - b^2])*d*e*Sqrt[e*Sin[c + d*x]]) - (b^2*Ellip
ticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]
])/((a^2 - b^2)*(a + Sqrt[a^2 - b^2])*d*e*Sqrt[e*Sin[c + d*x]]) - (2*a*Elli
pticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)*d*e^2*Sqrt[
Sin[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
```

```
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{3/2}}\,dx = -\int \frac{\cos(c+dx)}{(-b-a\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx$$

$$= \frac{2(b-a\cos(c+dx))}{(a^2-b^2)\,de\sqrt{e\sin(c+dx)}} + \frac{2\int \frac{\left(ab+\frac{1}{2}a^2\cos(c+dx)\right)\sqrt{e\sin(c+d}}{-b-a\cos(c+dx)}}{(a^2-b^2)\,e^2}$$

$$= \frac{2(b-a\cos(c+dx))}{(a^2-b^2)\,de\sqrt{e\sin(c+dx)}} - \frac{a\int\sqrt{e\sin(c+dx)}\,dx}{(a^2-b^2)\,e^2} + \frac{(ab)\int}{}$$

$$= \frac{2(b-a\cos(c+dx))}{(a^2-b^2)\,de\sqrt{e\sin(c+dx)}} + \frac{b^2\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}\right.}}{2\,(a^2-b^2)\,e}$$

$$= \frac{2(b-a\cos(c+dx))}{(a^2-b^2)\,de\sqrt{e\sin(c+dx)}} - \frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{e\sin(}}{(a^2-b^2)\,de^2\sqrt{\sin(c+dx}}$$

$$= \frac{2(b-a\cos(c+dx))}{(a^2-b^2)\,de\sqrt{e\sin(c+dx)}} - \frac{b^2\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right.\right.}{(a^2-b^2)\left(a-\sqrt{a^2-b^2}\right)d}$$

$$= \frac{\sqrt{a}\,b\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{(a^2-b^2)^{5/4}\,de^{3/2}} - \frac{\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\,s}}{\sqrt[4]{a^2-}}\right.}{(a^2-b^2)^{5/4}\,de}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 21.99, size = 711, normalized size = 1.65

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sec[c + d*x])*(e*Sin[c + d*x])^(3/2)),x]
```

```
[Out] ((b + a*Cos[c + d*x])*(-((Sqrt[a]*(b + a*Sqrt[Cos[c + d*x]^2])*Sec[c + d*x]
^2*Sin[c + d*x]^(3/2)*(Cos[c + d*x]*(3*Sqrt[2]*b*(-a^2 + b^2)^(3/4)*(2*ArcT
an[1 - (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[
1 + (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^
2 + b^2] - Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c
+ d*x]] + Log[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Si
n[c + d*x]] + a*Sin[c + d*x]]) + 8*a^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[
c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(3/2)) + (2 + 2*
```

```
I)*b*Sqrt[Cos[c + d*x]^2]*(3*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - ((1 + I)*Sqrt[
a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[a]*S
qrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[
a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]] + Log[Sqrt[a^2
- b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c +
 d*x]]) - (4 - 4*I)*Sqrt[a]*b*AppellF1[3/4, 1/2, 1, 7/4, Sin[c + d*x]^2, (a
^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(3/2))))/((a - b)*(a + b)*(b +
 a*Cos[c + d*x]))) + 24*(b - a*Cos[c + d*x])*Tan[c + d*x]))/(12*(a^2 - b^2)
*d*(a + b*Sec[c + d*x])*(e*Sin[c + d*x])^(3/2))
```

Maple [A]
time = 0.26, size = 689, normalized size = 1.60

method	result
default	$\frac{\frac{b\arctan\left(\frac{\sqrt{e\sin\left(dx+c\right)}}{\left(\frac{e^{2}\left(a^{2}-b^{2}\right)}{a^{2}}\right)^{\frac{1}{4}}}\right)}{e(a+b)(a-b)\left(\frac{e^{2}\left(a^{2}-b^{2}\right)}{a^{2}}\right)^{\frac{1}{4}}}-\frac{b\ln\left(\frac{\sqrt{e\sin\left(dx+c\right)}+\left(\frac{e^{2}\left(a^{2}-b^{2}\right)}{a^{2}}\right)^{\frac{1}{4}}}{\sqrt{e\sin\left(dx+c\right)}-\left(\frac{e^{2}\left(a^{2}-b^{2}\right)}{a^{2}}\right)^{\frac{1}{4}}}\right)}{2e(a+b)(a-b)\left(\frac{e^{2}\left(a^{2}-b^{2}\right)}{a^{2}}\right)^{\frac{1}{4}}}+\frac{2b}{e\left(a^{2}-b^{2}\right)\sqrt{e\sin\left(dx+c\right)}}-\frac{b^{2}\left(\sqrt{a^{2}-\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (1/e*b/(a+b)/(a-b)/(e^2*(a^2-b^2)/a^2)^(1/4)*arctan((e*sin(d*x+c))^(1/2)/(e
^2*(a^2-b^2)/a^2)^(1/4))-1/2/e*b/(a+b)/(a-b)/(e^2*(a^2-b^2)/a^2)^(1/4)*ln((
(e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*
(a^2-b^2)/a^2)^(1/4)))+2/e*b/(a^2-b^2)/(e*sin(d*x+c))^(1/2)-1/2*b^2*((a^2-b
^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*Ell
ipticPi((-sin(d*x+c)+1)^(1/2),-a/((a^2-b^2)^(1/2)-a),1/2*2^(1/2))-(a^2-b^2)
^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*Ellipt
icPi((-sin(d*x+c)+1)^(1/2),a/(a+(a^2-b^2)^(1/2)),1/2*2^(1/2))+4*(-sin(d*x+c
)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1
)^(1/2),1/2*2^(1/2))*a-2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d
*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a+(-sin(d*x+c)+1)^
(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*a*EllipticPi((-sin(d*x+c)+1)^
(1/2),-a/((a^2-b^2)^(1/2)-a),1/2*2^(1/2))+(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+
c)+2)^(1/2)*sin(d*x+c)^(1/2)*a*EllipticPi((-sin(d*x+c)+1)^(1/2),a/(a+(a^2-b
^2)^(1/2)),1/2*2^(1/2))-4*cos(d*x+c)^2*a)/e/(a+(a^2-b^2)^(1/2))/((a^2-b^2)^
(1/2)-a)/(a-b)/(a+b)/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(sin(d*x + c))/(a*cos(d*x + c)^2*e^(3/2) - a*e^(3/2) + (b*cos
(d*x + c)^2*e^(3/2) - b*e^(3/2))*sec(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e\sin\left(c+dx\right)\right)^{\frac{3}{2}}\left(a+b\sec\left(c+dx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(1/((e*sin(c + d*x))**(3/2)*(a + b*sec(c + d*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)*(e*sin(d*x + c))^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)}{{\left(e\,\sin\left(c+d\,x\right)\right)}^{3/2}\,\left(b+a\,\cos\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*sin(c + d*x))^(3/2)*(a + b/cos(c + d*x))),x)
```

```
[Out] int(cos(c + d*x)/((e*sin(c + d*x))^(3/2)*(b + a*cos(c + d*x))), x)
```

3.239 $\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=452

$$-\frac{a^{3/2}b\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\,\sqrt{e}}\right)}{\left(a^2-b^2\right)^{7/4}de^{5/2}}-\frac{a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\,\sqrt{e}}\right)}{\left(a^2-b^2\right)^{7/4}de^{5/2}}+\frac{2(b-a\cos(c+dx))}{3\left(a^2-b^2\right)de(e\sin(c+dx)}$$

```
[Out] -a^(3/2)*b*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/(a^
2-b^2)^(7/4)/d/e^(5/2)-a^(3/2)*b*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-
b^2)^(1/4)/e^(1/2))/(a^2-b^2)^(7/4)/d/e^(5/2)+2/3*(b-a*cos(d*x+c))/(a^2-b^2
)/d/e/(e*sin(d*x+c))^(3/2)-2/3*a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/
2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)
^(1/2)/(a^2-b^2)/d/e^2/(e*sin(d*x+c))^(1/2)-a*b^2*(sin(1/2*c+1/4*Pi+1/2*d*x
)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2
*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)/d/e^2/(a^2-b^2-a
*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-a*b^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^
(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a
+(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)/d/e^2/(a^2-b^2+a*(a^2
-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.76, antiderivative size = 452, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3957, 2945, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$\frac{2a\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{3de^2\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}+\frac{ab^2\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{de^2\left(a^2-b^2\right)\left(-a\sqrt{a^2-b^2}+a^2-b^2\right)\sqrt{e\sin(c+dx)}}+\frac{ab^2\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2a}{a+\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{de^2\left(a^2-b^2\right)\left(a\sqrt{a^2-b^2}+a^2-b^2\right)\sqrt{e\sin(c+dx)}}+\frac{2(b-a\cos(c+dx))}{3de\left(a^2-b^2\right)(e\sin(c+dx))^{3/2}}-\frac{a^{3/2}b\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{a^2-b^2}}\right)}{de^{5/2}\left(a^2-b^2\right)^{7/4}}-\frac{a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{a^2-b^2}}\right)}{de^{5/2}\left(a^2-b^2\right)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sec[c + d*x])*(e*Sin[c + d*x])^(5/2)),x]
```

```
[Out] -((a^(3/2)*b*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[
e])])/((a^2 - b^2)^(7/4)*d*e^(5/2))) - (a^(3/2)*b*ArcTanh[(Sqrt[a]*Sqrt[e*S
in[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/((a^2 - b^2)^(7/4)*d*e^(5/2)) +
 (2*(b - a*Cos[c + d*x]))/(3*(a^2 - b^2)*d*e*(e*Sin[c + d*x])^(3/2)) + (2*a
*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*(a^2 - b^2)*d*e^2*
Sqrt[e*Sin[c + d*x]]) + (a*b^2*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c -
 Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/((a^2 - b^2)*(a^2 - b^2 - a*Sqrt[a^2
 - b^2])*d*e^2*Sqrt[e*Sin[c + d*x]]) + (a*b^2*EllipticPi[(2*a)/(a + Sqrt[a^
2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/((a^2 - b^2)*(a^2 - b
^2 + a*Sqrt[a^2 - b^2])*d*e^2*Sqrt[e*Sin[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
```

```
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{5/2}}\,dx = -\int \frac{\cos(c+dx)}{(-b-a\cos(c+dx))(e\sin(c+dx))^{5/2}}\,dx$$

$$= \frac{2(b-a\cos(c+dx))}{3(a^2-b^2)\,de(e\sin(c+dx))^{3/2}} + \frac{2\int \frac{ab-\frac{1}{2}a^2\cos(c+dx)}{(-b-a\cos(c+dx))\sqrt{e\sin(c+}}}{3(a^2-b^2)\,e^2}$$

$$= \frac{2(b-a\cos(c+dx))}{3(a^2-b^2)\,de(e\sin(c+dx))^{3/2}} + \frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{3(a^2-b^2)\,e^2} + \frac{(ab}{}$$

$$= \frac{2(b-a\cos(c+dx))}{3(a^2-b^2)\,de(e\sin(c+dx))^{3/2}} + \frac{(ab^2)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2}\right.}}{2(a^2-b^2)^{3/}}$$

$$= \frac{2(b-a\cos(c+dx))}{3(a^2-b^2)\,de(e\sin(c+dx))^{3/2}} + \frac{2aF\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin}}{3(a^2-b^2)\,de^2\sqrt{e\sin(c+}}$$

$$= \frac{2(b-a\cos(c+dx))}{3(a^2-b^2)\,de(e\sin(c+dx))^{3/2}} + \frac{2aF\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin}}{3(a^2-b^2)\,de^2\sqrt{e\sin(c+}}$$

$$= -\frac{a^{3/2}b\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{(a^2-b^2)^{7/4}\,de^{5/2}} - \frac{a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e}}{\sqrt[4]{a^2}}\right.}{(a^2-b^2)^{7/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 30.44, size = 1233, normalized size = 2.73

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sec[c + d*x])*(e*Sin[c + d*x])^(5/2)),x]
```

```
[Out] -1/3*(a*(b + a*Cos[c + d*x])*Sec[c + d*x]*Sin[c + d*x]^(5/2)*((-2*a*Cos[c +
 d*x]^2*(b + a*Sqrt[1 - Sin[c + d*x]^2])*((b*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[a
]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[a]*S
qrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - Sqrt[2]*Sqr
t[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]] + Log[Sqrt[-a^
2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c
+ d*x]]))/(4*Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(3/4)) - (5*a*(a^2 - b^2)*AppellF
1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sqrt
```

```
[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((5*(a^2 - b^2)*AppellF1[1/4, -1/2
, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + 2*(2*a^2*Appe
llF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] +
 (-a^2 + b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^
2)/(a^2 - b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x]^2)))))/((b +
 a*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (4*b*Cos[c + d*x]*(b + a*Sqrt[1 -
Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[a]*(2*ArcTan[1 - ((1 + I)*Sqrt[a]*Sqrt
[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[a]*Sqrt[Sin
[c + d*x]])/(a^2 - b^2)^(1/4)] + Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[a]*(a^2
 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]] - Log[Sqrt[a^2 - b^2]
+ (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]])
)/(a^2 - b^2)^(3/4) + (5*b*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d
*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sin[c + d*x]])/(Sqrt[1 - Sin[
c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*
Sin[c + d*x]^2)/(a^2 - b^2)] + 2*(2*a^2*AppellF1[5/4, 1/2, 2, 9/4, Sin[c +
d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + (a^2 - b^2)*AppellF1[5/4, 3/2,
1, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)])*Sin[c + d*x]^2)*
(b^2 + a^2*(-1 + Sin[c + d*x]^2)))))/((b + a*Cos[c + d*x])*Sqrt[1 - Sin[c +
 d*x]^2])))/((a - b)*(a + b)*d*(a + b*Sec[c + d*x])*(e*Sin[c + d*x])^(5/2))
 - (2*(b - a*Cos[c + d*x])*(b + a*Cos[c + d*x])*Tan[c + d*x])/(3*(-a^2 + b^
2)*d*(a + b*Sec[c + d*x])*(e*Sin[c + d*x])^(5/2))
```

Maple [A]
time = 0.39, size = 657, normalized size = 1.45

method	result
default	$\frac{b\,a^2 \left(\frac{e^2(a^2-b^2)}{a^2}\right)^{\frac{1}{4}} \ln\left(\frac{\sqrt{e\sin(dx+c)}+\left(\frac{e^2(a^2-b^2)}{a^2}\right)^{\frac{1}{4}}}{\sqrt{e\sin(dx+c)}-\left(\frac{e^2(a^2-b^2)}{a^2}\right)^{\frac{1}{4}}}\right)}{2e(a+b)(a-b)\left(-a^2e^2+b^2e^2\right)} + \frac{b\,a^2 \left(\frac{e^2(a^2-b^2)}{a^2}\right)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{e\sin(dx+c)}}{\left(\frac{e^2(a^2-b^2)}{a^2}\right)^{\frac{1}{4}}}\right)}{e(a+b)(a-b)\left(-a^2e^2+b^2e^2\right)} + \frac{\cdots}{3e\left(a^2-b\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (1/2/e*b/(a+b)/(a-b)*a^2*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*ln(((
e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(
a^2-b^2)/a^2)^(1/4)))+1/e*b/(a+b)/(a-b)*a^2*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2
*e^2+b^2*e^2)*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))+2/3/e*
```

```
b/(a^2-b^2)/(e*sin(d*x+c))^(3/2)+(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*a/e^2*(1
/3/(a^2-b^2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(cos(d*x+c)^2-1)*((-sin(d*x+
c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(5/2)*EllipticF((-sin(d*x+c)+
1)^(1/2),1/2*2^(1/2))+2*cos(d*x+c)^2*sin(d*x+c))-1/(a-b)/(a+b)*b^2*(-1/2/(a
^2-b^2)^(1/2)/a*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/
2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin
(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+1/2/(a^2-b^2)^(1/2)/a
*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^
2*e*sin(d*x+c))^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2
),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 5010 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)*(e*sin(d*x + c))^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)}{(e\,\sin(c+d\,x))^{5/2}\,\left(b+a\,\cos(c+d\,x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*sin(c + d*x))^(5/2)*(a + b/cos(c + d*x))),x)
```

```
[Out] int(cos(c + d*x)/((e*sin(c + d*x))^(5/2)*(b + a*cos(c + d*x))), x)
```

3.240 $\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=511

$$\frac{a^{5/2}b\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{(a^2-b^2)^{9/4}de^{7/2}}-\frac{a^{5/2}b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{(a^2-b^2)^{9/4}de^{7/2}}+\frac{2(b-a\cos(c+dx))}{5(a^2-b^2)de(e\sin(c+dx))^{5/2}}$$

```
[Out] a^(5/2)*b*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/(a^2
-b^2)^(9/4)/d/e^(7/2)-a^(5/2)*b*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b
^2)^(1/4)/e^(1/2))/(a^2-b^2)^(9/4)/d/e^(7/2)+2/5*(b-a*cos(d*x+c))/(a^2-b^2)
/d/e/(e*sin(d*x+c))^(5/2)+2/5*(5*a^2*b-a*(3*a^2+2*b^2)*cos(d*x+c))/(a^2-b^2
)^2/d/e^3/(e*sin(d*x+c))^(1/2)+a^2*b^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/
sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a-(a^2-
b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^2/d/e^3/(a-(a^2-b^2)^(1/2))
/(e*sin(d*x+c))^(1/2)+a^2*b^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c
+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1/2
)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^2/d/e^3/(a+(a^2-b^2)^(1/2))/(e*sin(d
*x+c))^(1/2)+2/5*a*(3*a^2+2*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/
2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x
+c))^(1/2)/(a^2-b^2)^2/d/e^4/sin(d*x+c)^(1/2)
```

Rubi [A]

time = 0.99, antiderivative size = 511, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3957, 2945, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$-\frac{2a(3a^2+2b^2)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5de^4(a^2-b^2)^2\sqrt{\sin(c+dx)}}+\frac{2(5a^2b-a(3a^2+2b^2)\cos(c+dx))}{5de^3(a^2-b^2)^2\sqrt{e\sin(c+dx)}}-\frac{a^2b^2\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{de^3(a^2-b^2)^2\left(a-\sqrt{a^2-b^2}\right)\sqrt{e\sin(c+dx)}}-\frac{a^2b^2\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2a}{a+\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{de^3(a^2-b^2)^2\left(\sqrt{a^2-b^2}+a\right)\sqrt{e\sin(c+dx)}}+\frac{2(b-a\cos(c+dx))}{5de(a^2-b^2)(e\sin(c+dx))^{5/2}}+\frac{a^{5/2}b\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{de^{7/2}(a^2-b^2)^{9/4}}-\frac{a^{5/2}b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{de^{7/2}(a^2-b^2)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sec[c + d*x])*(e*Sin[c + d*x])^(7/2)),x]
```

```
[Out] (a^(5/2)*b*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e]
)])/((a^2 - b^2)^(9/4)*d*e^(7/2)) - (a^(5/2)*b*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[
c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/((a^2 - b^2)^(9/4)*d*e^(7/2)) + (2
*(b - a*Cos[c + d*x]))/(5*(a^2 - b^2)*d*e*(e*Sin[c + d*x])^(5/2)) + (2*(5*a
^2*b - a*(3*a^2 + 2*b^2)*Cos[c + d*x]))/(5*(a^2 - b^2)^2*d*e^3*Sqrt[e*Sin[c
 + d*x]]) - (a^2*b^2*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*
x)/2, 2]*Sqrt[Sin[c + d*x]])/((a^2 - b^2)^2*(a - Sqrt[a^2 - b^2])*d*e^3*Sqr
t[e*Sin[c + d*x]]) - (a^2*b^2*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c -
Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/((a^2 - b^2)^2*(a + Sqrt[a^2 - b^2])*
d*e^3*Sqrt[e*Sin[c + d*x]]) - (2*a*(3*a^2 + 2*b^2)*EllipticE[(c - Pi/2 + d*
x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(5*(a^2 - b^2)^2*d*e^4*Sqrt[Sin[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
```

```
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{7/2}}\,dx = -\int \frac{\cos(c+dx)}{(-b-a\cos(c+dx))(e\sin(c+dx))^{7/2}}\,dx$$

$$= \frac{2(b-a\cos(c+dx))}{5\left(a^2-b^2\right)de(e\sin(c+dx))^{5/2}} + \frac{2\int \frac{ab-\frac{3}{2}a^2\cos(c+dx)}{(-b-a\cos(c+dx))(e\sin(c+dx))^{3}}}{5\left(a^2-b^2\right)e^2}$$

$$= \frac{2(b-a\cos(c+dx))}{5\left(a^2-b^2\right)de(e\sin(c+dx))^{5/2}} + \frac{2(5a^2b-a(3a^2+2b^2)\cos(c}{5\left(a^2-b^2\right)^2de^3\sqrt{e\sin(c+}}$$

$$= \frac{2(b-a\cos(c+dx))}{5\left(a^2-b^2\right)de(e\sin(c+dx))^{5/2}} + \frac{2(5a^2b-a(3a^2+2b^2)\cos(c}{5\left(a^2-b^2\right)^2de^3\sqrt{e\sin(c+}}$$

$$= \frac{2(b-a\cos(c+dx))}{5\left(a^2-b^2\right)de(e\sin(c+dx))^{5/2}} + \frac{2(5a^2b-a(3a^2+2b^2)\cos(c}{5\left(a^2-b^2\right)^2de^3\sqrt{e\sin(c+}}$$

$$= \frac{2(b-a\cos(c+dx))}{5\left(a^2-b^2\right)de(e\sin(c+dx))^{5/2}} + \frac{2(5a^2b-a(3a^2+2b^2)\cos(c}{5\left(a^2-b^2\right)^2de^3\sqrt{e\sin(c+}}$$

$$= \frac{2(b-a\cos(c+dx))}{5\left(a^2-b^2\right)de(e\sin(c+dx))^{5/2}} + \frac{2(5a^2b-a(3a^2+2b^2)\cos(c}{5\left(a^2-b^2\right)^2de^3\sqrt{e\sin(c+}}$$

$$= \frac{a^{5/2}b\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{\left(a^2-b^2\right)^{9/4}de^{7/2}} - \frac{a^{5/2}b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e}}{\sqrt[4]{a^2}}\right.}{\left(a^2-b^2\right)^{9/4}d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 15.33, size = 797, normalized size = 1.56

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sec[c + d*x])*(e*Sin[c + d*x])^(7/2)),x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sin[c + d*x]^3*((-2*((a^2 - b^2)*(-b + a*Cos[c + d*x]
)*Csc[c + d*x]^2*Sec[c + d*x] + a*(3*a^2 + 2*b^2 - 5*a*b*Sec[c + d*x])))/(a
^2 - b^2)^2 - ((b + a*Sqrt[Cos[c + d*x]^2])*Sec[c + d*x]^2*Sqrt[Sin[c + d*x
]]*((3*a^3 + 2*a*b^2)*Cos[c + d*x]*(3*Sqrt[2]*b*(-a^2 + b^2)^(3/4)*(2*ArcTa
n[1 - (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1
```

```
 + (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2
 + b^2] - Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c +
 d*x]] + Log[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin
[c + d*x]] + a*Sin[c + d*x]]) + 8*a^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c
 + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(3/2)) + (2 + 2*I
)*a*b*(4*a^2 + b^2)*Sqrt[Cos[c + d*x]^2]*(3*(a^2 - b^2)^(3/4)*(2*ArcTan[1 -
 ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + ((1
 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2]
- (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]]
+ Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]
] + I*a*Sin[c + d*x]]) - (4 - 4*I)*Sqrt[a]*b*AppellF1[3/4, 1/2, 1, 7/4, Sin
[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(3/2))))/(12*Sq
rt[a]*(a - b)^2*(a + b)^2*(a^2 - b^2)*(b + a*Cos[c + d*x]))))/(5*d*(a + b*S
ec[c + d*x])*(e*Sin[c + d*x])^(7/2))
```

Maple [**A**]
time = 0.35, size = 924, normalized size = 1.81

method	result	size
default	Expression too large to display	924

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] (2/5/e*b/(a-b)/(a+b)/(e*sin(d*x+c))^(5/2)+2/e^3*b/(a+b)^2/(a-b)^2*a^2/(e*si
n(d*x+c))^(1/2)+1/e^3*b*a^2/(a+b)^2/(a-b)^2/(e^2*(a^2-b^2)/a^2)^(1/4)*arcta
n((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))-1/2/e^3*b*a^2/(a+b)^2/(a-
b)^2/(e^2*(a^2-b^2)/a^2)^(1/4)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)
^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))-1/10/e^3*b^2*(12*
(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticE((-s
in(d*x+c)+1)^(1/2),1/2*2^(1/2))*a^2+8*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2
)^(1/2)*sin(d*x+c)^(7/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*b^2-6
*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticF((-
sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a^2-4*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+
2)^(1/2)*sin(d*x+c)^(7/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*b^2+
5*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticPi(
(-sin(d*x+c)+1)^(1/2),-a/((a^2-b^2)^(1/2)-a),1/2*2^(1/2))*(a^2-b^2)^(1/2)*a
+5*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticPi
((-sin(d*x+c)+1)^(1/2),-a/((a^2-b^2)^(1/2)-a),1/2*2^(1/2))*a^2-5*(-sin(d*x+
c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticPi((-sin(d*x+c)
+1)^(1/2),a/(a+(a^2-b^2)^(1/2)),1/2*2^(1/2))*(a^2-b^2)^(1/2)*a+5*(-sin(d*x+
c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticPi((-sin(d*x+c)
+1)^(1/2),a/(a+(a^2-b^2)^(1/2)),1/2*2^(1/2))*a^2+12*a^2*cos(d*x+c)^4*sin(d*
x+c)+8*b^2*cos(d*x+c)^4*sin(d*x+c)-16*a^2*cos(d*x+c)^2*sin(d*x+c)-4*b^2*cos
```

```
(d*x+c)^2*sin(d*x+c))*a/sin(d*x+c)^3/(a+(a^2-b^2)^(1/2))/((a^2-b^2)^(1/2)-a
)/(a-b)^2/(a+b)^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(7/2),x, algorithm="maxima")
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(7/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))**(7/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate(1/((b*sec(d*x + c) + a)*(e*sin(d*x + c))^(7/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)}{{\left(e\,\sin\left(c+d\,x\right)\right)}^{7/2}\,\left(b+a\,\cos\left(c+d\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*sin(c + d*x))^(7/2)*(a + b/cos(c + d*x))),x)
[Out] int(cos(c + d*x)/((e*sin(c + d*x))^(7/2)*(b + a*cos(c + d*x))), x)
```

3.241 $\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=1070

$$-\frac{7b^3(a^2-b^2)^{3/4}e^{9/2}\mathrm{ArcTan}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{13/2}d}+\frac{2b(a^2-b^2)^{7/4}e^{9/2}\mathrm{ArcTan}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{13/2}d}+\cdots$$

[Out] -7/2*b^3*(a^2-b^2)^(3/4)*e^(9/2)*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^(13/2)/d+2*b*(a^2-b^2)^(7/4)*e^(9/2)*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^(13/2)/d+7/2*b^3*(a^2-b^2)^(3/4)*e^(9/2)*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^(13/2)/d-2*b*(a^2-b^2)^(7/4)*e^(9/2)*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^(13/2)/d-14/45*e^3*cos(d*x+c)*(e*sin(d*x+c))^(3/2)/a^2/d-7/15*b^2*e^3*(5*b-3*a*cos(d*x+c))*(e*sin(d*x+c))^(3/2)/a^5/d+4/15*b*e^3*(5*a^2-5*b^2+3*a*b*cos(d*x+c))*(e*sin(d*x+c))^(3/2)/a^5/d+4/7*b*e*(e*sin(d*x+c))^(7/2)/a^3/d-2/9*e*cos(d*x+c)*(e*sin(d*x+c))^(7/2)/a^2/d+b^2*e*(e*sin(d*x+c))^(7/2)/a^3/d/(b+a*cos(d*x+c))-7/2*b^4*(a^2-b^2)*e^5*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^7/d/(a-(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+2*b^2*(a^2-b^2)^2*e^5*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^7/d/(a-(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-7/2*b^4*(a^2-b^2)*e^5*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^7/d/(a+(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+2*b^2*(a^2-b^2)^2*e^5*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^7/d/(a+(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-14/15*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/a^2/d/sin(d*x+c)^(1/2)+7/5*b^2*(3*a^2-5*b^2)*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/a^6/d/sin(d*x+c)^(1/2)+4/5*b^2*(8*a^2-5*b^2)*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/a^6/d/sin(d*x+c)^(1/2)

Rubi [A]

time = 2.10, antiderivative size = 1070, normalized size of antiderivative = 1.00, number of steps used = 35, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3957, 2991, 2715, 2721, 2719, 2772, 2944, 2946, 2780, 2886, 2884, 335, 304, 211, 214, 2774}

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(9/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-7*b^3*(a^2 - b^2)^(3/4)*e^(9/2)*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a
^2 - b^2)^(1/4)*Sqrt[e])])/(2*a^(13/2)*d) + (2*b*(a^2 - b^2)^(7/4)*e^(9/2)*
ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(13/
2)*d) + (7*b^3*(a^2 - b^2)^(3/4)*e^(9/2)*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*
x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(2*a^(13/2)*d) - (2*b*(a^2 - b^2)^(7/4)*
e^(9/2)*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])]
)/(a^(13/2)*d) + (7*b^4*(a^2 - b^2)*e^5*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^
2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*a^7*(a - Sqrt[a^2 - b^2]
)*d*Sqrt[e*Sin[c + d*x]]) - (2*b^2*(a^2 - b^2)^2*e^5*EllipticPi[(2*a)/(a -
Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^7*(a - Sqrt
[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (7*b^4*(a^2 - b^2)*e^5*EllipticPi[(2
*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*a^
7*(a + Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) - (2*b^2*(a^2 - b^2)^2*e^5*
EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c +
 d*x]])/(a^7*(a + Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (14*e^4*Ellipt
icE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(15*a^2*d*Sqrt[Sin[c + d*x
]]) - (7*b^2*(3*a^2 - 5*b^2)*e^4*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Si
n[c + d*x]])/(5*a^6*d*Sqrt[Sin[c + d*x]]) - (4*b^2*(8*a^2 - 5*b^2)*e^4*Elli
pticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(5*a^6*d*Sqrt[Sin[c + d*
x]]) - (14*e^3*Cos[c + d*x]*(e*Sin[c + d*x])^(3/2))/(45*a^2*d) - (7*b^2*e^3
*(5*b - 3*a*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2))/(15*a^5*d) + (4*b*e^3*(5*
(a^2 - b^2) + 3*a*b*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2))/(15*a^5*d) + (4*b
*e*(e*Sin[c + d*x])^(7/2))/(7*a^3*d) - (2*e*Cos[c + d*x]*(e*Sin[c + d*x])^(
7/2))/(9*a^2*d) + (b^2*e*(e*Sin[c + d*x])^(7/2))/(a^3*d*(b + a*Cos[c + d*x]
))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
```

```
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2991

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)(e\sin(c+dx))^{9/2}}{(-b-a\cos(c+dx))^2}\,dx$$

$$= \int \left(\frac{(e\sin(c+dx))^{9/2}}{a^2} + \frac{b^2(e\sin(c+dx))^{9/2}}{a^2(b+a\cos(c+dx))^2} - \frac{2b(e\sin(c+dx))^{9/2}}{a^2(b+a\cos(c+dx))}\right)\,dx$$

$$= \frac{\int (e\sin(c+dx))^{9/2}\,dx}{a^2} - \frac{(2b)\int \frac{(e\sin(c+dx))^{9/2}}{b+a\cos(c+dx)}\,dx}{a^2} + \frac{b^2\int \frac{(e\sin(c+dx))^{9/2}}{(b+a\cos(c+dx))^2}\,dx}{a^2}$$

$$= \frac{4be(e\sin(c+dx))^{7/2}}{7a^3d} - \frac{2e\cos(c+dx)(e\sin(c+dx))^{7/2}}{9a^2d} + \frac{b^2e(e\sin(c+dx))^{7}}{a^3d(b+a\cos(c+d}$$

$$= -\frac{14e^3\cos(c+dx)(e\sin(c+dx))^{3/2}}{45a^2d} - \frac{7b^2e^3(5b-3a\cos(c+dx))(e\sin(c+dx}{15a^5d}$$

$$= -\frac{14e^3\cos(c+dx)(e\sin(c+dx))^{3/2}}{45a^2d} - \frac{7b^2e^3(5b-3a\cos(c+dx))(e\sin(c+dx}{15a^5d}$$

$$= \frac{14e^4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{15a^2d\sqrt{\sin(c+dx)}} - \frac{14e^3\cos(c+dx)(e\sin(c+dx))^{3/}}{45a^2d}$$

$$= \frac{14e^4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{15a^2d\sqrt{\sin(c+dx)}} - \frac{7b^2(3a^2-5b^2)\,e^4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+d\right.\right.}{5a^6d\sqrt{\sin(c+}}$$

$$= \frac{7b^4(a^2-b^2)\,e^5\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{2a^7\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} - \frac{2b^2(a^2-}{}$$

$$= -\frac{7b^3(a^2-b^2)^{3/4}\,e^{9/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{13/2}d} + \frac{2b(a^2-b^2)^{7/4}\,e^{9/2}\tan^{-}}{a^{1}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 24.25, size = 974, normalized size = 0.91

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sin[c + d*x])^(9/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*(e*Sin[c + d*x])^(9/2)*(((14*a^4 - 1
59*a^2*b^2 + 165*b^4)*Cos[c + d*x]^2*(3*Sqrt[2]*b*(-a^2 + b^2)^(3/4)*(2*Arc
Tan[1 - (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan
[1 + (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a
^2 + b^2] - Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c
 + d*x]] + Log[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[S
in[c + d*x]] + a*Sin[c + d*x]]) + 8*a^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin
[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(3/2))*(b + a*S
qrt[1 - Sin[c + d*x]^2]))/(12*a^(3/2)*(a^2 - b^2)*(b + a*Cos[c + d*x])*(1 -
 Sin[c + d*x]^2)) + (2*(-46*a^3*b + 66*a*b^3)*Cos[c + d*x]*(((1/8 + I/8)*(2
*ArcTan[1 - ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*Arc
Tan[1 + ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[
a^2 - b^2] - (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin
[c + d*x]] + Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[S
in[c + d*x]] + I*a*Sin[c + d*x]]))/(Sqrt[a]*(a^2 - b^2)^(1/4)) + (b*AppellF
1[3/4, 1/2, 1, 7/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c
 + d*x]^(3/2))/(3*(-a^2 + b^2)))*(b + a*Sqrt[1 - Sin[c + d*x]^2]))/((b + a*
Cos[c + d*x])*Sqrt[1 - Sin[c + d*x]^2])))/(30*a^5*d*(a + b*Sec[c + d*x])^2*
Sin[c + d*x]^(9/2)) + ((b + a*Cos[c + d*x])^2*Csc[c + d*x]^4*Sec[c + d*x]^2
*(e*Sin[c + d*x])^(9/2)*(-1/21*(b*(-37*a^2 + 56*b^2)*Sin[c + d*x])/a^5 + (a
^2*b^2*Sin[c + d*x] - b^4*Sin[c + d*x])/(a^5*(b + a*Cos[c + d*x])) - ((19*a
^2 - 54*b^2)*Sin[2*(c + d*x)])/(90*a^4) - (b*Sin[3*(c + d*x)])/(7*a^3) + Si
n[4*(c + d*x)]/(36*a^2)))/(d*(a + b*Sec[c + d*x])^2)
```

Maple [A]
time = 0.86, size = 2081, normalized size = 1.94

method	result	size
default	Expression too large to display	2081

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(9/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] (4/7*e*b/a^3*(e*sin(d*x+c))^(7/2)+4/3*e^3*b/a^3*(e*sin(d*x+c))^(3/2)-8/3*e^
3*b^3/a^5*(e*sin(d*x+c))^(3/2)+e^5*b^3/a^3*(e*sin(d*x+c))^(3/2)/(-a^2*cos(d
*x+c)^2*e^2+b^2*e^2)-e^5*b^5/a^5*(e*sin(d*x+c))^(3/2)/(-a^2*cos(d*x+c)^2*e^
2+b^2*e^2)+2*e^5*b/a^3/(e^2*(a^2-b^2)/a^2)^(1/4)*arctan((e*sin(d*x+c))^(1/2
)/(e^2*(a^2-b^2)/a^2)^(1/4))-e^5*b/a^3/(e^2*(a^2-b^2)/a^2)^(1/4)*ln(((e*sin
(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b
^2)/a^2)^(1/4)))-15/2*e^5*b^3/a^5/(e^2*(a^2-b^2)/a^2)^(1/4)*arctan((e*sin(d
*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))+15/4*e^5*b^3/a^5/(e^2*(a^2-b^2)/a^2
)^(1/4)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))
^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))+11/2*e^5*b^5/a^7/(e^2*(a^2-b^2)/a^2)^(1/
4)*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))-11/4*e^5*b^5/a^7/
(e^2*(a^2-b^2)/a^2)^(1/4)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4
```

```
))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))+(cos(d*x+c)^2*e*sin(d*
x+c))^(1/2)*e^5*(-1/45/a^6/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*(10*a^4*cos(d*
x+c)^6+42*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*Ell
ipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a^4-432*(-sin(d*x+c)+1)^(1/2)*(2*
sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^
(1/2))*a^2*b^2+450*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^
(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*b^4-21*(-sin(d*x+c)+1)^(
1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2
),1/2*2^(1/2))*a^4+216*b^2*a^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)
*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-225*b^4*(-si
n(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d
*x+c)+1)^(1/2),1/2*2^(1/2))-34*a^4*cos(d*x+c)^4+54*a^2*b^2*cos(d*x+c)^4+24*
a^4*cos(d*x+c)^2-54*a^2*b^2*cos(d*x+c)^2)-b^2*(3*a^4-10*a^2*b^2+7*b^4)/a^6*
(-1/2/a^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(co
s(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c
)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-1/2/a^2*(-sin(d*x+c)+1)^(1/
2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2
)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/
2)/a),1/2*2^(1/2)))+2*b^4*(a^4-2*a^2*b^2+b^4)/a^6*(-1/2*a^2/e/b^2/(a^2-b^2)
*sin(d*x+c)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-cos(d*x+c)^2*a^2+b^2)+1/2/b
^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/
(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/
2))-1/4/b^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+
c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),
1/2*2^(1/2))-1/4/b^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)
*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(a^2-b^2)^(1/2)/a)*E
llipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+3/8/(a
^2-b^2)/a^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(
cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x
+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-1/4/b^2/(a^2-b^2)*(-sin(d
*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(
d*x+c))^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(
a^2-b^2)^(1/2)/a),1/2*2^(1/2))+3/8/(a^2-b^2)/a^2*(-sin(d*x+c)+1)^(1/2)*(2*s
in(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(a
^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1
/2*2^(1/2))))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(9/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(9/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(9/2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(9/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*sin(d*x + c))^(9/2)/(b*sec(d*x + c) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^2\,{\left(e\,\sin\left(c+d\,x\right)\right)}^{9/2}}{{\left(b+a\,\cos\left(c+d\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(9/2)/(a + b/cos(c + d*x))^2,x)
```

```
[Out] int((cos(c + d*x)^2*(e*sin(c + d*x))^(9/2))/(b + a*cos(c + d*x))^2, x)
```

3.242 $\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=1101

$$\frac{5b^3\sqrt[4]{a^2-b^2}\,e^{7/2}\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\,\sqrt{e}}\right)}{2a^{11/2}d}-\frac{2b(a^2-b^2)^{5/4}\,e^{7/2}\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\,\sqrt{e}}\right)}{a^{11/2}d}+\frac{5b^3\cdots}{\cdots}$$

```
[Out] 5/2*b^3*(a^2-b^2)^(1/4)*e^(7/2)*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^
2)^(1/4)/e^(1/2))/a^(11/2)/d-2*b*(a^2-b^2)^(5/4)*e^(7/2)*arctan(a^(1/2)*(e*
sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^(11/2)/d+5/2*b^3*(a^2-b^2)^(1/
4)*e^(7/2)*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^
(11/2)/d-2*b*(a^2-b^2)^(5/4)*e^(7/2)*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(
a^2-b^2)^(1/4)/e^(1/2))/a^(11/2)/d+4/5*b*e*(e*sin(d*x+c))^(5/2)/a^3/d-2/7*e
*cos(d*x+c)*(e*sin(d*x+c))^(5/2)/a^2/d+b^2*e*(e*sin(d*x+c))^(5/2)/a^3/d/(b+
a*cos(d*x+c))-10/21*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*P
i+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/a^
2/d/(e*sin(d*x+c))^(1/2)+5/3*b^2*(a^2-3*b^2)*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)
^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(
1/2))*sin(d*x+c)^(1/2)/a^6/d/(e*sin(d*x+c))^(1/2)+4/3*b^2*(4*a^2-3*b^2)*e^4
*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(co
s(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/a^6/d/(e*sin(d*x+c))^(1/2
)+5/2*b^4*(a^2-b^2)*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*P
i+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(
1/2))*sin(d*x+c)^(1/2)/a^6/d/(a^2-b^2-a*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/
2)-2*b^2*(a^2-b^2)^2*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*
Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a-(a^2-b^2)^(1/2)),2^
(1/2))*sin(d*x+c)^(1/2)/a^6/d/(a^2-b^2-a*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1
/2)+5/2*b^4*(a^2-b^2)*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4
*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1/2)),2
^(1/2))*sin(d*x+c)^(1/2)/a^6/d/(a^2-b^2+a*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(
1/2)-2*b^2*(a^2-b^2)^2*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/
4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1/2)),
2^(1/2))*sin(d*x+c)^(1/2)/a^6/d/(a^2-b^2+a*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^
(1/2)-10/21*e^3*cos(d*x+c)*(e*sin(d*x+c))^(1/2)/a^2/d-5/3*b^2*e^3*(3*b-a*co
s(d*x+c))*(e*sin(d*x+c))^(1/2)/a^5/d+4/3*b*e^3*(3*a^2-3*b^2+a*b*cos(d*x+c))
*(e*sin(d*x+c))^(1/2)/a^5/d
```

Rubi [A]
time = 2.13, antiderivative size = 1101, normalized size of antiderivative = 1.00, number of steps used = 35, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3957, 2991, 2715, 2721, 2720, 2772, 2944, 2946, 2781, 2886, 2884, 335, 218,

214, 211, 2774}

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(7/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (5*b^3*(a^2 - b^2)^(1/4)*e^(7/2)*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^
2 - b^2)^(1/4)*Sqrt[e])])/(2*a^(11/2)*d) - (2*b*(a^2 - b^2)^(5/4)*e^(7/2)*A
rcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(11/2
)*d) + (5*b^3*(a^2 - b^2)^(1/4)*e^(7/2)*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x
]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(2*a^(11/2)*d) - (2*b*(a^2 - b^2)^(5/4)*e
^(7/2)*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])
/(a^(11/2)*d) + (10*e^4*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]]
)/(21*a^2*d*Sqrt[e*Sin[c + d*x]]) - (5*b^2*(a^2 - 3*b^2)*e^4*EllipticF[(c -
 Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*a^6*d*Sqrt[e*Sin[c + d*x]]) - (4*
b^2*(4*a^2 - 3*b^2)*e^4*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]]
)/(3*a^6*d*Sqrt[e*Sin[c + d*x]]) - (5*b^4*(a^2 - b^2)*e^4*EllipticPi[(2*a)/
(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*a^6*(a
^2 - b^2 - a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (2*b^2*(a^2 - b^2)^
2*e^4*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[S
in[c + d*x]])/(a^6*(a^2 - b^2 - a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]])
- (5*b^4*(a^2 - b^2)*e^4*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2
+ d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*a^6*(a^2 - b^2 + a*Sqrt[a^2 - b^2])*d*S
qrt[e*Sin[c + d*x]]) + (2*b^2*(a^2 - b^2)^2*e^4*EllipticPi[(2*a)/(a + Sqrt[
a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^6*(a^2 - b^2 + a
*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) - (10*e^3*Cos[c + d*x]*Sqrt[e*Sin
[c + d*x]])/(21*a^2*d) - (5*b^2*e^3*(3*b - a*Cos[c + d*x])*Sqrt[e*Sin[c + d
*x]])/(3*a^5*d) + (4*b*e^3*(3*(a^2 - b^2) + a*b*Cos[c + d*x])*Sqrt[e*Sin[c
+ d*x]])/(3*a^5*d) + (4*b*e*(e*Sin[c + d*x])^(5/2))/(5*a^3*d) - (2*e*Cos[c
+ d*x]*(e*Sin[c + d*x])^(5/2))/(7*a^2*d) + (b^2*e*(e*Sin[c + d*x])^(5/2))/(
a^3*d*(b + a*Cos[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
```

```
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2991

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)(e\sin(c+dx))^{7/2}}{(-b-a\cos(c+dx))^2}\,dx$$

$$= \int \left(\frac{(e\sin(c+dx))^{7/2}}{a^2} + \frac{b^2(e\sin(c+dx))^{7/2}}{a^2(b+a\cos(c+dx))^2} - \frac{2b(e\sin(c+dx))^{7/2}}{a^2(b+a\cos(c+dx))}\right) dx$$

$$= \frac{\int (e\sin(c+dx))^{7/2}\,dx}{a^2} - \frac{(2b)\int \frac{(e\sin(c+dx))^{7/2}}{b+a\cos(c+dx)}\,dx}{a^2} + \frac{b^2\int \frac{(e\sin(c+dx))^{7/2}}{(b+a\cos(c+dx))^2}\,dx}{a^2}$$

$$= \frac{4be(e\sin(c+dx))^{5/2}}{5a^3d} - \frac{2e\cos(c+dx)(e\sin(c+dx))^{5/2}}{7a^2d} + \frac{b^2e(e\sin(c+dx))^{5/2}}{a^3d(b+a\cos(c+dx)}$$

$$= -\frac{10e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21a^2d} - \frac{5b^2e^3(3b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}{3a^5d}$$

$$= -\frac{10e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21a^2d} - \frac{5b^2e^3(3b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}{3a^5d}$$

$$= \frac{10e^4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{21a^2d\sqrt{e\sin(c+dx)}} - \frac{10e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21a^2d} -$$

$$= \frac{10e^4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{21a^2d\sqrt{e\sin(c+dx)}} - \frac{5b^2(a^2-3b^2)\,e^4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right.}{3a^6d\sqrt{e\sin(c+dx)}}$$

$$= \frac{10e^4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{21a^2d\sqrt{e\sin(c+dx)}} - \frac{5b^2(a^2-3b^2)\,e^4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right.}{3a^6d\sqrt{e\sin(c+dx)}}$$

$$= \frac{5b^3\sqrt[4]{a^2-b^2}\,e^{7/2}\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\,\sqrt{e}}\right)}{2a^{11/2}d} - \frac{2b(a^2-b^2)^{5/4}\,e^{7/2}\tan^{-1}\left(\sqrt{\right.}}{a^{11/2}d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 45.36, size = 2095, normalized size = 1.90

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sin[c + d*x])^(7/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*(-1/42*((23*a^2 - 84*b^2)*Cos[c + d*x])/a^4 - (b^2*
(-a^2 + b^2))/(a^5*(b + a*Cos[c + d*x])) - (2*b*Cos[2*(c + d*x)])/(5*a^3) +
 Cos[3*(c + d*x)]/(14*a^2))*Csc[c + d*x]^3*Sec[c + d*x]^2*(e*Sin[c + d*x])^
(7/2))/(d*(a + b*Sec[c + d*x])^2) + ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*
(e*Sin[c + d*x])^(7/2)*((2*(50*a^4 - 273*a^2*b^2 + 105*b^4)*Cos[c + d*x]^2*
(b + a*Sqrt[1 - Sin[c + d*x]^2])*((b*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[a]*Sqrt[S
in[c + d*x]])/(-a^2 + b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[a]*Sqrt[Sin[
c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - Sqrt[2]*Sqrt[a]*(-a
^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^2]
 + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]])
)/(4*Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(3/4)) - (5*a*(a^2 - b^2)*AppellF1[1/4, -
1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sin[c +
 d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4
, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + 2*(2*a^2*AppellF1[5/4
, -1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + (-a^2 +
 b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2
- b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x]^2)))))/((b + a*Cos[c
 + d*x])*(1 - Sin[c + d*x]^2)) + (2*(-139*a^3*b + 210*a*b^3)*Cos[c + d*x]*(
b + a*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[a]*(2*ArcTan[1 - ((1 +
I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*S
qrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + Log[Sqrt[a^2 - b^2] - (1 +
I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]] - Log[S
qrt[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a
*Sin[c + d*x]]))/(a^2 - b^2)^(3/4) + (5*b*(a^2 - b^2)*AppellF1[1/4, 1/2, 1,
 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sin[c + d*x]])
/(Sqrt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c
+ d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + 2*(2*a^2*AppellF1[5/4, 1/2, 2
, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + (a^2 - b^2)*Appe
llF1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)])*S
in[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x]^2)))))/((b + a*Cos[c + d*x])*S
qrt[1 - Sin[c + d*x]^2]) + ((231*a^3*b - 420*a*b^3)*Cos[c + d*x]*Cos[2*(c +
 d*x)]*(b + a*Sqrt[1 - Sin[c + d*x]^2])*((((1/2 - I/2)*(a^2 - 2*b^2)*ArcTan[
1 - ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)])/(a^(3/2)*(a^2
- b^2)^(3/4)) - ((1/2 - I/2)*(a^2 - 2*b^2)*ArcTan[1 + ((1 + I)*Sqrt[a]*Sqrt
[Sin[c + d*x]])/(a^2 - b^2)^(1/4)])/(a^(3/2)*(a^2 - b^2)^(3/4)) + ((1/4 - I
/4)*(a^2 - 2*b^2)*Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*S
qrt[Sin[c + d*x]] + I*a*Sin[c + d*x]])/(a^(3/2)*(a^2 - b^2)^(3/4)) - ((1/4
- I/4)*(a^2 - 2*b^2)*Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4
)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]])/(a^(3/2)*(a^2 - b^2)^(3/4)) + (4*
Sqrt[Sin[c + d*x]])/a + (4*b*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (a^
2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(5/2))/(5*(a^2 - b^2)) + (10*b*
(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)
/(a^2 - b^2)]*Sqrt[Sin[c + d*x]])/(Sqrt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*
AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)
] + 2*(2*a^2*AppellF1[5/4, 1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2
```

)/(a^2 - b^2)] + (a^2 - b^2)*AppellF1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x]^2)))))/((b + a*Cos[c + d*x])*(1 - 2*Sin[c + d*x]^2)*Sqrt[1 - Sin[c + d*x]^2])))/(210*a^5*d*(a + b*Sec[c + d*x])^2*Sin[c + d*x]^(7/2))

Maple [A]
time = 0.97, size = 1943, normalized size = 1.76

method	result	size
default	Expression too large to display	1943

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(7/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] (4/5*e*b/a^3*(e*sin(d*x+c))^(5/2)+4*e^3*b/a^3*(e*sin(d*x+c))^(1/2)-8*e^3*b^3/a^5*(e*sin(d*x+c))^(1/2)+e^5*b^3/a^3*(e*sin(d*x+c))^(1/2)/(-a^2*cos(d*x+c)^2*e^2+b^2*e^2)-e^5*b^5/a^5*(e*sin(d*x+c))^(1/2)/(-a^2*cos(d*x+c)^2*e^2+b^2*e^2)+2*e^5*b/a*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))-13/2*e^5*b^3/a^3*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))+9/2*e^5*b^5/a^5*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))+e^5*b/a*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))-13/4*e^5*b^3/a^3*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))+9/4*e^5*b^5/a^5*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))+(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*e^4*(-1/21/a^6/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*(-6*a^4*cos(d*x+c)^4*sin(d*x+c)+5*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a^4-84*b^2*a^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+105*b^4*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+16*a^4*cos(d*x+c)^2*sin(d*x+c)-42*a^2*b^2*cos(d*x+c)^2*sin(d*x+c))+2*b^4*(a^4-2*a^2*b^2+b^4)/a^6*(-1/2*a^2/e/b^2/(a^2-b^2)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-cos(d*x+c)^2*a^2+b^2)-1/4/b^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-1/4/b^2/(a^2-b^2)^(3/2)*a*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+5/8/(a^2-b^2)^(3/2)/a*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+1/4/b^2/(a^2-b^2)^(3/2)*a*(-sin(d*x+c)+1)^(
```

```
1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1
/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(
1/2)/a),1/2*2^(1/2))-5/8/(a^2-b^2)^(3/2)/a*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x
+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(a^2-b^2
)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(
1/2)))-b^2/a^6*(3*a^4-10*a^2*b^2+7*b^4)*(-1/2/(a^2-b^2)^(1/2)/a*(-sin(d*x+c
)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+
c))^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-
b^2)^(1/2)/a),1/2*2^(1/2))+1/2/(a^2-b^2)^(1/2)/a*(-sin(d*x+c)+1)^(1/2)*(2*s
in(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(a
^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1
/2*2^(1/2))))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
[Out] e^(7/2)*integrate(sin(d*x + c)^(7/2)/(b*sec(d*x + c) + a)^2, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(7/2)/(a+b*sec(d*x+c))**2,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate((e*sin(d*x + c))^(7/2)/(b*sec(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^2\,{\left(e\,\sin\left(c+d\,x\right)\right)}^{7/2}}{{\left(b+a\,\cos\left(c+d\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(7/2)/(a + b/cos(c + d*x))^2,x)
[Out] int((cos(c + d*x)^2*(e*sin(c + d*x))^(7/2))/(b + a*cos(c + d*x))^2, x)
```

3.243 $\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=850

$$-\frac{3b^3e^{5/2}\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\,\sqrt{e}}\right)}{2a^{9/2}\sqrt[4]{a^2-b^2}\,d}+\frac{2b(a^2-b^2)^{3/4}\,e^{5/2}\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\,\sqrt{e}}\right)}{a^{9/2}d}+\frac{3b^3e^{5/2}\tanh}{2}$$

```
[Out] -3/2*b^3*e^(5/2)*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2
))/a^(9/2)/(a^2-b^2)^(1/4)/d+2*b*(a^2-b^2)^(3/4)*e^(5/2)*arctan(a^(1/2)*(e*
sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^(9/2)/d+3/2*b^3*e^(5/2)*arctan
h(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^(9/2)/(a^2-b^2)^(
1/4)/d-2*b*(a^2-b^2)^(3/4)*e^(5/2)*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^
2-b^2)^(1/4)/e^(1/2))/a^(9/2)/d+4/3*b*e*(e*sin(d*x+c))^(3/2)/a^3/d-2/5*e*co
s(d*x+c)*(e*sin(d*x+c))^(3/2)/a^2/d+b^2*e*(e*sin(d*x+c))^(3/2)/a^3/d/(b+a*c
os(d*x+c))-3/2*b^4*e^3*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi
+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(1
/2))*sin(d*x+c)^(1/2)/a^5/d/(a-(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+2*b^2*
(a^2-b^2)*e^3*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)
*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(
d*x+c)^(1/2)/a^5/d/(a-(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-3/2*b^4*e^3*(si
n(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1
/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^5/
d/(a+(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+2*b^2*(a^2-b^2)*e^3*(sin(1/2*c+1
/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*
Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^5/d/(a+(a^2
-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-6/5*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/
2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(
e*sin(d*x+c))^(1/2)/a^2/d/sin(d*x+c)^(1/2)+7*b^2*e^2*(sin(1/2*c+1/4*Pi+1/2*
d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x)
,2^(1/2))*(e*sin(d*x+c))^(1/2)/a^4/d/sin(d*x+c)^(1/2)
```

Rubi [A]

time = 1.56, antiderivative size = 850, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3957, 2991, 2715, 2721, 2719, 2772, 2946, 2780, 2886, 2884, 335, 304, 211, 214, 2774}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(5/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-3*b^3*e^(5/2)*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sq
rt[e])])/(2*a^(9/2)*(a^2 - b^2)^(1/4)*d) + (2*b*(a^2 - b^2)^(3/4)*e^(5/2)*A
rcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(9/2)
*d) + (3*b^3*e^(5/2)*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1
/4)*Sqrt[e])])/(2*a^(9/2)*(a^2 - b^2)^(1/4)*d) - (2*b*(a^2 - b^2)^(3/4)*e^(
5/2)*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(
a^(9/2)*d) + (3*b^4*e^3*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 +
 d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*a^5*(a - Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c
 + d*x]]) - (2*b^2*(a^2 - b^2)*e^3*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]),
(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^5*(a - Sqrt[a^2 - b^2])*d*Sqr
t[e*Sin[c + d*x]]) + (3*b^4*e^3*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c
- Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*a^5*(a + Sqrt[a^2 - b^2])*d*Sqrt
[e*Sin[c + d*x]]) - (2*b^2*(a^2 - b^2)*e^3*EllipticPi[(2*a)/(a + Sqrt[a^2 -
 b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^5*(a + Sqrt[a^2 - b^2
])*d*Sqrt[e*Sin[c + d*x]]) + (6*e^2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e
*Sin[c + d*x]])/(5*a^2*d*Sqrt[Sin[c + d*x]]) - (7*b^2*e^2*EllipticE[(c - Pi
/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(a^4*d*Sqrt[Sin[c + d*x]]) + (4*b*e*(
e*Sin[c + d*x])^(3/2))/(3*a^3*d) - (2*e*Cos[c + d*x]*(e*Sin[c + d*x])^(3/2)
)/(5*a^2*d) + (b^2*e*(e*Sin[c + d*x])^(3/2))/(a^3*d*(b + a*Cos[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
```

```
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
```

```
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2991

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\sec(c+dx))^2}\,dx &= \int \frac{\cos^2(c+dx)(e\sin(c+dx))^{5/2}}{(-b-a\cos(c+dx))^2}\,dx \\
&= \int \left(\frac{(e\sin(c+dx))^{5/2}}{a^2} + \frac{b^2(e\sin(c+dx))^{5/2}}{a^2(b+a\cos(c+dx))^2} - \frac{2b(e\sin(c+dx))^{5/2}}{a^2(b+a\cos(c+dx))}\right)dx \\
&= \frac{\int (e\sin(c+dx))^{5/2}\,dx}{a^2} - \frac{(2b)\int \frac{(e\sin(c+dx))^{5/2}}{b+a\cos(c+dx)}\,dx}{a^2} + \frac{b^2\int \frac{(e\sin(c+dx))^{5/2}}{(b+a\cos(c+dx))^2}\,dx}{a^2} \\
&= \frac{4be(e\sin(c+dx))^{3/2}}{3a^3d} - \frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5a^2d} + \frac{b^2e(e\sin(c+dx))^{3}}{a^3d(b+a\cos(c+d}\\
&= \frac{4be(e\sin(c+dx))^{3/2}}{3a^3d} - \frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5a^2d} + \frac{b^2e(e\sin(c+dx))^{3}}{a^3d(b+a\cos(c+d}\\
&= \frac{6e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5a^2d\sqrt{\sin(c+dx)}} + \frac{4be(e\sin(c+dx))^{3/2}}{3a^3d} - \frac{2e\cos(c+}{}\\
&= \frac{6e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5a^2d\sqrt{\sin(c+dx)}} - \frac{7b^2e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin}}{a^4d\sqrt{\sin(c+dx)}}\\
&= \frac{3b^4e^3\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{2a^5\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} - \frac{2b^2(a^2-b^2)e^3\Pi\Big(}{a^5\Big(}\\
&= -\frac{3b^3e^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{9/2}\sqrt[4]{a^2-b^2}\,d} + \frac{2b(a^2-b^2)^{3/4}e^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\,s}}{\sqrt[4]{a^2-}}\right.}{a^{9/2}d}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 23.92, size = 886, normalized size = 1.04

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sin[c + d*x])^(5/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -1/10*((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*(e*Sin[c + d*x])^(5/2)*(((-6*a
^2 + 35*b^2)*Cos[c + d*x]^2*(3*Sqrt[2]*b*(-a^2 + b^2)^(3/4)*(2*ArcTan[1 - (
Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + (Sqr
```

```
t[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2]
 - Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]]
+ Log[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*
x]] + a*Sin[c + d*x]]) + 8*a^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c + d*x]
^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(3/2))*(b + a*Sqrt[1 - S
in[c + d*x]^2]))/(12*a^(3/2)*(a^2 - b^2)*(b + a*Cos[c + d*x])*(1 - Sin[c +
d*x]^2)) + (28*a*b*Cos[c + d*x]*(((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[a
]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[a]*Sq
rt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[a
]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]] + Log[Sqrt[a^2 -
 b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c +
d*x]]))/(Sqrt[a]*(a^2 - b^2)^(1/4)) + (b*AppellF1[3/4, 1/2, 1, 7/4, Sin[c +
 d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(3/2))/(3*(-a^2 + b
^2)))*(b + a*Sqrt[1 - Sin[c + d*x]^2]))/((b + a*Cos[c + d*x])*Sqrt[1 - Sin[
c + d*x]^2])))/(a^3*d*(a + b*Sec[c + d*x])^2*Sin[c + d*x]^(5/2)) + ((b + a*
Cos[c + d*x])^2*Csc[c + d*x]^2*Sec[c + d*x]^2*(e*Sin[c + d*x])^(5/2)*((4*b*
Sin[c + d*x])/(3*a^3) + (b^2*Sin[c + d*x])/(a^3*(b + a*Cos[c + d*x])) - Sin
[2*(c + d*x)]/(5*a^2)))/(d*(a + b*Sec[c + d*x])^2)
```

Maple [A]
time = 0.70, size = 1644, normalized size = 1.93

method	result	size
default	Expression too large to display	1644

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(5/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] (4/3*b/a^3*e*(e*sin(d*x+c))^(3/2)+b^3/a^3*e^3*(e*sin(d*x+c))^(3/2)/(-a^2*co
s(d*x+c)^2*e^2+b^2*e^2)+2*b/a^3*e^3/(e^2*(a^2-b^2)/a^2)^(1/4)*arctan((e*sin
(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))-b/a^3*e^3/(e^2*(a^2-b^2)/a^2)^(1/
4)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2
)-(e^2*(a^2-b^2)/a^2)^(1/4)))-7/2*b^3/a^5*e^3/(e^2*(a^2-b^2)/a^2)^(1/4)*arc
tan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))+7/4*b^3/a^5*e^3/(e^2*(a
^2-b^2)/a^2)^(1/4)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*
sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))+(cos(d*x+c)^2*e*sin(d*x+c))^(
1/2)*e^3*(-1/5/a^2/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*(6*(-sin(d*x+c)+1)^(1/
2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),
1/2*2^(1/2))-3*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2
)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-2*cos(d*x+c)^4+2*cos(d*x+c)^
2)+3*b^2/a^4*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/
(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*(2*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^
(1/2))-EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2)))+2*b^4*(a^2-b^2)/a^4*(-
1/2*a^2/e/b^2/(a^2-b^2)*sin(d*x+c)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-cos(
d*x+c)^2*a^2+b^2)+1/2/b^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^
```

```
(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticE((-sin(d*
x+c)+1)^(1/2),1/2*2^(1/2))-1/4/b^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d
*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF
((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-1/4/b^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)
*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/
(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)
/a),1/2*2^(1/2))+3/8/(a^2-b^2)/a^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(
1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(a^2-b^2)^(1/2)/
a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-1/
4/b^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/
2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin
(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+3/8/(a^2-b^2)/a^2*(-s
in(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*
sin(d*x+c))^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/
(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2)))-b^2*(3*a^2-5*b^2)/a^4*(-1/2/a^2*(-sin(d
*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(
d*x+c))^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(
a^2-b^2)^(1/2)/a),1/2*2^(1/2))-1/2/a^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+
2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(a^2-b^2)^(1
/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2)
)))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
[Out] e^(5/2)*integrate(sin(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^2, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(5/2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*sin(d*x + c))^(5/2)/(b*sec(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^2\,{\left(e\,\sin\left(c+d\,x\right)\right)}^{5/2}}{{\left(b+a\,\cos\left(c+d\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(5/2)/(a + b/cos(c + d*x))^2,x)
```

```
[Out] int((cos(c + d*x)^2*(e*sin(c + d*x))^(5/2))/(b + a*cos(c + d*x))^2, x)
```

3.244 $\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=882

$$\frac{b^3e^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\,\sqrt{e}}\right)}{2a^{7/2}\left(a^2-b^2\right)^{3/4}d}-\frac{2b\sqrt[4]{a^2-b^2}\,e^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\,\sqrt{e}}\right)}{a^{7/2}d}+\frac{b^3e^{3/2}\tanh^{-1}\left(\cdots\right.}{2a^{7/2}\left(a\cdots\right.}$$

```
[Out] 1/2*b^3*e^(3/2)*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2)
)/a^(7/2)/(a^2-b^2)^(3/4)/d-2*b*(a^2-b^2)^(1/4)*e^(3/2)*arctan(a^(1/2)*(e*s
in(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^(7/2)/d+1/2*b^3*e^(3/2)*arctanh
(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^(7/2)/(a^2-b^2)^(3
/4)/d-2*b*(a^2-b^2)^(1/4)*e^(3/2)*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2
-b^2)^(1/4)/e^(1/2))/a^(7/2)/d-2/3*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/
sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(
d*x+c)^(1/2)/a^2/d/(e*sin(d*x+c))^(1/2)+5*b^2*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x
)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^
(1/2))*sin(d*x+c)^(1/2)/a^4/d/(e*sin(d*x+c))^(1/2)+1/2*b^4*e^2*(sin(1/2*c+1
/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*
Pi+1/2*d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^4/d/(a^2-b^
2-a*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-2*b^2*(a^2-b^2)*e^2*(sin(1/2*c+1/
4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*P
i+1/2*d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^4/d/(a^2-b^2
-a*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+1/2*b^4*e^2*(sin(1/2*c+1/4*Pi+1/2*
d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x
),2*a/(a+(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^4/d/(a^2-b^2+a*(a^2-b
^2)^(1/2))/(e*sin(d*x+c))^(1/2)-2*b^2*(a^2-b^2)*e^2*(sin(1/2*c+1/4*Pi+1/2*d
*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x)
,2*a/(a+(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^4/d/(a^2-b^2+a*(a^2-b^
2)^(1/2))/(e*sin(d*x+c))^(1/2)+4*b*e*(e*sin(d*x+c))^(1/2)/a^3/d-2/3*e*cos(d
*x+c)*(e*sin(d*x+c))^(1/2)/a^2/d+b^2*e*(e*sin(d*x+c))^(1/2)/a^3/d/(b+a*cos(
d*x+c))
```

Rubi [A]
time = 1.56, antiderivative size = 882, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3957, 2991, 2715, 2721, 2720, 2772, 2946, 2781, 2886, 2884, 335, 218, 214, 211, 2774}

$$-\frac{e^2\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}\,b^4}{2a^4\left(a^2-\sqrt{a^2-b^2}\,a-b^2\right)d\sqrt{e\sin(c+dx)}}-\frac{e^2\Pi\left(\frac{2a}{a+\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}\,b^4}{2a^4\left(a^2+\sqrt{a^2-b^2}\,a-b^2\right)d\sqrt{e\sin(c+dx)}}+\frac{e^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\,\sqrt{e}}\right)b^3}{2a^{7/2}\left(a^2-b^2\right)^{3/4}d}+\frac{e^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\,\sqrt{e}}\right)b^3}{2a^{7/2}\left(a^2-b^2\right)^{3/4}d}+\frac{e\sqrt{e\sin(c+dx)}\,b^2}{a^3d(b+a\cos(c+dx))}-\frac{5e^2F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}\,b^2}{a^4d\sqrt{e\sin(c+dx)}}+\frac{2\left(a^2-b^2\right)e^2\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}\,b^2}{a^4\left(a^2-\sqrt{a^2-b^2}\,a-b^2\right)d\sqrt{e\sin(c+dx)}}+\frac{2\left(a^2-b^2\right)e^2\Pi\left(\frac{2a}{a+\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}\,b^2}{a^4\left(a^2+\sqrt{a^2-b^2}\,a-b^2\right)d\sqrt{e\sin(c+dx)}}-\frac{2\sqrt[4]{a^2-b^2}\,e^{3/2}\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\,\sqrt{e}}\right)b}{a^{7/2}d}-\frac{2\sqrt[4]{a^2-b^2}\,e^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\,\sqrt{e}}\right)b}{a^{7/2}d}+\frac{4e\sqrt{e\sin(c+dx)}\,b}{a^3d}-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3a^2d}+\frac{2e^2F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}}{3a^2d\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(3/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (b^3*e^(3/2)*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[
e])])/(2*a^(7/2)*(a^2 - b^2)^(3/4)*d) - (2*b*(a^2 - b^2)^(1/4)*e^(3/2)*ArcT
an[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(7/2)*d)
 + (b^3*e^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*S
qrt[e])])/(2*a^(7/2)*(a^2 - b^2)^(3/4)*d) - (2*b*(a^2 - b^2)^(1/4)*e^(3/2)*
ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(7/
2)*d) + (2*e^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*a^2*
d*Sqrt[e*Sin[c + d*x]]) - (5*b^2*e^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[
Sin[c + d*x]])/(a^4*d*Sqrt[e*Sin[c + d*x]]) - (b^4*e^2*EllipticPi[(2*a)/(a
- Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*a^4*(a^2
- b^2 - a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (2*b^2*(a^2 - b^2)*e^2
*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c
+ d*x]])/(a^4*(a^2 - b^2 - a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) - (b^
4*e^2*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[S
in[c + d*x]])/(2*a^4*(a^2 - b^2 + a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]
) + (2*b^2*(a^2 - b^2)*e^2*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/
2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^4*(a^2 - b^2 + a*Sqrt[a^2 - b^2])*d*S
qrt[e*Sin[c + d*x]]) + (4*b*e*Sqrt[e*Sin[c + d*x]])/(a^3*d) - (2*e*Cos[c +
d*x]*Sqrt[e*Sin[c + d*x]])/(3*a^2*d) + (b^2*e*Sqrt[e*Sin[c + d*x]])/(a^3*d*
(b + a*Cos[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
```

```
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2991

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)(e\sin(c+dx))^{3/2}}{(-b-a\cos(c+dx))^2}\,dx$$

$$= \int \left(\frac{(e\sin(c+dx))^{3/2}}{a^2} + \frac{b^2(e\sin(c+dx))^{3/2}}{a^2(b+a\cos(c+dx))^2} - \frac{2b(e\sin(c+dx))^{3/2}}{a^2(b+a\cos(c+dx))}\right) dx$$

$$= \frac{\int (e\sin(c+dx))^{3/2}\,dx}{a^2} - \frac{(2b)\int \frac{(e\sin(c+dx))^{3/2}}{b+a\cos(c+dx)}\,dx}{a^2} + \frac{b^2\int \frac{(e\sin(c+dx))^{3/2}}{(b+a\cos(c+dx))^2}\,dx}{a^2}$$

$$= \frac{4be\sqrt{e\sin(c+dx)}}{a^3d} - \frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3a^2d} + \frac{b^2e\sqrt{e\sin(c+dx)}}{a^3d(b+a\cos(c+dx)}$$

$$= \frac{4be\sqrt{e\sin(c+dx)}}{a^3d} - \frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3a^2d} + \frac{b^2e\sqrt{e\sin(c+dx)}}{a^3d(b+a\cos(c+dx)}$$

$$= \frac{2e^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{3a^2d\sqrt{e\sin(c+dx)}} + \frac{4be\sqrt{e\sin(c+dx)}}{a^3d} - \frac{2e\cos(c+d}{}$$

$$= \frac{2e^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{3a^2d\sqrt{e\sin(c+dx)}} - \frac{5b^2e^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c}}{a^4d\sqrt{e\sin(c+dx)}}$$

$$= \frac{2e^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{3a^2d\sqrt{e\sin(c+dx)}} - \frac{5b^2e^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c}}{a^4d\sqrt{e\sin(c+dx)}}$$

$$= \frac{b^3e^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{7/2}(a^2-b^2)^{3/4}d} - \frac{2b\sqrt[4]{a^2-b^2}\,e^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c}}{\sqrt[4]{a^2-b^2}}\right.}{a^{7/2}d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 44.52, size = 2012, normalized size = 2.28

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sin[c + d*x])^(3/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*((-2*Cos[c + d*x])/(3*a^2) + b^2/(a^3*(b + a*Cos[c
+ d*x])))*Csc[c + d*x]*Sec[c + d*x]^2*(e*Sin[c + d*x])^(3/2))/(d*(a + b*Sec
[c + d*x])^2) - ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*(e*Sin[c + d*x])^(3/
```

2)*((2*(-2*a^2 + 3*b^2)*Cos[c + d*x]^2*(b + a*Sqrt[1 - Sin[c + d*x]^2])*((b
*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] +
2*ArcTan[1 + (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log
[Sqrt[-a^2 + b^2] - Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] +
a*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4
)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(3
/4)) - (5*a*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Si
n[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((5
*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^
2)/(a^2 - b^2)] + 2*(2*a^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (a^2
*Sin[c + d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin
[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*
(-1 + Sin[c + d*x]^2)))))/((b + a*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (8*
a*b*Cos[c + d*x]*(b + a*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[a]*(2
*ArcTan[1 - ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*Arc
Tan[1 + ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + Log[Sqrt[
a^2 - b^2] - (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin
[c + d*x]] - Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[S
in[c + d*x]] + I*a*Sin[c + d*x]]))/(a^2 - b^2)^(3/4) + (5*b*(a^2 - b^2)*App
ellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*S
qrt[Sin[c + d*x]])/(Sqrt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1
/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + 2*(2*a^2*Ap
pellF1[5/4, 1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]
+ (a^2 - b^2)*AppellF1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^
2)/(a^2 - b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x]^2)))))/((b +
a*Cos[c + d*x])*Sqrt[1 - Sin[c + d*x]^2]) - (6*a*b*Cos[c + d*x]*Cos[2*(c +
d*x)]*(b + a*Sqrt[1 - Sin[c + d*x]^2])*((((1/2 - I/2)*(a^2 - 2*b^2)*ArcTan[
1 - ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)])/(a^(3/2)*(a^2
- b^2)^(3/4)) - ((1/2 - I/2)*(a^2 - 2*b^2)*ArcTan[1 + ((1 + I)*Sqrt[a]*Sqrt
[Sin[c + d*x]])/(a^2 - b^2)^(1/4)])/(a^(3/2)*(a^2 - b^2)^(3/4)) + ((1/4 - I
/4)*(a^2 - 2*b^2)*Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*S
qrt[Sin[c + d*x]] + I*a*Sin[c + d*x]])/(a^(3/2)*(a^2 - b^2)^(3/4)) - ((1/4
- I/4)*(a^2 - 2*b^2)*Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4
)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]])/(a^(3/2)*(a^2 - b^2)^(3/4)) + (4*
Sqrt[Sin[c + d*x]])/a + (4*b*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (a^
2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(5/2))/(5*(a^2 - b^2)) + (10*b*
(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)
/(a^2 - b^2)]*Sqrt[Sin[c + d*x]])/(Sqrt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*
AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)
] + 2*(2*a^2*AppellF1[5/4, 1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2
)/(a^2 - b^2)] + (a^2 - b^2)*AppellF1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (a^
2*Sin[c + d*x]^2)/(a^2 - b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*
x]^2)))))/((b + a*Cos[c + d*x])*(1 - 2*Sin[c + d*x]^2)*Sqrt[1 - Sin[c + d*x
]^2])))/(6*a^3*d*(a + b*Sec[c + d*x])^2*Sin[c + d*x]^(3/2))

Maple [A]
time = 0.83, size = 1569, normalized size = 1.78

method	result	size
default	Expression too large to display	1569

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(3/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] (4*b/a^3*e*(e*sin(d*x+c))^(1/2)+b^3/a^3*e^3*(e*sin(d*x+c))^(1/2)/(-a^2*cos(
d*x+c)^2*e^2+b^2*e^2)+2*b/a*e^3*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2
)*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))-5/2*b^3/a^3*e^3*(e
^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*arctan((e*sin(d*x+c))^(1/2)/(e^2
*(a^2-b^2)/a^2)^(1/4))+b/a*e^3*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)
*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-
(e^2*(a^2-b^2)/a^2)^(1/4)))-5/4*b^3/a^3*e^3*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2
*e^2+b^2*e^2)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d
*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))+(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*
e^2*(-1/3/a^2/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*((-sin(d*x+c)+1)^(1/2)*(2*s
in(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(
1/2))+2*cos(d*x+c)^2*sin(d*x+c))+3*b^2/a^4*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x
+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((
-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+2*b^4*(a^2-b^2)/a^4*(-1/2*a^2/e/b^2/(a^2-
b^2)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-cos(d*x+c)^2*a^2+b^2)-1/4/b^2/(a^2
-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*
x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-1/4
/b^2/(a^2-b^2)^(3/2)*a*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x
+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticP
i((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+5/8/(a^2-b^2)^
(3/2)/a*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(
d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+
1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+1/4/b^2/(a^2-b^2)^(3/2)*a*(-s
in(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*
sin(d*x+c))^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/
(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-5/8/(a^2-b^2)^(3/2)/a*(-sin(d*x+c)+1)^(1
/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/
2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1
/2)/a),1/2*2^(1/2)))-b^2/a^4*(3*a^2-5*b^2)*(-1/2/(a^2-b^2)^(1/2)/a*(-sin(d*
x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d
*x+c))^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a
^2-b^2)^(1/2)/a),1/2*2^(1/2))+1/2/(a^2-b^2)^(1/2)/a*(-sin(d*x+c)+1)^(1/2)*(
2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1
+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a
),1/2*2^(1/2))))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
[Out] e^(3/2)*integrate(sin(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^2, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(3/2)/(a+b*sec(d*x+c))**2,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate((e*sin(d*x + c))^(3/2)/(b*sec(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^2\,{\left(e\,\sin\left(c+d\,x\right)\right)}^{3/2}}{{\left(b+a\,\cos\left(c+d\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(3/2)/(a + b/cos(c + d*x))^2,x)
[Out] int((cos(c + d*x)^2*(e*sin(c + d*x))^(3/2))/(b + a*cos(c + d*x))^2, x)
```

3.245 $\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=809

$$\frac{b^3\sqrt{e}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\,\sqrt{e}}\right)}{2a^{5/2}\left(a^2-b^2\right)^{5/4}d}+\frac{2b\sqrt{e}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\,\sqrt{e}}\right)}{a^{5/2}\sqrt[4]{a^2-b^2}\,d}-\frac{b^3\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{e\sin}}{\sqrt[4]{a^2-}}\right.}{2a^{5/2}\left(a^2-b^2\right)^{5/4}}$$

```
[Out] b^2*(e*sin(d*x+c))^(3/2)/a/(a^2-b^2)/d/e/(b+a*cos(d*x+c))+1/2*b^3*arctan(a^
(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))*e^(1/2)/a^(5/2)/(a^2-b^
2)^(5/4)/d+2*b*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))
*e^(1/2)/a^(5/2)/(a^2-b^2)^(1/4)/d-1/2*b^3*arctanh(a^(1/2)*(e*sin(d*x+c))^(
1/2)/(a^2-b^2)^(1/4)/e^(1/2))*e^(1/2)/a^(5/2)/(a^2-b^2)^(5/4)/d-2*b*arctanh
(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))*e^(1/2)/a^(5/2)/(a^2
-b^2)^(1/4)/d+2*b^2*e*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+
1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(1/
2))*sin(d*x+c)^(1/2)/a^3/d/(a-(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+1/2*b^4
*e*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi
(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2
)/a^3/(a^2-b^2)/d/(a-(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+2*b^2*e*(sin(1/2
*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+
1/4*Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^3/d/(a+
(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+1/2*b^4*e*(sin(1/2*c+1/4*Pi+1/2*d*x)^
2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a
/(a+(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^3/(a^2-b^2)/d/(a+(a^2-b^2)
^(1/2))/(e*sin(d*x+c))^(1/2)-2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*
c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c
))^(1/2)/a^2/d/sin(d*x+c)^(1/2)+b^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin
(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(
d*x+c))^(1/2)/a^2/(a^2-b^2)/d/sin(d*x+c)^(1/2)
```

Rubi [A]
time = 1.35, antiderivative size = 809, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3957, 2991, 2721, 2719, 2773, 2946, 2780, 2886, 2884, 335, 304, 211, 214}

$$-\frac{e\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}\,b^4}{2a^3\left(a^2-b^2\right)\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}-\frac{e\Pi\left(\frac{2a}{a+\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}\,b^4}{2a^3\left(a^2-b^2\right)\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}+\frac{\sqrt{e}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\,\sqrt{e}}\right)b^3}{2a^{5/2}\left(a^2-b^2\right)^{5/4}d}-\frac{\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\,\sqrt{e}}\right)b^3}{2a^{5/2}\left(a^2-b^2\right)^{5/4}d}+\frac{(e\sin(c+dx))^{3/2}b^2}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))}-\frac{E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}\,b^2}{a^2\left(a^2-b^2\right)d\sqrt{\sin(c+dx)}}-\frac{2e\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}\,b^2}{a^3\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}-\frac{2e\Pi\left(\frac{2a}{a+\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}\,b^2}{a^3\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}+\frac{2\sqrt{e}\,\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\,\sqrt{e}}\right)b}{a^{5/2}\sqrt[4]{a^2-b^2}\,d}-\frac{2\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\,\sqrt{e}}\right)b}{a^{5/2}\sqrt[4]{a^2-b^2}\,d}+\frac{2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{a^2d\sqrt{\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Sin[c + d*x]]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (b^3*Sqrt[e]*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[
e])])/(2*a^(5/2)*(a^2 - b^2)^(5/4)*d) + (2*b*Sqrt[e]*ArcTan[(Sqrt[a]*Sqrt[e
```

```
*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(5/2)*(a^2 - b^2)^(1/4)*d)
 - (b^3*Sqrt[e]*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*S
qrt[e])])/(2*a^(5/2)*(a^2 - b^2)^(5/4)*d) - (2*b*Sqrt[e]*ArcTanh[(Sqrt[a]*S
qrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(5/2)*(a^2 - b^2)^(1/
4)*d) - (2*b^2*e*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2
, 2]*Sqrt[Sin[c + d*x]])/(a^3*(a - Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]])
 - (b^4*e*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sq
rt[Sin[c + d*x]])/(2*a^3*(a^2 - b^2)*(a - Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c +
 d*x]]) - (2*b^2*e*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)
/2, 2]*Sqrt[Sin[c + d*x]])/(a^3*(a + Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]
]) - (b^4*e*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*
Sqrt[Sin[c + d*x]])/(2*a^3*(a^2 - b^2)*(a + Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c
 + d*x]]) + (2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(a^2*
d*Sqrt[Sin[c + d*x]]) - (b^2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c
+ d*x]])/(a^2*(a^2 - b^2)*d*Sqrt[Sin[c + d*x]]) + (b^2*(e*Sin[c + d*x])^(3/
2))/(a*(a^2 - b^2)*d*e*(b + a*Cos[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2991

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)\sqrt{e\sin(c+dx)}}{(-b-a\cos(c+dx))^2}\,dx$$

$$= \int \left(\frac{\sqrt{e\sin(c+dx)}}{a^2} + \frac{b^2\sqrt{e\sin(c+dx)}}{a^2(b+a\cos(c+dx))^2} - \frac{2b\sqrt{e\sin(c+dx)}}{a^2(b+a\cos(c+dx))}\right) dx$$

$$= \frac{\int \sqrt{e\sin(c+dx)}\,dx}{a^2} - \frac{(2b)\int \frac{\sqrt{e\sin(c+dx)}}{b+a\cos(c+dx)}\,dx}{a^2} + \frac{b^2\int \frac{\sqrt{e\sin(c+dx)}}{(b+a\cos(c+dx))^2}\,dx}{a^2}$$

$$= \frac{b^2(e\sin(c+dx))^{3/2}}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))} + \frac{b^2\int \frac{\left(-b-\frac{1}{2}a\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{b+a\cos(c+dx)}\,dx}{a^2\left(a^2-b^2\right)} + \cdots$$

$$= \frac{2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{a^2d\sqrt{\sin(c+dx)}} + \frac{b^2(e\sin(c+dx))^{3/2}}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))} - \cdots$$

$$= -\frac{2b^2e\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{a^3\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} - \frac{2b^2e\Pi\left(\frac{2a}{a+\sqrt{a^2-\cdots}}\right.}{a^3\left(a+\cdots\right.}$$

$$= \frac{2b\sqrt{e}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{5/2}\sqrt[4]{a^2-b^2}\,d} - \frac{2b\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{5/2}\sqrt[4]{a^2-b^2}\,d}$$

$$= \frac{2b\sqrt{e}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{5/2}\sqrt[4]{a^2-b^2}\,d} - \frac{2b\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{5/2}\sqrt[4]{a^2-b^2}\,d}$$

$$= \frac{b^3\sqrt{e}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{5/2}\left(a^2-b^2\right)^{5/4}d} + \frac{2b\sqrt{e}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{5/2}\sqrt[4]{a^2-b^2}\,d}$$

<u>**Mathematica**</u> [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 23.27, size = 717, normalized size = 0.89

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[e*Sin[c + d*x]]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^2*Sqrt[e*Sin[c + d*x]]*(24*b^2*Sin[c + d
*x]^(3/2) + ((b + a*Sqrt[Cos[c + d*x]^2])*Sec[c + d*x]*((2*a^2 - 3*b^2)*Cos
[c + d*x]*(3*Sqrt[2]*b*(-a^2 + b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[a]*Sq
rt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[a]*Sqrt[
Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - Sqrt[2]*Sqrt[a]
*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]] + Log[Sqrt[-a^2 +
b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*
x]]) + 8*a^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c + d*x]^2, (a^2*Sin[c + d
*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(3/2)) - (2 + 2*I)*a^2*b*Sqrt[Cos[c + d*x]
^2]*(3*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])
/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^
2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*S
qrt[Sin[c + d*x]] + I*a*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[
a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]]) - (4 - 4*I)*Sq
rt[a]*b*AppellF1[3/4, 1/2, 1, 7/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^
2 - b^2)]*Sin[c + d*x]^(3/2))))/(a^(3/2)*(a - b)*(a + b))))/(24*a*(a^2 - b^
2)*d*(a + b*Sec[c + d*x])^2*Sqrt[Sin[c + d*x]])
```

Maple [A]
time = 0.61, size = 1499, normalized size = 1.85

method	result	size
default	Expression too large to display	1499

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(1/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] (e*b^3/a/(a^2-b^2)*(e*sin(d*x+c))^(3/2)/(-a^2*cos(d*x+c)^2*e^2+b^2*e^2)-e*b
/a/(a^2-b^2)/(e^2*(a^2-b^2)/a^2)^(1/4)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b
^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))+3/4*e*b^3
/a^3/(a^2-b^2)/(e^2*(a^2-b^2)/a^2)^(1/4)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2
-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))+2*e*b/a
/(a^2-b^2)/(e^2*(a^2-b^2)/a^2)^(1/4)*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-
b^2)/a^2)^(1/4))-3/2*e*b^3/a^3/(a^2-b^2)/(e^2*(a^2-b^2)/a^2)^(1/4)*arctan((
e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))+(cos(d*x+c)^2*e*sin(d*x+c))^
(1/2)*e*(-1/a^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/
2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*(2*EllipticE((-sin(d*x+c)+1)^(1/2),1/2
*2^(1/2))-EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2)))+2*b^4/a^2*(-1/2*a^2
/e/b^2/(a^2-b^2)*sin(d*x+c)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-cos(d*x+c)^
2*a^2+b^2)+1/2/b^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*s
in(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticE((-sin(d*x+c)+1)
^(1/2),1/2*2^(1/2))-1/4/b^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2
)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((-sin(
d*x+c)+1)^(1/2),1/2*2^(1/2))-1/4/b^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin
(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(a^2
```

```
-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2
*2^(1/2))+3/8/(a^2-b^2)/a^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*si
n(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(a^2-b^2)^(1/2)/a)*Elli
pticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-1/4/b^2/(
a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos
(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)
+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+3/8/(a^2-b^2)/a^2*(-sin(d*x+
c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x
+c))^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2
-b^2)^(1/2)/a),1/2*2^(1/2)))-3/a^2*b^2*(-1/2/a^2*(-sin(d*x+c)+1)^(1/2)*(2*s
in(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(a
^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1
/2*2^(1/2))-1/2/a^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)
^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((
-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))))/cos(d*x+c)/(e*s
in(d*x+c))^(1/2))/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
[Out] e^(1/2)*integrate(sqrt(sin(d*x + c))/(b*sec(d*x + c) + a)^2, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\sin{(c+dx)}}}{(a+b\sec{(c+dx)})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(1/2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(sqrt(e*sin(c + d*x))/(a + b*sec(c + d*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*sin(d*x + c))/(b*sec(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^2\,\sqrt{e\,\sin\left(c+d\,x\right)}}{{\left(b+a\,\cos\left(c+d\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(1/2)/(a + b/cos(c + d*x))^2,x)
```

```
[Out] int((cos(c + d*x)^2*(e*sin(c + d*x))^(1/2))/(b + a*cos(c + d*x))^2, x)
```

3.246 $\int \frac{1}{(a+b\sec(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx$

Optimal. Leaf size=838

$$-\frac{3b^3\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{3/2}\left(a^2-b^2\right)^{7/4}d\sqrt{e}}-\frac{2b\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{3/2}\left(a^2-b^2\right)^{3/4}d\sqrt{e}}-\frac{3b^3\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right.}{2a^{3/2}\left(a^2-b^2\right)^{7/4}d\sqrt{e}}$$

```
[Out] -3/2*b^3*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^(3/
2)/(a^2-b^2)^(7/4)/d/e^(1/2)-2*b*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b
^2)^(1/4)/e^(1/2))/a^(3/2)/(a^2-b^2)^(3/4)/d/e^(1/2)-3/2*b^3*arctanh(a^(1/2
)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^(3/2)/(a^2-b^2)^(7/4)/d/e
^(1/2)-2*b*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^
(3/2)/(a^2-b^2)^(3/4)/d/e^(1/2)-2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1
/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c
)^(1/2)/a^2/d/(e*sin(d*x+c))^(1/2)-b^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/
sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(
d*x+c)^(1/2)/a^2/(a^2-b^2)/d/(e*sin(d*x+c))^(1/2)-2*b^2*(sin(1/2*c+1/4*Pi+1
/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*
d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^2/d/(a^2-b^2-a*(a^
2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-3/2*b^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1
/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a-(
a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^2/(a^2-b^2)/d/(a^2-b^2-a*(a^2-b
^2)^(1/2))/(e*sin(d*x+c))^(1/2)-2*b^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/s
in(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2-b
^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^2/d/(a^2-b^2+a*(a^2-b^2)^(1/2))/(e*s
in(d*x+c))^(1/2)-3/2*b^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*
Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1/2)),2^
(1/2))*sin(d*x+c)^(1/2)/a^2/(a^2-b^2)/d/(a^2-b^2+a*(a^2-b^2)^(1/2))/(e*sin(
d*x+c))^(1/2)+b^2*(e*sin(d*x+c))^(1/2)/a/(a^2-b^2)/d/e/(b+a*cos(d*x+c))
```

Rubi [A]
time = 1.41, antiderivative size = 838, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3957, 2991, 2721, 2720, 2773, 2946, 2781, 2886, 2884, 335, 218, 214, 211}

$$\frac{3\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}\,b^4}{2a^2\left(a^2-b^2\right)\left(a^2-\sqrt{a^2-b^2}\,a-b^2\right)d\sqrt{e\sin(c+dx)}}+\frac{3\Pi\left(\frac{2a}{a+\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}\,b^4}{2a^2\left(a^2-b^2\right)\left(a^2+\sqrt{a^2-b^2}\,a-b^2\right)d\sqrt{e\sin(c+dx)}}-\frac{3\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b^3}{2a^{3/2}\left(a^2-b^2\right)^{7/4}d\sqrt{e}}-\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b^3}{2a^{3/2}\left(a^2-b^2\right)^{7/4}d\sqrt{e}}+\frac{\sqrt{e\sin(c+dx)}\,b^2}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))}+\frac{F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}\,b^2}{a^2\left(a^2-b^2\right)d\sqrt{e\sin(c+dx)}}+\frac{2\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}\,b^2}{a^2\left(a^2-\sqrt{a^2-b^2}\,a-b^2\right)d\sqrt{e\sin(c+dx)}}+\frac{2\Pi\left(\frac{2a}{a+\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}\,b^2}{a^2\left(a^2+\sqrt{a^2-b^2}\,a-b^2\right)d\sqrt{e\sin(c+dx)}}-\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b}{a^{3/2}\left(a^2-b^2\right)^{3/4}d\sqrt{e}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b}{a^{3/2}\left(a^2-b^2\right)^{3/4}d\sqrt{e}}+\frac{2F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}}{a^2d\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sec[c + d*x])^2*Sqrt[e*Sin[c + d*x]]),x]
```

```
[Out] (-3*b^3*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])
/(2*a^(3/2)*(a^2 - b^2)^(7/4)*d*Sqrt[e]) - (2*b*ArcTan[(Sqrt[a]*Sqrt[e*Sin[
```

```
c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(3/2)*(a^2 - b^2)^(3/4)*d*Sqrt[
e]) - (3*b^3*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt
[e])])/(2*a^(3/2)*(a^2 - b^2)^(7/4)*d*Sqrt[e]) - (2*b*ArcTanh[(Sqrt[a]*Sqrt
[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(3/2)*(a^2 - b^2)^(3/4)*
d*Sqrt[e]) + (2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^2*d
*Sqrt[e*Sin[c + d*x]]) + (b^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c +
 d*x]])/(a^2*(a^2 - b^2)*d*Sqrt[e*Sin[c + d*x]]) + (2*b^2*EllipticPi[(2*a)/
(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^2*(a^2
 - b^2 - a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (3*b^4*EllipticPi[(2*
a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*a^2
*(a^2 - b^2)*(a^2 - b^2 - a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (2*b
^2*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[
c + d*x]])/(a^2*(a^2 - b^2 + a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (
3*b^4*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[S
in[c + d*x]])/(2*a^2*(a^2 - b^2)*(a^2 - b^2 + a*Sqrt[a^2 - b^2])*d*Sqrt[e*S
in[c + d*x]]) + (b^2*Sqrt[e*Sin[c + d*x]])/(a*(a^2 - b^2)*d*e*(b + a*Cos[c
+ d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
```

```
^2, 0]
```

Rule 2991

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx = \int \frac{\cos^2(c+dx)}{(-b-a\cos(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx$$

$$= \int \left(\frac{1}{a^2\sqrt{e\sin(c+dx)}} + \frac{b^2}{a^2(-b-a\cos(c+dx))^2\sqrt{e\sin(c+ \cdots}\right.$$

$$= \frac{\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{a^2} + \frac{(2b)\int \frac{1}{(-b-a\cos(c+dx))\sqrt{e\sin(c+dx)}} \cdots}{a^2}$$

$$= \frac{b^2\sqrt{e\sin(c+dx)}}{a(a^2-b^2)\,de(b+a\cos(c+dx))} + \frac{b^2\int \frac{b-\frac{1}{2}a\cos(c+dx)}{(-b-a\cos(c+dx))\sqrt{e\sin(\cdots}}}{a^2(a^2-b^2)}$$

$$= \frac{2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{\sin(c+dx)}}{a^2d\sqrt{e\sin(c+dx)}} + \frac{b^2\sqrt{e\sin(c+\cdots}}{a(a^2-b^2)\,de(b+a\,\mathrm{c}\cdots}$$

$$= \frac{2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{\sin(c+dx)}}{a^2d\sqrt{e\sin(c+dx)}} + \frac{2b^2\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\cdots\right.}{a^2\left(a^2-b^2-a\sqrt{\cdots}\right.}$$

$$= -\frac{2b\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{3/2}(a^2-b^2)^{3/4}d\sqrt{e}} - \frac{2b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(\cdots}}{\sqrt[4]{a^2-b^{\cdots}}}\right.}{a^{3/2}(a^2-b^2)^{3/4}d\cdots}$$

$$= -\frac{2b\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{3/2}(a^2-b^2)^{3/4}d\sqrt{e}} - \frac{2b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(\cdots}}{\sqrt[4]{a^2-b^{\cdots}}}\right.}{a^{3/2}(a^2-b^2)^{3/4}d\cdots}$$

$$= -\frac{3b^3\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{3/2}(a^2-b^2)^{7/4}d\sqrt{e}} - \frac{2b\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(\cdots}}{\sqrt[4]{a^2-b^{\cdots}}}\right.}{a^{3/2}(a^2-b^2)^{3/4}\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 30.72, size = 1246, normalized size = 1.49

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sec[c + d*x])^2*Sqrt[e*Sin[c + d*x]]),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*Sqrt[Sin[c + d*x]]*((2*(-2*a^2 + b^2
)*Cos[c + d*x]^2*(b + a*Sqrt[1 - Sin[c + d*x]^2])*((b*(-2*ArcTan[1 - (Sqrt[
2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*
Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - Sq
rt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]] + Log
[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] +
 a*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(3/4)) - (5*a*(a^2 - b^2
)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b
^2)]*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((5*(a^2 - b^2)*AppellF1[
1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + 2*(2
*a^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2
- b^2)] + (-a^2 + b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Sin[
c + d*x]^2)/(a^2 - b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x]^2))
)))/((b + a*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (4*a*b*Cos[c + d*x]*(b +
a*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[a]*(2*ArcTan[1 - ((1 + I)*S
qrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[
a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + Log[Sqrt[a^2 - b^2] - (1 + I)*S
qrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]] - Log[Sqrt[
a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin
[c + d*x]]))/(a^2 - b^2)^(3/4) + (5*b*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4
, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sin[c + d*x]])/(Sq
rt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*
x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + 2*(2*a^2*AppellF1[5/4, 1/2, 2, 9/
4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + (a^2 - b^2)*AppellF1
[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)])*Sin[c
 + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x]^2)))))/((b + a*Cos[c + d*x])*Sqrt[
1 - Sin[c + d*x]^2])))/(2*a*(-a + b)*(a + b)*d*(a + b*Sec[c + d*x])^2*Sqrt[
e*Sin[c + d*x]]) + (b^2*(b + a*Cos[c + d*x])*Sec[c + d*x]*Tan[c + d*x])/(a*
(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2*Sqrt[e*Sin[c + d*x]])
```

Maple [A]
time = 0.75, size = 1476, normalized size = 1.76

method	result	size
default	Expression too large to display	1476

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (b^3/a*e/(a^2-b^2)*(e*sin(d*x+c))^(1/2)/(-a^2*cos(d*x+c)^2*e^2+b^2*e^2)+2*b
*a*e/(a^2-b^2)*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*arctan((e*sin(d
*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))-1/2*b^3/a*e/(a^2-b^2)*(e^2*(a^2-b^2
)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/
a^2)^(1/4))+b*a*e/(a^2-b^2)*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*ln
(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^
```

```
2*(a^2-b^2)/a^2)^(1/4)))-1/4*b^3/a*e/(a^2-b^2)*(e^2*(a^2-b^2)/a^2)^(1/4)/(-
a^2*e^2+b^2*e^2)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*si
n(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))+(cos(d*x+c)^2*e*sin(d*x+c))^(1/
2)*(-1/a^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(c
os(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2)
)+2/a^2*b^4*(-1/2*a^2/e/b^2/(a^2-b^2)*(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(-c
os(d*x+c)^2*a^2+b^2)-1/4/b^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+
2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*EllipticF((-sin
(d*x+c)+1)^(1/2),1/2*2^(1/2))-1/4/b^2/(a^2-b^2)^(3/2)*a*(-sin(d*x+c)+1)^(1/
2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2
)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/
2)/a),1/2*2^(1/2))+5/8/(a^2-b^2)^(3/2)/a*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c
)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(a^2-b^2)^
(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/
2))+1/4/b^2/(a^2-b^2)^(3/2)*a*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*
sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(a^2-b^2)^(1/2)/a)*El
lipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-5/8/(a^
2-b^2)^(3/2)/a*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2
)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(
d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2)))-3/a^2*b^2*(-1/2/(a^2-
b^2)^(1/2)/a*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/
(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*
x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+1/2/(a^2-b^2)^(1/2)/a*(-
sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e
*sin(d*x+c))^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1
/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{e \sin(c + dx)} (a + b \sec(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**2/(e*sin(d*x+c))**(1/2),x)
[Out] Integral(1/(sqrt(e*sin(c + d*x))*(a + b*sec(c + d*x))**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*sec(d*x + c) + a)^2*sqrt(e*sin(d*x + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c + d\,x)^2}{\sqrt{e\,\sin(c + d\,x)}\,(b + a\,\cos(c + d\,x))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*sin(c + d*x))^(1/2)*(a + b/cos(c + d*x))^2),x)
[Out] int(cos(c + d*x)^2/((e*sin(c + d*x))^(1/2)*(b + a*cos(c + d*x))^2), x)
```

3.247 $\int \frac{1}{(a+b\sec(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=1054

$$\frac{5b^3\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2\sqrt{a}\left(a^2-b^2\right)^{9/4}de^{3/2}}+\frac{2b\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{\sqrt{a}\left(a^2-b^2\right)^{5/4}de^{3/2}}-\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2\sqrt{a}\left(a^2-b^2\right)^{9/4}de^{3/2}}$$

```
[Out] 5/2*b^3*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/(a^2-b
^2)^(9/4)/d/e^(3/2)/a^(1/2)+2*b*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^
2)^(1/4)/e^(1/2))/(a^2-b^2)^(5/4)/d/e^(3/2)/a^(1/2)-5/2*b^3*arctanh(a^(1/2)
*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/(a^2-b^2)^(9/4)/d/e^(3/2)/a^
(1/2)-2*b*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/(a^
2-b^2)^(5/4)/d/e^(3/2)/a^(1/2)-2*cos(d*x+c)/a^2/d/e/(e*sin(d*x+c))^(1/2)+b^
2/a/(a^2-b^2)/d/e/(b+a*cos(d*x+c))/(e*sin(d*x+c))^(1/2)+4*b*(a-b*cos(d*x+c)
)/a^2/(a^2-b^2)/d/e/(e*sin(d*x+c))^(1/2)+b^2*(5*a*b-(3*a^2+2*b^2)*cos(d*x+c
))/a^2/(a^2-b^2)^2/d/e/(e*sin(d*x+c))^(1/2)+5/2*b^4*(sin(1/2*c+1/4*Pi+1/2*d
*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x)
,2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a/(a^2-b^2)^2/d/e/(a-(a^
2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+2*b^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2
)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a-(a^
2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a/(a^2-b^2)/d/e/(a-(a^2-b^2)^(1/2))
/(e*sin(d*x+c))^(1/2)+5/2*b^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c
+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1/2
)),2^(1/2))*sin(d*x+c)^(1/2)/a/(a^2-b^2)^2/d/e/(a+(a^2-b^2)^(1/2))/(e*sin(d
*x+c))^(1/2)+2*b^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2
*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1/2)),2^(1/2))
*sin(d*x+c)^(1/2)/a/(a^2-b^2)/d/e/(a+(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+
2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(c
os(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/a^2/d/e^2/sin(d*x+c)
^(1/2)+4*b^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*
EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/a^2/(a^2-
b^2)/d/e^2/sin(d*x+c)^(1/2)+b^2*(3*a^2+2*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)
^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2
))*(e*sin(d*x+c))^(1/2)/a^2/(a^2-b^2)^2/d/e^2/sin(d*x+c)^(1/2)
```

Rubi [A]

time = 1.98, antiderivative size = 1054, normalized size of antiderivative = 1.00, number of steps used = 33, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3957, 2991, 2716, 2721, 2719, 2773, 2945, 2946, 2780, 2886, 2884, 335, 304, 211, 214, 2775}

$$-\frac{5\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}\,b^4}{2a\left(a^2-b^2\right)^2\left(a-\sqrt{a^2-b^2}\right)de\sqrt{e\sin(c+dx)}}-\frac{5\Pi\left(\frac{2a}{a+\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}\,b^4}{2a\left(a^2-b^2\right)^2\left(a+\sqrt{a^2-b^2}\right)de\sqrt{e\sin(c+dx)}}+\frac{5\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b^3}{2\sqrt{a}\left(a^2-b^2\right)^{9/4}de^{3/2}}-\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b^3}{2\sqrt{a}\left(a^2-b^2\right)^{9/4}de^{3/2}}-\frac{\left(3a^2+2b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}\,b^2}{a^2\left(a^2-b^2\right)^2de^2\sqrt{\sin(c+dx)}}-\frac{4E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}\,b^2}{a^2\left(a^2-b^2\right)de^2\sqrt{\sin(c+dx)}}+\frac{\left(5ab-\left(3a^2+2b^2\right)\cos(c+dx)\right)b^2}{a^2\left(a^2-b^2\right)^2de\sqrt{e\sin(c+dx)}}-\frac{2\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}\,b^2}{a\left(a^2-b^2\right)\left(a-\sqrt{a^2-b^2}\right)de\sqrt{e\sin(c+dx)}}-\frac{2\Pi\left(\frac{2a}{a+\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}\,b^2}{a\left(a^2-b^2\right)\left(a+\sqrt{a^2-b^2}\right)de\sqrt{e\sin(c+dx)}}+\frac{b^2}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))\sqrt{e\sin(c+dx)}}+\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b}{\sqrt{a}\left(a^2-b^2\right)^{5/4}de^{3/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)b}{\sqrt{a}\left(a^2-b^2\right)^{5/4}de^{3/2}}+\frac{4(a-b\cos(c+dx))b}{a^2\left(a^2-b^2\right)de\sqrt{e\sin(c+dx)}}-\frac{2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{a^2de^2\sqrt{\sin(c+dx)}}-\frac{2\cos(c+dx)}{a^2de\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sec[c + d*x])^2*(e*Sin[c + d*x])^(3/2)),x]
```

```
[Out] (5*b^3*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/
(2*Sqrt[a]*(a^2 - b^2)^(9/4)*d*e^(3/2)) + (2*b*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c
 + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(Sqrt[a]*(a^2 - b^2)^(5/4)*d*e^(3/2
)) - (5*b^3*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[
e])])/(2*Sqrt[a]*(a^2 - b^2)^(9/4)*d*e^(3/2)) - (2*b*ArcTanh[(Sqrt[a]*Sqrt[
e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(Sqrt[a]*(a^2 - b^2)^(5/4)*d
*e^(3/2)) - (2*Cos[c + d*x])/(a^2*d*e*Sqrt[e*Sin[c + d*x]]) + b^2/(a*(a^2 -
 b^2)*d*e*(b + a*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]]) + (4*b*(a - b*Cos[c +
d*x]))/(a^2*(a^2 - b^2)*d*e*Sqrt[e*Sin[c + d*x]]) + (b^2*(5*a*b - (3*a^2 +
2*b^2)*Cos[c + d*x]))/(a^2*(a^2 - b^2)^2*d*e*Sqrt[e*Sin[c + d*x]]) - (5*b^4
*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c
+ d*x]])/(2*a*(a^2 - b^2)^2*(a - Sqrt[a^2 - b^2])*d*e*Sqrt[e*Sin[c + d*x]])
 - (2*b^2*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sq
rt[Sin[c + d*x]])/(a*(a^2 - b^2)*(a - Sqrt[a^2 - b^2])*d*e*Sqrt[e*Sin[c + d
*x]]) - (5*b^4*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2,
2]*Sqrt[Sin[c + d*x]])/(2*a*(a^2 - b^2)^2*(a + Sqrt[a^2 - b^2])*d*e*Sqrt[e*
Sin[c + d*x]]) - (2*b^2*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 +
 d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a*(a^2 - b^2)*(a + Sqrt[a^2 - b^2])*d*e*Sq
rt[e*Sin[c + d*x]]) - (2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*
x]])/(a^2*d*e^2*Sqrt[Sin[c + d*x]]) - (4*b^2*EllipticE[(c - Pi/2 + d*x)/2,
2]*Sqrt[e*Sin[c + d*x]])/(a^2*(a^2 - b^2)*d*e^2*Sqrt[Sin[c + d*x]]) - (b^2*
(3*a^2 + 2*b^2)*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(a^2
*(a^2 - b^2)^2*d*e^2*Sqrt[Sin[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
```

```
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2775

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
```

```
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2991

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx = \int \frac{\cos^2(c+dx)}{(-b-a\cos(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx$$

$$= \int \left(\frac{1}{a^2(e\sin(c+dx))^{3/2}} + \frac{b^2}{a^2(-b-a\cos(c+dx))^2(e\sin(c+\right.$$

$$= \frac{\int \frac{1}{(e\sin(c+dx))^{3/2}}\,dx}{a^2} + \frac{(2b)\int \frac{1}{(-b-a\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx}{a^2} + \frac{b}{}$$

$$= -\frac{2\cos(c+dx)}{a^2de\sqrt{e\sin(c+dx)}} + \frac{b^2}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))\sqrt{}}$$

$$= -\frac{2\cos(c+dx)}{a^2de\sqrt{e\sin(c+dx)}} + \frac{b^2}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))\sqrt{}}$$

$$= -\frac{2\cos(c+dx)}{a^2de\sqrt{e\sin(c+dx)}} + \frac{b^2}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))\sqrt{}}$$

$$= -\frac{2\cos(c+dx)}{a^2de\sqrt{e\sin(c+dx)}} + \frac{b^2}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))\sqrt{}}$$

$$= -\frac{2\cos(c+dx)}{a^2de\sqrt{e\sin(c+dx)}} + \frac{b^2}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))\sqrt{}}$$

$$= \frac{2b\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{\sqrt{a}\left(a^2-b^2\right)^{5/4}de^{3/2}} - \frac{2b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(}}{\sqrt[4]{a^2-b^2}}\right.}{\sqrt{a}\left(a^2-b^2\right)^{5/4}de}$$

$$= \frac{5b^3\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2\sqrt{a}\left(a^2-b^2\right)^{9/4}de^{3/2}} + \frac{2b\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c}}{\sqrt[4]{a^2-b^2}}\right.}{\sqrt{a}\left(a^2-b^2\right)^{5/4}d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 15.47, size = 772, normalized size = 0.73

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sec[c + d*x])^2*(e*Sin[c + d*x])^(3/2)),x]
```

```
[Out] ((b + a*Cos[c + d*x])*(-(((a^2 - b^2)*(b + a*Sqrt[Cos[c + d*x]^2])*Sec[c +
d*x]^3*Sin[c + d*x]^(3/2)*((2*a^3 + 3*a*b^2)*Cos[c + d*x]*(3*Sqrt[2]*b*(-a^
2 + b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b
^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)
^(1/4)] - Log[Sqrt[-a^2 + b^2] - Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Si
n[c + d*x]] + a*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^
2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]]) + 8*a^(5/2)*AppellF1[3
/4, -1/2, 1, 7/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c +
 d*x]^(3/2)) + (1 + I)*a*(6*a^2*b + 4*b^3)*Sqrt[Cos[c + d*x]^2]*(3*(a^2 - b
^2)^(3/4)*(2*ArcTan[1 - ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1
/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)]
 - Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x
]] + I*a*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^
(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]]) - (4 - 4*I)*Sqrt[a]*b*AppellF
1[3/4, 1/2, 1, 7/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c
 + d*x]^(3/2))))/(a^(3/2)*(a - b)^2*(a + b)^2)) + 24*(-2*(b + a*Cos[c + d*x
])*(-2*a*b + (a^2 + b^2)*Cos[c + d*x])*Csc[c + d*x] + a*b^2*Sin[c + d*x])*T
an[c + d*x]^2))/(24*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^2*(e*Sin[c + d*x])
^(3/2))
```

Maple [A]
time = 0.63, size = 1623, normalized size = 1.54

method	result	size
default	Expression too large to display	1623

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (4/e*b*a/(a^2-b^2)^2/(e*sin(d*x+c))^(1/2)+1/e*b^3*a/(a+b)^2/(a-b)^2*(e*sin(
d*x+c))^(3/2)/(-a^2*cos(d*x+c)^2*e^2+b^2*e^2)+2/e*b*a/(a+b)^2/(a-b)^2/(e^2*
(a^2-b^2)/a^2)^(1/4)*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))
-1/e*b*a/(a+b)^2/(a-b)^2/(e^2*(a^2-b^2)/a^2)^(1/4)*ln(((e*sin(d*x+c))^(1/2)
+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)
))+1/2/e*b^3/a/(a+b)^2/(a-b)^2/(e^2*(a^2-b^2)/a^2)^(1/4)*arctan((e*sin(d*x+
c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))-1/4/e*b^3/a/(a+b)^2/(a-b)^2/(e^2*(a^2-
b^2)/a^2)^(1/4)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin
(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))+(cos(d*x+c)^2*e*sin(d*x+c))^(1/2
```

```
)/e*((a^2+b^2)/(a^2-b^2)^2*(2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*
sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-(-sin(d*x+c)+
1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^
(1/2),1/2*2^(1/2))-2*cos(d*x+c)^2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)+2*b^4/
(a+b)/(a-b)*(-1/2*a^2/e/b^2/(a^2-b^2)*sin(d*x+c)*(cos(d*x+c)^2*e*sin(d*x+c)
)^(1/2)/(-cos(d*x+c)^2*a^2+b^2)+1/2/b^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*
sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*Elli
pticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-1/4/b^2/(a^2-b^2)*(-sin(d*x+c)+1)^
(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(
1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-1/4/b^2/(a^2-b^2)*(-sin(d
*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(
d*x+c))^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(
a^2-b^2)^(1/2)/a),1/2*2^(1/2))+3/8/(a^2-b^2)/a^2*(-sin(d*x+c)+1)^(1/2)*(2*s
in(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(a
^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1
/2*2^(1/2))-1/4/b^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*
sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(a^2-b^2)^(1/2)/a)*El
lipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+3/8/(a^
2-b^2)/a^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(c
os(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+
c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2)))-b^2*(3*a^2-b^2)/(a+b)^2/(
a-b)^2*(-1/2/a^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1
/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-si
n(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-1/2/a^2*(-sin(d*x+c)
+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c
))^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b
^2)^(1/2)/a),1/2*2^(1/2))))/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**2/(e*sin(d*x+c))**(3/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(1/((b*sec(d*x + c) + a)^2*(e*sin(d*x + c))^(3/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^2}{{\left(e\,\sin\left(c+d\,x\right)\right)}^{3/2}\,{\left(b+a\,\cos\left(c+d\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*sin(c + d*x))^(3/2)*(a + b/cos(c + d*x))^2),x)
[Out] int(cos(c + d*x)^2/((e*sin(c + d*x))^(3/2)*(b + a*cos(c + d*x))^2), x)
```

3.248 $\int \frac{1}{(a+b\sec(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=1089

$$-\frac{7\sqrt{a}\,b^3\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\,\sqrt{e}}\right)}{2\left(a^2-b^2\right)^{11/4}de^{5/2}}-\frac{2\sqrt{a}\,b\text{ArcTan}\left(\frac{\sqrt{a}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\,\sqrt{e}}\right)}{\left(a^2-b^2\right)^{7/4}de^{5/2}}-\frac{7\sqrt{a}\,b^3\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\ldots}}{\sqrt[4]{\ldots}}\right.}{2\left(a^2-b^2\right)^{11/\ldots}}$$

```
[Out] -2/3*cos(d*x+c)/a^2/d/e/(e*sin(d*x+c))^(3/2)+b^2/a/(a^2-b^2)/d/e/(b+a*cos(d
*x+c))/(e*sin(d*x+c))^(3/2)+4/3*b*(a-b*cos(d*x+c))/a^2/(a^2-b^2)/d/e/(e*sin
(d*x+c))^(3/2)+1/3*b^2*(7*a*b-(5*a^2+2*b^2)*cos(d*x+c))/a^2/(a^2-b^2)^2/d/e
/(e*sin(d*x+c))^(3/2)-7/2*b^3*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)
^(1/4)/e^(1/2))*a^(1/2)/(a^2-b^2)^(11/4)/d/e^(5/2)-2*b*arctan(a^(1/2)*(e*si
n(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))*a^(1/2)/(a^2-b^2)^(7/4)/d/e^(5/2)-
7/2*b^3*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))*a^(1/
2)/(a^2-b^2)^(11/4)/d/e^(5/2)-2*b*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2
-b^2)^(1/4)/e^(1/2))*a^(1/2)/(a^2-b^2)^(7/4)/d/e^(5/2)-2/3*(sin(1/2*c+1/4*P
i+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/
2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/a^2/d/e^2/(e*sin(d*x+c))^(1/2)-4/3*b^2*(si
n(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/
2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/a^2/(a^2-b^2)/d/e^2/(e*sin(d*
x+c))^(1/2)-1/3*b^2*(5*a^2+2*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1
/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c
)^(1/2)/a^2/(a^2-b^2)^2/d/e^2/(e*sin(d*x+c))^(1/2)-7/2*b^4*(sin(1/2*c+1/4*P
i+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1
/2*d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^2/d/e^2
/(a^2-b^2-a*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-2*b^2*(sin(1/2*c+1/4*Pi+1
/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*
d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)/d/e^2/(a^2
-b^2-a*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-7/2*b^4*(sin(1/2*c+1/4*Pi+1/2*
d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x
),2*a/(a+(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^2/d/e^2/(a^2-
b^2+a*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-2*b^2*(sin(1/2*c+1/4*Pi+1/2*d*x
)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2
*a/(a+(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)/d/e^2/(a^2-b^2+a
*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 2.07, antiderivative size = 1089, normalized size of antiderivative = 1.00, number of steps used = 33, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {3957, 2991, 2716, 2721, 2720, 2773, 2945, 2946, 2781, 2886, 2884, 335, 218, 214, 211, 2775}

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sec[c + d*x])^2*(e*Sin[c + d*x])^(5/2)),x]
```

```
[Out] (-7*Sqrt[a]*b^3*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sq
rt[e])])/(2*(a^2 - b^2)^(11/4)*d*e^(5/2)) - (2*Sqrt[a]*b*ArcTan[(Sqrt[a]*Sq
rt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/((a^2 - b^2)^(7/4)*d*e^(5
/2)) - (7*Sqrt[a]*b^3*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(
1/4)*Sqrt[e])])/(2*(a^2 - b^2)^(11/4)*d*e^(5/2)) - (2*Sqrt[a]*b*ArcTanh[(Sq
rt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/((a^2 - b^2)^(7/4
)*d*e^(5/2)) - (2*Cos[c + d*x])/(3*a^2*d*e*(e*Sin[c + d*x])^(3/2)) + b^2/(a
*(a^2 - b^2)*d*e*(b + a*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2)) + (4*b*(a - b
*Cos[c + d*x]))/(3*a^2*(a^2 - b^2)*d*e*(e*Sin[c + d*x])^(3/2)) + (b^2*(7*a*
b - (5*a^2 + 2*b^2)*Cos[c + d*x]))/(3*a^2*(a^2 - b^2)^2*d*e*(e*Sin[c + d*x]
)^(3/2)) + (2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*a^2*d
*e^2*Sqrt[e*Sin[c + d*x]]) + (4*b^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[S
in[c + d*x]])/(3*a^2*(a^2 - b^2)*d*e^2*Sqrt[e*Sin[c + d*x]]) + (b^2*(5*a^2
+ 2*b^2)*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*a^2*(a^2 -
 b^2)^2*d*e^2*Sqrt[e*Sin[c + d*x]]) + (7*b^4*EllipticPi[(2*a)/(a - Sqrt[a^2
 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*(a^2 - b^2)^2*(a^2
- b^2 - a*Sqrt[a^2 - b^2])*d*e^2*Sqrt[e*Sin[c + d*x]]) + (2*b^2*EllipticPi[
(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/((a
^2 - b^2)*(a^2 - b^2 - a*Sqrt[a^2 - b^2])*d*e^2*Sqrt[e*Sin[c + d*x]]) + (7*
b^4*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin
[c + d*x]])/(2*(a^2 - b^2)^2*(a^2 - b^2 + a*Sqrt[a^2 - b^2])*d*e^2*Sqrt[e*S
in[c + d*x]]) + (2*b^2*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 +
d*x)/2, 2]*Sqrt[Sin[c + d*x]])/((a^2 - b^2)*(a^2 - b^2 + a*Sqrt[a^2 - b^2])
*d*e^2*Sqrt[e*Sin[c + d*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2775

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
```

```
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2991

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
```

```
tQ[m, 0] || IntegerQ[n])
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx = \int \frac{\cos^2(c+dx)}{(-b-a\cos(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx$$

$$= \int \left(\frac{1}{a^2(e\sin(c+dx))^{5/2}} + \frac{b^2}{a^2(-b-a\cos(c+dx))^2(e\sin(c+\right.$$

$$= \frac{\int \frac{1}{(e\sin(c+dx))^{5/2}}\,dx}{a^2} + \frac{(2b)\int \frac{1}{(-b-a\cos(c+dx))(e\sin(c+dx))^{5/2}}\,dx}{a^2} + \frac{b}{}$$

$$= -\frac{2\cos(c+dx)}{3a^2de(e\sin(c+dx))^{3/2}} + \frac{b^2}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))(}$$

$$= -\frac{2\cos(c+dx)}{3a^2de(e\sin(c+dx))^{3/2}} + \frac{b^2}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))(}$$

$$= -\frac{2\cos(c+dx)}{3a^2de(e\sin(c+dx))^{3/2}} + \frac{b^2}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))(}$$

$$= -\frac{2\cos(c+dx)}{3a^2de(e\sin(c+dx))^{3/2}} + \frac{b^2}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))(}$$

$$= -\frac{2\cos(c+dx)}{3a^2de(e\sin(c+dx))^{3/2}} + \frac{b^2}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))(}$$

$$= -\frac{2\sqrt{a}\,b\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\,\sqrt{e}}\right)}{\left(a^2-b^2\right)^{7/4}de^{5/2}} - \frac{2\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{}}{}\right.}{\left(a^2-b^2\right.}$$

$$= -\frac{7\sqrt{a}\,b^3\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\,\sqrt{e}}\right)}{2\left(a^2-b^2\right)^{11/4}de^{5/2}} - \frac{2\sqrt{a}\,b\tan^{-1}\left(\frac{\sqrt{}}{}\right.}{\left(a^2-b\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 30.95, size = 1320, normalized size = 1.21

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sec[c + d*x])^2*(e*Sin[c + d*x])^(5/2)),x]
```

```
[Out] -1/6*((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*Sin[c + d*x]^(5/2)*((2*(-2*a^3
- 5*a*b^2)*Cos[c + d*x]^2*(b + a*Sqrt[1 - Sin[c + d*x]^2])*((b*(-2*ArcTan[1
 - (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + 2*ArcTan[1 +
(Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 +
b^2] - Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*
x]] + Log[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c
+ d*x]] + a*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(3/4)) - (5*a*(
a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)
/(a^2 - b^2)]*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((5*(a^2 - b^2)*
AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2
)] + 2*(2*a^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]
^2)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2,
(a^2*Sin[c + d*x]^2)/(a^2 - b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c +
 d*x]^2)))))/((b + a*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (2*(10*a^2*b + 4
*b^3)*Cos[c + d*x]*(b + a*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[a]*
(2*ArcTan[1 - ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*A
rcTan[1 + ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + Log[Sqr
t[a^2 - b^2] - (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*S
in[c + d*x]] - Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt
[Sin[c + d*x]] + I*a*Sin[c + d*x]]))/(a^2 - b^2)^(3/4) + (5*b*(a^2 - b^2)*A
ppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]
*Sqrt[Sin[c + d*x]])/(Sqrt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4,
 1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + 2*(2*a^2*
AppellF1[5/4, 1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)
] + (a^2 - b^2)*AppellF1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x
]^2)/(a^2 - b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x]^2)))))/((b
 + a*Cos[c + d*x])*Sqrt[1 - Sin[c + d*x]^2])))/((a - b)^2*(a + b)^2*d*(a +
b*Sec[c + d*x])^2*(e*Sin[c + d*x])^(5/2)) + ((b + a*Cos[c + d*x])^2*((a*b^2
)/((-a^2 + b^2)^2*(b + a*Cos[c + d*x])) - (2*(-2*a*b + a^2*Cos[c + d*x] + b
^2*Cos[c + d*x])*Csc[c + d*x]^2)/(3*(-a^2 + b^2)^2))*Sin[c + d*x]*Tan[c + d
*x]^2)/(d*(a + b*Sec[c + d*x])^2*(e*Sin[c + d*x])^(5/2))
```

Maple [A]
time = 0.92, size = 1594, normalized size = 1.46

method	result	size

default	Expression too large to display	1594

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (4/3/e*b*a/(a^2-b^2)^2/(e*sin(d*x+c))^(3/2)+1/e*b^3*a/(a+b)^2/(a-b)^2*(e*si
n(d*x+c))^(1/2)/(-a^2*cos(d*x+c)^2*e^2+b^2*e^2)+1/e*b*a^3/(a+b)^2/(a-b)^2*(
e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(
a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))+3/4/
e*b^3*a/(a+b)^2/(a-b)^2*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*ln(((e
*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a
^2-b^2)/a^2)^(1/4)))+2/e*b*a^3/(a+b)^2/(a-b)^2*(e^2*(a^2-b^2)/a^2)^(1/4)/(-
a^2*e^2+b^2*e^2)*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))+3/2
/e*b^3*a/(a+b)^2/(a-b)^2*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*arcta
n((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))+(cos(d*x+c)^2*e*sin(d*x+c
))^(1/2)/e^2*(1/3*(a^2+b^2)/(a^2-b^2)^2/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(
cos(d*x+c)^2-1)*((-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(5
/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+2*cos(d*x+c)^2*sin(d*x+c))
-b^2*(3*a^2-b^2)/(a+b)^2/(a-b)^2*(-1/2/(a^2-b^2)^(1/2)/a*(-sin(d*x+c)+1)^(1
/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/
2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1
/2)/a),1/2*2^(1/2))+1/2/(a^2-b^2)^(1/2)/a*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+
c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(a^2-b^2)
^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1
/2)))+2*b^4/(a+b)/(a-b)*(-1/2*a^2/e/b^2/(a^2-b^2)*(cos(d*x+c)^2*e*sin(d*x+c
))^(1/2)/(-cos(d*x+c)^2*a^2+b^2)-1/4/b^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2
*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)*Ell
ipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-1/4/b^2/(a^2-b^2)^(3/2)*a*(-sin(d
*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(
d*x+c))^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(
a^2-b^2)^(1/2)/a),1/2*2^(1/2))+5/8/(a^2-b^2)^(3/2)/a*(-sin(d*x+c)+1)^(1/2)*
(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(
1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/
a),1/2*2^(1/2))+1/4/b^2/(a^2-b^2)^(3/2)*a*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+
c)+2)^(1/2)*sin(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(a^2-b^2)
^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1
/2))-5/8/(a^2-b^2)^(3/2)/a*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin
(d*x+c)^(1/2)/(cos(d*x+c)^2*e*sin(d*x+c))^(1/2)/(1+(a^2-b^2)^(1/2)/a)*Ellip
ticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))))/cos(d*x+
c)/(e*sin(d*x+c))^(1/2))/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**2/(e*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^2*(e*sin(d*x + c))^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^2}{(e\,\sin(c+d\,x))^{5/2}\,(b+a\,\cos(c+d\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*sin(c + d*x))^(5/2)*(a + b/cos(c + d*x))^2),x)
```

```
[Out] int(cos(c + d*x)^2/((e*sin(c + d*x))^(5/2)*(b + a*cos(c + d*x))^2), x)
```

3.249 $\int \sqrt{a+b\sec(e+fx)}\,dx$

Optimal. Leaf size=125

$$-\frac{2\cot(e+fx)\Pi\left(\frac{a}{a+b};\text{ArcSin}\left(\frac{\sqrt{a+b}}{\sqrt{a+b\sec(e+fx)}}\right)|\frac{a-b}{a+b}\right)\sqrt{-\frac{b(1-\sec(e+fx))}{a+b\sec(e+fx)}}\sqrt{\frac{b(1+\sec(e+fx))}{a+b\sec(e+fx)}}}{\sqrt{a+b}\,f}$$

```
[Out] -2*cot(f*x+e)*EllipticPi((a+b)^(1/2)/(a+b*sec(f*x+e))^(1/2),a/(a+b),((a-b)/
(a+b))^(1/2))*(a+b*sec(f*x+e))*(-b*(1-sec(f*x+e))/(a+b*sec(f*x+e)))^(1/2)*(
b*(1+sec(f*x+e))/(a+b*sec(f*x+e)))^(1/2)/f/(a+b)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {3865}

$$-\frac{2\cot(e+fx)\sqrt{-\frac{b(1-\sec(e+fx))}{a+b\sec(e+fx)}}\sqrt{\frac{b(\sec(e+fx)+1)}{a+b\sec(e+fx)}}(a+b\sec(e+fx))\Pi\left(\frac{a}{a+b};\text{ArcSin}\left(\frac{\sqrt{a+b}}{\sqrt{a+b\sec(e+fx)}}\right)|\frac{a-b}{a+b}\right)}{f\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] (-2*Cot[e + f*x]*EllipticPi[a/(a + b), ArcSin[Sqrt[a + b]/Sqrt[a + b*Sec[e
+ f*x]]], (a - b)/(a + b)]*Sqrt[-((b*(1 - Sec[e + f*x]))/(a + b*Sec[e + f*x
]))]*Sqrt[(b*(1 + Sec[e + f*x]))/(a + b*Sec[e + f*x])]*(a + b*Sec[e + f*x])
)/(Sqrt[a + b]*f)
```

Rule 3865

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*((a + b
*Csc[c + d*x])/(d*Rt[a + b, 2]*Cot[c + d*x]))*Sqrt[b*((1 + Csc[c + d*x])/(a
 + b*Csc[c + d*x]))]*Sqrt[(-b)*((1 - Csc[c + d*x])/(a + b*Csc[c + d*x]))]*E
llipticPi[a/(a + b), ArcSin[Rt[a + b, 2]/Sqrt[a + b*Csc[c + d*x]]], (a - b)
/(a + b)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\sec(e+fx)}\,dx = -\frac{2\cot(e+fx)\Pi\left(\frac{a}{a+b};\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a+b\sec(e+fx)}}\right)|\frac{a-b}{a+b}\right)\sqrt{-\frac{b(1-\sec(e+fx))}{a+b\sec(e+fx)}}}{\sqrt{a+b}\,f}$$

Mathematica [A]
time = 0.19, size = 151, normalized size = 1.21

$$\frac{4\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}\left((-a+b)F\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{a-b}{a+b}\right)+2a\Pi\left(-1;\text{ArcSin}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{a-b}{a+b}\right)\right)\sqrt{a+b\sec(e+fx)}}{f(b+a\cos(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] (4*Cos[(e + f*x)/2]^2*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[(b + a*Cos
[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*((-a + b)*EllipticF[ArcSin[Tan[(e
+ f*x)/2]], (a - b)/(a + b)] + 2*a*EllipticPi[-1, ArcSin[Tan[(e + f*x)/2]],
 (a - b)/(a + b)])*Sqrt[a + b*Sec[e + f*x]])/(f*(b + a*Cos[e + f*x]))
```

Maple [A]
time = 1.46, size = 215, normalized size = 1.72

method	result
default	$-\frac{2\sqrt{\frac{a\cos(fx+e)+b}{\cos(fx+e)}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\sqrt{\frac{a\cos(fx+e)+b}{(\cos(fx+e)+1)(a+b)}}(\cos(fx+e)+1)^2(-1+\cos(fx+e))\left(\text{EllipticF}\left(\frac{-1+\cos(fx+e)}{\sin(fx+e)},\sqrt{\right.\right.}{f(a\cos(fx+e)+b)\sin(fx+e)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/f*((a*cos(f*x+e)+b)/cos(f*x+e))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*
((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*(cos(f*x+e)+1)^2*(-1+cos(f*x+
e))*(EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a-EllipticF(
(-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*b-2*a*EllipticPi((-1+cos(f*
x+e))/sin(f*x+e),-1,((a-b)/(a+b))^(1/2)))/(a*cos(f*x+e)+b)/sin(f*x+e)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sec{\left(e + fx \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(1/2),x)
[Out] Integral(sqrt(a + b*sec(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+\frac{b}{\cos\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^(1/2),x)
[Out] int((a + b/cos(e + f*x))^(1/2), x)
```

3.250 $\int \csc^2(e+fx)\sqrt{a+b\sec(e+fx)}\,dx$

Optimal. Leaf size=121

$$\frac{\sqrt{a+b}\cot(e+fx)F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{f}$$

```
[Out] cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2)
)*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2
)/f-cot(f*x+e)*(a+b*sec(f*x+e))^(1/2)/f
```

Rubi [A]
time = 0.08, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3960, 3917}

$$\frac{\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{f}-\frac{\cot(e+fx)\sqrt{a+b\sec(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] (Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a
+ b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 +
 Sec[e + f*x]))/(a - b))])/f - (Cot[e + f*x]*Sqrt[a + b*Sec[e + f*x]])/f
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)/cos[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[Tan[e + f*x]*((a + b*Csc[e + f*x])^m/f), x] + Dist[b*m, I
nt[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
m}, x]
```

Rubi steps

$$\int \csc^2(e+fx)\sqrt{a+b\sec(e+fx)}\,dx = -\frac{\cot(e+fx)\sqrt{a+b\sec(e+fx)}}{f} + \frac{1}{2}b\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,d$$

$$= \frac{\sqrt{a+b}\,\cot(e+fx)F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(}{}}}{f}$$

Mathematica [A]
time = 0.90, size = 120, normalized size = 0.99

$$\frac{-\left((b+a\cos(e+fx))\csc(e+fx)\sqrt{\frac{1}{1+\sec(e+fx)}}\right)+bF\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{a-b}{a+b}\right)\sqrt{\frac{a+b\sec(e+fx)}{(a+b)(1+\sec(e+fx))}}}{f\sqrt{\frac{1}{1+\sec(e+fx)}}\sqrt{a+b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] (-((b + a*Cos[e + f*x])*Csc[e + f*x]*Sqrt[(1 + Sec[e + f*x])^(-1)]) + b*Ell
ipticF[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]*Sqrt[(a + b*Sec[e + f*x])
/((a + b)*(1 + Sec[e + f*x]))])/(f*Sqrt[(1 + Sec[e + f*x])^(-1)]*Sqrt[a + b
*Sec[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(110) = 220$.
time = 0.23, size = 264, normalized size = 2.18

method	result
default	$-\frac{(-1+\cos(fx+e))^2\left(\cos(fx+e)\,\text{EllipticF}\left(\frac{-1+\cos(fx+e)}{\sin(fx+e)},\sqrt{\frac{a-b}{a+b}}\right)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\sqrt{\frac{a\cos(fx+e)+b}{(\cos(fx+e)+1)(a+b)}}\sin(fx+e)b+\sqrt{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*(-1+cos(f*x+e))^2*(cos(f*x+e)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a
-b)/(a+b))^(1/2))*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(
f*x+e)+1)/(a+b))^(1/2)*sin(f*x+e)*b+(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*c
os(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+
e),((a-b)/(a+b))^(1/2))*b*sin(f*x+e)+cos(f*x+e)^2*a+cos(f*x+e)*b)*(cos(f*x+
e)+1)^2*((a*cos(f*x+e)+b)/cos(f*x+e))^(1/2)/(a*cos(f*x+e)+b)/sin(f*x+e)^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e) + a)*csc(f*x + e)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(f*x + e) + a)*csc(f*x + e)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sec{\left(e + f x \right)}} \csc^{2}{\left(e + f x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*sec(f*x+e))**(1/2),x)
[Out] Integral(sqrt(a + b*sec(e + f*x))*csc(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e) + a)*csc(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)}}}{{\sin\left(e+f\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^(1/2)/sin(e + f*x)^2,x)
[Out] int((a + b/cos(e + f*x))^(1/2)/sin(e + f*x)^2, x)
```

3.251 $\int (a+b\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=309

$$-\frac{2(a-b)\sqrt{a+b}\cot(e+fx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\text{se}}{a}}}{f}$$

```
[Out] -2*(a-b)*cot(f*x+e)*EllipticE((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-
b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a
-b))^(1/2)/f+2*(2*a-b)*cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1
/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+
sec(f*x+e))/(a-b))^(1/2)/f-2*a*cot(f*x+e)*EllipticPi((a+b*sec(f*x+e))^(1/2)
/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+
b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/f
```

Rubi [A]

time = 0.16, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3866, 4006, 3869, 3917, 4089}

$$\frac{2(2a-b)\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{f}-\frac{2(a-b)\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{f}-\frac{2a\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*Cot[e + f*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[e + f*
x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqr
t[-((b*(1 + Sec[e + f*x]))/(a - b))])/f + (2*(2*a - b)*Sqrt[a + b]*Cot[e +
f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b
)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a -
b))])/f - (2*a*Sqrt[a + b]*Cot[e + f*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a
 + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]
))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/f
```

Rule 3866

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(3/2), x_Symbol] :> Int[(a^2 + b
*(2*a - b)*Csc[c + d*x])/Sqrt[a + b*Csc[c + d*x]], x] + Dist[b^2, Int[Csc[c
 + d*x]*((1 + Csc[c + d*x])/Sqrt[a + b*Csc[c + d*x]]), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
```

```
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+b\sec(e+fx))^{3/2}\,dx &= b^2\int \frac{\sec(e+fx)(1+\sec(e+fx))}{\sqrt{a+b\sec(e+fx)}}\,dx+\int \frac{a^2+(2a-b)b\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx\\ &= -\frac{2(a-b)\sqrt{a+b}\,\cot(e+fx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\cdots}{\cdots}}\cdots}{f}\\ &= -\frac{2(a-b)\sqrt{a+b}\,\cot(e+fx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\cdots}{\cdots}}\cdots}{f}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.08, size = 882, normalized size = 2.85

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*b*Cos[e + f*x]*(a + b*Sec[e + f*x])^(3/2)*Sin[e + f*x])/(f*(b + a*Cos[e
+ f*x])) + (2*(a + b*Sec[e + f*x])^(3/2)*(a*b*Sqrt[(-a + b)/(a + b)]*Tan[(e
 + f*x)/2] + b^2*Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2] - 2*a*b*Sqrt[(-a +
 b)/(a + b)]*Tan[(e + f*x)/2]^3 + a*b*Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/
2]^5 - b^2*Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2]^5 + (2*I)*a^2*EllipticPi
[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2]], (a
 + b)/(a - b)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[(a + b - a*Tan[(e + f*x)/2
]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] + (2*I)*a^2*EllipticPi[-((a + b)/(a -
b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2]], (a + b)/(a - b)]*T
an[(e + f*x)/2]^2*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[(a + b - a*Tan[(e + f*x
)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] - I*(a - b)*b*EllipticE[I*ArcSinh[S
qrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(e +
 f*x)/2]^2]*(1 + Tan[(e + f*x)/2]^2)*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b
*Tan[(e + f*x)/2]^2)/(a + b)] - I*(a - b)^2*EllipticF[I*ArcSinh[Sqrt[(-a +
b)/(a + b)]*Tan[(e + f*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(e + f*x)/2]^2
]*(1 + Tan[(e + f*x)/2]^2)*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e +
f*x)/2]^2)/(a + b)]))/(Sqrt[(-a + b)/(a + b)]*f*(b + a*Cos[e + f*x])^(3/2)*
Sec[e + f*x]^(3/2)*Sqrt[(1 - Tan[(e + f*x)/2]^2)^(-1)]*(-1 + Tan[(e + f*x)/
2]^2)*(1 + Tan[(e + f*x)/2]^2)^(3/2)*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b
*Tan[(e + f*x)/2]^2)/(1 + Tan[(e + f*x)/2]^2)])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1198 vs. $2(282) = 564$.
time = 0.57, size = 1199, normalized size = 3.88

method	result	size
default	Expression too large to display	1199

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*((a*cos(f*x+e)+b)/cos(f*x+e))^(1/2)*(cos(f*x+e)+1)^2*(-1+cos(f*x+e))^2*
(EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*cos(f*x+e)*a^2*(
cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1
/2)*sin(f*x+e)-2*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*
cos(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+
1)/(a+b))^(1/2)*sin(f*x+e)*a*b-EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/
(a+b))^(1/2))*cos(f*x+e)*b^2*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+
e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*sin(f*x+e)+EllipticE((-1+cos(f*x+e))/sin(
f*x+e),((a-b)/(a+b))^(1/2))*cos(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((
a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*sin(f*x+e)*a*b+EllipticE((-1+co
s(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*cos(f*x+e)*b^2*(cos(f*x+e)/(cos(f
```

```
*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*sin(f*x+e)-2*
EllipticPi((-1+cos(f*x+e))/sin(f*x+e),-1,((a-b)/(a+b))^(1/2))*cos(f*x+e)*a^
2*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))
^(1/2)*sin(f*x+e)+(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(
f*x+e)+1)/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(
1/2))*a^2*sin(f*x+e)-2*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/
(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+
b))^(1/2))*a*b*sin(f*x+e)-(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+
b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/
(a+b))^(1/2))*b^2*sin(f*x+e)+EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a
+b))^(1/2))*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)
+1)/(a+b))^(1/2)*sin(f*x+e)*a*b+EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)
/(a+b))^(1/2))*b^2*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos
(f*x+e)+1)/(a+b))^(1/2)*sin(f*x+e)-2*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),
-1,((a-b)/(a+b))^(1/2))*a^2*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e
)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*sin(f*x+e)-cos(f*x+e)^2*a*b+cos(f*x+e)*a*b
-cos(f*x+e)*b^2+b^2)/sin(f*x+e)^5/(a*cos(f*x+e)+b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(3/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \sec (e + fx))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((a + b*sec(e + f*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+\frac{b}{\cos\left(e+f\,x\right)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^(3/2),x)
[Out] int((a + b/cos(e + f*x))^(3/2), x)
```

3.252 $\int \csc^2(e+fx)(a+b\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=228

$$-\frac{3(a-b)\sqrt{a+b}\cot(e+fx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec}{a-}}}{f}$$

```
[Out] -cot(f*x+e)*(a+b*sec(f*x+e))^(3/2)/f-3*(a-b)*cot(f*x+e)*EllipticE((a+b*sec(
f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e)
)/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/f+3*(a-b)*cot(f*x+e)*Ellipti
cF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(
1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/f
```

Rubi [A]
time = 0.17, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3960, 3914, 3917, 4089}

$$\frac{3(a-b)\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{f}-\frac{3(a-b)\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{f}-\frac{\cot(e+fx)(a+b\sec(e+fx))^{3/2}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(a + b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-3*(a - b)*Sqrt[a + b]*Cot[e + f*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[e + f*
x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqr
t[-((b*(1 + Sec[e + f*x]))/(a - b))])/f + (3*(a - b)*Sqrt[a + b]*Cot[e + f*
x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]
*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b)
)])/f - (Cot[e + f*x]*(a + b*Sec[e + f*x])^(3/2))/f
```

Rule 3914

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[a - b, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[b, Int[Csc[e + f*x]*((1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x],
x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)/cos[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[Tan[e + f*x]*((a + b*Csc[e + f*x])^m/f), x] + Dist[b*m, I
nt[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
m}, x]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\begin{aligned}\int \csc^2(e+fx)(a+b\sec(e+fx))^{3/2}\,dx &= -\frac{\cot(e+fx)(a+b\sec(e+fx))^{3/2}}{f}+\frac{1}{2}(3b)\int \sec(e+fx)\sqrt{a-} \\ &= -\frac{\cot(e+fx)(a+b\sec(e+fx))^{3/2}}{f}+\frac{1}{2}(3(a-b)b)\int \frac{\sec(e}{\sqrt{a+b\,\mathrm{se}}} \\ &= -\frac{3(a-b)\sqrt{a+b}\,\cot(e+fx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)\right.}{f}\end{aligned}$$

Mathematica [**A**]
time = 9.01, size = 276, normalized size = 1.21

$$\frac{\cos(e+fx)(a+b\sec(e+fx))^{3/2}((-b-a\cos(e+fx))\csc(e+fx)+3b\sin(e+fx))}{f(b+a\cos(e+fx))}+\frac{3b(a+b\sec(e+fx))^{3/2}\left(-\frac{(a+b)\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}\left(E\left(\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{a-b}{a+b}\right)-F\left(\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{a-b}{a+b}\right)\right)}{\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}}-(b+a\cos(e+fx))\tan\left(\frac{1}{2}(e+fx)\right)\right)}{f(b+a\cos(e+fx))^2\sqrt{\sec^2\left(\frac{1}{2}(e+fx)\right)}\sec^{\frac{3}{2}}(e+fx)\sqrt{\cos^2\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*(a + b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (Cos[e + f*x]*(a + b*Sec[e + f*x])^(3/2)*((-b - a*Cos[e + f*x])*Csc[e + f*x
] + 3*b*Sin[e + f*x]))/(f*(b + a*Cos[e + f*x])) + (3*b*(a + b*Sec[e + f*x])
^(3/2)*(-(((a + b)*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*
(EllipticE[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)] - EllipticF[ArcSin[Ta
n[(e + f*x)/2]], (a - b)/(a + b)]))/Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])])
```

- (b + a*Cos[e + f*x])*Tan[(e + f*x)/2]))/(f*(b + a*Cos[e + f*x])^2*Sqrt[Se
c[(e + f*x)/2]^2]*Sec[e + f*x]^(3/2)*Sqrt[Cos[(e + f*x)/2]^2*Sec[e + f*x]])

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 848 vs. $2(208) = 416$.
time = 0.24, size = 849, normalized size = 3.72

method	result
default	$\dfrac{(-1+\cos(fx+e))^2 \Bigl(3\,\mathrm{EllipticE}\Bigl(\frac{-1+\cos(fx+e)}{\sin(fx+e)}, \sqrt{\frac{a-b}{a+b}}\Bigr)\cos(fx+e)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\,\sqrt{\frac{a\cos(fx+e)+b}{(\cos(fx+e)+1)(a+b)}}\,\sin(fx+e)ab+3\,\mathrm{E}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(f*x+e)^2*(a+b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/f*(-1+cos(f*x+e))^2*(3*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))
^(1/2))*cos(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos
(f*x+e)+1)/(a+b))^(1/2)*sin(f*x+e)*a*b+3*EllipticE((-1+cos(f*x+e))/sin(f*x+
e),((a-b)/(a+b))^(1/2))*cos(f*x+e)*b^2*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((
a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*sin(f*x+e)-3*EllipticF((-1+cos(
f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*cos(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*sin(f*x+e)*a*b-3*El
lipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*cos(f*x+e)*b^2*(cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)
*sin(f*x+e)+3*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*(co
s(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2
)*sin(f*x+e)*a*b+3*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2)
)*b^2*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a
+b))^(1/2)*sin(f*x+e)-3*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)
/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a
+b))^(1/2))*a*b*sin(f*x+e)-3*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+
e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-
b)/(a+b))^(1/2))*b^2*sin(f*x+e)-cos(f*x+e)^2*a^2-3*cos(f*x+e)^2*a*b+cos(f*x
+e)*a*b-3*cos(f*x+e)*b^2+2*b^2)*(cos(f*x+e)+1)^2*((a*cos(f*x+e)+b)/cos(f*x+
e))^(1/2)/(a*cos(f*x+e)+b)/sin(f*x+e)^5

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e))^(3/2),x, algorithm="maxima")

[Out] integrate((b*sec(f*x + e) + a)^(3/2)*csc(f*x + e)^2, x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e))^(3/2),x, algorithm="fricas")
[Out] integral((b*csc(f*x + e)^2*sec(f*x + e) + a*csc(f*x + e)^2)*sqrt(b*sec(f*x
+ e) + a), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*sec(f*x+e))**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e) + a)^(3/2)*csc(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(e+f\,x)}\right)^{3/2}}{\sin\left(e+f\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^(3/2)/sin(e + f*x)^2,x)
[Out] int((a + b/cos(e + f*x))^(3/2)/sin(e + f*x)^2, x)
```

3.253 $\int \frac{1}{\sqrt{a+b\sec(e+fx)}}\,dx$

Optimal. Leaf size=106

$$-\frac{2\sqrt{a+b}\,\cot(e+fx)\Pi\left(\frac{a+b}{a};\mathrm{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{af}$$

```
[Out] -2*cot(f*x+e)*EllipticPi((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/
(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))
/(a-b))^(1/2)/a/f
```

Rubi [A]
time = 0.02, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {3869}

$$-\frac{2\sqrt{a+b}\,\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\mathrm{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{af}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] (-2*Sqrt[a + b]*Cot[e + f*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[e
+ f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]
*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(a*f)
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}}\,dx = -\frac{2\sqrt{a+b}\,\cot(e+fx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{af}$$

Mathematica [A]
time = 0.15, size = 138, normalized size = 1.30

$$-\frac{4\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}\left(F\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{a-b}{a+b}\right)-2\Pi\left(-1;\text{ArcSin}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{a-b}{a+b}\right)\right)\sec(e+fx)}{f\sqrt{a+b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] (-4*Cos[(e + f*x)/2]^2*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[(b + a*Co
s[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*(EllipticF[ArcSin[Tan[(e + f*x)/2
]], (a - b)/(a + b)] - 2*EllipticPi[-1, ArcSin[Tan[(e + f*x)/2]], (a - b)/(
a + b)])*Sec[e + f*x])/(f*Sqrt[a + b*Sec[e + f*x]])
```

Maple [A]
time = 0.23, size = 178, normalized size = 1.68

method	result
default	$-\frac{2\sqrt{\frac{a\cos(fx+e)+b}{\cos(fx+e)}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\sqrt{\frac{a\cos(fx+e)+b}{(\cos(fx+e)+1)(a+b)}}(\cos(fx+e)+1)^2\left(\text{EllipticF}\left(\frac{-1+\cos(fx+e)}{\sin(fx+e)},\sqrt{\frac{a-b}{a+b}}\right)-2\,\text{Ell}\right.}{f(a\cos(fx+e)+b)\sin(fx+e)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/f*((a*cos(f*x+e)+b)/cos(f*x+e))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*
((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*(cos(f*x+e)+1)^2*(EllipticF((
-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))-2*EllipticPi((-1+cos(f*x+e))
/sin(f*x+e),-1,((a-b)/(a+b))^(1/2)))*(-1+cos(f*x+e))/(a*cos(f*x+e)+b)/sin(f
*x+e)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sec(f*x + e) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b*sec(f*x + e) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b\sec{(e + fx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sec(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sec(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a + \frac{b}{\cos{(e + f\,x)}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(e + f*x))^(1/2),x)
```

```
[Out] int(1/(a + b/cos(e + f*x))^(1/2), x)
```

3.254 $\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx$

Optimal. Leaf size=255

$$\frac{\cot(e+fx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{\sqrt{a+b}\,f}-\frac{\cot(}{}$$

```
[Out] cot(f*x+e)*EllipticE((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2)
)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/f/(a+b)^(1
/2)-cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(
1/2))*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/f/(a+b
)^(1/2)-cot(f*x+e)/f/(a+b*sec(f*x+e))^(1/2)+b^2*tan(f*x+e)/(a^2-b^2)/f/(a+b
*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3960, 3918, 21, 3914, 3917, 4089}

$$\frac{b^2\tan(e+fx)}{f(a^2-b^2)\sqrt{a+b\sec(e+fx)}}-\frac{\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{f\sqrt{a+b}}+\frac{\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{f\sqrt{a+b}}-\frac{\cot(e+fx)}{f\sqrt{a+b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] (Cot[e + f*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a +
b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x
]))/(a - b))])/(Sqrt[a + b]*f) - (Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*
Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a
 + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(Sqrt[a + b]*f) - Cot[e + f
*x]/(f*Sqrt[a + b*Sec[e + f*x]]) + (b^2*Tan[e + f*x])/((a^2 - b^2)*f*Sqrt[a
 + b*Sec[e + f*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3914

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[a - b, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + D
```

```
ist[b, Int[Csc[e + f*x]*((1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x],
x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3918

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*C
sc[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)/cos[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[Tan[e + f*x]*((a + b*Csc[e + f*x])^m/f), x] + Dist[b*m, I
nt[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
m}, x]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx = -\frac{\cot(e+fx)}{f\sqrt{a+b\sec(e+fx)}} - \frac{1}{2}b\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx$$

$$= -\frac{\cot(e+fx)}{f\sqrt{a+b\sec(e+fx)}} + \frac{b^2\tan(e+fx)}{(a^2-b^2)\,f\sqrt{a+b\sec(e+fx)}} + \frac{b\int \frac{\sec(e+fx)(-\frac{a}{2}\cdots}{\sqrt{a+b\sec\cdots}}}{a^2-\cdots}$$

$$= -\frac{\cot(e+fx)}{f\sqrt{a+b\sec(e+fx)}} + \frac{b^2\tan(e+fx)}{(a^2-b^2)\,f\sqrt{a+b\sec(e+fx)}} - \frac{b\int \sec(e+fx)\cdots}{2\cdots}$$

$$= -\frac{\cot(e+fx)}{f\sqrt{a+b\sec(e+fx)}} + \frac{b^2\tan(e+fx)}{(a^2-b^2)\,f\sqrt{a+b\sec(e+fx)}} - \frac{b\int \frac{\sec(e+\cdots}{\sqrt{a+b\sec(\cdots}}}{2(a+\cdots}$$

$$= \frac{\cot(e+fx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{\cdots}}{\sqrt{a+b}\,f}$$

Mathematica [A]
time = 4.56, size = 259, normalized size = 1.02

$$\frac{\sqrt{\sec(e+fx)}\left(\frac{(b+a\cos(e+fx))(-a+b\cos(e+fx))\csc(e+fx)}{(a^2-b^2)\sqrt{\sec(e+fx)}} + \frac{b\left(-\frac{(a+b)\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}\left(E\left(\operatorname{ArcSin}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{a-b}{a+b}\right)-F\left(\operatorname{ArcSin}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{a-b}{a+b}\right)\right)}{\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}}-(b+a\cos(e+fx))\tan\left(\frac{1}{2}(e+fx)\right)\right)}{(-a^2+b^2)\sqrt{\sec^2\left(\frac{1}{2}(e+fx)\right)}\sqrt{\cos^2\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)}}\right)}{f\sqrt{a+b\sec(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] (Sqrt[Sec[e + f*x]]*(((b + a*Cos[e + f*x])*(-a + b*Cos[e + f*x])*Csc[e + f*
x])/((a^2 - b^2)*Sqrt[Sec[e + f*x]]) + (b*(-(((a + b)*Sqrt[(b + a*Cos[e + f
*x])/((a + b)*(1 + Cos[e + f*x]))]*(EllipticE[ArcSin[Tan[(e + f*x)/2]], (a
- b)/(a + b)] - EllipticF[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]))/Sqrt
[Cos[e + f*x]/(1 + Cos[e + f*x])]) - (b + a*Cos[e + f*x])*Tan[(e + f*x)/2])
)/((-a^2 + b^2)*Sqrt[Sec[(e + f*x)/2]^2]*Sqrt[Cos[(e + f*x)/2]^2*Sec[e + f*
x]])))/(f*Sqrt[a + b*Sec[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 851 vs. $2(233) = 466$.
time = 0.24, size = 852, normalized size = 3.34

method	result
default	$-\frac{(-1+\cos(fx+e))^2\left(-\text{EllipticF}\left(\frac{-1+\cos(fx+e)}{\sin(fx+e)},\sqrt{\frac{a-b}{a+b}}\right)\cos(fx+e)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\sqrt{\frac{a\cos(fx+e)+b}{(\cos(fx+e)+1)(a+b)}}\sin(fx+e)ab-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(f*x+e)^2/(a+b*sec(f*x+e))^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/f*(-1+cos(f*x+e))^2*(-EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*cos(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*sin(f*x+e)*a*b-EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*cos(f*x+e)*b^2*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*sin(f*x+e)+EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*cos(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*sin(f*x+e)*a*b+EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*cos(f*x+e)*b^2*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*sin(f*x+e)-(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a*b*sin(f*x+e)-(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*b^2*sin(f*x+e)+EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*sin(f*x+e)*a*b+EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*b^2*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*sin(f*x+e)+cos(f*x+e)^2*a^2-cos(f*x+e)^2*a*b+cos(f*x+e)*a*b-cos(f*x+e)*b^2)*(cos(f*x+e)+1)^2*((a*cos(f*x+e)+b)/cos(f*x+e))^(1/2)/(a*cos(f*x+e)+b)/sin(f*x+e)^5/(a-b)/(a+b)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate(csc(f*x + e)^2/sqrt(b*sec(f*x + e) + a), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral(csc(f*x + e)^2/sqrt(b*sec(f*x + e) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*sec(f*x+e))**(1/2),x)
[Out] Integral(csc(e + f*x)**2/sqrt(a + b*sec(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(csc(f*x + e)^2/sqrt(b*sec(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sin\left(e+f\,x\right)^2\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^2*(a + b/cos(e + f*x))^(1/2)),x)
[Out] int(1/(sin(e + f*x)^2*(a + b/cos(e + f*x))^(1/2)), x)
```

3.255 $\int \frac{1}{(a+b\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=347

$$\frac{2\cot(e+fx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{a\sqrt{a+b}\,f}-\frac{2\cot}{}$$

```
[Out] 2*cot(f*x+e)*EllipticE((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/
2))*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/a/f/(a+b
)^(1/2)-2*cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a
-b))^(1/2))*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/
a/f/(a+b)^(1/2)-2*cot(f*x+e)*EllipticPi((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),
(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b
*(1+sec(f*x+e))/(a-b))^(1/2)/a^2/f+2*b^2*tan(f*x+e)/a/(a^2-b^2)/f/(a+b*sec(
f*x+e))^(1/2)
```

Rubi [A]

time = 0.23, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3870, 4143, 4006, 3869, 3917, 4089}

$$-\frac{2\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{a^2f}+\frac{2b^2\tan(e+fx)}{af(a^2-b^2)\sqrt{a+b\sec(e+fx)}}-\frac{2\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{af\sqrt{a+b}}+\frac{2\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{af\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x])^(-3/2),x]
```

```
[Out] (2*Cot[e + f*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a
+ b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f
*x]))/(a - b))])/(a*Sqrt[a + b]*f) - (2*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[
a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(a*Sqrt[a + b]*f) - (
2*Sqrt[a + b]*Cot[e + f*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[e +
f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(a^2*f) + (2*b^2*Tan[e + f*x])/(a*(
a^2 - b^2)*f*Sqrt[a + b*Sec[e + f*x]])
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(e+fx))^{3/2}}\,dx = \frac{2b^2\tan(e+fx)}{a\left(a^2-b^2\right)f\sqrt{a+b\sec(e+fx)}} - \frac{2\int \frac{\frac{1}{2}\left(-a^2+b^2\right)+\frac{1}{2}ab\sec(e+fx)+\frac{1}{2}b^2\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx}{a\left(a^2-b^2\right)}$$

$$= \frac{2b^2\tan(e+fx)}{a\left(a^2-b^2\right)f\sqrt{a+b\sec(e+fx)}} - \frac{2\int \frac{\frac{1}{2}\left(-a^2+b^2\right)+\left(\frac{ab}{2}-\frac{b^2}{2}\right)\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx}{a\left(a^2-b^2\right)} - \frac{b^2\int}{}$$

$$= \frac{2\cot(e+fx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}}{a\sqrt{a+b}\,f}$$

$$= \frac{2\cot(e+fx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}}{a\sqrt{a+b}\,f}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.11, size = 1249, normalized size = 3.60

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x])^(-3/2),x]
```

```
[Out] ((b + a*Cos[e + f*x])^2*Sec[e + f*x]^2*((2*b*Sin[e + f*x])/(a*(-a^2 + b^2))
 + (2*b^2*Sin[e + f*x])/(a*(a^2 - b^2)*(b + a*Cos[e + f*x]))))/(f*(a + b*Se
c[e + f*x])^(3/2)) + (2*(b + a*Cos[e + f*x])^(3/2)*Sec[e + f*x]^(3/2)*Sqrt[
(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(1 + Tan[(e + f*x)/2]
^2)]*(a*b*Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2] + b^2*Sqrt[(-a + b)/(a +
b)]*Tan[(e + f*x)/2] - 2*a*b*Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2]^3 + a*
b*Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2]^5 - b^2*Sqrt[(-a + b)/(a + b)]*Ta
n[(e + f*x)/2]^5 - (2*I)*a^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[
(-a + b)/(a + b)]*Tan[(e + f*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(e + f*x
)/2]^2]*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)]
 + (2*I)*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)
]*Tan[(e + f*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[(a
+ b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] - (2*I)*a^2*Ell
ipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/
2]], (a + b)/(a - b)]*Tan[(e + f*x)/2]^2*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[
(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] + (2*I)*b^2*
EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(e + f*
x)/2]], (a + b)/(a - b)]*Tan[(e + f*x)/2]^2*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sq
rt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] - I*(a -
```

```
b)*b*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2]], (a + b)/
(a - b)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*(1 + Tan[(e + f*x)/2]^2)*Sqrt[(a + b
- a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] + I*(a^2 + a*b - 2*
b^2)*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2]], (a + b)/
(a - b)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*(1 + Tan[(e + f*x)/2]^2)*Sqrt[(a + b
- a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)]))/(a*Sqrt[(-a + b)/
(a + b)]*(a^2 - b^2)*f*(a + b*Sec[e + f*x])^(3/2)*(-1 + Tan[(e + f*x)/2]^2)
*Sqrt[(1 + Tan[(e + f*x)/2]^2)/(1 - Tan[(e + f*x)/2]^2)]*(a*(-1 + Tan[(e +
f*x)/2]^2) - b*(1 + Tan[(e + f*x)/2]^2)))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1208 vs. $2(318) = 636$.
time = 0.31, size = 1209, normalized size = 3.48

method	result	size
default	Expression too large to display	1209

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*4^(1/2)*((a*cos(f*x+e)+b)/cos(f*x+e))^(1/2)*(EllipticF((-1+cos(f*x+e))/
sin(f*x+e),((a-b)/(a+b))^(1/2))*cos(f*x+e)*a^2*(cos(f*x+e)/(cos(f*x+e)+1))^
(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*sin(f*x+e)+EllipticF((-
1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*cos(f*x+e)*(cos(f*x+e)/(cos(f
*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*sin(f*x+e)*a*
b-EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*cos(f*x+e)*(cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)
*sin(f*x+e)*a*b-EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*c
os(f*x+e)*b^2*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+
e)+1)/(a+b))^(1/2)*sin(f*x+e)-2*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),-1,((
a-b)/(a+b))^(1/2))*cos(f*x+e)*a^2*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos
(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*sin(f*x+e)+2*(cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*sin(f*x+e)*cos(f*x
+e)*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),-1,((a-b)/(a+b))^(1/2))*b^2+(cos(
f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*
EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a^2*sin(f*x+e)+(c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/
2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a*b*sin(f*x+e)
-EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*(cos(f*x+e)/(cos
(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*sin(f*x+e)*
a*b-EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*b^2*(cos(f*x+
e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*sin(
f*x+e)-2*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),-1,((a-b)/(a+b))^(1/2))*a^2*
(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(
1/2)*sin(f*x+e)+2*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(
```

```
f*x+e)+1)/(a+b))^(1/2)*sin(f*x+e)*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),-1,
((a-b)/(a+b))^(1/2))*b^2+cos(f*x+e)^2*a*b-cos(f*x+e)^2*b^2-cos(f*x+e)*a*b+c
os(f*x+e)*b^2)/(a*cos(f*x+e)+b)/sin(f*x+e)/a/(a+b)/(a-b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e) + a)^(-3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(f*x + e) + a)/(b^2*sec(f*x + e)^2 + 2*a*b*sec(f*x + e)
+ a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \sec{\left(e + f x \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e))**(3/2),x)
[Out] Integral((a + b*sec(e + f*x))**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(f*x + e) + a)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+\frac{b}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(e + f*x))^(3/2),x)
[Out] int(1/(a + b/cos(e + f*x))^(3/2), x)
```

3.256 $\int \frac{\csc^2(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=318

$$\frac{4a\cot(e+fx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{(a-b)(a+b)^{3/2}f}-\frac{(3a}{}$$

```
[Out] -cot(f*x+e)/f/(a+b*sec(f*x+e))^(3/2)+4*a*cot(f*x+e)*EllipticE((a+b*sec(f*x+
e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(
-b*(1+sec(f*x+e))/(a-b))^(1/2)/(a-b)/(a+b)^(3/2)/f-(3*a-b)*cot(f*x+e)*Ellip
ticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(f*x+
e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/(a-b)/(a+b)^(3/2)/f+b^2*ta
n(f*x+e)/(a^2-b^2)/f/(a+b*sec(f*x+e))^(3/2)+4*a*b^2*tan(f*x+e)/(a^2-b^2)^2/
f/(a+b*sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3960, 3918, 4088, 4090, 3917, 4089}

$$\frac{4ab^2\tan(e+fx)}{f\left(a^2-b^2\right)^2\sqrt{a+b\sec(e+fx)}}+\frac{b^2\tan(e+fx)}{f\left(a^2-b^2\right)(a+b\sec(e+fx))^{3/2}}-\frac{(3a-b)\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{f(a-b)(a+b)^{3/2}}+\frac{4a\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{f(a-b)(a+b)^{3/2}}-\frac{\cot(e+fx)}{f(a+b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/(a + b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (4*a*Cot[e + f*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (
a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e +
 f*x]))/(a - b))])/((a - b)*(a + b)^(3/2)*f) - ((3*a - b)*Cot[e + f*x]*Elli
pticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(
b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/((a
 - b)*(a + b)^(3/2)*f) - Cot[e + f*x]/(f*(a + b*Sec[e + f*x])^(3/2)) + (b^2
*Tan[e + f*x])/((a^2 - b^2)*f*(a + b*Sec[e + f*x])^(3/2)) + (4*a*b^2*Tan[e
+ f*x])/((a^2 - b^2)^2*f*Sqrt[a + b*Sec[e + f*x]])
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3918

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*C
sc[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)/cos[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[Tan[e + f*x]*((a + b*Csc[e + f*x])^m/f), x] + Dist[b*m, I
nt[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
m}, x]
```

Rule 4088

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1
/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[
(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx = -\frac{\cot(e+fx)}{f(a+b\sec(e+fx))^{3/2}} - \frac{1}{2}(3b)\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))^{5/2}}\,dx$$

$$= -\frac{\cot(e+fx)}{f(a+b\sec(e+fx))^{3/2}} + \frac{b^2\tan(e+fx)}{(a^2-b^2)\,f(a+b\sec(e+fx))^{3/2}} + \frac{b\int \frac{\sec(e+fx)(-}{(a+b\,\mathrm{se}}}{a^2}$$

$$= -\frac{\cot(e+fx)}{f(a+b\sec(e+fx))^{3/2}} + \frac{b^2\tan(e+fx)}{(a^2-b^2)\,f(a+b\sec(e+fx))^{3/2}} + \frac{4ab^2\,\mathrm{t}}{(a^2-b^2)^2\,f\sqrt{}}$$

$$= -\frac{\cot(e+fx)}{f(a+b\sec(e+fx))^{3/2}} + \frac{b^2\tan(e+fx)}{(a^2-b^2)\,f(a+b\sec(e+fx))^{3/2}} + \frac{4ab^2\,\mathrm{t}}{(a^2-b^2)^2\,f\sqrt{}}$$

$$= \frac{4a\cot(e+fx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}}{(a-b)(a+b)^{3/2}f}$$

Mathematica [A]
time = 5.07, size = 259, normalized size = 0.81

$$\frac{-((a-b)((3a-b)b+a(a-3b)\cos(e+fx))\csc(e+fx))+8ab(a+b)\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}E\left(\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{a-b}{a+b}\right)\sec(e+fx)\sqrt{\frac{1}{1+\sec(e+fx)}}-2b(3a^2+4ab+b^2)\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}F\left(\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{a-b}{a+b}\right)\sec(e+fx)\sqrt{\frac{1}{1+\sec(e+fx)}}}{(a^2-b^2)^2 f\sqrt{a+b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/(a + b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-((a - b)*((3*a - b)*b + a*(a - 3*b)*Cos[e + f*x])*Csc[e + f*x]) + 8*a*b*(
a + b)*Cos[(e + f*x)/2]^2*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e + f
*x]))]*EllipticE[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]*Sec[e + f*x]*Sq
rt[(1 + Sec[e + f*x])^(-1)] - 2*b*(3*a^2 + 4*a*b + b^2)*Cos[(e + f*x)/2]^2*
Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*EllipticF[ArcSin[Ta
n[(e + f*x)/2]], (a - b)/(a + b)]*Sec[e + f*x]*Sqrt[(1 + Sec[e + f*x])^(-1)
])/((a^2 - b^2)^2*f*Sqrt[a + b*Sec[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1064 vs. 2(294) = 588.
time = 0.20, size = 1065, normalized size = 3.35

method	result	size
default	Expression too large to display	1065

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(a+b*sec(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*(-3*cos(f*x+e)*sin(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(
f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),
((a-b)/(a+b))^(1/2))*a^2*b-4*cos(f*x+e)*sin(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticF((-1+cos(f
*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a*b^2-cos(f*x+e)*sin(f*x+e)*(cos(f*x
+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*Ell
ipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*b^3+4*cos(f*x+e)*sin
(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/
(a+b))^(1/2)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a^2*
b+4*cos(f*x+e)*sin(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+
b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/
(a+b))^(1/2))*a*b^2-3*sin(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(
f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),
((a-b)/(a+b))^(1/2))*a^2*b-4*sin(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(
(a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(
f*x+e),((a-b)/(a+b))^(1/2))*a*b^2-sin(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1
/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))
/sin(f*x+e),((a-b)/(a+b))^(1/2))*b^3+4*sin(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1
))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticE((-1+cos(f*
x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a^2*b+4*sin(f*x+e)*(cos(f*x+e)/(cos(f
*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticE((-1
+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a*b^2+cos(f*x+e)^2*a^3-4*cos(f
*x+e)^2*a^2*b+3*cos(f*x+e)^2*a*b^2+3*cos(f*x+e)*a^2*b-4*cos(f*x+e)*a*b^2+co
s(f*x+e)*b^3)*((a*cos(f*x+e)+b)/cos(f*x+e))^(1/2)*4^(1/2)/(a*cos(f*x+e)+b)/
sin(f*x+e)/(a-b)^2/(a+b)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate(csc(f*x + e)^2/(b*sec(f*x + e) + a)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(f*x + e) + a)*csc(f*x + e)^2/(b^2*sec(f*x + e)^2 + 2*a*
b*sec(f*x + e) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(e + fx)}{(a + b\sec(e + fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*sec(f*x+e))**(3/2),x)
[Out] Integral(csc(e + f*x)**2/(a + b*sec(e + f*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate(csc(f*x + e)^2/(b*sec(f*x + e) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sin(e + f\,x)^2\,\left(a + \frac{b}{\cos(e+f\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^2*(a + b/cos(e + f*x))^(3/2)),x)
[Out] int(1/(sin(e + f*x)^2*(a + b/cos(e + f*x))^(3/2)), x)
```

3.257 $\int (a+b\sec(c+dx))^3(e\sin(c+dx))^m\,dx$

Optimal. Leaf size=249

$$\frac{a^3\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)\sqrt{\cos^2(c+dx)}}+\frac{3a^2b\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)(e\sin}{de(1+m)}$$

```
[Out] 3*a^2*b*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*(e*sin(d*x+c))^(
1+m)/d/e/(1+m)+b^3*hypergeom([2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*(e*si
n(d*x+c))^(1+m)/d/e/(1+m)+a^3*cos(d*x+c)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/
2*m],sin(d*x+c)^2)*(e*sin(d*x+c))^(1+m)/d/e/(1+m)/(cos(d*x+c)^2)^(1/2)+3*a*
b^2*hypergeom([3/2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*sec(d*x+c)*(e*sin(
d*x+c))^(1+m)*(cos(d*x+c)^2)^(1/2)/d/e/(1+m)
```

Rubi [A]
time = 0.26, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3957, 2991, 2722, 2644, 371, 2657}

$$\frac{a^3\cos(c+dx)(e\sin(c+dx))^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{de(m+1)\sqrt{\cos^2(c+dx)}}+\frac{3a^2b(e\sin(c+dx))^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{de(m+1)}+\frac{3ab^2\sqrt{\cos^2(c+dx)}\sec(c+dx)(e\sin(c+dx))^{m+1}\,{}_2F_1\left(\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{de(m+1)}+\frac{b^3(e\sin(c+dx))^{m+1}\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{de(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^3*(e*Sin[c + d*x])^m,x]
```

```
[Out] (a^3*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]
^2]*(e*Sin[c + d*x])^(1 + m))/(d*e*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (3*a^2*b
*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x]
)^(1 + m))/(d*e*(1 + m)) + (b^3*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2,
Sin[c + d*x]^2]*(e*Sin[c + d*x])^(1 + m))/(d*e*(1 + m)) + (3*a*b^2*Sqrt[Cos
[c + d*x]^2]*Hypergeometric2F1[3/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*S
ec[c + d*x]*(e*Sin[c + d*x])^(1 + m))/(d*e*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2991

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int (a+b\sec(c+dx))^3(e\sin(c+dx))^m\,dx &= -\int (-b-a\cos(c+dx))^3\sec^3(c+dx)(e\sin(c+dx))^m\,dx \\
&= -\int \left(-a^3(e\sin(c+dx))^m-3a^2b\sec(c+dx)(e\sin(c+dx))^m-\right. \\
&= a^3\int (e\sin(c+dx))^m\,dx+\left(3a^2b\right)\int \sec(c+dx)(e\sin(c+dx))^m \\
&= \frac{a^3\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)\sqrt{\cos^2(c+dx)}} \\
&= \frac{a^3\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)\sqrt{\cos^2(c+dx)}}
\end{aligned}
$$

Mathematica [A]
time = 0.21, size = 182, normalized size = 0.73

$$\frac{\left(a^3 \sqrt{\cos^2(c+dx)}\ {}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)+b\left(3a^2\cos(c+dx)\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)+b\left(3a\sqrt{\cos^2(c+dx)}\ {}_2F_1\left(\frac{3}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)+b\cos(c+dx)\,{}_2F_1\left(2,\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)\right)\right)\right)(e\sin(c+dx))^m\tan(c+dx)}{d(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^3*(e*Sin[c + d*x])^m,x]
```

```
[Out] ((a^3*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin
[c + d*x]^2] + b*(3*a^2*Cos[c + d*x]*Hypergeometric2F1[1, (1 + m)/2, (3 + m
)/2, Sin[c + d*x]^2] + b*(3*a*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[3/2, (
1 + m)/2, (3 + m)/2, Sin[c + d*x]^2] + b*Cos[c + d*x]*Hypergeometric2F1[2,
(1 + m)/2, (3 + m)/2, Sin[c + d*x]^2])))*(e*Sin[c + d*x])^m*Tan[c + d*x])/(
d*(1 + m))
```

Maple [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int \left(a+b\sec\left(dx+c\right)\right)^{3} \left(e\sin\left(dx+c\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3*(e*sin(d*x+c))^m,x)
```

```
[Out] int((a+b*sec(d*x+c))^3*(e*sin(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(e*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3*(e*sin(d*x + c))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(e*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c
) + a^3)*(e*sin(d*x + c))^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e \sin (c + dx))^m (a + b \sec (c + dx))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3*(e*sin(d*x+c))**m,x)
[Out] Integral((e*sin(c + d*x))**m*(a + b*sec(c + d*x))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(e*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^3*(e*sin(d*x + c))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (e \sin (c + d x))^m \left(a + \frac{b}{\cos (c + d x)}\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^m*(a + b/cos(c + d*x))^3,x)
[Out] int((e*sin(c + d*x))^m*(a + b/cos(c + d*x))^3, x)
```

3.258 $\int (a+b\sec(c+dx))^2(e\sin(c+dx))^m\,dx$

Optimal. Leaf size=190

$$\frac{a^2\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)\sin(c+dx)(e\sin(c+dx))^m}{d(1+m)\sqrt{\cos^2(c+dx)}}+\frac{2ab\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx\right.}{de(1+m)}$$

```
[Out] 2*a*b*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*(e*sin(d*x+c))^(1+
m)/d/e/(1+m)+a^2*cos(d*x+c)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+
c)^2)*sin(d*x+c)*(e*sin(d*x+c))^m/d/(1+m)/(cos(d*x+c)^2)^(1/2)+b^2*hypergeo
m([3/2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*(e*sin(d*x+c))^m*(cos(d*x+c)^2
)^(1/2)*tan(d*x+c)/d/(1+m)
```

Rubi [A]
time = 0.60, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3957, 2990, 2644, 371, 4483, 4486, 2722, 2657}

$$\frac{a^2\sin(c+dx)\cos(c+dx)(e\sin(c+dx))^m\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{d(m+1)\sqrt{\cos^2(c+dx)}}+\frac{2ab(e\sin(c+dx))^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{de(m+1)}+\frac{b^2\sqrt{\cos^2(c+dx)}\,\tan(c+dx)(e\sin(c+dx))^m\,{}_2F_1\left(\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2*(e*Sin[c + d*x])^m,x]
```

```
[Out] (a^2*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]
^2]*Sin[c + d*x]*(e*Sin[c + d*x])^m)/(d*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (2*
a*b*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d
*x])^(1 + m))/(d*e*(1 + m)) + (b^2*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[3
/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])^m*Tan[c + d*x])/
(d*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2990

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[2*a*(b/d), I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 4483

```
Int[(u_.)*((a_)*(v_))^(p_), x_Symbol] :> With[{uu = ActivateTrig[u], vv = A
ctivateTrig[v]}, Dist[a^IntPart[p]*((a*vv)^FracPart[p]/vv^FracPart[p]), Int
[uu*vv^p, x], x]] /; FreeQ[{a, p}, x] &&  !IntegerQ[p] &&  !InertTrigFreeQ[
v]
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\int (a+b\sec(c+dx))^2(e\sin(c+dx))^m\,dx = \int (-b-a\cos(c+dx))^2\sec^2(c+dx)(e\sin(c+dx))^m\,dx$$
$$= (2ab)\int \sec(c+dx)(e\sin(c+dx))^m\,dx + \int \left(b^2+a^2\cos^2(c+\right.$$
$$= \frac{(2ab)\text{Subst}\left(\int \frac{x^m}{1-\frac{x^2}{e^2}}\,dx, x, e\sin(c+dx)\right)}{de} + \left(\sin^{-m}(c+dx)(e\sin\right.$$
$$= \frac{2ab\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)} + \left(\sin^{-m}\right.$$
$$= \frac{2ab\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)} + \left(a^2\sin\right.$$
$$= \frac{a^2\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)\sin(c+dx)(e\sin(}{d(1+m)\sqrt{\cos^2(c+dx)}}$$

Mathematica [A]
time = 0.17, size = 134, normalized size = 0.71

$$\frac{(e\sin(c+dx))^m\left(2ab\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)\sin(c+dx)+\sqrt{\cos^2(c+dx)}\left(a^2\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)+b^2\,{}_2F_1\left(\frac{3}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)\right)\tan(c+dx)\right)}{d(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2*(e*Sin[c + d*x])^m,x]
```

```
[Out] ((e*Sin[c + d*x])^m*(2*a*b*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[c
 + d*x]^2]*Sin[c + d*x] + Sqrt[Cos[c + d*x]^2]*(a^2*Hypergeometric2F1[1/2,
(1 + m)/2, (3 + m)/2, Sin[c + d*x]^2] + b^2*Hypergeometric2F1[3/2, (1 + m)/
2, (3 + m)/2, Sin[c + d*x]^2])*Tan[c + d*x]))/(d*(1 + m))
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int (a+b\sec(dx+c))^2(e\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*(e*sin(d*x+c))^m,x)
```

```
[Out] int((a+b*sec(d*x+c))^2*(e*sin(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(e*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2*(e*sin(d*x + c))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(e*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*(e*sin(d*x + c))^m
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e \sin (c + dx))^m (a + b \sec (c + dx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*(e*sin(d*x+c))**m,x)
```

```
[Out] Integral((e*sin(c + d*x))**m*(a + b*sec(c + d*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(e*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2*(e*sin(d*x + c))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (e \sin (c + d x))^m \left(a + \frac{b}{\cos (c + d x)}\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^m*(a + b/cos(c + d*x))^2,x)
```

```
[Out] int((e*sin(c + d*x))^m*(a + b/cos(c + d*x))^2, x)
```

3.259 $\int (a+b\sec(c+dx))(e\sin(c+dx))^m\,dx$

Optimal. Leaf size=119

$$\frac{a\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)\sqrt{\cos^2(c+dx)}}+\frac{b\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)(e\sin(c+$$

```
[Out] b*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*(e*sin(d*x+c))^(1+m)/d
/e/(1+m)+a*cos(d*x+c)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*
(e*sin(d*x+c))^(1+m)/d/e/(1+m)/(cos(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3957, 2917, 2644, 371, 2722}

$$\frac{a\cos(c+dx)(e\sin(c+dx))^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{de(m+1)\sqrt{\cos^2(c+dx)}}+\frac{b(e\sin(c+dx))^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{de(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])*(e*Sin[c + d*x])^m,x]
```

```
[Out] (a*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2
]*(e*Sin[c + d*x])^(1 + m))/(d*e*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (b*Hyperge
ometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])^(1 + m
))/(d*e*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
```

```
&&  !IntegerQ[2*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int (a+b\sec(c+dx))(e\sin(c+dx))^m\,dx &= -\int (-b-a\cos(c+dx))\sec(c+dx)(e\sin(c+dx))^m\,dx\\ &= a\int (e\sin(c+dx))^m\,dx + b\int \sec(c+dx)(e\sin(c+dx))^m\,dx\\ &= \frac{a\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)\sqrt{\cos^2(c+dx)}} +\\ &= \frac{a\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)\sqrt{\cos^2(c+dx)}} +\end{aligned}$$

Mathematica [A]
time = 0.07, size = 98, normalized size = 0.82

$$\frac{\left(a\sqrt{\cos^2(c+dx)}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)+b\cos(c+dx)\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)\right)(e\sin(c+dx))^m\tan(c+dx)}{d(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])*(e*Sin[c + d*x])^m,x]
```

```
[Out] ((a*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c
 + d*x]^2] + b*Cos[c + d*x]*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[
c + d*x]^2])*(e*Sin[c + d*x])^m*Tan[c + d*x])/(d*(1 + m))
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int (a+b\sec(dx+c))(e\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*(e*sin(d*x+c))^m,x)
```

```
[Out] int((a+b*sec(d*x+c))*(e*sin(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(e*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(e*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e\sin{(c+dx)})^m (a+b\sec{(c+dx)})\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(e*sin(d*x+c))**m,x)
```

```
[Out] Integral((e*sin(c + d*x))**m*(a + b*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(e*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int (e\sin (c+d\,x))^m\,\left(a+\frac{b}{\cos (c+d\,x)}\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^m*(a + b/cos(c + d*x)),x)
[Out] int((e*sin(c + d*x))^m*(a + b/cos(c + d*x)), x)
```

3.260 $\int \frac{(e\sin(c+dx))^m}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=232

$$-\frac{beF_1\left(1-m;\frac{1-m}{2},\frac{1-m}{2};2-m;-\frac{a-b}{b+a\cos(c+dx)},\frac{a+b}{b+a\cos(c+dx)}\right)\left(-\frac{a(1-\cos(c+dx))}{b+a\cos(c+dx)}\right)^{\frac{1-m}{2}}\left(\frac{a(1+\cos(c+dx))}{b+a\cos(c+dx)}\right)^{\frac{1-m}{2}}(e\sin(}{a^2d(1-m)}$$

```
[Out] -b*e*AppellF1(1-m,1/2-1/2*m,1/2-1/2*m,2-m,(-a+b)/(b+a*cos(d*x+c)),(a+b)/(b+
a*cos(d*x+c)))*(-a*(1-cos(d*x+c))/(b+a*cos(d*x+c)))^(1/2-1/2*m)*(a*(1+cos(d
*x+c))/(b+a*cos(d*x+c)))^(1/2-1/2*m)*(e*sin(d*x+c))^(-1+m)/a^2/d/(1-m)+cos(
d*x+c)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*(e*sin(d*x+c))^
(1+m)/a/d/e/(1+m)/(cos(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3957, 2946, 2722, 2782}

$$\frac{\cos(c+dx)(e\sin(c+dx))^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{ade(m+1)\sqrt{\cos^2(c+dx)}}-\frac{be(e\sin(c+dx))^{m-1}\left(-\frac{a(1-\cos(c+dx))}{a\cos(c+dx)+b}\right)^{\frac{1-m}{2}}\left(\frac{a(\cos(c+dx)+1)}{a\cos(c+dx)+b}\right)^{\frac{1-m}{2}}F_1\left(1-m;\frac{1-m}{2},\frac{1-m}{2};2-m;-\frac{a-b}{b+a\cos(c+dx)},\frac{a+b}{b+a\cos(c+dx)}\right)}{a^2d(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^m/(a + b*Sec[c + d*x]),x]
```

```
[Out] -((b*e*AppellF1[1 - m, (1 - m)/2, (1 - m)/2, 2 - m, -((a - b)/(b + a*Cos[c
+ d*x])), (a + b)/(b + a*Cos[c + d*x])]*(-((a*(1 - Cos[c + d*x]))/(b + a*Co
s[c + d*x])))^((1 - m)/2)*((a*(1 + Cos[c + d*x]))/(b + a*Cos[c + d*x]))^((1
 - m)/2)*(e*Sin[c + d*x])^(-1 + m))/(a^2*d*(1 - m))) + (Cos[c + d*x]*Hyperg
eometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])^(1
+ m))/(a*d*e*(1 + m)*Sqrt[Cos[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2782

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p)*((-b)*((1 - Sin[e + f*x])/(a + b*Sin[e + f*x])))^((
p - 1)/2)*(b*((1 + Sin[e + f*x])/(a + b*Sin[e + f*x])))^((p - 1)/2)))*Appel
lF1[-p - m, (1 - p)/2, (1 - p)/2, 1 - p - m, (a + b)/(a + b*Sin[e + f*x]),
```

```
(a - b)/(a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && ILtQ[m, 0] &&  !IGtQ[m + p + 1, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{(e\sin(c+dx))^m}{a+b\sec(c+dx)}\,dx &= -\int \frac{\cos(c+dx)(e\sin(c+dx))^m}{-b-a\cos(c+dx)}\,dx\\ &= \frac{\int (e\sin(c+dx))^m\,dx}{a}+\frac{b\int \frac{(e\sin(c+dx))^m}{-b-a\cos(c+dx)}\,dx}{a}\\ &= -\frac{beF_1\left(1-m;\frac{1-m}{2},\frac{1-m}{2};2-m;-\frac{a-b}{b+a\cos(c+dx)},\frac{a+b}{b+a\cos(c+dx)}\right)\left(-\frac{a(1-\cos(c+dx))}{b+a\cos(c+dx)}\right)^{\frac{1-m}{2}}\left(\cdots\right.}{a^2d(1-m)}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 687 vs. 2(232) = 464.
time = 4.08, size = 687, normalized size = 2.96

$$\frac{2\left(-bF_1\left(\frac{1+m}{2};m,1;\frac{3+m}{2};-\tan^2\left(\frac{1}{2}(c+dx)\right),\frac{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)}{a+b}\right)+(a+b)\,{}_2F_1\left(\frac{1+m}{2},1+m;\frac{3+m}{2};-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\right)(e\sin(c+dx))^m\tan\left(\frac{1}{2}(c+dx)\right)}{d(a+b\sec(c+dx))\left(\left(-bF_1\left(\frac{1+m}{2};m,1;\frac{3+m}{2};-\tan^2\left(\frac{1}{2}(c+dx)\right),\frac{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)}{a+b}\right)+(a+b)\,{}_2F_1\left(\frac{1+m}{2},1+m;\frac{3+m}{2};-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\right)\sec^2\left(\frac{1}{2}(c+dx)\right)+2m\cot(c+dx)\left(-bF_1\left(\frac{1+m}{2};m,1;\frac{3+m}{2};-\tan^2\left(\frac{1}{2}(c+dx)\right),\frac{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)}{a+b}\right)+(a+b)\,{}_2F_1\left(\frac{1+m}{2},1+m;\frac{3+m}{2};-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\right)\tan\left(\frac{1}{2}(c+dx)\right)+2m\left(-bF_1\left(\frac{1+m}{2};m,1;\frac{3+m}{2};-\tan^2\left(\frac{1}{2}(c+dx)\right),\frac{(a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)}{a+b}\right)+(a+b)\,{}_2F_1\left(\frac{1+m}{2},1+m;\frac{3+m}{2};-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+\text{[illegible]}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sin[c + d*x])^m/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*(-(b*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*
Tan[(c + d*x)/2]^2)/(a + b)]) + (a + b)*Hypergeometric2F1[(1 + m)/2, 1 + m,
 (3 + m)/2, -Tan[(c + d*x)/2]^2])*(e*Sin[c + d*x])^m*Tan[(c + d*x)/2])/(d*(
a + b*Sec[c + d*x])*((-(b*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*
x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) + (a + b)*Hypergeometric2F1
```

```
[(1 + m)/2, 1 + m, (3 + m)/2, -Tan[(c + d*x)/2]^2])*Sec[(c + d*x)/2]^2 + 2*
m*Cot[c + d*x]*(-(b*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^
2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) + (a + b)*Hypergeometric2F1[(1 +
m)/2, 1 + m, (3 + m)/2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2] + 2*m*(-(b*A
ppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c +
d*x)/2]^2)/(a + b)]) + (a + b)*Hypergeometric2F1[(1 + m)/2, 1 + m, (3 + m)/
2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2 + ((1 + m)*Sec[(c + d*x)/2]^2*(
-((a + b)^2*(Hypergeometric2F1[(1 + m)/2, 1 + m, (3 + m)/2, -Tan[(c + d*x)/
2]^2] - (Sec[(c + d*x)/2]^2)^(-1 - m))) + (2*b*((-a + b)*AppellF1[(3 + m)/2
, m, 2, (5 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b
)] + (a + b)*m*AppellF1[(3 + m)/2, 1 + m, 1, (5 + m)/2, -Tan[(c + d*x)/2]^2
, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])*Tan[(c + d*x)/2]^2)/(3 + m)))/(a +
 b)))
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(e\sin(dx+c))^m}{a+b\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^m/(a+b*sec(d*x+c)),x)
[Out] int((e*sin(d*x+c))^m/(a+b*sec(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate((e*sin(d*x + c))^m/(b*sec(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c)),x, algorithm="fricas")
[Out] integral((e*sin(d*x + c))^m/(b*sec(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\sin(c+dx))^m}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**m/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((e*sin(c + d*x))**m/(a + b*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*sin(d*x + c))^m/(b*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)\,\left(e\,\sin(c+d\,x)\right)^m}{b+a\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^m/(a + b/cos(c + d*x)),x)
```

```
[Out] int((cos(c + d*x)*(e*sin(c + d*x))^m)/(b + a*cos(c + d*x)), x)
```

3.261 $\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=405

$$-\frac{2beF_1\left(1-m;\frac{1-m}{2},\frac{1-m}{2};2-m;-\frac{a-b}{b+a\cos(c+dx)},\frac{a+b}{b+a\cos(c+dx)}\right)\left(-\frac{a(1-\cos(c+dx))}{b+a\cos(c+dx)}\right)^{\frac{1-m}{2}}\left(\frac{a(1+\cos(c+dx))}{b+a\cos(c+dx)}\right)^{\frac{1-m}{2}}(e\sin}{a^3d(1-m)}$$

```
[Out] -2*b*e*AppellF1(1-m,1/2-1/2*m,1/2-1/2*m,2-m,(-a+b)/(b+a*cos(d*x+c)),(a+b)/(
b+a*cos(d*x+c)))*(-a*(1-cos(d*x+c))/(b+a*cos(d*x+c)))^(1/2-1/2*m)*(a*(1+cos
(d*x+c))/(b+a*cos(d*x+c)))^(1/2-1/2*m)*(e*sin(d*x+c))^(-1+m)/a^3/d/(1-m)+b^
2*e*AppellF1(2-m,1/2-1/2*m,1/2-1/2*m,3-m,(-a+b)/(b+a*cos(d*x+c)),(a+b)/(b+a
*cos(d*x+c)))*(-a*(1-cos(d*x+c))/(b+a*cos(d*x+c)))^(1/2-1/2*m)*(a*(1+cos(d*
x+c))/(b+a*cos(d*x+c)))^(1/2-1/2*m)*(e*sin(d*x+c))^(-1+m)/a^3/d/(2-m)/(b+a*
cos(d*x+c))+cos(d*x+c)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)
*(e*sin(d*x+c))^(1+m)/a^2/d/e/(1+m)/(cos(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 405, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3957, 2991, 2722, 2782}

$$\frac{b^2e(e\sin(c+dx))^{m-1}\left(-\frac{a(1-\cos(c+dx))}{a\cos(c+dx)+b}\right)^{\frac{1-m}{2}}\left(\frac{a(\cos(c+dx)+1)}{a\cos(c+dx)+b}\right)^{\frac{1-m}{2}}F_1\left(2-m;\frac{1-m}{2},\frac{1-m}{2};3-m;-\frac{a-b}{b+a\cos(c+dx)},\frac{a+b}{b+a\cos(c+dx)}\right)}{a^3d(2-m)(a\cos(c+dx)+b)}-\frac{2be(e\sin(c+dx))^{m-1}\left(-\frac{a(1-\cos(c+dx))}{a\cos(c+dx)+b}\right)^{\frac{1-m}{2}}\left(\frac{a(\cos(c+dx)+1)}{a\cos(c+dx)+b}\right)^{\frac{1-m}{2}}F_1\left(1-m;\frac{1-m}{2},\frac{1-m}{2};2-m;-\frac{a-b}{b+a\cos(c+dx)},\frac{a+b}{b+a\cos(c+dx)}\right)}{a^3d(1-m)}+\frac{\cos(c+dx)(e\sin(c+dx))^{m+1}{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{a^2de(m+1)\sqrt{\cos^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^m/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-2*b*e*AppellF1[1 - m, (1 - m)/2, (1 - m)/2, 2 - m, -((a - b)/(b + a*Cos[c
 + d*x])), (a + b)/(b + a*Cos[c + d*x])]*(-((a*(1 - Cos[c + d*x]))/(b + a*C
os[c + d*x])))^((1 - m)/2)*((a*(1 + Cos[c + d*x]))/(b + a*Cos[c + d*x]))^((
1 - m)/2)*(e*Sin[c + d*x])^(-1 + m))/(a^3*d*(1 - m)) + (b^2*e*AppellF1[2 -
m, (1 - m)/2, (1 - m)/2, 3 - m, -((a - b)/(b + a*Cos[c + d*x])), (a + b)/(b
 + a*Cos[c + d*x])]*(-((a*(1 - Cos[c + d*x]))/(b + a*Cos[c + d*x])))^((1 -
m)/2)*((a*(1 + Cos[c + d*x]))/(b + a*Cos[c + d*x]))^((1 - m)/2)*(e*Sin[c +
d*x])^(-1 + m))/(a^3*d*(2 - m)*(b + a*Cos[c + d*x])) + (Cos[c + d*x]*Hyperg
eometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])^(1
+ m))/(a^2*d*e*(1 + m)*Sqrt[Cos[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2782

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p)*((-b)*((1 - Sin[e + f*x])/(a + b*Sin[e + f*x])))^((
p - 1)/2)*(b*((1 + Sin[e + f*x])/(a + b*Sin[e + f*x])))^((p - 1)/2)))*Appel
lF1[-p - m, (1 - p)/2, (1 - p)/2, 1 - p - m, (a + b)/(a + b*Sin[e + f*x]),
(a - b)/(a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && ILtQ[m, 0] &&  !IGtQ[m + p + 1, 0]
```

Rule 2991

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^2}\,dx &= \int \frac{\cos^2(c+dx)(e\sin(c+dx))^m}{(-b-a\cos(c+dx))^2}\,dx \\
&= \int \left(\frac{(e\sin(c+dx))^m}{a^2}+\frac{b^2(e\sin(c+dx))^m}{a^2(b+a\cos(c+dx))^2}-\frac{2b(e\sin(c+dx))^m}{a^2(b+a\cos(c+dx))}\right)dx \\
&= \frac{\int (e\sin(c+dx))^m\,dx}{a^2}-\frac{(2b)\int \frac{(e\sin(c+dx))^m}{b+a\cos(c+dx)}\,dx}{a^2}+\frac{b^2\int \frac{(e\sin(c+dx))^m}{(b+a\cos(c+dx))^2}\,dx}{a^2} \\
&= -\frac{2beF_1\left(1-m;\frac{1-m}{2},\frac{1-m}{2};2-m;-\frac{a-b}{b+a\cos(c+dx)},\frac{a+b}{b+a\cos(c+dx)}\right)\left(-\frac{a(1-\cos(c+dx))}{b+a\cos(c+dx)}\right)^{1}}{a^3d(1-m)}
\end{aligned}
$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 1433 vs. $2(405) = 810$.
time = 9.85, size = 1433, normalized size = 3.54

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sin[c + d*x])^m/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-4*b*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Ta
n[(c + d*x)/2]^2)/(a + b)]*(b + a*Cos[c + d*x])*Sec[c + d*x]^2*(e*Sin[c + d
*x])^m*Tan[(c + d*x)/2])/(a^2*d*(a + b*Sec[c + d*x])^2*(AppellF1[(1 + m)/2,
 m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)
]*Sec[(c + d*x)/2]^2 + 2*m*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d
*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*Cot[c + d*x]*Tan[(c + d*x)/
2] + 2*m*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)
*Tan[(c + d*x)/2]^2)/(a + b)]*Tan[(c + d*x)/2]^2 - (2*(1 + m)*((-a + b)*App
ellF1[(3 + m)/2, m, 2, (5 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*
x)/2]^2)/(a + b)] + (a + b)*m*AppellF1[(3 + m)/2, 1 + m, 1, (5 + m)/2, -Tan
[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])*Sec[(c + d*x)/2]^2*
Tan[(c + d*x)/2]^2)/((a + b)*(3 + m)))) + (2*b^2*((a + b)*AppellF1[(1 + m)/
2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a +
b)] - 2*a*AppellF1[(1 + m)/2, m, 2, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b
)*Tan[(c + d*x)/2]^2)/(a + b)])*Sec[c + d*x]^2*(e*Sin[c + d*x])^m*Tan[(c +
d*x)/2])/(a^2*d*(a + b*Sec[c + d*x])^2*(((a + b)*AppellF1[(1 + m)/2, m, 1,
(3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] - 2*a
*AppellF1[(1 + m)/2, m, 2, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c
+ d*x)/2]^2)/(a + b)])*Sec[(c + d*x)/2]^2 + 2*m*((a + b)*AppellF1[(1 + m)/2
, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b
)] - 2*a*AppellF1[(1 + m)/2, m, 2, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)
*Tan[(c + d*x)/2]^2)/(a + b)])*Cot[c + d*x]*Tan[(c + d*x)/2] + 2*m*((a + b)
*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c
+ d*x)/2]^2)/(a + b)] - 2*a*AppellF1[(1 + m)/2, m, 2, (3 + m)/2, -Tan[(c +
d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])*Tan[(c + d*x)/2]^2 - (2*(
1 + m)*((-a^2 + b^2)*AppellF1[(3 + m)/2, m, 2, (5 + m)/2, -Tan[(c + d*x)/2]
^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + 4*a*(a - b)*AppellF1[(3 + m)/2,
 m, 3, (5 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)
] + (a + b)*m*((a + b)*AppellF1[(3 + m)/2, 1 + m, 1, (5 + m)/2, -Tan[(c + d
*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] - 2*a*AppellF1[(3 + m)/2, 1
 + m, 2, (5 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a +
b)]))*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2)/((a + b)*(3 + m)))) - ((b + a*
Cos[c + d*x])^2*Hypergeometric2F1[1/2, (1 - m)/2, 3/2, Cos[c + d*x]^2]*(e*S
in[c + d*x])^m*(Sin[c + d*x]^2)^((-1 - m)/2)*Tan[c + d*x])/(a^2*d*(a + b*Se
c[c + d*x])^2)
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(e\sin (dx+c))^m}{(a+b\sec (dx+c))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^2,x)
```

```
[Out] int((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
[Out] integrate((e*sin(d*x + c))^m/(b*sec(d*x + c) + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
[Out] integral((e*sin(d*x + c))^m/(b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\sin{(c+dx)})^m}{(a+b\sec{(c+dx)})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**m/(a+b*sec(d*x+c))**2,x)
[Out] Integral((e*sin(c + d*x))**m/(a + b*sec(c + d*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate((e*sin(d*x + c))^m/(b*sec(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos{(c+d\,x)}^2\,{(e\,\sin{(c+d\,x)})}^m}{{(b+a\,\cos{(c+d\,x)})}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^m/(a + b/cos(c + d*x))^2,x)
[Out] int((cos(c + d*x)^2*(e*sin(c + d*x))^m)/(b + a*cos(c + d*x))^2, x)
```

3.262 $\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=580

$$-\frac{3beF_1\left(1-m;\frac{1-m}{2},\frac{1-m}{2};2-m;-\frac{a-b}{b+a\cos(c+dx)},\frac{a+b}{b+a\cos(c+dx)}\right)\left(-\frac{a(1-\cos(c+dx))}{b+a\cos(c+dx)}\right)^{\frac{1-m}{2}}\left(\frac{a(1+\cos(c+dx))}{b+a\cos(c+dx)}\right)^{\frac{1-m}{2}}(e\sin(}{a^4d(1-m)}$$

```
[Out] -3*b*e*AppellF1(1-m,1/2-1/2*m,1/2-1/2*m,2-m,(-a+b)/(b+a*cos(d*x+c)),(a+b)/(
b+a*cos(d*x+c)))*(-a*(1-cos(d*x+c))/(b+a*cos(d*x+c)))^(1/2-1/2*m)*(a*(1+cos
(d*x+c))/(b+a*cos(d*x+c)))^(1/2-1/2*m)*(e*sin(d*x+c))^(-1+m)/a^4/d/(1-m)-b^
3*e*AppellF1(3-m,1/2-1/2*m,1/2-1/2*m,4-m,(-a+b)/(b+a*cos(d*x+c)),(a+b)/(b+a
*cos(d*x+c)))*(-a*(1-cos(d*x+c))/(b+a*cos(d*x+c)))^(1/2-1/2*m)*(a*(1+cos(d*
x+c))/(b+a*cos(d*x+c)))^(1/2-1/2*m)*(e*sin(d*x+c))^(-1+m)/a^4/d/(3-m)/(b+a*
cos(d*x+c))^2+3*b^2*e*AppellF1(2-m,1/2-1/2*m,1/2-1/2*m,3-m,(-a+b)/(b+a*cos(
d*x+c)),(a+b)/(b+a*cos(d*x+c)))*(-a*(1-cos(d*x+c))/(b+a*cos(d*x+c)))^(1/2-1
/2*m)*(a*(1+cos(d*x+c))/(b+a*cos(d*x+c)))^(1/2-1/2*m)*(e*sin(d*x+c))^(-1+m)
/a^4/d/(2-m)/(b+a*cos(d*x+c))+cos(d*x+c)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/
2*m],sin(d*x+c)^2)*(e*sin(d*x+c))^(1+m)/a^3/d/e/(1+m)/(cos(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.40, antiderivative size = 580, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3957, 2991, 2722, 2782}

$$-\frac{b^3e(e\sin(c+dx))^{m-1}\left(-\frac{a(1-\cos(c+dx))}{a\cos(c+dx)+b}\right)^{\frac{1-m}{2}}\left(\frac{a(\cos(c+dx)+1)}{a\cos(c+dx)+b}\right)^{\frac{1-m}{2}}F_1\left(3-m;\frac{1-m}{2},\frac{1-m}{2};4-m;-\frac{a-b}{b+a\cos(c+dx)},\frac{a+b}{b+a\cos(c+dx)}\right)}{a^4d(3-m)(a\cos(c+dx)+b)^2}+\frac{3b^2e(e\sin(c+dx))^{m-1}\left(-\frac{a(1-\cos(c+dx))}{a\cos(c+dx)+b}\right)^{\frac{1-m}{2}}\left(\frac{a(\cos(c+dx)+1)}{a\cos(c+dx)+b}\right)^{\frac{1-m}{2}}F_1\left(2-m;\frac{1-m}{2},\frac{1-m}{2};3-m;-\frac{a-b}{b+a\cos(c+dx)},\frac{a+b}{b+a\cos(c+dx)}\right)}{a^4d(2-m)(a\cos(c+dx)+b)}-\frac{3be(e\sin(c+dx))^{m-1}\left(-\frac{a(1-\cos(c+dx))}{a\cos(c+dx)+b}\right)^{\frac{1-m}{2}}\left(\frac{a(\cos(c+dx)+1)}{a\cos(c+dx)+b}\right)^{\frac{1-m}{2}}F_1\left(1-m;\frac{1-m}{2},\frac{1-m}{2};2-m;-\frac{a-b}{b+a\cos(c+dx)},\frac{a+b}{b+a\cos(c+dx)}\right)}{a^4d(1-m)}+\frac{\cos(c+dx)(e\sin(c+dx))^{m+1}{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{a^3de(m+1)\sqrt{\cos^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^m/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (-3*b*e*AppellF1[1 - m, (1 - m)/2, (1 - m)/2, 2 - m, -((a - b)/(b + a*Cos[c
 + d*x])), (a + b)/(b + a*Cos[c + d*x])]*(-((a*(1 - Cos[c + d*x]))/(b + a*C
os[c + d*x])))^((1 - m)/2)*((a*(1 + Cos[c + d*x]))/(b + a*Cos[c + d*x]))^((
1 - m)/2)*(e*Sin[c + d*x])^(-1 + m))/(a^4*d*(1 - m)) - (b^3*e*AppellF1[3 -
m, (1 - m)/2, (1 - m)/2, 4 - m, -((a - b)/(b + a*Cos[c + d*x])), (a + b)/(b
 + a*Cos[c + d*x])]*(-((a*(1 - Cos[c + d*x]))/(b + a*Cos[c + d*x])))^((1 -
m)/2)*((a*(1 + Cos[c + d*x]))/(b + a*Cos[c + d*x]))^((1 - m)/2)*(e*Sin[c +
d*x])^(-1 + m))/(a^4*d*(3 - m)*(b + a*Cos[c + d*x])^2) + (3*b^2*e*AppellF1[
2 - m, (1 - m)/2, (1 - m)/2, 3 - m, -((a - b)/(b + a*Cos[c + d*x])), (a + b
)/(b + a*Cos[c + d*x])]*(-((a*(1 - Cos[c + d*x]))/(b + a*Cos[c + d*x])))^((
1 - m)/2)*((a*(1 + Cos[c + d*x]))/(b + a*Cos[c + d*x]))^((1 - m)/2)*(e*Sin[
c + d*x])^(-1 + m))/(a^4*d*(2 - m)*(b + a*Cos[c + d*x])) + (Cos[c + d*x]*Hy
pergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])
^(1 + m))/(a^3*d*e*(1 + m)*Sqrt[Cos[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2782

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p)*((-b)*((1 - Sin[e + f*x])/(a + b*Sin[e + f*x])))^((
p - 1)/2)*(b*((1 + Sin[e + f*x])/(a + b*Sin[e + f*x])))^((p - 1)/2)))*Appel
lF1[-p - m, (1 - p)/2, (1 - p)/2, 1 - p - m, (a + b)/(a + b*Sin[e + f*x]),
(a - b)/(a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && ILtQ[m, 0] &&  !IGtQ[m + p + 1, 0]
```

Rule 2991

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^3}\,dx = -\int \frac{\cos^3(c+dx)(e\sin(c+dx))^m}{(-b-a\cos(c+dx))^3}\,dx$$

$$= -\int \left(-\frac{(e\sin(c+dx))^m}{a^3} + \frac{b^3(e\sin(c+dx))^m}{a^3(b+a\cos(c+dx))^3} - \frac{3b^2(e\sin(c+dx))^m}{a^3(b+a\cos(c+dx))^2} +\right.$$

$$= \frac{\int (e\sin(c+dx))^m\,dx}{a^3} - \frac{(3b)\int \frac{(e\sin(c+dx))^m}{b+a\cos(c+dx)}\,dx}{a^3} + \frac{(3b^2)\int \frac{(e\sin(c+dx))^m}{(b+a\cos(c+dx))^2}\,dx}{a^3} - \frac{b^3}{}$$

$$= -\frac{3beF_1\left(1-m;\frac{1-m}{2},\frac{1-m}{2};2-m;-\frac{a-b}{b+a\cos(c+dx)},\frac{a+b}{b+a\cos(c+dx)}\right)\left(-\frac{a(1-\cos(c+dx))}{b+a\cos(c+dx)}\right)}{a^4d(1-m)}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2700 vs. $2(580) = 1160$.
time = 13.75, size = 2700, normalized size = 4.66

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sin[c + d*x])^m/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (-6*b*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Ta
n[(c + d*x)/2]^2)/(a + b)]*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^3*(e*Sin[c +
 d*x])^m*Tan[(c + d*x)/2])/(a^3*d*(a + b*Sec[c + d*x])^3*(AppellF1[(1 + m)/
2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a +
b)]*Sec[(c + d*x)/2]^2 + 2*m*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c +
 d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*Cot[c + d*x]*Tan[(c + d*x
)/2] + 2*m*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a -
b)*Tan[(c + d*x)/2]^2)/(a + b)]*Tan[(c + d*x)/2]^2 - (2*(1 + m)*((-a + b)*A
ppellF1[(3 + m)/2, m, 2, (5 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c +
d*x)/2]^2)/(a + b)] + (a + b)*m*AppellF1[(3 + m)/2, 1 + m, 1, (5 + m)/2, -T
an[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])*Sec[(c + d*x)/2]^
2*Tan[(c + d*x)/2]^2)/((a + b)*(3 + m)))) + (6*b^2*((a + b)*AppellF1[(1 + m
)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a
+ b)] - 2*a*AppellF1[(1 + m)/2, m, 2, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a -
 b)*Tan[(c + d*x)/2]^2)/(a + b)])*(b + a*Cos[c + d*x])*Sec[c + d*x]^3*(e*Si
n[c + d*x])^m*Tan[(c + d*x)/2])/(a^3*d*(a + b*Sec[c + d*x])^3*(((a + b)*App
ellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*
x)/2]^2)/(a + b)] - 2*a*AppellF1[(1 + m)/2, m, 2, (3 + m)/2, -Tan[(c + d*x)
/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])*Sec[(c + d*x)/2]^2 + 2*m*((a
+ b)*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan
[(c + d*x)/2]^2)/(a + b)] - 2*a*AppellF1[(1 + m)/2, m, 2, (3 + m)/2, -Tan[(
c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])*Cot[c + d*x]*Tan[(c +
 d*x)/2] + 2*m*((a + b)*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)
/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] - 2*a*AppellF1[(1 + m)/2, m, 2
, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])*Ta
n[(c + d*x)/2]^2 - (2*(1 + m)*((-a^2 + b^2)*AppellF1[(3 + m)/2, m, 2, (5 +
m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + 4*a*(a -
 b)*AppellF1[(3 + m)/2, m, 3, (5 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[
(c + d*x)/2]^2)/(a + b)] + (a + b)*m*((a + b)*AppellF1[(3 + m)/2, 1 + m, 1,
 (5 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] - 2*
a*AppellF1[(3 + m)/2, 1 + m, 2, (5 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Ta
n[(c + d*x)/2]^2)/(a + b)]))*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2)/((a + b
)*(3 + m)))) - (2*b^3*((a + b)^2*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[
(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] - 4*a*(a + b)*AppellF
1[(1 + m)/2, m, 2, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2
]^2)/(a + b)] + 4*a^2*AppellF1[(1 + m)/2, m, 3, (3 + m)/2, -Tan[(c + d*x)/2
```

```
]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])*Sec[c + d*x]^3*(e*Sin[c + d*x])
^m*Tan[(c + d*x)/2])/(a^3*d*(a + b*Sec[c + d*x])^3*(((a + b)^2*AppellF1[(1
+ m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/
(a + b)] - 4*a*(a + b)*AppellF1[(1 + m)/2, m, 2, (3 + m)/2, -Tan[(c + d*x)/
2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + 4*a^2*AppellF1[(1 + m)/2, m,
3, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])*S
ec[(c + d*x)/2]^2 + 2*m*((a + b)^2*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Ta
n[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] - 4*a*(a + b)*Appel
lF1[(1 + m)/2, m, 2, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)
/2]^2)/(a + b)] + 4*a^2*AppellF1[(1 + m)/2, m, 3, (3 + m)/2, -Tan[(c + d*x)
/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])*Cot[c + d*x]*Tan[(c + d*x)/2]
 + 2*m*((a + b)^2*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2,
 ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] - 4*a*(a + b)*AppellF1[(1 + m)/2, m,
 2, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] +
 4*a^2*AppellF1[(1 + m)/2, m, 3, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*T
an[(c + d*x)/2]^2)/(a + b)])*Tan[(c + d*x)/2]^2 + (2*(1 + m)*((a + b)^2*((a
 - b)*AppellF1[(3 + m)/2, m, 2, (5 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Ta
n[(c + d*x)/2]^2)/(a + b)] - (a + b)*m*AppellF1[(3 + m)/2, 1 + m, 1, (5 + m
)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) - 4*a*(a +
 b)*(2*(a - b)*AppellF1[(3 + m)/2, m, 3, (5 + m)/2, -Tan[(c + d*x)/2]^2, ((
a - b)*Tan[(c + d*x)/2]^2)/(a + b)] - (a + b)*m*AppellF1[(3 + m)/2, 1 + m,
2, (5 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) +
 4*a^2*(3*(a - b)*AppellF1[(3 + m)/2, m, 4, (5 + m)/2, -Tan[(c + d*x)/2]^2,
 ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] - (a + b)*m*AppellF1[(3 + m)/2, 1 +
m, 3, (5 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]
))*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2)/((a + b)*(3 + m)))) - ((b + a*Cos
[c + d*x])^3*Hypergeometric2F1[1/2, (1 - m)/2, 3/2, Cos[c + d*x]^2]*Sec[c +
 d*x]*(e*Sin[c + d*x])^m*(Sin[c + d*x]^2)^((-1 - m)/2)*Tan[c + d*x])/(a^3*d
*(a + b*Sec[c + d*x])^3)
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\left(e \sin \left(d x +c \right)\right)^{m}}{\left(a +b \sec \left(d x +c \right)\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^3,x)
[Out] int((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((e*sin(d*x + c))^m/(b*sec(d*x + c) + a)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((e*sin(d*x + c))^m/(b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 +
3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\sin{(c+dx)})^m}{(a+b\sec{(c+dx)})^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**m/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((e*sin(c + d*x))**m/(a + b*sec(c + d*x))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((e*sin(d*x + c))^m/(b*sec(d*x + c) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos{(c+d\,x)}^3\,(e\,\sin{(c+d\,x)})^m}{(b+a\,\cos{(c+d\,x)})^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^m/(a + b/cos(c + d*x))^3,x)
```

```
[Out] int((cos(c + d*x)^3*(e*sin(c + d*x))^m)/(b + a*cos(c + d*x))^3, x)
```

3.263 $\int (a+b\sec(c+dx))^{3/2}(e\sin(c+dx))^m\,dx$

Optimal. Leaf size=28

$$\mathrm{Int}\left((a+b\sec(c+dx))^{3/2}(e\sin(c+dx))^m,x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^(3/2)*(e*sin(d*x+c))^m,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+b\sec(c+dx))^{3/2}(e\sin(c+dx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(3/2)*(e*Sin[c + d*x])^m,x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^(3/2)*(e*Sin[c + d*x])^m, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^{3/2}(e\sin(c+dx))^m\,dx = \int (a+b\sec(c+dx))^{3/2}(e\sin(c+dx))^m\,dx$$

Mathematica [A]
time = 10.49, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^{3/2}(e\sin(c+dx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(3/2)*(e*Sin[c + d*x])^m,x]
[Out] Integrate[(a + b*Sec[c + d*x])^(3/2)*(e*Sin[c + d*x])^m, x]
```

Maple [A]
time = 0.11, size = 0, normalized size = 0.00

$$\int \left(a+b\sec\left(dx+c\right)\right)^{\frac{3}{2}} \left(e\sin\left(dx+c\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)*(e*sin(d*x+c))^m,x)
[Out] int((a+b*sec(d*x+c))^(3/2)*(e*sin(d*x+c))^m,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(e*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*(e*sin(d*x + c))^m, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(e*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^(3/2)*(e*sin(d*x + c))^m, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)*(e*sin(d*x+c))**m,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6437 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(e*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*(e*sin(d*x + c))^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int (e\,\sin\left(c+d\,x\right))^m\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^m*(a + b/cos(c + d*x))^(3/2),x)
[Out] int((e*sin(c + d*x))^m*(a + b/cos(c + d*x))^(3/2), x)
```

3.264 $\int \sqrt{a+b\sec(c+dx)}\,(e\sin(c+dx))^m\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\sqrt{a+b\sec(c+dx)}\,(e\sin(c+dx))^m,x\right)$$

```
[Out] Unintegrable((e*sin(d*x+c))^m*(a+b*sec(d*x+c))^(1/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt{a+b\sec(c+dx)}\,(e\sin(c+dx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]*(e*Sin[c + d*x])^m,x]
[Out] Defer[Int][Sqrt[a + b*Sec[c + d*x]]*(e*Sin[c + d*x])^m, x]
```

Rubi steps

$$\int \sqrt{a+b\sec(c+dx)}\,(e\sin(c+dx))^m\,dx=\int \sqrt{a+b\sec(c+dx)}\,(e\sin(c+dx))^m\,dx$$

Mathematica [A]
time = 0.30, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec(c+dx)}\,(e\sin(c+dx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]*(e*Sin[c + d*x])^m,x]
[Out] Integrate[Sqrt[a + b*Sec[c + d*x]]*(e*Sin[c + d*x])^m, x]
```

Maple [A]
time = 0.11, size = 0, normalized size = 0.00

$$\int \left(e\sin\left(dx+c\right)\right)^{m}\sqrt{a+b\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^m*(a+b*sec(d*x+c))^(1/2),x)
[Out] int((e*sin(d*x+c))^m*(a+b*sec(d*x+c))^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e\sin (c+dx))^m \sqrt{a+b\sec (c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**m*(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral((e*sin(c + d*x))**m*sqrt(a + b*sec(c + d*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int (e\,\sin (c+d\,x))^m\,\sqrt{a+\frac{b}{\cos (c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^m*(a + b/cos(c + d*x))^(1/2),x)
[Out] int((e*sin(c + d*x))^m*(a + b/cos(c + d*x))^(1/2), x)
```

3.265 $\int \frac{(e\sin(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=28

$$\mathrm{Int}\left(\frac{(e\sin(c+dx))^m}{\sqrt{a+b\sec(c+dx)}},x\right)$$

```
[Out] Unintegrable((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(e\sin(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(e*Sin[c + d*x])^m/Sqrt[a + b*Sec[c + d*x]],x]
[Out] Defer[Int][(e*Sin[c + d*x])^m/Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{(e\sin(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Mathematica [A]
time = 2.18, size = 0, normalized size = 0.00

$$\int \frac{(e\sin(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e*Sin[c + d*x])^m/Sqrt[a + b*Sec[c + d*x]],x]
[Out] Integrate[(e*Sin[c + d*x])^m/Sqrt[a + b*Sec[c + d*x]], x]
```

Maple [A]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{(e\sin(dx+c))^m}{\sqrt{a+b\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] int((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*sin(d*x + c))^m/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e*sin(d*x + c))^m/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e \sin(c + dx))^m}{\sqrt{a + b \sec(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**m/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((e*sin(c + d*x))**m/sqrt(a + b*sec(c + d*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*sin(d*x + c))^m/sqrt(b*sec(d*x + c) + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(e \sin (c+d x))^m}{\sqrt{a+\frac{b}{\cos (c+d x)}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^m/(a + b/cos(c + d*x))^(1/2),x)
[Out] int((e*sin(c + d*x))^m/(a + b/cos(c + d*x))^(1/2), x)
```

3.266 $\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=28

$$\operatorname{Int}\left(\frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^{3/2}},x\right)$$

```
[Out] Unintegrable((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(e*Sin[c + d*x])^m/(a + b*Sec[c + d*x])^(3/2),x]
[Out] Defer[Int][(e*Sin[c + d*x])^m/(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Mathematica [A]
time = 3.36, size = 0, normalized size = 0.00

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e*Sin[c + d*x])^m/(a + b*Sec[c + d*x])^(3/2),x]
[Out] Integrate[(e*Sin[c + d*x])^m/(a + b*Sec[c + d*x])^(3/2), x]
```

Maple [A]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{(e\sin(dx+c))^m}{(a+b\sec(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x)
[Out] int((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((e*sin(d*x + c))^m/(b*sec(d*x + c) + a)^(3/2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*(e*sin(d*x + c))^m/(b^2*sec(d*x + c)^2 +
2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e \sin (c + dx))^m}{(a + b \sec (c + dx))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**m/(a+b*sec(d*x+c))**(3/2),x)
[Out] Integral((e*sin(c + d*x))**m/(a + b*sec(c + d*x))**(3/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((e*sin(d*x + c))^m/(b*sec(d*x + c) + a)^(3/2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(e\,\sin(c+d\,x))^m}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^m/(a + b/cos(c + d*x))^(3/2),x)
[Out] int((e*sin(c + d*x))^m/(a + b/cos(c + d*x))^(3/2), x)
```

3.267 $\int (a+b\sec(c+dx))^n(e\sin(c+dx))^m\,dx$

Optimal. Leaf size=26

$$\text{Int}((a+b\sec(c+dx))^n(e\sin(c+dx))^m,x)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^n*(e*sin(d*x+c))^m,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+b\sec(c+dx))^n(e\sin(c+dx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^n*(e*Sin[c + d*x])^m,x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^n*(e*Sin[c + d*x])^m, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^n(e\sin(c+dx))^m\,dx = \int (a+b\sec(c+dx))^n(e\sin(c+dx))^m\,dx$$

Mathematica [A]
time = 2.22, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^n(e\sin(c+dx))^m\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^n*(e*Sin[c + d*x])^m,x]
[Out] Integrate[(a + b*Sec[c + d*x])^n*(e*Sin[c + d*x])^m, x]
```

Maple [A]
time = 0.11, size = 0, normalized size = 0.00

$$\int \left(a+b\sec\left(dx+c\right)\right)^{n}\left(e\sin\left(dx+c\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n*(e*sin(d*x+c))^m,x)
[Out] int((a+b*sec(d*x+c))^n*(e*sin(d*x+c))^m,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*(e*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n*(e*sin(d*x + c))^m, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*(e*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^n*(e*sin(d*x + c))^m, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (e\sin{(c+dx)})^m (a+b\sec{(c+dx)})^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n*(e*sin(d*x+c))**m,x)
[Out] Integral((e*sin(c + d*x))**m*(a + b*sec(c + d*x))**n, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*(e*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^n*(e*sin(d*x + c))^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int {\left(e\,\sin\left(c+d\,x\right)\right)}^m\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^m*(a + b/cos(c + d*x))^n,x)
[Out] int((e*sin(c + d*x))^m*(a + b/cos(c + d*x))^n, x)
```

3.268 $\int (a+b\sec(c+dx))^n \sin^5(c+dx)\,dx$

Optimal. Leaf size=150

$$\frac{b\,{}_2F_1\left(2,1+n;2+n;1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{a^2d(1+n)}-\frac{2b^3\,{}_2F_1\left(4,1+n;2+n;1+\frac{b\sec(c+dx)}{a}\right)(a+b\mathrm{se}}{a^4d(1+n)}$$

```
[Out] b*hypergeom([2, 1+n],[2+n],1+b*sec(d*x+c)/a)*(a+b*sec(d*x+c))^(1+n)/a^2/d/(
1+n)-2*b^3*hypergeom([4, 1+n],[2+n],1+b*sec(d*x+c)/a)*(a+b*sec(d*x+c))^(1+n
)/a^4/d/(1+n)+b^5*hypergeom([6, 1+n],[2+n],1+b*sec(d*x+c)/a)*(a+b*sec(d*x+c
))^(1+n)/a^6/d/(1+n)
```

Rubi [A]
time = 0.09, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3959, 186, 67}

$$\frac{b^5(a+b\sec(c+dx))^{n+1}\,{}_2F_1\left(6,n+1;n+2;\frac{b\sec(c+dx)}{a}+1\right)}{a^6d(n+1)}-\frac{2b^3(a+b\sec(c+dx))^{n+1}\,{}_2F_1\left(4,n+1;n+2;\frac{b\sec(c+dx)}{a}+1\right)}{a^4d(n+1)}+\frac{b(a+b\sec(c+dx))^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{b\sec(c+dx)}{a}+1\right)}{a^2d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^5,x]
```

```
[Out] (b*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*Sec[c + d*x])/a]*(a + b*Sec[c
+ d*x])^(1 + n))/(a^2*d*(1 + n)) - (2*b^3*Hypergeometric2F1[4, 1 + n, 2 + n
, 1 + (b*Sec[c + d*x])/a]*(a + b*Sec[c + d*x])^(1 + n))/(a^4*d*(1 + n)) + (
b^5*Hypergeometric2F1[6, 1 + n, 2 + n, 1 + (b*Sec[c + d*x])/a]*(a + b*Sec[c
 + d*x])^(1 + n))/(a^6*d*(1 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 3959

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> Dist[-f^(-1), Subst[Int[(-1 + x)^((p - 1)/2)*(1 + x)^((p -
```

```
1)/2)*((a + b*x)^m/x^(p + 1)), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, e,
f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+b\sec(c+dx))^n \sin^5(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{(-1+x)^2(1+x)^2(a-bx)^n}{x^6}\,dx, x, -\sec(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{(a-bx)^n}{x^6} - \frac{2(a-bx)^n}{x^4} + \frac{(a-bx)^n}{x^2}\right) dx, x, -\sec(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a-bx)^n}{x^6}\,dx, x, -\sec(c+dx)\right)}{d} - \frac{\text{Subst}\left(\int \frac{(a-bx)^n}{x^2}\,dx, x, -\right.}{d}$$

$$= \frac{b\,{}_2F_1\left(2,1+n;2+n;1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{a^2d(1+n)} - \frac{2b^3\,{}_2F}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 562 vs. 2(150) = 300.
time = 5.61, size = 562, normalized size = 3.75

$$-\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\cos(c+dx)\left(192a^3(-1+n)(b+a\cos(c+dx))^2-240a^3(-1+n)(b+a\cos(c+dx))^2\sec^2\left(\frac{1}{2}(c+dx)\right)-24a^2(2a-b(-4+n))(-1+n)(b+a\cos(c+dx))^2\sec^2\left(\frac{1}{2}(c+dx)\right)+40a^2(2a-b(-3+n))(-1+n)(b+a\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)+a(1-n)\left(96a^2+4ab(6-4n)-4b^2(12-7n+n^2)\right)(b+a\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)-10a\left((-1+n)\left(-14a^2+2ab(-1+n)+b^2(6-5n+n^2)\right)(b+a\cos(c+dx))^2+b\left(24a^3+12a^2b(-1+n)-4ab^2(2-3n+n^2)-b^3(-6+11n-6n^2+n^3)\right){}_2F_1\left(2,1-n;2-n;\frac{a\cos(c+dx)}{b+a\cos(c+dx)}\right)\right)\sec^6\left(\frac{1}{2}(c+dx)\right)+\left((-1+n)\left(-84a^3+2a^2b(18-7n)+4ab^2(9-9n+2n^2)+b^3(-24+26n-9n^2+n^3)\right)(b+a\cos(c+dx))^2+b\left(120a^4+120a^3b(-1+n)-10ab^3(-6+11n-6n^2+n^3)-b^4(24-50n+35n^2-10n^3+n^4)\right){}_2F_1\left(2,1-n;2-n;\frac{a\cos(c+dx)}{b+a\cos(c+dx)}\right)\right)\sec^6\left(\frac{1}{2}(c+dx)\right)\right)(a+b\sec(c+dx))^n}{120a^4d(-1+n)(b+a\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^5,x]
```

```
[Out] -1/120*(Cos[(c + d*x)/2]^6*Cos[c + d*x]*(192*a^3*(-1 + n)*(b + a*Cos[c + d*
x])^2 - 240*a^3*(-1 + n)*(b + a*Cos[c + d*x])^2*Sec[(c + d*x)/2]^2 - 24*a^2
*(2*a - b*(-4 + n))*(-1 + n)*(b + a*Cos[c + d*x])^2*Sec[(c + d*x)/2]^2 + 40
*a^2*(2*a - b*(-3 + n))*(-1 + n)*(b + a*Cos[c + d*x])^2*Sec[(c + d*x)/2]^4
+ a*(1 - n)*(96*a^2 + 4*a*b*(6 - 4*n) - 4*b^2*(12 - 7*n + n^2))*(b + a*Cos[
c + d*x])^2*Sec[(c + d*x)/2]^4 - 10*a*((-1 + n)*(-14*a^2 + 2*a*b*(-1 + n) +
 b^2*(6 - 5*n + n^2))*(b + a*Cos[c + d*x])^2 + b*(24*a^3 + 12*a^2*b*(-1 + n
) - 4*a*b^2*(2 - 3*n + n^2) - b^3*(-6 + 11*n - 6*n^2 + n^3))*Hypergeometric
2F1[2, 1 - n, 2 - n, (a*Cos[c + d*x])/(b + a*Cos[c + d*x])])*Sec[(c + d*x)/
2]^6 + ((-1 + n)*(-84*a^3 + 2*a^2*b*(18 - 7*n) + 4*a*b^2*(9 - 9*n + 2*n^2)
+ b^3*(-24 + 26*n - 9*n^2 + n^3))*(b + a*Cos[c + d*x])^2 + b*(120*a^4 + 120
*a^3*b*(-1 + n) - 10*a*b^3*(-6 + 11*n - 6*n^2 + n^3) - b^4*(24 - 50*n + 35*
n^2 - 10*n^3 + n^4))*Hypergeometric2F1[2, 1 - n, 2 - n, (a*Cos[c + d*x])/(b
 + a*Cos[c + d*x])])*Sec[(c + d*x)/2]^6)*(a + b*Sec[c + d*x])^n)/(a^4*d*(-1
 + n)*(b + a*Cos[c + d*x]))
```

Maple [F]
time = 0.35, size = 0, normalized size = 0.00

$$\int (a+b\sec(dx+c))^n \left(\sin^5(dx+c)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n*sin(d*x+c)^5,x)
```

```
[Out] int((a+b*sec(d*x+c))^n*sin(d*x+c)^5,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*sin(d*x + c)^5, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] integral((cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*(b*sec(d*x + c) + a)^n*sin
(d*x + c), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n*sin(d*x+c)**5,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^5,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*sin(d*x + c)^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(c+d\,x)^5\left(a+\frac{b}{\cos(c+d\,x)}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5*(a + b/cos(c + d*x))^n,x)
```

```
[Out] int(sin(c + d*x)^5*(a + b/cos(c + d*x))^n, x)
```

3.269 $\int (a+b\sec(c+dx))^n \sin^3(c+dx)\,dx$

Optimal. Leaf size=121

$$\frac{b(6a^2-b^2(2-3n+n^2))\,{}_2F_1\left(2,1+n;2+n;1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{6a^4d(1+n)}+\frac{\cos^3(c+dx)(a+b\sec$$

```
[Out] 1/6*b*(6*a^2-b^2*(n^2-3*n+2))*hypergeom([2, 1+n],[2+n],1+b*sec(d*x+c)/a)*(a
+b*sec(d*x+c))^(1+n)/a^4/d/(1+n)+1/6*cos(d*x+c)^3*(a+b*sec(d*x+c))^(1+n)*(2
*a-b*(2-n)*sec(d*x+c))/a^2/d
```

Rubi [A]
time = 0.07, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3959, 150, 67}

$$\frac{\cos^3(c+dx)(2a-b(2-n)\sec(c+dx))(a+b\sec(c+dx))^{n+1}}{6a^2d}+\frac{b(6a^2-b^2(n^2-3n+2))(a+b\sec(c+dx))^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{b\sec(c+dx)}{a}+1\right)}{6a^4d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^3,x]
```

```
[Out] (b*(6*a^2 - b^2*(2 - 3*n + n^2))*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*
Sec[c + d*x])/a]*(a + b*Sec[c + d*x])^(1 + n))/(6*a^4*d*(1 + n)) + (Cos[c +
 d*x]^3*(a + b*Sec[c + d*x])^(1 + n)*(2*a - b*(2 - n)*Sec[c + d*x]))/(6*a^2
*d)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 150

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(
n + 2) - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h)
 + d*e*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(
f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)/(
b^2*(b*c - a*d)^2*(m + 1)*(m + 2)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x]
 + Dist[f*(h/b^2) - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1
) - d*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)
))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), Int[(a + b*x)^(m + 2)*(c + d*x)^n,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[
```

```
m + n + 3, 0] &&  !LtQ[n, -2]))
```

Rule 3959

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> Dist[-f^(-1), Subst[Int[(-1 + x)^((p - 1)/2)*(1 + x)^((p -
 1)/2)*((a + b*x)^m/x^(p + 1)), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, e,
 f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+b\sec(c+dx))^n\sin^3(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{(-1+x)(1+x)(a-bx)^n}{x^4}\,dx, x, -\sec(c+dx)\right)}{d}$$
$$= \frac{\cos^3(c+dx)(a+b\sec(c+dx))^{1+n}(2a-b(2-n)\sec(c+dx))}{6a^2d} - \frac{(6$$
$$= \frac{b(6a^2-b^2(2-3n+n^2))\ {}_2F_1\left(2,1+n;2+n;1+\frac{b\sec(c+dx)}{a}\right)(a+b\,s}{6a^4d(1+n)}$$

Mathematica [A]
time = 1.21, size = 155, normalized size = 1.28

$$\frac{\cos(c+dx)\left(-\frac{2(2a-b(-2+n))(b+a\cos(c+dx))^2}{a}+8\cos^2\left(\frac{1}{2}(c+dx)\right)(b+a\cos(c+dx))^2-\frac{2b(-6a^2+b^2(2-3n+n^2))\ {}_2F_1\left(2,1-n;2-n;\frac{a\cos(c+dx)}{b+a\cos(c+dx)}\right)}{a(-1+n)}\right)(a+b\sec(c+dx))^n}{12ad(b+a\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^3,x]
```

```
[Out] (Cos[c + d*x]*((-2*(2*a - b*(-2 + n))*(b + a*Cos[c + d*x])^2)/a + 8*Cos[(c
+ d*x)/2]^2*(b + a*Cos[c + d*x])^2 - (2*b*(-6*a^2 + b^2*(2 - 3*n + n^2))*Hy
pergeometric2F1[2, 1 - n, 2 - n, (a*Cos[c + d*x])/(b + a*Cos[c + d*x])])/(a
*(-1 + n)))*(a + b*Sec[c + d*x])^n)/(12*a*d*(b + a*Cos[c + d*x]))
```

Maple [F]
time = 0.34, size = 0, normalized size = 0.00

$$\int (a+b\sec(dx+c))^n\left(\sin^3(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n*sin(d*x+c)^3,x)
```

```
[Out] int((a+b*sec(d*x+c))^n*sin(d*x+c)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^3,x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n*sin(d*x + c)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^3,x, algorithm="fricas")
[Out] integral(-(cos(d*x + c)^2 - 1)*(b*sec(d*x + c) + a)^n*sin(d*x + c), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n*sin(d*x+c)**3,x)
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^3,x, algorithm="giac")
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Evaluation time: 0.45sym2poly/r2sym(const gen & e,const i
ndex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(c+d\,x\right)^3\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3*(a + b/cos(c + d*x))^n,x)
[Out] int(sin(c + d*x)^3*(a + b/cos(c + d*x))^n, x)
```

3.270 $\int (a+b\sec(c+dx))^n \sin(c+dx)\,dx$

Optimal. Leaf size=48

$$\frac{b\,{}_2F_1\left(2,1+n;2+n;1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{a^2d(1+n)}$$

```
[Out] b*hypergeom([2, 1+n],[2+n],1+b*sec(d*x+c)/a)*(a+b*sec(d*x+c))^(1+n)/a^2/d/(
1+n)
```

Rubi [A]
time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3959, 67}

$$\frac{b(a+b\sec(c+dx))^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{b\sec(c+dx)}{a}+1\right)}{a^2d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^n*Sin[c + d*x],x]
```

```
[Out] (b*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*Sec[c + d*x])/a]*(a + b*Sec[c
+ d*x])^(1 + n))/(a^2*d*(1 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 3959

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> Dist[-f^(-1), Subst[Int[(-1 + x)^((p - 1)/2)*(1 + x)^((p -
 1)/2)*((a + b*x)^m/x^(p + 1)), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, e,
 f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+b\sec(c+dx))^n\sin(c+dx)\,dx &= -\frac{\text{Subst}\left(\int\frac{(a-bx)^n}{x^2}\,dx,x,-\sec(c+dx)\right)}{d}\\ &= \frac{b\,{}_2F_1\left(2,1+n;2+n;1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{a^2d(1+n)}\end{aligned}$$

Mathematica [A]
time = 0.36, size = 72, normalized size = 1.50

$$\frac{b\cos(c+dx)\,{}_2F_1\left(2,1-n;2-n;\frac{a\cos(c+dx)}{b+a\cos(c+dx)}\right)(a+b\sec(c+dx))^n}{d(-1+n)(b+a\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^n*Sin[c + d*x],x]
[Out] (b*Cos[c + d*x]*Hypergeometric2F1[2, 1 - n, 2 - n, (a*Cos[c + d*x])/(b + a*
Cos[c + d*x])]*(a + b*Sec[c + d*x])^n)/(d*(-1 + n)*(b + a*Cos[c + d*x]))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int (a+b\sec(dx+c))^n \sin(dx+c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n*sin(d*x+c),x)
[Out] int((a+b*sec(d*x+c))^n*sin(d*x+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n*sin(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^n*sin(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^n \sin(c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n*sin(d*x+c),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**n*sin(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*sin(d*x + c), x)
```

Mupad [B]
time = 1.39, size = 73, normalized size = 1.52

$$\frac{\cos\left(c+d\,x\right)\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^n\,{{}}_2{\mathrm{F}}_1\left(1-n,-n;\ 2-n;\ -\frac{a\,\cos\left(c+d\,x\right)}{b}\right)}{d\,{\left(\frac{a\,\cos\left(c+d\,x\right)}{b}+1\right)}^n\,\left(n-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a + b/cos(c + d*x))^n,x)
```

```
[Out] (cos(c + d*x)*(a + b/cos(c + d*x))^n*hypergeom([1 - n, -n], 2 - n, -(a*cos(
c + d*x))/b))/(d*((a*cos(c + d*x))/b + 1)^n*(n - 1))
```

3.271 $\int \csc(c+dx)(a+b\sec(c+dx))^n\,dx$

Optimal. Leaf size=115

$$\frac{{}_2F_1\left(1,1+n;2+n;\frac{a+b\sec(c+dx)}{a-b}\right)(a+b\sec(c+dx))^{1+n}}{2(a-b)d(1+n)}-\frac{{}_2F_1\left(1,1+n;2+n;\frac{a+b\sec(c+dx)}{a+b}\right)(a+b\sec(c+dx}{2(a+b)d(1+n)}$$

```
[Out] 1/2*hypergeom([1, 1+n],[2+n],(a+b*sec(d*x+c))/(a-b))*(a+b*sec(d*x+c))^(1+n)
/(a-b)/d/(1+n)-1/2*hypergeom([1, 1+n],[2+n],(a+b*sec(d*x+c))/(a+b))*(a+b*se
c(d*x+c))^(1+n)/(a+b)/d/(1+n)
```

Rubi [A]
time = 0.08, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3959, 88, 70}

$$\frac{(a+b\sec(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a+b\sec(c+dx)}{a-b}\right)}{2d(n+1)(a-b)}-\frac{(a+b\sec(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a+b\sec(c+dx)}{a+b}\right)}{2d(n+1)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + b*Sec[c + d*x])^n,x]
```

```
[Out] (Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Sec[c + d*x])/(a - b)]*(a + b*Se
c[c + d*x])^(1 + n))/(2*(a - b)*d*(1 + n)) - (Hypergeometric2F1[1, 1 + n, 2
 + n, (a + b*Sec[c + d*x])/(a + b)]*(a + b*Sec[c + d*x])^(1 + n))/(2*(a + b
)*d*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 3959

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> Dist[-f^(-1), Subst[Int[(-1 + x)^((p - 1)/2)*(1 + x)^((p -
 1)/2)*((a + b*x)^m/x^(p + 1)), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, e,
```

```
 f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \csc(c+dx)(a+b\sec(c+dx))^n\,dx &= -\frac{\text{Subst}\left(\int \frac{(a-bx)^n}{(-1+x)(1+x)}\,dx, x, -\sec(c+dx)\right)}{d}\\
&= -\frac{\text{Subst}\left(\int \frac{(a-bx)^n}{-1+x^2}\,dx, x, -\sec(c+dx)\right)}{d}\\
&= -\frac{\text{Subst}\left(\int \left(-\frac{(a-bx)^n}{2(1-x)}-\frac{(a-bx)^n}{2(1+x)}\right)\,dx, x, -\sec(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{(a-bx)^n}{1-x}\,dx, x, -\sec(c+dx)\right)}{2d}+\frac{\text{Subst}\left(\int \frac{(a-bx)^n}{1+x}\,dx, x, -\text{se}\right.}{2d}\\
&= \frac{{}_2F_1\left(1,1+n;2+n;\frac{a+b\sec(c+dx)}{a-b}\right)(a+b\sec(c+dx))^{1+n}}{2(a-b)d(1+n)}-\frac{{}_2F_1\left(1,\right.}{}
\end{aligned}
$$

Mathematica [A]
time = 0.66, size = 132, normalized size = 1.15

$$
\frac{\left({}_2F_1\left(1,-n;1-n;\frac{(a+b)\cos(c+dx)}{b+a\cos(c+dx)}\right)-2^n\,{}_2F_1\left(-n,-n;1-n;\frac{(-a+b)\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)}{2b}\right)\left(\frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{b}\right)^{-n}\right)(a+b\sec(c+dx))^n}{2dn}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*(a + b*Sec[c + d*x])^n,x]
```

```
[Out] ((Hypergeometric2F1[1, -n, 1 - n, ((a + b)*Cos[c + d*x])/(b + a*Cos[c + d*x
])] - (2^n*Hypergeometric2F1[-n, -n, 1 - n, ((-a + b)*Cos[c + d*x]*Sec[(c +
 d*x)/2]^2)/(2*b)])/(((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/b)^n)*(a + b
*Sec[c + d*x])^n)/(2*d*n)
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$
\int \csc\left(dx+c\right)\left(a+b\sec\left(dx+c\right)\right)^{n}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+b*sec(d*x+c))^n,x)
```

```
[Out] int(csc(d*x+c)*(a+b*sec(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n*csc(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^n*csc(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec (c + dx))^n \csc (c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c))**n,x)
[Out] Integral((a + b*sec(c + d*x))**n*csc(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^n*csc(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + \frac{b}{\cos(c+d\,x)}\right)^n}{\sin (c + d\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^n/sin(c + d*x),x)
[Out] int((a + b/cos(c + d*x))^n/sin(c + d*x), x)
```

3.272 $\int \csc^3(c+dx)(a+b\sec(c+dx))^n\,dx$

Optimal. Leaf size=231

$$\frac{{}_2F_1\left(1,1+n;2+n;\frac{a+b\sec(c+dx)}{a-b}\right)(a+b\sec(c+dx))^{1+n}}{4(a-b)d(1+n)}-\frac{{}_2F_1\left(1,1+n;2+n;\frac{a+b\sec(c+dx)}{a+b}\right)(a+b\sec(c+d}{4(a+b)d(1+n)}$$

```
[Out] 1/4*hypergeom([1, 1+n],[2+n],(a+b*sec(d*x+c))/(a-b))*(a+b*sec(d*x+c))^(1+n)
/(a-b)/d/(1+n)-1/4*hypergeom([1, 1+n],[2+n],(a+b*sec(d*x+c))/(a+b))*(a+b*se
c(d*x+c))^(1+n)/(a+b)/d/(1+n)+1/4*b*hypergeom([2, 1+n],[2+n],(a+b*sec(d*x+c
))/(a-b))*(a+b*sec(d*x+c))^(1+n)/(a-b)^2/d/(1+n)+1/4*b*hypergeom([2, 1+n],[
2+n],(a+b*sec(d*x+c))/(a+b))*(a+b*sec(d*x+c))^(1+n)/(a+b)^2/d/(1+n)
```

Rubi [A]

time = 0.14, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3959, 186, 70, 726}

$$\frac{(a+b\sec(c+dx))^{n+1}{}_2F_1\left(1,n+1;n+2;\frac{a+b\sec(c+dx)}{a-b}\right)}{4d(n+1)(a-b)}-\frac{(a+b\sec(c+dx))^{n+1}{}_2F_1\left(1,n+1;n+2;\frac{a+b\sec(c+dx)}{a+b}\right)}{4d(n+1)(a+b)}+\frac{b(a+b\sec(c+dx))^{n+1}{}_2F_1\left(2,n+1;n+2;\frac{a+b\sec(c+dx)}{a-b}\right)}{4d(n+1)(a-b)^2}+\frac{b(a+b\sec(c+dx))^{n+1}{}_2F_1\left(2,n+1;n+2;\frac{a+b\sec(c+dx)}{a+b}\right)}{4d(n+1)(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*(a + b*Sec[c + d*x])^n,x]
```

```
[Out] (Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Sec[c + d*x])/(a - b)]*(a + b*Se
c[c + d*x])^(1 + n))/(4*(a - b)*d*(1 + n)) - (Hypergeometric2F1[1, 1 + n, 2
 + n, (a + b*Sec[c + d*x])/(a + b)]*(a + b*Sec[c + d*x])^(1 + n))/(4*(a + b
)*d*(1 + n)) + (b*Hypergeometric2F1[2, 1 + n, 2 + n, (a + b*Sec[c + d*x])/(
a - b)]*(a + b*Sec[c + d*x])^(1 + n))/(4*(a - b)^2*d*(1 + n)) + (b*Hypergeo
metric2F1[2, 1 + n, 2 + n, (a + b*Sec[c + d*x])/(a + b)]*(a + b*Sec[c + d*x
])^(1 + n))/(4*(a + b)^2*d*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 186

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 726

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] &
& NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m]
```

Rule 3959

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> Dist[-f^(-1), Subst[Int[(-1 + x)^((p - 1)/2)*(1 + x)^((p -
 1)/2)*((a + b*x)^m/x^(p + 1)), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, e,
 f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \csc^3(c+dx)(a+b\sec(c+dx))^n\,dx = -\frac{\text{Subst}\left(\int \frac{x^2(a-bx)^n}{(-1+x)^2(1+x)^2}\,dx, x, -\sec(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{(a-bx)^n}{4(-1+x)^2}+\frac{(a-bx)^n}{4(1+x)^2}+\frac{(a-bx)^n}{2(-1+x^2)}\right)dx, x, -\sec(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a-bx)^n}{(-1+x)^2}\,dx, x, -\sec(c+dx)\right)}{4d} - \frac{\text{Subst}\left(\int \frac{(a-bx)^n}{(1+x)^2}\,dx, x, -\right.}{4d}$$

$$= \frac{b\,{}_2F_1\left(2,1+n;2+n;\frac{a+b\sec(c+dx)}{a-b}\right)(a+b\sec(c+dx))^{1+n}}{4(a-b)^2d(1+n)} + \frac{b\,{}_2F_1\left(2\right.}{}$$

$$= \frac{b\,{}_2F_1\left(2,1+n;2+n;\frac{a+b\sec(c+dx)}{a-b}\right)(a+b\sec(c+dx))^{1+n}}{4(a-b)^2d(1+n)} + \frac{b\,{}_2F_1\left(2\right.}{}$$

$$= \frac{{}_2F_1\left(1,1+n;2+n;\frac{a+b\sec(c+dx)}{a-b}\right)(a+b\sec(c+dx))^{1+n}}{4(a-b)d(1+n)} - \frac{{}_2F_1\left(1,1\right.}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 513 vs. 2(231) = 462.
time = 13.80, size = 513, normalized size = 2.22

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]^3*(a + b*Sec[c + d*x])^n,x]
```

```
[Out] ((Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n*(a + b*Sec[c + d*x])^n*((1 - Tan[(c +
d*x)/2]^2)^(-1))^n*(1 - Tan[(c + d*x)/2]^4)^n*(2*(a + b + b*n)*Hypergeometr
ic2F1[1, -n, 1 - n, ((a + b)*Cos[c + d*x])/(b + a*Cos[c + d*x])]*(1 - Tan[(
```

```
c + d*x)/2]^2)^n + (-2*(a + b + b*n)*Hypergeometric2F1[-n, -n, 1 - n, ((a -
 b)*(-1 + Tan[(c + d*x)/2]^2))/(2*b)]*(2 - 2*Tan[(c + d*x)/2]^2)^n + (n*(b
+ a*Cos[c + d*x])*Csc[(c + d*x)/2]^2*(1 - Tan[(c + d*x)/2]^2)^n*(-2*a*Hyper
geometric2F1[n, 1 + n, 2 + n, ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(2*
b)]*(-(((a - b)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/b^2))
^n*Tan[(c + d*x)/2]^2 + 2^n*(a - b)*(1 + n)*(((b + a*Cos[c + d*x])*Sec[(c +
 d*x)/2]^2)/b)^n*(-1 + Tan[(c + d*x)/2]^2)))/(2^n*(a - b)*(1 + n)))/(((b +
a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/b)^n))/(8*(a + b)*d*n*(Cos[c + d*x]*Sec
[(c + d*x)/2]^4)^n*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n*(1 - Tan[(c + d*x)/2
]^2)^(2*n))
```

Maple [**F**]
time = 0.10, size = 0, normalized size = 0.00

$$\int \left(\csc^{3}\left(dx+c\right)\right) \left(a+b\sec\left(dx+c\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(a+b*sec(d*x+c))^n,x)
```

```
[Out] int(csc(d*x+c)^3*(a+b*sec(d*x+c))^n,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*sec(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*csc(d*x + c)^3, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*sec(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^n*csc(d*x + c)^3, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(a+b*sec(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*sec(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*csc(d*x + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^n}{\sin\left(c+d\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^n/sin(c + d*x)^3,x)
```

```
[Out] int((a + b/cos(c + d*x))^n/sin(c + d*x)^3, x)
```

3.273 $\int (a+b\sec(c+dx))^n \sin^4(c+dx)\,dx$

Optimal. Leaf size=24

$$\text{Int}\left((a+b\sec(c+dx))^n\sin^4(c+dx),x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^n*sin(d*x+c)^4,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+b\sec(c+dx))^n\sin^4(c+dx)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^4,x]
```

```
[Out] Defer[Int][(a + b*Sec[c + d*x])^n*Sin[c + d*x]^4, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^n\sin^4(c+dx)\,dx = \int (a+b\sec(c+dx))^n\sin^4(c+dx)\,dx$$

Mathematica [A]
time = 7.53, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^n\sin^4(c+dx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^4,x]
```

```
[Out] Integrate[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^4, x]
```

Maple [A]
time = 0.37, size = 0, normalized size = 0.00

$$\int \left(a+b\sec\left(dx+c\right)\right)^n\left(\sin^4\left(dx+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n*sin(d*x+c)^4,x)
[Out] int((a+b*sec(d*x+c))^n*sin(d*x+c)^4,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^4,x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n*sin(d*x + c)^4, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^4,x, algorithm="fricas")
[Out] integral((cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*(b*sec(d*x + c) + a)^n, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n*sin(d*x+c)**4,x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^4,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^n*sin(d*x + c)^4, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \sin\left(c+d\,x\right)^4\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4*(a + b/cos(c + d*x))^n,x)
[Out] int(sin(c + d*x)^4*(a + b/cos(c + d*x))^n, x)
```

3.274 $\int (a+b\sec(c+dx))^n \sin^2(c+dx)\,dx$

Optimal. Leaf size=24

$$\mathrm{Int}\left((a+b\sec(c+dx))^n\sin^2(c+dx),x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^n*sin(d*x+c)^2,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+b\sec(c+dx))^n\sin^2(c+dx)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^2,x]
```

```
[Out] Defer[Int][(a + b*Sec[c + d*x])^n*Sin[c + d*x]^2, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^n\sin^2(c+dx)\,dx = \int (a+b\sec(c+dx))^n\sin^2(c+dx)\,dx$$

Mathematica [A]
time = 2.02, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^n\sin^2(c+dx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^2,x]
```

```
[Out] Integrate[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^2, x]
```

Maple [A]
time = 0.28, size = 0, normalized size = 0.00

$$\int (a+b\sec(dx+c))^n\left(\sin^2(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n*sin(d*x+c)^2,x)
[Out] int((a+b*sec(d*x+c))^n*sin(d*x+c)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^2,x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n*sin(d*x + c)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^2,x, algorithm="fricas")
[Out] integral(-(cos(d*x + c)^2 - 1)*(b*sec(d*x + c) + a)^n, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec (c + dx))^n \sin^2 (c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n*sin(d*x+c)**2,x)
[Out] Integral((a + b*sec(c + d*x))**n*sin(c + d*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^2,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^n*sin(d*x + c)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \sin (c + d\,x)^2 \left(a + \frac{b}{\cos (c + d\,x)}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2*(a + b/cos(c + d*x))^n,x)
[Out] int(sin(c + d*x)^2*(a + b/cos(c + d*x))^n, x)
```

3.275 $\int \csc^2(c+dx)(a+b\sec(c+dx))^n\,dx$

Optimal. Leaf size=136

$$-\frac{\cot(c+dx)(a+b\sec(c+dx))^n}{d}+\frac{\sqrt{2}\,bnF_1\left(\frac{1}{2};\frac{1}{2},1-n;\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+}{(a+b)d\sqrt{1+\sec(c+dx)}}$$

```
[Out] -cot(d*x+c)*(a+b*sec(d*x+c))^n/d+b*n*AppellF1(1/2,1-n,1/2,3/2,b*(1-sec(d*x+
c))/(a+b),1/2-1/2*sec(d*x+c))*(a+b*sec(d*x+c))^n*2^(1/2)*tan(d*x+c)/(a+b)/d
/(((a+b*sec(d*x+c))/(a+b))^n)/(1+sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3960, 3919, 144, 143}

$$\frac{\sqrt{2}\,bn\tan(c+dx)(a+b\sec(c+dx))^n\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{-n}F_1\left(\frac{1}{2};\frac{1}{2},1-n;\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d(a+b)\sqrt{\sec(c+dx)+1}}-\frac{\cot(c+dx)(a+b\sec(c+dx))^n}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + b*Sec[c + d*x])^n,x]
```

```
[Out] -((Cot[c + d*x]*(a + b*Sec[c + d*x])^n)/d) + (Sqrt[2]*b*n*AppellF1[1/2, 1/2
, 1 - n, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*(a + b*
Sec[c + d*x])^n*Tan[c + d*x])/((a + b)*d*Sqrt[1 + Sec[c + d*x]]*((a + b*Sec
[c + d*x])/(a + b))^n)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 3919

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*m]
```

Rule 3960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)/cos[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[Tan[e + f*x]*((a + b*Csc[e + f*x])^m/f), x] + Dist[b*m, I
nt[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
m}, x]
```

Rubi steps

$$\int \csc^2(c+dx)(a+b\sec(c+dx))^n\,dx = -\frac{\cot(c+dx)(a+b\sec(c+dx))^n}{d} + (bn)\int \sec(c+dx)(a+b\sec($$

$$= -\frac{\cot(c+dx)(a+b\sec(c+dx))^n}{d} - \frac{(bn\tan(c+dx))\text{Subst}\left(\int \frac{}{\sqrt{1}}\right.}{d\sqrt{1-\sec(c+dx}}$$

$$= -\frac{\cot(c+dx)(a+b\sec(c+dx))^n}{d} - \frac{\left(bn(a+b\sec(c+dx))^n\left(-\frac{a-}{}\right.\right.}{}$$

$$= -\frac{\cot(c+dx)(a+b\sec(c+dx))^n}{d} + \frac{\sqrt{2}\,bnF_1\left(\frac{1}{2};\frac{1}{2},1-n;\frac{3}{2};\frac{1}{2}(1-\right.}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 3614 vs. $2(136) = 272$.
time = 16.13, size = 3614, normalized size = 26.57

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]^2*(a + b*Sec[c + d*x])^n,x]
[Out] ((b + a*Cos[c + d*x])^n*Cot[(c + d*x)/2]*Csc[c + d*x]^2*Sec[c + d*x]^n*(a +
 b*Sec[c + d*x])^n*(-((AppellF1[-1/2, n, -n, 1/2, Tan[(c + d*x)/2]^2, ((a -
```

b)*Tan[(c + d*x)/2]^2)/(a + b)]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n)/(((b
+ a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b))^n) + (3*(a + b)*AppellF1[1/2
, n, -n, 3/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*Tan
[(c + d*x)/2]^2)/(3*(a + b)*AppellF1[1/2, n, -n, 3/2, Tan[(c + d*x)/2]^2, (
(a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + 2*n*((-a + b)*AppellF1[3/2, n, 1 - n
, 5/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*
AppellF1[3/2, 1 + n, -n, 5/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]
^2)/(a + b)])*Tan[(c + d*x)/2]^2)))/(2*d*(-1/4*((b + a*Cos[c + d*x])^n*Csc[
(c + d*x)/2]^2*Sec[c + d*x]^n*(-((AppellF1[-1/2, n, -n, 1/2, Tan[(c + d*x)/
2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*(Cos[c + d*x]*Sec[(c + d*x)/2]^
2)^n)/(((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b))^n) + (3*(a + b)*A
ppellF1[1/2, n, -n, 3/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(
a + b)]*Tan[(c + d*x)/2]^2)/(3*(a + b)*AppellF1[1/2, n, -n, 3/2, Tan[(c + d
*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + 2*n*((-a + b)*AppellF1[3/
2, n, 1 - n, 5/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]
 + (a + b)*AppellF1[3/2, 1 + n, -n, 5/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(
c + d*x)/2]^2)/(a + b)])*Tan[(c + d*x)/2]^2))) - (a*n*(b + a*Cos[c + d*x])^
(-1 + n)*Cot[(c + d*x)/2]*Sec[c + d*x]^n*Sin[c + d*x]*(-((AppellF1[-1/2, n,
 -n, 1/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*(Cos[c
+ d*x]*Sec[(c + d*x)/2]^2)^n)/(((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a
 + b))^n) + (3*(a + b)*AppellF1[1/2, n, -n, 3/2, Tan[(c + d*x)/2]^2, ((a -
b)*Tan[(c + d*x)/2]^2)/(a + b)]*Tan[(c + d*x)/2]^2)/(3*(a + b)*AppellF1[1/2
, n, -n, 3/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + 2
n((-a + b)*AppellF1[3/2, n, 1 - n, 5/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[
(c + d*x)/2]^2)/(a + b)] + (a + b)*AppellF1[3/2, 1 + n, -n, 5/2, Tan[(c + d
*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])*Tan[(c + d*x)/2]^2)))/2 +
(n*(b + a*Cos[c + d*x])^n*Cot[(c + d*x)/2]*Sec[c + d*x]^(1 + n)*Sin[c + d*x
]*(-((AppellF1[-1/2, n, -n, 1/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)
/2]^2)/(a + b)]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n)/(((b + a*Cos[c + d*x])
*Sec[(c + d*x)/2]^2)/(a + b))^n) + (3*(a + b)*AppellF1[1/2, n, -n, 3/2, Tan
[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*Tan[(c + d*x)/2]^2)/
(3*(a + b)*AppellF1[1/2, n, -n, 3/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c +
d*x)/2]^2)/(a + b)] + 2*n*((-a + b)*AppellF1[3/2, n, 1 - n, 5/2, Tan[(c + d
*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*AppellF1[3/2, 1 +
 n, -n, 5/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])*Tan
[(c + d*x)/2]^2)))/2 + ((b + a*Cos[c + d*x])^n*Cot[(c + d*x)/2]*Sec[c + d*x
]^n*(-(((Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n*(((a - b)*n*AppellF1[1/2, n, 1
- n, 3/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*Sec[(c
+ d*x)/2]^2*Tan[(c + d*x)/2])/(a + b) - n*AppellF1[1/2, 1 + n, -n, 3/2, Tan
[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*Sec[(c + d*x)/2]^2*T
an[(c + d*x)/2]))/(((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b))^n) -
(n*AppellF1[-1/2, n, -n, 1/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]
^2)/(a + b)]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(-1 + n)*(-(Sec[(c + d*x)/2]
^2*Sin[c + d*x]) + Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(((b
+ a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b))^n + n*AppellF1[-1/2, n, -n,

```
1/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*(Cos[c + d*x
]*Sec[(c + d*x)/2]^2)^n*(((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b))
^(-1 - n)*(-((a*Sec[(c + d*x)/2]^2*Sin[c + d*x])/(a + b)) + ((b + a*Cos[c +
 d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(a + b)) + (3*(a + b)*AppellF1[
1/2, n, -n, 3/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*
Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(3*(a + b)*AppellF1[1/2, n, -n, 3/2, T
an[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + 2*n*((-a + b)*Ap
pellF1[3/2, n, 1 - n, 5/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)
/(a + b)] + (a + b)*AppellF1[3/2, 1 + n, -n, 5/2, Tan[(c + d*x)/2]^2, ((a -
 b)*Tan[(c + d*x)/2]^2)/(a + b)])*Tan[(c + d*x)/2]^2) + (3*(a + b)*Tan[(c +
 d*x)/2]^2*(-1/3*((a - b)*n*AppellF1[3/2, n, 1 - n, 5/2, Tan[(c + d*x)/2]^2
, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]
)/(a + b) + (n*AppellF1[3/2, 1 + n, -n, 5/2, Tan[(c + d*x)/2]^2, ((a - b)*T
an[(c + d*x)/2]^2)/(a + b)]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/3))/(3*(a
+ b)*AppellF1[1/2, n, -n, 3/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2
]^2)/(a + b)] + 2*n*((-a + b)*AppellF1[3/2, n, 1 - n, 5/2, Tan[(c + d*x)/2]
^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*AppellF1[3/2, 1 + n, -n
, 5/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])*Tan[(c +
d*x)/2]^2) - (3*(a + b)*AppellF1[1/2, n, -n, 3/2, Tan[(c + d*x)/2]^2, ((a -
 b)*Tan[(c + d*x)/2]^2)/(a + b)]*Tan[(c + d*x)/...
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \left(\csc^2\left(dx+c\right)\right)\left(a+b\sec\left(dx+c\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+b*sec(d*x+c))^n,x)
[Out] int(csc(d*x+c)^2*(a+b*sec(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*sec(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*sec(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec(c + dx))^n \csc^2(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+b*sec(d*x+c))**n,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**n*csc(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*sec(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^n}{\sin\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^n/sin(c + d*x)^2,x)
```

```
[Out] int((a + b/cos(c + d*x))^n/sin(c + d*x)^2, x)
```

3.276 $\int \csc^4(c+dx)(a+b\sec(c+dx))^n\,dx$

Optimal. Leaf size=424

$$-\frac{3F_1\left(-\frac{1}{2};\frac{5}{2},-n;\frac{1}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\cot(c+dx)\sqrt{1+\sec(c+dx)}\,(a+b\sec(c+dx))^n\left(\frac{a}{\ldots}\right.}{2\sqrt{2}\,d}$$

[Out] -1/12*AppellF1(-3/2,-n,5/2,-1/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*cot(d*x+c)^3*(1+sec(d*x+c))^(3/2)*(a+b*sec(d*x+c))^n/d/(((a+b*sec(d*x+c))/(a+b))^n)*2^(1/2)-3/4*AppellF1(-1/2,-n,5/2,1/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*cot(d*x+c)*(a+b*sec(d*x+c))^n*(1+sec(d*x+c))^(1/2)/d/(((a+b*sec(d*x+c))/(a+b))^n)*2^(1/2)+1/2*AppellF1(1/2,-n,3/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*(a+b*sec(d*x+c))^n*tan(d*x+c)/d/(((a+b*sec(d*x+c))/(a+b))^n)*2^(1/2)/(1+sec(d*x+c))^(1/2)+1/4*AppellF1(1/2,-n,5/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*(a+b*sec(d*x+c))^n*tan(d*x+c)/d/(((a+b*sec(d*x+c))/(a+b))^n)*2^(1/2)/(1+sec(d*x+c))^(1/2)

Rubi [F]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^n\,dx$$

Verification is not applicable to the result.

[In] Int[Csc[c + d*x]^4*(a + b*Sec[c + d*x])^n,x]

[Out] Defer[Int][Csc[c + d*x]^4*(a + b*Sec[c + d*x])^n, x]

Rubi steps

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^n\,dx = \int \csc^4(c+dx)(a+b\sec(c+dx))^n\,dx$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 5928 vs. 2(424) = 848.
time = 21.38, size = 5928, normalized size = 13.98

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]^4*(a + b*Sec[c + d*x])^n,x]
```

```
[Out] Result too large to show
```

Maple [**F**]
time = 0.10, size = 0, normalized size = 0.00

$$\int \left(\csc^{4}\left(dx+c\right)\right)\left(a+b\sec\left(dx+c\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(a+b*sec(d*x+c))^n,x)
```

```
[Out] int(csc(d*x+c)^4*(a+b*sec(d*x+c))^n,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*sec(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*csc(d*x + c)^4, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*sec(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^n*csc(d*x + c)^4, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(a+b*sec(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*sec(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*csc(d*x + c)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^n}{\sin\left(c+d\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^n/sin(c + d*x)^4,x)
```

```
[Out] int((a + b/cos(c + d*x))^n/sin(c + d*x)^4, x)
```

3.277 $\int (a+b\sec(c+dx))^n \sin^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=26

$$\text{Int}\left((a+b\sec(c+dx))^n \sin^{\frac{3}{2}}(c+dx),x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^n*sin(d*x+c)^(3/2),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+b\sec(c+dx))^n \sin^{\frac{3}{2}}(c+dx)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^(3/2),x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^n*Sin[c + d*x]^(3/2), x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^n \sin^{\frac{3}{2}}(c+dx)\,dx = \int (a+b\sec(c+dx))^n \sin^{\frac{3}{2}}(c+dx)\,dx$$

Mathematica [A]
time = 2.67, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^n \sin^{\frac{3}{2}}(c+dx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^(3/2),x]
[Out] Integrate[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^(3/2), x]
```

Maple [A]
time = 0.09, size = 0, normalized size = 0.00

$$\int (a+b\sec(dx+c))^n \left(\sin^{\frac{3}{2}}(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n*sin(d*x+c)^(3/2),x)
[Out] int((a+b*sec(d*x+c))^n*sin(d*x+c)^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n*sin(d*x + c)^(3/2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^(3/2),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^n*sin(d*x + c)^(3/2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n*sin(d*x+c)**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6437 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^n*sin(d*x + c)^(3/2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \sin(c+d\,x)^{3/2}\left(a+\frac{b}{\cos(c+d\,x)}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^(3/2)*(a + b/cos(c + d*x))^n,x)
[Out] int(sin(c + d*x)^(3/2)*(a + b/cos(c + d*x))^n, x)
```

3.278 $\int (a+b\sec(c+dx))^n \sqrt{\sin(c+dx)}\, dx$

Optimal. Leaf size=26

$$\text{Int}\left((a+b\sec(c+dx))^n \sqrt{\sin(c+dx)},x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^n*sin(d*x+c)^(1/2),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+b\sec(c+dx))^n \sqrt{\sin(c+dx)}\, dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^n*Sqrt[Sin[c + d*x]],x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^n*Sqrt[Sin[c + d*x]], x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^n \sqrt{\sin(c+dx)}\, dx = \int (a+b\sec(c+dx))^n \sqrt{\sin(c+dx)}\, dx$$

Mathematica [A]
time = 4.46, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^n \sqrt{\sin(c+dx)}\, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^n*Sqrt[Sin[c + d*x]],x]
[Out] Integrate[(a + b*Sec[c + d*x])^n*Sqrt[Sin[c + d*x]], x]
```

Maple [A]
time = 0.09, size = 0, normalized size = 0.00

$$\int (a+b\sec(dx+c))^n \left(\sqrt{\sin}\,(dx+c)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n*sin(d*x+c)^(1/2),x)
```

```
[Out] int((a+b*sec(d*x+c))^n*sin(d*x+c)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*sqrt(sin(d*x + c)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^n*sqrt(sin(d*x + c)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec{(c + dx)})^n \sqrt{\sin{(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n*sin(d*x+c)**(1/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**n*sqrt(sin(c + d*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*sqrt(sin(d*x + c)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \sqrt{\sin\left(c+d\,x\right)}\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^(1/2)*(a + b/cos(c + d*x))^n,x)
[Out] int(sin(c + d*x)^(1/2)*(a + b/cos(c + d*x))^n, x)
```

3.279 $\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\sin(c+dx)}} dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{(a+b\sec(c+dx))^n}{\sqrt{\sin(c+dx)}},x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^n/sin(d*x+c)^(1/2),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\sin(c+dx)}} dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^n/Sqrt[Sin[c + d*x]],x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^n/Sqrt[Sin[c + d*x]], x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\sin(c+dx)}} dx = \int \frac{(a+b\sec(c+dx))^n}{\sqrt{\sin(c+dx)}} dx$$

Mathematica [A]
time = 3.26, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\sin(c+dx)}} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^n/Sqrt[Sin[c + d*x]],x]
[Out] Integrate[(a + b*Sec[c + d*x])^n/Sqrt[Sin[c + d*x]], x]
```

Maple [A]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(dx+c))^n}{\sqrt{\sin(dx+c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n/sin(d*x+c)^(1/2),x)
```

```
[Out] int((a+b*sec(d*x+c))^n/sin(d*x+c)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n/sin(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n/sqrt(sin(d*x + c)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n/sin(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^n/sqrt(sin(d*x + c)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\sec{(c + dx)})^{n}}{\sqrt{\sin{(c + dx)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n/sin(d*x+c)**(1/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**n/sqrt(sin(c + d*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n/sin(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n/sqrt(sin(d*x + c)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^n}{\sqrt{\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^n/sin(c + d*x)^(1/2),x)
[Out] int((a + b/cos(c + d*x))^n/sin(c + d*x)^(1/2), x)
```

3.280 $\int \frac{(a+b\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{(a+b\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)},x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^n/sin(d*x+c)^(3/2),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^n/Sin[c + d*x]^(3/2),x]
```

```
[Out] Defer[Int][(a + b*Sec[c + d*x])^n/Sin[c + d*x]^(3/2), x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(a+b\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx$$

Mathematica [A]
time = 3.85, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^n/Sin[c + d*x]^(3/2),x]
```

```
[Out] Integrate[(a + b*Sec[c + d*x])^n/Sin[c + d*x]^(3/2), x]
```

Maple [A]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(dx+c))^n}{\sin(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n/sin(d*x+c)^(3/2),x)
```

```
[Out] int((a+b*sec(d*x+c))^n/sin(d*x+c)^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n/sin(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n/sin(d*x + c)^(3/2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n/sin(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(b*sec(d*x + c) + a)^n*sqrt(sin(d*x + c))/(cos(d*x + c)^2 - 1), x
)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \sec{(c + dx)})^n}{\sin^{\frac{3}{2}}{(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n/sin(d*x+c)**(3/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**n/sin(c + d*x)**(3/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n/sin(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n/sin(d*x + c)^(3/2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^n}{\sin\left(c+d\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^n/sin(c + d*x)^(3/2),x)
[Out] int((a + b/cos(c + d*x))^n/sin(c + d*x)^(3/2), x)
```

3.281 $\int (e\csc(c+dx))^{5/2}(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=190

$$-\frac{2ae^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d}-\frac{2ae^2\csc(c+dx)\sqrt{e\csc(c+dx)}}{3d}+\frac{ae^2\text{ArcTan}\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc}}{d}$$

```
[Out] -2/3*a*e^2*cot(d*x+c)*(e*csc(d*x+c))^(1/2)/d-2/3*a*e^2*csc(d*x+c)*(e*csc(d*
x+c))^(1/2)/d+a*e^2*arctan(sin(d*x+c)^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c
)^(1/2)/d+a*e^2*arctanh(sin(d*x+c)^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(
1/2)/d-2/3*a*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d
*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d
*x+c)^(1/2)/d
```

Rubi [A]
time = 0.11, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3963, 3957, 2917, 2644, 331, 335, 218, 212, 209, 2716, 2720}

$$\frac{ae^2\sqrt{\sin(c+dx)}\,\text{ArcTan}\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}}{d}-\frac{2ae^2\csc(c+dx)\sqrt{e\csc(c+dx)}}{3d}-\frac{2ae^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d}+\frac{ae^2\sqrt{\sin(c+dx)}\,\sqrt{e\csc(c+dx)}\,\tanh^{-1}\left(\sqrt{\sin(c+dx)}\right)}{d}+\frac{2ae^2\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\csc(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Csc[c + d*x])^(5/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] (-2*a*e^2*Cot[c + d*x]*Sqrt[e*Csc[c + d*x]])/(3*d) - (2*a*e^2*Csc[c + d*x]*
Sqrt[e*Csc[c + d*x]])/(3*d) + (a*e^2*ArcTan[Sqrt[Sin[c + d*x]]]*Sqrt[e*Csc[
c + d*x]]*Sqrt[Sin[c + d*x]])/d + (a*e^2*ArcTanh[Sqrt[Sin[c + d*x]]]*Sqrt[e
*Csc[c + d*x]]*Sqrt[Sin[c + d*x]])/d + (2*a*e^2*Sqrt[e*Csc[c + d*x]]*Ellipt
icF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
```

```
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3963

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int (e\csc(c+dx))^{5/2}(a+a\sec(c+dx))\,dx = \left(e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{a+a\sec(c+dx)}{\sin^{\frac{5}{2}}(c+dx)}\,dx$$

$$= -\left(\left(e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{(-a-a\cos(c+dx}{\sin^{\frac{5}{2}}(c+}\right.$$

$$= \left(ae^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sin^{\frac{5}{2}}(c+dx)}\,dx + \left(ae\right.$$

$$= -\frac{2ae^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d} + \frac{1}{3}\left(ae^2\sqrt{e\csc(c+dx)}\right.$$

$$= -\frac{2ae^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d} - \frac{2ae^2\csc(c+dx)\sqrt{e\csc(}}{3d}$$

$$= -\frac{2ae^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d} - \frac{2ae^2\csc(c+dx)\sqrt{e\csc(}}{3d}$$

$$= -\frac{2ae^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d} - \frac{2ae^2\csc(c+dx)\sqrt{e\csc(}}{3d}$$

$$= -\frac{2ae^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d} - \frac{2ae^2\csc(c+dx)\sqrt{e\csc(}}{3d}$$

Mathematica [A]
time = 0.99, size = 135, normalized size = 0.71

$$-\frac{a(e\csc(c+dx))^{5/2}\left(6\mathrm{ArcTan}\left(\sqrt{\csc(c+dx)}\right)+4\cot\left(\frac{1}{2}(c+dx)\right)\sqrt{\csc(c+dx)}+3\log\left(1-\sqrt{\csc(c+dx)}\right)-3\log\left(1+\sqrt{\csc(c+dx)}\right)+4\sqrt{\csc(c+dx)}\,F\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)\sqrt{\sin(c+dx)}\right)}{6d\csc^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Csc[c + d*x])^(5/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/6*(a*(e*Csc[c + d*x])^(5/2)*(6*ArcTan[Sqrt[Csc[c + d*x]]] + 4*Cot[(c + d
*x)/2]*Sqrt[Csc[c + d*x]] + 3*Log[1 - Sqrt[Csc[c + d*x]]] - 3*Log[1 + Sqrt[
Csc[c + d*x]]] + 4*Sqrt[Csc[c + d*x]]*EllipticF[(-2*c + Pi - 2*d*x)/4, 2]*S
qrt[Sin[c + d*x]]))/(d*Csc[c + d*x]^(5/2))
```

Maple [C] Result contains complex when optimal does not.
time = 1.62, size = 679, normalized size = 3.57

method	result
default	$\frac{a(-1+\cos(dx+c))\left(4i\,\mathrm{EllipticF}\left(\sqrt{\frac{i\cos(dx+c)+\sin(dx+c)-i}{\sin(dx+c)}},\frac{\sqrt{2}}{2}\right)\sin(dx+c)\sqrt{\frac{-i\cos(dx+c)+\sin(dx+c)+i}{\sin(dx+c)}}\sqrt{\frac{i\cos(dx+c)+\cdots}{\sin(\cdots}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*csc(d*x+c))^(5/2)*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*a/d*(-1+cos(d*x+c))*(4*I*EllipticF(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x
+c))^(1/2),1/2*2^(1/2))*sin(d*x+c)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c)
)^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/
sin(d*x+c))^(1/2)-3*I*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(
1/2),1/2+1/2*I,1/2*2^(1/2))*sin(d*x+c)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*
x+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+
c))/sin(d*x+c))^(1/2)-3*I*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c
))^(1/2),1/2-1/2*I,1/2*2^(1/2))*sin(d*x+c)*((-I*cos(d*x+c)+sin(d*x+c)+I)/si
n(d*x+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-I*(-1+cos(
d*x+c))/sin(d*x+c))^(1/2)-3*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x
+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(d*x+c)*((-I*cos(d*x+c)+sin(d*x+c)+I)/
sin(d*x+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-I*(-1+co
s(d*x+c))/sin(d*x+c))^(1/2)+3*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d
*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*sin(d*x+c)*((-I*cos(d*x+c)+sin(d*x+c)+I
)/sin(d*x+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-I*(-1+
cos(d*x+c))/sin(d*x+c))^(1/2)+2*2^(1/2))*(1+cos(d*x+c))^2*(e/sin(d*x+c))^(5
/2)/sin(d*x+c)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(5/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] e^(5/2)*integrate((a*sec(d*x + c) + a)*csc(d*x + c)^(5/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(5/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))**(5/2)*(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(5/2)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*csc(d*x + c))^(5/2)*(a*sec(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)\,{\left(\frac{e}{\sin\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))*(e/sin(c + d*x))^(5/2),x)
```

```
[Out] int((a + a/cos(c + d*x))*(e/sin(c + d*x))^(5/2), x)
```

3.282 $\int (e\csc(c+dx))^{3/2}(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=169

$$-\frac{2ae\sqrt{e\csc(c+dx)}}{d}-\frac{2ae\cos(c+dx)\sqrt{e\csc(c+dx)}}{d}-\frac{ae\text{ArcTan}\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}\sqrt{\sin(}}{d}$$

[Out] -2*a*e*(e*csc(d*x+c))^(1/2)/d-2*a*e*cos(d*x+c)*(e*csc(d*x+c))^(1/2)/d-a*e*arctan(sin(d*x+c)^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/d+a*e*arctanh(sin(d*x+c)^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/d+2*a*e*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/d

Rubi [A]
time = 0.12, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$,
Rules used = {3963, 3957, 2917, 2644, 331, 335, 304, 209, 212, 2716, 2719}

$$-\frac{ae\sqrt{\sin(c+dx)}\,\text{ArcTan}\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}}{d}-\frac{2ae\sqrt{e\csc(c+dx)}}{d}-\frac{2ae\cos(c+dx)\sqrt{e\csc(c+dx)}}{d}+\frac{ae\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\tanh^{-1}\left(\sqrt{\sin(c+dx)}\right)}{d}-\frac{2ae\sqrt{\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\csc(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Csc[c + d*x])^(3/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] (-2*a*e*Sqrt[e*Csc[c + d*x]])/d - (2*a*e*Cos[c + d*x]*Sqrt[e*Csc[c + d*x]])
/d - (a*e*ArcTan[Sqrt[Sin[c + d*x]]]*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]
])/d + (a*e*ArcTanh[Sqrt[Sin[c + d*x]]]*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d
*x]])/d - (2*a*e*Sqrt[e*Csc[c + d*x]]*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt
[Sin[c + d*x]])/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
```

```
/b, 0]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
```

```
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3963

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int (e\csc(c+dx))^{3/2}(a+a\sec(c+dx))\,dx = \left(e\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{a+a\sec(c+dx)}{\sin^{\frac{3}{2}}(c+dx)}\,dx$$

$$= -\left(\left(e\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{(-a-a\cos(c+dx))\,\mathrm{s}}{\sin^{\frac{3}{2}}(c+dx}\right.$$

$$= \left(ae\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sin^{\frac{3}{2}}(c+dx)}\,dx + \left(ae\sqrt{\right.$$

$$= -\frac{2ae\cos(c+dx)\sqrt{e\csc(c+dx)}}{d} - \left(ae\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c\right.$$

$$= -\frac{2ae\sqrt{e\csc(c+dx)}}{d} - \frac{2ae\cos(c+dx)\sqrt{e\csc(c+dx)}}{d} - \frac{2ae}{}$$

$$= -\frac{2ae\sqrt{e\csc(c+dx)}}{d} - \frac{2ae\cos(c+dx)\sqrt{e\csc(c+dx)}}{d} - \frac{2ae}{}$$

$$= -\frac{2ae\sqrt{e\csc(c+dx)}}{d} - \frac{2ae\cos(c+dx)\sqrt{e\csc(c+dx)}}{d} - \frac{2ae}{}$$

$$= -\frac{2ae\sqrt{e\csc(c+dx)}}{d} - \frac{2ae\cos(c+dx)\sqrt{e\csc(c+dx)}}{d} - \frac{ae\,\mathrm{t}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.86, size = 146, normalized size = 0.86

$$\frac{a(e\csc(c+dx))^{3/2}\left(2\mathrm{ArcTan}\left(\sqrt{\csc(c+dx)}\right)-4(1+\cos(c+dx))\sqrt{\csc(c+dx)}-\log\left(1-\sqrt{\csc(c+dx)}\right)+\log\left(1+\sqrt{\csc(c+dx)}\right)+\frac{2\csc^{\frac{3}{2}}(c+dx)\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};\csc^2(c+dx)\right)\sin(2(c+dx))}{\sqrt{-\cot^2(c+dx)}}\right)}{2d\csc^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Csc[c + d*x])^(3/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*(e*Csc[c + d*x])^(3/2)*(2*ArcTan[Sqrt[Csc[c + d*x]]] - 4*(1 + Cos[c + d*
x])*Sqrt[Csc[c + d*x]] - Log[1 - Sqrt[Csc[c + d*x]]] + Log[1 + Sqrt[Csc[c +
 d*x]]] + (2*Csc[c + d*x]^(3/2)*Hypergeometric2F1[-1/4, 1/2, 3/4, Csc[c + d
*x]^2]*Sin[2*(c + d*x)])/Sqrt[-Cot[c + d*x]^2]))/(2*d*Csc[c + d*x]^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.27, size = 1522, normalized size = 9.01

method	result	size
default	Expression too large to display	1522

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*csc(d*x+c))^(3/2)*(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a/d*(I*cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(d*x+c)+
sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(
1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2+1/2*I,1/
2*2^(1/2))-I*cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(d*x+c
)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))
^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2-1/2*I,
1/2*2^(1/2))+cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(d*x+c
)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))
^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2+1/2*I,
1/2*2^(1/2))+cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(d*x+c
)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))
^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2-1/2*I,
1/2*2^(1/2))+4*cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(d*x
+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c
))^(1/2)*EllipticE(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/
2))-2*cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(d*x+c)+sin(d
*x+c)-I)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*
EllipticF(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))+I*((I
*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/s
in(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),
1/2+1/2*I,1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)-I*((I*cos(d*x+
c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c)
)^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2-1/2*I
,1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)+((I*cos(d*x+c)+sin(d*x+
c)-I)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*Ell
ipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2
))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)+((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d
*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticPi(((I
*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-I*(-1+
cos(d*x+c))/sin(d*x+c))^(1/2)+4*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1
/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticE(((I*cos(d*x+c
```

```
)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+
c))^(1/2)-2*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-
sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(
d*x+c))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)-4*2^(1/2))
*(e/sin(d*x+c))^(3/2)*sin(d*x+c)*2^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(3/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] e^(3/2)*integrate((a*sec(d*x + c) + a)*csc(d*x + c)^(3/2), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.06, size = 186, normalized size = 1.10

$$-\frac{4\,\sqrt{2i}\,ae^{\frac{3}{2}}\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos{(dx+c)}+i\,\sin{(dx+c)}))+4\,\sqrt{-2i}\,ae^{\frac{3}{2}}\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos{(dx+c)}-i\,\sin{(dx+c)}))+2\,a\arctan\left(\frac{\sin(dx+c)-1}{2\,\sqrt{\sin{(dx+c)}}}\right)e^{\frac{3}{2}}-ae^{\frac{3}{2}}\log\left(\frac{\cos(dx+c)^2+\frac{4\,\left(\cos(dx+c)^2-\sin(dx+c)-1\right)}{\sqrt{\sin{(dx+c)}}}-6\,\sin(dx+c)-2}{\cos(dx+c)^2+2\,\sin(dx+c)-2}\right)+\frac{8\,\left(a\cos(dx+c)e^{\frac{3}{2}}+ae^{\frac{3}{2}}\right)}{\sqrt{\sin{(dx+c)}}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(3/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(4*sqrt(2*I)*a*e^(3/2)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0,
 cos(d*x + c) + I*sin(d*x + c))) + 4*sqrt(-2*I)*a*e^(3/2)*weierstrassZeta(4
, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*a*arctan
(1/2*(sin(d*x + c) - 1)/sqrt(sin(d*x + c)))*e^(3/2) - a*e^(3/2)*log((cos(d*
x + c)^2 + 4*(cos(d*x + c)^2 - sin(d*x + c) - 1)/sqrt(sin(d*x + c)) - 6*sin
(d*x + c) - 2)/(cos(d*x + c)^2 + 2*sin(d*x + c) - 2)) + 8*(a*cos(d*x + c)*e
^(3/2) + a*e^(3/2))/sqrt(sin(d*x + c)))/d
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))**(3/2)*(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(3/2)*(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((e*csc(d*x + c))^(3/2)*(a*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)\,{\left(\frac{e}{\sin\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))*(e/sin(c + d*x))^(3/2),x)
[Out] int((a + a/cos(c + d*x))*(e/sin(c + d*x))^(3/2), x)
```

3.283 $\int \sqrt{e \csc(c+dx)}\,(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=121

$$\frac{a\text{ArcTan}\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}{d}+\frac{a\tanh^{-1}\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}\sqrt{\sin}}{d}$$

```
[Out] a*arctan(sin(d*x+c)^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/d+a*arctan
h(sin(d*x+c)^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/d-2*a*(sin(1/2*c+
1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*
Pi+1/2*d*x),2^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.10, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3963, 3957, 2917, 2644, 335, 218, 212, 209, 2720}

$$\frac{a\sqrt{\sin(c+dx)}\,\text{ArcTan}\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}}{d}+\frac{a\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\tanh^{-1}\left(\sqrt{\sin(c+dx)}\right)}{d}+\frac{2a\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\csc(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Csc[c + d*x]]*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*ArcTan[Sqrt[Sin[c + d*x]]]*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]])/d +
(a*ArcTanh[Sqrt[Sin[c + d*x]]]*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]])/d +
 (2*a*Sqrt[e*Csc[c + d*x]]*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*
x]])/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3963

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sqrt{e\csc(c+dx)}\,(a+a\sec(c+dx))\,dx = \left(\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{a+a\sec(c+dx)}{\sqrt{\sin(c+dx)}}\,dx$$

$$= -\left(\left(\left(\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{(-a-a\cos(c+dx))\,\mathrm{se}}{\sqrt{\sin(c+dx)}}\right.\right.$$

$$= \left(a\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx + \left(a\sqrt{\ }\right.$$

$$= \frac{2a\sqrt{e\csc(c+dx)}\,F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{d} + \frac{\left(a\sqrt{e}\right.}{}$$

$$= \frac{2a\sqrt{e\csc(c+dx)}\,F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{d} + \frac{\left(2a\sqrt{\ }\right.}{}$$

$$= \frac{2a\sqrt{e\csc(c+dx)}\,F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{d} + \frac{\left(a\sqrt{e}\right.}{}$$

$$= \frac{a\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}}{d} + \frac{a\tan}{}$$

Mathematica [A]
time = 0.57, size = 111, normalized size = 0.92

$$-\frac{a\sqrt{e\csc(c+dx)}\left(2\text{ArcTan}\left(\sqrt{\csc(c+dx)}\right)+\log\left(1-\sqrt{\csc(c+dx)}\right)-\log\left(1+\sqrt{\csc(c+dx)}\right)+4\sqrt{\csc(c+dx)}\,F\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)\sqrt{\sin(c+dx)}\right)}{2d\sqrt{\csc(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Csc[c + d*x]]*(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/2*(a*Sqrt[e*Csc[c + d*x]]*(2*ArcTan[Sqrt[Csc[c + d*x]]] + Log[1 - Sqrt[C
sc[c + d*x]]] - Log[1 + Sqrt[Csc[c + d*x]]] + 4*Sqrt[Csc[c + d*x]]*Elliptic
F[(-2*c + Pi - 2*d*x)/4, 2]*Sqrt[Sin[c + d*x]]))/(d*Sqrt[Csc[c + d*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.18, size = 291, normalized size = 2.40

method	result
default	$-\frac{a\sqrt{2}\,\sqrt{\frac{e}{\sin(dx+c)}}\,(-1+\cos(dx+c))\sqrt{\frac{i\cos(dx+c)+\sin(dx+c)-i}{\sin(dx+c)}}\,\sqrt{-\frac{i\cos(dx+c)-\sin(dx+c)-i}{\sin(dx+c)}}\,\sqrt{-\frac{i(-1+\cos(dx+c))}{\sin(dx+c)}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(e*csc(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/d*2^(1/2)*(e/sin(d*x+c))^(1/2)*(-1+cos(d*x+c))*((I*cos(d*x+c)+sin(d*
x+c)-I)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(
-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(I*EllipticPi(((I*cos(d*x+c)+sin(d*x+c
)-I)/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+I*EllipticPi(((I*cos(d*x+c)+s
in(d*x+c)-I)/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+EllipticPi(((I*cos(d*
x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-EllipticPi(((I*
cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2)))/sin(d*x+
c)^2*(1+cos(d*x+c))^2
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*csc(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(1/2)*integrate((a*sec(d*x + c) + a)*sqrt(csc(d*x + c)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*csc(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sqrt{e\csc\left(c+dx\right)}\,dx+\int\sqrt{e\csc\left(c+dx\right)}\,\sec\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*csc(d*x+c))**(1/2),x)
```

```
[Out] a*(Integral(sqrt(e*csc(c + d*x)), x) + Integral(sqrt(e*csc(c + d*x))*sec(c
+ d*x), x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*csc(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*csc(d*x + c))*(a*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)\,\sqrt{\frac{e}{\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))*(e/sin(c + d*x))^(1/2),x)
[Out] int((a + a/cos(c + d*x))*(e/sin(c + d*x))^(1/2), x)
```

3.284 $\int \frac{a+a\sec(c+dx)}{\sqrt{e\csc(c+dx)}}\,dx$

Optimal. Leaf size=122

$$-\frac{a\text{ArcTan}\left(\sqrt{\sin(c+dx)}\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}+\frac{a\tanh^{-1}\left(\sqrt{\sin(c+dx)}\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}+\frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

```
[Out] -a*arctan(sin(d*x+c)^(1/2))/d/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)+a*arcta
nh(sin(d*x+c)^(1/2))/d/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)-2*a*(sin(1/2*c
+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4
*Pi+1/2*d*x),2^(1/2))/d/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3963, 3957, 2917, 2644, 335, 304, 209, 212, 2719}

$$-\frac{a\text{ArcTan}\left(\sqrt{\sin(c+dx)}\right)}{d\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}+\frac{a\tanh^{-1}\left(\sqrt{\sin(c+dx)}\right)}{d\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}+\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{d\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/Sqrt[e*Csc[c + d*x]],x]
```

```
[Out] -((a*ArcTan[Sqrt[Sin[c + d*x]]])/(d*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]]
)) + (a*ArcTanh[Sqrt[Sin[c + d*x]]])/(d*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d
*x]]) + (2*a*EllipticE[(c - Pi/2 + d*x)/2, 2])/(d*Sqrt[e*Csc[c + d*x]]*Sqrt
[Sin[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
```

```
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3963

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{a+a\sec(c+dx)}{\sqrt{e\csc(c+dx)}}\,dx = \frac{\int (a+a\sec(c+dx))\sqrt{\sin(c+dx)}\,dx}{\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$
$$= -\frac{\int (-a-a\cos(c+dx))\sec(c+dx)\sqrt{\sin(c+dx)}\,dx}{\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$
$$= \frac{a\int \sqrt{\sin(c+dx)}\,dx}{\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a\int \sec(c+dx)\sqrt{\sin(c+dx)}\,dx}{\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$
$$= \frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a\text{Subst}\left(\int \frac{\sqrt{x}}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$
$$= \frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{(2a)\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, \sqrt{\sin(c+dx)}\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$
$$= \frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\sin(c+dx)}\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{a\text{Sul}}{d}$$
$$= -\frac{a\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a\tanh^{-1}\left(\sqrt{\sin(c+dx)}\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{2aE}{d\sqrt{e\,\csc}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.57, size = 130, normalized size = 1.07

$$\frac{a\left(-4\cot(c+dx)\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};\csc^2(c+dx)\right)+\sqrt{-\cot^2(c+dx)}\sqrt{\csc(c+dx)}\left(2\text{ArcTan}\left(\sqrt{\csc(c+dx)}\right)-\log\left(1-\sqrt{\csc(c+dx)}\right)+\log\left(1+\sqrt{\csc(c+dx)}\right)\right)\right)}{2d\sqrt{-\cot^2(c+dx)}\sqrt{e\csc(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])/Sqrt[e*Csc[c + d*x]],x]
```

```
[Out] (a*(-4*Cot[c + d*x]*Hypergeometric2F1[-1/4, 1/2, 3/4, Csc[c + d*x]^2] + Sqr
t[-Cot[c + d*x]^2]*Sqrt[Csc[c + d*x]]*(2*ArcTan[Sqrt[Csc[c + d*x]]] - Log[1
 - Sqrt[Csc[c + d*x]]] + Log[1 + Sqrt[Csc[c + d*x]]])))/(2*d*Sqrt[-Cot[c +
d*x]^2]*Sqrt[e*Csc[c + d*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.21, size = 1535, normalized size = 12.58

method	result	size
default	Expression too large to display	1535

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))/(e*csc(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/2*a/d*(I*cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-I*cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-4*cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticE(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))+2*cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))+I*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)-I*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)+((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)+((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)-4*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticE(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)+2*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)-2*2^(1/2)*cos(d*x+c)+2*2^(1/2))/(e/sin(d*x+c))^(1/2)/sin(d*x+c)*2^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-1/2)*integrate((a*sec(d*x + c) + a)/sqrt(csc(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.42, size = 206, normalized size = 1.69

$$\frac{\left(4\sqrt{2i}\,a\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c)))+4\sqrt{-2i}\,a\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(dx+c)-i\sin(dx+c)))+2\,a\arctan\left(\frac{76\cos(dx+c)^2+\frac{425\left(\cos(dx+c)^2+\sin(dx+c)-1\right)}{\sqrt{\sin(dx+c)}}-152\sin(dx+c)-152}{2\left(16\cos(dx+c)^2+393\sin(dx+c)-32\right)}\right)+a\log\left(\frac{\cos(dx+c)^2+\frac{4\left(\cos(dx+c)^2-\sin(dx+c)-1\right)}{\sqrt{\sin(dx+c)}}-6\sin(dx+c)-2}{\cos(dx+c)^2+2\sin(dx+c)-2}\right)\right)e^{\left(-\frac{1}{2}\right)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(4*sqrt(2*I)*a*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x
+ c) + I*sin(d*x + c))) + 4*sqrt(-2*I)*a*weierstrassZeta(4, 0, weierstrassP
Inverse(4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*a*arctan(1/2*(76*cos(d*x
+ c)^2 + 425*(cos(d*x + c)^2 + sin(d*x + c) - 1)/sqrt(sin(d*x + c)) - 152*s
in(d*x + c) - 152)/(16*cos(d*x + c)^2 + 393*sin(d*x + c) - 32)) + a*log((co
s(d*x + c)^2 + 4*(cos(d*x + c)^2 - sin(d*x + c) - 1)/sqrt(sin(d*x + c)) - 6
*sin(d*x + c) - 2)/(cos(d*x + c)^2 + 2*sin(d*x + c) - 2)))*e^(-1/2)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\frac{1}{\sqrt{e\csc(c+dx)}}\,dx+\int\frac{\sec(c+dx)}{\sqrt{e\csc(c+dx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))**(1/2),x)
```

```
[Out] a*(Integral(1/sqrt(e*csc(c + d*x)), x) + Integral(sec(c + d*x)/sqrt(e*csc(c
 + d*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)/sqrt(e*csc(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{a+\frac{a}{\cos(c+d\,x)}}{\sqrt{\frac{e}{\sin(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/(e/sin(c + d*x))^(1/2),x)
[Out] int((a + a/cos(c + d*x))/(e/sin(c + d*x))^(1/2), x)
```

3.285 $\int \frac{a+a\sec(c+dx)}{(e\csc(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=182

$$-\frac{2a}{de\sqrt{e\csc(c+dx)}}-\frac{2a\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}}+\frac{a\text{ArcTan}\left(\sqrt{\sin(c+dx)}\right)}{de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}+\frac{a\tanh^{-1}\left(\sqrt{\sin(c+dx)}\right)}{de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

```
[Out] -2*a/d/e/(e*csc(d*x+c))^(1/2)-2/3*a*cos(d*x+c)/d/e/(e*csc(d*x+c))^(1/2)+a*a
rctan(sin(d*x+c)^(1/2))/d/e/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)+a*arctanh
(sin(d*x+c)^(1/2))/d/e/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)-2/3*a*(sin(1/2
*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1
/4*Pi+1/2*d*x),2^(1/2))/d/e/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3963, 3957, 2917, 2644, 327, 335, 218, 212, 209, 2715, 2720}

$$\frac{a\text{ArcTan}\left(\sqrt{\sin(c+dx)}\right)}{de\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}-\frac{2a}{de\sqrt{e\csc(c+dx)}}-\frac{2a\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}}+\frac{a\tanh^{-1}\left(\sqrt{\sin(c+dx)}\right)}{de\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}+\frac{2aF\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{3de\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/(e*Csc[c + d*x])^(3/2),x]
```

```
[Out] (-2*a)/(d*e*Sqrt[e*Csc[c + d*x]]) - (2*a*Cos[c + d*x])/(3*d*e*Sqrt[e*Csc[c
 + d*x]]) + (a*ArcTan[Sqrt[Sin[c + d*x]]])/(d*e*Sqrt[e*Csc[c + d*x]]*Sqrt[Si
n[c + d*x]]) + (a*ArcTanh[Sqrt[Sin[c + d*x]]])/(d*e*Sqrt[e*Csc[c + d*x]]*Sq
rt[Sin[c + d*x]]) + (2*a*EllipticF[(c - Pi/2 + d*x)/2, 2])/(3*d*e*Sqrt[e*Cs
c[c + d*x]]*Sqrt[Sin[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
```

```
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3963

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+a\sec(c+dx)}{(e\csc(c+dx))^{3/2}}\,dx &= \frac{\int (a+a\sec(c+dx))\sin^{\frac{3}{2}}(c+dx)\,dx}{e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} \\
&= -\frac{\int (-a-a\cos(c+dx))\sec(c+dx)\sin^{\frac{3}{2}}(c+dx)\,dx}{e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} \\
&= \frac{a\int \sin^{\frac{3}{2}}(c+dx)\,dx}{e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a\int \sec(c+dx)\sin^{\frac{3}{2}}(c+dx)\,dx}{e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} \\
&= -\frac{2a\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}} + \frac{a\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{3e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a\text{Subst}\left(\int \frac{x^{3/2}}{1-x^2}\,dx,\right.}{de\sqrt{e\csc(c+dx)}} \\
&= -\frac{2a}{de\sqrt{e\csc(c+dx)}} - \frac{2a\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}} + \frac{2aF\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{3de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} \\
&= -\frac{2a}{de\sqrt{e\csc(c+dx)}} - \frac{2a\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}} + \frac{2aF\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{3de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} \\
&= -\frac{2a}{de\sqrt{e\csc(c+dx)}} - \frac{2a\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}} + \frac{2aF\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{3de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} \\
&= -\frac{2a}{de\sqrt{e\csc(c+dx)}} - \frac{2a\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}} + \frac{a\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)}{de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}
\end{aligned}$$

Mathematica [A]
time = 9.20, size = 135, normalized size = 0.74

$$-\frac{a\left(12+4\cos(c+dx)+6\text{ArcTan}\left(\sqrt{\csc(c+dx)}\right)\sqrt{\csc(c+dx)}+3\sqrt{\csc(c+dx)}\log\left(1-\sqrt{\csc(c+dx)}\right)-3\sqrt{\csc(c+dx)}\log\left(1+\sqrt{\csc(c+dx)}\right)+\frac{4F\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)}{\sqrt{\sin(c+dx)}}\right)}{6de\sqrt{e\csc(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])/(e*Csc[c + d*x])^(3/2),x]
```

```
[Out] -1/6*(a*(12 + 4*Cos[c + d*x] + 6*ArcTan[Sqrt[Csc[c + d*x]]]*Sqrt[Csc[c + d*
x]] + 3*Sqrt[Csc[c + d*x]]*Log[1 - Sqrt[Csc[c + d*x]]] - 3*Sqrt[Csc[c + d*x
]]*Log[1 + Sqrt[Csc[c + d*x]]] + (4*EllipticF[(-2*c + Pi - 2*d*x)/4, 2])/Sq
rt[Sin[c + d*x]]))/(d*e*Sqrt[e*Csc[c + d*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.20, size = 695, normalized size = 3.82

method	result
default	$-\frac{a\left(3i\,\mathrm{EllipticPi}\left(\sqrt{\frac{i\cos(dx+c)+\sin(dx+c)-i}{\sin(dx+c)}},\frac{1}{2}+\frac{i}{2},\frac{\sqrt{2}}{2}\right)\sin(dx+c)\sqrt{\frac{-i\cos(dx+c)+\sin(dx+c)+i}{\sin(dx+c)}}\,\sqrt{\frac{i\cos(dx+c)+\sin(dx-}{\sin(dx+c)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/(e*csc(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*a/d*(3*I*sin(d*x+c)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-
I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(
d*x+c)-I)/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(
d*x+c))^(1/2)+3*I*sin(d*x+c)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)
*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+
sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/
sin(d*x+c))^(1/2)-4*I*sin(d*x+c)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(
1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+
c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x
+c))^(1/2)+3*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2+
1/2*I,1/2*2^(1/2))*sin(d*x+c)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/
2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/sin(d
*x+c))^(1/2)-3*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/
2-1/2*I,1/2*2^(1/2))*sin(d*x+c)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(
1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/sin
(d*x+c))^(1/2)+2*cos(d*x+c)^2*2^(1/2)+4*2^(1/2)*cos(d*x+c)-6*2^(1/2))/(-1+c
os(d*x+c))/(e/sin(d*x+c))^(3/2)/sin(d*x+c)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))^(3/2),x, algorithm="maxima")
```

[Out] e^(-3/2)*integrate((a*sec(d*x + c) + a)/csc(d*x + c)^(3/2), x)

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{1}{(e\csc(c+dx))^{\frac{3}{2}}}\,dx+\int \frac{\sec(c+dx)}{(e\csc(c+dx))^{\frac{3}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))**(3/2),x)
```

```
[Out] a*(Integral((e*csc(c + d*x))**(-3/2), x) + Integral(sec(c + d*x)/(e*csc(c +
 d*x))**(3/2), x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)/(e*csc(d*x + c))^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+\frac{a}{\cos(c+d\,x)}}{{\left(\frac{e}{\sin(c+d\,x)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/(e/sin(c + d*x))^(3/2),x)
```

```
[Out] int((a + a/cos(c + d*x))/(e/sin(c + d*x))^(3/2), x)
```

3.286 $\int \frac{a+a\sec(c+dx)}{(e\csc(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=197

$$-\frac{a\text{ArcTan}\left(\sqrt{\sin(c+dx)}\right)}{de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}+\frac{a\tanh^{-1}\left(\sqrt{\sin(c+dx)}\right)}{de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}+\frac{6aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{5de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

```
[Out] -2/3*a*sin(d*x+c)/d/e^2/(e*csc(d*x+c))^(1/2)-2/5*a*cos(d*x+c)*sin(d*x+c)/d/
e^2/(e*csc(d*x+c))^(1/2)-a*arctan(sin(d*x+c)^(1/2))/d/e^2/(e*csc(d*x+c))^(1
/2)/sin(d*x+c)^(1/2)+a*arctanh(sin(d*x+c)^(1/2))/d/e^2/(e*csc(d*x+c))^(1/2)
/sin(d*x+c)^(1/2)-6/5*a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*P
i+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))/d/e^2/(e*csc(d*x+c)
)^(1/2)/sin(d*x+c)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3963, 3957, 2917, 2644, 327, 335, 304, 209, 212, 2715, 2719}

$$-\frac{a\text{ArcTan}\left(\sqrt{\sin(c+dx)}\right)}{de^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}-\frac{2a\sin(c+dx)}{3de^2\sqrt{e\csc(c+dx)}}-\frac{2a\sin(c+dx)\cos(c+dx)}{5de^2\sqrt{e\csc(c+dx)}}+\frac{a\tanh^{-1}\left(\sqrt{\sin(c+dx)}\right)}{de^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}+\frac{6aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{5de^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/(e*Csc[c + d*x])^(5/2),x]
```

```
[Out] -((a*ArcTan[Sqrt[Sin[c + d*x]]])/(d*e^2*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d
*x]])) + (a*ArcTanh[Sqrt[Sin[c + d*x]]])/(d*e^2*Sqrt[e*Csc[c + d*x]]*Sqrt[S
in[c + d*x]]) + (6*a*EllipticE[(c - Pi/2 + d*x)/2, 2])/(5*d*e^2*Sqrt[e*Csc[
c + d*x]]*Sqrt[Sin[c + d*x]]) - (2*a*Sin[c + d*x])/(3*d*e^2*Sqrt[e*Csc[c +
d*x]]) - (2*a*Cos[c + d*x]*Sin[c + d*x])/(5*d*e^2*Sqrt[e*Csc[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
```

```
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3963

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{a+a\sec(c+dx)}{(e\csc(c+dx))^{5/2}}\,dx = \frac{\int (a+a\sec(c+dx))\sin^{\frac{5}{2}}(c+dx)\,dx}{e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= -\frac{\int (-a-a\cos(c+dx))\sec(c+dx)\sin^{\frac{5}{2}}(c+dx)\,dx}{e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{a\int \sin^{\frac{5}{2}}(c+dx)\,dx}{e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a\int \sec(c+dx)\sin^{\frac{5}{2}}(c+dx)\,dx}{e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= -\frac{2a\cos(c+dx)\sin(c+dx)}{5de^2\sqrt{e\csc(c+dx)}} + \frac{(3a)\int \sqrt{\sin(c+dx)}\,dx}{5e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a\text{Subst}\left(\int \frac{x^{5/}}{1-}\right.}{de^2\sqrt{e\csc(c}}$$

$$= \frac{6aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{5de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{2a\sin(c+dx)}{3de^2\sqrt{e\csc(c+dx)}} - \frac{2a\cos(c+dx)\sin(}{5de^2\sqrt{e\csc(c+}}$$

$$= \frac{6aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{5de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{2a\sin(c+dx)}{3de^2\sqrt{e\csc(c+dx)}} - \frac{2a\cos(c+dx)\sin(}{5de^2\sqrt{e\csc(c+}}$$

$$= \frac{6aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{5de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{2a\sin(c+dx)}{3de^2\sqrt{e\csc(c+dx)}} - \frac{2a\cos(c+dx)\sin(}{5de^2\sqrt{e\csc(c+}}$$

$$= -\frac{a\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)}{de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a\tanh^{-1}\left(\sqrt{\sin(c+dx)}\right)}{de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{}{5de^2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.98, size = 165, normalized size = 0.84

$$\frac{a\left(-72\cot(c+dx)\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};\csc^2(c+dx)\right)-2\sqrt{-\cot^2(c+dx)}\left(-30\text{ArcTan}\left(\sqrt{\csc(c+dx)}\right)\sqrt{\csc(c+dx)}+15\sqrt{\csc(c+dx)}\left(\log\left(1-\sqrt{\csc(c+dx)}\right)-\log\left(1+\sqrt{\csc(c+dx)}\right)\right)+20\sin(c+dx)+6\sin(2(c+dx))\right)\right)}{60de^2\sqrt{-\cot^2(c+dx)}\sqrt{e\csc(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])/(e*Csc[c + d*x])^(5/2),x]
```

```
[Out] (a*(-72*Cot[c + d*x]*Hypergeometric2F1[-1/4, 1/2, 3/4, Csc[c + d*x]^2] - 2*
Sqrt[-Cot[c + d*x]^2]*(-30*ArcTan[Sqrt[Csc[c + d*x]]]*Sqrt[Csc[c + d*x]] +
15*Sqrt[Csc[c + d*x]]*(Log[1 - Sqrt[Csc[c + d*x]]] - Log[1 + Sqrt[Csc[c + d
*x]]]) + 20*Sin[c + d*x] + 6*Sin[2*(c + d*x)])))/(60*d*e^2*Sqrt[-Cot[c + d*
x]^2]*Sqrt[e*Csc[c + d*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.19, size = 1529, normalized size = 7.76

method	result	size
default	Expression too large to display	1529

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/(e*csc(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/30*a/d*(-15*I*cos(d*x+c)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(
(-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+si
n(d*x+c)-I)/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/si
n(d*x+c))^(1/2)+15*I*cos(d*x+c)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1
/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+
c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-I*(-1+cos(d*x+c
))/sin(d*x+c))^(1/2)+18*cos(d*x+c)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))
^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*
x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d
*x+c))^(1/2)+15*cos(d*x+c)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(
(-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+si
n(d*x+c)-I)/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/si
n(d*x+c))^(1/2)+15*cos(d*x+c)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2
)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)
+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-I*(-1+cos(d*x+c))
/sin(d*x+c))^(1/2)-36*cos(d*x+c)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(
1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticE(((I*cos(d*x+
c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x
+c))^(1/2)-15*I*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x
+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)
/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(
1/2)+15*I*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+si
n(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d
*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)+6
*cos(d*x+c)^3*2^(1/2)+18*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-
I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)+sin(d
*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/
```

```
2)+15*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*
x+c)+I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c
))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)+15*((
I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/
sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)
,1/2-1/2*I,1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)-36*((I*cos(d*
x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+
c))^(1/2)*EllipticE(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1
/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)+10*cos(d*x+c)^2*2^(1/2)-24*2^(1/
2)*cos(d*x+c)+8*2^(1/2))/(e/sin(d*x+c))^(5/2)/sin(d*x+c)^3*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] e^(-5/2)*integrate((a*sec(d*x + c) + a)/csc(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.31, size = 252, normalized size = 1.28

$$\frac{\left(36\sqrt{2i}\,a\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c)))+36\sqrt{-2i}\,a\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)-i\sin(dx+c)))+30\,a\arctan\left(\frac{76\cos(dx+c)^2+\frac{425\left(\cos(dx+c)^2+\sin(dx+c)-1\right)}{\sqrt{\sin(dx+c)}}-152\sin(dx+c)-152}{2\left(16\cos(dx+c)^2+393\sin(dx+c)-32\right)}\right)+15\,a\log\left(\frac{\cos(dx+c)^2+\frac{4\left(\cos(dx+c)^2-\sin(dx+c)-1\right)}{\sqrt{\sin(dx+c)}}-6\sin(dx+c)-2}{\cos(dx+c)^2+2\sin(dx+c)-2}\right)+\frac{8\left(3a\cos(dx+c)^3+5a\cos(dx+c)^2-3a\cos(dx+c)-5a\right)}{\sqrt{\sin(dx+c)}}\right)e^{\left(-\frac{5}{2}\right)}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/60*(36*sqrt(2*I)*a*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*
x + c) + I*sin(d*x + c))) + 36*sqrt(-2*I)*a*weierstrassZeta(4, 0, weierstra
ssPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c))) + 30*a*arctan(1/2*(76*cos(
d*x + c)^2 + 425*(cos(d*x + c)^2 + sin(d*x + c) - 1)/sqrt(sin(d*x + c)) - 1
52*sin(d*x + c) - 152)/(16*cos(d*x + c)^2 + 393*sin(d*x + c) - 32)) + 15*a*
log((cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - sin(d*x + c) - 1)/sqrt(sin(d*x +
c)) - 6*sin(d*x + c) - 2)/(cos(d*x + c)^2 + 2*sin(d*x + c) - 2)) + 8*(3*a*c
os(d*x + c)^3 + 5*a*cos(d*x + c)^2 - 3*a*cos(d*x + c) - 5*a)/sqrt(sin(d*x +
 c)))*e^(-5/2)/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)/(e*csc(d*x + c))^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+\frac{a}{\cos(c+d\,x)}}{{\left(\frac{e}{\sin(c+d\,x)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/(e/sin(c + d*x))^(5/2),x)
[Out] int((a + a/cos(c + d*x))/(e/sin(c + d*x))^(5/2), x)
```

3.287 $\int (e\csc(c+dx))^{5/2}(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=270

$$-\frac{2a^2e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d}-\frac{4a^2e^2\csc(c+dx)\sqrt{e\csc(c+dx)}}{3d}-\frac{2a^2e^2\csc(c+dx)\sqrt{e\csc(c+dx)}\,\mathrm{se}}{3d}$$

```
[Out] -2/3*a^2*e^2*cot(d*x+c)*(e*csc(d*x+c))^(1/2)/d-4/3*a^2*e^2*csc(d*x+c)*(e*cs
c(d*x+c))^(1/2)/d-2/3*a^2*e^2*csc(d*x+c)*sec(d*x+c)*(e*csc(d*x+c))^(1/2)/d+
2*a^2*e^2*arctan(sin(d*x+c)^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/d+
2*a^2*e^2*arctanh(sin(d*x+c)^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/d
-7/3*a^2*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*
EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c
)^(1/2)/d+5/3*a^2*e^2*(e*csc(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.23, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3963, 3957, 2952, 2716, 2720, 2644, 331, 335, 218, 212, 209, 2650, 2651}

$$\frac{2a^2e^2\sqrt{\sin(c+dx)}\,\text{ArcTan}\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}}{d}-\frac{4a^2e^2\csc(c+dx)\sqrt{e\csc(c+dx)}}{3d}-\frac{2a^2e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d}+\frac{5a^2e^2\tan(c+dx)\sqrt{e\csc(c+dx)}}{3d}-\frac{2a^2e^2\csc(c+dx)\sec(c+dx)\sqrt{e\csc(c+dx)}}{3d}+\frac{2a^2e^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\tanh^{-1}\left(\sqrt{\sin(c+dx)}\right)}{d}+\frac{7a^2e^2\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\csc(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Csc[c + d*x])^(5/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-2*a^2*e^2*Cot[c + d*x]*Sqrt[e*Csc[c + d*x]])/(3*d) - (4*a^2*e^2*Csc[c + d
*x]*Sqrt[e*Csc[c + d*x]])/(3*d) - (2*a^2*e^2*Csc[c + d*x]*Sqrt[e*Csc[c + d*
x]]*Sec[c + d*x])/(3*d) + (2*a^2*e^2*ArcTan[Sqrt[Sin[c + d*x]]]*Sqrt[e*Csc[
c + d*x]]*Sqrt[Sin[c + d*x]])/d + (2*a^2*e^2*ArcTanh[Sqrt[Sin[c + d*x]]]*Sq
rt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]])/d + (7*a^2*e^2*Sqrt[e*Csc[c + d*x]]*
EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*d) + (5*a^2*e^2*Sqr
t[e*Csc[c + d*x]]*Tan[c + d*x])/(3*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2650

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2651

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(b*Sin[e + f*x])^(n + 1))*((a*Cos[e + f*x])^(m + 1)
/(a*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x]
)^n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
```

```
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3963

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int (e\csc(c+dx))^{5/2}(a+a\sec(c+dx))^2\,dx &= \left(e^2\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^2}{\sin^{\frac{5}{2}}(c+dx)}\,dx \\
&= \left(e^2\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{(-a-a\cos(c+dx))^2}{\sin^{\frac{5}{2}}(c+dx} \\
&= \left(e^2\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \left(\frac{a^2}{\sin^{\frac{5}{2}}(c+dx)}+\frac{2a^2\,\mathrm{s}}{\sin}\right. \\
&= \left(a^2e^2\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sin^{\frac{5}{2}}(c+dx)}\,dx+\Big(\\
&= -\frac{2a^2e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d}-\frac{2a^2e^2\csc(c+dx)\sqrt{e\,\mathrm{cs}}}{3} \\
&= -\frac{2a^2e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d}-\frac{4a^2e^2\csc(c+dx)\sqrt{e\,\mathrm{cs}}}{3d} \\
&= -\frac{2a^2e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d}-\frac{4a^2e^2\csc(c+dx)\sqrt{e\,\mathrm{cs}}}{3d} \\
&= -\frac{2a^2e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d}-\frac{4a^2e^2\csc(c+dx)\sqrt{e\,\mathrm{cs}}}{3d} \\
&= -\frac{2a^2e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d}-\frac{4a^2e^2\csc(c+dx)\sqrt{e\,\mathrm{cs}}}{3d}
\end{aligned}
$$

Mathematica [**A**]
time = 27.53, size = 244, normalized size = 0.90

$$
-\frac{a^2e\cos^4\left(\frac{1}{2}(c+dx)\right)(e\csc(c+dx))^{3/2}\left(-7\cot^2(c+dx)F\left(\mathrm{ArcSin}\left(\sqrt{\csc(c+dx)}\right)\middle|-1\right)+\sqrt{-\cot^2(c+dx)}\,\sqrt{\csc(c+dx)}\left(-7+6\,\mathrm{ArcTan}\left(\sqrt{\csc(c+dx)}\right)\sqrt{\cos^2(c+dx)}\,\sqrt{\csc(c+dx)}+4\left(1+\sqrt{\cos^2(c+dx)}\right)\csc^2(c+dx)+3\sqrt{\cos^2(c+dx)}\,\sqrt{\csc(c+dx)}\left(\log\left(1-\sqrt{\csc(c+dx)}\right)-\log\left(1+\sqrt{\csc(c+dx)}\right)\right)\right)\right)\sec(c+dx)\sec^4\left(\frac{1}{2}\csc^{-1}(\csc(c+dx))\right)}{3d\sqrt{-\cot^2(c+dx)}\,\csc^{\frac{5}{2}}(c+dx)}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Csc[c + d*x])^(5/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/3*(a^2*e*Cos[(c + d*x)/2]^4*(e*Csc[c + d*x])^(3/2)*(-7*Cot[c + d*x]^2*El
lipticF[ArcSin[Sqrt[Csc[c + d*x]]], -1] + Sqrt[-Cot[c + d*x]^2]*Sqrt[Csc[c
+ d*x]]*(-7 + 6*ArcTan[Sqrt[Csc[c + d*x]]]*Sqrt[Cos[c + d*x]^2]*Sqrt[Csc[c
+ d*x]] + 4*(1 + Sqrt[Cos[c + d*x]^2])*Csc[c + d*x]^2 + 3*Sqrt[Cos[c + d*x]
^2]*Sqrt[Csc[c + d*x]]*(Log[1 - Sqrt[Csc[c + d*x]]] - Log[1 + Sqrt[Csc[c +
d*x]]])))*Sec[c + d*x]*Sec[ArcCsc[Csc[c + d*x]]/2]^4)/(d*Sqrt[-Cot[c + d*x]
^2]*Csc[c + d*x]^(5/2))
```

Maple [**C**] Result contains complex when optimal does not.
time = 0.22, size = 745, normalized size = 2.76

method	result
default	$-\frac{a^2(-1+\cos(dx+c))\left(6i\cos(dx+c)\sin(dx+c)\,\text{EllipticPi}\left(\sqrt{\frac{i\cos(dx+c)+\sin(dx+c)-i}{\sin(dx+c)}},\frac{1}{2}+\frac{i}{2},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{i\cos(dx+c)+\sin(dx+c)-}{\sin(dx+c)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*csc(d*x+c))^(5/2)*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*a^2/d*(-1+cos(d*x+c))*(6*I*cos(d*x+c)*sin(d*x+c)*EllipticPi(((I*cos(d*
x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((I*cos(d*x+c)+
sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(I*
cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)+6*I*cos(d*x+c)*sin(d*x+c)*Ellipt
icPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*
((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/sin(d*x+
c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)-5*I*cos(d*x+c)*si
n(d*x+c)*EllipticF(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/
2))*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/sin(
d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)+6*cos(d*x+c)*
sin(d*x+c)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2+1/
2*I,1/2*2^(1/2))*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-I*(-1+cos
(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)-
6*cos(d*x+c)*sin(d*x+c)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))
^(1/2),1/2-1/2*I,1/2*2^(1/2))*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2
)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d
*x+c))^(1/2)-7*2^(1/2)*cos(d*x+c)+3*2^(1/2))*(e/sin(d*x+c))^(5/2)*(1+cos(d*
x+c))^2/sin(d*x+c)/cos(d*x+c)*2^(1/2)
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))**(5/2)*(a+a*sec(d*x+c))**2,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate((e*csc(d*x + c))^(5/2)*(a*sec(d*x + c) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^2\,{\left(\frac{e}{\sin\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2*(e/sin(c + d*x))^(5/2),x)
[Out] int((a + a/cos(c + d*x))^2*(e/sin(c + d*x))^(5/2), x)
```

3.288 $\int (e\csc(c+dx))^{3/2}(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=240

$$-\frac{4a^2e\sqrt{e\csc(c+dx)}}{d}-\frac{2a^2e\cos(c+dx)\sqrt{e\csc(c+dx)}}{d}-\frac{2a^2e\sqrt{e\csc(c+dx)}\,\sec(c+dx)}{d}-\frac{2a^2e\mathrm{ArcTan}\Big(}{}$$

```
[Out] -4*a^2*e*(e*csc(d*x+c))^(1/2)/d-2*a^2*e*cos(d*x+c)*(e*csc(d*x+c))^(1/2)/d-2
*a^2*e*sec(d*x+c)*(e*csc(d*x+c))^(1/2)/d-2*a^2*e*arctan(sin(d*x+c)^(1/2))*(
e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/d+2*a^2*e*arctanh(sin(d*x+c)^(1/2))*(e
*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/d+5*a^2*e*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)
^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2
))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/d+3*a^2*e*sin(d*x+c)*(e*csc(d*x+c)
)^(1/2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.23, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3963, 3957, 2952, 2716, 2719, 2644, 331, 335, 304, 209, 212, 2650, 2651}

$$-\frac{2a^2e\sqrt{\sin(c+dx)}\,\mathrm{ArcTan}\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}}{d}-\frac{4a^2e\sqrt{e\csc(c+dx)}}{d}-\frac{2a^2e\cos(c+dx)\sqrt{e\csc(c+dx)}}{d}-\frac{2a^2e\sec(c+dx)\sqrt{e\csc(c+dx)}}{d}+\frac{3a^2e\sin(c+dx)\tan(c+dx)\sqrt{e\csc(c+dx)}}{d}+\frac{2a^2e\sqrt{\sin(c+dx)}\,\sqrt{e\csc(c+dx)}\,\tanh^{-1}\left(\sqrt{\sin(c+dx)}\right)}{d}-\frac{5a^2e\sqrt{\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\csc(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Csc[c + d*x])^(3/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-4*a^2*e*Sqrt[e*Csc[c + d*x]])/d - (2*a^2*e*Cos[c + d*x]*Sqrt[e*Csc[c + d*
x]])/d - (2*a^2*e*Sqrt[e*Csc[c + d*x]]*Sec[c + d*x])/d - (2*a^2*e*ArcTan[Sq
rt[Sin[c + d*x]]]*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]])/d + (2*a^2*e*Arc
Tanh[Sqrt[Sin[c + d*x]]]*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]])/d - (5*a^
2*e*Sqrt[e*Csc[c + d*x]]*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]
])/d + (3*a^2*e*Sqrt[e*Csc[c + d*x]]*Sin[c + d*x]*Tan[c + d*x])/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2650

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2651

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(b*Sin[e + f*x])^(n + 1))*((a*Cos[e + f*x])^(m + 1)
/(a*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x]
)^n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
```

```
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3963

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int (e\csc(c+dx))^{3/2}(a+a\sec(c+dx))^2\,dx = \left(e\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^2}{\sin^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \left(e\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{(-a-a\cos(c+dx))^2\,\mathrm{s}}{\sin^{\frac{3}{2}}(c+dx)}$$

$$= \left(e\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \left(\frac{a^2}{\sin^{\frac{3}{2}}(c+dx)} + \frac{2a^2\,\mathrm{se}}{\sin^{\frac{3}{2}}}\right.$$

$$= \left(a^2e\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sin^{\frac{3}{2}}(c+dx)}\,dx + \left(a\right.$$

$$= -\frac{2a^2e\cos(c+dx)\sqrt{e\csc(c+dx)}}{d} - \frac{2a^2e\sqrt{e\csc(c+dx)}\,\mathrm{se}}{d}$$

$$= -\frac{4a^2e\sqrt{e\csc(c+dx)}}{d} - \frac{2a^2e\cos(c+dx)\sqrt{e\csc(c+dx)}}{d} -$$

$$= -\frac{4a^2e\sqrt{e\csc(c+dx)}}{d} - \frac{2a^2e\cos(c+dx)\sqrt{e\csc(c+dx)}}{d} -$$

$$= -\frac{4a^2e\sqrt{e\csc(c+dx)}}{d} - \frac{2a^2e\cos(c+dx)\sqrt{e\csc(c+dx)}}{d} -$$

$$= -\frac{4a^2e\sqrt{e\csc(c+dx)}}{d} - \frac{2a^2e\cos(c+dx)\sqrt{e\csc(c+dx)}}{d} -$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 28.00, size = 307, normalized size = 1.28

$$\frac{\left(1+\cos\left(2\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\right)^2\cos(c+dx)(e\csc(c+dx))^{3/2}\left(-1+\csc^2(c+dx)\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2\left(-\frac{2\left(2\sqrt{\csc(c+dx)}+\frac{\csc^{\frac{5}{2}}(c+dx)\sqrt{1-\sin^2(c+dx)}}{2-2\csc^2(c+dx)}\right)}{d}-\frac{-6\operatorname{ArcTan}\left(\sqrt{\csc(c+dx)}\right)+3\log\left(1-\sqrt{\csc(c+dx)}\right)-3\log\left(1+\sqrt{\csc(c+dx)}\right)-\frac{5\csc^{\frac{5}{2}}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};\csc^2(c+dx)\right)\sqrt{1-\sin^2(c+dx)}}{\sqrt{1-\csc^2(c+dx)}}}{3d}\right)}{4\left(1+\cos\left(2\left(\frac{c}{2}+\frac{1}{2}\left(-c+\csc^{-1}(\csc(c+dx))\right)\right)\right)\right)^2\csc^{\frac{7}{2}}(c+dx)\sqrt{1-\sin^2(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Csc[c + d*x])^(3/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] ((1 + Cos[2*(c/2 + (d*x)/2)])^2*Cos[c + d*x]*(e*Csc[c + d*x])^(3/2)*(-1 + C
sc[c + d*x]^2)*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*((-2*(2*Sqrt[Csc
[c + d*x]] + (Csc[c + d*x]^(5/2)*Sqrt[1 - Sin[c + d*x]^2])/(2 - 2*Csc[c + d
*x]^2)))/d - (-6*ArcTan[Sqrt[Csc[c + d*x]]] + 3*Log[1 - Sqrt[Csc[c + d*x]]]
 - 3*Log[1 + Sqrt[Csc[c + d*x]]] - (5*Csc[c + d*x]^(5/2)*Hypergeometric2F1[
1/2, 3/4, 7/4, Csc[c + d*x]^2]*Sqrt[1 - Sin[c + d*x]^2])/Sqrt[1 - Csc[c + d
```

```
*x]^2])/(3*d)))/(4*(1 + Cos[2*(c/2 + (-c + ArcCsc[Csc[c + d*x]])/2)])^2*Csc
[c + d*x]^(7/2)*Sqrt[1 - Sin[c + d*x]^2])
```

Maple [C] Result contains complex when optimal does not.
time = 0.20, size = 1559, normalized size = 6.50

method	result	size
default	Expression too large to display	1559

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*csc(d*x+c))^(3/2)*(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a^2/d*(2*I*cos(d*x+c)^2*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(
d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*
x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2-1
/2*I,1/2*2^(1/2))-2*I*cos(d*x+c)^2*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((
I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/
sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)
,1/2+1/2*I,1/2*2^(1/2))-2*cos(d*x+c)^2*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2
)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)
+I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(
1/2),1/2-1/2*I,1/2*2^(1/2))-2*cos(d*x+c)^2*(-I*(-1+cos(d*x+c))/sin(d*x+c))^
(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*
x+c)+I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c
))^(1/2),1/2+1/2*I,1/2*2^(1/2))-10*cos(d*x+c)^2*(-I*(-1+cos(d*x+c))/sin(d*x
+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+s
in(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticE(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d
*x+c))^(1/2),1/2*2^(1/2))+5*cos(d*x+c)^2*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1
/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+
c)+I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^
(1/2),1/2*2^(1/2))+2*I*cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I
*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/s
in(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),
1/2-1/2*I,1/2*2^(1/2))-2*I*cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)
*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+
I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1
/2),1/2+1/2*I,1/2*2^(1/2))-2*cos(d*x+c)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*
x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticPi(((I*
cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-I*(-1+c
os(d*x+c))/sin(d*x+c))^(1/2)-2*cos(d*x+c)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(
d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticPi(((
I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-I*(-1
+cos(d*x+c))/sin(d*x+c))^(1/2)-10*cos(d*x+c)*((I*cos(d*x+c)+sin(d*x+c)-I)/s
in(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticE(
((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(d*x
```

```
+c))/sin(d*x+c))^(1/2)+5*cos(d*x+c)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c)
)^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d
*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(
d*x+c))^(1/2)+9*2^(1/2)*cos(d*x+c)-2^(1/2))*(e/sin(d*x+c))^(3/2)*sin(d*x+c)
/cos(d*x+c)*2^(1/2)
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.27, size = 247, normalized size = 1.03

$$-\frac{5\sqrt{2i}\,a^2\cos(dx+c)\,e^{\frac{3}{2}}\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)+i\,\sin(dx+c)))+5\sqrt{-2i}\,a^2\cos(dx+c)\,e^{\frac{3}{2}}\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)-i\,\sin(dx+c)))+2\,a^2\arctan\left(\frac{\sin(dx+c)-1}{2\sqrt{\sin(dx+c)}}\right)\cos(dx+c)\,e^{\frac{3}{2}}-a^2\cos(dx+c)\,e^{\frac{3}{2}}\log\left(\frac{\cos(dx+c)^2+\frac{4\left(\cos(dx+c)^2-\sin(dx+c)-1\right)}{\sqrt{\sin(dx+c)}}-6\,\sin(dx+c)-2}{\cos(dx+c)^2+2\,\sin(dx+c)-2}\right)+\frac{2\left(5\,a^2\cos(dx+c)^2e^{\frac{3}{2}}+4\,a^2\cos(dx+c)e^{\frac{3}{2}}-a^2e^{\frac{3}{2}}\right)}{\sqrt{\sin(dx+c)}}}{2\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(5*sqrt(2*I)*a^2*cos(d*x + c)*e^(3/2)*weierstrassZeta(4, 0, weierstras
sPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) + 5*sqrt(-2*I)*a^2*cos(d*x
+ c)*e^(3/2)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) -
 I*sin(d*x + c))) + 2*a^2*arctan(1/2*(sin(d*x + c) - 1)/sqrt(sin(d*x + c)))
*cos(d*x + c)*e^(3/2) - a^2*cos(d*x + c)*e^(3/2)*log((cos(d*x + c)^2 + 4*(c
os(d*x + c)^2 - sin(d*x + c) - 1)/sqrt(sin(d*x + c)) - 6*sin(d*x + c) - 2)/
(cos(d*x + c)^2 + 2*sin(d*x + c) - 2)) + 2*(5*a^2*cos(d*x + c)^2*e^(3/2) +
4*a^2*cos(d*x + c)*e^(3/2) - a^2*e^(3/2))/sqrt(sin(d*x + c)))/(d*cos(d*x +
c))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))**(3/2)*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*csc(d*x + c))^(3/2)*(a*sec(d*x + c) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^2\,\left(\frac{e}{\sin\left(c+d\,x\right)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2*(e/sin(c + d*x))^(3/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^2*(e/sin(c + d*x))^(3/2), x)
```

3.289 $\int \sqrt{e \csc(c+dx)}\,(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=154

$$\frac{2a^2\operatorname{ArcTan}\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}{d}+\frac{2a^2\tanh^{-1}\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}}{d}$$

```
[Out] 2*a^2*arctan(sin(d*x+c)^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/d+2*a^
2*arctanh(sin(d*x+c)^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/d-3*a^2*(
sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(
1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/d+a^2*
(e*csc(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A]

time = 0.18, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3963, 3957, 2952, 2720, 2644, 335, 218, 212, 209, 2651}

$$\frac{2a^2\sqrt{\sin(c+dx)}\operatorname{ArcTan}\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}}{d}+\frac{a^2\tan(c+dx)\sqrt{e\csc(c+dx)}}{d}+\frac{2a^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\tanh^{-1}\left(\sqrt{\sin(c+dx)}\right)}{d}+\frac{3a^2\sqrt{\sin(c+dx)}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\csc(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Csc[c + d*x]]*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*a^2*ArcTan[Sqrt[Sin[c + d*x]]]*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]])/
d + (2*a^2*ArcTanh[Sqrt[Sin[c + d*x]]]*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*
x]])/d + (3*a^2*Sqrt[e*Csc[c + d*x]]*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[
Sin[c + d*x]])/d + (a^2*Sqrt[e*Csc[c + d*x]]*Tan[c + d*x])/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2651

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(b*Sin[e + f*x])^(n + 1))*((a*Cos[e + f*x])^(m + 1)
/(a*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x]
)^n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3963

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{e\csc(c+dx)}\,(a+a\sec(c+dx))^2\,dx &= \left(\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\sin(c+dx)}}\,dx \\
&= \left(\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{(-a-a\cos(c+dx))^2\sec}{\sqrt{\sin(c+dx)}} \\
&= \left(\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \left(\frac{a^2}{\sqrt{\sin(c+dx)}}+\frac{2a^2\,\mathrm{se}}{\sqrt{\sin}}\right. \\
&= \left(a^2\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx+\Big(a \\
&= \frac{2a^2\sqrt{e\csc(c+dx)}\,F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{\sin(c+dx)}}{d}+\frac{a^2}{} \\
&= \frac{3a^2\sqrt{e\csc(c+dx)}\,F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{\sin(c+dx)}}{d}+\frac{a^2}{} \\
&= \frac{3a^2\sqrt{e\csc(c+dx)}\,F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right)\sqrt{\sin(c+dx)}}{d}+\frac{a^2}{} \\
&= \frac{2a^2\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}}{d}+
\end{aligned}
$$

Mathematica [A]
time = 23.10, size = 216, normalized size = 1.40

$$
\frac{16a^2\cos^4\left(\frac{1}{2}(c+dx)\right)\csc^{\frac{5}{2}}(c+dx)\sqrt{e\csc(c+dx)}\left(3\cot^2(c+dx)F\left(\mathrm{ArcSin}\left(\sqrt{\csc(c+dx)}\right)\middle|-1\right)+\sqrt{-\cot^2(c+dx)}\,\sqrt{\csc(c+dx)}\left(1-2\mathrm{ArcTan}\left(\sqrt{\csc(c+dx)}\right)\sqrt{\cos^2(c+dx)}\,\sqrt{\csc(c+dx)}+\sqrt{\cos^2(c+dx)}\,\sqrt{\csc(c+dx)}\left(-\log\left(1-\sqrt{\csc(c+dx)}\right)+\log\left(1+\sqrt{\csc(c+dx)}\right)\right)\right)\right)\sec(c+dx)\sin^4\left(\frac{1}{2}\csc^{-1}(\csc(c+dx))\right)}{d\sqrt{-\cot^2(c+dx)}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[e*Csc[c + d*x]]*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (16*a^2*Cos[(c + d*x)/2]^4*Csc[c + d*x]^(5/2)*Sqrt[e*Csc[c + d*x]]*(3*Cot[c
 + d*x]^2*EllipticF[ArcSin[Sqrt[Csc[c + d*x]]], -1] + Sqrt[-Cot[c + d*x]^2]
*Sqrt[Csc[c + d*x]]*(1 - 2*ArcTan[Sqrt[Csc[c + d*x]]]*Sqrt[Cos[c + d*x]^2]*
Sqrt[Csc[c + d*x]] + Sqrt[Cos[c + d*x]^2]*Sqrt[Csc[c + d*x]]*(-Log[1 - Sqrt
[Csc[c + d*x]]] + Log[1 + Sqrt[Csc[c + d*x]]])))*Sec[c + d*x]*Sin[ArcCsc[Cs
c[c + d*x]]/2]^4)/(d*Sqrt[-Cot[c + d*x]^2])
```

Maple [C] Result contains complex when optimal does not.
time = 0.24, size = 744, normalized size = 4.83

method	result
default	$\frac{a^2(-1+\cos(dx+c))\left(i\cos(dx+c)\sin(dx+c)\,\mathrm{EllipticF}\left(\sqrt{\frac{i\cos(dx+c)+\sin(dx+c)-i}{\sin(dx+c)}},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{i\cos(dx+c)+\sin(dx+c)-i}{\sin(dx+c)}}\,\sqrt{-\frac{i}{}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(e*csc(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a^2/d*(-1+cos(d*x+c))*(I*cos(d*x+c)*sin(d*x+c)*((I*cos(d*x+c)+sin(d*x+c
)-I)/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c
)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)+sin(d*x+c)-I)/si
n(d*x+c))^(1/2),1/2*2^(1/2))-2*I*cos(d*x+c)*sin(d*x+c)*((I*cos(d*x+c)+sin(d
*x+c)-I)/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d
*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-
I)/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-2*I*cos(d*x+c)*sin(d*x+c)*((I*c
os(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(
1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x
+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-2*cos(d*x+c)*sin
(d*x+c)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2+1/2*I
,1/2*2^(1/2))*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*
x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)+2*c
os(d*x+c)*sin(d*x+c)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1
/2),1/2-1/2*I,1/2*2^(1/2))*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(
-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+
c))^(1/2)+2^(1/2)*cos(d*x+c)-2^(1/2))*(1+cos(d*x+c))^2*(e/sin(d*x+c))^(1/2)
/cos(d*x+c)/sin(d*x+c)^3*2^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*csc(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(1/2)*integrate((a*sec(d*x + c) + a)^2*sqrt(csc(d*x + c)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*csc(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int \sqrt{e\csc(c+dx)}\,dx + \int 2\sqrt{e\csc(c+dx)}\sec(c+dx)\,dx + \int \sqrt{e\csc(c+dx)}\sec^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(e*csc(d*x+c))**(1/2),x)
[Out] a**2*(Integral(sqrt(e*csc(c + d*x)), x) + Integral(2*sqrt(e*csc(c + d*x))*s
ec(c + d*x), x) + Integral(sqrt(e*csc(c + d*x))*sec(c + d*x)**2, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*csc(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*csc(d*x + c))*(a*sec(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos(c+d\,x)}\right)^2\sqrt{\frac{e}{\sin(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2*(e/sin(c + d*x))^(1/2),x)
[Out] int((a + a/cos(c + d*x))^2*(e/sin(c + d*x))^(1/2), x)
```

3.290 $\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\csc(c+dx)}}\,dx$

Optimal. Leaf size=153

$$-\frac{2a^2\mathrm{ArcTan}\left(\sqrt{\sin(c+dx)}\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}+\frac{2a^2\tanh^{-1}\left(\sqrt{\sin(c+dx)}\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}+\frac{a^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}+\frac{a^2}{d\sqrt{}}$$

```
[Out] -2*a^2*arctan(sin(d*x+c)^(1/2))/d/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)+2*a
^2*arctanh(sin(d*x+c)^(1/2))/d/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)-a^2*(s
in(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1
/2*c+1/4*Pi+1/2*d*x),2^(1/2))/d/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)+a^2*t
an(d*x+c)/d/(e*csc(d*x+c))^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3963, 3957, 2952, 2719, 2644, 335, 304, 209, 212, 2651}

$$-\frac{2a^2\mathrm{ArcTan}\left(\sqrt{\sin(c+dx)}\right)}{d\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}+\frac{a^2\tan(c+dx)}{d\sqrt{e\csc(c+dx)}}+\frac{2a^2\tanh^{-1}\left(\sqrt{\sin(c+dx)}\right)}{d\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}+\frac{a^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{d\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/Sqrt[e*Csc[c + d*x]],x]
```

```
[Out] (-2*a^2*ArcTan[Sqrt[Sin[c + d*x]]])/(d*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*
x]]) + (2*a^2*ArcTanh[Sqrt[Sin[c + d*x]]])/(d*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin
[c + d*x]]) + (a^2*EllipticE[(c - Pi/2 + d*x)/2, 2])/(d*Sqrt[e*Csc[c + d*x]
]*Sqrt[Sin[c + d*x]]) + (a^2*Tan[c + d*x])/(d*Sqrt[e*Csc[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
```

```
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2651

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(b*Sin[e + f*x])^(n + 1))*((a*Cos[e + f*x])^(m + 1)
/(a*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x]
)^n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3963

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
```

```
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\csc(c+dx)}}\,dx = \frac{\int (a+a\sec(c+dx))^2\sqrt{\sin(c+dx)}\,dx}{\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}}$$

$$= \frac{\int (-a-a\cos(c+dx))^2\sec^2(c+dx)\sqrt{\sin(c+dx)}\,dx}{\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}}$$

$$= \frac{\int \left(a^2\sqrt{\sin(c+dx)}+2a^2\sec(c+dx)\sqrt{\sin(c+dx)}+a^2\sec^2(c+dx)\sqrt{\sin(c-\cdots}\right.}{\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}}$$

$$= \frac{a^2\int\sqrt{\sin(c+dx)}\,dx}{\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}} + \frac{a^2\int\sec^2(c+dx)\sqrt{\sin(c+dx)}\,dx}{\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}} + \frac{(2a^2)\int\cdots}{\sqrt{e\cdots}}$$

$$= \frac{2a^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{d\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}} + \frac{a^2\tan(c+dx)}{d\sqrt{e\csc(c+dx)}} - \frac{a^2\int\sqrt{\sin(c+dx}\cdots}{2\sqrt{e\csc(c+dx)}\,\sqrt{\sin\cdots}}$$

$$= \frac{a^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{d\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}} + \frac{a^2\tan(c+dx)}{d\sqrt{e\csc(c+dx)}} + \frac{(4a^2)\,\mathrm{Subst}\left(\int\frac{x^2}{1-x^4}\,dx,\cdots\right.}{d\sqrt{e\csc(c+dx)}\cdots}$$

$$= \frac{a^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{d\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}} + \frac{a^2\tan(c+dx)}{d\sqrt{e\csc(c+dx)}} + \frac{(2a^2)\,\mathrm{Subst}\left(\int\frac{1}{1-x^2}\,dx,\cdots\right.}{d\sqrt{e\csc(c+dx)}\cdots}$$

$$= -\frac{2a^2\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)}{d\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}} + \frac{2a^2\tanh^{-1}\left(\sqrt{\sin(c+dx)}\right)}{d\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}} + \frac{a^2\cdots}{d\sqrt{e\,\mathrm{c}\cdots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 27.70, size = 288, normalized size = 1.88

$$\frac{\left(1+\cos\left(2\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\right)^2\cos(c+dx)\left(-1+\csc^2(c+dx)\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2\left(-\frac{-2+\csc^2(c+dx)}{d\csc^{\frac{3}{2}}(c+dx)\sqrt{1-\sin^2(c+dx)}}-\frac{-6\mathrm{ArcTan}\left(\sqrt{\csc(c+dx)}\right)+3\log\left(1-\sqrt{\csc(c+dx)}\right)-3\log\left(1+\sqrt{\csc(c+dx)}\right)+\frac{\csc^{\frac{5}{2}}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};\csc^2(c+dx)\right)\sqrt{1-\sin^2(c+dx)}}{\sqrt{1-\csc^2(c+dx)}}}{3d}\right)}{4\left(1+\cos\left(2\left(\frac{c}{2}+\frac{1}{2}\left(-c+\csc^{-1}(\csc(c+dx))\right)\right)\right)\right)^2\csc^{\frac{3}{2}}(c+dx)\sqrt{e\csc(c+dx)}\,\sqrt{1-\sin^2(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/Sqrt[e*Csc[c + d*x]],x]
```

```
[Out] ((1 + Cos[2*(c/2 + (d*x)/2)])^2*Cos[c + d*x]*(-1 + Csc[c + d*x]^2)*Sec[c/2
+ (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(-((-2 + Csc[c + d*x]^2)/(d*Csc[c + d*x
```

```
]^(3/2)*Sqrt[1 - Sin[c + d*x]^2])) - (-6*ArcTan[Sqrt[Csc[c + d*x]]] + 3*Log
[1 - Sqrt[Csc[c + d*x]]] - 3*Log[1 + Sqrt[Csc[c + d*x]]] + (Csc[c + d*x]^(5
/2)*Hypergeometric2F1[1/2, 3/4, 7/4, Csc[c + d*x]^2]*Sqrt[1 - Sin[c + d*x]^
2])/Sqrt[1 - Csc[c + d*x]^2])/(3*d)))/(4*(1 + Cos[2*(c/2 + (-c + ArcCsc[Csc
[c + d*x]])/2)])^2*Csc[c + d*x]^(3/2)*Sqrt[e*Csc[c + d*x]]*Sqrt[1 - Sin[c +
 d*x]^2])
```

Maple [C] Result contains complex when optimal does not.
time = 0.23, size = 1605, normalized size = 10.49

method	result	size
default	Expression too large to display	1605

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a^2/d*(2*I*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+
c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/
sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)^2*(-I*(-1+cos(d*x+c))/s
in(d*x+c))^(1/2)-2*I*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-(I*co
s(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+
c)-I)/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)^2*(-I*(-1+cos(d*x
+c))/sin(d*x+c))^(1/2)+2*I*cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)
*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-
I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1
/2),1/2+1/2*I,1/2*2^(1/2))-2*I*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/
2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c
)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)*(-I*(-1
+cos(d*x+c))/sin(d*x+c))^(1/2)+2*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(
1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x
+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)^2*(-I
*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)+2*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c
))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos
(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)^2
*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)-2*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d
*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticE(((I*
cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)^2*(-I*(-
1+cos(d*x+c))/sin(d*x+c))^(1/2)+((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1
/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c
)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)^2*(-I*(-1+cos(d*x
+c))/sin(d*x+c))^(1/2)+2*cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(
(I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)
/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2
),1/2+1/2*I,1/2*2^(1/2))+2*cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)
*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-
```

```
I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1
/2),1/2-1/2*I,1/2*2^(1/2))-2*cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/
2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c
)-I)/sin(d*x+c))^(1/2)*EllipticE(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(
1/2),1/2*2^(1/2))+cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(
d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*
x+c))^(1/2)*EllipticF(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^
(1/2))-2*cos(d*x+c)^2*2^(1/2)+2^(1/2)*cos(d*x+c)+2^(1/2))/sin(d*x+c)/cos(d*
x+c)/(e/sin(d*x+c))^(1/2)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-1/2)*integrate((a*sec(d*x + c) + a)^2/sqrt(csc(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.04, size = 272, normalized size = 1.78

$$\frac{\left(\sqrt{2i}\,a^2\cos(dx+c)\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)+i\,\sin(dx+c)))+\sqrt{-2i}\,a^2\cos(dx+c)\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)-i\,\sin(dx+c)))+2\,a^2\arctan\left(\frac{76\cos(dx+c)^2+\frac{425\left(\cos(dx+c)^2+\sin(dx+c)-1\right)}{\sqrt{\sin(dx+c)}}-152\sin(dx+c)-152}{2\left(16\cos(dx+c)^2+393\sin(dx+c)-32\right)}\right)\cos(dx+c)+a^2\cos(dx+c)\log\left(\frac{\cos(dx+c)^2+\frac{4\left(\cos(dx+c)^2-\sin(dx+c)-1\right)}{\sqrt{\sin(dx+c)}}-6\sin(dx+c)-2}{\cos(dx+c)^2+2\sin(dx+c)-2}\right)-\frac{2\left(a^2\cos(dx+c)^2-a^2\right)}{\sqrt{\sin(dx+c)}}\right)e^{\left(-\frac{1}{2}\right)}}{2\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2*I)*a^2*cos(d*x + c)*weierstrassZeta(4, 0, weierstrassPInverse(4
, 0, cos(d*x + c) + I*sin(d*x + c))) + sqrt(-2*I)*a^2*cos(d*x + c)*weierstr
assZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2
*a^2*arctan(1/2*(76*cos(d*x + c)^2 + 425*(cos(d*x + c)^2 + sin(d*x + c) - 1
)/sqrt(sin(d*x + c)) - 152*sin(d*x + c) - 152)/(16*cos(d*x + c)^2 + 393*sin
(d*x + c) - 32))*cos(d*x + c) + a^2*cos(d*x + c)*log((cos(d*x + c)^2 + 4*(c
os(d*x + c)^2 - sin(d*x + c) - 1)/sqrt(sin(d*x + c)) - 6*sin(d*x + c) - 2)/
(cos(d*x + c)^2 + 2*sin(d*x + c) - 2)) - 2*(a^2*cos(d*x + c)^2 - a^2)/sqrt(
sin(d*x + c)))*e^(-1/2)/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int \frac{1}{\sqrt{e\csc\left(c+dx\right)}}\,dx+\int \frac{2\sec\left(c+dx\right)}{\sqrt{e\csc\left(c+dx\right)}}\,dx+\int \frac{\sec^{2}\left(c+dx\right)}{\sqrt{e\csc\left(c+dx\right)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2/(e*csc(d*x+c))**(1/2),x)
```

```
[Out] a**2*(Integral(1/sqrt(e*csc(c + d*x)), x) + Integral(2*sec(c + d*x)/sqrt(e*
csc(c + d*x)), x) + Integral(sec(c + d*x)**2/sqrt(e*csc(c + d*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/sqrt(e*csc(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^2}{\sqrt{\frac{e}{\sin\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/(e/sin(c + d*x))^(1/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^2/(e/sin(c + d*x))^(1/2), x)
```

3.291 $\int \frac{(a+a\sec(c+dx))^2}{(e\csc(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=222

$$-\frac{4a^2}{de\sqrt{e\csc(c+dx)}}-\frac{2a^2\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}}+\frac{a^2\sec(c+dx)}{de\sqrt{e\csc(c+dx)}}+\frac{2a^2\operatorname{ArcTan}\left(\sqrt{\sin(c+dx)}\right)}{de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}+\frac{2a^2\operatorname{ta}}{de\sqrt{e\operatorname{c}}}$$

```
[Out] -4*a^2/d/e/(e*csc(d*x+c))^(1/2)-2/3*a^2*cos(d*x+c)/d/e/(e*csc(d*x+c))^(1/2)
+a^2*sec(d*x+c)/d/e/(e*csc(d*x+c))^(1/2)+2*a^2*arctan(sin(d*x+c)^(1/2))/d/e
/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)+2*a^2*arctanh(sin(d*x+c)^(1/2))/d/e/
(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)+1/3*a^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)
^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2
))/d/e/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3963, 3957, 2952, 2715, 2720, 2644, 327, 335, 218, 212, 209, 2646}

$$\frac{2a^2\operatorname{ArcTan}\left(\sqrt{\sin(c+dx)}\right)}{de\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}-\frac{4a^2}{de\sqrt{e\csc(c+dx)}}-\frac{2a^2\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}}+\frac{a^2\sec(c+dx)}{de\sqrt{e\csc(c+dx)}}+\frac{2a^2\tanh^{-1}\left(\sqrt{\sin(c+dx)}\right)}{de\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}-\frac{a^2F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{3de\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/(e*Csc[c + d*x])^(3/2),x]
```

```
[Out] (-4*a^2)/(d*e*Sqrt[e*Csc[c + d*x]]) - (2*a^2*Cos[c + d*x])/(3*d*e*Sqrt[e*Cs
c[c + d*x]]) + (a^2*Sec[c + d*x])/(d*e*Sqrt[e*Csc[c + d*x]]) + (2*a^2*ArcTa
n[Sqrt[Sin[c + d*x]]])/(d*e*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]]) + (2*a
^2*ArcTanh[Sqrt[Sin[c + d*x]]])/(d*e*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]
]) - (a^2*EllipticF[(c - Pi/2 + d*x)/2, 2])/(3*d*e*Sqrt[e*Csc[c + d*x]]*Sqr
t[Sin[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2646

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Sin[e + f*
x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3963

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2}{(e\csc(c+dx))^{3/2}}\,dx = \frac{\int (a+a\sec(c+dx))^2\sin^{\frac{3}{2}}(c+dx)\,dx}{e\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}}$$

$$= \frac{\int (-a-a\cos(c+dx))^2\sec^2(c+dx)\sin^{\frac{3}{2}}(c+dx)\,dx}{e\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}}$$

$$= \frac{\int \left(a^2\sin^{\frac{3}{2}}(c+dx)+2a^2\sec(c+dx)\sin^{\frac{3}{2}}(c+dx)+a^2\sec^2(c+dx)\sin^{\frac{3}{2}}(c+d\right.}{e\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}}$$

$$= \frac{a^2\int \sin^{\frac{3}{2}}(c+dx)\,dx}{e\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}} + \frac{a^2\int \sec^2(c+dx)\sin^{\frac{3}{2}}(c+dx)\,dx}{e\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}} + \frac{(2a^2)}{e\sqrt{e}}$$

$$= -\frac{2a^2\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}} + \frac{a^2\sec(c+dx)}{de\sqrt{e\csc(c+dx)}} + \frac{a^2\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{3e\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx}}$$

$$= -\frac{4a^2}{de\sqrt{e\csc(c+dx)}} - \frac{2a^2\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}} + \frac{a^2\sec(c+dx)}{de\sqrt{e\csc(c+dx)}} - \frac{a^2 F}{3de\sqrt{e\,c}}$$

$$= -\frac{4a^2}{de\sqrt{e\csc(c+dx)}} - \frac{2a^2\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}} + \frac{a^2\sec(c+dx)}{de\sqrt{e\csc(c+dx)}} - \frac{a^2 F}{3de\sqrt{e\,c}}$$

$$= -\frac{4a^2}{de\sqrt{e\csc(c+dx)}} - \frac{2a^2\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}} + \frac{a^2\sec(c+dx)}{de\sqrt{e\csc(c+dx)}} - \frac{a^2 F}{3de\sqrt{e\,c}}$$

$$= -\frac{4a^2}{de\sqrt{e\csc(c+dx)}} - \frac{2a^2\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}} + \frac{a^2\sec(c+dx)}{de\sqrt{e\csc(c+dx)}} + \frac{2a^2\,\mathrm{ta}}{de\sqrt{e\csc}}$$

<u>Mathematica</u> [A]
time = 30.01, size = 302, normalized size = 1.36

$$\frac{\left(1+\cos\left(2\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\right)^2\cos(c+dx)\left(-1+\csc^2(c+dx)\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2\left(-\frac{6\mathrm{ArcTan}\left(\sqrt{\csc(c+dx)}\right)+3\log\left(1-\sqrt{\csc(c+dx)}\right)-3\log\left(1+\sqrt{\csc(c+dx)}\right)+\frac{\csc(c+dx)F\left(\mathrm{ArcSin}\left(\sqrt{\csc(c+dx)}\right)\middle|-1\right)\sqrt{1-\sin^2(c+dx)}}{\sqrt{1-\csc^2(c+dx)}}}{3d}-\frac{-2+\csc^2(c+dx)\left(-1+12\sqrt{1-\sin^2(c+dx)}\right)}{3d\csc^{\frac{5}{2}}(c+dx)\sqrt{1-\sin^2(c+dx)}}\right)}{4\left(1+\cos\left(2\left(\frac{c}{2}+\frac{1}{2}\left(-c+\csc^{-1}(\csc(c+dx))\right)\right)\right)\right)^2\sqrt{\csc(c+dx)}\,(e\csc(c+dx))^{3/2}\sqrt{1-\sin^2(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/(e*Csc[c + d*x])^(3/2),x]
```

```
[Out] ((1 + Cos[2*(c/2 + (d*x)/2)])^2*Cos[c + d*x]*(-1 + Csc[c + d*x]^2)*Sec[c/2
+ (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(-1/3*(6*ArcTan[Sqrt[Csc[c + d*x]]] + 3
*Log[1 - Sqrt[Csc[c + d*x]]] - 3*Log[1 + Sqrt[Csc[c + d*x]]] + (Csc[c + d*x
]*EllipticF[ArcSin[Sqrt[Csc[c + d*x]]], -1]*Sqrt[1 - Sin[c + d*x]^2])/Sqrt[
1 - Csc[c + d*x]^2])/d - (-2 + Csc[c + d*x]^2*(-1 + 12*Sqrt[1 - Sin[c + d*x
```

```
]^2]))/(3*d*Csc[c + d*x]^(5/2)*Sqrt[1 - Sin[c + d*x]^2])))/(4*(1 + Cos[2*(c
/2 + (-c + ArcCsc[Csc[c + d*x]])/2)])^2*Sqrt[Csc[c + d*x]]*(e*Csc[c + d*x])
^(3/2)*Sqrt[1 - Sin[c + d*x]^2])
```

Maple [C] Result contains complex when optimal does not.
time = 0.19, size = 763, normalized size = 3.44

method	result
default	$-\frac{a^2\left(6i\cos(dx+c)\sin(dx+c)\,\mathrm{EllipticPi}\left(\sqrt{\frac{i\cos(dx+c)+\sin(dx+c)-i}{\sin(dx+c)}},\frac{1}{2}+\frac{i}{2},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{i\cos(dx+c)+\sin(dx+c)-i}{\sin(dx+c)}}\sqrt{-\frac{i(-1+c}{\sin(}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*a^2/d*(6*I*cos(d*x+c)*sin(d*x+c)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-
I)/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((I*cos(d*x+c)+sin(d*x+c)-I)/si
n(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d
*x+c)-I)/sin(d*x+c))^(1/2)+6*I*cos(d*x+c)*sin(d*x+c)*EllipticPi(((I*cos(d*x
+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((I*cos(d*x+c)+s
in(d*x+c)-I)/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(I*c
os(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)-13*I*cos(d*x+c)*sin(d*x+c)*((I*co
s(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1
/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c
)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))+6*cos(d*x+c)*sin(d*x+c)*Elli
pticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2)
)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/sin(d*
x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)-6*cos(d*x+c)*si
n(d*x+c)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2-1/2*
I,1/2*2^(1/2))*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-I*(-1+cos(d
*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)+2*
cos(d*x+c)^3*2^(1/2)+10*cos(d*x+c)^2*2^(1/2)-15*2^(1/2)*cos(d*x+c)+3*2^(1/2
))/(-1+cos(d*x+c))/cos(d*x+c)/(e/sin(d*x+c))^(3/2)/sin(d*x+c)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] e^(-3/2)*integrate((a*sec(d*x + c) + a)^2/csc(d*x + c)^(3/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int \frac{1}{(e\csc(c+dx))^{\frac{3}{2}}}\,dx+\int \frac{2\sec(c+dx)}{(e\csc(c+dx))^{\frac{3}{2}}}\,dx+\int \frac{\sec^2(c+dx)}{(e\csc(c+dx))^{\frac{3}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2/(e*csc(d*x+c))**(3/2),x)
```

```
[Out] a**2*(Integral((e*csc(c + d*x))**(-3/2), x) + Integral(2*sec(c + d*x)/(e*cs
c(c + d*x))**(3/2), x) + Integral(sec(c + d*x)**2/(e*csc(c + d*x))**(3/2),
x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/(e*csc(d*x + c))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}{\left(\frac{e}{\sin(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/(e/sin(c + d*x))^(3/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^2/(e/sin(c + d*x))^(3/2), x)
```

3.292 $\int \frac{(a+a\sec(c+dx))^2}{(e\csc(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=236

$$-\frac{2a^2\text{ArcTan}\left(\sqrt{\sin(c+dx)}\right)}{de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}+\frac{2a^2\tanh^{-1}\left(\sqrt{\sin(c+dx)}\right)}{de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}-\frac{9a^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{5de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

```
[Out] -4/3*a^2*sin(d*x+c)/d/e^2/(e*csc(d*x+c))^(1/2)-2/5*a^2*cos(d*x+c)*sin(d*x+c
)/d/e^2/(e*csc(d*x+c))^(1/2)-2*a^2*arctan(sin(d*x+c)^(1/2))/d/e^2/(e*csc(d*
x+c))^(1/2)/sin(d*x+c)^(1/2)+2*a^2*arctanh(sin(d*x+c)^(1/2))/d/e^2/(e*csc(d
*x+c))^(1/2)/sin(d*x+c)^(1/2)+9/5*a^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/s
in(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))/d/e^2
/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)+a^2*tan(d*x+c)/d/e^2/(e*csc(d*x+c))^
(1/2)
```

Rubi [A]

time = 0.22, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {3963, 3957, 2952, 2715, 2719, 2644, 327, 335, 304, 209, 212, 2646}

$$-\frac{2a^2\text{ArcTan}\left(\sqrt{\sin(c+dx)}\right)}{de^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}-\frac{4a^2\sin(c+dx)}{3de^2\sqrt{e\csc(c+dx)}}+\frac{a^2\tan(c+dx)}{de^2\sqrt{e\csc(c+dx)}}-\frac{2a^2\sin(c+dx)\cos(c+dx)}{5de^2\sqrt{e\csc(c+dx)}}+\frac{2a^2\tanh^{-1}\left(\sqrt{\sin(c+dx)}\right)}{de^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}-\frac{9a^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{5de^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/(e*Csc[c + d*x])^(5/2),x]
```

```
[Out] (-2*a^2*ArcTan[Sqrt[Sin[c + d*x]]])/(d*e^2*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c
+ d*x]]) + (2*a^2*ArcTanh[Sqrt[Sin[c + d*x]]])/(d*e^2*Sqrt[e*Csc[c + d*x]]*
Sqrt[Sin[c + d*x]]) - (9*a^2*EllipticE[(c - Pi/2 + d*x)/2, 2])/(5*d*e^2*Sqr
t[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]]) - (4*a^2*Sin[c + d*x])/(3*d*e^2*Sqrt[
e*Csc[c + d*x]]) - (2*a^2*Cos[c + d*x]*Sin[c + d*x])/(5*d*e^2*Sqrt[e*Csc[c
+ d*x]]) + (a^2*Tan[c + d*x])/(d*e^2*Sqrt[e*Csc[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2646

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Sin[e + f*
x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3963

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2}{(e\csc(c+dx))^{5/2}}\,dx = \frac{\int (a+a\sec(c+dx))^2\sin^{\frac{5}{2}}(c+dx)\,dx}{e^2\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}}$$
$$= \frac{\int (-a-a\cos(c+dx))^2\sec^2(c+dx)\sin^{\frac{5}{2}}(c+dx)\,dx}{e^2\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}}$$
$$= \frac{\int \left(a^2\sin^{\frac{5}{2}}(c+dx)+2a^2\sec(c+dx)\sin^{\frac{5}{2}}(c+dx)+a^2\sec^2(c+dx)\sin^{\frac{5}{2}}(c+d\right.}{e^2\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}}$$
$$= \frac{a^2\int \sin^{\frac{5}{2}}(c+dx)\,dx}{e^2\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}} + \frac{a^2\int \sec^2(c+dx)\sin^{\frac{5}{2}}(c+dx)\,dx}{e^2\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}} + \frac{(2a^2)}{e^2\sqrt{}}$$
$$= -\frac{2a^2\cos(c+dx)\sin(c+dx)}{5de^2\sqrt{e\csc(c+dx)}} + \frac{a^2\tan(c+dx)}{de^2\sqrt{e\csc(c+dx)}} + \frac{(3a^2)\int \sqrt{\sin(c+}}{5e^2\sqrt{e\csc(c+dx)}\,\sqrt{s}}$$
$$= -\frac{9a^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{5de^2\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}} - \frac{4a^2\sin(c+dx)}{3de^2\sqrt{e\csc(c+dx)}} - \frac{2a^2\cos(c+d}{5de^2\sqrt{e\,cs}}$$
$$= -\frac{9a^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{5de^2\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}} - \frac{4a^2\sin(c+dx)}{3de^2\sqrt{e\csc(c+dx)}} - \frac{2a^2\cos(c+d}{5de^2\sqrt{e\,cs}}$$
$$= -\frac{9a^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{5de^2\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}} - \frac{4a^2\sin(c+dx)}{3de^2\sqrt{e\csc(c+dx)}} - \frac{2a^2\cos(c+d}{5de^2\sqrt{e\,cs}}$$
$$= -\frac{2a^2\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)}{de^2\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}} + \frac{2a^2\tanh^{-1}\left(\sqrt{\sin(c+dx)}\right)}{de^2\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}} - \frac{}{5}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 31.63, size = 322, normalized size = 1.36

$$\frac{\left(1+\cos\left(2\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\right)^2\cos(c+dx)\sqrt{\csc(c+dx)}\left(-1+\csc^2(c+dx)\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a+a\sec(c+dx))^2\left(-\frac{-10\operatorname{ArcTan}\left(\sqrt{\csc(c+dx)}\right)+5\log\left(1-\sqrt{\csc(c+dx)}\right)-5\log\left(1+\sqrt{\csc(c+dx)}\right)-\frac{3\csc^{\frac{5}{2}}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};\csc^2(c+dx)\right)\sqrt{1-\sin^2(c+dx)}}{\sqrt{1-\csc^2(c+dx)}}}{5d}-\frac{-6-27\csc^4(c+dx)+\csc^2(c+dx)\left(18+20\sqrt{1-\sin^2(c+dx)}\right)}{15d\csc^{\frac{7}{2}}(c+dx)\sqrt{1-\sin^2(c+dx)}}\right)}{4\left(1+\cos\left(2\left(\frac{c}{2}+\frac{1}{2}\left(-c+\csc^{-1}(\csc(c+dx))\right)\right)\right)\right)^2(e\csc(c+dx))^{5/2}\sqrt{1-\sin^2(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/(e*Csc[c + d*x])^(5/2),x]
```

```
[Out] ((1 + Cos[2*(c/2 + (d*x)/2)])^2*Cos[c + d*x]*Sqrt[Csc[c + d*x]]*(-1 + Csc[c
 + d*x]^2)*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(-1/5*(-10*ArcTan[Sq
rt[Csc[c + d*x]]] + 5*Log[1 - Sqrt[Csc[c + d*x]]] - 5*Log[1 + Sqrt[Csc[c +
d*x]]] - (3*Csc[c + d*x]^(5/2)*Hypergeometric2F1[1/2, 3/4, 7/4, Csc[c + d*x
```

```
]^2]*Sqrt[1 - Sin[c + d*x]^2])/Sqrt[1 - Csc[c + d*x]^2])/d - (-6 - 27*Csc[c
 + d*x]^4 + Csc[c + d*x]^2*(18 + 20*Sqrt[1 - Sin[c + d*x]^2]))/(15*d*Csc[c
+ d*x]^(7/2)*Sqrt[1 - Sin[c + d*x]^2])))/(4*(1 + Cos[2*(c/2 + (-c + ArcCsc[
Csc[c + d*x]])/2)])^2*(e*Csc[c + d*x])^(5/2)*Sqrt[1 - Sin[c + d*x]^2])
```

Maple [C] Result contains complex when optimal does not.
time = 0.20, size = 1636, normalized size = 6.93

method	result	size
default	Expression too large to display	1636

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/30*a^2/d*(30*I*cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(d
*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x
+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2+1/
2*I,1/2*2^(1/2))+30*I*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-(I*c
os(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x
+c)-I)/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)^2*(-I*(-1+cos(d*
x+c))/sin(d*x+c))^(1/2)+30*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(
-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+si
n(d*x+c)-I)/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(d*x+c)^2*(-I*(-1+c
os(d*x+c))/sin(d*x+c))^(1/2)+30*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1
/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+
c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)^2*(-I*
(-1+cos(d*x+c))/sin(d*x+c))^(1/2)+54*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c
))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticE(((I*cos(
d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)^2*(-I*(-1+co
s(d*x+c))/sin(d*x+c))^(1/2)-27*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/
2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)
+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)^2*(-I*(-1+cos(d*x+
c))/sin(d*x+c))^(1/2)-30*I*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(
-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+si
n(d*x+c)-I)/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)*(-I*(-1+cos
(d*x+c))/sin(d*x+c))^(1/2)-30*I*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1
/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+
c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(d*x+c)^2*(-I*
(-1+cos(d*x+c))/sin(d*x+c))^(1/2)+6*cos(d*x+c)^4*2^(1/2)+30*cos(d*x+c)*(-I*
(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^
(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*
x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+30*cos(d*x+c)*(
-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c
))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos
(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+54*cos(d*x+c
```

```
)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*
x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticE(((I*c
os(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))-27*cos(d*x+c)*(-I*(-
1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1
/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c
)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))+20*cos(d*x+c)^3*2^(1/2)+6*co
s(d*x+c)^2*2^(1/2)-47*2^(1/2)*cos(d*x+c)+15*2^(1/2))/cos(d*x+c)/(e/sin(d*x+
c))^(5/2)/sin(d*x+c)^3*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] e^(-5/2)*integrate((a*sec(d*x + c) + a)^2/csc(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.21, size = 313, normalized size = 1.33

$$-\frac{\left(27\,\sqrt{2i}\,a^2\cos\left(dx+c\right)\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))+27\,\sqrt{-2i}\,a^2\cos\left(dx+c\right)\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))-30\,a^2\arctan\left(\frac{76\,\cos(dx+c)^2+\frac{425\,\left(\cos(dx+c)^2+\sin(dx+c)-1\right)}{\sqrt{\sin\left(dx+c\right)}}-152\,\sin(dx+c)-152}{2\,\left(16\,\cos(dx+c)^2+393\,\sin(dx+c)-32\right)}\right)\cos\left(dx+c\right)-15\,a^2\cos\left(dx+c\right)\log\left(\frac{\cos(dx+c)^2+\frac{4\,\left(\cos(dx+c)^2-\sin(dx+c)-1\right)}{\sqrt{\sin\left(dx+c\right)}}-6\,\sin(dx+c)-2}{\cos(dx+c)^2+2\,\sin(dx+c)-2}\right)-\frac{2\,\left(6\,a^2\cos(dx+c)^4+20\,a^2\cos(dx+c)^3-21\,a^2\cos(dx+c)^2-20\,a^2\cos(dx+c)+15\,a^2\right)}{\sqrt{\sin\left(dx+c\right)}}\right)e^{\left(-\frac{5}{2}\right)}}{30\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/30*(27*sqrt(2*I)*a^2*cos(d*x + c)*weierstrassZeta(4, 0, weierstrassPInve
rse(4, 0, cos(d*x + c) + I*sin(d*x + c))) + 27*sqrt(-2*I)*a^2*cos(d*x + c)*
weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x +
c))) - 30*a^2*arctan(1/2*(76*cos(d*x + c)^2 + 425*(cos(d*x + c)^2 + sin(d*x
 + c) - 1)/sqrt(sin(d*x + c)) - 152*sin(d*x + c) - 152)/(16*cos(d*x + c)^2
+ 393*sin(d*x + c) - 32))*cos(d*x + c) - 15*a^2*cos(d*x + c)*log((cos(d*x +
 c)^2 + 4*(cos(d*x + c)^2 - sin(d*x + c) - 1)/sqrt(sin(d*x + c)) - 6*sin(d*
x + c) - 2)/(cos(d*x + c)^2 + 2*sin(d*x + c) - 2)) - 2*(6*a^2*cos(d*x + c)^
4 + 20*a^2*cos(d*x + c)^3 - 21*a^2*cos(d*x + c)^2 - 20*a^2*cos(d*x + c) + 1
5*a^2)/sqrt(sin(d*x + c)))*e^(-5/2)/(d*cos(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2/(e*csc(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\text{could not integrate}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^2/(e*csc(d*x + c))^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}{\left(\frac{e}{\sin(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/(e/sin(c + d*x))^(5/2),x)
[Out] int((a + a/cos(c + d*x))^2/(e/sin(c + d*x))^(5/2), x)
```

3.293 $\int \frac{(e \csc(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=155

$$-\frac{4e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{21ad}+\frac{2e^2\cot(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{7ad}-\frac{2e^2\csc^3(c+dx)\sqrt{e\csc(c}{7ad}$$

```
[Out] -4/21*e^2*cot(d*x+c)*(e*csc(d*x+c))^(1/2)/a/d+2/7*e^2*cot(d*x+c)*csc(d*x+c)
^2*(e*csc(d*x+c))^(1/2)/a/d-2/7*e^2*csc(d*x+c)^3*(e*csc(d*x+c))^(1/2)/a/d-4
/21*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ellip
ticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/
2)/a/d
```

Rubi [A]
time = 0.16, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3963, 3957, 2918, 2644, 30, 2647, 2716, 2720}

$$-\frac{2e^2\csc^3(c+dx)\sqrt{e\csc(c+dx)}}{7ad}+\frac{2e^2\cot(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{7ad}-\frac{4e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{21ad}+\frac{4e^2\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\csc(c+dx)}}{21ad}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Csc[c + d*x])^(5/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-4*e^2*Cot[c + d*x]*Sqrt[e*Csc[c + d*x]])/(21*a*d) + (2*e^2*Cot[c + d*x]*C
sc[c + d*x]^2*Sqrt[e*Csc[c + d*x]])/(7*a*d) - (2*e^2*Csc[c + d*x]^3*Sqrt[e*
Csc[c + d*x]])/(7*a*d) + (4*e^2*Sqrt[e*Csc[c + d*x]]*EllipticF[(c - Pi/2 +
d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(21*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
```

```
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3963

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\int \frac{(e\csc(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx = \left(e^2\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{1}{(a+a\sec(c+dx))\sin^{\frac{5}{2}}(c+dx)}\,dx$$

$$= -\left(\left(e^2\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{\cos(c+dx)}{(-a-a\cos(c+dx))\sin^{\frac{5}{2}}(c+dx)}\,dx\right.$$

$$= \frac{\left(e^2\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{\cos(c+dx)}{\sin^{\frac{9}{2}}(c+dx)}\,dx}{a} - \frac{\left(e^2\sqrt{e\csc(c+dx)}\,\sqrt{\sin(\right.}{a}$$

$$= \frac{2e^2\cot(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{7ad} + \frac{\left(2e^2\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+d\right.}{7a}$$

$$= -\frac{4e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{21ad} + \frac{2e^2\cot(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{7ad}$$

$$= -\frac{4e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{21ad} + \frac{2e^2\cot(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{7ad}$$

Mathematica [A]
time = 0.68, size = 131, normalized size = 0.85

$$-\frac{\csc^2\left(\frac{1}{2}(c+dx)\right)(e\csc(c+dx))^{5/2}\sec^4\left(\frac{1}{2}(c+dx)\right)\left((2+\cos(c+dx)-2\cos(2(c+dx))-\cos(3(c+dx)))F\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)+2(4+2\cos(c+dx)+\cos(2(c+dx)))\sqrt{\sin(c+dx)}\right)\sin^{\frac{5}{2}}(c+dx)}{168ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Csc[c + d*x])^(5/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/168*(Csc[(c + d*x)/2]^2*(e*Csc[c + d*x])^(5/2)*Sec[(c + d*x)/2]^4*((2 +
Cos[c + d*x] - 2*Cos[2*(c + d*x)] - Cos[3*(c + d*x)])*EllipticF[(-2*c + Pi
- 2*d*x)/4, 2] + 2*(4 + 2*Cos[c + d*x] + Cos[2*(c + d*x)])*Sqrt[Sin[c + d*x
]])*Sin[c + d*x]^(5/2))/(a*d)
```

Maple [C] Result contains complex when optimal does not.
time = 0.19, size = 465, normalized size = 3.00

method	result
default	$-\frac{(-1+\cos(dx+c))^3\left(2i\sin(dx+c)\left(\cos^2(dx+c)\right)\sqrt{-\frac{i(-1+\cos(dx+c))}{\sin(dx+c)}}\,\sqrt{\frac{i\cos(dx+c)+\sin(dx+c)-i}{\sin(dx+c)}}\,\sqrt{\frac{-i\cos(dx+c)+\sin(dx}{\sin(dx+c)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/21/a/d*(-1+cos(d*x+c))^3*(2*I*sin(d*x+c)*cos(d*x+c)^2*(-I*(-1+cos(d*x+c)
)/sin(d*x+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos
```

```
(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)+sin(d*x+c)
-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))+4*I*cos(d*x+c)*((-I*cos(d*x+c)+sin(d*x+c
)+I)/sin(d*x+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-I*(
-1+cos(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*EllipticF(((I*cos(d*x+c)+sin(d*
x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))+2*I*((-I*cos(d*x+c)+sin(d*x+c)+I)/si
n(d*x+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-I*(-1+cos(
d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*EllipticF(((I*cos(d*x+c)+sin(d*x+c)-I)
/sin(d*x+c))^(1/2),1/2*2^(1/2))-2*cos(d*x+c)^2*2^(1/2)-2*2^(1/2)*cos(d*x+c)
-3*2^(1/2))*(1+cos(d*x+c))^2*(e/sin(d*x+c))^(5/2)/sin(d*x+c)^5*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] e^(5/2)*integrate(csc(d*x + c)^(5/2)/(a*sec(d*x + c) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.17, size = 148, normalized size = 0.95

$$-\frac{2\left(\sqrt{2i}\left(i\cos\left(dx+c\right)e^{\frac{5}{2}}+ie^{\frac{5}{2}}\right)\sin\left(dx+c\right)\mathrm{weierstrassPInverse}(4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right))+\sqrt{-2i}\left(-i\cos\left(dx+c\right)e^{\frac{5}{2}}-ie^{\frac{5}{2}}\right)\sin\left(dx+c\right)\mathrm{weierstrassPInverse}(4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right))+\frac{2\cos(dx+c)^2e^{\frac{5}{2}}+2\cos(dx+c)e^{\frac{5}{2}}+3e^{\frac{5}{2}}}{\sqrt{\sin\left(dx+c\right)}}\right)}{21\left(ad\cos\left(dx+c\right)+ad\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] -2/21*(sqrt(2*I)*(I*cos(d*x + c)*e^(5/2) + I*e^(5/2))*sin(d*x + c)*weierstr
assPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(-2*I)*(-I*cos(d*x +
 c)*e^(5/2) - I*e^(5/2))*sin(d*x + c)*weierstrassPInverse(4, 0, cos(d*x + c
) - I*sin(d*x + c)) + (2*cos(d*x + c)^2*e^(5/2) + 2*cos(d*x + c)*e^(5/2) +
3*e^(5/2))/sqrt(sin(d*x + c)))/((a*d*cos(d*x + c) + a*d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))**(5/2)/(a+a*sec(d*x+c)),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 8011 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*csc(d*x + c))^(5/2)/(a*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)\,{\left(\frac{e}{\sin\left(c+d\,x\right)}\right)}^{5/2}}{a\,\left(\cos\left(c+d\,x\right)+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/sin(c + d*x))^(5/2)/(a + a/cos(c + d*x)),x)
```

```
[Out] int((cos(c + d*x)*(e/sin(c + d*x))^(5/2))/(a*(cos(c + d*x) + 1)), x)
```

3.294 $\int \frac{(e\csc(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=145

$$-\frac{4e\cos(c+dx)\sqrt{e\csc(c+dx)}}{5ad}+\frac{2e\cot(c+dx)\csc(c+dx)\sqrt{e\csc(c+dx)}}{5ad}-\frac{2e\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{5ad}$$

```
[Out] -4/5*e*cos(d*x+c)*(e*csc(d*x+c))^(1/2)/a/d+2/5*e*cot(d*x+c)*csc(d*x+c)*(e*c
sc(d*x+c))^(1/2)/a/d-2/5*e*csc(d*x+c)^2*(e*csc(d*x+c))^(1/2)/a/d+4/5*e*(sin
(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2
*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/a/d
```

Rubi [A]
time = 0.16, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3963, 3957, 2918, 2644, 30, 2647, 2716, 2719}

$$-\frac{2e\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{5ad}-\frac{4e\cos(c+dx)\sqrt{e\csc(c+dx)}}{5ad}+\frac{2e\cot(c+dx)\csc(c+dx)\sqrt{e\csc(c+dx)}}{5ad}-\frac{4e\sqrt{\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\csc(c+dx)}}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Csc[c + d*x])^(3/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-4*e*Cos[c + d*x]*Sqrt[e*Csc[c + d*x]])/(5*a*d) + (2*e*Cot[c + d*x]*Csc[c
+ d*x]*Sqrt[e*Csc[c + d*x]])/(5*a*d) - (2*e*Csc[c + d*x]^2*Sqrt[e*Csc[c + d
*x]])/(5*a*d) - (4*e*Sqrt[e*Csc[c + d*x]]*EllipticE[(c - Pi/2 + d*x)/2, 2]*
Sqrt[Sin[c + d*x]])/(5*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
```

```
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3963

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{(e\csc(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = \left(e\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{1}{(a+a\sec(c+dx))\sin^{\frac{3}{2}}(c+dx)}\,dx$$

$$= -\left(\left(e\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{\cos(c+dx)}{(-a-a\cos(c+dx))\sin^{\frac{3}{2}}(c+dx)}\,dx\right)$$

$$= \frac{\left(e\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{\cos(c+dx)}{\sin^{\frac{7}{2}}(c+dx)}\,dx}{a} - \frac{\left(e\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+\right.}{a}$$

$$= \frac{2e\cot(c+dx)\csc(c+dx)\sqrt{e\csc(c+dx)}}{5ad} + \frac{\left(2e\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)}{5a}$$

$$= -\frac{4e\cos(c+dx)\sqrt{e\csc(c+dx)}}{5ad} + \frac{2e\cot(c+dx)\csc(c+dx)\sqrt{e\csc(c+dx)}}{5ad} - $$

$$= -\frac{4e\cos(c+dx)\sqrt{e\csc(c+dx)}}{5ad} + \frac{2e\cot(c+dx)\csc(c+dx)\sqrt{e\csc(c+dx)}}{5ad} - $$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.90, size = 230, normalized size = 1.59

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)(e\csc(c+dx))^{3/2}\left(\frac{8\sqrt{2}\,e^{i(c-dx)}\sqrt{\frac{ie^{i(c+dx)}}{-1+e^{2i(c+dx)}}}\left(3-3e^{2i(c+dx)}+e^{2idx}(1+e^{2ic})\sqrt{1-e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};e^{2i(c+dx)}\right)\right)\sec(c+dx)}{d(1+e^{2ic})\csc^{\frac{3}{2}}(c+dx)}-\frac{6\left(4\cos(dx)\sec(c)+\sec^2\left(\frac{1}{2}(c+dx)\right)\right)\tan(c+dx)}{d}\right)}{15a(1+\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Csc[c + d*x])^(3/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(e*Csc[c + d*x])^(3/2)*((8*Sqrt[2]*E^(I*(c - d*x))*Sqrt
[(I*E^(I*(c + d*x)))/(-1 + E^((2*I)*(c + d*x)))]*(3 - 3*E^((2*I)*(c + d*x))
 + E^((2*I)*d*x)*(1 + E^((2*I)*c))*Sqrt[1 - E^((2*I)*(c + d*x))]*Hypergeome
tric2F1[1/2, 3/4, 7/4, E^((2*I)*(c + d*x))])*Sec[c + d*x])/(d*(1 + E^((2*I)
*c))*Csc[c + d*x]^(3/2)) - (6*(4*Cos[d*x]*Sec[c] + Sec[(c + d*x)/2]^2)*Tan[
c + d*x])/d))/(15*a*(1 + Sec[c + d*x]))
```

Maple [C] Result contains complex when optimal does not.
time = 0.20, size = 799, normalized size = 5.51

method	result
default	$-\frac{(-1+\cos(dx+c))\left(4\sqrt{\frac{i\cos(dx+c)+\sin(dx+c)-i}{\sin(dx+c)}}\,\sqrt{-\frac{i\cos(dx+c)-\sin(dx+c)-i}{\sin(dx+c)}}\,\mathrm{EllipticE}\left(\sqrt{\frac{i\cos(dx+c)+\sin(dx+c)-i}{\sin(dx+c)}},\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/a/d*(-1+cos(d*x+c))*(4*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*
(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticE(((I*cos(d*x+c)+si
n(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)^2*(-I*(-1+cos(d*x+c))
/sin(d*x+c))^(1/2)-2*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-(I*co
s(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)+sin(d*x+c
)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)^2*(-I*(-1+cos(d*x+c))/sin(d*
x+c))^(1/2)+8*cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(d*x+
c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c)
)^(1/2)*EllipticE(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2
))-4*cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(d*x+c)+sin(d*
x+c)-I)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*E
llipticF(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))+4*((I*
cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/si
n(d*x+c))^(1/2)*EllipticE(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/
2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)-2*((I*cos(d*x+c)+sin(d*x+c
)-I)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*Elli
pticF(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-I*(-1+c
os(d*x+c))/sin(d*x+c))^(1/2)-2*2^(1/2)*cos(d*x+c)-3*2^(1/2))*(e/sin(d*x+c))
^(3/2)/sin(d*x+c)*2^(1/2)
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.95, size = 126, normalized size = 0.87

$$-\frac{2\left(\sqrt{2i}\left(\cos\left(dx+c\right)e^{\frac{3}{2}}+e^{\frac{3}{2}}\right)\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)))+\sqrt{-2i}\left(\cos\left(dx+c\right)e^{\frac{3}{2}}+e^{\frac{3}{2}}\right)\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)))+\frac{2\cos(dx+c)^{2}e^{\frac{3}{2}}+2\cos(dx+c)e^{\frac{3}{2}}+e^{\frac{3}{2}}}{\sqrt{\sin\left(dx+c\right)}}\right)}{5\left(ad\cos\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -2/5*(sqrt(2*I)*(cos(d*x + c)*e^(3/2) + e^(3/2))*weierstrassZeta(4, 0, weie
rstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) + sqrt(-2*I)*(cos(d*x
```

```
 + c)*e^(3/2) + e^(3/2))*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, co
s(d*x + c) - I*sin(d*x + c))) + (2*cos(d*x + c)^2*e^(3/2) + 2*cos(d*x + c)*
e^(3/2) + e^(3/2))/sqrt(sin(d*x + c)))/(a*d*cos(d*x + c) + a*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(e\csc(c+dx))^{\frac{3}{2}}}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))**(3/2)/(a+a*sec(d*x+c)),x)
[Out] Integral((e*csc(c + d*x))**(3/2)/(sec(c + d*x) + 1), x)/a
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((e*csc(d*x + c))^(3/2)/(a*sec(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)\,\left(\frac{e}{\sin(c+d\,x)}\right)^{3/2}}{a\,\left(\cos(c+d\,x)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/sin(c + d*x))^(3/2)/(a + a/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e/sin(c + d*x))^(3/2))/(a*(cos(c + d*x) + 1)), x)
```

3.295 $\int \frac{\sqrt{e \csc(c+dx)}}{a+a \sec(c+dx)} \, dx$

Optimal. Leaf size=105

$$\frac{2 \cot(c+dx)\sqrt{e \csc(c+dx)}}{3ad}-\frac{2 \csc(c+dx)\sqrt{e \csc(c+dx)}}{3ad}+\frac{4\sqrt{e \csc(c+dx)}\,F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{3ad}$$

[Out] 2/3*cot(d*x+c)*(e*csc(d*x+c))^(1/2)/a/d-2/3*csc(d*x+c)*(e*csc(d*x+c))^(1/2)/a/d-4/3*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/a/d

Rubi [A]
time = 0.14, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3963, 3957, 2918, 2644, 30, 2647, 2720}

$$-\frac{2 \csc(c+dx)\sqrt{e \csc(c+dx)}}{3ad}+\frac{2 \cot(c+dx)\sqrt{e \csc(c+dx)}}{3ad}+\frac{4\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e \csc(c+dx)}}{3ad}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[e*Csc[c + d*x]]/(a + a*Sec[c + d*x]),x]

[Out] (2*Cot[c + d*x]*Sqrt[e*Csc[c + d*x]])/(3*a*d) - (2*Csc[c + d*x]*Sqrt[e*Csc[c + d*x]])/(3*a*d) + (4*Sqrt[e*Csc[c + d*x]]*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*a*d)

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
```

```
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3963

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{e\csc(c+dx)}}{a+a\sec(c+dx)}\,dx &= \left(\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{1}{(a+a\sec(c+dx))\sqrt{\sin(c+dx)}}\,dx \\
&= -\left(\left(\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{\cos(c+dx)}{(-a-a\cos(c+dx))\sqrt{\sin(c+dx)}}\,dx\right) \\
&= \frac{\left(\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{\cos(c+dx)}{\sin^{\frac{5}{2}}(c+dx)}\,dx}{a} - \frac{\left(\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right.}{a} \\
&= \frac{2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3ad} + \frac{\left(2\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+d}}}{3a} \\
&= \frac{2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3ad} - \frac{2\csc(c+dx)\sqrt{e\csc(c+dx)}}{3ad} + \frac{4\sqrt{e\csc(c+dx)}}{}
\end{aligned}$$

Mathematica [A]
time = 0.25, size = 60, normalized size = 0.57

$$\frac{2(e\csc(c+dx))^{3/2}\left(-1+\cos(c+dx)-2F\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)\sin^{\frac{3}{2}}(c+dx)\right)}{3ade}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Csc[c + d*x]]/(a + a*Sec[c + d*x]),x]
```

```
[Out] (2*(e*Csc[c + d*x])^(3/2)*(-1 + Cos[c + d*x] - 2*EllipticF[(-2*c + Pi - 2*d
*x)/4, 2]*Sin[c + d*x]^(3/2)))/(3*a*d*e)
```

Maple [C] Result contains complex when optimal does not.
time = 0.18, size = 326, normalized size = 3.10

method	result
default	$\frac{\sqrt{\frac{e}{\sin(dx+c)}}\,(1+\cos(dx+c))^2(-1+\cos(dx+c))^2\left(2i\cos(dx+c)\sin(dx+c)\,\text{EllipticF}\left(\sqrt{\frac{i\cos(dx+c)+\sin(dx+c)-i}{\sin(dx+c)}},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{i}{}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*csc(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/a/d*(e/sin(d*x+c))^(1/2)*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^2*(2*I*cos(d*
x+c)*sin(d*x+c)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-I*(-1+cos(
d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*E
llipticF(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))+2*I*El
lipticF(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))*((I*cos
(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/
2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*sin(d*x+c)+2^(1/2)*cos(d
*x+c)-2^(1/2))/sin(d*x+c)^5*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] e^(1/2)*integrate(sqrt(csc(d*x + c))/(a*sec(d*x + c) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.80, size = 103, normalized size = 0.98

$$-\frac{2\left(\sqrt{2i}\left(i\cos\left(dx+c\right)e^{\frac{1}{2}}+i\,e^{\frac{1}{2}}\right)\text{weierstrassPInverse}(4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right))+\sqrt{-2i}\left(-i\cos\left(dx+c\right)e^{\frac{1}{2}}-i\,e^{\frac{1}{2}}\right)\text{weierstrassPInverse}(4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right))+e^{\frac{1}{2}}\sqrt{\sin\left(dx+c\right)}\right)}{3\left(ad\cos\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -2/3*(sqrt(2*I)*(I*cos(d*x + c)*e^(1/2) + I*e^(1/2))*weierstrassPInverse(4,
 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(-2*I)*(-I*cos(d*x + c)*e^(1/2) -
I*e^(1/2))*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) + e^(1/
2)*sqrt(sin(d*x + c)))/(a*d*cos(d*x + c) + a*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{e \csc (c+dx)}}{\sec (c+dx)+1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))**(1/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(sqrt(e*csc(c + d*x))/(sec(c + d*x) + 1), x)/a
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*csc(d*x + c))/(a*sec(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos (c+d\,x)\,\sqrt{\frac{e}{\sin (c+d\,x)}}}{a\,\left(\cos (c+d\,x)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/sin(c + d*x))^(1/2)/(a + a/cos(c + d*x)),x)
```

```
[Out] int((cos(c + d*x)*(e/sin(c + d*x))^(1/2))/(a*(cos(c + d*x) + 1)), x)
```

3.296 $\int \frac{1}{\sqrt{e \csc(c+dx)}\,(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=99

$$\frac{2\cot(c+dx)}{ad\sqrt{e\csc(c+dx)}}-\frac{2\csc(c+dx)}{ad\sqrt{e\csc(c+dx)}}+\frac{4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{ad\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

```
[Out] 2*cot(d*x+c)/a/d/(e*csc(d*x+c))^(1/2)-2*csc(d*x+c)/a/d/(e*csc(d*x+c))^(1/2)
-4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(
cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))/a/d/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2
)
```

Rubi [A]
time = 0.16, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3963, 3957, 2918, 2644, 30, 2647, 2719}

$$-\frac{2\csc(c+dx)}{ad\sqrt{e\csc(c+dx)}}+\frac{2\cot(c+dx)}{ad\sqrt{e\csc(c+dx)}}+\frac{4E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{ad\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Csc[c + d*x]]*(a + a*Sec[c + d*x])),x]
```

```
[Out] (2*Cot[c + d*x])/(a*d*Sqrt[e*Csc[c + d*x]]) - (2*Csc[c + d*x])/(a*d*Sqrt[e*
Csc[c + d*x]]) + (4*EllipticE[(c - Pi/2 + d*x)/2, 2])/(a*d*Sqrt[e*Csc[c + d
*x]]*Sqrt[Sin[c + d*x]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
```

```
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3963

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\sqrt{e\csc(c+dx)}\,(a+a\sec(c+dx))}\,dx = \frac{\int \frac{\sqrt{\sin(c+dx)}}{a+a\sec(c+dx)}\,dx}{\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}}$$

$$= -\frac{\int \frac{\cos(c+dx)\sqrt{\sin(c+dx)}}{-a-a\cos(c+dx)}\,dx}{\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}}$$

$$= \frac{\int \frac{\cos(c+dx)}{\sin^{\frac{3}{2}}(c+dx)}\,dx}{a\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}} - \frac{\int \frac{\cos^2(c+dx)}{\sin^{\frac{3}{2}}(c+dx)}\,dx}{a\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+}$$

$$= \frac{2\cot(c+dx)}{ad\sqrt{e\csc(c+dx)}} + \frac{2\int \sqrt{\sin(c+dx)}\,dx}{a\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}} + \frac{\text{Subst}}{ad\sqrt{e}$$

$$= \frac{2\cot(c+dx)}{ad\sqrt{e\csc(c+dx)}} - \frac{2\csc(c+dx)}{ad\sqrt{e\csc(c+dx)}} + \frac{4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+\right.\right.}{ad\sqrt{e\csc(c+dx)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.42, size = 95, normalized size = 0.96

$$\frac{6(2i+\cot(c+dx)-\csc(c+dx))-4\sqrt{1-e^{2i(c+dx)}}\,(i+\cot(c+dx))\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};e^{2i(c+dx)}\right)}{3ad\sqrt{e\csc(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[e*Csc[c + d*x]]*(a + a*Sec[c + d*x])),x]
```

```
[Out] (6*(2*I + Cot[c + d*x] - Csc[c + d*x]) - 4*Sqrt[1 - E^((2*I)*(c + d*x))]*(I
 + Cot[c + d*x])*Hypergeometric2F1[1/2, 3/4, 7/4, E^((2*I)*(c + d*x))])/(3*
a*d*Sqrt[e*Csc[c + d*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.17, size = 536, normalized size = 5.41

method	result
default	$-\frac{\left(4\cos(dx+c)\sqrt{-\frac{i(-1+\cos(dx+c))}{\sin(dx+c)}}\sqrt{\frac{i\cos(dx+c)+\sin(dx+c)-i}{\sin(dx+c)}}\sqrt{-\frac{i\cos(dx+c)-\sin(dx+c)-i}{\sin(dx+c)}}\,\mathrm{EllipticE}\left(\sqrt{\frac{i\cos(dx\text{-}}{}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))/(e*csc(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a/d*(4*cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(d*x+c)+s
in(d*x+c)-I)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1
/2)*EllipticE(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))-2
*cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)
-I)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*Ellip
ticF(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))+4*((I*cos(
d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*
x+c))^(1/2)*EllipticE(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^
(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)-2*((I*cos(d*x+c)+sin(d*x+c)-I)
/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*Elliptic
F(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(d
*x+c))/sin(d*x+c))^(1/2)+2^(1/2)*cos(d*x+c)-2^(1/2))/(e/sin(d*x+c))^(1/2)/s
in(d*x+c)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*csc(d*x+c))^(1/2),x, algorithm="maxima")
[Out] e^(-1/2)*integrate(1/((a*sec(d*x + c) + a)*sqrt(csc(d*x + c))), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.88, size = 78, normalized size = 0.79

$$\frac{2\left(\sqrt{2i}\ \mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))+\sqrt{-2i}\ \mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))+\frac{\cos(dx+c)-1}{\sqrt{\sin\left(dx+c\right)}}\right)e^{\left(-\frac{1}{2}\right)}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*csc(d*x+c))^(1/2),x, algorithm="fricas")
[Out] 2*(sqrt(2*I)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) +
 I*sin(d*x + c))) + sqrt(-2*I)*weierstrassZeta(4, 0, weierstrassPInverse(4,
 0, cos(d*x + c) - I*sin(d*x + c))) + (cos(d*x + c) - 1)/sqrt(sin(d*x + c))
)*e^(-1/2)/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sqrt{e\csc\left(c+dx\right)}\sec\left(c+dx\right)+\sqrt{e\csc\left(c+dx\right)}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*csc(d*x+c))**(1/2),x)
[Out] Integral(1/(sqrt(e*csc(c + d*x))*sec(c + d*x) + sqrt(e*csc(c + d*x))), x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*csc(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(e*csc(d*x + c))*(a*sec(d*x + c) + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)}{a\,\sqrt{\frac{e}{\sin\left(c+d\,x\right)}}\,\left(\cos\left(c+d\,x\right)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(c + d*x))*(e/sin(c + d*x))^(1/2)),x)
[Out] int(cos(c + d*x)/(a*(e/sin(c + d*x))^(1/2)*(cos(c + d*x) + 1)), x)
```

3.297 $\int \frac{1}{(e\csc(c+dx))^{3/2}(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=106

$$\frac{2}{ade\sqrt{e\csc(c+dx)}}-\frac{2\cos(c+dx)}{3ade\sqrt{e\csc(c+dx)}}-\frac{4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{3ade\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

```
[Out] 2/a/d/e/(e*csc(d*x+c))^(1/2)-2/3*cos(d*x+c)/a/d/e/(e*csc(d*x+c))^(1/2)+4/3*
(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos
(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))/a/d/e/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3963, 3957, 2918, 2644, 30, 2649, 2720}

$$\frac{2}{ade\sqrt{e\csc(c+dx)}}-\frac{2\cos(c+dx)}{3ade\sqrt{e\csc(c+dx)}}-\frac{4F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{3ade\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Csc[c + d*x])^(3/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] 2/(a*d*e*Sqrt[e*Csc[c + d*x]]) - (2*Cos[c + d*x])/(3*a*d*e*Sqrt[e*Csc[c + d
*x]]) - (4*EllipticF[(c - Pi/2 + d*x)/2, 2])/(3*a*d*e*Sqrt[e*Csc[c + d*x]]*
Sqrt[Sin[c + d*x]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2649

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(b*Sin[e + f*x])^(n + 1)*((a*Cos[e + f*x])^(m - 1)/
(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3963

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{(e\csc(c+dx))^{3/2}(a+a\sec(c+dx))}\,dx = \frac{\int \frac{\sin^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx}{e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$
$$= -\frac{\int \frac{\cos(c+dx)\sin^{\frac{3}{2}}(c+dx)}{-a-a\cos(c+dx)}\,dx}{e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$
$$= \frac{\int \frac{\cos(c+dx)}{\sqrt{\sin(c+dx)}}\,dx}{ae\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{\int \frac{\cos^2(c+dx)}{\sqrt{\sin(c+dx)}}\,dx}{ae\sqrt{e\csc(c+dx)}\sqrt{\sin(c}$$
$$= -\frac{2\cos(c+dx)}{3ade\sqrt{e\csc(c+dx)}} - \frac{2\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{3ae\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} +$$
$$= \frac{2}{ade\sqrt{e\csc(c+dx)}} - \frac{2\cos(c+dx)}{3ade\sqrt{e\csc(c+dx)}} - \frac{4F\left(\frac{1}{2}(c\right.}{3ade\sqrt{e\csc(c+}$$

Mathematica [A]
time = 0.26, size = 70, normalized size = 0.66

$$\frac{4F\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)-2(-3+\cos(c+dx))\sqrt{\sin(c+dx)}}{3ad(e\csc(c+dx))^{3/2}\sin^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Csc[c + d*x])^(3/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (4*EllipticF[(-2*c + Pi - 2*d*x)/4, 2] - 2*(-3 + Cos[c + d*x])*Sqrt[Sin[c +
 d*x]])/(3*a*d*(e*Csc[c + d*x])^(3/2)*Sin[c + d*x]^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.17, size = 194, normalized size = 1.83

method	result
default	$-\frac{\left(-2i\,\mathrm{EllipticF}\left(\sqrt{\frac{i\cos(dx+c)+\sin(dx+c)-i}{\sin(dx+c)}},\frac{\sqrt{2}}{2}\right)\sin(dx+c)\sqrt{\frac{-i\cos(dx+c)+\sin(dx+c)+i}{\sin(dx+c)}}\,\sqrt{\frac{i\cos(dx+c)+\sin(dx+c)-}{\sin(dx+c)}}\right.}{3ad(-1+\cos(dx+c))\left(\frac{e}{\sin(dx+c)}\right)^{\frac{3}{2}}\sin(dx+c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a/d*(-2*I*sin(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(d*x
+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c
))^(1/2)*EllipticF(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/
2))+cos(d*x+c)^2*2^(1/2)-4*2^(1/2)*cos(d*x+c)+3*2^(1/2))/(-1+cos(d*x+c))/(e
/sin(d*x+c))^(3/2)/sin(d*x+c)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] e^(-3/2)*integrate(1/((a*sec(d*x + c) + a)*csc(d*x + c)^(3/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.73, size = 74, normalized size = 0.70

$$-\frac{2\left((\cos(dx+c)-3)\sqrt{\sin(dx+c)}-i\sqrt{2i}\,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{-2i}\,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)-i\sin(dx+c))\right)e^{\left(-\frac{3}{2}\right)}}{3\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -2/3*((cos(d*x + c) - 3)*sqrt(sin(d*x + c)) - I*sqrt(2*I)*weierstrassPInver
se(4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(-2*I)*weierstrassPInverse(
4, 0, cos(d*x + c) - I*sin(d*x + c)))*e^(-3/2)/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{(e\csc(c+dx))^{\frac{3}{2}}\sec(c+dx)+(e\csc(c+dx))^{\frac{3}{2}}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))**(3/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(1/((e*csc(c + d*x))**(3/2)*sec(c + d*x) + (e*csc(c + d*x))**(3/2))
, x)/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*csc(d*x + c))^(3/2)*(a*sec(d*x + c) + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)}{a\left(\frac{e}{\sin(c+d\,x)}\right)^{3/2}\,(\cos(c+d\,x)+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(c + d*x))*(e/sin(c + d*x))^(3/2)),x)
```

```
[Out] int(cos(c + d*x)/(a*(e/sin(c + d*x))^(3/2)*(cos(c + d*x) + 1)), x)
```

3.298 $\int \frac{1}{(e\csc(c+dx))^{5/2}(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=120

$$-\frac{4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{5ade^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}+\frac{2\sin(c+dx)}{3ade^2\sqrt{e\csc(c+dx)}}-\frac{2\cos(c+dx)\sin(c+dx)}{5ade^2\sqrt{e\csc(c+dx)}}$$

```
[Out] 2/3*sin(d*x+c)/a/d/e^2/(e*csc(d*x+c))^(1/2)-2/5*cos(d*x+c)*sin(d*x+c)/a/d/e
^2/(e*csc(d*x+c))^(1/2)+4/5*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1
/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))/a/d/e^2/(e*csc(
d*x+c))^(1/2)/sin(d*x+c)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3963, 3957, 2918, 2644, 30, 2649, 2719}

$$\frac{2\sin(c+dx)}{3ade^2\sqrt{e\csc(c+dx)}}-\frac{2\sin(c+dx)\cos(c+dx)}{5ade^2\sqrt{e\csc(c+dx)}}-\frac{4E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{5ade^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Csc[c + d*x])^(5/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (-4*EllipticE[(c - Pi/2 + d*x)/2, 2])/(5*a*d*e^2*Sqrt[e*Csc[c + d*x]]*Sqrt[
Sin[c + d*x]]) + (2*Sin[c + d*x])/(3*a*d*e^2*Sqrt[e*Csc[c + d*x]]) - (2*Cos
[c + d*x]*Sin[c + d*x])/(5*a*d*e^2*Sqrt[e*Csc[c + d*x]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2649

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(b*Sin[e + f*x])^(n + 1)*((a*Cos[e + f*x])^(m - 1)/
(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
```

```
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3963

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(e\csc(c+dx))^{5/2}(a+a\sec(c+dx))}\,dx &= \frac{\int \frac{\sin^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx}{e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\
&= -\frac{\int \frac{\cos(c+dx)\sin^{\frac{5}{2}}(c+dx)}{-a-a\cos(c+dx)}\,dx}{e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\
&= \frac{\int \cos(c+dx)\sqrt{\sin(c+dx)}\,dx}{ae^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{\int \cos^2(c+dx)\sqrt{\sin(c+}}{ae^2\sqrt{e\csc(c+dx)}\sqrt{\sin(}}\\
&= -\frac{2\cos(c+dx)\sin(c+dx)}{5ade^2\sqrt{e\csc(c+dx)}} - \frac{2\int\sqrt{\sin(c+dx)}\,dx}{5ae^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx}}\\
&= -\frac{4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{5ade^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{2\sin(c+dx)}{3ade^2\sqrt{e\csc(c+dx)}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.55, size = 100, normalized size = 0.83

$$\frac{8\sqrt{1-e^{2i(c+dx)}}\,(i+\cot(c+dx))\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};e^{2i(c+dx)}\right)+20\sin(c+dx)-6(4i+\sin(2(c+dx)))}{30ade^2\sqrt{e\csc(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Csc[c + d*x])^(5/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (8*Sqrt[1 - E^((2*I)*(c + d*x))]*(I + Cot[c + d*x])*Hypergeometric2F1[1/2,
3/4, 7/4, E^((2*I)*(c + d*x))] + 20*Sin[c + d*x] - 6*(4*I + Sin[2*(c + d*x)
]))/(30*a*d*e^2*Sqrt[e*Csc[c + d*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.19, size = 551, normalized size = 4.59

method	result
default	$\frac{\left(12\cos(dx+c)\sqrt{\frac{i\cos(dx+c)+\sin(dx+c)-i}{\sin(dx+c)}}\,\sqrt{\frac{-i\cos(dx+c)+\sin(dx+c)+i}{\sin(dx+c)}}\,\mathrm{EllipticE}\left(\sqrt{\frac{i\cos(dx+c)+\sin(dx+c)-i}{\sin(dx+c)}},\frac{\sqrt{2}}{2}\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15/a/d*(12*cos(d*x+c)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I
*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticE(((I*cos(d*x+c)+sin(d*
x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2
)-6*cos(d*x+c)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+
c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)+sin(d*x+c)-I)/s
in(d*x+c))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)+3*cos(d
*x+c)^3*2^(1/2)+12*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(
d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticE(((I*cos(d*x+c)+sin(d*x+c)-
I)/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)-6*(
(I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)
/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)
,1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)-5*cos(d*x+c)^2*2^(1/2)+
3*2^(1/2)*cos(d*x+c)-2^(1/2))/(e/sin(d*x+c))^(5/2)/sin(d*x+c)^3*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] e^(-5/2)*integrate(1/((a*sec(d*x + c) + a)*csc(d*x + c)^(5/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.61, size = 103, normalized size = 0.86

$$-\frac{2\left(3\sqrt{2i}\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)+i\,\sin(dx+c)))+3\sqrt{-2i}\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)-i\,\sin(dx+c)))-\frac{3\cos(dx+c)^3-5\cos(dx+c)^2-3\cos(dx+c)+5}{\sqrt{\sin(dx+c)}}\right)e^{\left(-\frac{5}{2}\right)}}{15\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -2/15*(3*sqrt(2*I)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x
+ c) + I*sin(d*x + c))) + 3*sqrt(-2*I)*weierstrassZeta(4, 0, weierstrassPIn
verse(4, 0, cos(d*x + c) - I*sin(d*x + c))) - (3*cos(d*x + c)^3 - 5*cos(d*x
 + c)^2 - 3*cos(d*x + c) + 5)/sqrt(sin(d*x + c)))*e^(-5/2)/(a*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))**(5/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*csc(d*x + c))^(5/2)*(a*sec(d*x + c) + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)}{a\,{\left(\frac{e}{\sin(c+d\,x)}\right)}^{5/2}\,\left(\cos(c+d\,x)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(c + d*x))*(e/sin(c + d*x))^(5/2)),x)
```

```
[Out] int(cos(c + d*x)/(a*(e/sin(c + d*x))^(5/2)*(cos(c + d*x) + 1)), x)
```

3.299 $\int \frac{1}{(e\csc(c+dx))^{7/2}(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=149

$$-\frac{2\cos(c+dx)}{21ade^3\sqrt{e\csc(c+dx)}}+\frac{2\cos^3(c+dx)}{7ade^3\sqrt{e\csc(c+dx)}}-\frac{4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{21ade^3\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}+\frac{2\sin^2(c+dx)}{5ade^3\sqrt{e\csc(c+dx)}}$$

```
[Out] -2/21*cos(d*x+c)/a/d/e^3/(e*csc(d*x+c))^(1/2)+2/7*cos(d*x+c)^3/a/d/e^3/(e*c
sc(d*x+c))^(1/2)+2/5*sin(d*x+c)^2/a/d/e^3/(e*csc(d*x+c))^(1/2)+4/21*(sin(1/
2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+
1/4*Pi+1/2*d*x),2^(1/2))/a/d/e^3/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3963, 3957, 2918, 2644, 30, 2648, 2649, 2720}

$$\frac{2\cos^3(c+dx)}{7ade^3\sqrt{e\csc(c+dx)}}-\frac{2\cos(c+dx)}{21ade^3\sqrt{e\csc(c+dx)}}+\frac{2\sin^2(c+dx)}{5ade^3\sqrt{e\csc(c+dx)}}-\frac{4F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{21ade^3\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Csc[c + d*x])^(7/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (-2*Cos[c + d*x])/(21*a*d*e^3*Sqrt[e*Csc[c + d*x]]) + (2*Cos[c + d*x]^3)/(7
*a*d*e^3*Sqrt[e*Csc[c + d*x]]) - (4*EllipticF[(c - Pi/2 + d*x)/2, 2])/(21*a
*d*e^3*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]]) + (2*Sin[c + d*x]^2)/(5*a*d
*e^3*Sqrt[e*Csc[c + d*x]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
```

```
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2649

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(b*Sin[e + f*x])^(n + 1)*((a*Cos[e + f*x])^(m - 1)/
(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3963

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{(e\csc(c+dx))^{7/2}(a+a\sec(c+dx))}\,dx = \frac{\int \frac{\sin^{\frac{7}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx}{e^3\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= -\frac{\int \frac{\cos(c+dx)\sin^{\frac{7}{2}}(c+dx)}{-a-a\cos(c+dx)}\,dx}{e^3\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{\int \cos(c+dx)\sin^{\frac{3}{2}}(c+dx)\,dx}{ae^3\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{\int \cos^2(c+dx)\sin^{\frac{3}{2}}(c\cdot}{ae^3\sqrt{e\csc(c+dx)}\sqrt{\mathrm{si}}}$$

$$= \frac{2\cos^3(c+dx)}{7ade^3\sqrt{e\csc(c+dx)}} - \frac{\int \frac{\cos^2(c+dx)}{\sqrt{\sin(c+dx)}}\,dx}{7ae^3\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= -\frac{2\cos(c+dx)}{21ade^3\sqrt{e\csc(c+dx)}} + \frac{2\cos^3(c+dx)}{7ade^3\sqrt{e\csc(c+dx)}} + \frac{2\sin}{5ade^3\sqrt{}}$$

$$= -\frac{2\cos(c+dx)}{21ade^3\sqrt{e\csc(c+dx)}} + \frac{2\cos^3(c+dx)}{7ade^3\sqrt{e\csc(c+dx)}} - \frac{}{21ade^3}$$

Mathematica [A]
time = 0.38, size = 91, normalized size = 0.61

$$\frac{\sqrt{e\csc(c+dx)}\left(80F\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)\sqrt{\sin(c+dx)}+126\sin(c+dx)+10\sin(2(c+dx))-42\sin(3(c+dx))+15\sin(4(c+dx))\right)}{420ade^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Csc[c + d*x])^(7/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (Sqrt[e*Csc[c + d*x]]*(80*EllipticF[(-2*c + Pi - 2*d*x)/4, 2]*Sqrt[Sin[c + 
d*x]] + 126*Sin[c + d*x] + 10*Sin[2*(c + d*x)] - 42*Sin[3*(c + d*x)] + 15*S
in[4*(c + d*x)]))/(420*a*d*e^4)
```

Maple [C] Result contains complex when optimal does not.
time = 0.17, size = 221, normalized size = 1.48

method	result
default	$\frac{\left(10i\,\mathrm{EllipticF}\left(\sqrt{\frac{i\cos(dx+c)+\sin(dx+c)-i}{\sin(dx+c)}},\frac{\sqrt{2}}{2}\right)\sin(dx+c)\sqrt{\frac{-i\cos(dx+c)+\sin(dx+c)+i}{\sin(dx+c)}}\sqrt{\frac{i\cos(dx+c)+\sin(dx+c)-i}{\sin(dx+c)}}\right.}{105ad(-1+\cos(dx+c))}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*csc(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/105/a/d*(10*I*sin(d*x+c)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(
(-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)+sin
(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(
1/2)+15*cos(d*x+c)^4*2^(1/2)-36*cos(d*x+c)^3*2^(1/2)+16*cos(d*x+c)^2*2^(1/2
)+26*2^(1/2)*cos(d*x+c)-21*2^(1/2))/(-1+cos(d*x+c))/(e/sin(d*x+c))^(7/2)/si
n(d*x+c)^3*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] e^(-7/2)*integrate(1/((a*sec(d*x + c) + a)*csc(d*x + c)^(7/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.77, size = 96, normalized size = 0.64

$$\frac{2\left(\left(15\cos\left(dx+c\right)^{3}-21\cos\left(dx+c\right)^{2}-5\cos\left(dx+c\right)+21\right)\sqrt{\sin\left(dx+c\right)}+5i\sqrt{2i}\,\mathrm{weierstrassPInverse}(4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right))-5i\sqrt{-2i}\,\mathrm{weierstrassPInverse}(4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right))\right)e^{\left(-\frac{7}{2}\right)}}{105\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 2/105*((15*cos(d*x + c)^3 - 21*cos(d*x + c)^2 - 5*cos(d*x + c) + 21)*sqrt(s
in(d*x + c)) + 5*I*sqrt(2*I)*weierstrassPInverse(4, 0, cos(d*x + c) + I*sin
(d*x + c)) - 5*I*sqrt(-2*I)*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(
d*x + c)))*e^(-7/2)/(a*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))**(7/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] integrate(1/((e*csc(d*x + c))^(7/2)*(a*sec(d*x + c) + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)}{a\,{\left(\frac{e}{\sin\left(c+d\,x\right)}\right)}^{7/2}\,\left(\cos\left(c+d\,x\right)+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(c + d*x))*(e/sin(c + d*x))^(7/2)),x)
[Out] int(cos(c + d*x)/(a*(e/sin(c + d*x))^(7/2)*(cos(c + d*x) + 1)), x)
```

3.300 $\int \frac{(e\csc(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=268

$$-\frac{4e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{231a^2d}+\frac{16e^2\cot(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{77a^2d}-\frac{2e^2\cot^3(c+dx)\csc^2(c+dx)}{11a^2d}$$

```
[Out] -4/231*e^2*cot(d*x+c)*(e*csc(d*x+c))^(1/2)/a^2/d+16/77*e^2*cot(d*x+c)*csc(d
*x+c)^2*(e*csc(d*x+c))^(1/2)/a^2/d-2/11*e^2*cot(d*x+c)^3*csc(d*x+c)^2*(e*cs
c(d*x+c))^(1/2)/a^2/d-4/7*e^2*csc(d*x+c)^3*(e*csc(d*x+c))^(1/2)/a^2/d-2/11*
e^2*cot(d*x+c)*csc(d*x+c)^4*(e*csc(d*x+c))^(1/2)/a^2/d+4/11*e^2*csc(d*x+c)^
5*(e*csc(d*x+c))^(1/2)/a^2/d-4/231*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/
sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*c
sc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/a^2/d
```

Rubi [A]
time = 0.36, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3963, 3957, 2954, 2952, 2647, 2716, 2720, 2644, 14}

$$\frac{4e^2\csc^5(c+dx)\sqrt{e\csc(c+dx)}}{11a^2d}-\frac{4e^2\csc^3(c+dx)\sqrt{e\csc(c+dx)}}{7a^2d}-\frac{2e^2\cot^3(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{11a^2d}-\frac{2e^2\cot(c+dx)\csc^4(c+dx)\sqrt{e\csc(c+dx)}}{11a^2d}+\frac{16e^2\cot(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{77a^2d}-\frac{4e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{231a^2d}+\frac{4e^2\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\csc(c+dx)}}{231a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Csc[c + d*x])^(5/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-4*e^2*Cot[c + d*x]*Sqrt[e*Csc[c + d*x]])/(231*a^2*d) + (16*e^2*Cot[c + d*
x]*Csc[c + d*x]^2*Sqrt[e*Csc[c + d*x]])/(77*a^2*d) - (2*e^2*Cot[c + d*x]^3*
Csc[c + d*x]^2*Sqrt[e*Csc[c + d*x]])/(11*a^2*d) - (4*e^2*Csc[c + d*x]^3*Sqr
t[e*Csc[c + d*x]])/(7*a^2*d) - (2*e^2*Cot[c + d*x]*Csc[c + d*x]^4*Sqrt[e*Cs
c[c + d*x]])/(11*a^2*d) + (4*e^2*Csc[c + d*x]^5*Sqrt[e*Csc[c + d*x]])/(11*a
^2*d) + (4*e^2*Sqrt[e*Csc[c + d*x]]*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[S
in[c + d*x]])/(231*a^2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3963

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
```

```
FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{(e\csc(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx = \left(e^2\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{1}{(a+a\sec(c+dx))^2\sin^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \left(e^2\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{\cos^2(c+dx)}{(-a-a\cos(c+dx))^2\sin^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{\left(e^2\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{\sin^{\frac{13}{2}}(c+dx)}\,dx}{a^4}$$

$$= \frac{\left(e^2\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \left(\frac{a^2\cos^2(c+dx)}{\sin^{\frac{13}{2}}(c+dx)} - \frac{2a^2\cos^3(c+dx)}{\sin^{\frac{13}{2}}(c+dx)} + \frac{a^2\cos^4(c+dx)}{\sin^{\frac{13}{2}}(c+dx)}\right.}{a^4}$$

$$= \frac{\left(e^2\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{\cos^2(c+dx)}{\sin^{\frac{13}{2}}(c+dx)}\,dx}{a^2} + \frac{\left(e^2\sqrt{e\csc(c+dx)}\,\sqrt{\sin\right.}{}$$

$$= -\frac{2e^2\cot^3(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{11a^2d} - \frac{2e^2\cot(c+dx)\csc^4(c+dx)\sqrt{}}{11a^2d}$$

$$= \frac{16e^2\cot(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{77a^2d} - \frac{2e^2\cot^3(c+dx)\csc^2(c+dx)\sqrt{}}{11a^2d}$$

$$= -\frac{4e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{231a^2d} + \frac{16e^2\cot(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+d}}{77a^2d}$$

$$= -\frac{4e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{231a^2d} + \frac{16e^2\cot(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+d}}{77a^2d}$$

Mathematica [A]
time = 0.73, size = 115, normalized size = 0.43

$$-\frac{e^3\csc^2\left(\frac{1}{2}(c+dx)\right)\sec^6\left(\frac{1}{2}(c+dx)\right)\left(52+97\cos(c+dx)+4\cos(2(c+dx))+\cos(3(c+dx))+\csc^4\left(\frac{1}{2}(c+dx)\right)F\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)\sin^{\frac{11}{2}}(c+dx)\right)}{3696a^2d\sqrt{e\csc(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Csc[c + d*x])^(5/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/3696*(e^3*Csc[(c + d*x)/2]^2*Sec[(c + d*x)/2]^6*(52 + 97*Cos[c + d*x] +
4*Cos[2*(c + d*x)] + Cos[3*(c + d*x)] + Csc[(c + d*x)/2]^4*EllipticF[(-2*c
+ Pi - 2*d*x)/4, 2]*Sin[c + d*x]^(11/2)))/(a^2*d*Sqrt[e*Csc[c + d*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.21, size = 621, normalized size = 2.32

method	result
default	$\frac{(-1+\cos(dx+c))^4 \left(2i(\cos^3(dx+c)) \sin(dx+c) \sqrt{-\frac{i(-1+\cos(dx+c))}{\sin(dx+c)}} \sqrt{\frac{i\cos(dx+c)+\sin(dx+c)-i}{\sin(dx+c)}} \sqrt{-\frac{i\cos(dx+c)-\sin(dx+}{\sin(dx+c)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/231/a^2/d*(-1+cos(d*x+c))^4*(2*I*cos(d*x+c)^3*sin(d*x+c)*(-I*(-1+cos(d*x+
c))/sin(d*x+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-(I*c
os(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)+sin(d*x+
c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))+6*I*cos(d*x+c)^2*sin(d*x+c)*(-I*(-1+co
s(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*
(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)+si
n(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))+6*I*cos(d*x+c)*sin(d*x+c)*((I*co
s(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1
/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c
)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))+2*I*EllipticF(((I*cos(d*x+c)
+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))*((I*cos(d*x+c)+sin(d*x+c)-I)/
sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin
(d*x+c)-I)/sin(d*x+c))^(1/2)*sin(d*x+c)-2*cos(d*x+c)^3*2^(1/2)-4*cos(d*x+c)
^2*2^(1/2)-47*2^(1/2)*cos(d*x+c)-24*2^(1/2))*(1+cos(d*x+c))^2*(e/sin(d*x+c)
)^(5/2)/sin(d*x+c)^7*2^(1/2)
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.52, size = 202, normalized size = 0.75

$$-\frac{2\left(\sqrt{2i}\left(i\cos\left(dx+c\right)^2e^{\frac{5}{2}}+2i\cos\left(dx+c\right)e^{\frac{5}{2}}+ie^{\frac{5}{2}}\right)\sin\left(dx+c\right)\mathrm{weierstrassPInverse}(4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right))+\sqrt{-2i}\left(-i\cos\left(dx+c\right)^2e^{\frac{5}{2}}-2i\cos\left(dx+c\right)e^{\frac{5}{2}}-ie^{\frac{5}{2}}\right)\sin\left(dx+c\right)\mathrm{weierstrassPInverse}(4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right))+\frac{2\cos(dx+c)^3e^{\frac{5}{2}}+4\cos(dx+c)^2e^{\frac{5}{2}}+47\cos(dx+c)e^{\frac{5}{2}}+24e^{\frac{5}{2}}}{\sqrt{\sin\left(dx+c\right)}}\right)}{231\left(a^2d\cos\left(dx+c\right)^2+2a^2d\cos\left(dx+c\right)+a^2d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -2/231*(sqrt(2*I)*(I*cos(d*x + c)^2*e^(5/2) + 2*I*cos(d*x + c)*e^(5/2) + I*
e^(5/2))*sin(d*x + c)*weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x +
```

```
c)) + sqrt(-2*I)*(-I*cos(d*x + c)^2*e^(5/2) - 2*I*cos(d*x + c)*e^(5/2) - I*
e^(5/2))*sin(d*x + c)*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x +
c)) + (2*cos(d*x + c)^3*e^(5/2) + 4*cos(d*x + c)^2*e^(5/2) + 47*cos(d*x + c
)*e^(5/2) + 24*e^(5/2))/sqrt(sin(d*x + c)))/((a^2*d*cos(d*x + c)^2 + 2*a^2*
d*cos(d*x + c) + a^2*d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))**(5/2)/(a+a*sec(d*x+c))**2,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 8011 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate((e*csc(d*x + c))^(5/2)/(a*sec(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^2\,{\left(\frac{e}{\sin\left(c+d\,x\right)}\right)}^{5/2}}{a^2\,{\left(\cos\left(c+d\,x\right)+1\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/sin(c + d*x))^(5/2)/(a + a/cos(c + d*x))^2,x)
[Out] int((cos(c + d*x)^2*(e/sin(c + d*x))^(5/2))/(a^2*(cos(c + d*x) + 1)^2), x)
```

3.301 $\int \frac{(e\csc(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=250

$$-\frac{4e\cos(c+dx)\sqrt{e\csc(c+dx)}}{15a^2d}+\frac{16e\cot(c+dx)\csc(c+dx)\sqrt{e\csc(c+dx)}}{45a^2d}-\frac{2e\cot^3(c+dx)\csc(c+dx)}{9a^2d}$$

```
[Out] -4/15*e*cos(d*x+c)*(e*csc(d*x+c))^(1/2)/a^2/d+16/45*e*cot(d*x+c)*csc(d*x+c)
*(e*csc(d*x+c))^(1/2)/a^2/d-2/9*e*cot(d*x+c)^3*csc(d*x+c)*(e*csc(d*x+c))^(1
/2)/a^2/d-4/5*e*csc(d*x+c)^2*(e*csc(d*x+c))^(1/2)/a^2/d-2/9*e*cot(d*x+c)*cs
c(d*x+c)^3*(e*csc(d*x+c))^(1/2)/a^2/d+4/9*e*csc(d*x+c)^4*(e*csc(d*x+c))^(1/
2)/a^2/d+4/15*e*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*
x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*
x+c)^(1/2)/a^2/d
```

Rubi [A]
time = 0.34, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3963, 3957, 2954, 2952, 2647, 2716, 2719, 2644, 14}

$$\frac{4e\csc^4(c+dx)\sqrt{e\csc(c+dx)}}{9a^2d}-\frac{4e\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{5a^2d}-\frac{4e\cos(c+dx)\sqrt{e\csc(c+dx)}}{15a^2d}-\frac{2e\cot^3(c+dx)\csc(c+dx)\sqrt{e\csc(c+dx)}}{9a^2d}-\frac{2e\cot(c+dx)\csc^3(c+dx)\sqrt{e\csc(c+dx)}}{9a^2d}+\frac{16e\cot(c+dx)\csc(c+dx)\sqrt{e\csc(c+dx)}}{45a^2d}-\frac{4e\sqrt{\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\csc(c+dx)}}{15a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Csc[c + d*x])^(3/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-4*e*Cos[c + d*x]*Sqrt[e*Csc[c + d*x]])/(15*a^2*d) + (16*e*Cot[c + d*x]*Cs
c[c + d*x]*Sqrt[e*Csc[c + d*x]])/(45*a^2*d) - (2*e*Cot[c + d*x]^3*Csc[c + d
*x]*Sqrt[e*Csc[c + d*x]])/(9*a^2*d) - (4*e*Csc[c + d*x]^2*Sqrt[e*Csc[c + d*
x]])/(5*a^2*d) - (2*e*Cot[c + d*x]*Csc[c + d*x]^3*Sqrt[e*Csc[c + d*x]])/(9*
a^2*d) + (4*e*Csc[c + d*x]^4*Sqrt[e*Csc[c + d*x]])/(9*a^2*d) - (4*e*Sqrt[e*
Csc[c + d*x]]*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(15*a^2*
d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3963

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
```

```
FreeQ[{a, b, e, f, g, m, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e\csc(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx &= \left(e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{1}{(a+a\sec(c+dx))^2\sin^{\frac{3}{2}}(c+dx)}\,dx\\
&= \left(e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{\cos^2(c+dx)}{(-a-a\cos(c+dx))^2\sin^{\frac{3}{2}}(c+dx)}\,dx\\
&= \frac{\left(e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{\sin^{\frac{11}{2}}(c+dx)}\,dx}{a^4}\\
&= \frac{\left(e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \left(\frac{a^2\cos^2(c+dx)}{\sin^{\frac{11}{2}}(c+dx)}-\frac{2a^2\cos^3(c+dx)}{\sin^{\frac{11}{2}}(c+dx)}+\frac{a^2\cos^4(c+dx)}{\sin^{\frac{11}{2}}(c+dx)}\right.}{a^4}\\
&= \frac{\left(e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{\cos^2(c+dx)}{\sin^{\frac{11}{2}}(c+dx)}\,dx}{a^2}+\frac{\left(e\sqrt{e\csc(c+dx)}\sqrt{\sin\right.}{}\\
&= -\frac{2e\cot^3(c+dx)\csc(c+dx)\sqrt{e\csc(c+dx)}}{9a^2d}-\frac{2e\cot(c+dx)\csc^3(c+dx)\sqrt{}}{9a^2d}\\
&= \frac{16e\cot(c+dx)\csc(c+dx)\sqrt{e\csc(c+dx)}}{45a^2d}-\frac{2e\cot^3(c+dx)\csc(c+dx)\sqrt{e}}{9a^2d}\\
&= -\frac{4e\cos(c+dx)\sqrt{e\csc(c+dx)}}{15a^2d}+\frac{16e\cot(c+dx)\csc(c+dx)\sqrt{e\csc(c+dx)}}{45a^2d}\\
&= -\frac{4e\cos(c+dx)\sqrt{e\csc(c+dx)}}{15a^2d}+\frac{16e\cot(c+dx)\csc(c+dx)\sqrt{e\csc(c+dx)}}{45a^2d}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.15, size = 247, normalized size = 0.99

$$\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)(e\csc(c+dx))^{3/2}\sec(c+dx)\left(\frac{16\sqrt{2}\,e^{i(c-dx)}\sqrt{\frac{ie^{i(c+dx)}}{-1+e^{2i(c+dx)}}}\left(3-3e^{2i(c+dx)}+e^{2idx}(1+e^{2ic})\sqrt{1-e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};e^{2i(c+dx)}\right)\right)\sec(c+dx)}{d(1+e^{2ic})\csc^{\frac{3}{2}}(c+dx)}-\frac{2(24\cos(dx)\sec(c)+(8+13\cos(c+dx))\sec^4\left(\frac{1}{2}(c+dx)\right))\tan(c+dx)}{d}\right)}{45a^2(1+\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Csc[c + d*x])^(3/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]^4*(e*Csc[c + d*x])^(3/2)*Sec[c + d*x]*((16*Sqrt[2]*E^(I*(
c - d*x))*Sqrt[(I*E^(I*(c + d*x)))/(-1 + E^((2*I)*(c + d*x)))]*(3 - 3*E^((2
```

```
*I)*(c + d*x)) + E^((2*I)*d*x)*(1 + E^((2*I)*c))*Sqrt[1 - E^((2*I)*(c + d*x
))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((2*I)*(c + d*x))])*Sec[c + d*x])/(d
*(1 + E^((2*I)*c))*Csc[c + d*x]^(3/2)) - (2*(24*Cos[d*x]*Sec[c] + (8 + 13*C
os[c + d*x])*Sec[(c + d*x)/2]^4)*Tan[c + d*x])/d))/(45*a^2*(1 + Sec[c + d*x
])^2)
```

Maple [C] Result contains complex when optimal does not.
time = 0.20, size = 1044, normalized size = 4.18

method	result	size
default	Expression too large to display	1044

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/45/a^2/d*(-1+cos(d*x+c))^2*(12*cos(d*x+c)^3*(-I*(-1+cos(d*x+c))/sin(d*x+c
))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin
(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticE(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x
+c))^(1/2),1/2*2^(1/2))-6*cos(d*x+c)^3*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2
)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)
+I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1
/2),1/2*2^(1/2))+36*cos(d*x+c)^2*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*
cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/si
n(d*x+c))^(1/2)*EllipticE(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/
2*2^(1/2))-18*cos(d*x+c)^2*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(d*
x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+
c))^(1/2)*EllipticF(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1
/2))+36*cos(d*x+c)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(
d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticE(((I*cos(d*x+c)+sin(d*x+c)-
I)/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)-18*
cos(d*x+c)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+s
in(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d
*x+c))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)+12*((I*cos(
d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*
x+c))^(1/2)*EllipticE(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^
(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)-6*((I*cos(d*x+c)+sin(d*x+c)-I)
/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*Elliptic
F(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(d
*x+c))/sin(d*x+c))^(1/2)-6*cos(d*x+c)^2*2^(1/2)-25*2^(1/2)*cos(d*x+c)-14*2^
(1/2))*(e/sin(d*x+c))^(3/2)/sin(d*x+c)^3*2^(1/2)
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.52, size = 184, normalized size = 0.74

$$-\frac{2\left(3\sqrt{2i}\left(\cos\left(dx+c\right)^{2}e^{\frac{3}{2}}+2\cos\left(dx+c\right)e^{\frac{3}{2}}+e^{\frac{3}{2}}\right)\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)))+3\sqrt{-2i}\left(\cos\left(dx+c\right)^{2}e^{\frac{3}{2}}+2\cos\left(dx+c\right)e^{\frac{3}{2}}+e^{\frac{3}{2}}\right)\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)))+\frac{6\cos(dx+c)^{3}e^{\frac{3}{2}}+12\cos(dx+c)^{2}e^{\frac{3}{2}}+19\cos(dx+c)e^{\frac{3}{2}}+8e^{\frac{3}{2}}}{\sqrt{\sin\left(dx+c\right)}}\right)}{45\left(a^{2}d\cos\left(dx+c\right)^{2}+2a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -2/45*(3*sqrt(2*I)*(cos(d*x + c)^2*e^(3/2) + 2*cos(d*x + c)*e^(3/2) + e^(3/
2))*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*
x + c))) + 3*sqrt(-2*I)*(cos(d*x + c)^2*e^(3/2) + 2*cos(d*x + c)*e^(3/2) +
e^(3/2))*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*s
in(d*x + c))) + (6*cos(d*x + c)^3*e^(3/2) + 12*cos(d*x + c)^2*e^(3/2) + 19*
cos(d*x + c)*e^(3/2) + 8*e^(3/2))/sqrt(sin(d*x + c)))/(a^2*d*cos(d*x + c)^2
 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(e\csc(c+dx))^{\frac{3}{2}}}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))**(3/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral((e*csc(c + d*x))**(3/2)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)
/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*csc(d*x + c))^(3/2)/(a*sec(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^2\,{\left(\frac{e}{\sin\left(c+d\,x\right)}\right)}^{3/2}}{a^2\,{\left(\cos\left(c+d\,x\right)+1\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/sin(c + d*x))^(3/2)/(a + a/cos(c + d*x))^2,x)
[Out] int((cos(c + d*x)^2*(e/sin(c + d*x))^(3/2))/(a^2*(cos(c + d*x) + 1)^2), x)
```

3.302 $\int \frac{\sqrt{e\csc(c+dx)}}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=201

$$\frac{16\cot(c+dx)\sqrt{e\csc(c+dx)}}{21a^2d}-\frac{2\cot^3(c+dx)\sqrt{e\csc(c+dx)}}{7a^2d}-\frac{4\csc(c+dx)\sqrt{e\csc(c+dx)}}{3a^2d}-\frac{2\cot(c+}{}$$

```
[Out] 16/21*cot(d*x+c)*(e*csc(d*x+c))^(1/2)/a^2/d-2/7*cot(d*x+c)^3*(e*csc(d*x+c))
^(1/2)/a^2/d-4/3*csc(d*x+c)*(e*csc(d*x+c))^(1/2)/a^2/d-2/7*cot(d*x+c)*csc(d
*x+c)^2*(e*csc(d*x+c))^(1/2)/a^2/d+4/7*csc(d*x+c)^3*(e*csc(d*x+c))^(1/2)/a^
2/d-20/21*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ell
ipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(
1/2)/a^2/d
```

Rubi [A]

time = 0.31, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3963, 3957, 2954, 2952, 2647, 2716, 2720, 2644, 14}

$$\frac{4\csc^3(c+dx)\sqrt{e\csc(c+dx)}}{7a^2d}-\frac{4\csc(c+dx)\sqrt{e\csc(c+dx)}}{3a^2d}-\frac{2\cot^3(c+dx)\sqrt{e\csc(c+dx)}}{7a^2d}-\frac{2\cot(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{7a^2d}+\frac{16\cot(c+dx)\sqrt{e\csc(c+dx)}}{21a^2d}+\frac{20\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\csc(c+dx)}}{21a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Csc[c + d*x]]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (16*Cot[c + d*x]*Sqrt[e*Csc[c + d*x]])/(21*a^2*d) - (2*Cot[c + d*x]^3*Sqrt[
e*Csc[c + d*x]])/(7*a^2*d) - (4*Csc[c + d*x]*Sqrt[e*Csc[c + d*x]])/(3*a^2*d
) - (2*Cot[c + d*x]*Csc[c + d*x]^2*Sqrt[e*Csc[c + d*x]])/(7*a^2*d) + (4*Csc
[c + d*x]^3*Sqrt[e*Csc[c + d*x]])/(7*a^2*d) + (20*Sqrt[e*Csc[c + d*x]]*Elli
pticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(21*a^2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3963

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sqrt{e\csc(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \left(\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{1}{(a+a\sec(c+dx))^2\sqrt{\sin(c+dx)}}\,dx$$

$$= \left(\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{\cos^2(c+dx)}{(-a-a\cos(c+dx))^2\sqrt{\sin(c+dx)}}\,dx$$

$$= \frac{\left(\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{\sin^{\frac{9}{2}}(c+dx)}\,dx}{a^4}$$

$$= \frac{\left(\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \left(\frac{a^2\cos^2(c+dx)}{\sin^{\frac{9}{2}}(c+dx)} - \frac{2a^2\cos^3(c+dx)}{\sin^{\frac{9}{2}}(c+dx)} + \frac{a^2\cos^4(c+dx)}{\sin^{\frac{9}{2}}(c+dx)}\right.}{a^4}$$

$$= \frac{\left(\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{\cos^2(c+dx)}{\sin^{\frac{9}{2}}(c+dx)}\,dx}{a^2} + \frac{\left(\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c-\right.}{a^2}$$

$$= -\frac{2\cot^3(c+dx)\sqrt{e\csc(c+dx)}}{7a^2d} - \frac{2\cot(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{7a^2d}$$

$$= \frac{16\cot(c+dx)\sqrt{e\csc(c+dx)}}{21a^2d} - \frac{2\cot^3(c+dx)\sqrt{e\csc(c+dx)}}{7a^2d} - \frac{2\cot(c+a}{}$$

$$= \frac{16\cot(c+dx)\sqrt{e\csc(c+dx)}}{21a^2d} - \frac{2\cot^3(c+dx)\sqrt{e\csc(c+dx)}}{7a^2d} - \frac{4\csc(c+a}{}$$

Mathematica [A]
time = 0.45, size = 82, normalized size = 0.41

$$-\frac{4\csc^3(c+dx)\sqrt{e\csc(c+dx)}\left(2(8+11\cos(c+dx))\sin^4\left(\frac{1}{2}(c+dx)\right)+5F\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)\sin^{\frac{7}{2}}(c+dx)\right)}{21a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Csc[c + d*x]]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-4*Csc[c + d*x]^3*Sqrt[e*Csc[c + d*x]]*(2*(8 + 11*Cos[c + d*x])*Sin[(c + d
*x)/2]^4 + 5*EllipticF[(-2*c + Pi - 2*d*x)/4, 2]*Sin[c + d*x]^(7/2)))/(21*a
^2*d)
```

Maple [C] Result contains complex when optimal does not.
time = 0.20, size = 474, normalized size = 2.36

method	result

default	$-\frac{\sqrt{\frac{e}{\sin(dx+c)}}\,(1+\cos(dx+c))^2(-1+\cos(dx+c))^3\left(10i(\cos^2(dx+c))\sin(dx+c)\sqrt{-\frac{i(-1+\cos(dx+c))}{\sin(dx+c)}}\sqrt{\frac{i\cos(dx+c)+\sin(dx+c}{\sin(dx+c)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*csc(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/21/a^2/d*(e/sin(d*x+c))^(1/2)*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^3*(10*I*s
in(d*x+c)*cos(d*x+c)^2*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(d*x+c)
+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^
(1/2)*EllipticF(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))
+20*I*cos(d*x+c)*sin(d*x+c)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*
(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x
+c))^(1/2)*EllipticF(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(
1/2))+10*I*EllipticF(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(
1/2))*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/si
n(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*sin(d*x+c)+
11*cos(d*x+c)^2*2^(1/2)-3*2^(1/2)*cos(d*x+c)-8*2^(1/2))/sin(d*x+c)^7*2^(1/2
)
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.47, size = 160, normalized size = 0.80

$$-\frac{2\left(5\sqrt{2i}\left(i\cos(dx+c)^2e^{\frac{1}{2}}+2i\cos(dx+c)e^{\frac{1}{2}}+ie^{\frac{1}{2}}\right)\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c))+5\sqrt{-2i}\left(-i\cos(dx+c)^2e^{\frac{1}{2}}-2i\cos(dx+c)e^{\frac{1}{2}}-ie^{\frac{1}{2}}\right)\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)-i\sin(dx+c))+\left(11\cos(dx+c)e^{\frac{1}{2}}+8e^{\frac{1}{2}}\right)\sqrt{\sin(dx+c)}\right)}{21\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -2/21*(5*sqrt(2*I)*(I*cos(d*x + c)^2*e^(1/2) + 2*I*cos(d*x + c)*e^(1/2) + I
*e^(1/2))*weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt
(-2*I)*(-I*cos(d*x + c)^2*e^(1/2) - 2*I*cos(d*x + c)*e^(1/2) - I*e^(1/2))*w
eierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) + (11*cos(d*x + c)*
e^(1/2) + 8*e^(1/2))*sqrt(sin(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*co
s(d*x + c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{e \csc(c+dx)}}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))**(1/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(sqrt(e*csc(c + d*x))/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a*
*2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*csc(d*x + c))/(a*sec(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^2\,\sqrt{\frac{e}{\sin(c+d\,x)}}}{a^2\,\left(\cos(c+d\,x)+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/sin(c + d*x))^(1/2)/(a + a/cos(c + d*x))^2,x)
```

```
[Out] int((cos(c + d*x)^2*(e/sin(c + d*x))^(1/2))/(a^2*(cos(c + d*x) + 1)^2), x)
```

3.303 $\int \frac{1}{\sqrt{e\csc(c+dx)}\,(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=199

$$\frac{16\cot(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}}-\frac{2\cot^3(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}}-\frac{4\csc(c+dx)}{a^2d\sqrt{e\csc(c+dx)}}-\frac{2\cot(c+dx)\csc^2(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}}+\frac{4\csc^3(c-}{5a^2d\sqrt{e\csc(}$$

```
[Out] 16/5*cot(d*x+c)/a^2/d/(e*csc(d*x+c))^(1/2)-2/5*cot(d*x+c)^3/a^2/d/(e*csc(d*
x+c))^(1/2)-4*csc(d*x+c)/a^2/d/(e*csc(d*x+c))^(1/2)-2/5*cot(d*x+c)*csc(d*x+
c)^2/a^2/d/(e*csc(d*x+c))^(1/2)+4/5*csc(d*x+c)^3/a^2/d/(e*csc(d*x+c))^(1/2)
-28/5*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ellipti
cE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))/a^2/d/(e*csc(d*x+c))^(1/2)/sin(d*x+c)
^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3963, 3957, 2954, 2952, 2647, 2716, 2719, 2644, 14}

$$\frac{4\csc^3(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}}-\frac{4\csc(c+dx)}{a^2d\sqrt{e\csc(c+dx)}}-\frac{2\cot^3(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}}-\frac{2\cot(c+dx)\csc^2(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}}+\frac{16\cot(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}}+\frac{28E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{5a^2d\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Csc[c + d*x]]*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (16*Cot[c + d*x])/(5*a^2*d*Sqrt[e*Csc[c + d*x]]) - (2*Cot[c + d*x]^3)/(5*a^
2*d*Sqrt[e*Csc[c + d*x]]) - (4*Csc[c + d*x])/(a^2*d*Sqrt[e*Csc[c + d*x]]) -
 (2*Cot[c + d*x]*Csc[c + d*x]^2)/(5*a^2*d*Sqrt[e*Csc[c + d*x]]) + (4*Csc[c
+ d*x]^3)/(5*a^2*d*Sqrt[e*Csc[c + d*x]]) + (28*EllipticE[(c - Pi/2 + d*x)/2
, 2])/(5*a^2*d*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3963

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\sqrt{e\csc(c+dx)}\,(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sqrt{\sin(c+dx)}}{(a+a\sec(c+dx))^2}\,dx}{\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}}$$

$$= \frac{\int \frac{\cos^2(c+dx)\sqrt{\sin(c+dx)}}{(-a-a\cos(c+dx))^2}\,dx}{\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}}$$

$$= \frac{\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{\sin^{\frac{7}{2}}(c+dx)}\,dx}{a^4\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}}$$

$$= \frac{\int \left(\frac{a^2\cos^2(c+dx)}{\sin^{\frac{7}{2}}(c+dx)} - \frac{2a^2\cos^3(c+dx)}{\sin^{\frac{7}{2}}(c+dx)} + \frac{a^2\cos^4(c+dx)}{\sin^{\frac{7}{2}}(c+dx)}\right)dx}{a^4\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}}$$

$$= \frac{\int \frac{\cos^2(c+dx)}{\sin^{\frac{7}{2}}(c+dx)}\,dx}{a^2\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}} + \frac{\int \frac{\cos^4(c+dx)}{\sin^{\frac{7}{2}}(c+dx)}\,dx}{a^2\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c}$$

$$= -\frac{2\cot^3(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}} - \frac{2\cot(c+dx)\csc^2(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}} - \frac{}{5a^2\sqrt{e}}$$

$$= \frac{16\cot(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}} - \frac{2\cot^3(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}} - \frac{2\cot(c+dx)\,c}{5a^2d\sqrt{e\csc}}$$

$$= \frac{16\cot(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}} - \frac{2\cot^3(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}} - \frac{4\csc(c+d}{a^2d\sqrt{e\csc(c+}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.05, size = 252, normalized size = 1.27

$$\frac{4\cos^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\csc(c+dx)}\,\sec^2(c+dx)\left(-\frac{28\sqrt{2}\,e^{i(c-dx)}\sqrt{\frac{ie^{i(c+dx)}}{-1+e^{2i(c+dx)}}}\left(3-3e^{2i(c+dx)}+e^{2idx}(1+e^{2ic})\sqrt{1-e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};e^{2i(c+dx)}\right)\right)}{1+e^{2ic}} - 3\sqrt{\csc(c+dx)}\left((-23+5\cos(2c))\cos(dx)\sec(c) - 2\left(-10+\sec^2\left(\frac{1}{2}(c+dx)\right)+5\sin(c)\sin(dx)\right)\right)\right)}{15a^2d\sqrt{e\csc(c+dx)}\,(1+\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[e*Csc[c + d*x]]*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (4*Cos[(c + d*x)/2]^4*Sqrt[Csc[c + d*x]]*Sec[c + d*x]^2*((-28*Sqrt[2]*E^(I*
(c - d*x))*Sqrt[(I*E^(I*(c + d*x)))/(-1 + E^((2*I)*(c + d*x)))]*(3 - 3*E^((
2*I)*(c + d*x)) + E^((2*I)*d*x)*(1 + E^((2*I)*c))*Sqrt[1 - E^((2*I)*(c + d*
x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((2*I)*(c + d*x))]))/(1 + E^((2*I)*
c)) - 3*Sqrt[Csc[c + d*x]]*((-23 + 5*Cos[2*c])*Cos[d*x]*Sec[c] - 2*(-10 + S
```

```
ec[(c + d*x)/2]^2 + 5*Sin[c]*Sin[d*x]])))/(15*a^2*d*Sqrt[e*Csc[c + d*x]]*(1
 + Sec[c + d*x])^2)
```

Maple [C] Result contains complex when optimal does not.
time = 0.20, size = 811, normalized size = 4.08

method	result
default	$\frac{(-1+\cos(dx+c))\left(28\sqrt{\frac{i\cos(dx+c)+\sin(dx+c)-i}{\sin(dx+c)}}\,\sqrt{-\frac{i\cos(dx+c)-\sin(dx+c)-i}{\sin(dx+c)}}\,\mathrm{EllipticE}\left(\sqrt{\frac{i\cos(dx+c)+\sin(dx+c)-i}{\sin(dx+c)}},\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5/a^2/d*(-1+cos(d*x+c))*(28*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2
)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticE(((I*cos(d*x+c)+
sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)^2*(-I*(-1+cos(d*x+c
))/sin(d*x+c))^(1/2)-14*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-(I
*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)+sin(d*
x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)^2*(-I*(-1+cos(d*x+c))/sin
(d*x+c))^(1/2)+56*cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(
d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*
x+c))^(1/2)*EllipticE(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^
(1/2))-28*cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(d*x+c)+s
in(d*x+c)-I)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1
/2)*EllipticF(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))+2
8*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)
-I)/sin(d*x+c))^(1/2)*EllipticE(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1
/2),1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)-14*((I*cos(d*x+c)+si
n(d*x+c)-I)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/
2)*EllipticF(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-
I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)+5*cos(d*x+c)^2*2^(1/2)+2^(1/2)*cos(d*x+
c)-6*2^(1/2))/(e/sin(d*x+c))^(1/2)/sin(d*x+c)^3*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-1/2)*integrate(1/((a*sec(d*x + c) + a)^2*sqrt(csc(d*x + c))), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.96, size = 123, normalized size = 0.62

$$\frac{2\left(7\sqrt{2i}\left(\cos\left(dx+c\right)+1\right)\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)))+7\sqrt{-2i}\left(\cos\left(dx+c\right)+1\right)\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)))+\frac{9\cos(dx+c)^2-\cos(dx+c)-8}{\sqrt{\sin\left(dx+c\right)}}\right)}{5\left(a^2d\cos\left(dx+c\right)e^{\frac{1}{2}}+a^2de^{\frac{1}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/5*(7*sqrt(2*I)*(cos(d*x + c) + 1)*weierstrassZeta(4, 0, weierstrassPInver
se(4, 0, cos(d*x + c) + I*sin(d*x + c))) + 7*sqrt(-2*I)*(cos(d*x + c) + 1)*
weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x +
c))) + (9*cos(d*x + c)^2 - cos(d*x + c) - 8)/sqrt(sin(d*x + c)))/(a^2*d*cos
(d*x + c)*e^(1/2) + a^2*d*e^(1/2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sqrt{e\csc\left(c+dx\right)}\sec^2\left(c+dx\right)+2\sqrt{e\csc\left(c+dx\right)}\sec\left(c+dx\right)+\sqrt{e\csc\left(c+dx\right)}}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**2/(e*csc(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(e*csc(c + d*x))*sec(c + d*x)**2 + 2*sqrt(e*csc(c + d*x))*s
ec(c + d*x) + sqrt(e*csc(c + d*x))), x)/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(e*csc(d*x + c))*(a*sec(d*x + c) + a)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cos\left(c+d\,x\right)}^2}{a^2\,\sqrt{\frac{e}{\sin\left(c+d\,x\right)}}\,{\left(\cos\left(c+d\,x\right)+1\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(c + d*x))^2*(e/sin(c + d*x))^(1/2)),x)
```

```
[Out] int(cos(c + d*x)^2/(a^2*(e/sin(c + d*x))^(1/2)*(cos(c + d*x) + 1)^2), x)
```

3.304 $\int \frac{1}{(e\csc(c+dx))^{3/2}(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=213

$$\frac{4}{a^2de\sqrt{e\csc(c+dx)}}-\frac{4\cos(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}}-\frac{2\cos(c+dx)\cot^2(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}}-\frac{2\cot(c+dx)\csc(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}}+\frac{4}{3a^2d}$$

```
[Out] 4/a^2/d/e/(e*csc(d*x+c))^(1/2)-4/3*cos(d*x+c)/a^2/d/e/(e*csc(d*x+c))^(1/2)-
2/3*cos(d*x+c)*cot(d*x+c)^2/a^2/d/e/(e*csc(d*x+c))^(1/2)-2/3*cot(d*x+c)*csc
(d*x+c)/a^2/d/e/(e*csc(d*x+c))^(1/2)+4/3*csc(d*x+c)^2/a^2/d/e/(e*csc(d*x+c)
)^(1/2)+4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ell
ipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))/a^2/d/e/(e*csc(d*x+c))^(1/2)/sin(
d*x+c)^(1/2)
```

Rubi [A]

time = 0.32, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3963, 3957, 2954, 2952, 2647, 2720, 2644, 14, 2649}

$$\frac{4\csc^2(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}}+\frac{4}{a^2de\sqrt{e\csc(c+dx)}}-\frac{4\cos(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}}-\frac{2\cot(c+dx)\csc(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}}-\frac{2\cos(c+dx)\cot^2(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}}-\frac{4F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{a^2de\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Csc[c + d*x])^(3/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] 4/(a^2*d*e*Sqrt[e*Csc[c + d*x]]) - (4*Cos[c + d*x])/(3*a^2*d*e*Sqrt[e*Csc[c
 + d*x]]) - (2*Cos[c + d*x]*Cot[c + d*x]^2)/(3*a^2*d*e*Sqrt[e*Csc[c + d*x]]
) - (2*Cot[c + d*x]*Csc[c + d*x])/(3*a^2*d*e*Sqrt[e*Csc[c + d*x]]) + (4*Csc
[c + d*x]^2)/(3*a^2*d*e*Sqrt[e*Csc[c + d*x]]) - (4*EllipticF[(c - Pi/2 + d*
x)/2, 2])/(a^2*d*e*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
```

```
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2649

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(b*Sin[e + f*x])^(n + 1)*((a*Cos[e + f*x])^(m - 1)/
(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3963

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{(e\csc(c+dx))^{3/2}(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sin^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{e\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}}$$

$$= \frac{\int \frac{\cos^2(c+dx)\sin^{\frac{3}{2}}(c+dx)}{(-a-a\cos(c+dx))^2}\,dx}{e\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}}$$

$$= \frac{\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{\sin^{\frac{5}{2}}(c+dx)}\,dx}{a^4e\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}}$$

$$= \frac{\int \left(\frac{a^2\cos^2(c+dx)}{\sin^{\frac{5}{2}}(c+dx)} - \frac{2a^2\cos^3(c+dx)}{\sin^{\frac{5}{2}}(c+dx)} + \frac{a^2\cos^4(c+dx)}{\sin^{\frac{5}{2}}(c+dx)}\right)dx}{a^4e\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}}$$

$$= \frac{\int \frac{\cos^2(c+dx)}{\sin^{\frac{5}{2}}(c+dx)}\,dx}{a^2e\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}} + \frac{\int \frac{\cos^4(c+dx)}{\sin^{\frac{5}{2}}(c+dx)}\,d}{a^2e\sqrt{e\csc(c+dx)}\,\sqrt{s}}$$

$$= -\frac{2\cos(c+dx)\cot^2(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}} - \frac{2\cot(c+dx)\csc(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}} - \frac{}{3}$$

$$= -\frac{4\cos(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}} - \frac{2\cos(c+dx)\cot^2(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}} - \frac{2\,\mathrm{co}}{3a}$$

$$= \frac{4}{a^2de\sqrt{e\csc(c+dx)}} - \frac{4\cos(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}} - \frac{2\cos(c+d}{3a^2de\sqrt{e}}$$

Mathematica [A]
time = 0.38, size = 101, normalized size = 0.47

$$\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\left(12(1+\cos(c+dx))F\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)+(15+10\cos(c+dx)-\cos(2(c+dx)))\sqrt{\sin(c+dx)}\right)}{6a^2d(e\csc(c+dx))^{3/2}\sin^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Csc[c + d*x])^(3/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (Sec[(c + d*x)/2]^2*(12*(1 + Cos[c + d*x])*EllipticF[(-2*c + Pi - 2*d*x)/4,
 2] + (15 + 10*Cos[c + d*x] - Cos[2*(c + d*x)])*Sqrt[Sin[c + d*x]]))/(6*a^2
*d*(e*Csc[c + d*x])^(3/2)*Sin[c + d*x]^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.18, size = 327, normalized size = 1.54

method	result
default	$-\frac{\left(6i\cos(dx+c)\sqrt{\frac{-i\cos(dx+c)+\sin(dx+c)+i}{\sin(dx+c)}}\,\sqrt{\frac{i\cos(dx+c)+\sin(dx+c)-i}{\sin(dx+c)}}\,\sqrt{-\frac{i(-1+\cos(dx+c))}{\sin(dx+c)}}\,\sin(dx+c)\,\mathrm{EllipticF}\left(\sqrt{\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a^2/d*(6*I*sin(d*x+c)*cos(d*x+c)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+
c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos
(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/si
n(d*x+c))^(1/2)+6*I*sin(d*x+c)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/
2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)
+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c
))^(1/2)-cos(d*x+c)^3*2^(1/2)+6*cos(d*x+c)^2*2^(1/2)+3*2^(1/2)*cos(d*x+c)-8
*2^(1/2))/(e/sin(d*x+c))^(3/2)/sin(d*x+c)^3*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] e^(-3/2)*integrate(1/((a*sec(d*x + c) + a)^2*csc(d*x + c)^(3/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.79, size = 119, normalized size = 0.56

$$-\frac{2\left(3\sqrt{2i}\left(-i\cos\left(dx+c\right)-i\right)\mathrm{weierstrassPInverse}(4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right))+3\sqrt{-2i}\left(i\cos\left(dx+c\right)+i\right)\mathrm{weierstrassPInverse}(4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right))+\left(\cos\left(dx+c\right)^2-5\cos\left(dx+c\right)-8\right)\sqrt{\sin\left(dx+c\right)}\right)}{3\left(a^2d\cos\left(dx+c\right)e^{\frac{3}{2}}+a^2de^{\frac{3}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -2/3*(3*sqrt(2*I)*(-I*cos(d*x + c) - I)*weierstrassPInverse(4, 0, cos(d*x +
 c) + I*sin(d*x + c)) + 3*sqrt(-2*I)*(I*cos(d*x + c) + I)*weierstrassPInver
se(4, 0, cos(d*x + c) - I*sin(d*x + c)) + (cos(d*x + c)^2 - 5*cos(d*x + c)
- 8)*sqrt(sin(d*x + c)))/(a^2*d*cos(d*x + c)*e^(3/2) + a^2*d*e^(3/2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\left(e\csc\left(c+dx\right)\right)^{\frac{3}{2}}\sec^2\left(c+dx\right)+2\left(e\csc\left(c+dx\right)\right)^{\frac{3}{2}}\sec\left(c+dx\right)+\left(e\csc\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))**(3/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(1/((e*csc(c + d*x))**(3/2)*sec(c + d*x)**2 + 2*(e*csc(c + d*x))**(
3/2)*sec(c + d*x) + (e*csc(c + d*x))**(3/2)), x)/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*csc(d*x + c))^(3/2)*(a*sec(d*x + c) + a)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^2}{a^2\,{\left(\frac{e}{\sin\left(c+d\,x\right)}\right)}^{3/2}\,{\left(\cos\left(c+d\,x\right)+1\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(c + d*x))^2*(e/sin(c + d*x))^(3/2)),x)
```

```
[Out] int(cos(c + d*x)^2/(a^2*(e/sin(c + d*x))^(3/2)*(cos(c + d*x) + 1)^2), x)
```

3.305 $\int \frac{1}{(e\csc(c+dx))^{5/2}(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=215

$$-\frac{2\cot(c+dx)}{a^2de^2\sqrt{e\csc(c+dx)}}-\frac{2\cos^2(c+dx)\cot(c+dx)}{a^2de^2\sqrt{e\csc(c+dx)}}+\frac{4\csc(c+dx)}{a^2de^2\sqrt{e\csc(c+dx)}}-\frac{44E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{5a^2de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c}}$$

```
[Out] -2*cot(d*x+c)/a^2/d/e^2/(e*csc(d*x+c))^(1/2)-2*cos(d*x+c)^2*cot(d*x+c)/a^2/
d/e^2/(e*csc(d*x+c))^(1/2)+4*csc(d*x+c)/a^2/d/e^2/(e*csc(d*x+c))^(1/2)+4/3*
sin(d*x+c)/a^2/d/e^2/(e*csc(d*x+c))^(1/2)-12/5*cos(d*x+c)*sin(d*x+c)/a^2/d/
e^2/(e*csc(d*x+c))^(1/2)+44/5*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c
+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))/a^2/d/e^2/(e*
csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3963, 3957, 2954, 2952, 2647, 2719, 2644, 14, 2649}

$$\frac{4\csc(c+dx)}{a^2de^2\sqrt{e\csc(c+dx)}}-\frac{2\cot(c+dx)}{a^2de^2\sqrt{e\csc(c+dx)}}+\frac{4\sin(c+dx)}{3a^2de^2\sqrt{e\csc(c+dx)}}-\frac{2\cos^2(c+dx)\cot(c+dx)}{a^2de^2\sqrt{e\csc(c+dx)}}-\frac{12\sin(c+dx)\cos(c+dx)}{5a^2de^2\sqrt{e\csc(c+dx)}}-\frac{44E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{5a^2de^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Csc[c + d*x])^(5/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (-2*Cot[c + d*x])/(a^2*d*e^2*Sqrt[e*Csc[c + d*x]]) - (2*Cos[c + d*x]^2*Cot[
c + d*x])/(a^2*d*e^2*Sqrt[e*Csc[c + d*x]]) + (4*Csc[c + d*x])/(a^2*d*e^2*Sq
rt[e*Csc[c + d*x]]) - (44*EllipticE[(c - Pi/2 + d*x)/2, 2])/(5*a^2*d*e^2*Sq
rt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]]) + (4*Sin[c + d*x])/(3*a^2*d*e^2*Sqrt
[e*Csc[c + d*x]]) - (12*Cos[c + d*x]*Sin[c + d*x])/(5*a^2*d*e^2*Sqrt[e*Csc[
c + d*x]])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2649

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(b*Sin[e + f*x])^(n + 1)*((a*Cos[e + f*x])^(m - 1)/
(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3963

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{(e\csc(c+dx))^{5/2}(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sin^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{e^2\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}}$$

$$= \frac{\int \frac{\cos^2(c+dx)\sin^{\frac{5}{2}}(c+dx)}{(-a-a\cos(c+dx))^2}\,dx}{e^2\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}}$$

$$= \frac{\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{\sin^{\frac{3}{2}}(c+dx)}\,dx}{a^4e^2\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}}$$

$$= \frac{\int \left(\frac{a^2\cos^2(c+dx)}{\sin^{\frac{3}{2}}(c+dx)} - \frac{2a^2\cos^3(c+dx)}{\sin^{\frac{3}{2}}(c+dx)} + \frac{a^2\cos^4(c+dx)}{\sin^{\frac{3}{2}}(c+dx)}\right)dx}{a^4e^2\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}}$$

$$= \frac{\int \frac{\cos^2(c+dx)}{\sin^{\frac{3}{2}}(c+dx)}\,dx}{a^2e^2\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}} + \frac{\int \frac{\cos^4(c+dx)}{\sin^{\frac{3}{2}}(c+dx)}\,d}{a^2e^2\sqrt{e\csc(c+dx)}\,\sqrt{s}}$$

$$= -\frac{2\cot(c+dx)}{a^2de^2\sqrt{e\csc(c+dx)}} - \frac{2\cos^2(c+dx)\cot(c+dx)}{a^2de^2\sqrt{e\csc(c+dx)}} - \frac{}{a^2e^2\sqrt{}}$$

$$= -\frac{2\cot(c+dx)}{a^2de^2\sqrt{e\csc(c+dx)}} - \frac{2\cos^2(c+dx)\cot(c+dx)}{a^2de^2\sqrt{e\csc(c+dx)}} - \frac{}{a^2de^2}$$

$$= -\frac{2\cot(c+dx)}{a^2de^2\sqrt{e\csc(c+dx)}} - \frac{2\cos^2(c+dx)\cot(c+dx)}{a^2de^2\sqrt{e\csc(c+dx)}} + \frac{4}{a^2de^2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.39, size = 125, normalized size = 0.58

$$\frac{-123\cot(c+dx) + 88\sqrt{1-e^{2i(c+dx)}}\,(i+\cot(c+dx))\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};e^{2i(c+dx)}\right) + \csc(c+dx)(140-20\cos(2(c+dx))+3\cos(3(c+dx))-264i\sin(c+dx))}{30a^2de^2\sqrt{e\csc(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Csc[c + d*x])^(5/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (-123*Cot[c + d*x] + 88*Sqrt[1 - E^((2*I)*(c + d*x))]*(I + Cot[c + d*x])*Hy
pergeometric2F1[1/2, 3/4, 7/4, E^((2*I)*(c + d*x))] + Csc[c + d*x]*(140 - 2
0*Cos[2*(c + d*x)] + 3*Cos[3*(c + d*x)] - (264*I)*Sin[c + d*x]))/(30*a^2*d*
e^2*Sqrt[e*Csc[c + d*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.17, size = 551, normalized size = 2.56

method	result
default	$\frac{\left(-66\cos(dx+c)\sqrt{\frac{i\cos(dx+c)+\sin(dx+c)-i}{\sin(dx+c)}}\ \sqrt{\frac{-i\cos(dx+c)+\sin(dx+c)+i}{\sin(dx+c)}}\ \mathrm{EllipticF}\left(\sqrt{\frac{i\cos(dx+c)+\sin(dx+c)-i}{\sin(dx+c)}}, \frac{\sqrt{2}}{2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/15/a^2/d*(-66*cos(d*x+c)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(
(-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)+sin
(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(
1/2)+132*cos(d*x+c)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos
(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticE(((I*cos(d*x+c)+sin(d*x+c)
-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)+3*
cos(d*x+c)^3*2^(1/2)-66*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I
*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)+sin(d*
x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2
)+132*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*
x+c)+I)/sin(d*x+c))^(1/2)*EllipticE(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c)
)^(1/2),1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)-10*cos(d*x+c)^2*
2^(1/2)+33*2^(1/2)*cos(d*x+c)-26*2^(1/2))/(e/sin(d*x+c))^(5/2)/sin(d*x+c)^3
*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] e^(-5/2)*integrate(1/((a*sec(d*x + c) + a)^2*csc(d*x + c)^(5/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.96, size = 103, normalized size = 0.48

$$-\frac{2\left(33\sqrt{2i}\ \mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c)))+33\sqrt{-2i}\ \mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)-i\sin(dx+c)))-\frac{3\cos(dx+c)^3-10\cos(dx+c)^2-33\cos(dx+c)+40}{\sqrt{\sin(dx+c)}}\right)e^{\left(-\frac{5}{2}\right)}}{15\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -2/15*(33*sqrt(2*I)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x
 + c) + I*sin(d*x + c))) + 33*sqrt(-2*I)*weierstrassZeta(4, 0, weierstrassP
```

```
Inverse(4, 0, cos(d*x + c) - I*sin(d*x + c))) - (3*cos(d*x + c)^3 - 10*cos(
d*x + c)^2 - 33*cos(d*x + c) + 40)/sqrt(sin(d*x + c)))*e^(-5/2)/(a^2*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))**(5/2)/(a+a*sec(d*x+c))**2,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 5009 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate(1/((e*csc(d*x + c))^(5/2)*(a*sec(d*x + c) + a)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^2}{a^2\,{\left(\frac{e}{\sin\left(c+d\,x\right)}\right)}^{5/2}\,{\left(\cos\left(c+d\,x\right)+1\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(c + d*x))^2*(e/sin(c + d*x))^(5/2)),x)
[Out] int(cos(c + d*x)^2/(a^2*(e/sin(c + d*x))^(5/2)*(cos(c + d*x) + 1)^2), x)
```

3.306 $\int \frac{1}{(e\csc(c+dx))^{7/2}(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=172

$$-\frac{4}{a^2de^3\sqrt{e\csc(c+dx)}}+\frac{26\cos(c+dx)}{21a^2de^3\sqrt{e\csc(c+dx)}}+\frac{2\cos^3(c+dx)}{7a^2de^3\sqrt{e\csc(c+dx)}}+\frac{52F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\,2\right.}{21a^2de^3\sqrt{e\csc(c+dx)}\,\sqrt{\sin}}$$

```
[Out] -4/a^2/d/e^3/(e*csc(d*x+c))^(1/2)+26/21*cos(d*x+c)/a^2/d/e^3/(e*csc(d*x+c))
^(1/2)+2/7*cos(d*x+c)^3/a^2/d/e^3/(e*csc(d*x+c))^(1/2)+4/5*sin(d*x+c)^2/a^2
/d/e^3/(e*csc(d*x+c))^(1/2)-52/21*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1
/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))/a^2/d/e^3
/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3963, 3957, 2954, 2952, 2649, 2720, 2644, 14}

$$-\frac{4}{a^2de^3\sqrt{e\csc(c+dx)}}+\frac{2\cos^3(c+dx)}{7a^2de^3\sqrt{e\csc(c+dx)}}+\frac{26\cos(c+dx)}{21a^2de^3\sqrt{e\csc(c+dx)}}+\frac{4\sin^2(c+dx)}{5a^2de^3\sqrt{e\csc(c+dx)}}+\frac{52F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{21a^2de^3\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Csc[c + d*x])^(7/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] -4/(a^2*d*e^3*Sqrt[e*Csc[c + d*x]]) + (26*Cos[c + d*x])/(21*a^2*d*e^3*Sqrt[
e*Csc[c + d*x]]) + (2*Cos[c + d*x]^3)/(7*a^2*d*e^3*Sqrt[e*Csc[c + d*x]]) +
(52*EllipticF[(c - Pi/2 + d*x)/2, 2])/(21*a^2*d*e^3*Sqrt[e*Csc[c + d*x]]*Sq
rt[Sin[c + d*x]]) + (4*Sin[c + d*x]^2)/(5*a^2*d*e^3*Sqrt[e*Csc[c + d*x]])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2649

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(b*Sin[e + f*x])^(n + 1)*((a*Cos[e + f*x])^(m - 1)/
```

```
(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3963

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{(e\csc(c+dx))^{7/2}(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sin^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{e^3\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}}$$

$$= \frac{\int \frac{\cos^2(c+dx)\sin^{\frac{7}{2}}(c+dx)}{(-a-a\cos(c+dx))^2}\,dx}{e^3\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}}$$

$$= \frac{\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{\sqrt{\sin(c+dx)}}\,dx}{a^4e^3\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}}$$

$$= \frac{\int \left(\frac{a^2\cos^2(c+dx)}{\sqrt{\sin(c+dx)}} - \frac{2a^2\cos^3(c+dx)}{\sqrt{\sin(c+dx)}} + \frac{a^2\cos^4(c+dx)}{\sqrt{\sin(c+dx)}}\right)dx}{a^4e^3\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}}$$

$$= \frac{\int \frac{\cos^2(c+dx)}{\sqrt{\sin(c+dx)}}\,dx}{a^2e^3\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}} + \frac{\int \frac{\cos^4(c+dx)}{\sqrt{\sin(c+d\cdots}}}{a^2e^3\sqrt{e\csc(c+dx)}\,\cdots}$$

$$= \frac{2\cos(c+dx)}{3a^2de^3\sqrt{e\csc(c+dx)}} + \frac{2\cos^3(c+dx)}{7a^2de^3\sqrt{e\csc(c+dx)}} + \frac{2\cdots}{3a^2e^3\sqrt{\cdots}}$$

$$= \frac{26\cos(c+dx)}{21a^2de^3\sqrt{e\csc(c+dx)}} + \frac{2\cos^3(c+dx)}{7a^2de^3\sqrt{e\csc(c+dx)}} + \frac{\cdots}{3a^2de^3\cdots}$$

$$= -\frac{4}{a^2de^3\sqrt{e\csc(c+dx)}} + \frac{26\cos(c+dx)}{21a^2de^3\sqrt{e\csc(c+dx)}} + \frac{2\cdots}{7a^2de\cdots}$$

Mathematica [A]
time = 1.43, size = 94, normalized size = 0.55

$$\frac{\sqrt{e\csc(c+dx)}\left(-520F\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)+(-756+305\cos(c+dx)-84\cos(2(c+dx))+15\cos(3(c+dx)))\sqrt{\sin(c+dx)}\right)\sqrt{\sin(c+dx)}}{210a^2de^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Csc[c + d*x])^(7/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (Sqrt[e*Csc[c + d*x]]*(-520*EllipticF[(-2*c + Pi - 2*d*x)/4, 2] + (-756 + 3
05*Cos[c + d*x] - 84*Cos[2*(c + d*x)] + 15*Cos[3*(c + d*x)])*Sqrt[Sin[c + d
*x]])*Sqrt[Sin[c + d*x]])/(210*a^2*d*e^4)
```

Maple [C] Result contains complex when optimal does not.
time = 0.17, size = 221, normalized size = 1.28

method	result
default	$-\frac{\left(130i\,\mathrm{EllipticF}\left(\sqrt{\frac{i\cos(dx+c)+\sin(dx+c)-i}{\sin(dx+c)}},\frac{\sqrt{2}}{2}\right)\sin(dx+c)\sqrt{\frac{-i\cos(dx+c)+\sin(dx+c)+i}{\sin(dx+c)}}\sqrt{\frac{i\cos(dx+c)+\sin(dx+c)-i}{\sin(dx+c)}}\right.}{105a^2d(-1+\cos(dx+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*csc(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/105/a^2/d*(130*I*sin(d*x+c)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/
2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)
+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c
))^(1/2)-15*cos(d*x+c)^4*2^(1/2)+57*cos(d*x+c)^3*2^(1/2)-107*cos(d*x+c)^2*2
^(1/2)+233*2^(1/2)*cos(d*x+c)-168*2^(1/2))/(-1+cos(d*x+c))/(e/sin(d*x+c))^(
7/2)/sin(d*x+c)^3*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] e^(-7/2)*integrate(1/((a*sec(d*x + c) + a)^2*csc(d*x + c)^(7/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.73, size = 96, normalized size = 0.56

$$\frac{2\left(\left(15\cos(dx+c)^3-42\cos(dx+c)^2+65\cos(dx+c)-168\right)\sqrt{\sin(dx+c)}-65i\sqrt{2i}\,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)+i\sin(dx+c))+65i\sqrt{-2i}\,\mathrm{weierstrassPInverse}(4,0,\cos(dx+c)-i\sin(dx+c))\right)e^{\left(-\frac{7}{2}\right)}}{105\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 2/105*((15*cos(d*x + c)^3 - 42*cos(d*x + c)^2 + 65*cos(d*x + c) - 168)*sqrt
(sin(d*x + c)) - 65*I*sqrt(2*I)*weierstrassPInverse(4, 0, cos(d*x + c) + I*
sin(d*x + c)) + 65*I*sqrt(-2*I)*weierstrassPInverse(4, 0, cos(d*x + c) - I*
sin(d*x + c)))*e^(-7/2)/(a^2*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))**(7/2)/(a+a*sec(d*x+c))**2,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate(1/((e*csc(d*x + c))^(7/2)*(a*sec(d*x + c) + a)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^2}{a^2\,{\left(\frac{e}{\sin\left(c+d\,x\right)}\right)}^{7/2}\,{\left(\cos\left(c+d\,x\right)+1\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(c + d*x))^2*(e/sin(c + d*x))^(7/2)),x)
[Out] int(cos(c + d*x)^2/(a^2*(e/sin(c + d*x))^(7/2)*(cos(c + d*x) + 1)^2), x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                        Small rewrite of logic in main function to make it*)
(*                        match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
      return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022 Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```